Higher Order Compact Explicit Jump Immersed Interface Methods for Incompressible Viscous Flows: Application and Development

Ph.D. Thesis

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Abstract

This study is primarily focused on the development of explicit jump high-order compact finite difference immersed interface approaches for the purpose of solving incompressible viscous flows that are governed by the Navier-Stokes (N-S) equation on uniform and non-uniform grids on a Cartesian mesh. In all, three basic schemes have been developed in the process: one for elliptic problems and the steady state of N-S equations with discontinuities in the solutions, source terms, and coefficients across the interface; the next one is the transient counterpart of the previously developed one uniform grids; and lastly, a discrete level-set approach on non-uniform grids with complex interfaces. The overall accuracy of the scheme is four in space and two in time. Throughout the whole physical domain, a nine-point compact stencil is maintained by incorporating the jump conditions into the right-hand side of the matrix equation $Ax = b$ resulting from discretization of the concerned equations. We use the streamfunction- vorticity (ψ-ζ) formulation of the N-S equation, and the jump conditions for ψ and ζ at the irregular point across the interface are taken into account by using a new method based on Lagrangian interpolation.

The main emphasis of the work is the simulation of flow around bluff bodies by employing the above mentioned schemes, which has many applications in industries and day to day life. During the process, both the grid independence of the computed solutions and the numerical rate of convergence of the schemes are determined on a circular and star-shaped interface. High-quality simulations are carried out over a wide range of Reynolds numbers (Re) between 10 and 3000 for the stationary bluff bodies. Detailed analyses of the fluid flow are performed for flow past stationary circular, ellipse and cactus-shaped cylinders, NACA 0012 airfoil, a swimming fish and oscillating circular cylinders. Furthermore, more specifically, the proposed approaches have been applied to comprehensively investigate heat transfer owing to forced convection from a heated elliptical cylinder in a uniform free stream with angle of inclination.

In all the cases, the proposed schemes were seen to very efficiently capture all the flow and heat characteristics associated with the problems under consideration. Apart from aptly resolving the flow separation leading to secondary and tertiary vortex phenomena, the von *Kármán* vortex's streets, a typical feature of periodic flows under different flow situations have also been accurately captured by our simulation. Computations were performed for not only flow past single bodies, but also those involving multiple and moving bodies, thus exemplifying the robustness of the proposed approaches. For the selected test cases where numerical and experimental results are already available, our computational results were extremely close to all of them.