Approximation Algorithms for Sweep Coverage in Wireless Sensor Networks

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by

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CERTIFICATE

This is to certify that this thesis entitled "**Approximation Algorithms for Sweep Coverage in Wireless Sensor Networks**" being submitted by Mr. Barun Gorain to the Department of Mathematics, Indian Institute of Technology Guwahati, is a record of bona fide research work under my supervision and is worthy of consideration for the award of the degree of Doctor of Philosophy of the Institute.

The results contained in this thesis have not been submitted in part or full to any other university or institute for the award of any degree or diploma.

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ABSTRACT

Coverage is one of the most important issues in wireless sensor networks (WSNs) and it is a well studied research area. To ensure coverage, a set of sensors continuously monitor the subject to be covered. Sweep coverage is a recent development for the applications of WSNs where periodic monitoring by a set of mobile sensors is sufficient instead of continuous one like traditional coverage. In this thesis, we remark on the flaw of the approximation algorithms [41] for sweep coverage for a set of points. We propose a 3-approximation algorithm to guarantee sweep coverage for the vertices of a weighted graph, where vertices are considered as the set of points. When vertices of the graph have different sweep periods and processing times, we propose a $O(\log \rho)$-approximation algorithm as a solution, where ρ is the ratio of the maximum and minimum sweep periods among the vertices. If speeds of the mobile sensors are different, we prove that it is impossible to design any constant factor approximation algorithm to solve the problem, unless P=NP.

Energy is an important issue for the sensors. To incorporate it, an energy efficient sweep coverage problem is proposed, where a combination of static and mobile sensors are used. The problem is NP-hard and cannot be approximated within a factor of 2, unless P=NP. An 8-approximation algorithm is proposed to solve it. A 2-approximation algorithm is also proposed for a special case of the problem, which is the best possible approximation factor. Energy utilization for the mobile sensors is restricted as they are battery-powered. Considering the limitation, an energy restricted sweep coverage problem is defined and a $(5 + \frac{2}{\alpha})$-approximation algorithm is proposed to solve this NP-hard problem. Periodic patrol inspection is important to detect illegal movement across national border. A portion of border can be modeled as a finite length curve. With this model, we introduce barrier sweep coverage concept for periodic monitoring of finite length curves in two dimensional plane. A solution is proposed to sweep cover a finite length curve with optimal number of mobile sensors. An energy restricted barrier sweep coverage problem is introduced and proposed a $\frac{13}{3}$-approximation algorithm for a finite length curve. We prove that finding minimum number of mobile sensors to sweep coverage for a set of finite length curves is NP-hard and cannot be approximated within a factor of 2. For that a 5-approximation algorithm is proposed. A 2-approximation

algorithm is also proposed to solve the problem for a special case, where each mobile sensor must visit all points of each curve. We formulate a data gathering problem by a set of data mules and prove that the problem is NP-hard. A 3-approximation algorithm is proposed to solve it. Through simulation, performance of the algorithm for multiple finite length curves is investigated. We introduce area sweep coverage problem and prove that the problem is NP-hard. A $2\sqrt{2}$-approximation algorithm is proposed to solve the problem for a square region. For arbitrary bounder region $\mathcal{A}$ the approximation factor is $2\left(\sqrt{2}+\frac{2r\mathcal{P}}{Area(\mathcal{A})}\right)$, where $\mathcal{P}$ is the perimeter of $\mathcal{A}$ and r is sensing range of the mobile sensors. Through simulation, we analyze performance of the algorithm for area bounded by arbitrary simple polygons. We propose a distributed sweep coverage problem, where static sensors are placed at the location of PoIs, one for each. In our proposed algorithm, the static sensors themselves decide number with initial positions as well as movement strategy of the mobile sensors through message passing. The mobile sensors visit the static sensors once in a given time period to sweep cover the PoIs. The proposed algorithm achieved an approximation factor of 4. A 2-approximation algorithm is also proposed for a special case of the problem.

Contents

Chapter 1

Introduction

In last few decades, immense achievements in wireless sensor networks (WSNs) took place due to their wide range of potential applications. WSNs are used in almost every aspects of real life applications such as general engineering, agriculture & environmental monitoring, civil engineering, military applications, health monitoring & surgery, etc. A sensor node or in short, sensor is a battery-powered small autonomous device consisting of a sensing unit, small processor, memory and transceiver. It is capable of sensing physical or environmental conditions within its sensing range. A sensor can communicate with other sensors through radio frequency channels within its communication range. Once sensors are deployed over a region of interest, they form a wireless network with their self-organization capacities. Sensors monitor physical or environmental conditions like light, temperature, humidity, motion of objects, etc., and cooperatively send data through the network to a sink. The sink provides a user interface to interact with the user. The WSN provides an opportunity to monitor objects remotely. There are several issues of designing hardware of a sensor such as power source, memory capacity, processor speed, range of communication, size of a sensor node, manufacturing cost, etc. In spite of having sensors with the given specifications, there are several challenges on theoretical modeling, design of energy-efficient protocols, capacity/throughput of a network, localization, routing, coverage, channel access and scheduling, connectivity, quality of service, security, implementation, etc. Out of them, coverage is one of the most important challenge of the WSNs. The fundamental question of the coverage problem is "how well do the sensors observe physical space?". The main objective of coverage problem is to cover the subjects by the sensing range of the sensors. For example, in forest monitoring [1], every location of the forest must be covered by at least one sensor in order to detect any unusual activities like forest fire, activities of poachers, etc. immediately. Similarly, covering boundary of the forest allows controlling and elimination of the poaching activities, and illegal entry through the boundary. A similar notion of coverage is termed as k-coverage [33, 36, 40], where instead of one sensor, the subjects of interest must be within the sensing range of at least k sensors.

There are different aspects that are needed to be considered for designing suitable coverage algorithms for different applications. Mainly, there are three types of coverage problem found in literature. The first one is *point coverage* or *target coverage* [7, 9, 11, 67, 69], where a set of points, called targets are to be covered by a set of sensors. The second one is the *area coverage* [3, 44, 52, 57, 62, 63], where each and every point inside a bounded region is to be covered by a set of sensors. The other type of problem is called *barrier coverage* [33, 35, 36, 40, 42, 49, 50, 51, 54], where the objective is to secured the boundary of a region by covering a set of sensors.

Coverage problems are studied in the area of computational geometry for many years before the devolvement of WSNs. Some of the most popular problems are the art gallery problem, packing and covering of geometric objects, etc. The objective of art gallery problem [21, 26] is to determine the number of guards and their locations to cover an art gallery such that every point of the gallery is visible by at least one guard. There are several variations of this problem depending on the restrictions of placement of the guards provided the art gallery is of a polygonal structure. These variations are named as vertex guard, edge guard and point guard. In vertex guard, the guards are to be placed on the vertices of the polygonal gallery. In edge guard, the guards are to be placed on the edges of the polygonal gallery and in the point guard, the guards are to be placed on any point inside the polygonal gallery. All these variations of the problem are known to be NP-hard. The packing and covering problems [27] are studied in computational geometry for last six or seven decades. Objective of the packing problems is to pack a given area with minimum number of objects of a given shape like circle, square or hexagon without overlapping. In the covering problem, the objects can be overlapped and the objective is to cover the given area with minimum number of objects.

For the aforementioned problems, where coverage are maintained continuously, termed as *static coverage* [41]. But there are typical applications where only periodic patrol inspections are sufficient for a certain set of points of interest instead of continuous monitoring like static coverage. This type of coverage is termed as *sweep coverage*. First, theoretical aspects of the sweep coverage problem is studied by Li et al. [41], where periodic patrol inspections are required for a given set of points of interest (PoIs) by a set of mobile sensors. Unlike static coverage, in sweep coverage, the coverage requirement for the PoIs are time variant. Therefore, the solutions of static coverage problems cannot be applied for sweep coverage as they may lead to an unnecessary extra overhead and poor efficiency.

As per the model [41] for sweep coverage, the mobile sensors are assumed point sensors. So, the mobile sensor should reach at the position of a point of interest (PoI) to cover it at a time instance. A point is said to be *t-sweep covered* if and only if at least one mobile sensor visits the point within every t time period, where t is called *sweep period* of the point. The objective of the sweep coverage problem is to find minimum number of mobile sensors to guarantee sweep coverage for a given set of PoIs. Sweep coverage is useful for many real life applications like periodic health checkup, periodic automobile servicing, periodic inspection of boilers/pressure vessels in industry, periodic monitoring to secure target locations, periodic data gathering for agriculture and environmental monitoring, etc. Let us consider a practical scenario for data gathering, where a set of static sensors are deployed over a region of interest to collect information from the region. If the size of the data is high, it is expensive to send the data through message passing to the sink and it is more useful to collect data from the individual sensors. A mobile sensor can collect the data stored at a static sensor and deliver it to the sink. Also, having limited storage capacity of the static sensors, a mobile sensor must visit the static sensors frequently and collect data to avoid significant data loss. In this scenario, sweep coverage is useful. In this case, a static sensor is visited by a mobile sensor at least once in every predefined time period and the mobile sensors collect data from the static sensors.

As we have already mentioned, there are three types of static coverage problems depending on the subject to be covered by the sensors. Designing suitable sweep coverage algorithms for those variations are challenging problems. On sweep coverage, there are many works exist in literature for a set of discrete points. Most of the existing works [14, 19, 41, 55, 61, 64, 66, 68] focus on designing heuristics. Li et al. [41] formally defined the problem for a set of points and analyzed it from a theoretical point of view. They proved that the sweep coverage problem is NP-hard and cannot be approximated within a factor of 2, unless P=NP. The authors proposed constant factor approximation algorithms to solve the problem. But there is a serious flaw in the approximation algorithm [41]. This motivates us to design a constant factor approximation algorithm for sweep coverage problem by correcting the flaw of the previous work. Further, it will be interesting to investigate the sweep coverage problem in the context of barrier coverage, area coverage problems. It also might be interesting to design sweep coverage for distributed setting. There are several challenges for designing sweep coverage algorithms. These are fault tolerance [6], energy efficiency [9], self-stabilization [5, 47], scalability [43], etc.

1.1 Scope of the Thesis

In this thesis, we introduce and analyze several sweep coverage problems depending on various theoretical and practical aspects. Most of the sweep coverage problems we consider are NP-hard. So, our focus is to design suitable approximation algorithms for those problems.

1.1.1 Sweep coverage for a set of points

In Chapter 3, we remark on the flaw of the approximation algorithm proposed in the paper [41] for a set of PoIs and propose a 3-approximation algorithm to guarantee sweep coverage. As the previous approximation algorithm is not correct, to the best of our knowledge, we are the first to design constant factor approximation algorithm for the sweep coverage problem. A 2-approximation algorithm is proposed for a special case of the problem, where every PoI must be visited by each of the mobile sensors. A solution for sweep coverage in presence of obstacles is also proposed.

1.1.2 Sweep coverage with heterogeneity

In Chapter 4, we propose an algorithm to solve the sweep coverage problem when different PoIs are having different sweep periods and processing times. Our proposed algorithm achieves approximation factor $O(\log \rho)$, where ρ is the ratio of maximum and minimum sweep periods. We also investigate the problem when speed of the mobile sensors are different. We prove that it is impossible to design any constant factor approximation algorithm for sweep coverage problem with mobile sensors having different speeds, unless P=NP.

1.1.3 Solving energy issues for sweep coverage

In Chapter 5, we introduce an energy efficient sweep coverage problem where a combination of static and mobile sensors are used to sweep cover a given set of PoIs. An 8-approximation algorithm is proposed to solve this NP-hard problem. A 2-approximation algorithm is proposed for a special case of the problem where all mobile sensors visit the same subset of PoIs, while for the remaining PoIs, static sensors are used. Then we introduce another variation of sweep coverage, called energy restricted sweep coverage problem and propose a $(5 + \frac{2}{\alpha})$-approximation algorithm to solve this NP-hard problem.

1.1.4 Area sweep coverage

The sweep coverage problem for a bounded area is introduced in Chapter 6. We prove that the area sweep coverage problem is NP-hard. A $2\sqrt{2}$-approximation algorithm is proposed for a square region. The approximation factor is further improved for rectangular region. For arbitrary bounded region, the approximation factor of our proposed algorithm is a function of area, perimeter of the region and the sensing range of the mobile sensors. Through simulation, we analyze performance of the algorithm for randomly generated simple polygons.

1.1.5 Barrier sweep coverage

In Chapter 7, we introduce barrier sweep coverage problem, where a barrier is represented by a finite length continuous curve. We propose optimal solution for sweep coverage of a single curve. We introduce an energy restricted sweep coverage problem for single curve and propose a $\frac{13}{3}$-approximation algorithm. For multiple curves in a plane, we propose a 5-approximation algorithm for barrier sweep coverage problem and a 2-approximation algorithm for a special case of the problem. As an application of barrier sweep coverage problem, we formulate a data gathering problem and propose a 3-approximation algorithm to solve it.

1.1.6 Distributed sweep coverage

Li et al. [41] commented on the impossibility of designing distributed sweep coverage algorithm for a given set of PoIs. In Chapter 8, we propose a distributed sweep coverage in a different context. We assume static sensors are placed at the location of the PoIs, one for each. The set of static sensors forms a network. The static sensors decide the initial positions and movement strategy of the mobile sensors through message passing. A set of mobile sensors visits the static sensors once in a given time period to sweep cover the static sensors. We introduce a distributed sweep coverage problem and propose a 4-approximation algorithm. A 2-approximation algorithm is also proposed for a special case of the problem.

Chapter 2

Review of Related Works

2.1 Introduction

Coverage is a well-studied area of research in WSNs. But time-variant coverage in WSN is developed recently. Based on the dimension of time, coverage can be classified into two categories such as *static coverage* and *sweep coverage*. In static coverage, generally subjects are monitored by a set of sensors all along without any interruption whereas in sweep coverage subjects are monitored by a set of mobile sensors periodically. In this chapter, we briefly describe recent contributions that addressed different coverage problems in the context of our contributions in this thesis.

2.2 Static coverage

In static coverage, sensors continuously monitor subjects after deployment. Depending on the types of the subjects to be covered, the static coverage is classified into three categories in literature. Those are *point coverage*, *area coverage*, and *barrier coverage*. Each category is reviewed separately in the following sections.

2.2.1 Point coverage

Most of the works found in literature on point or target coverage problems are focused on maximizing network lifetime. Since the sensors are equipped with limited battery, recharging or replacing the battery may not always viable for applications in remote areas. In order to extend lifetime of the network and hence lifetime of the sensors, the set of sensors are divided into several disjoint subsets. The subsets are made in such a way that each PoI is in the sensing range of several sensors belonging to several subsets. The network lifetime can be extended with proper sleep-wake-up scheduling among the subsets of the sensors.

In paper [9], authors model the discrete target coverage problem as a disjoint set cover problem and proved that the problem is NP-complete. The disjoint set cover problem is transformed into a maximum flow problem. A heuristic is proposed using mixed integer programming to solve the maximum flow problem. Cardei et al. [8] improved the solution of [9] by choosing the set covers which are not necessarily disjoint. In this paper, the point coverage problem is modeled as a maximum set cover problem and proved the problem is NP-complete. Two efficient greedy heuristics based on linear programming are proposed to solve the problem. Zorbas et al. [69] proposed a centralized greedy heuristic to produce non-disjoint sets of sensors for energy efficient monitoring of the targets. Each of these individual sets can monitor all the targets. The sensors are included in a set depending on a cost function that takes into account the monitoring capabilities and remaining battery life of the sensors. Chaudhary et al. [11] proposed a centralized energy efficient approach to produce non-disjoint sets of sensors. The sets are made based on maximum residual battery life of the sensors, which can cover at least one object. Zamanifar et al. [67] discussed a variant of target coverage problem called target connected coverage, where all the sensors are connected to the base station with the help of relay sensors. Cardei et al. [7] proposed connected set cover problem for target coverage. The sensors are divided into disjoint sets such that each set covers all the targets and each sensor is connected to the base station. The objective of the problem is to find maximum number of sets among the set of sensors. The authors proposed three solutions; an integer programming based solution, a greedy approach and a distributed heuristic to solve the problem. Mini et al. [45] used the artificial ant colony algorithm [28, 29, 30, 31] to solve target coverage problem. Udgata et al. [60] proposed an algorithm for sensor deployment problem in irregular terrain, where number of points is more compared to the number of the deployed sensors. The goal of the proposed solution is to minimize the sensing range of the deployed sensors for efficient energy utilization. In paper [16], the authors considered a geometric version of the point coverage problem called unit disk cover problem. They discussed the computational complexity of the problem which is NP-hard. A constant factor approximation algorithm is proposed to solve the problem.

2.2.2 Area coverage

In area coverage problem, each point in the area is covered by a set of sensing disks of the respective sensors. Depending on the mobility of the sensors, the objective of the area coverage problems is classified into two categories. In the first category [57, 59], a large numbers of static sensors are deployed to cover the area of interest. The objective is to divide the sensors into disjoint sets such that each of which can guarantee full coverage of the area. The network lifetime can be extended with proper scheduling by turning off and on different sets alternatively. In the second category [3, 44, 52, 62, 63], a set of mobile sensors are used. The objective is to improve coverage by proper movement of the mobile sensors. Slijepcevic et al. [57] introduced 'k-set cover problem' with static sensors, which is an NP-complete problem. One heuristic is presented to find different set of sensors to cover the area. Lifetime of the network is extended by activating different sets periodically. In paper [59], the authors proposed an energy efficient area coverage algorithm for maintaining trade-off among network connectivity, coverage, and lifetime of the sensors. An extension of connected dominating set model is used to solve the area coverage problem that ensures load balancing among the active sensors. Initial random deployment of sensors over an area of interest may not guarantee complete coverage. Mobile sensors can improve the coverage by reducing overlaps with the neighboring sensors and moving towards the uncovered region. Wang et al. [63] proposed three movement assisted sensor deployment algorithms, which are VECtor-based (VEC), VORonoi-based (VOR) and Minimax. In this paper the authors used Voronoi diagram with respect to positions of the sensors to identify coverage holes. Then in VEC algorithm, the sensors, which do not cover their respective Voronoi cell, are pushed to fill the coverage holes. The sensors are moved to the farthest Voronoi vertex in VOR and minimize the coverage hole. Minimax algorithm moves the sensors to close to the farthest Voronoi vertex avoiding the generation of new holes. There is another approach proposed by Cao et al. [62] which is based on Voronoi diagram. A network of static and mobile sensors are considered in this work. Initially, static and mobile sensors are randomly deployed over a region. The static sensors find presence of holes by computing their local Voronoi cell. Then the static sensors on the boundary of holes bid for mobile sensors to move at the farthest Voronoi point. The authors proposed two kinds of bidding protocols, one is distance based and other is price based. In the distance based protocol, the static sensors on hole boundary bid for mobile sensors which is closest to the corresponding static sensor. In the price based protocol, the static sensors bid for a mobile sensor which is the cheapest with respect to coverage improvement. Mobile sensors can move from one hole to another if there is any coverage improvement. Another proxy based protocol is defined in the same paper where sensors first calculate their final locations then move once to their final location. The ATRI algorithm [44], makes the overall deployment layout close to equilateral triangulation by moving sensors after initial random deployment. It is assumed that the network is completely connected. The sensing disk of every sensor is divided into six equal sectors. In every iteration of the algorithm, each sensor adjusts its position with the nearest neighbor of each sector. One centralized approach of sensor movement is proposed by Saravi et al. [52]. In this approach, the given area is delaunay triangulated, where each triangle is equilateral with side $\sqrt{3}r$. Each of the sensors has information of the delaunay vertices of the triangulation. After random deployment of the sensors, they move to their closest delaunay vertex. Bartolini et al. [3] proposed an heuristic, Push & Pull for complete coverage in a bounded region with mobile sensors. In this approach mobile sensors make an equilateral triangular tiling in a plane by their movements. With respect to the location of an initiator, a regular hexagonal structure is formed with an arbitrary choice of six neighboring sensors and their appropriate movements. The initiator is located at the center of the hexagon. A sensor whose hexagonal structure is already formed is called a snapped otherwise unsnapped. If some unsnapped sensors are located inside the hexagon of a snapped sensor, then the snapped sensor Push these unsnapped sensors to the lower density area of the plane. If some snapped sensors detect any coverage hole adjacent to their hexagon, they send hole trigger messages for attracting unsnapped sensors to fill the coverage hole. The process of message sending and attracting sensors is called Pull activity. If two different clusters having different tiling orientation then tiling marge activity is applied to marge into a common tiling.

