Abstract

The study of state transfer in quantum communication networks received a lot of attention over the past few decades. A quantum network can be modelled by a graph with the adjacency matrix of the graph as the Hamiltonian of such system. In that case, we identify a quantum system by its underlying graph. If there is no external dynamic control over the system then some physical properties of the quantum system depend only on the underlying graph. In this thesis, we are concerned with two such properties: perfect state transfer and pretty good state transfer. We find some major classes of graphs exhibiting either of those properties.

Let G be a graph with adjacency matrix A. The transition matrix of G relative to A is defined by $H(t) := \exp(-itA)$, $t \in \mathbb{R}$. The graph G is said to have perfect state transfer from a vertex u to another vertex v if there exists $\tau\,(\neq 0) \in \mathbb{R}$ such that the uv-th entry of $H(\tau)$ has unit modulus. In case $u = v$, we say that G is periodic at the vertex u at time τ. The graph G is said to be periodic if it is periodic at all vertices at the same time. Perfect state transfer is a rare phenomena so we also consider an approximation called pretty good state transfer. The graph G is said to admit pretty good state transfer between a pair of vertices u and v if there exists a sequence of real numbers $\{t_k\}$ and a complex number γ of unit modulus such that $\lim\limits_{k\to\infty} H(t_k)\mathbf{e}_u = \gamma\mathbf{e}_v$. In Chapter 1, we introduce these topics in detail. Along with that we discuss some relevant definitions and basic results.

We mainly consider two classes of graphs. One is NEPS (non-complete extended P-sum) of the path on three vertices and the other one is Cayley graph. In both classes, we investigate graphs for perfect state transfer and pretty good state transfer.

It is well known that a path on three vertices exhibits perfect state transfer and so we investigate some NEPS of the path on three vertices in Chapter 2. A sufficient condition is found for a NEPS of path on three vertices to have perfect state transfer. Using these NEPS, some other graphs are also constructed that admit perfect state transfer. The results of this chapter are published in [40].

In Chapter 3, we also find that NEPS of the path on three vertices whose basis contains tuples with hamming weights of both parities do not exhibit perfect state transfer. But these NEPS admit pretty good state transfer with an additional condition. Further, we investigate pretty good state transfer on Cartesian product of graphs and we find that a graph can have PGST from a vertex u to two different vertices v and w. The results of this chapter are published in [42].

A gcd-graph is a Cayley graph over a finite abelian group defined by greatest common divisors. In Chapter 4, we establish a sufficient condition for a gcd-graph to have periodicity and PST. Using this we deduce that there exists gcd-graph having PST over any abelian group of order divisible by 4. Also, we find a necessary and sufficient condition for a class of gcd-graphs to be periodic at π. Using this, we characterize a class of gcd-graphs not exhibiting PST at $\frac{\pi}{2^k}$ for any positive integer k. The results of this chapter appear in [39].

In Chapter 5, we find that pretty good state transfer occurs in a cycle on n vertices if and only if n is a power of two and it occurs between every pair of antipodal vertices. In addition, we look for pretty good state transfer in more general circulant graphs. We prove that union (edge disjoint) of an integral circulant graph with a cycle, each on 2^k ($k \geq 3$) vertices, admits pretty good state transfer. The complement of such union also admits pretty good state transfer. This enables us to find some non-circulant graphs admitting pretty good state transfer. Among the complement of cycles we also find a class of graphs not exhibiting pretty good state transfer. The results of this chapter appear in [41].

In Chapter 6, we describe a few directions for future research based on the work of this thesis.
