Local Geometry of Curve Graphs of Closed Surfaces

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Let S_g denote a closed, orientable surface of genus $g \geq 2$. Let $\mathcal{C}(S_g)$ be the associated curve graph and d be the associated path metric. Let α and β be curves on S_g and $T_\beta(\alpha)$ be the Dehn twist of α about β.

If $d(\alpha, \beta) = 3$, we show that $d(\alpha, T_\beta(\alpha)) = 4$. This produces many tractable examples of distance 4 vertices in $\mathcal{C}(S_g)$. As an application we show that the minimum intersection number of any two curves at a distance 4 on S_g is at most $(2g-1)^2$.

Let $d(\alpha, \beta) = 4$. We fix the vertex α and apply T_β to it in an attempt to create pairs of curves at a distance 5 apart. We give a necessary and sufficient topological condition for $d(\alpha, T_\beta(\alpha))$ to be 4. We then characterise the pairs of α and β for which $5 \leq d(\alpha, T_\beta(\alpha)) \leq 6$. Lastly, we give an example of a pair of curves on S_2 which represent vertices at a distance 5 in $\mathcal{C}(S_2)$ with intersection number 144. This example gives that $i_{min}(2, 5) \leq 144$.

Our proofs majorly rely on cut and paste techniques.