2.2.3 Barrier coverage

The concept of barrier coverage comes from the notion of intruder detection in WSNs. The purpose of intruder detection is to detect any intruder that attempts to penetrate the region of interest. In many real life applications like military surveillance, border protection, protection of significant important infrastructures, dangerous substance monitoring, etc., intruder detection is essential. In literature, the concept of barrier coverage first appeared in the context of robotics [23]. Kumar et al. [36] introduced the notion of k-barrier coverage for WSNs and proposed an efficient algorithm to determine whether a belt region is k-barrier covered or not. Two probabilistic barrier coverage, namely weak barrier coverage and strong barrier coverage are introduced in the paper. A centralized optimal sleep-wake-up algorithm [35] is proposed to prolong lifetime of the barrier coverage. Chen et al. [12] designed some localized sleep-wake-up algorithms that provide near-optimal solutions for local barrier coverage problem. Liu et al. [42] proposed an efficient distributed algorithm to construct multiple disjoint barriers for strong barrier coverage in a randomly deployed sensor network over a long irregular strip region. Saipulla et al. [49] studied barrier coverage for the line-based deployment. A tight lower bound for the existence of barrier coverage is proposed. Li et al. [40] discussed the weak k-barrier coverage and analyzed probability of weak k-barrier coverage. A lower bound is derived for the probability of weak k-barrier coverage with and without considering the border effect, respectively. There are many works [33, 50, 51, 54] focused on the improvement of coverage performance using mobile sensors. Shen et al. [54] discussed an energy efficient centralized relocation technique of the mobile sensors such that the mobile sensors form a barrier after relocation. The k-barrier coverage problem is associated with the classical kinetic theory of gas molecules in physics by Keung et al. [33]. The authors derived a relationship between barrier coverage and a set of parameters including sensor density, sensor and intruder density. Saipulla et al. [50] proposed a greedy algorithm to find barrier gaps. The authors adopted maximum flow algorithm to fill the gaps by relocation of the mobile sensors. Another type of barrier coverage problems are proposed by Dash et al. [17], where the objective is to cover a set of line segments in a plane. A line segment is said to be covered by a sensor if the line segment intersects sensing disk of the sensor. The authors addressed the problem of covering a set of line segments by minimum number of sensors with uniform sensing range of the sensors. Two constant factor approximation algorithms and a PTAS are proposed for axis parallel line segments. A constant factor approximation algorithm is proposed for k-covering axis parallel line segments maintaining minimum separation between them. In [18], the authors introduced two new metrics 'smallest k-covered line segment' and 'longest k-uncovered line segment' in order to measure the quality of coverage of a set of line segments in a plane. Different deterministic algorithms are proposed to solve the above problems for a set of line segments with different characteristics.

2.3 Sweep coverage

The concept of sweep coverage initially comes from the context of robotics [4, 23]. In [4], the authors considered a dynamic sensor coverage problem using mobile robots in absence of global localization information. The sensors are mobilized by mounting them on the mobile robots. The robots explore an unexplored area by deploying small communication beacons. The robots decide direction of movements during the exploration using local markers with the beacons. There are several works [14, 19, 41, 55, 61, 64, 66] on sweep coverage in WSNs. Most of the works focused on designing suitable heuristics. In [64], the authors considered a network consisting static and mobile sensors. The static sensors gather information from the environment. The mobile sensors move to a static sensor to collect or download gathered information periodically. A mobile sensor can download data from a static sensor if it is inside the communication range of the static sensor. In this paper, the authors considered two different problems. In the first problem, the objective is to minimize number of mobile sensors that can guarantee data download from every static sensor in a given time period with high probability. In the second problem, the objective is to guide mobile sensors for moving towards static sensors without any centralized control. For the first problem, a relation is established among the delay bound, the information access probability, and the required number of mobile sensors. For the second problem, a distributed heuristic is proposed using a virtual 3D map in the static sensor networks. A periodic data gathering problem similar to the first problem of [64] is considered in [61]. A path planning heuristic is proposed in order to minimize the number of mobile sensors. Du et al. [19] proposed two different heuristics for different movement constraints on the mobile sensors. In the first heuristic (*MinExpand*), mobile sensors move in the same path in every time period. In the second heuristic (*OSweep*), the mobile sensors move in different paths in different time periods. Hung et al. [14] considered a sweep coverage problem where nonuniform deployment of a set of PoIs is made over an area of interest. The area is divided into smaller sub-areas. Then mobile sensors are deployed over the sub-areas, one for each, to guarantee sweep coverage of all PoIs in the respective sub-areas. Due to unequal number of PoIs in different sub-areas, sweep periods (patrol times) of the mobile sensors may not be same. Objective of the proposed heuristic is to make the patrol times approximately same for all mobile sensors. In [55], the authors considered a problem where sweep periods of the PoIs are different. A scheme is proposed based on periodic vehicle routing problem to minimize number of unnecessary visits of a PoI by a mobile sensor. To extend lifetime of sweep coverage, Yang et al. [66] utilized base station as a power source for periodical refueling or replacing battery of the mobile sensors. The authors proposed two heuristics with one base station and multiple base stations, respectively.

Theoretical aspects of the sweep coverage problem is studied by Li et al. [41]. The authors proved that finding minimum number of mobile sensors to sweep cover a set of PoIs is NP-hard. It is proved that the problem cannot be approximated less than a factor of 2, unless P=NP. A $(2+\epsilon)$-approximation and a 3-approximation algorithms are proposed to solve the problem. The authors remarked on impossibility of design distributed local algorithm to guarantee sweep coverage of all PoIs, i.e., a mobile sensor cannot locally determine whether all PoIs are sweep covered without global information. But there is a flaw of the approximation algorithms. This motivates us to design a constant factor approximation algorithm for the problem by correcting the flaw.

There are similar patrolling problems [15, 20, 32] like sweep coverage with different objectives. Objective of the problems is to minimize time between two consecutive visits of any point while monitoring a given road network or boundary of a region by a set of mobile agents having different speeds. In [15], the authors proposed two strategies; partition based strategy and cycle based strategy to obtain movement schedules of the mobile agents. The authors proved that the strategy obtains optimal solution when number of agents is less than or equal to two for partition strategy and number of agents is less than or equal to four for cycle strategy. Kawamura et al. [32] proved that the partition strategy proposed in the paper [15] achieves optimal solution for number of agents less than or equal to three.

Chapter 3

Sweep Coverage for a Set of Points

3.1 Introduction

There are applications in WSNs where a set of discrete points are needed to be monitored periodically. For example, monitoring health of a structure such as bridge, building, level of flammable gas in mines, condition of machineries, temperature of boiler in industry, etc., where periodic monitoring is required for certain important locations, i.e., points of interest (PoIs). Sweep coverage for the set of PoIs provides solution for the aforementioned applications.

Sweep coverage is formally introduced by Li et al. [41] for a set of PoIs. According to their work [41], the definition of sweep coverage is as follows.

Definition 3.1.1 (t-sweep coverage)**.** Let $U = \{u_1,\ u_2,\ \cdots,\ u_n\}$ be a set of PoIs in a two dimensional plane, and $M = \{m_1, m_2, \cdots, m_n\}$ be a set of mobile sensors. Let v be the uniform speed of the mobile sensors. A PoI u_i is said to be t-sweep covered if and only if at least one mobile sensor m_j visits u_i in every t time period.

Definition 3.1.2 (Sweep coverage problem)**.** Let $U = \{u_1,\ u_2,\ \cdots,\ u_n\}$ be a set of PoIs in a two dimensional plane, and $M = \{m_1, m_2, \cdots, m_n\}$ be a set of mobile sensors. Let v be the uniform speed of the mobile sensors. For a given $t > 0$, find the minimum number of mobile sensors such that each PoI in U is t-sweep covered.

Li et al. [41] proved that the sweep coverage problem is NP-hard and cannot be approximated within a factor of 2, unless P=NP. The authors proposed a $(2+\epsilon)$-approximation algorithm to solve the problem. The algorithm computes approximate TSP tour through all the PoIs and divides the tour into parts of length at most $\frac{vt}{2}$. Then one mobile sensor is deployed in every partition and let the mobile sensors move back and forth to sweep cover all PoIs of the corresponding partitions. But there is a serious flaw in the approximation algorithm [41] as explained below. Assume there are only two PoIs in a plane, the distance between them is 100 meter and vt is 20 meter. Therefore, the length of the TSP tour is 200 meter and according to the algorithm [41], total number of mobile sensors needed is $200/\frac{vt}{2}$= 200/10=20. But practically it is sufficient to place only two mobile sensors to monitor two PoIs respectively and thus two mobile sensors can guarantee sweep coverage. Hence the algorithms proposed by Li et al. [41] does not provide a solution which achieves approximation factor $(2+\epsilon)$.

3.1.1 Contribution

In this chapter our contributions on sweep coverage problem are given below.

- We remark on the flaw of the approximation algorithm proposed in the paper [41] for sweep coverage and propose a 2-approximation algorithm to guarantee sweep coverage of a set of PoIs for a special case.
- A 3-approximation algorithm is proposed for the vertices of an arbitrary weighted graph where vertices are considered as a set of PoIs.

3.2 2-Approximation Algorithm for a Special Case

In this section, we consider a special case of the sweep coverage problem, where each mobile sensor visits all PoIs during its movement. In other words, all the mobile sensors must move along the same movement path. In the optimal solution, every mobile sensor must move along the optimal TSP tour among the PoIs.

Consider a given set of PoIs U=$\{u_1, u_2, \cdots, u_n\}$ in a two dimensional plane. Let $G=(U,E,w)$ be the complete weighted graph with each PoI as a vertex and the line segment joining two PoIs in the plane as edge. The weight $w(e)$ of an edge $e \in E$ is equal to the Euclidean distance between the vertices and it is denoted by $w(e)$. Our proposed Algorithm 1 (PointSweepCoverage) finds an approximate TSP tour L using the 2-approximation algorithm for metric TSP [46] by the method of finding Euler tour from a minimum spanning tree (MST) of G. Let L_{opt} be the optimal TSP tour of G. Hence, $w(L) \leq 2w(L_{opt})$. Partition L into $\left\lceil \frac{w(L)}{vt} \right\rceil$ parts of length vt each as shown in Fig. 3.1. Then deploy $\left\lceil \frac{w(L)}{vt} \right\rceil$ mobile sensors at all partitioning points, one for each. To cover each PoI periodically, each mobile sensor starts moving at the same time along L in the same direction, either clockwise or anti-clockwise.

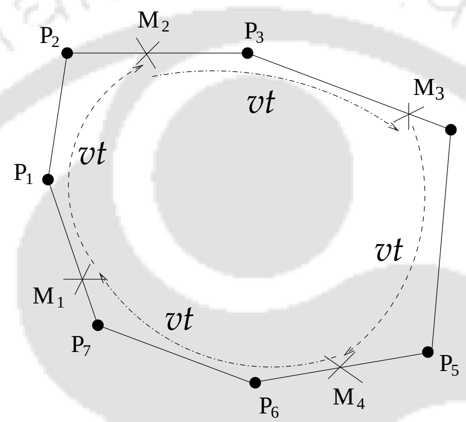


Figure 3.1: Showing TSP tour for PoIs, P_1, P_2, $\cdots$, P_7, and initial positions of the mobile sensors M_1, M_2, M_3, M_4

Algorithm 1 PointSweepCoverage

1: Find a tour L of G by the method of finding Euler tour from a MST of G using the 2-approximation algorithm for metric TSP [46].

2: Partition L into $\left\lceil \frac{w(L)}{vt} \right\rceil$ parts and deploy $\left\lceil \frac{w(L)}{vt} \right\rceil$ mobile sensors at all partitioning points, one for each.

3: Each mobile sensor then starts moving at the same time along L in same direction.

Following Lemma 3.2.1 proves correctness of the Algorithm 1.

Lemma 3.2.1. *Each point on L can be visited by at least one mobile sensor in every time period t.*

Proof. Let us consider a point p on L and let t_0 be the time when p is visited by a mobile sensor last time. Now we have to prove that the point p must be visited by at least one mobile sensor within $t_0 + t$ time. According to the deployment strategy of the mobile sensors any two consecutive mobile sensors are within a distance of at most vt at any time. So, when p is visited by a mobile sensor at t_0, another mobile sensor was on the way to p and within the distance of vt along L. Hence p will be again visited by another mobile sensor within next t time. $\square$

Theorem 3.2.2. *The number of mobile sensors required for the Algorithm 1 is no more than twice the number needed for the optimal solution.*

Proof. Let L_{opt} be the optimal TSP tour for the graph G. Let L be the tour calculated by the Algorithm 1. Then by 2-approximation algorithm for metric TSP [46], $w(L) \leq 2w(L_{opt})$. Let N_{opt} be the number of mobile sensors required for optimal solution. Then $N_{opt} \times vt \geq w(L_{opt})$, i.e., $N_{opt} \geq \left\lceil \frac{w(L_{opt})}{vt} \right\rceil$. Let N be the number of mobile sensors calculated by Algorithm 1. Then $N = \left\lceil \frac{w(L)}{vt} \right\rceil$. Therefore, the approximation ratio of the Algorithm 1 is equal to $\frac{N}{N_{opt}} \leq \left\lceil \frac{2w(L_{opt})}{vt} \right\rceil \Big/ \left\lceil \frac{w(L_{opt})}{vt} \right\rceil \leq 2$. $\square$

Remark 3.2.3. The running time of Algorithm 1 is not polynomial. All the other steps expect the partitioning of the tour L can be done in polynomial time. The MST of the complete graph G can be computed in $O(n^2)$ time using Prim's algorithm, where n is the number of vertices of G. The edges of the MST can be doubled in $O(n)$ time and the Euler tour on it can be found in $O(n)$ time [22]. The tour L can be computed in $O(n)$ time from the Euler tour. But computing the partitioning points on L will take $O(\frac{L}{vt})$ time. Hence time complexity of Algorithm 1 is $O(n^2 + \frac{L}{vt})$ which is not polynomial in n.

3.3 Sweep Coverage on a Weighted Graph

In this section, we propose solution for sweep coverage problem on a weighted undirected graph. The vertices of the graph are considered as PoIs. The mobile sensors can move along the edges of the graph with a uniform speed v. Let $G = (U, E, w)$ be a weighted graph, where weight of an edge (u_i, u_j) for $u_i, u_j \in U$ is denoted by $w(u_i, u_j)$. Let n be the total number of vertices in G. For any subgraph H of G, we denote $w(H)$ for the sum of the edge weights of H. We define sweep coverage for the vertices of a graph as follows.

Definition 3.3.1 (t-sweep coverage)**.** Let $U = \{u_1,\ u_2,\ \cdots,\ u_n\}$ be the vertices of a weighted graph $G = (U, E, w)$, and $M = \{m_1, m_2, \cdots, m_n\}$ be the set of mobile sensors. The mobile sensors move with a uniform speed v along edges of the graph. A vertex u_i is said to be t-sweep covered if and only if at least one mobile sensor m_j visits u_i in every t time period.

Problem 1 (Sweep coverage problem on graph)**.** Let $U = \{u_1,\ u_2,\ \cdots,\ u_n\}$ be the vertices of a weighted graph $G = (U, E, w)$ and $M = \{m_1, m_2, \cdots, m_n\}$ be the set of mobile sensors. The mobile sensors move with a uniform speed v along the edges of the graph. For a given $t > 0$, find the minimum number of mobile sensors such that each vertex of G is t-sweep covered.

We propose Algorithm 2 (GraphSweepCoverage) to find minimum number of mobile sensors for the sweep coverage problem on graph. From step 1 to step 11 of the algorithm execute in n iterations for finding the best possible solution i.e., number of mobile sensors. In kth iteration ($1 \leq k \leq n$), the minimum spanning forest F_k with k connected components $C_1, C_2, \cdots, C_k$ is computed. After that k disjoint tours $T_1, T_2, \cdots, T_k$ are found by doubling all edges of each component. Partition each T_i into $\left\lceil \frac{w(T_i)}{vt} \right\rceil$ parts of weight at most vt. If T_i contains only one vertex, number of partition on T_i is one. Total number of partitions for kth iteration denoted by N_k and which is equal to the number of mobile sensors required for the iteration. Minimum over the number of mobile sensors of all iterations is chosen as the output of Algorithm 2. Initial positions and the movement schedule of the mobile sensors are calculated within step 13 to step 21 of the algorithm.

3.3.1 Analysis

Theorem 3.3.2. *Algorithm 2 guarantees t-sweep coverage for each vertex of G.*

Proof. For any vertex u_i we want to show that it is t-sweep covered. According to Algorithm 2, there are two following cases.

Case 1: (**u_i is in a component with more than one vertex**) Let u_i is visited by a mobile sensor m_j at time t_0. According to the algorithm, mobile sensors are

Algorithm 2 GraphSweepCoverage

for $k = 1$ *to* n **do**
  Find the minimum spanning forest F_k on G with $n-k$ edges. Let $C_1, C_2, \cdots, C_k$ be the connected components of F_k.
  $N_k = 0$.
  for $j = 1$ *to* k **do**
    if C_j is a component having more than one vertex **then**
      $N_k = N_k + \left\lceil \frac{2w(C_i)}{vt} \right\rceil$.
    else
      $N_k = N_k + 1$.
    end if
  end for
end for
Let J be the index $\in \{1, 2, \cdots, n\}$ such that $N_J = \min\{N_1, N_2, \cdots, N_n\}$.
Let $C_1, C_2, \cdots, C_J$ be the connected components of F_J.
for $i = 1$ to J **do**
  if C_j is a component having more than one vertex **then**
    Find a Eulerian tour T_i on C_i by doubling each edge of C_j. Partition the tour into $\left\lceil \frac{w(T_i)}{vt} \right\rceil$ parts and deploy one mobile sensor at each of the partitioning points.
  else
    Deploy one mobile sensor at the vertex of C_i.
  end if
end for
All mobile sensors start moving at the same time along the respective tours having more than one vertex in same direction. If a mobile sensor is deployed on a tour containing only one vertex then it periodically monitors the vertex.

initially deployed within vt distance apart. Then the mobile sensors start moving with same speed v in the same direction. So u_i will be again visited by the next mobile sensor of m_j within time $t + t_0$.

Case 2: **(u_i is in a component with no other vertices)** In this case the statement of the theorem is trivially true as one mobile sensor is deployed at u_i and which periodically covers it.

□

Lemma 3.3.3. *Let opt be the number of mobile sensors needed in the optimal solution. Let opt′ be the minimum number of paths of weight $\leq vt$ which span U on G. Then*

$opt \geq opt'$.

Proof. We want to prove it by the method of contradiction. Let us assume $opt < opt'$. Consider the paths of movements by the mobile sensors in the optimal solution during any time period $[t_0, t_0 + t]$. Let $P_1, P_2, \cdots, P_{opt}$ be the movement paths of the mobile sensors with $w(P_i) \leq vt$. Since each vertex is visited by a mobile sensor at least once in time period t therefore $\bigcup_{i=1}^{opt} P_i$ spans all the vertices of U. Hence, $\{P_1, P_2, \cdots, P_{opt}\}$ is a collection of paths with $w(P_i) \leq vt$ that spans U, which contradicts the fact that $opt < opt'$. Therefore $opt \geq opt'$. □

Theorem 3.3.4. *The Algorithm 2 is a 3-approximation algorithm.*

Proof. Let opt be the minimum number of mobile sensors required in the optimal solution. Let opt' be the minimum number of paths of weight $\leq vt$, which span U on G and Min_path be the sum of the weights of all such paths. Then by Lemma 3.3.3,

$$opt' \leq opt \tag{3.3.1}$$

and

$$Min\_path \leq opt' \times vt \tag{3.3.2}$$

Again, these opt' number of paths of weights $\leq vt$ forms a spanning forest with opt' disjoint connected components. As $F_{opt'}$ is the minimum spanning forest with opt' connected components, therefore,

$$w(F_{opt'}) \leq Min\_path. \tag{3.3.3}$$

Algorithm 2 chooses the minimum over all N_k for $k = 1$ to n. Let us consider the iteration of the algorithm when $k = opt'$. Consider a connected component C_i of F_k. If C_i is a component with more than one vertex, then number of partitions on C_i is $\left\lceil \frac{2w(C_i)}{vt} \right\rceil$ which is $\leq \left(\frac{2w(C_i)}{vt} + 1 \right)$. If C_i is a component with one vertex, then $w(C_i) = 0$ and number of partition on C_i is 1 which is equal to $\left(\frac{2w(C_i)}{vt} + 1 \right)$. Therefore, total number of partitions in this iteration is given by,

$$\begin{aligned}
N_k &\leq \sum_{i=1}^{k}\left(\frac{2w(C_i)}{vt}+1\right) \\
&= \frac{2w(F_k)}{vt}+k \\
&\leq \frac{2Min_path}{vt}+k && \text{from Equation (3.3.3)} \\
&\leq 2k+k && \text{from Equation (3.3.2)} \\
&= 3k \\
&\leq 3opt && \text{from Equation (3.3.1)}
\end{aligned}$$

Since $N \leq N_k$, therefore, $N \leq 3opt$. Therefore the approximation factor of the Algorithm 2 is 3. □

Theorem 3.3.5. *The time complexity of Algorithm 2 is $O(n^2 \log n)$.*

Proof. Selection of first k edges of a graph in Kruskal's algorithm gives the minimum spanning forest with $n-k$ components. For each k, depth first search can be applied to identify the connected components. Hence time complexity for computing the step 1-13 is $O(n^2 \log n + n^2)$. In step 16, $O(n)$ time is required to find partitioning points of the Eulerian tours after doubling each of the components. Therefore overall time complexity of the Algorithm 2 is $O(n^2 \log n)$. □

3.4 Sweep Coverage in Presence of Obstacles

A mobile sensor cannot move straight from a PoI u_i to another PoI u_j in presence of an obstacle on the path. An alternative path is required to avoid the obstacle. To calculate an alternative path between any pair of PoIs we use the idea of visibility graph [34, 58]. There is an edge between two vertices of a visibility graph if line joining the vertices does not intersect any obstacle. For simplicity we assume obstacles are polygonal shape. Position of the polygons and their vertices are known. The construction of the visibility graph G_v for a given set of PoIs and obstacles is given below.

The vertex set of G_v is the union of PoIs and the vertices of all obstacles. There is an edge between two vertices of G_v if they are visible to each other, i.e., no obstacle between their line of sight path. The weight of an edge is Euclidean distance between respective vertices. In presence of obstacles on the direct path of u_i and u_j, an alternative path is calculated through the vertices of polygons to avoid collision with the obstacles. All pair shortest path among the PoIs can be found on G_v using Floyd-Warshall algorithm. We construct a complete graph G' for the set of vertices corresponding to the set of PoIs only with edge weight w, where w is defined as $w(u_i, u_j)$ = length of the shortest path between vertices u_i and u_j in G_v. Each edge (u_i, u_j) of G' corresponds one path between u_i and u_j in G_v. Apply Algorithm 2 on G' to sweep cover the set of PoIs.

