INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

This thesis address some problems on frames and their generalizations viz Hilbert space valued Gabor frames and Hilbert-Schmidt frames. Mainly, we analyze Gabor frames on amalgam spaces, obtain solution of a Feichtinger problem and establish Balian–Low type theorems on $L^2(\mathbb{C})$. Thesis is divided into six chapters.

In Chapter 1, we give a brief introduction of frame theory, discuss some well-known results, basic definitions and provide a literature survey.

In Chapter 2, we prove the convergence of Gabor expansions to identity operator in the operator norm as well as weak* sense on $W(L^p, L^q)$ as the sampling density tends to infinity. Using it we show the validity of the Janssen's representation and the Wexler-Raz biorthogonality condition for Gabor frame operator on $W(L^p, L^q)$.

Let $\mathbb{H}$ be a separable Hilbert space. In Chapter 3, we establish a generalization of Walnut's representation and Janssen's representation of the $\mathbb{H}-$valued Gabor frame operator on $\mathbb{H}-$valued weighted amalgam spaces $W_{\mathbb{H}}(L^p, L^q_v)$, $1 \leq p, q \leq \infty$. Also we show that the frame operator is invertible on $W_{\mathbb{H}}(L^p, L^q_v)$, $1 \leq p, q \leq \infty$, if the window function is in the Wiener amalgam space $W_{\mathbb{H}}(L^\infty, L^1_w)$. Further, we obtain the Walnut's representation and invertibility of the frame operator corresponding to Gabor superframes and multi-window Gabor frames on $W_{\mathbb{H}}(L^p, L^q_v)$, $1 \leq p, q \leq \infty$, as a special case by choosing the appropriate Hilbert space $\mathbb{H}$.

In Chapter 4, we study the Hilbert−Schmidt frame (HS-frame) theory for separable Hilbert spaces. We first present some characterizations of HS-frames and prove that HS-frames share many important properties with frames. Then we show how the inverse of the HS-frame operator can be approximated using finite-dimensional methods. Also we present a classical perturbation result and prove that HS-frames are stable under small perturbations. Further as an application we establish Parseval type identities and inequalities for HS-frames.

In Chapter 5, we answer the open question of Feichtinger's regarding integral and pseudodifferential operators on modulation spaces. Also, we discuss the solutions of several reformulated problems inspired by the original Feichtinger's question in Hilbert space operator theory that was posed by Heil and Larson. These results provide some connections between operator theory and the theory of modulation spaces.

In Chapter 6, we show that $\|Zg\|_2$ and $\|\bar{Z}g\|_2$ cannot both be simultaneously finite if the twisted Gabor frame generated by $g \in L^2(\mathbb{C})$ forms an orthonormal basis or an exact frame for $L^2(\mathbb{C})$. The operators $Z = \frac{d}{dz} + \frac{1}{2}\bar{z}$ and $\bar{Z} = \frac{d}{d\bar{z}} - \frac{1}{2}z$ are associated with the special Hermite operator $L = -\Delta_z + \frac{1}{4}|z|^2 - i\left(x\frac{d}{dy} - y\frac{d}{dx}\right)$ on $\mathbb{C}$, where Δ_z is the standard Laplacian on $\mathbb{C}$ and $z = x+iy$. Also the amalgam version of BLT is proved using Weyl transform and the distinction between BLT and amalgam BLT is illustrated by examples. The twisted Zak transform is introduced and using it several versions of the Balian-Low type theorems on $L^2(\mathbb{C})$ are established.