

## Abstract

The main objective of this thesis is to study *a priori* error analysis of the two-scale composite finite element (CFE) method for parabolic initial-boundary value problems (IBVPs) in two-dimensional convex and nonconvex polygonal domains. When the physical domain is non-convex or very complicated, the standard finite element method (FEM) requires to generate finite element mesh that resolves the domain boundary. As a result, the degrees of freedom of such finite element space are distributed in a nonoptimal way with respect to the approximation quality which drastically increases the minimal dimension of the finite element space. Whereas, the CFE discretizations are based on the two-scale grid refinement: In the interior of the domain at a proper distance from the boundary, the solution is approximated by a coarse-scale parameter  $H$  whereas the near-boundary triangles are discretized by a fine-scale parameter  $h$  which approximates the Dirichlet boundary conditions. The coarse-scale grid  $\mathcal{T}_H$  contains the degrees of freedom whereas the fine-scale grid  $\mathcal{T}_h$  adaptively resolves the boundary  $\Gamma$  that contains the slave nodes only. The basic idea of the CFE procedure is to work with fewer degrees of freedom by allowing finite element mesh to resolve the domain boundaries and to preserve the asymptotic order convergence on coarse-scale mesh. In contrast to the standard finite elements, CFE method uses the minimal dimension of the approximation space. These new class of finite elements can be thought of as a generalization of standard finite elements by allowing the approximation of the domain boundaries in a flexible adaptive manner. This is especially very advantageous for problems on complicated domains.

The present dissertation is commenced with general introduction to the respective field along with the detailed overview of CFE discretizations. Additionally, the background knowledge and motivations of *a priori* error analysis for the two-scale CFE approximation of parabolic IBVPs in two-dimensional convex and nonconvex polygonal domains are discussed. To begin with, we first study *a priori* estimates in both  $L^\infty(L^2)$  and  $L^\infty(H^1)$ -norms for the spatially semidiscrete and fully discrete CFE approximation of the linear parabolic problem in a convex domain with smooth initial data. The estimates of the associated elliptic or Ritz projection in the framework of CFE method play an important role in the error analysis. We derive optimal order convergence (up to logarithmic terms) for smooth initial data. Then, we concentrate on the nonsmooth data error analysis for the homogeneous linear parabolic equa-

tion in a convex domain. Optimal order error estimates (up to logarithmic terms) with respect to space discretization are shown to hold for positive time even for nonsmooth initial data. The eigenfunctions expansion related to the elliptic operator and the rational approximation for exponential are the key technical tools used in nonsmooth data error analysis.

We next proceed to the *a priori* error analysis for the spatially semidiscrete and fully discrete CFE approximation of linear parabolic problem in a nonconvex domain for both smooth and nonsmooth initial data. Compared to the error estimates for convex domain, the rates of convergence in nonconvex domain is reduced for both smooth and nonsmooth data. Furthermore, we consider the CFE error analysis for nonlinear parabolic problems in nonconvex domains. Both the spatially semidiscrete and fully discrete schemes are analyzed and the related error estimates are derived. Finally, we focus on *a priori* error estimates in the  $L^2(L^2)$ -norm for the spatially semidiscrete and fully discrete CFE approximation of parabolic problem with measure data in time for both convex and nonconvex domains. The solution of this kind of problem exhibits low regularity which introduces some difficulties in both theory and numerics of CFE method. An effort has been made to investigate the CFE error analysis and derive convergence properties for both convex and nonconvex domains.

We perform extensive numerical experiments of the proposed CFE method for both convex and nonconvex domains to support our theoretical results. It is shown that the number of degrees of freedom for CFE method is much less compared to the standard FEM, which substantially reduces the computational effort.