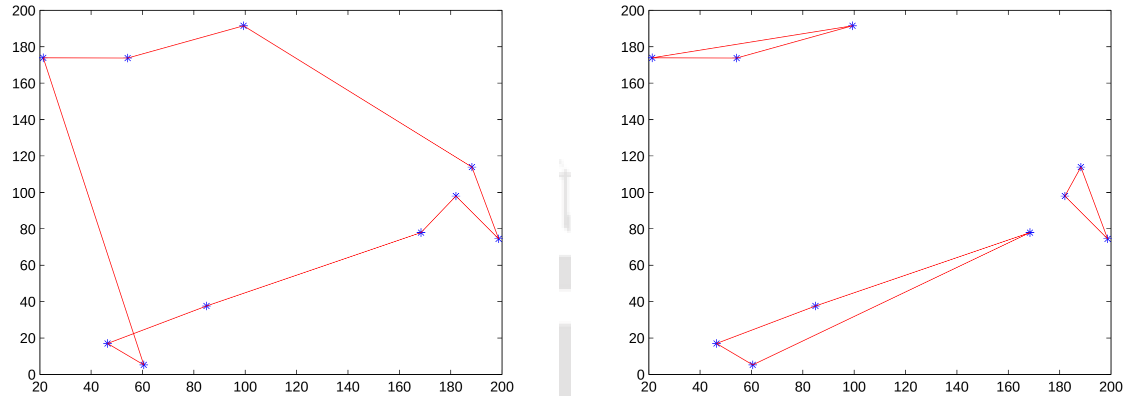
Theorem 3.4.1. *The time complexity of Algorithm 2 is $O((n+k)^3)$, where n is the number of PoIs and k is total number of vertices of the polygonal obstacles.*

Proof. The total number of vertices of the visibility graph G_v is $(n+k)$, since G_v is constructed with all PoIs and vertices of the polygonal obstacles. The computation of G_v can be done in $O(n^2k)$ time, since intersection of a line segment with an obstacle can be found in $O(k)$ time. All pair shortest paths for the PoIs can be found in $O(n+k)^3$ time using Floyd-Warshall algorithm. The running time of Algorithm 2 is $O\left((n+k)^2 \log(n+k)\right)$. Hence when polygonal obstacles are present in the plane with total k vertices, the time complexity of Algorithm 2 is $O((n+k)^3)$. $\square$

3.5 Simulation

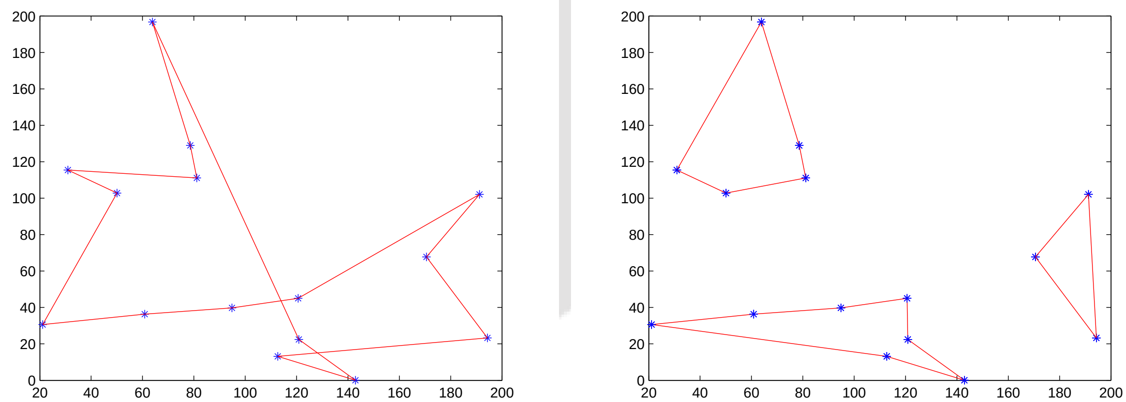
In this section, we study performance of our proposed Algorithm 1 for sweep coverage problem through simulation. We compare Algorithm 1 with *MinExpand* algorithm proposed by Du et al. [19]. The idea of the *MinExpand* algorithm is to find disjoint cycles having subset of PoIs of length less than or equal to vt. Then, one mobile sensor is assigned for one cycle to guarantee t-sweep coverage of the given set of PoIs. We implement both of the algorithms, Algorithm 1 and *MinExpand* for comparison study. We generate different set of PoIs randomly inside a 200 meter by 200 meter square region in a two dimensional plane. The PoIs are the input for both of the above algorithms.

Throughout the simulation we have taken 1 meter/second as uniform speed of the mobile sensors. From Fig. 3.2 to Fig. 3.4 showing tour outputs for different number of PoIs when sweep period $t = 300$ second and $vt = 300$ meter.



(a) Tour by Algorithm 1 (b) Tour Cycles by MinExpand

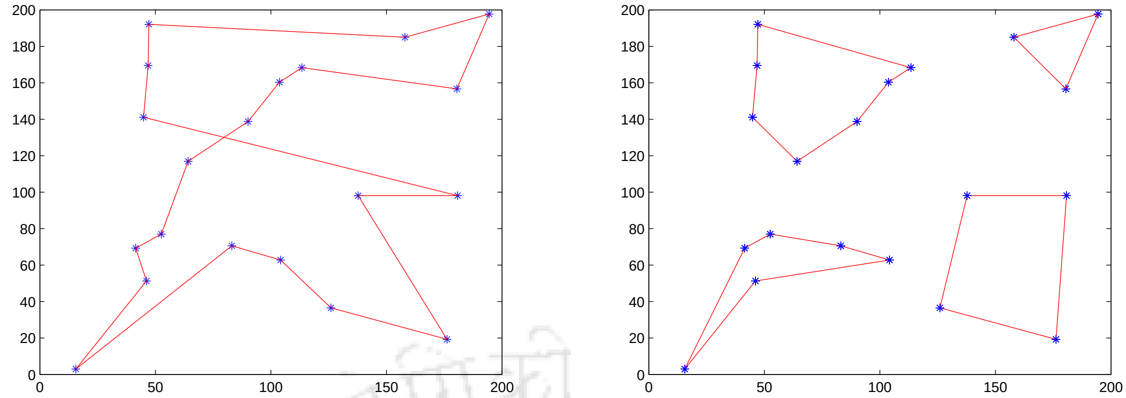
Figure 3.2: An example of sweep coverage for 10 PoIs



(a) Tour by Algorithm 1 (b) Tour Cycles by MinExpand

Figure 3.3: An example of sweep coverage for 15 PoIs

An example of tours of mobile sensors for 10 PoIs is shown in Fig. 3.2. Fig. 3.2a shows tour output by Algorithm 1, where length of the tour is 468.9 meter and required number of mobile sensors is 2. Fig. 3.2b shows three disjoint tour cycles by *MinExpand* of lengths 284.9 meter, 161.3 meter and 86.6 meter respectively. The required number of mobile sensors is 3. An example of tours of mobile sensors for 15 PoIs is shown in



(a) Tour by Algorithm 1 (b) Tour Cycles by MinExpand

Figure 3.4: An example of sweep coverage for 20 PoIs

No. of PoIs	Number of Mobile Sensors	
	MinExpand [19]	Algorithm 1
10	3	3
20	4	4
30	5	4
40	6	5
50	7	5
60	7	6
70	8	6
80	9	7
90	10	7
100	11	8

Table 3.1: Comparison of the number of mobile sensors to achieve sweep coverage varying with number of PoIs

Fig. 3.3. Fig. 3.3a shows tour output by Algorithm 1, where length of the tour is 543.7 meter and required number of mobile sensors is 2. Fig. 3.3b shows three disjoint tour cycles by *MinExpand* of lengths 280.8 meter, 230.2 meter and 169.3 meter respectively. The required number of mobile sensors is 3. An example of tours of mobile sensors for 20 PoIs is shown in Fig. 3.4. Fig. 3.4a shows tour output by Algorithm 1, where length of the tour is 892.1 meter and required number of mobile sensors is 3. Fig. 3.4b shows four disjoint tour cycles by *MinExpand* of lengths 254.6 meter, 237.7 meter, 224.17 meter and 118.1 meter respectively. The required number of mobile sensors is 4.

Table 3.1 shows comparison of average number of mobile sensors to achieve sweep

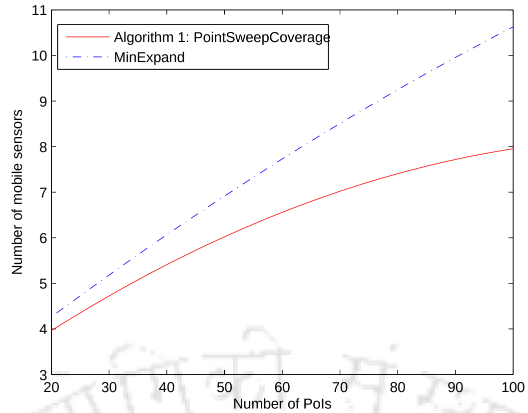


Figure 3.5: Comparison of the number of mobile sensors to achieve sweep coverage varying with number of PoIs

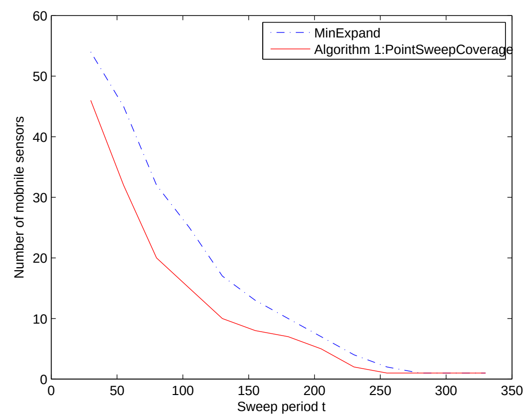


Figure 3.6: Comparison of the number of mobile sensors to achieve sweep coverage varying with sweep period t (in second)

coverage varying with number of PoIs. The average number of mobile sensor is calculated for 100 several executions of the algorithms with sweep period 200 second. In each execution, PoIs are randomly generated in the plane. A graphical representation of the Table 3.1 is illustrated in the Fig. 3.5. The Table 3.1 and Fig. 3.5 show that with increasing number of PoIs, our Algorithm 1 performs better than *MinExpand* with respect to required number of mobile sensors.

Table 3.2 shows comparison of average number of mobile sensors to achieve sweep coverage varying with sweep period. We have taken fix number of randomly generated PoIs and which is 50 for this comparison. The average number of mobile sensors is

Sweep period (in second)	Number of Mobile Sensors	
	MinExpand [19]	Algorithm 1
30	54	46
55	45	32
80	32	20
105	25	15
130	17	10
155	13	8
180	10	6
205	7	5
230	4	2
255	2	1
280	2	1
305	1	1
330	1	1

Table 3.2: Comparison of the number of mobile sensors to achieve sweep coverage varying with sweep period (in second)

calculated for 100 several executions of the algorithms for a fixed sweep period. A graphical representation of the Table 3.2 is illustrated in the Fig. 3.6. The Table 3.2 and Fig. 3.6 show that for sweep period less than 255 second, less number of mobile sensors are required for our Algorithm 1 compare to *MinExpand*, which implies our algorithm performs better than *MinExpand*.

3.6 Conclusion

In this chapter we have remarked on the flaw of the previous solution [41] for sweep coverage of a set of PoIs. We have proposed a 2-approximation algorithm for a special case of the problem which is the best possible approximation factor. We have proposed a 3-approximation algorithm for sweep coverage for vertices of an arbitrary weighted graph. Simulation results show that the proposed algorithm performs better in terms of the number of mobile sensors to achieve sweep coverage compared to the existing algorithm.

Chapter 4

Sweep Coverage with Heterogeneity

4.1 Introduction

In Chapter 3, we consider the sweep coverage problem where parameters are homogeneous i.e., sweep periods of all PoIs are same and speeds of all mobile sensors are also same. But there are different applications where heterogeneity may present. For example, monitoring machineries of an industry, where different machines may have different importance. A few machines must be monitored more frequently than the others. So, in terms of sweep coverage, sweep periods of all machines are not same. Similarly, mobile sensors may not have same speed. In this chapter, we investigate two different sweep coverage problems considering two different contexts of heterogeneity. In the first problem, sweep periods of the PoIs are different and in the second problem, the speeds of the mobile sensors are different. In practice some processing time is required for a mobile sensor to process some tasks such as monitoring, sampling or exchanging data at each of the PoIs during its visit. Some finite amount of processing delays/times for the mobile sensors at the PoIs are included in the first problem.

4.1.1 Contribution

In this chapter, our contributions are as follows:

- We propose an approximation algorithm to solve the sweep coverage problem for a weighted graph when vertices of the graph have different sweep periods and processing times. The approximation factor of the algorithm is $O(\log \rho)$, where $\rho = \frac{t_{max}}{t_{min}}$, t_{min} and t_{max} are the minimum and maximum sweep periods among the vertices.

- If speeds of the mobile sensors are different, we prove that it is impossible to design any constant factor approximation algorithm to solve the sweep coverage problem, unless P=NP.

4.2 Sweep Coverage with Different Sweep Periods and Processing Times

Definition of the sweep coverage [41] for different sweep periods of the PoIs is given below.

Definition 4.2.1. Let $U = \{u_1, u_2, \cdots, u_n\}$ be a set of PoIs, and $\{t_1, t_2, \cdots, t_n\}$ be the set of sweep periods. Let $M = \{m_1, m_2, \cdots, m_n\}$ be the set of mobile sensors which can move with a uniform speed v. A PoI u_i is said to be t_i-sweep covered iff a mobile sensor visits u_i at least once in every t_i time period.

The objective of the sweep coverage problem is to find minimum number of mobile sensor such that each u_i is t_i sweep covered. Li et al. [41] proposed a 3-approximation algorithm for the problem. But there is a serious flaw in the algorithm. The flaw can be shown by the same example described in the introduction of the previous Chapter 3. Here we propose an algorithm to solve the sweep coverage problem called *DSSweep coverage problem*, where sweep periods for the vertices of a weighted graph are different. The vertices of the graph are considered as the set of PoIs. The mobile sensors use some finite amount of processing time during their visit at each of the vertices.

Problem 2 (DSSweep Coverage Problem)**.** Let $U = \{u_1, u_2, \cdots, u_n\}$ be the vertex set of a weighted graph $G = (U, E, w)$. Let t_i and τ_i be the sweep period and processing time for u_i, respectively for $i = 1, \cdots, n$. Let $M = \{m_1, m_2, \cdots, m_n\}$ be the set of mobile sensors which can move with a uniform speed v along the edges of the graph. Find the minimum number of mobile sensors such that each u_i is t_i-sweep covered.

4.2.1 Proposed algorithm

Our proposed algorithm solves Problem 2 in two phases. First phase finds number of mobile sensors and second phase calculates movement strategy of the mobile sensors.

Finding number of mobile sensors

For $G = (U, E, w)$, we construct complete graph $G' = (U, E', w')$. Define weight of an edge $(u_i, u_j) \in E'$ as $w'(u_i, u_j) = d(u_i, u_j) + \frac{v}{2}(\tau_i + \tau_j)$ for $i, j = 1$ to n, where $d(u_i, u_j)$ is the weight of the shortest path between u_i and u_j in G. Let $\rho = \frac{t_{max}}{t_{min}}$, where t_{min} and t_{max} are the minimum and maximum sweep periods among the vertices. We partition U into subsets $U_1, U_2, \cdots, U_{\lceil \log \rho \rceil}$, where U_i=$\{u_j \in U | 2^{i-1} \cdot t_{min} \leq t_j < 2^i \cdot t_{min}\}$. Let G'_i be the induced subgraph of G' for the vertex set U_i. For $i = 1$ to $\lceil \log \rho \rceil$, we apply step 1 to step 12 of the Algorithm 2 (chapter 3) on G'_i to find the number of mobile sensors for $(2^{i-1} \cdot t_{min})$-sweep coverage of all vertices in U_i.

Movement strategy of the mobile sensors

Now we explain how to deploy the mobile sensors to ensure sweep coverage for all the vertices of G. Deployments of the mobile sensors are explained below for U_i, $i = 1$ to $\log \lceil \rho \rceil$. Let C_1, $C_2, \cdots, C_{J_i}$ be the connected components of G'_i computed in step 13 of Algorithm 2. Find a tour T_k by doubling all edges of C_k for $k = 1, 2, \cdots, J_i$. If T_k contains only one vertex, one mobile sensor is deployed at the vertex for periodical monitoring. If there are more than one vertex in T_k, we compute a tour T'_k by replacing vertices and edges of T_k as explained below. Let $u_{i_1}, u_{i_2}, \cdots, u_{i_l} \in U_i$ be the vertices along T_k in the clockwise direction. Replace each vertex u_{i_h} in T_k by two vertices u'_{i_h} and u''_{i_h} and introduce an edge (u'_{i_h}, u''_{i_h}) with edge weight $w'(u'_{i_h}, u''_{i_h})$=$v\tau_{i_h}$. The edges $(u_{i_1}, u_{i_2}), (u_{i_2}, u_{i_3}), \cdots, (u_{i_{l-1}}, u_{i_l}), (u_{i_l}, u_1)$ of T_k are replaced by the edges (u''_{i_1}, u'_{i_2}), $(u''_{i_2}, u'_{i_3}), \cdots, (u''_{i_{l-1}}, u'_{i_l})$, (u''_{i_l}, u'_{i_1}) respectively. The weight of the edge $(u''_{i_j}, u'_{i_{j+1}})$ is given by $w'(u''_{i_j}, u'_{i_{j+1}}) = w'(u_{i_j}, u_{i_{j+1}}) - \frac{v}{2}(\tau_{i_j} + \tau_{i_{j+1}})$.

An example of a tour T_k on G'_i is shown in Fig. 4.1 and corresponding construction of T'_k from T_k is shown in Fig. 4.2.

Partition T'_k into $\left\lceil \frac{w'(T'_k)}{v \cdot 2^{i-1} t_{min}} \right\rceil$ parts of weight at most $v \cdot 2^{i-1} t_{min}$. According to the

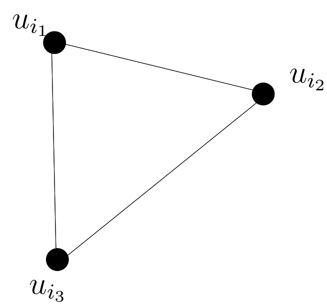


Figure 4.1: Tour T_k on G'_i

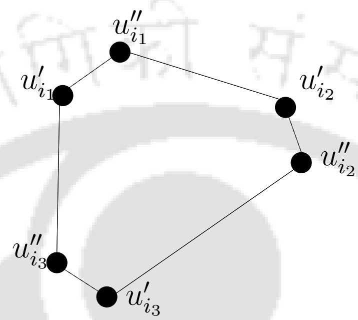


Figure 4.2: Construction of T'_k from T_k

positions of the partitioning points, mobile sensors are deployed on the original graph G as follows. If the position of a partitioning point is on the edge $(u'_{i_j}, u''_{i_j}) \in T'_k$ for some j, then one mobile sensor is deployed at the vertex u_{i_j} in G. If the position of a partitioning point is on the edge $(u''_{i_p}, u'_{i_q}) \in T'_k$ for $p \neq q$, then one mobile sensor is deployed at the corresponding position on the edge (u_{i_p}, u_{i_q}) in G.

After deployment, all mobile sensors move around the tour T'_k in the same direction and the movement of the mobile sensors are reflected in the original graph G as explained below. If position of a mobile sensor is on the edge $(u'_{i_j}, u''_{i_j}) \in T'_k$ for some j, then it waits at the vertex u_{i_j} on G for the time which is equal to the time taken by the mobile sensor to move from its current position to u''_{i_j} on the tour T'_k with uniform speed v. Otherwise it continues its movement with uniform speed v along the respective edge in G.

An example is shown in Fig. 4.3 to explain deployment and movement strategy of the mobile sensors. The partitioning points are shown by the crossed marks on the tour T'_k. Corresponding to the partitioning point P on T'_k, a mobile sensor is deployed at the vertex u_{i_1} in G. The mobile sensor waits at u_{i_1} in G for the time it takes to move from

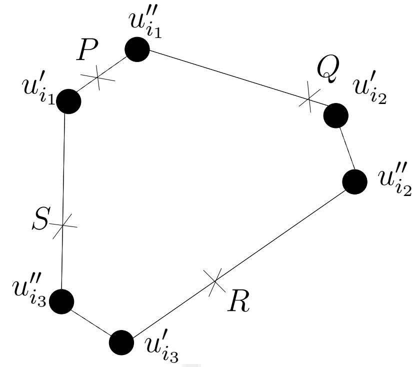


Figure 4.3: Partitioning a tour T'_k and initial deployment of mobile sensors

P to u''_{i_1} along the tour T'_k. Then it moves to the next vertex u_{i_2} along the shortest path from from u_{i_1} to u_{i_2} in G with uniform speed v. Corresponding to the partitioning point Q, a mobile sensor is deployed at the same position on respective edge in G. Then it moves to the next vertex in the tour along the shortest path of the two vertices with uniform speed v.

