INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

In this thesis, we study p-adic analogues of certain classical hypergeometric identities, relations of p-adic hypergeometric functions to algebraic varieties and modular forms, and find certain special values of the p-adic hypergeometric functions. Firstly, we establish two transformations for the p-adic hypergeometric function, which can be described as analogues of a transformation of Euler and a transformation of Clausen. We derive some more transformations and find certain special values of p-adic hypergeometric functions. Next, we study relationships between p-adic hypergeometric functions and the number of points on diagonal hypersurfaces over a finite field. We also express the trace of Frobenius of elliptic curves in terms of special values of the p-adic hypergeometric functions. We then derive summation identities for these functions. As an application of the summation identities, we prove identities for the trace of Frobenius endomorphism on certain families of elliptic curves. Finally, we study relationships between p-adic hypergeometric functions and modular forms. We prove p-adic analogues of certain classical hypergeometric identities, and using these identities we express the p-th Fourier coefficient of certain weight three newforms in terms of special values of p-adic hypergeometric functions. Rodriguez-Villegas conjectured certain supercongruences between values of truncated hypergeometric series and the p-th Fourier coefficients of these newforms. As a consequence of our main results, we obtain another proof of these supercongruences conjectured by Rodriguez-Villegas.