Abstract

This thesis studies arithmetic properties of ℓ-regular overpartitions, Andrews' singular overpartitions, overpartitions into odd parts, cubic and overcubic partition pairs, and Andrews' integer partitions with even parts below odd parts. We use various dissections of Ramanujan's theta functions to find infinite families of arithmetic identities and Ramanujan-type congruences for ℓ-regular overpartitions and overpartitions into odd parts. We find certain congruences satisfied by $\overline{A}_\ell(n)$ for $\ell = 4, 8$ and 9, where $\overline{A}_\ell(n)$ denotes the number of ℓ-regular overpartitions of n. We find several infinite families of congruences including some Ramanujan-type congruences satisfied by $\overline{A}_{2\ell}(n)$ and $\overline{A}_{4\ell}(n)$ for any $\ell \geq 1$. We next prove several congruences for $\overline{p}_o(n)$ modulo 8 and 16, where $\overline{p}_o(n)$ denotes the number of overpartitions of n into odd parts. We also obtain the generating functions for $\overline{p}_o(16n+2)$, $\overline{p}_o(16n+6)$, and $\overline{p}_o(16n+10)$; and some new p-dissection formulas.

In a very recent paper, Andrews introduced the partition function $\mathcal{EO}(n)$ which counts the number of partitions of n where every even part is less that each odd part. He denoted by $\overline{\mathcal{EO}}(n)$, the number of partitions counted by $\mathcal{EO}(n)$ in which *only* the largest even part appears an odd number of times. We use arithmetic properties of modular forms and eta-quotients to study distribution of Andrews' singular overpartitions, cubic and overcubic partition pairs, and Andrews' integer partitions with even parts below odd parts. We use q-series manipulations and

Radu's algorithm on modular forms to derive certain congruences satisfied by cubic partition pairs, overcubic partition pairs and Andrews' integer partitions with even parts below odd parts. Along the way, we affirm two conjectures on Andrews' singular overpartitions and cubic partition pairs. We find two infinite families of congruences for $\overline{\mathcal{EO}}(n)$ using the theory of Hecke eigenforms. We also prove that there are infinitely many integers N in every arithmetic progression for which $\overline{\mathcal{EO}}(N)$ is even; and that there are infinitely many integers M in every arithmetic progression for which $\overline{\mathcal{EO}}(M)$ is odd so long as there is at least one.