4.2.2 Analysis

Theorem 4.2.2. *According to the movement strategy of the mobile sensors each vertex $u_i \in U$ is t_i-sweep covered with processing time τ_i.*

Proof. If u_i belongs to a component with one vertex, then t_i sweep coverage is trivial by the mobile sensor deployed at u_i. Now if u_i belongs to a component with more than one vertex, then according to the proposed algorithm for Problem 2, $u_i \in U_j$ for some $j = 1$ to $\lceil \log \rho \rceil$. By Theorem 3.3.2, u_i is sweep covered with sweep period $2^{j-1} \cdot t_{min} \leq t_i$. Therefore u_i is t_i sweep covered. Also, when a mobile sensor visits u_i, it waits at u_i for τ_i time which follows from the construction of the tour explained in Section 4.2.1. □

Theorem 4.2.3. *The approximation factor of the proposed algorithm for Problem 2 is $6 \lceil \log \rho \rceil$.*

Proof. Let OPT be the optimal solution of the Problem 2 on the graph G and OPT_1, OPT_2, $\cdots$ $OPT_{\lceil \log \rho \rceil}$ be the optimal solutions for G'_1, G'_2, $\cdots$ $G'_{\lceil \log \rho \rceil}$ respectively (ref. Section 4.2.1). Then $OPT_i \leq OPT$ for $i = 1$ to $\lceil \log \rho \rceil$, as the sweep coverage of all the vertices of G ensures sweep coverage for all the vertices in G'_i. According to the algorithm for Problem 2 let OUT_1, OUT_2, $\cdots$, $OUT_{\lceil \log \rho \rceil}$ be the number of mobile sensors for G'_1, G'_2, $\cdots$ $G'_{\lceil \log \rho \rceil}$ respectively. Then $OUT_i \leq 6OPT_i$, as the length of the partition in OPT_i is at most twice of the length of the partition in OUT_i and the Algorithm 2 is a 3-approximation algorithm. Hence, total number of mobile sensors needed is equals to $\sum_{j=1}^{\lceil \log \rho \rceil} OUT_j \leq 6 \sum_{j=1}^{\lceil \log \rho \rceil} OPT_j \leq 6 \lceil \log \rho \rceil OPT$. Therefore, the algorithm for Problem 2 is $6 \lceil \log \rho \rceil$-approximation algorithm. □

Theorem 4.2.4. *The running time of the algorithm for Problem 2 is* $O(n^2 \log n \log \rho)$.

Proof. The time complexity of the Algorithm 2 is $O(n^2 \log n)$. The algorithm for Problem 2 uses the Algorithm 2 as a subroutine for $O(\log \rho)$ disjoint set of of vertices. Hence the running time is $O(n^2 \log n \log \rho)$. □

Remark 4.2.5. Although it seems that the time complexity of the algorithm for Problem 2 is pseudo polynomial but according to the decomposition strategy of the vertices of the graph there can be at most n non-empty disjoint vertex sets. Hence the complexity of the algorithm is $O(n^3 \log n)$ which is polynomial in n.

4.3 Inapproximability of Sweep Coverage with Mobile Sensors having Different Speeds

In this section, we consider the sweep coverage problem called *DVSweep coverage problem*, where speeds of the mobile sensors may be different. Let $m_1, m_2, \cdots, m_n$ be the mobile sensors and $v_1, v_2, \cdots v_n$ be their respective speeds, where $v_i \geq 0$ for all i= 1 to n. The problem is to find minimum number of mobile sensors such that every vertex of a given graph G is t-sweep covered.

Problem 3 (DVSweep Coverage Problem)**.** Let $U = \{u_1, u_2, \cdots, u_n\}$ be the vertex set of a weighted graph $G = (U, E, w)$. Let $m_1, m_2, \cdots, m_n$ be the mobile sensors and $v_1, v_2, \cdots v_n$ be their respective speeds, where $v_i \geq 0$ for all i= 1 to n. For a given $t > 0$, find minimum number of mobile sensors such that each u_i is t-sweep covered.

We prove that no polynomial time constant factor approximation algorithm exists for the above problem, unless P=NP.

Theorem 4.3.1. *No polynomial time constant factor approximation algorithm exists to solve the sweep coverage problem by mobile sensors with different speeds, unless P=NP.*

Proof. We prove the theorem by the method of contradiction. For the purpose, we propose a polynomial time reduction from metric TSP problem, which is known to be NP-hard, to an instance of Problem 3.

Suppose there exists a k-approximation algorithm $\mathcal{A}$ for Problem 3. Without loss of generality we assume k is a positive integer. We consider an instance (G_1, L) of metric TSP, where $G_1 = (U_1, E_1, w_1)$ is a complete weighted graph with n $(> k)$ vertices and weights of the edges of G_1 are integer and satisfy triangle inequality. We construct a complete graph $G_2 = (U_2, E_2, w_2)$ with n^2 vertices from G_1 as follows. For each vertex $u_i \in U_1$, we consider n vertices $u_i^1, u_i^2, \cdots u_i^n$ in U_2 for $i = 1, 2, \cdots, n$. We define weight of an edge $(u_i^l, u_j^k) \in E_2$ as $w_2(u_i^l, u_j^k) = w_1(u_i, u_j) - \frac{1}{(n+1)^2}$ for $i \neq j$ and $w_2(u_i^l, u_i^k) = \frac{1}{(n-1)(n+1)^2}$ of G_2.

Next, we prove that the graph G_1 has a tour with weight at most l iff G_2 has a tour with weight at most l. To prove this, let $T_1 : u_{i_1}, u_{i_2}, \cdots, u_{i_n}, u_{i_1}$ be a tour in G_1 having weight at most l, i.e., $w_1(T_1) \leq l$. Now, we compute a tour T_2 on G_2 as $u_{i_1}^1, u_{i_1}^2, \cdots, u_{i_1}^n, u_{i_2}^1, u_{i_2}^2, \cdots, u_{i_2}^n, \cdots, u_{i_n}^1, u_{i_n}^2, \cdots, u_{i_n}^n, u_{i_1}^1$. The weight of T_2 i.e., $w_2(T_2)$ is equal to $\sum_{i=1}^{n} \sum_{j=1}^{n-1} w_2(u_i^j, u_i^{j+1}) + w_1(T_1) - \frac{n}{(n+1)^2} \leq \frac{n}{(n+1)^2} + l - \frac{n}{(n+1)^2} = l$.

Conversely, let $T_2' : x_1, x_2, \cdots, x_{n^2}, x_1$ be a tour in G_2 of weight at most l, where $x_1, x_2, \cdots, x_{n^2} \in U_2$ are distinct vertices. We construct a tour T_1' from T_2' as follows. First we remove all the edges of type (u_i^k, u_i^l) from T_2'. The remaining all edges of T_2' are of the form (u_i^k, u_j^l) for $i \neq j$. For each and every such edge (u_i^k, u_j^l) of T_2' we consider corresponding edge (u_i, u_j) in G_1 and construct a graph $\mathcal{E}$. For example, if edges $(u_i^{k_1}, u_j^{l_1})$ and $(u_i^{k_2}, u_j^{l_2})$ for $k_1 \neq k_2$ and $l_1 \neq l_2$ are in T_2', then we consider the edge (u_i, u_j) twice in $\mathcal{E}$. One can easily verify that the graph $\mathcal{E}$ with all vertices U_1 form an Eulerian graph. We construct a tour T_1' on G_1 by short cutting the edges of $\mathcal{E}$. Since the edge weights of G_1 follow triangle inequality, $w_1(T_1') \leq w_1(\mathcal{E}) \leq w_2(T_2') + \frac{n^2}{(n+1)^2} < l + 1$. Therefore, $w_1(T_1') \leq l$ as edge weights in G_1 are integers.

We consider an instance of the sweep coverage problem by mobile sensors with different speeds as follows: We take G_2 as the input graph, sweep period $t = 1$, and a set of n^2 mobile sensors, among them speed of one mobile sensor is L and speeds of remaining

$n^2 - 1$ mobile sensors are zero. Let y be the number of mobile sensors returned by $\mathcal{A}$ on G_2.

Case 1: $(y \leq k)$ Among the k mobile sensors, speed of one mobile sensor is L and speeds of remaining $y-1$ mobile sensors are zero. Therefore, the mobile sensor with speed L moves along a tour covering $n^2 - y + 1$ vertices and remaining $y - 1$ mobile sensors cover remaining $y - 1$ vertices one for each. Since $n > k$ therefore $n^2 - y + 1 > n^2 - n$. Hence at least one vertex from each of the set $\{u_i^1, u_i^2, \cdots, u_i^n\}$ for different i is visited by the mobile sensor with speed L and weight of the tour is at most L. From this tour, a tour T of weight at most L of G_1 can be constructed in the same way by constructing an Eulerian graph of G_1 and short cutting as explained before. Hence there is a tour on G_1 with weight at most L that covers all the vertices of G_1.

Case 2: $(y > k)$ As $\mathcal{A}$ is a k-approximation algorithm, we can say that there is no tour on G_2 with weight at most L that covers all the vertices of G_2. This implies that there is no tour on G_1 with weight at most L that covers all the vertices of G_1.

Hence, TSP can be decided in polynomial time by applying $\mathcal{A}$ on G_2, which is a contradiction, unless P=NP. Hence the statement of the theorem follows. $\square$

4.4 Conclusion

In this chapter, we have discussed the sweep coverage problem with two different context of heterogeneity. In the first problem, the sweep periods of the PoIs are different. We have remarked on the flaw of the previous solution [41] of the problem. Our proposed algorithm achieves approximation factor $O(\log \rho)$, where $\rho = \frac{t_{max}}{t_{min}}$, t_{min} and t_{max} are the minimum and maximum sweep periods among the vertices. For the second problem, the speeds of the mobile sensors are different. For this problem, we have proved that it is impossible to design any constant factor approximation algorithm, unless P=NP.

Chapter 5

Solving Energy Issues for Sweep Coverage

5.1 Introduction

Energy is a very important resource which is needed to be considered while designing efficient algorithms for WSNs. Since the sensors have limited battery as energy source, proper energy utilization can extend their lifetime. In this chapter, we consider an energy efficient sweep coverage problem with mobile and static sensors by minimizing the total energy utilization. Activity of a sensor is restricted by the capacity of its battery unless the battery is recharged or replaced. It is also recommendable to recharge or replace the battery just before (at a limiting condition) exceeding its capacity for optimal resource utilization. It may happen that during some activities of a mobile sensor, capacity of its battery is at the limiting condition. In that case the sensor stops its activities until the battery is recharged or replaced. So, with the restriction on the maximum energy utilization, how to guarantee sweep coverage with minimum number of mobile sensors is a challenging problem. In this chapter we consider an energy restricted sweep coverage problem where the total energy used by a mobile sensor in a given time period is bounded.

5.1.1 Contribution

In this chapter, our contributions are as follows:

- We propose a variation of sweep coverage problem, called energy efficient sweep coverage problem or ESweep coverage problem. The problem is NP-hard and cannot be approximated within a factor of 2, unless P=NP. We propose an 8-approximation algorithm to solve it.

- A 2-approximation algorithm is proposed for a special case of the above problem, which is the best possible approximation factor.

- We introduce another variation of sweep coverage, called energy restricted sweep coverage problem or ERSweep coverage problem and propose a $(5+\frac{2}{\alpha})$-approximation algorithm to solve this NP-hard problem.

5.2 Energy Efficient Sweep Coverage Problem

Instead of using mobile sensors, sometimes using both static and mobile sensors for sweep coverage can be more efficient. For example, let us consider the scenario where there are three points u_1, u_2, u_3 in a plane. The distance between the points are as follows: $d(u_1, u_2) = 5$, $d(u_2, u_3) = 20$, $d(u_3, u_1) = 20$. Let the value of vt is 10. According to Algorithm 2 in Chapter 3, a mobile sensor is deployed to visit u_1 and u_2 and another mobile sensor is deployed at u_3. Now, since energy utilization of a static sensor is much lesser than that of a mobile sensor, using one static sensor at u_3 is more effective than the use of a mobile sensor. Motivated by the above example, we redefine the sweep coverage problem as follows.

Definition 5.2.1 (t-GSweep coverage)**.** Let $\mathcal{U} = \{u_1, u_2, \cdots, u_n\}$ be a set of PoIs, $S = \{s_1, s_2, \cdots, s_n\}$ be a set of static sensors and $M = \{m_1, m_2, \cdots, m_n\}$ be a set of mobile sensors. Let v be the uniform speed of the mobile sensors. For a given time period $t \geq 0$, a PoI u_i is said to be t-GSweep covered if and only if either of the following two cases happens:

1. A static sensor s_j is deployed at u_i which continuously monitors u_i.

2. At least one mobile sensor m_j visits u_i in every t time period.

The time period t is called the *sweep period* of the set of points $\mathcal{U}$.

We define the following problem, called energy efficient sweep coverage problem, in short ESweep coverage problem.

Problem 4 (ESweep Coverage Problem)**.** Let $\mathcal{U}$=$\{u_1,\ u_2,\ \cdots,\ u_n\}$ be a set of PoIs, $S = \{s_1, s_2, \cdots, s_p\}$ be a set of static sensors and $M = \{m_1, m_2, \cdots, m_q\}$ be a set of mobile sensors. Let v be the uniform speed of the mobile sensors, λ, μ be the energy consumptions per unit time by a static and a mobile sensors, respectively. For given $t > 0$, find number of sensors, combination of static and mobile sensors such that t-GSweep coverage of the set of PoIs can be guaranteed and total energy consumption per unit of time is minimized.

The following theorem gives the complexity result for the ESweep coverage problem.

Theorem 5.2.2. *The ESweep coverage problem is NP-hard and cannot be approximated within a factor of 2, unless P=NP.*

Proof. For $\lambda = \mu$, the ESweep coverage problem reduces to the sweep coverage problem presented in [41], where the objective is to minimize total number of sensors. According to Theorem 1 in [41], the sweep coverage problem is NP-hard and cannot be approximated within a factor of 2 unless $P = NP$. Therefore, ESweep coverage problem is also NP-hard and cannot be approximated within a factor of 2, unless P=NP. □

To get the solution to ESweep coverage problem, we use solutions of k-TSP [25] and k-MST [25, 48] problems.

Definition 5.2.3 (k-MST problem [25])**.** Let $G = (V, E, w)$ be a weighed graph, where the edge weights are positive real numbers and k be a given positive integer. Find a minimum weighted tree of G that spans any k vertices of G.

Definition 5.2.4 (k-TSP problem [25])**.** Let $G = (V, E, w)$ be a weighed graph, where the edge weights are positive real numbers and k be a given positive integer. Find a minimum weighted tour of G visiting any k vertices of G.

Both of the above problems are NP-hard and the best known solutions for these problems are proposed by Garg in [25] where 2-approximation algorithms for both of the problems are provided.

5.2.1 Algorithm and Analysis

In this section, we propose an approximation algorithm for the ESweep coverage problem with approximation factor 8. Our algorithm use the 2-approximation algorithm for k-MST [25] as a subroutine. Consider a set of PoIs $\mathcal{U}$=$\{u_1, u_2, \cdots, u_n\}$ in two dimensional plane. Let $G = (U, E, w)$ be the complete weighted graph with each PoI as a vertex and the line segment joining two PoIs in the plane as edge. The weight of an edge $e \in E$ is equal to the Euclidean distance between the vertices and it is denoted by $w(e)$. For any subgraph H of G, we denote $w(H)$ as the sum of the edge weights of H. We denote $V(H)$ and $E(H)$ as the vertex and edge set of H respectively.

The inputs of the proposed Algorithm 3 (ESweepCoverage) are the graph G, speed v, sweep period t, energy consumption per unit time for static and mobile senors λ and μ, respectively. Output of the algorithm is the number of mobile and static sensors with the deployment locations for static sensors and movement schedule for mobile sensors. The formal description of the algorithm is as follows. At the beginning of the algorithm a graph $G' = (U, E, w')$ from $G = (U, E, w)$ is constructed by changing weight of every edge $e \in E$ as follows. For all $e \in E$, if $w(e) \leq vt$ then assign $w'(e) = \frac{w(e)}{vt}$, otherwise assign $w'(e) = 1$. Next, for each k, $1 \leq k \leq n$, we do the following steps.

We compute a tree $MST_k^{G'}$ which spans any k vertices of G' using the 2-approximation algorithm [25] for k-MST. Let MST_k^G be the corresponding tree of $MST_k^{G'}$ in G. Now, we delete all the edges of weight more than vt from MST_k^G. This may split MST_k^G into several connected components. Find tours on each of the components by doubling the edges and short cutting. Then partition the tours into several parts of length vt. Compute the value of $f_k = (\lambda \cdot t)X_k + (\mu \cdot t)Y_k$, where Y_k is the total number of partitions over all tours and $X_k = n - k$. Let $k = k_0$ for which the value of f_k, $1 \leq k \leq n$, be minimum. We select the number of static sensors $X = X_{k_0}$ and number of mobile sensors $Y = Y_{k_0}$ as the output of our algorithm. The deployment locations of static sensors and the movement schedules of mobile sensors are calculated as follows. There are $n-k_0$ vertices, which are not covered by $MST^G_{k_0}$, so $n-k_0$ static sensors are deployed at $n-k_0$ vertices, one for each. For deployment of mobile sensors, assume that $MST^G_{k_0}$ is split into h components $C_1, C_2, \cdots, C_h$. Let $T_1, T_2, \cdots, T_h$ be the corresponding tours on the components after doubling the edges and short cutting. For each j, $1 \leq j \leq h$, partition the tour T_j into $\left\lceil \frac{w(T_j)}{vt} \right\rceil$ parts of weight at most vt. Then deploy one mobile sensor at each of the partitioning points on T_j. All mobile sensors deployed on T_j start their movements along T_j at the same time in the same direction.

Algorithm 3 ESweepCoverage

1: Construct graph $G' = (U, E, w')$ from $G = (U, E, w)$
2: **for** $k = 1$ **to** n **do**
3:     Find a tree $MST^{G'}_k$ which spans any k vertices of G' using the 2-approximation algorithm for k-MST [25] .
4:     Remove all the edges with weight $> vt$ from MST^G_k. Assume that after removal of the edges MST^G_k is split into k_c components $C_1, C_2, \cdots, C_{k_c}$.
5:     Find tours $T_1, T_2, \cdots, T_{k_c}$ by doubling the edges of each of the respective components and short cutting.
6:     Set $Y_k = \sum_{i=1}^{k_c} \left\lceil \frac{w(T_i)}{vt} \right\rceil$ and $X_k = n - k$ then define $f_k = (\lambda \cdot t)X_k + (\mu \cdot t)Y_k$.
7: **end for**
8: $f = \min\{f_1, f_2, \cdots, f_k\}$ and k_0 is the index such that $f = f_{k_0}$.
9: For $k = k_0$ partition $T_1, T_2, \cdots T_h$ into $\sum_{i=1}^{h} \left\lceil \frac{w(T_i)}{vt} \right\rceil$ parts of weight vt (found in step 5).
10: Deploy one mobile sensor at each of the partitioning points of every tour and deploy one static sensor at each of the remaining $n - k_0$ vertices.
11: The mobile sensors start their movements at the same time along the corresponding tours in the same direction.

Theorem 5.2.5. *Algorithm 3 ensures t-GSweep coverage for each of the PoIs.*

Proof. To prove it we consider any vertex u_i and show that it is t-GSweep covered. According to the Algorithm 3, either of the following two cases happens for u_i.

Case 1. One static sensor is deployed at u_i. In this case the statement is trivially true.

Case 2. u_i is visited by some mobile sensors. Let u_i is visited by a mobile sensor at time t_0. According to the algorithm, the mobile sensors are deployed within distance at most vt along the tour in which u_i belongs. So u_i will be visited by the next mobile sensor within time $t + t_0$. □

To find the approximation factor of the proposed algorithm, let mst^k_{opt} be the minimum weighted tree on G' that spans any k vertices of G'. Let X_{opt} and Y_{opt} be the number of static and mobile sensors in the optimal solution of ESweep coverage problem for the graph G. Let Y_{opt} mobile sensors visit k_{opt} vertices of G and remaining $n - k_{opt}$ vertices are sweep covered by $X_{opt} = n - k_{opt}$ static sensors. Following lemma establishes a relation between $w'(mst^{k_{opt}}_{opt})$ and Y_{opt}.

Lemma 5.2.6. $\left\lceil w'(mst^{k_{opt}}_{opt}) \right\rceil \leq 2Y_{opt}$.

Proof. Let us consider movement of the mobile sensors in optimal solution on G. Let $P_1, P_2, \cdots, P_{Y_{opt}}$ be the movement paths of Y_{opt} mobile sensors in a time interval $[t_0, t+t_0]$. Since a mobile sensor can move at most vt distance during the time interval, no P_i contains any edge with weight more that vt. Therefore, in G', $w'(P_i) = \frac{w(P_i)}{vt} \leq 1$ and thus $\sum_{i=1}^{Y_{opt}} w'(P_i) \leq Y_{opt}$.

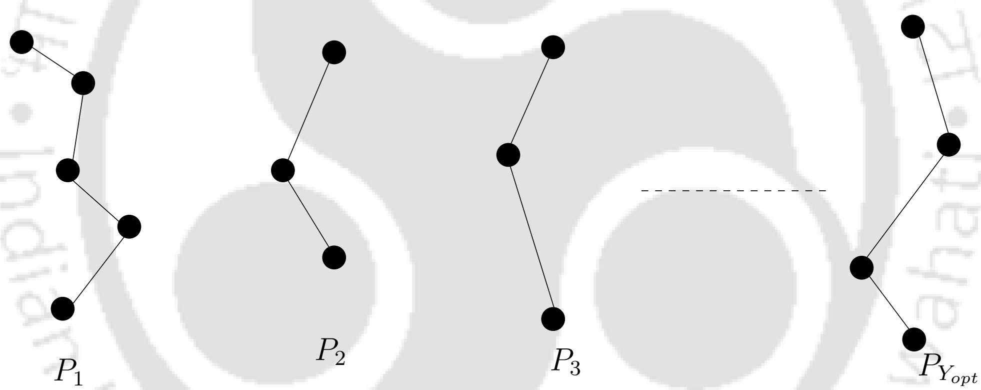


Figure 5.1: Movement of the mobile sensors in the optimal solution where $w(P_i) \leq vt$ for $i = 1, 2, \cdots, Y_{opt}$.

