INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

The main objective of this thesis is to study adaptive finite element methods (AFEMs) for parabolic interface problems in a bounded convex polygonal domain in R^2. Interface problems arise in a wide variety of applications in science and engineering such as material sciences and fluid dynamics when two or more distinct materials or fluids with different conductivities or densities or diffusions are interacting across the interface. Due to the discontinuity of the coefficients along the interface, the analytic solutions are rarely available for the interface problems. Therefore, numerical approximation is the only way to proceed with such problems. Even if the solution is smooth in each individual subdomain, the global regularity of the solution of such a problem is very low. As a result, it is very challenging to achieve higher order accuracy in the finite element method (FEM). Therefore, much attention has been paid in the recent years to the study of both theory and numerics of time-dependent interface problems. It is known that AFEMs are widely used numerical techniques to enhance the accuracy and efficiency of the finite element method. The key to the success of AFEMs relies on a posteriori error analysis, which provides error indicators for the design of adaptive algorithms. The adaptive method reduces the computational efforts and ensures higher density nodes in a particular area of the given domain where the solution is very difficult to approximate.

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This thesis investigates a posteriori error analysis and develops adaptive algorithms for various FEMs for solving parabolic interface problems. More precisely, AFEMs for the conforming FEM, non-conforming FEM, immersed FEM, non-conforming immersed FEM for linear parabolic interface problems have been analyzed. Lastly, an extension to the semilinear parabolic interface problem using the conforming finite element method is considered and analyzed. For all these methods, new error indicators are introduced and adaptive algorithms are presented. Our first problem focuses on the linear parabolic interface problems with nonzero flux jump across the interface using conforming finite elements. A residual-based a posteriori error estimate for the fully discrete approximation is considered and analyzed. Both global upper and local lower bounds for the error are derived. Among the crucial technical tools used in the analysis include the Cauchy-Schwarz inequality, the approximation properties of the Cle´ment interpolation operator, the properties of bubble functions, and energy arguments. An adaptive space-time algorithm is presented using the derived error indicators. Numerical results are displayed to study the performance of the error estimators. Our next attempt is to investigate an adaptive immersed finite element method for parabolic interface problems with nonzero flux jump. The interface is assumed to be smooth, and the finite element meshes do not lie on the interface. We derive both upper and lower bounds for the error using energy arguments. A space-time adaptive algorithm is provided and supportive numerical results are presented. Our next focus is on residual-based a posteriori error estimation for nonconforming finite element approximation to the parabolic interface problem. The reliability of the estimator is analyzed without using the Helmholtz decomposition. We derive both upper and lower bounds for the error. The representation of the error equation, the approximation properties of the modified Cle´ment interpolation operator, the trace inequality, and energy arguments are key ingredients used in the analysis. Numerical results are presented to demonstrate the behavior of the derived estimators. Next, we turn our attention to the a posteriori error analysis and adaptive mesh refinement for the parabolic interface problem using nonconforming immersed finite element method. The residual-based a posteriori error estimates are derived using energy arguments. Both upper and lower bounds for the error are established. A space-time adaptive algorithm is proposed and numerical results are provided.

Finally, we extend our analysis for the linear case to treat the semi-linear parabolic interface problems. The residual-based a posteriori error analysis for the fully discrete backward Euler method is presented. Our strategy is to avoid solving the nonlinear system by considering a modified linearized fully discrete scheme. The properties of the Cle´ment interpolation operator and energy arguments are used to derive an upper bound for the a posteriori error. The bubble function technique is used to derive a posteriori lower bound for the error. A space-time adaptive algorithm is presented using the derived estimators. Numerical results are provided to illustrate the behavior of the derived estimators.

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