Abstract

The main objective of this thesis is to investigate the numerical analysis of finite element method for the subdiffusion equations with memory. Both smooth and singular kernels are considered covering smooth and nonsmooth initial data.

Prior to the development of numerical schemes, we first study the existence, uniqueness, and stability results of the continuous solution with respect to various Sobolev regularity assumptions on the problem data, i.e., the initial data and the source function. For both smooth and singular kernel cases, we show that the solution corresponding to the homogeneous problem is infinitely differentiable with respect to time variable when the initial data is simply an element of $L^2(\Omega)$. In each case, we study the semidiscrete as well as the fully discrete schemes and carry out the convergence analysis for both smooth and nonsmooth initial data. Error bounds are expressed directly in terms of the problem data. We discretize the spatial variable based on Galerkin finite element method (GFEM) by using piecewise linear functions. The temporal discretization is performed by convolution quadrature with the generating function given by the backward Euler (BE) and the second-order backward difference (SBD) schemes and the L1 scheme. For the smooth kernel case, we derive optimal order error bounds for smooth initial data and almost optimal order for nonsmooth data in both $L^2(\Omega)$- and $H^1(\Omega)$-norms for the spatially discrete scheme. For a particular choice of the kernel operator in the memory term, we have achieved optimal error estimates for nonsmooth initial data. In addition, we study the convergence analysis with nonsmooth data by energy arguments which enables us to recover optimal order error bounds even for a more general kernel in the memory term. For the temporal discretization, we demonstrate error bounds of first-order and second-order accuracy in time for the BE and SBD schemes, respectively. In the singular kernel case, we prove optimal error bounds for the GFEM to the spatial discretization and an error estimate of order $O(k)$ in time for the L1 scheme. In all cases, we validate our theoretical convergence rates of the approximate solutions by extensive numerical experiments.