Consider one vertex $v_i \in P_i$ for all $i = 1$ to Y_{opt}. Let H' be the subgraph of G' such that $V(H') = U$ and $E(H') = \bigcup_{i=1}^{Y_{opt}} E(P_i) \bigcup \{(v_j, v_{j+1}) |\ j = 1 \ \ to \ \ Y_{opt} - 1\}$. An example of the movement paths of the mobile sensors in the optimal solution is shown in Fig. 5.1 and the formation of the graph H' is shown in Fig. 5.2. Note that H' is a tree which spans k_{opt} vertices of G' and $w'(H) = \sum_{i=1}^{Y_{opt}} w'(P_i) + \sum_{i=1}^{Y_{opt}-1} w'(v_i, v_{i+1}) \leq Y_{opt} + (Y_{opt} - 1) \leq 2Y_{opt} - 1$. Since $mst^{k_{opt}}_{opt}$ is the minimum weighted tree on G' that spans k_{opt} vertices of G', therefore $w'(mst^{k_{opt}}_{opt}) \leq w'(H) \leq 2Y_{opt} - 1$.
Hence $\left\lceil w'(mst^{k_{opt}}_{opt}) \right\rceil \leq w'(mst^{k_{opt}}_{opt}) + 1 \leq 2Y_{opt}$. $\square$

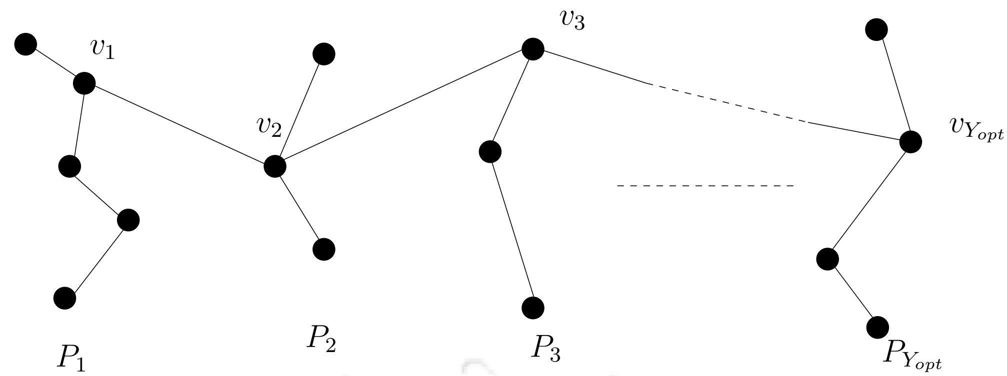


Figure 5.2: The tree H' spans k_{opt} vertices of G'.

Theorem 5.2.7. *The approximation factor of the Algorithm 3 is 8.*

Proof. The Algorithm 3 finds $MST_{k_{opt}}^{G'}$ for the iteration $k = k_{opt}$ using 2-approximation algorithm of k-MST [25] on G'. Then $w'(MST_{k_{opt}}^{G'}) \leq 2w'(mst_{opt}^{k_{opt}})$. After computing $MST_{k_{opt}}^{G'}$, all the edges of weight more than vt are deleted from $MST_{k_{opt}}^{G}$. Let m be the number of deleted edges. First we consider the case when at least one edge is deleted, i.e., $m \neq 0$. So, $MST_{k_{opt}}^{G}$ splits into $m+1$ connected components $C_1, C_2, \cdots, C_{m+1}$ in G. The corresponding tours $T_1, T_2, \cdots, T_{m+1}$ are computed after doubling the edges of each component and short cutting. According to the step 6 of the Algorithm 3, total number of mobile sensor needed is $Y_{k_{opt}}$, which is
$Y_{k_{opt}} = \sum_{i=1}^{m+1} \left\lceil \frac{w(T_i)}{vt} \right\rceil \leq \sum_{i=1}^{m+1} \frac{2w(C_i)}{vt} + (m+1)$.
Also, $w'(MST_{k_{opt}}^{G'}) = \sum_{i=1}^{m+1} \frac{w(C_i)}{vt} + m$. Therefore,

$$\begin{aligned} Y_{k_{opt}} &\leq \sum_{i=1}^{m+1} \frac{2w(C_i)}{vt} + (m+1) \\ &\leq \sum_{i=1}^{m+1} \frac{2w(C_i)}{vt} + 2m \\ &\leq 2w'(MST_{k_{opt}}^{G'}) \\ &\leq 4w'(mst_{opt}^{k_{opt}}) \\ &\leq 8Y_{opt} \qquad \text{(Lemma 5.2.6).} \end{aligned}$$

Now, we consider the case when no edge from $MST_{k_{opt}}^{G}$ is deleted i.e., $m = 0$. Then $Y_{k_{opt}} \leq 2 \left\lceil \frac{w\left(MST_{k_{opt}}^{G}\right)}{vt} \right\rceil \leq 4 \left\lceil w'(mst_{opt}^{k_{opt}}) \right\rceil \leq 8Y_{opt}$.

Therefore from the above two cases we have

$\lambda X_{k_{opt}} + \mu Y_{k_{opt}} \leq \lambda X_{opt} + 8\mu Y_{opt} \leq 8(\lambda X_{opt} + \mu Y_{opt})$. Hence, Algorithm 3 is an 8-approximation algorithm. □

5.2.2 A special case of ESweep coverage problem

A special case of the ESweep coverage problem is named as SESweep coverage problem, which is presented in this section. In this special case all mobile sensors visit same subset of PoIs to guarantee t-GSweep coverage while for the remaining PoIs, static sensors are used to guarantee t-GSweep coverage. We define the SESweep coverage problem as follows.

Problem 5 (SESweep Coverage Problem)**.** Let $\mathcal{U}$=$\{u_1, u_2, \cdots, u_n\}$ be a set of PoIs, $S = \{s_1, s_2, \cdots, s_p\}$ be a set of static sensors and $M = \{m_1, m_2, \cdots, m_q\}$ be a set of mobile sensors. Let v be the uniform speed of the mobile sensors and λ and μ be the energy consumption in every unit of time for a static sensor and mobile sensor, respectively. Find X and Y, such that

1. X static sensors guarantee t-GSweep coverage of X PoIs,
2. remaining $n-X$ PoIs are t-GSweep covered by Y mobile sensor such that each mobile sensor visits all $n - X$ PoIs and
3. $\lambda X + \mu Y$ is minimum.

Note that the SESweep coverage problem is NP-hard and cannot be approximated within a factor of 2, unless P=NP. The hardness proof follows from the proof of Theorem 1 in [41] for $\lambda = \mu$. Algorithm 4 (SESweepCoverage) is proposed to solve the SESweep coverage problem which uses the 2-approximation algorithm for k-TSP [25] as a subroutine. The inputs of the Algorithm 4 are the graph G, speed v, sweep period t, and energy consumption per unit time for static and mobile senors λ and μ, respectively. Output of the algorithm is the number of mobile and static sensors with the deployment locations for the static sensors and the movement schedule for the mobile sensors.

According to the Algorithm 4, all mobile sensors move along a single cycle which cover a subset of PoIs of G. To cover the remaining PoIs, static sensors are deployed.

Algorithm 4 SESweepCoverage

for $k = 1$ **to** n **do**
  Find a tour TSP_k^G of G visiting k vertices using the k-TSP 2-approximation algorithm [25].
  Set $Y_k = \left\lceil \frac{w(TSP_k^G)}{vt} \right\rceil$ and $X_k = n - k$ then define $f_k = \lambda X_k + \mu Y_k$.
end for
$f = \min\{f_1, f_2, \cdots, f_k\}$ and k_0 is the index such that $f = f_{k_0}$
For $k = k_0$, partition the tour $TSP_{k_0}^G$ into $\left\lceil \frac{w(TSP_{k_0}^G)}{vt} \right\rceil$ parts of length vt
Deploy one mobile sensor at each of the partitioning points of the tour and deploy one static sensor at each of the remaining $n - k_0$ vertices.
The mobile sensors start their movement at the same time along the tour in the same direction.

Therefore, by the similar argument given in Theorem 5.2.5, each PoI in $\mathcal{U}$ are t-GSweep covered.

Theorem 5.2.8. *The Algorithm 4 is a 2-approximation algorithm.*

Proof. Let X_{opt} and Y_{opt} be the number of static and mobile sensors in the optimal solution. Let Y_{opt} mobile sensors visit k_{opt} vertices of G and remaining $n - k_{opt}$ vertices are sweep covered by $X_{opt} = n - k_{opt}$ static sensors. Let $tsp_{opt}^{k_{opt}}$ be the minimum weighted tour in G covering k_{opt} vertices. The tour TSP_k^G of G visiting k vertices is computed using the k-TSP 2-approximation algorithm [25]. Then $w(TSP_{k_{opt}}^G) \leq 2w(tsp_{opt}^{k_{opt}})$. Since Y_{opt} mobile sensors visit k_{opt} vertices of G and all mobile sensors visit all the k_{opt} vertices, therefore,
$Y_{opt} \cdot vt \geq w(tsp_{opt}^{k_{opt}})$, which is equivalent to $\left\lceil \frac{w(tsp_{opt}^{k_{opt}})}{vt} \right\rceil \leq Y_{opt}$. According to the Algorithm 4, when $k = k_{opt}$, $Y_{k_{opt}} = \left\lceil \frac{w(TSP_{k_{opt}}^G)}{vt} \right\rceil \leq 2 \left\lceil \frac{w(tsp_{opt}^{k_{opt}})}{vt} \right\rceil \leq 2Y_{opt}$.
Therefore, $\lambda X_{k_{opt}} + \mu Y_{k_{opt}} \leq \lambda X_{opt} + 2\mu Y_{opt} \leq 2(\lambda X_{opt} + \mu Y_{opt})$. Hence the Algorithm 4 is a 2-approximation algorithm. □

5.3 Energy Restricted Sweep Coverage Problem

Generally sensors are equipped with limited power battery. For long running applications, without recharging or replacing batteries it is not possible to continue sweep coverage for long time. Also, it is not possible to recharge or replace batteries frequently. After a certain time interval it can be done. Until then, the sensors have to use their existing power of the batteries to continue activities. We introduce a variation of sweep coverage problem, called Energy Restricted Sweep (ERSweep) coverage problem, where there is an upper bound $\mathcal{Z}$ on the energy utilization by a mobile sensor for every time period Γ. For example a mobile sensor consumes maximum energy $\mathcal{Z}$ in every time period $[\Gamma' + k\Gamma, \Gamma' + (k+1)\Gamma]$, where Γ' is the time when the mobile sensor starts its movement and $k \geq 0$. If a mobile sensor consumes $\mathcal{Z}$ energy before completing a time period then it stops its activities until the forthcoming time period. Here activity means movement and visiting the PoIs. The problem definition is given below.

Problem 6 (ERSweep Coverage Problem)**.** Let U=$\{u_1, u_2, \cdots, u_n\}$ be a set of PoIs and $M = \{m_1, m_2, \cdots, m_q\}$ be a set of mobile sensors. Let v be the uniform speed of the mobile sensors and $\mathcal{Z}$ be the upper bound of the energy consumption by a mobile sensor in every Γ time period. Let $\mu_i \geq 0$ be the energy consumption by a mobile sensor to visit u_i for $i = 1$ to n and μ be the energy consumption by a mobile sensor for its movement per unit time. For a given $t > 0$, find the minimum number of mobile sensors such that t-sweep coverage for every PoI in U is guaranteed.

If a mobile sensor has less than μ_i energy just before visiting u_i, then it partially completes the visit and wait until the starting of next time period.

Theorem 5.3.1. *ERSweep Coverage Problem is NP-hard and cannot be approximated within a factor of 2 unless $P = NP$.*

Proof. In a particular case if there is no visiting cost at u_i, i.e., $\mu_i = 0$ for $i = 1$ to n and $t = \Gamma$, $\mu \cdot t = \mathcal{Z}$, the energy consumption by a mobile sensor in time t is $\mathcal{Z}$, which is the energy consumption due to its mobility. In this case the problem is equivalent to find the minimum number of mobile sensors to guarantee sweep coverage for the set of PoIs, which is NP-hard and cannot be approximated within a factor of 2, unless P=NP according to Theorem 1 of [41]. Therefore, ERSweep Coverage Problem is NP-hard and cannot be approximated within a factor of 2 unless $P = NP$. $\square$

5.3.1 Algorithm and Analysis

Let $G = (U, E)$ be the complete weighted graph with each PoI as a vertex and the line segment joining between two PoIs on the plane as an edge. We define two weight functions $w_1 : E \longrightarrow \mathbb{R}^+$ and $w_2 : E \cup U \longrightarrow \mathbb{R}^+$ as follows. For any edge $e = (u_i, u_j) \in E$, $w_1(e)$ = the Euclidean distance between u_i and u_j. For the edge $e = (u_i, u_j) \in E$, $w_2(e) = \frac{w_1(e)}{v} \cdot \mu$ and for each vertex $u_i \in U$, $w_2(u_i) = \mu_i$. For a subgraph H of G, $w_1(H) = \sum_{e \in E(H)} w_1(e)$ and $w_2(H) = \sum_{v \in V(H)} w_2(v) + \sum_{e \in E(H)} w_2(e)$. Basically, for any path P in G, $w_1(P)$ gives the total distance a mobile sensor must travel to cover each point on P and $w_2(P)$ gives the total energy loss by a mobile sensor for moving along P. The Algorithm 5 (ERSweepCoverage) is proposed to solve the ERSweep coverage problem which returns the number of mobile sensors needed to guarantee t-sweep coverage of the given set of PoIs by maintaining the energy restriction of the mobile sensors.

Algorithm 5 ERSweepCoverage

1: **for** $k = 1$ to n **do**
2: Find the minimum spanning forest F_k with k components on G with respect to the weight function w_1. Let $C_1, C_2, \cdots C_k$ be the connected components of F_k.
3: Find tours $T_1, T_2, \cdots, T_k$ by doubling the edges of $C_1, C_2, \cdots C_k$, respectively and short cutting.
4: **for** $i = 1$ to k **do**
5: **if** $\Gamma \geq t$ **then**
6: Set x_i=PartitionTour1$(T_i, t, \Gamma, \mathcal{Z})$
7: **else**
8: Set x_i=PartitionTour2$(T_i, t, \Gamma, \mathcal{Z})$
9: **end if**
10: **end for**
11: $N_k = \sum_{j=1}^{k} x_j$
12: **end for**
13: $N = \min\{N_1, N_2, \cdots, N_n\}$, and let J be the index such that $N_J = N$
14: Repeat step 2 to step 11 for $k = J$.
15: Deploy one mobile sensor at each of the partitioning points on each tour. The mobile sensors start moving at the same time along the respective tours in same direction.

To find the best possible solution, from step 2 to step 11 of the Algorithm 5 is executed in n iterations. In kth iteration ($1 \leq k \leq n$), the minimum spanning forest F_k with k connected components with respect to w_1 is computed. After that k disjoint tours $T_1, T_2, \cdots, T_k$ are found by doubling all edges of the components $C_1, C_2, \cdots, C_k$, respectively and short cutting. Depending on the values of Γ and t, each tour T_i is partitioned using one of the following two partitioning strategies.

PARTITIONTOUR1($T_i, t, \Gamma, \mathcal{Z}$): This partitioning strategy is applied when $\Gamma \geq t$. For each tour T_i, we partition T_i into $\left\lceil \frac{w_2(T_i)}{\mathcal{Z}} \right\rceil$ parts with weight at most $\mathcal{Z}$ with respect to w_2. Let $Pr_1, Pr_2, \cdots, Pr_{\left\lceil \frac{w_2(T_i)}{\mathcal{Z}} \right\rceil}$ be the partitions. Now, partition Pr_j into $\left\lceil \frac{w_1(Pr_j)}{vt} \right\rceil$ parts with weight at most vt with respect to w_1. Finally, this partitioning strategy returns total number of partitioning points $\sum_{j=1}^{\left\lceil \frac{w_2(T_i)}{\mathcal{Z}} \right\rceil} \left\lceil \frac{w_1(Pr_j)}{vt} \right\rceil$ for all partitions.

PARTITIONTOUR2($T_i, t, \Gamma, \mathcal{Z}$): This partitioning strategy is applied when $\Gamma < t$. Let α be the positive integer such that $\alpha \cdot \Gamma \leq t < (\alpha + 1) \cdot \Gamma$. Partition T_i into $\left\lceil \frac{w_1(T_i)}{vt} \right\rceil$ parts with weight at most vt with respect to the weight function w_1. Let $Pr_1, Pr_2, \cdots, Pr_{\left\lceil \frac{w_1(T_i)}{vt} \right\rceil}$ be the partitions. Now, partition Pr_j into $\left\lceil \frac{w_2(Pr_j)}{\alpha \cdot \mathcal{Z}} \right\rceil$ parts with weight at most $\alpha \cdot \mathcal{Z}$ with respect to w_2. Finally, this partitioning strategy returns total number of partitioning points $\sum_{j=1}^{\left\lceil \frac{w_1(T_i)}{vt} \right\rceil} \left\lceil \frac{w_1(Pr_j)}{\alpha \cdot \mathcal{Z}} \right\rceil$ for all partitions.

Minimum over the number of partitions of all iterations is chosen as the output of the Algorithm 5.

Theorem 5.3.2. *The Algorithm 5 ensures t-sweep coverage of each PoI and energy constraint of each mobile sensor.*

Proof. Let $u_i \in U$ be visited by a mobile sensor m_j at time t_0. According to Algorithm 5, mobile sensors are initially deployed within vt distance apart, after that they start moving with speed v in the same direction. So u_i will be again visited by the next mobile sensor within time $t + t_0$. Also the partitioning strategies guarantee each mobile sensor to consume at most $\mathcal{Z}$ energy in any time interval $[j\Gamma + \Gamma', (j + 1)\Gamma + \Gamma']$, where j is a positive integer. □

Let *opt* be the number of mobile sensor in the optimal solution. The following lemmas give lower bounds on *opt*.

Lemma 5.3.3. *$opt \geq \frac{w_1(F_{opt})}{vt}$, where F_{opt} is the minimum spanning forest of G with respect to w_1 having opt number of components.*

Proof. Consider movements of the mobile sensors in optimal solution during the time interval $[t_0, t+t_0]$. Let $P_1, P_2, \cdots, P_{opt}$ be the movement paths of the mobile sensors. Then $w_1(P_i) \leq vt$. $P_1, P_2, \cdots, P_{opt}$ form a forest with *opt* number of components that spans all the vertices of G. Since F_{opt} is the minimum spanning forest with *opt* number of components, $opt \cdot vt \geq w_1(F_{opt})$, i.e., $opt \geq \frac{w_1(F_{opt})}{vt}$. □

Lemma 5.3.4. *If* $\Gamma \geq t$ *then* $opt \geq \frac{w_2(F_{opt})}{\mathcal{Z}}$.

Proof. Let $P_1, P_2, \cdots, P_{opt}$ be the movement paths of the mobile sensors during the time interval $[t_0, t+t_0]$. Then, $w_2(P_i) \leq \mathcal{Z}$ for $i = 1$ to *opt*. Now,

$$\begin{aligned}
w_2(P_1) + w_2(P_2) + \cdots + w_2(P_{opt}) &= \sum_{i=1}^{opt}\left(\sum_{u\in V(P_i)} w_2(u) + \sum_{e\in E(P_i)} w_2(e)\right) \\
&= \sum_{u\in V(G)} w_2(u) + \sum_{e\in E(\bigcup_{i=1}^{opt} P_i)} w_2(e) \\
&= \sum_{u\in V(G)} w_2(u) + \frac{\mu}{v}\cdot \sum_{e\in E(\bigcup_{i=1}^{opt} P_i)} w_1(e) \\
&\geq \sum_{u\in V(G)} w_2(u) + \frac{\mu}{v}\cdot \sum_{e\in E(F_{opt})} w_1(e) \\
&= w_2(F_{opt}).
\end{aligned}$$

Therefore, $\mathcal{Z} \cdot opt \geq w_2(F_{opt})$, i.e., $opt \geq \frac{w_2(F_{opt})}{\mathcal{Z}}$. □

Lemma 5.3.5. *If* $\Gamma < t$ *then* $opt \geq \frac{w_2(F_{opt})}{(\alpha+1)\cdot\mathcal{Z}}$, *where* α *is the integer such that* $\alpha \cdot \Gamma \leq t < (\alpha+1)\cdot\Gamma$.

Proof. Let $P_1, P_2, \cdots, P_{opt}$ be the movement paths of the mobile sensors in the optimal solution during the time interval $[t_0, t+t_0]$. Since $\alpha\cdot\Gamma \leq t < (\alpha+1)\cdot\Gamma$, the maximum energy consumption by a mobile sensor for its activities in the time interval $[t_0, t+t_0]$ is less than $(\alpha+1)\cdot\mathcal{Z}$. Therefore, $w_2(P_i) \leq (\alpha+1)\cdot\mathcal{Z}$, for $i = 1$ to *opt*. Again, from the proof of Lemma 5.3.4, we have $w_2(P_1) + w_2(P_2) + \cdots + w_2(P_{opt}) \geq w_2(F_{opt})$. Therefore, $(\alpha+1)\cdot\mathcal{Z}\cdot opt \geq w_2(F_{opt})$, i.e., $opt \geq \frac{w_2(F_{opt})}{(\alpha+1)\cdot\mathcal{Z}}$. □

Lemma 5.3.6. *The number of partition x_i on the tour T_i is given by*

$$\begin{aligned} x_i &\le \frac{w_2(T_i)}{\mathcal{Z}} + \frac{w_1(T_i)}{vt} + 1 \qquad if\ \Gamma \ge t \\ &\le \frac{w_2(T_i)}{\alpha \cdot \mathcal{Z}} + \frac{w_1(T_i)}{vt} + 1 \qquad if\ \Gamma < t \end{aligned}$$

Proof. For $\Gamma \ge t$ each tour T_i is partitioned into parts with weight $\le \mathcal{Z}$ with respect to w_2. Let $Pr_1, Pr_2, \cdots, Pr_{\lceil \frac{w_2(T_i)}{\mathcal{Z}} \rceil}$ be the partitions with $w_2(Pr_i) \le \mathcal{Z}$. According to PartitionTour1$(T_i, t, \Gamma, \mathcal{Z})$, total number of partitions on T_i is $x_i = \sum_{j=1}^{\lceil \frac{w_2(T_i)}{\mathcal{Z}} \rceil} \lceil \frac{w_1(Pr_j)}{vt} \rceil \le \sum_{j=1}^{\lceil \frac{w_2(T_i)}{\mathcal{Z}} \rceil} \frac{w_1(Pr_j)}{vt} + \lceil \frac{w_2(T_i)}{\mathcal{Z}} \rceil \le \frac{w_1(T_i)}{vt} + \lceil \frac{w_2(T_i)}{\mathcal{Z}} \rceil \le \frac{w_1(T_i)}{vt} + \frac{w_2(T_i)}{\mathcal{Z}} + 1$.

