INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

Fluctuating geometries occur in a variety of macroscopic non-equilibrium phenomena in Nature. Examples start from trajectories of Brownian particles, structure of sea shores, landscapes, mountains, islands, rivers, sediments, and even the surface geometry of thin films grown in the laboratory. Dynamics of these non-equilibrium phenomena being stochastic in nature, their statistical properties are of interest in modern trends of research.

In this thesis, we focus mainly on surface growth dynamics and fluid turbulence, which are of great value due to their practical importance. In particular, we study a few normalized cumulants of distribution functions (height distribution for surface growth and distribution of velocity derivative for fluid turbulence), namely, skewness, kurtosis, hyper-skewness. The non-zero value of those cumulants confirm that the corresponding probability distributions are non-Gaussian in nature. There are various types of surface growth, which can be described by different continuum equations. The most generic continuum nonlinear equation for surface growth is Kardar-Parisi-Zhang (KPZ) equation driven by a Gaussian white noise. The equation does not respect the up-down symmetry of the surface height because of the nonlinear term in the equation. First, we study one-dimensional KPZ equation by employing a perturbative renormalization scheme. We calculate the second and third order cumulants of the height-distribution from the renormalized Feynman loop diagrams obtaining the skewness value S = 0.3237 which is independent of any model parameter.

We also study the fourth and fifth order cumulants of the one-dimensional KPZ height-distribution. Following diagrammatic renormalization schemes, we obtain the kurtosis Q = 0.1523 and heperskewness $\tilde{S} = 0.0835$. These values are also independent of model parameters signifying their universality. However, these values do not agree with the Baik-Rains values because of dominance of the Gaussian fluctuations in one dimension.

We further study the third and fourth cumulants of height distribution governed by (2+1)-dimensional KPZ equation. We follow a diagrammatic scheme to derive the expressions for renormalized cumulants up to fourth order. Assuming a value for the roughness exponent χ from reliable numerical prediction, we calculate the second, third and fourth order cumulants, yielding skewness S = 0.2879 and kurtosis Q = 1995, that agree well with existing numerical estimates. The conserved dynamics of height fluctuations, for example, in the process of molecular beam epitaxy, is described by the VLDS equation driven by a non-conserved Gaussian white noise. Using a diagrammatic method, we calculate the renormalized second and third order cumulants in the case of 1+1 dimensional VLDS dynamics. Our obtained skewness value S = - 0.0441 is consistent with the numerical prediction of Das Sarma et al. who obtained S = - 0.1±0.15 in the steady state and suggested that it is likely to be negative although they did not exclude a zero or slightly positive value. Our calculated skewness value asserts that the probability distribution function is negatively skewed.

Finally, we consider the problem of fluid turbulence governed by the Navier–Stokes equation. The second and third order cumulants of the velocity derivative are calculated from the renormalized loop diagrams within the phenomenological frameworks of Kolmogorov and Pao. Evaluating the loop integrals, we obtain the velocity derivative skewness as S = − 0.647 in the inertial range and $\mathcal{S}$ = - 0.682 when the dissipation range is included. These estimated values are comparable with other theoretical and experimental estimates. We observe that our theoretically predicted values agree better with experimental and numerical estimations when the dimensionality of the system increases. This is expected because the role of Gaussian fluctuations coming from the stochastic noise term diminishes when the system dimensionality increases.