INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

Name of the Student	:	**Shamik Das**
Roll Number	:	**166123003**
Programme of Study	:	**Ph.D.**
Thesis Title	:	**Congruent Number and Related Topics**
Name of Thesis Supervisor(s)	:	**Anupam Saikia**
Thesis Submitted to the Department/ Center	:	**Mathematics**
Date of completion of Thesis Viva-Voce Exam	:	**24th September, 2021**
Key words for description of Thesis Work	:	Congruent Numbers and Their Generalizations

---

SHORT ABSTRACT

A positive integer n is called a **congruent number** if it is equal to the area of a right triangle with rational sides. Determining whether a given positive number is congruent or not is known as the **congruent number problem**. It is well-known that n is a congruent number if and only if the rank of the Mordell-Weil group consisting of all rational points on the **congruent number elliptic curve** $E_n : y^2=x^3-n^2x$ is positive. Although congruent numbers have been studied for centuries, the problem of providing a complete classification still remains elusive. Various mathematicians constructed infinite families of congruent and non-congruent numbers with prime factors that satisfy certain congruence conditions.

We begin the thesis by introducing congruent number and its generalization followed by inclusion of certain preliminaries from basic algebra, number theory and elliptic curves that we need to elaborate our results. Then we show a construction of infinite families non-congruent numbers with arbitrarily many pairs of prime factors generalizing results using the method of complete 2-descent on congruent number elliptic curve. Further, we construct families of highly composite non-congruent numbers by considering Monsky's matrices too.

The notion of theta-congruent number is a generalization of congruent number, where one considers the area of a triangle with all possible angles theta such that cosine of theta is rational rather than just theta= pi/2. Here, we prove a criterion for a natural number to be a theta-congruent number over certain classes of real number fields.

Next, we prove a divisibility result for the class number of complex quadratic field having discriminant $-pq$, where p and q are distinct primes congruent to 5 and 7 modulo 8 respectively and pq is a congruent number. Rather than modular forms of half-integral weight, we exploit the method of complete 2-descent.

At the end, we include our finding concerning the period of the regular continued fraction of square root of pq where q is a bigger prime and both of them are congruent to 3 modulo 4. We prove that the length of the period is divisible by 4 when q is a quadratic non-residue modulo p and is of the form $4k+2$ when q is a quadratic residue modulo p. We further examine the parity of the the central term in the palindromic part of the period of square root of pq.