For $\Gamma < t$ each tour T_i is partitioned into parts with weight $\le vt$ with respect to w_1. Let $Pr_1, Pr_2, \cdots, Pr_{\lceil \frac{w_1(T_i)}{vt} \rceil}$ be the partitions with $w_1(Pr_i) \le vt$. According to PartitionTour2$(T_i, t, \Gamma, \mathcal{Z})$, total number of partitions on T_i is $x_i = \sum_{j=1}^{\lceil \frac{w_1(T_i)}{vt} \rceil} \lceil \frac{w_2(Pr_j)}{\alpha \cdot \mathcal{Z}} \rceil \le \sum_{j=1}^{\lceil \frac{w_1(T_i)}{vt} \rceil} \frac{w_2(Pr_j)}{\alpha \cdot \mathcal{Z}} + \lceil \frac{w_1(T_i)}{vt} \rceil \le \frac{w_2(T_i)}{\alpha \mathcal{Z}} + \lceil \frac{w_1(T_i)}{vt} \rceil \le \frac{w_1(T_i)}{vt} + \frac{w_2(T_i)}{\alpha \cdot \mathcal{Z}} + 1$. □

Theorem 5.3.7. *The Algorithm 5 is a 5-approximation algorithm for $\Gamma \ge t$.*

Proof. Algorithm 5 chooses the minimum over all N_k for $k = 1$ to n. Let us consider the iteration of the algorithm when $k = opt$. For $\Gamma \ge t$, the total number of mobile sensors required is given by

$$\begin{aligned} \sum_{i=1}^{k} x_i &\le \sum_{i=1}^{k} \frac{w_2(T_i)}{\mathcal{Z}} + \sum_{i=1}^{k} \frac{w_1(T_i)}{vt} + k \qquad (Lemma\ 5.3.6) \\ &\le 2\sum_{i=1}^{k} \frac{w_2(C_i)}{\mathcal{Z}} + 2\sum_{i=1}^{k} \frac{w_1(C_i)}{vt} + k \\ &= 2\frac{w_2(F_k)}{\mathcal{Z}} + 2\frac{w_1(F_k)}{vt} + k \\ &\le 2k + 2k + k \qquad (Lemma\ 5.3.4 \text{ and } Lemma\ 5.3.3) \\ &= 5opt. \end{aligned}$$

Therefore if $\Gamma \ge t$, the Algorithm 5 is a 5-approximation algorithm. □

Theorem 5.3.8. *Algorithm 5 is a $(5+\frac{2}{\alpha})$-approximation algorithm for $\Gamma < t$, where α is the integer such that $\alpha \cdot \Gamma \leq t < (\alpha+1)\cdot\Gamma$.*

Proof. The Algorithm 5 chooses the minimum over all N_k for $k = 1$ to n. Let us consider the iteration of the algorithm when $k = opt$. For $\Gamma < t$, the total number of mobile sensors required is given by

$$\begin{aligned}
\sum_{i=1}^{k} x_i &\leq \sum_{i=1}^{k} \frac{w_2(T_i)}{\alpha \mathcal{Z}} + \sum_{i=1}^{k} \frac{w_1(T_i)}{vt} + k \qquad (\textit{Lemma } 5.3.6)\\
&\leq 2\sum_{i=1}^{k} \frac{w_2(C_i)}{\alpha \mathcal{Z}} + 2\sum_{i=1}^{k} \frac{w_1(C_i)}{vt} + k\\
&= 2\frac{w_2(F_k)}{\alpha \mathcal{Z}} + 2\frac{w_1(F_k)}{vt} + k\\
&\leq 2k\left(\frac{\alpha+1}{\alpha}\right) + 2k + k \qquad (\textit{Lemma } 5.3.5 \text{ and } \textit{Lemma } 5.3.3)\\
&= \left(5+\frac{2}{\alpha}\right)k\\
&= \left(5+\frac{2}{\alpha}\right) opt.
\end{aligned}$$

Therefore if $\Gamma < t$, the Algorithm 5 is a $(5+\frac{2}{\alpha})$-approximation algorithm. □

5.4 Conclusion

In this chapter we have introduced two sweep coverage problems, one is energy efficient and other is energy restricted. Objective of the first problem is to guarantee sweep coverage for a given set of points by a set of sensors (static and/or mobile) with minimum energy consumption per unit time. We have proposed an 8-approximation algorithm to solve the problem. Another solution for a special case is proposed, which achieves the best possible approximation factor 2. Objective of the second problem is to find minimum number of mobile sensors to guarantee sweep coverage where the maximum energy utilization by a mobile sensor is bounded. We have proposed a $(5+\frac{2}{\alpha})$-approximation algorithm to solve the problem.

Chapter 6

Area Sweep Coverage

6.1 Introduction

Area coverage is a well studied research area in WSNs [3, 44, 52, 57, 62, 63]. It has several applications on environmental monitoring, surveillance, military applications, forest fire detection, etc., where a set of sensors is deployed over a geographical region to monitor continuously. But there are applications where periodic monitoring is sufficient instead of continuous one. For example in forest fire detection, periodical monitoring of the region of a forest is sufficient to detect occurrence of fire. If each point of the forest is monitored once in a time period, the location of occurrence of fire can be detected. In general, sweep period depends on the frequency of occurrence of an event for a specific application. As there are uncountably many points inside a bounded region, unlike problems discussed in the previous chapters, point sensors cannot be used. Homogeneous mobile sensors with sensing range r are used for area coverage. The sensor is typically modeled as a disk in the two dimensional plane. The location of the sensor is at the center of the disk. The disk with radius r is termed as *sensing disk*. The main objective of the area sweep coverage is to find minimum number of mobile sensors to guarantee sweep coverage of each point inside a bounded area.

6.1.1 Contribution

Our contribution in this chapter are as follows:

- We introduce the area sweep coverage problem and prove that the problem is NP-hard.
- A $2\sqrt{2}$-approximation algorithm is proposed for a square region. An improvement of the approximation factor is made for any rectangular region.
- The approximation factor of the proposed algorithm is a function of area, perimeter of the region and sensing range of the mobile sensors for arbitrary bounded region. Performance of the algorithm is investigated for arbitrary simple polygonal regions.

6.2 Area Sweep Coverage Problem

Definition of the area sweep coverage is given below.

Definition 6.2.1. Let $\mathcal{A}$ be a given bounded area of interest (AoI) and $M = \{m_1, m_2, \cdots, m_p\}$ be a set of mobile sensors. $\mathcal{A}$ is said to be t-area sweep covered if and only if each point of $\mathcal{A}$ is in the sensing disk of at least one mobile sensor in every t time period. The time period t is said to be *sweep period* of the area $\mathcal{A}$.

Problem 7 (Area Sweep Coverage Problem)**.** Let $\mathcal{A}$ be a given bounded area of interest (AoI) and $M = \{m_1, m_2, \cdots, m_p\}$ be a set of mobile sensors. Let v be the uniform speed of the mobile sensors. For a given time period $t \geq 0$, find minimum number of mobile sensors such that $\mathcal{A}$ is t-sweep covered.

Theorem 6.2.2. *The area sweep coverage problem is NP-hard.*

Proof. Li et al. [39] have shown that covering a bounded region with minimum number of static sensor is NP-hard. Covering a bounded region is a special instance of the area sweep coverage problem when the sweep period $t = 0$. Hence the area sweep coverage problem is also NP-hard. ☐

6.3 Solution of Area Sweep Coverage

Although the area sweep coverage is different from sweep coverage for a set of PoIs, but it can be solved using a solution of the sweep coverage for points. Let us consider a point set $P = \{p_1, p_2, \cdots, p_k\}$, where each $p_i \in \mathcal{A}$ such that placement of static sensors at each point p_i covers $\mathcal{A}$.

Lemma 6.3.1. *The sweep coverage of the set of points P on $\mathcal{A}$ guarantees the area sweep coverage of $\mathcal{A}$.*

Proof. Suppose we have a solution of sweep coverage for the set of points P. Therefore at least one mobile sensor visits each p_i at least once in every t time period. Since placement of one static sensor at each p_i gives complete coverage of $\mathcal{A}$ according to above assumption, therefore each point of $\mathcal{A}$ is inside at least one sensing disk in every t time period. Hence the solution of sweep coverage problem for P provides a solution of area sweep coverage problem for $\mathcal{A}$. □

We propose Algorithm 6 (AreaSweepCoverage) for area sweep coverage problem, where $\mathcal{A}$ is a square region. Assume side of the square is divisible by $\sqrt{2}r$. Divide the area into square grids of side $\sqrt{2}r$. Let P be the set of center points of each grid cell. Clearly, deploying one static sensor with sensing radius r at each point of P covers $\mathcal{A}$. Therefore by Lemma 6.3.1, the solution of point sweep coverage problem for P on $\mathcal{A}$ provides solution of the area sweep coverage problem for $\mathcal{A}$.

Algorithm 6 AreaSweepCoverage

1: Divide the area $\mathcal{A}$ into square grids of side $\sqrt{2}r$.
2: Let P be the set of center points of each square grid.
3: Apply Algorithm 2 to compute number of mobile sensors for sweep coverage of P.

6.3.1 Analysis

Lemma 6.3.2. *Area covered by a mobile sensor with sensing range r due to its movement is maximum when it moves along a straight line.*

Proof. Let us consider a straight line l and an arbitrary open curve c (not a straight line) of same length ϵ, where $\epsilon < 2r$. Place one sensing disk at the starting point and other sensing disk at the ending point of l and c respectively. Clearly, the overlap between the sensing disks of c is more than the overlap between the sensing disks of l, as the distance between two end points of c is less than ϵ. Hence the proof. □

Lemma 6.3.3. *If N_{opt} is the number of mobile sensors for the optimal solution of area sweep coverage problem then $N_{opt} \geq \frac{Area(\mathcal{A})}{2rvt}$.*

Proof. The total path moved by a mobile sensor in t time is vt. According to Lemma 6.3.2, the coverage area is maximum when a mobile sensor moves in straight line. Let at time t_0 the mobile sensor be at I and at time $t_0 + t$ the mobile sensor be at F on the straight line IF as shown in Fig. 6.1. At time t_0, the mobile sensor covers the area

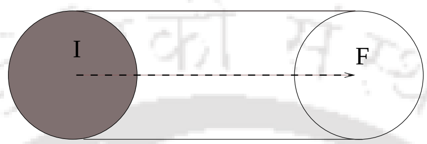


Figure 6.1: The area covered by a mobile sensor in time t shown in the white area. The colored disk is the intial position of the sensing disk

πr^2, which is shown by shaded disk in Fig. 6.1. The remanning area $2rvt$ is covered due to movement of the mobile sensor along the line IF, which is shown by white area in the same figure. So, the total area covered by the mobile sensor in time t is $2rvt$. If N_{opt} is the number of mobile sensors in the optimal solution to sweep cover $\mathcal{A}$, then $2rvt \times N_{opt} \geq Area(\mathcal{A})$, implies $N_{opt} \geq \frac{Area(\mathcal{A})}{2rvt}$. □

Theorem 6.3.4. *The approximation ratio of the Algorithm 6 is $2\sqrt{2}$.*

Proof. Total number of points in the set P is $\frac{Area(\mathcal{A})}{2r^2}$. Now let us consider the graph with the set of points in P as vertices and the edges are the line joining the centers of the adjacent cells. Each edge of this graph is of length $\sqrt{2}r$. As each edge of this graph is $\sqrt{2}r$, the cost of the minimum spanning tree is $\leq \frac{Area(\mathcal{A})}{2r^2} \times \sqrt{2}r$. According to Algorithm 2, the total length of the tour of the mobile sensors is less than or equal to twice the length of the MST of G. Therefore, the length of the tour computed by Algorithm 2 is $\leq \frac{Area(\mathcal{A})}{2r^2} \times \sqrt{2}r \times 2$. The number of mobile sensors computed by Algorithm 6 is $\leq \frac{Area(\mathcal{A})}{2r^2} \times \sqrt{2}r \times \frac{2}{vt}$. Therefore the approximation ratio of Algorithm 6 is $\leq \frac{Area(\mathcal{A})}{2r^2} \times \sqrt{2}r \times \frac{2}{N_{opt} \times vt} \leq \frac{Area(\mathcal{A})}{2r^2} \times \sqrt{2}r \times \frac{2}{\frac{Area(\mathcal{A})}{2rvt} \times vt} = 2\sqrt{2}$. □

Theorem 6.3.5. *Time complexity of Algorithm 6 is $O(m^2 \log m)$, where $m = \frac{a^2}{2r^2}$, a is side of the square region and r is sensing radius of the mobile sensors.*

Proof. We partition the square area $a \times a$ into $\sqrt{2}r \times \sqrt{2}r$ grid cells. So, total number of grid cells is $\frac{a^2}{2r^2} = m$. Therefore, by Theorem 3.3.5, the time complexity of Algorithm 6 is $O(m^2 \log m)$, where $m = \frac{a^2}{2r^2}$. □

6.4 Improved Solution for Rectangular Region

In this section, we propose an improved solution of the problem for a rectangular region. Let $\mathcal{R}$ be a rectangular region with sides a and b such that the sides are divisible by $\sqrt{2}r$. Divide $\mathcal{R}$ into square grids of side $\sqrt{2}r$. Let $m = \frac{a}{\sqrt{2}r}$ and $n = \frac{b}{\sqrt{2}r}$. Let $\mathcal{P}$ be the set of center points of the grid cells. Let $G_{m,n}$ be the grid graph corresponding to the grid cells in P. Property of a grid graph $G_{m,n}$ is given below as mentioned by Skiena [56].

Property 1. *$G_{m,n}$ is Hamiltonian if either the number of rows or columns is even.*

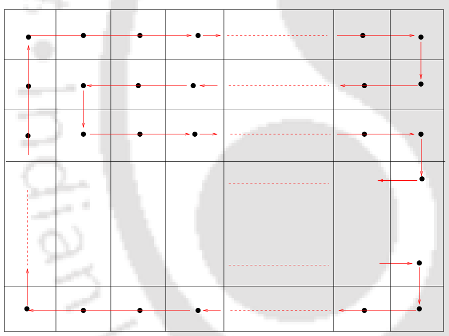


Figure 6.2: The movement path of the mobile sensors for m is even

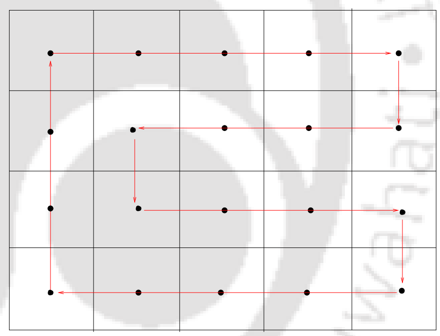


Figure 6.3: The movement path of the mobile sensors for $m = 4$

Based on the above property, we consider following two cases to find a tour of $G_{m,n}$:

Case 1 *Either m or n is even*: Without loss of generality assume m is even. According to Property 1, a hamiltonian cycle can be found in this case. We draw the hamiltonian cycle with length L (say), which is shown in Fig. 6.2. One particular example is also shown in Fig. 6.3 for $m = 4$. In this case $L = \sqrt{2}r \times mn$.

Case 2 *Both m and n are odd*: We draw a cycle of length L (say), as shown in Fig. 6.4, which is a hamiltonian cycle for the corresponding complete graph G of $G_{m,n}$.

One particular example of the cycle is also shown in Fig. 6.5. Note that the cycle contains only one edge which is not belongs to $G_{m,n}$. In this case $L = \sqrt{2}r(mn-1)+2r$.

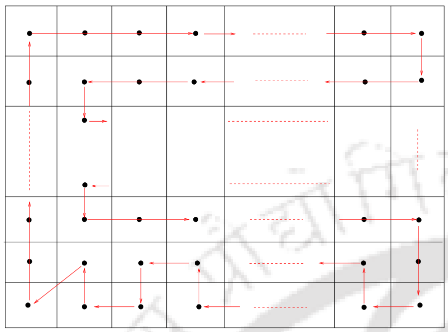


Figure 6.4: The movement path of the mobile sensors for odd m and n

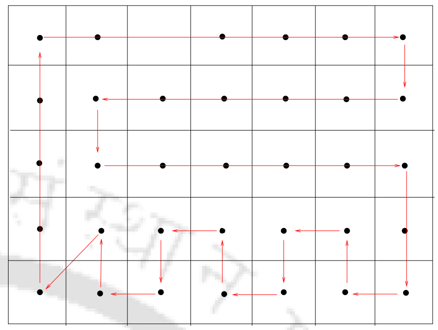


Figure 6.5: The movement path of the mobile sensors for $m = 5$ and $n = 7$

After finding the movement cycles in the both of the above cases, partition the cycles into $\lceil \frac{L}{vt} \rceil$ parts and deploy $\lceil \frac{L}{vt} \rceil$ mobile sensors at all partitioning points, one for each. Each mobile sensor starts moving at the same time along L in the same direction for sweep coverage of $\mathcal{R}$.

Theorem 6.4.1. *The approximation factor of area sweep coverage algorithm for a grid graph of even number of vertices is* $\sqrt{2}$ *and for odd number of vertices is* $\left(\sqrt{2}+\frac{2-\sqrt{2}}{mn}\right)$.

Proof. Since length of the tour for case 1 is $\sqrt{2}r \times mn$, the total number of mobile sensors required for sweep coverage of $\mathcal{R}$ is $\lceil \frac{mn\times\sqrt{2}r}{vt} \rceil$. The approximation factor for this case is $\frac{mn\times\sqrt{2}r}{vt} \Big/ \frac{2mnr^2}{2rvt} = \sqrt{2}$. Similarly, the approximation factor for case 2 is $\frac{(mn-1)\times\sqrt{2}r+2r}{vt} \Big/ \frac{2mnr^2}{2rvt}$ $= \sqrt{2}+\frac{2-\sqrt{2}}{mn}$. □

6.5 Solution for Arbitrary Bounded Region

In this section, we consider area sweep coverage problem for an arbitrary bounded region. Let $\mathcal{A}$ be an arbitrary bounded region. We divide the region $\mathcal{A}$ into square grids of side $\sqrt{2}r$. Since shape of the region is arbitrary, the grid cells, which are adjacent to the perimeter $\mathcal{P}$ of the region may not be complete squares. The maximum number of incomplete square grid cells is $\leq \frac{\mathcal{P}}{\sqrt{2}r}$, whereas $\frac{Area(\mathcal{A})}{2r^2}$ is the maximum number of square grid cells. Now we apply Algorithm 6 for sweep coverage of the region $\mathcal{A}$.

Theorem 6.5.1. *The approximation factor of the Algorithm 6 for an arbitrary bounded region is* $2\left(\sqrt{2}+\frac{2r\mathcal{P}}{Area(\mathcal{A})}\right)$.

Proof. The number of grid points in $\mathcal{A}$ is $\leq \frac{Area(\mathcal{A})}{2r^2}+\frac{\mathcal{P}}{\sqrt{2}r}$, Therefore the number of nodes required for sweep coverage of P is $\leq \left(\frac{Area(\mathcal{A})}{2r^2}+\frac{\mathcal{P}}{\sqrt{2}r}\right)\times\sqrt{2}r\times\frac{2}{vt}$. Hence the approximation factor of Algorithm 6 is
$\leq \left(\left(\frac{Area(\mathcal{A})}{2r^2}+\frac{\mathcal{P}}{\sqrt{2}r}\right)\times\sqrt{2}r\times\frac{2}{vt}\right)\Big/\left(\frac{Area(\mathcal{A})}{2rvt}\right)=2\left(\sqrt{2}+\frac{2r\mathcal{P}}{Area(\mathcal{A})}\right)$. $\square$

6.6 Simulation

As approximation factor of Algorithm 6 for arbitrary bounded region is $2\left(\sqrt{2}+\frac{2r\mathcal{P}}{Area(\mathcal{A})}\right)$, which is a function of area of the region, perimeter of the region and sensing radius. It is not easy to guess the value of the factor. For getting the idea about the value we simulate it for randomly generated simple polygonal area on Matlab platform. For example we illustrate two randomly generated simple polygons as shown in Fig. 6.6 and Fig. 6.7 with number of vertices 75 and 91, respectively.

