Abstract

Studying the behavior of a dynamical system and the dependency of its solution on the initial state or initial condition began in the late 1880s. Describing the dynamics of dynamical systems as a function of time on the state space can generate a differential equation. Thus, the theory of dynamical systems may be said to be a special and important topic in the theory of differential equations. It falls under the qualitative theory which is mainly concerned with properties which are not quantified. The study of qualitative theory leads to a better understanding of the dynamical systems.

This thesis mainly focuses on the stability analysis of dynamical and control systems. We basically deal with two types of stability treatment: (i) the Ulam-Hyers stability of the solution of an impulsive fractional-order integro-differential equation involving Caputo fractional-order derivatives, and also the Ulam-Hyers stability of finite difference equations corresponding to first- and second-order differential equations, (ii) the Lyapunov stability for a class of fractional-order differential equations involving a non-singular kernel fractional-order derivative called Caputo-Fabrizio derivative and discuss the various concepts such as the existence of a periodic solution, stabilization, and asymptotic stability of Caputo-Fabrizio fractional-order semilinear evolution equations.

In the beginning of this study, we establish the existence of Ulam-Hyers and generalized Ulam-Hyers-Rassias stability results of the mild solutions of Caputo fractional non-instantaneous impulsive integro-differential equations in the form of two separate problems. The main results are established by using Banach fixed point theorem under appropriate assumptions. For the second problem, we use our results to estimate the bound for the difference between the fractional-order and the integer-order non-instantaneous impulsive RLC circuit current and show that the bound mainly depends on the bandwidth of the RLC circuit.

In the next part of this thesis, we consider the qualitative analysis as introduced by Lyapunov for different classes of fractional-order differential equations involving Caputo-Fabrizio fractional-order derivatives containing a non-singular kernel. First, we revisit the Lyapunov stability of an equilibrium point of an autonomous Caputo-Fabrizio fractional-order system and show that all isolated equilibrium points of an autonomous system are asymptotically stable, and we find that only constant solutions exist for autonomous systems. Further, we study the Lyapunov stability for the intermediate value Caputo-

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Fabrizio linear and nonlinear autonomous systems and derive the condition required for the equilibrium point for such systems to be asymptotically stable. A suitable example is presented at the end to illustrate the result of the existence of such a stability. In another problem, the existence of a periodic solution of the Caputo-Fabrizio fractional-order system is considered. Under a similar assumption as the one for an integer-order differential system, and by using the properties of the Caputo-Fabrizio derivative, the existence of a periodic solution of a non-autonomous Caputo-Fabrizio fractional-order differential system is established. As an application, we derive a periodic solution of a fractional-order Gunn diode oscillator under a periodic input voltage, and observe that the diameter of the periodic orbit keeps reducing as the fractional-order continuously increases. Further, by constructing a suitable linear feedback control, the solution of a linear non-homogeneous fractional-order system is stabilized to a periodic solution, and an example is presented to support the obtained result.

In another problem, we discuss the asymptotic stability of fractional-order linear and semilinear evolution equations involving a Caputo-Fabrizio fractional derivative for fractional-order $\alpha \in (0,1)$, and introduce a new concept of a global solution for the Caputo-Fabrizio system. Laplace transform and Grönwall inequality are used to derive the local and global asymptotic stability conditions. By constructing a suitable linear feedback control and using our main results, we stabilize the Caputo-Fabrizio fractional-order linear and semilinear evolution equations. In the end, by using the stabilization result, we stabilize a fractional-order chaotic system to support the obtained results.

The last problem of the thesis is considered in a different direction. By using the Ulam-Hyers stability results for the linear recurrence relation, we establish the Ulam-Hyers stability of a second-order convergent finite difference equation corresponding to the first- and second-order non-homogeneous linear differential equations with constant coefficients. Further, as per the location of the roots of the characteristic polynomial of the equivalent recurrence relation, the minimum Ulam-Hyers constant is determined, and a suitable example is presented to support the obtained result.

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