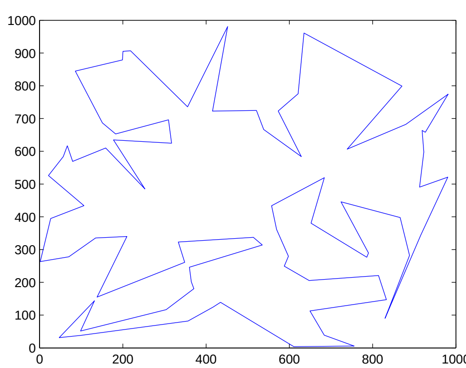


Figure 6.6: Randomly generated polygon with 75 vertices

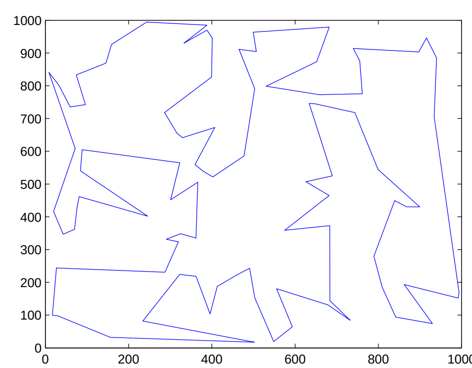


Figure 6.7: Randomly generated polygon with 91 vertices

Fig. 6.8 is showing average value of the approximation factor $2\left(\sqrt{2}+\frac{2r\mathcal{P}}{Area(\mathcal{A})}\right)$ with respect to number of vertices of the simple polygons. Average value of the approximation factor is computed by randomly generating 100 simple polygons for fixed number of

Number of vertices for polygons	20	40	60	80	100
Approximation factor	2.8753	2.8882	2.8981	2.9076	2.9154
Growth ($\mathcal{G}(i)$)	0.0005	0.0008	0.0004	0.0014	0.0014

Table 6.1: Showing value of the approximation factor and corresponding growth for different number of polygonal vertices

vertices. The polygons are generated inside a square region of size $1000m \times 1000m$. The radius of the sensing disks is taken as $1m$. As shown in the Table 6.1, the approximation factor is increasing from 2.87 to 2.91 for the number of vertices ranging from 10 to 100. To observe growth of the approximation factor we define it as: $\mathcal{G}(i) = |AF(i) - AF(i-1)|$, where $AF(i)$ is the value of the approximation factor for a region bounded by a simple polygon with i vertices, *i.e.*, $AF(i) = 2\left(\sqrt{2} + \frac{2r\mathcal{P}(i)}{Area(\mathcal{A}(i))}\right)$, $\mathcal{P}(i)$ is the perimeter of a polygon with i vertices and $\mathcal{A}(i)$ is the corresponding region bounded by the polygon. The value of the growth is very small as shown in the Table 6.1.

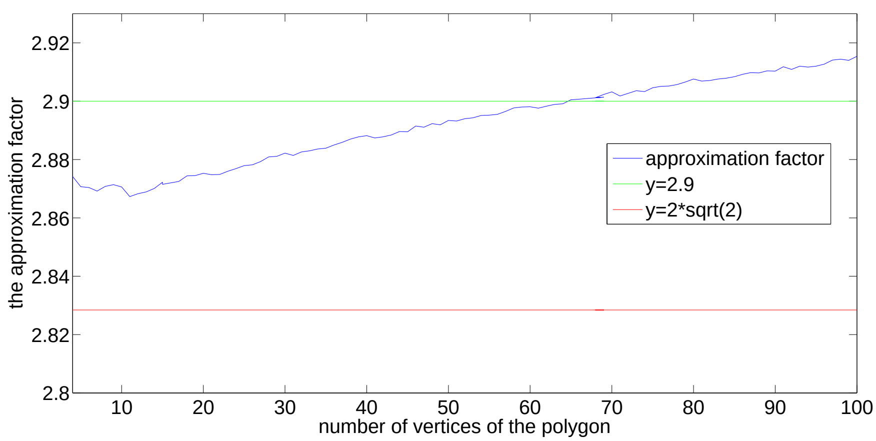


Figure 6.8: Number of vertices of the polygon vs approximation factor

Fig. 6.8 shows that the approximation factor is increasing very slowly with respect to increasing number of vertices of the polygons. As shown in Fig. 6.9, the growth is almost fixed, which is approximately a straight line with respect to increasing number of vertices. The reason why the approximation factor is increasing in our simulation is that we are increasing the number of vertices of the polygon by generating more vertices inside a fixed square region of size $1000m \times 1000m$. Therefore, the perimeter of the polygon increases as number of edges increase, but the area of the polygon does not

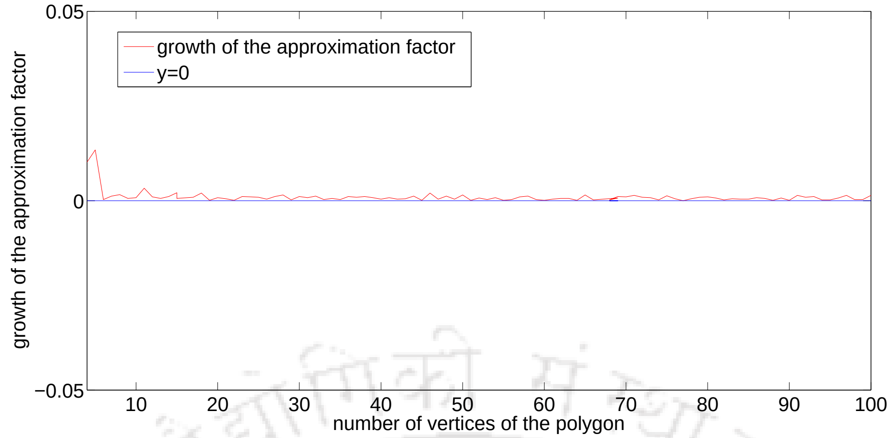


Figure 6.9: Graph for number of vertices of the polygon vs growth of the approximation factor

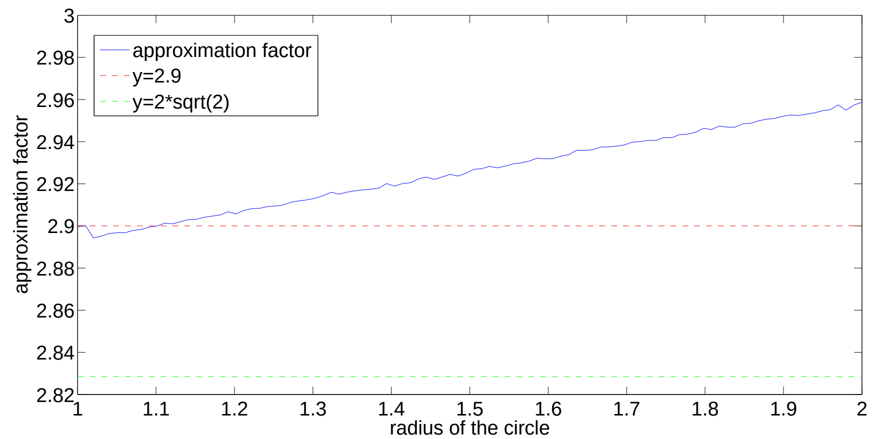


Figure 6.10: Graph for the approximation factor vs sensing radius

increase in that proportion. The nature of the approximation factor is also investigated with respect to the sensing radius of the sensors. Fig. 6.10 shows that the approximation factor is increasing very slowly with respect to increasing sensing radius for fixed number of vertices of the polygons.

6.7 Conclusion

In this chapter, we have introduced area sweep coverage problem and we have proved that the problem is NP-hard. A $2\sqrt{2}-$approximation algorithm is proposed for a square region. The approximation factor is improved for rectangular region. We have solved sweep coverage for arbitrary bounded regions, where the approximation factor of the solution is a function of perimeter, area of the region and sensing radius. With simulation, we have investigated how the approximation factor varies for arbitrary region, bounded by simple polygon.

Chapter 7

Barrier Sweep Coverage

7.1 Introduction

Periodic monitoring is important to protect our national borders from unauthorized immigration, terrorist activities, illegal movement of weapons, drugs, etc. It is also important to detect spread of pollutants, chemicals, etc, in an industrial township. In most of the previous works on barrier coverage [12, 36, 42, 49, 65], static sensors are deployed for continuous monitoring of the borders or boundaries. But for the aforementioned types of applications, deployment of static sensors at the borders are not cost effective in terms of resource utilization. Coverage requirement for those applications is time-variant. An efficient solution can be provided by utilizing less number of mobile sensors with appropriate movement strategy. The cost involvement aspect of this solution is mobility and storage capacity of the mobile sensors. In this chapter we study barrier sweep coverage with mobile sensors where a barrier is considered as a finite length continuous curve in a plane. The coverage at every point on a curve is time-variant. Throughout this chapter, by *finite curve* we mean a continuous curve of finite length.

7.1.1 Contribution

Our contributions in this chapter are as follows:

- We introduce barrier sweep coverage concept for covering finite curves in a plane. The problem is solved optimally with minimum number of mobile sensors.

- An energy restricted barrier sweep coverage problem is defined and a $\frac{13}{3}$-approximation algorithm is proposed to solve it for a finite curve.

- Barrier sweep coverage problem for multiple finite curves (BSCMC) is also proposed and proved that the problem is NP-hard and cannot be approximated within a factor of 2, unless P=NP. A 2-approximation algorithm is proposed to solve the problem for a special case. A 5-approximation algorithm is proposed to solve BSCMC.

- We formulate a data gathering problem, where a set of data mules is used for gathering data and prove that the problem is NP-hard. A 3-approximation algorithm is proposed to solve it.

- Through simulation, we investigate performance of the proposed algorithms for multiple finite curves.

7.2 Problem Definitions

Let $\mathcal{C}$ be a finite curve on a two dimensional plane. $\mathcal{C}$ is said to be *covered* by a set of sensors if and only if each point on $\mathcal{C}$ is covered by at least one sensor. Based on this coverage metric, we give the definitions of barrier sweep coverage as follows.

Definition 7.2.1. (**t-barrier sweep coverage**) Let $\mathcal{C}$ be any finite curve on a two dimensional plane and M={m_1,m_2,$\cdots$, m_p} be a set of mobile sensors. $\mathcal{C}$ is said to be t-barrier sweep covered if and only if each point of $\mathcal{C}$ is visited by at least one mobile sensor in every time period t.

Problem 8. (**Barrier sweep coverage problem for single finite curve**) Let $\mathcal{C}$ be a finite curve and M={m_1,m_2,$\cdots$, m_p} be a set of mobile sensors. Let v be the uniform speed of the mobile sensors. For a given $t > 0$, find the minimum number of mobile sensors such that $\mathcal{C}$ is t-barrier sweep covered.

Problem 9. (**Barrier sweep coverage problem for multiple finite curves** (BSCMC)) Let $\mathcal{X} = \{\mathcal{C}_1, \mathcal{C}_2, \cdots, \mathcal{C}_n\}$ be a set of finite curves and M={m_1,m_2,$\cdots$, m_p} be a set of mobile sensors. Let v be the uniform speed of the mobile sensors. For a given $t > 0$, find the minimum number of mobile sensors such that each $\mathcal{C}_i$ for $i = 1, 2, \cdots, n$ is t-barrier sweep covered.

In general sensors are equipped with limited battery power. In order to continue sweep coverage for long time, each mobile sensor must visits an energy source to recharge or replace its battery. Let Γ be the maximum travel time for a mobile sensor starting with full powered battery maintaining uniform speed v till battery power goes off. Let e be an energy source in the plane and every mobile sensor can recharge or replace its battery by visiting e. We define energy restricted barrier sweep coverage problem for a finite curve $\mathcal{C}$ as given below.

Problem 10. (**Energy restricted barrier sweep coverage problem**) Let $\mathcal{C}$ be a finite curve and M=$\{m_1$,m_2,$\cdots$, $m_p\}$ be a set of mobile sensors. Let v be the uniform speed of the mobile sensors. Let e be an energy source in the plane. For given $t, \Gamma > 0$, find the minimum number of mobile sensors such that $\mathcal{C}$ is t-barrier sweep covered and each mobile sensor visits e once in every Γ time period.

7.3 Barrier Sweep Coverage for a finite Curve

In this section, first we propose a solution for finding the optimal number of mobile sensors to sweep cover a finite curve. Later, we give an approximate solution for the energy restricted barrier sweep coverage problem for a finite curve.

7.3.1 Optimal solution for a finite curve

The barrier sweep coverage problem for a finite curve can be optimally solved using the following strategy. Let $|\mathcal{C}|$ be the length of the finite curve $\mathcal{C}$. If $\mathcal{C}$ is a closed curve, then partition $\mathcal{C}$ into $\left\lceil \frac{|\mathcal{C}|}{vt} \right\rceil$ equal parts of length vt and deploy $\left\lceil \frac{|\mathcal{C}|}{vt} \right\rceil$ mobile sensors at the partitioning points, one for each. Then all mobile sensors start moving at same time along $\mathcal{C}$ in the same direction to ensure t-barrier sweep coverage of $\mathcal{C}$. If $\mathcal{C}$ is an open curve, then join the end points of $\mathcal{C}$ to make it close and apply the strategy for closed curve.

7.3.2 Energy restricted barrier sweep coverage

In this section, we propose an algorithm for the energy restricted barrier sweep coverage problem. The approximation factor of the proposed algorithm is $\frac{13}{3}$ though it is not known whether the problem is NP-hard or not. Let e be the energy source in the plane. To make the problem feasible, we assume that the distance of any point on $\mathcal{C}$ from e is less than $\frac{v\Gamma}{2}$.

Definition 7.3.1. (e**-tour**) A tour, denoted by $\{e, p, q, e\}$ is called an e-tour if it starts from e, visits $arc(pq)$ from p to q along $\mathcal{C}$ and then returns to e such that total length of the tour is at most $v\Gamma$, where p and q are two points on $\mathcal{C}$.

For example $\{e, i_1, i_2, e\}$ is an e-tour as shown in Fig. 7.1, provided the length of the tour is at most $v\Gamma$.

The objective of our technique is to find a tour through e and $\mathcal{C}$, which is concatenation of multiple number of e-tours. Let $d(a, b)$ be the Euclidean distance between two points a and b. Let $d_c(p, q)$ be the distance between two points p and q along $\mathcal{C}$ in the clockwise direction, where p and q are two points on $\mathcal{C}$. So, $d_c(p, q)$ is equal to the length of the $arc(pq)$.

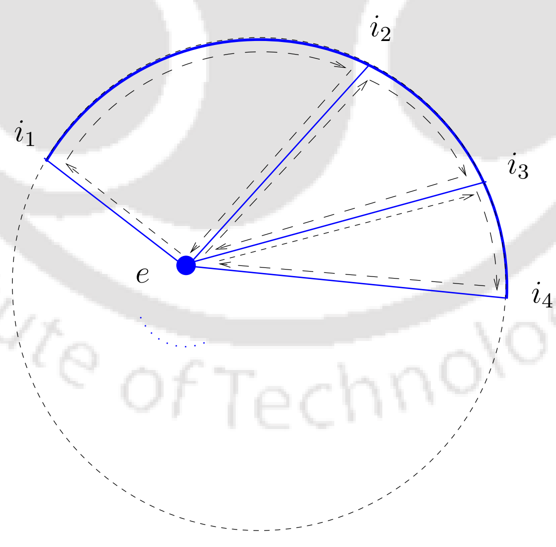


Figure 7.1: Showing selection of e-tours $\{e, i_1, i_2, e\}$, $\{e, i_2, i_3, e\}$, $\{e, i_3, i_4, e\}$, $\cdots$

First we consider $\mathcal{C}$ is a finite 'closed' curve and solve Problem 10 for it. Later we describe how to solve the problem for an 'open' curve. We choose any point i_1 on

$\mathcal{C}$. Find a point i_2 on $\mathcal{C}$ in the clockwise direction from i_1 (Ref. Fig. 7.1) such that

Algorithm 7 ENERGYRESTRICTEDBSC

Choose any point i_1 on $\mathcal{C}$.
$\mathcal{C}' = \mathcal{C}$, $n' = 1$.
while $\mathcal{C}' \neq \phi$ **do**
  if $d(e, i_{n'}) + |\mathcal{C}'| \leq \frac{v\Gamma}{2}$ **then**
    $h = i_1$.
  else
    Select a point h on $\mathcal{C}$ in a clockwise direction from $i_{n'}$ such that $d(i_{n'}, h) = \frac{v\Gamma}{2} - d(e, i_{n'})$.
  end if
  if $n' \neq 1$ and $d(e, i_{n'-1}) + d_c(i_{n'-1}, h) + d(e, h) \leq v\Gamma$ **then**
    $\mathcal{C}' = \mathcal{C}' \setminus arc(i_{n'}h)$.
    $i_{n'} = h$, $\mathcal{T}_{n'-1} = \{e, i_{n'-1}, i_{n'}, e\}$.
  else
    $i_{n'+1} = h$, $\mathcal{T}_{n'} = \{e, i_{n'}, i_{n'+1}, e\}$.
    $\mathcal{C}' = \mathcal{C}' \setminus arc(i_{n'}i_{n'+1})$.
    $n' = n' + 1$.
  end if
  $\mathcal{C}' = \mathcal{C}' \setminus arc(i_{n'}i_{n'+1})$.
end while
$APPRX = \mathcal{T}_1 \cdot \mathcal{T}_2 \cdot \mathcal{T}_2 \cdot \cdots \cdot \mathcal{T}_{n'}$, where '$\cdot$' is denoted as concatenation operation.
Divide $APPRX$ into equal parts of length vt and deploy one mobile sensor at each of the partitioning points.
All mobile sensor start moving along $APPRX$ at same time in same direction.

$d_c(i_1, i_2) = \frac{v\Gamma}{2} - d(e, i_1)$. Here $\mathcal{T}_1 = \{e, i_1, i_2, e\}$ is an e-tour, since length of $\mathcal{T}_1$ is equal to $d(e, i_1) + d_c(i_1, i_2) + d(i_2, e) \leq d(e, i_1) + \frac{v\Gamma}{2} - d(e, i_1) + \frac{v\Gamma}{2} = v\Gamma$, as shown in the Fig. 7.1. Next e-tour is selected as $\{e, i_2, i_3, e\}$, where i_3 is a point on $\mathcal{C}$ in the clockwise direction from i_2 such that $d_c(i_2, i_3) = \frac{v\Gamma}{2} - d(e, i_2)$. Once the second tour is selected, we check whether the combination of the previous tour and current tour together, i.e., $\{e, i_1, i_3, e\}$ forms an e-tour or not. If the combined tour $\{e, i_1, i_3, e\}$ is a valid e-tour, then the previous tour $\{e, i_1, i_2, e\}$ is updated into $\{e, i_1, i_3, e\}$ and we proceed to select the next e-tour. This updating process continues until combined tour violates the length constraint of e-tour. In general, after computing an e-tour $\mathcal{T}_j = \{e, i_j, i_{j+1}, e\}$, we select next point i_{j+2} on $\mathcal{C}$ such that $d_c(i_{j+1}, i_{j+2}) = \frac{v\Gamma}{2} - d(e, i_{j+1})$. Then if $\{e, i_j, i_{j+2}, e\}$ this is a valid e-tour, we update the tour $\mathcal{T}_j$ as $\mathcal{T}_j = \{e, i_j, i_{j+2}, e\}$. Otherwise the tour $\mathcal{T}_{j+1}$

is selected as $\mathcal{T}_{j+1} = \{e, i_{j+1}, i_{j+2}, e\}$. Finally $\mathcal{C}$ is decomposed into multiple number of e-tours such that every point on $\mathcal{C}$ is included in some e-tour.

If $\mathcal{C}$ is an 'open' curve, we use the above technique to decompose $\mathcal{C}$ into multiple number of e-tours considering one end point of $\mathcal{C}$ as i_1 and continue till the other end point. Note that according to the construction of the e-tours, length of the combined tour of any two consecutive e-tours is always greater than $v\Gamma$.

Let $APPRX$ be the total tour after concatenation of the e-tours, $\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \cdots$ one after another in order of obtained by the above technique. For example $\mathcal{T}_1 = \{e, i_1, i_2, e\}$, $\mathcal{T}_2 = \{e, i_2, i_3, e\}$, $\mathcal{T}_3 = \{e, i_3, i_4, e\}, \cdots$ are e-tours as shown in the Fig. 7.1, where concatenation of those tours is $\mathcal{T}_1 \cdot \mathcal{T}_2 \cdot \mathcal{T}_3 \cdots = \{e, i_1, i_2, e\} \cdot \{e, i_2, i_3, e\} \cdot \{e, i_3, i_4, e\} \cdots$ $= \{e, i_1, i_2, e, i_2, i_3, e, i_3, i_4, e, \cdots\}$, where '$\cdot$' is denoted as the concatenation operation of the e-tours. Let $|APPRX|$ be the length of $APPRX$. Divide $APPRX$ into equal parts of length vt and deploy one mobile sensor at each of the partitioning points. The mobile sensors then start moving along $APPRX$ in the same direction to ensure sweep coverage of $\mathcal{C}$. The Algorithm 7 (EnergyRestrictedBSC) for energy restricted barrier sweep coverage is proposed for a closed curve. But as explained above it is also applicable for an open curve.

Theorem 7.3.2. *According to Algorithm 7, each mobile sensor visits e in every Γ time period and each point on $\mathcal{C}$ is visited by at least one mobile sensor in every t time period.*

Proof. Mobile sensors move along the tour $APPRX$ according to movement strategy of Algorithm 7. As length of each e-tour is less than or equals to $v\Gamma$, the mobile sensors visit e after traveling at most $v\Gamma$ distance since its last visit of e. Therefore, each mobile sensor visits e once in every Γ time period. Again, the mobile sensors are deployed at every partitioning points of $APPRX$ and two consecutive partitioning points are within vt distance apart. The relative distance between any two consecutive mobile sensors is at most vt at any time as they are moving in same speed v and same direction. So, when any point p on $\mathcal{C}$ is visited by a mobile sensor at t_0, another mobile sensor was on the way to p and within the distance of vt along $APPRX$. Hence p will be again visited by another mobile sensor within next t time. □

To analyze the approximation factor of Algorithm 7, we consider some special points on $\mathcal{C}$. Let i_p^1, i_p^2 be two special points on the $arc(i_p i_{p+1})$ of $\mathcal{C}$ such that the $arc(i_p i_{p+1})$ is partitioned into three equal parts, i.e., the length of $arc(i_p i_p^1)$ equal to the length of $arc(i_p^1 i_p^2)$ equal to the length of $arc(i_p^2 i_{p+1})$. We define a set of points, $\mathcal{I} = \{i_j, i_j^1, i_j^2 | j = 1 \text{ to } n'\}$. Following two lemmas give an upper bound of the length of the tour $APPRX$ and a lower bound of the length of the optimal tour respectively.

Lemma 7.3.3. $|APPRX| \leq \frac{1}{3}\left(2\sum_j \left(d(e,i_j) + d(e,i_j^1) + d(e,i_j^2)\right) + 5|\mathcal{C}|\right)$.

Proof. According to Algorithm 7, total length of the tour $APPRX$ is

$$\begin{aligned} |APPRX| &= d(e,i_1) + d_c(i_1,i_2) + d(i_2,e) + d(e,i_2) + d_c(i_2,i_3) \\ &\quad + d(i_3,e) + \cdots + d(e,i_1) \\ &= |\mathcal{C}| + 2\sum_j d(e,i_j). \end{aligned} \tag{7.3.1}$$

Now by triangle inequality,

$$\begin{aligned} d(e,i_j) &\leq d(e,i_j^1) + d_c(i_j,i_j^1) \text{ and} \\ d(e,i_j) &\leq d(e,i_j^2) + d_c(i_j,i_j^2). \end{aligned}$$

Therefore,

$$|APPRX| \leq |\mathcal{C}| + 2\sum_j \left(d(e,i_j^1) + d_c(i_j,i_j^1)\right). \tag{7.3.2}$$

$$|APPRX| \leq |\mathcal{C}| + 2\sum_j \left(d(e,i_j^2) + d_c(i_j,i_j^2)\right). \tag{7.3.3}$$

From the above Equation 7.3.1, 7.3.2 and 7.3.3, we can write,

$$\begin{aligned} 3|APPRX| &\leq 3|\mathcal{C}| + 2\sum_j d(e,i_j) + 2\sum_j \left(d(e,i_j^1) + d_c(i_j,i_j^1)\right) \\ &\quad + 2\sum_j \left(d(e,i_j^2) + d_c(i_j,i_j^2)\right). \end{aligned} \tag{7.3.4}$$

As the points i_j^1 and i_j^2 divide length of the $arc(i_j i_{j+1})$ into three equal parts, therefore, $\sum_j d_c(i_j, i_j^1) = \frac{|\mathcal{C}|}{3}$ and $\sum_j d_c(i_j, i_j^2) = \frac{2|\mathcal{C}|}{3}$.

Using these two results, Equation 7.3.4 can be written as:

$$\begin{aligned}
3|APPRX| &\leq 3|\mathcal{C}| + 2\sum_j \left(d(e, i_j) + d(e, i_j^1) + d(e, i_j^2)\right) + 2\frac{|\mathcal{C}|}{3} + \frac{4|\mathcal{C}|}{3} \\
|APPRX| &\leq \frac{1}{3}\left(2\sum_j \left(d(e, i_j) + d(e, i_j^1) + d(e, i_j^2)\right) + 5|\mathcal{C}|\right).
\end{aligned}$$

$\square$

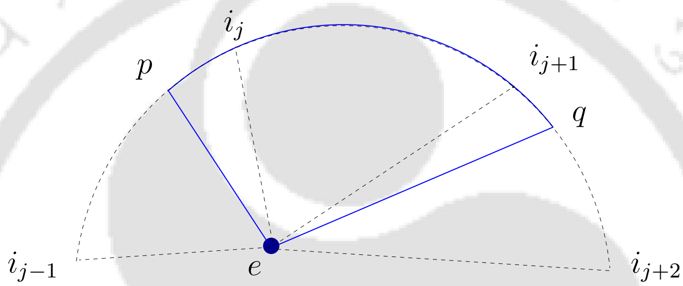


Figure 7.2: Showing one e-tour $OPT_l = \{e, p, q, e\}$ of the optimal tour OPT

Lemma 7.3.4. *Let OPT be the optimal tour of the mobile sensors and $|OPT|$ be length of OPT, then* $|OPT| \geq \max\left\{\frac{1}{4}\sum_j \left(d(e, i_j) + d(e, i_j^1) + d(e, i_j^2)\right), |\mathcal{C}|\right\}$.

Proof. Let $OPT_l = \{e, p, q, e\}$ be an e-tour of the optimal tour OPT (Ref. Fig. 7.2). We claim that at most one $arc(i_j i_{j+1})$, part of an e-tour computed using Algorithm 7 for some j, can be completely contained in $arc(pq)$, where $arc(pq)$ is a part of OPT_l. To prove this, let us assume that there are two such arcs $arc(i_{j-1} i_j)$ and $arc(i_j i_{j+1})$ completely contained in $arc(p, q)$. As length of combined tour of two consecutive e-tours is always greater than $v\Gamma$ according to step 12 of Algorithm 7, therefore,

$$(d(e, i_{j-1}) + d_c(i_{j-1}, i_{j+1}) + d(e, i_{j+1})) > v\Gamma. \tag{7.3.5}$$

Now,

$$\begin{aligned}|OPT_l| &= d(e,p) + d_c(p, i_{j-1}) + d_c(i_{j-1}, i_j) + d_c(i_j, i_{j+1}) + d_c(i_{j+1}, q) + d(e,q)\\ &\geq d(e, i_{j-1}) + d_c(i_{j-1}, i_{j+1}) + d(e, i_{j+1})\\ &> v\Gamma \text{ (from Equation 7.3.5).}\end{aligned}$$

This contradicts the fact that OPT_l is an e-tour. Therefore, at most one $arc(i_j i_{j+1})$ for some j, can be completely contained in $arc(pq)$. Hence, at most one complete $arc(i_j i_{j+1})$ and two $arc(i_{j-1} i_j)$ and $arc(i_{j+1} i_{j+2})$ partially contained in $arc(pq)$ as shown in Fig. 7.2. Therefore, maximum eight points from the set $\mathcal{I}$ may belong to the $arc(pq)$, since there are four points i_j, i_j^1, i_j^2, i_{j+1} for the $arc(i_j i_{j+1})$ and at most four special points, i_{j-1}^1, i_{j-1}^2 and i_{j+1}^1, i_{j+1}^2 for the $arc(i_{j-1} i_j)$ and $arc(i_{j+1} i_{j+2})$ respectively.

Now, for any point x on $arc(pq)$ of OPT_l implies $|OPT_l| \geq 2d(e,x)$. As there are at most eight points of $\mathcal{I}$ in $arc(pq)$, which implies

$$|OPT_l| \geq \frac{2\sum_{x \,\in\, \mathcal{I} \cap arc(pq)} d(e,x)}{8}. \tag{7.3.6}$$

Since all the points in $\mathcal{I}$ are on $arc(pq)$ for some OPT_l, where OPT_l is a part of OPT, therefore,

$|OPT| \geq \sum_l |OPT_l| \geq \dfrac{2\sum_j \left(d(e,i_j) + d(e,i_j^1) + d(e,i_j^2)\right)}{8}$. Also, $|OPT| \geq |\mathcal{C}|$.

Hence, $|OPT| \geq \max\left\{\dfrac{1}{4}\sum_j \left(d(e,i_j) + d(e,i_j^1) + d(e,i_j^2)\right), |\mathcal{C}|\right\}$. □

Theorem 7.3.5. *The approximation factor of Algorithm 7 is* $\dfrac{13}{3}$.

Proof. Let N be the number of mobile sensors needed in our solution and N_{opt} be the number of mobile sensors in the optimal solution.

Then $N = \left\lceil \dfrac{|APPRX|}{vt} \right\rceil$ and $N_{opt} \geq \dfrac{|OPT|}{vt}$. From Lemma 7.3.3 and Lemma 7.3.4, we have

$\dfrac{N}{N_{opt}} \leq \dfrac{|APPRX|}{|OPT|} \leq \dfrac{8}{3} + \dfrac{5}{3} = \dfrac{13}{3}$. Hence the approximation factor of our proposed Algorithm 7 is $\dfrac{13}{3}$. □

7.4 Barrier Sweep Coverage for Multiple finite Curves

Hardness of the barrier sweep coverage problem for multiple finite curves (BSCMC) is established with the following theorem.

Theorem 7.4.1. *BSCMC is NP-hard and cannot be approximated within a factor of* 2*, unless P=NP.*

Proof. Finding minimum number of mobile sensors with uniform speed to guarantee sweep coverage for a set of points in two dimensional plane is NP-hard and it cannot be approximated within a factor of 2, unless P=NP, as proved in the paper [41] by Li et al. The sweep coverage problem proposed in [41] is a special case of BSCMC when all curves are points. Therefore, BSCMC is NP-hard and cannot be approximated within a factor of 2, unless P=NP. □

7.4.1 2-approximation solution for a special case

In this section, we propose a solution for a special case of BSCMC where each mobile sensor must visits each point of all the curves. Initially, we propose an algorithm where each curve is a line segment. Later we explain how the same idea works for any set of finite curves. Let $\mathcal{L} = \{l_1, l_2, \cdots, l_n\}$ be a set of line segments on a two dimensional plane.

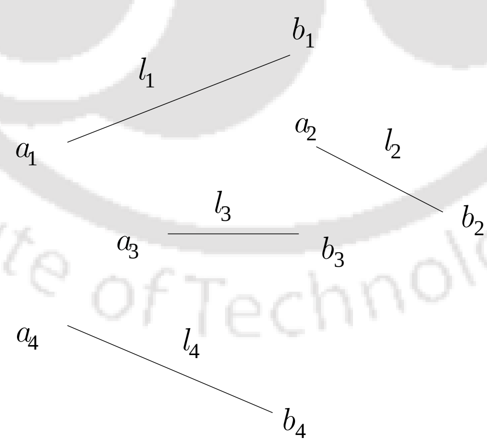


Figure 7.3: Set of line segments $\mathcal{L}$

Let S be the set of shortest distance line s_{ij} between every pair of line segments (l_i, l_j) for $i \neq j$. We define a complete weighted graph $G = (V, E)$, where $V = \{v_1, v_2, \cdots, v_n\}$

is the set of vertices. The vertex v_i represents line segment l_i for $i = 1$ to n. $E = V \times V$ is the set of edges, where the edge (v_i, v_j) represents $s_{ij} \in S$. For any subgraph H of G, we denote $|H|$ be total sum of lengths of the edges of H. Let T be a minimum spanning tree (MST) of G. T can be represented as $T_{\mathcal{L}}$, where $T_{\mathcal{L}} = \mathcal{L} \cup \{s_{ij} : (v_i, v_j) \in T\}$. An illustration is shown from Fig. 7.3 to Fig. 7.6, where a set of line segments is shown in Fig. 7.3, corresponding complete graph G is shown in Fig. 7.4, an MST T of G is shown in Fig. 7.5 and the representation $T_{\mathcal{L}}$ of T is shown in Fig. 7.6.

We construct a graph $G_{\mathcal{L}}$ from $T_{\mathcal{L}}$ by introducing vertices at the end points of each line segment in $T_{\mathcal{L}}$, which may split each l_i into several smaller line segments. According to Fig. 7.7, vertices of $G_{\mathcal{L}}$ are $\{a_1, p, b_1,\ a_2, q, b_2,\ a_3, b_3,\ a_4, r, b_4\}$. The vertex p splits line segment (a_1, b_1) into two smaller line segments (a_1, p) and (p, b_1). Similarly, vertices q and r split (a_2, b_2) and (a_4, b_4) into (a_2, q), (q, b_2) and (a_4, r), (r, b_4) respectively, whereas the line segment (a_3, b_3) remains same. Each of these line segments and the lines corresponding to the edges of T together are the edges of the graph $G_{\mathcal{L}}$. According to Fig. 7.7, edges of $G_{\mathcal{L}}$ are $\{(a_1, p)$, (p, b_1), (a_2, q), (q, b_2), (a_3, b_3), (a_4, r), (r, b_4), (p, a_3), (a_3, r), $(b_3, q)\}$.

The graph $G_{\mathcal{L}}$ is a tree and sum of the edge weights of the graph is $|G_{\mathcal{L}}| = |T| + \sum_{i=1}^{n} l_i$, where $|T|$ is sum of the edge weights of T. Following Algorithm 8 (BarrierSweepCoverage) computes a tour on $G_{\mathcal{L}}$ and finds number of mobile sensors and their movement paths.

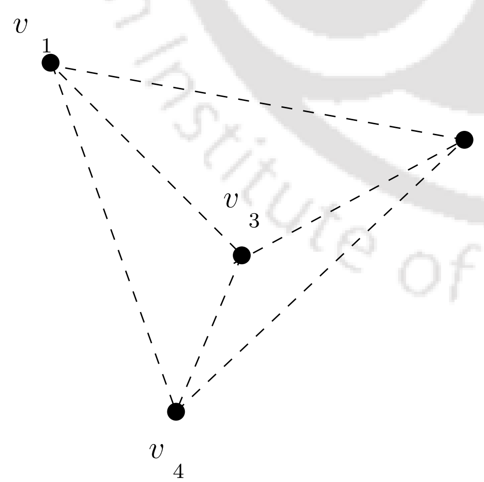


Figure 7.4: Complete graph G

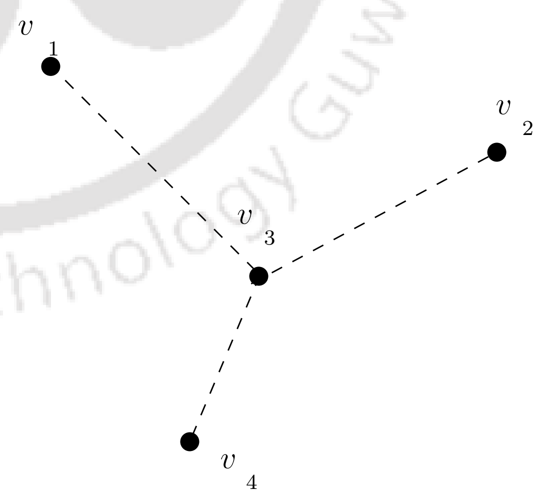


Figure 7.5: MST T of G

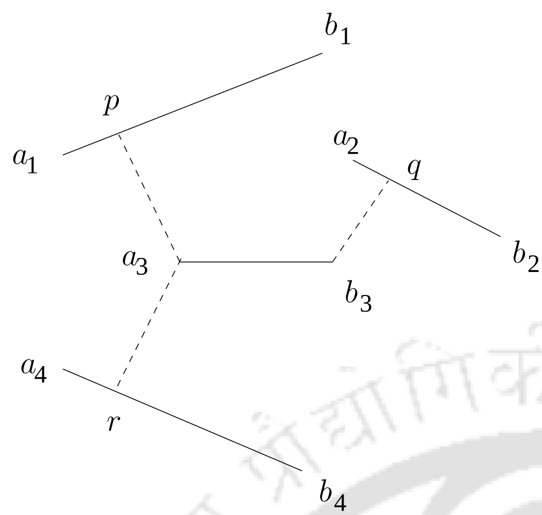


Figure 7.6: $T_{\mathcal{L}}$

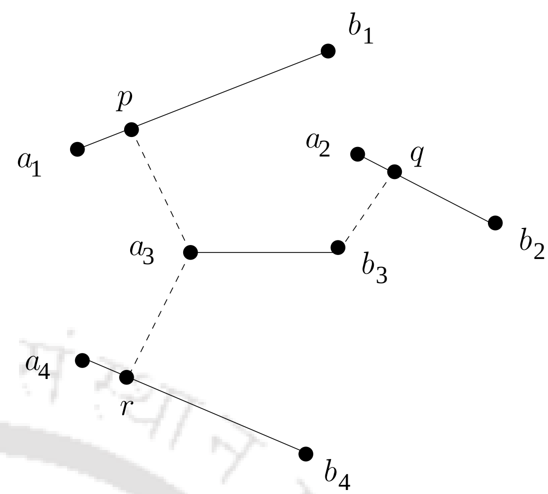


Figure 7.7: $G_{\mathcal{L}}$

Lemma 7.4.2. *According to Algorithm 8 each point on l_i can be visited by at least one mobile sensor in every time period t for $i = 1, 2, \cdots n$.*

Proof. Let us consider any point p on a line segment l_i and let t' be the time when a mobile sensor visited p last time. Now we have to prove that the point p must be visited by at least one mobile sensor in $t' + t$ time. According the deployment strategy of mobile sensors any two consecutive mobile sensors are within the distance of vt at any time. So, when a mobile sensor visited p at t' another mobile sensor is on the way to p and within the distance of vt along $\mathcal{E}$. Hence p will be again visited by another mobile sensor within next t time. $\square$

Algorithm 8 BarrierSweepCoverage

1: Construct complete weighted graph G from the given set of line segments $\mathcal{L}$.
2: Find an MST T of G.
3: Construct $G_{\mathcal{L}}$.
4: Find Eulerian graph after doubling each edge of $G_{\mathcal{L}}$.
5: Find an Eulerian tour $\mathcal{E}$ on the Eulerian graph. Let $|\mathcal{E}|$ be the length of $\mathcal{E}$.
6: Partition $\mathcal{E}$ into $\left\lceil \frac{|\mathcal{E}|}{vt} \right\rceil$ parts and deploy $\left\lceil \frac{|\mathcal{E}|}{vt} \right\rceil$ mobile sensors at all partitioning points, one for each.
7: Each mobile sensor then starts moving at the same time along $\mathcal{E}$ in same direction.

Lemma 7.4.3. *If L_{opt} is the length of the optimal TSP tour for visiting all points of every line segment in $\mathcal{L}$, then $|T| + \sum_{i=1}^{n} l_i \leq L_{opt}$.*

Proof. The optimal TSP tour L_{opt} contains two types of movement paths; movement paths along the line segments of $\mathcal{L}$ and movement paths between the line segments. Let total length of the movement paths along the line segments be L_{along}. Since L_{opt} is the optimal tour for visiting all points of each line segment $l_i \in \mathcal{L}$, therefore,

$$L_{along} \geq \sum_{i=1}^{n} l_i. \tag{7.4.1}$$

Let L_G be the optimal TSP tour on G. Then $|T| \leq L_G$. Let total length of the movement paths between the line segments be $L_{between}$. Since L_G is the optimal TSP tour on G and the weights of all edges of G are taken to be the shortest distance between respective line segments, therefore,

$$L_{between} \geq L_G. \tag{7.4.2}$$

Now, from Equation 7.4.1 and 7.4.2, $L_G + \sum_{i=1}^{n} l_i \leq L_{opt}$. Hence $|T| + \sum_{i=1}^{n} l_i \leq L_{opt}$. $\square$

Theorem 7.4.4. *The approximation factor of Algorithm 8 is 2.*

Proof. The total edge weights of $G_{\mathcal{L}}$ is $|T| + \sum_{i=1}^{n} l_i$. Now, $|\mathcal{E}| = 2(|T| + \sum_{i=1}^{n} l_i)$, since Eulerian tour $\mathcal{E}$ found by Algorithm 8 after doubling each edges of $G_{\mathcal{L}}$. By Lemma 7.4.3, $|\mathcal{E}| \leq 2L_{opt}$. Let N_{opt} be the number of mobile sensors required for optimal solution. Then $N_{opt} \times vt \geq L_{opt}$, *i.e.*, $N_{opt} \geq \left\lceil \frac{L_{opt}}{vt} \right\rceil$. The number of mobile sensors calculated by Algorithm 8 is $\left\lceil \frac{|\mathcal{E}|}{vt} \right\rceil$ (=N, say). Therefore, the approximation factor of Algorithm 8 is equal to $\frac{N}{N_{opt}} \leq \left\lceil \frac{2L_{opt}}{vt} \right\rceil \Big/ \left\lceil \frac{L_{opt}}{vt} \right\rceil \leq 2$. $\square$

7.4.2 Solution for BSCMC

In this section, we propose an algorithm for the BSCMC problem, where each curve is a line segment. The description of the algorithm is given below.

For $k = 1$ to n following calculations are performed. Compute a minimum spanning forest F_k of G with k components. Let $C^1, C^2, \cdots, C^k$ be the connected components of F_k. Let $C^i_{\mathcal{L}}$ be a representation of C^i on the set of line segments $\mathcal{L}$ for $i = 1$ to

k, where $C^i_{\mathcal{L}} = \{l_j | v_j \in C^i\} \cup \{s_{ij} | (v_i, v_j) \in C^i\}$. Construct graph $G^i_{\mathcal{L}}$ from $C^i_{\mathcal{L}}$ in the same way $G_{\mathcal{L}}$ is constructed from $T_{\mathcal{L}}$ in Section 7.4.1. Clearly, $\sum_{i=1}^{k} |G^i_{\mathcal{L}}| = \sum_{j=1}^{n} l_j + |F_k|$. Find Eulerian tours $\mathcal{E}^1_{\mathcal{L}}$, $\mathcal{E}^2_{\mathcal{L}}$, $\cdots$, $\mathcal{E}^k_{\mathcal{L}}$ after doubling the edges of $G^1_{\mathcal{L}}$, $G^2_{\mathcal{L}}$, $\cdots$, $G^k_{\mathcal{L}}$ respectively. Partition each $\mathcal{E}^k_{\mathcal{L}}$ into $\left\lceil \frac{|\mathcal{E}^k_{\mathcal{L}}|}{vt} \right\rceil$ parts of length vt. Let N_k be the total number of partitioning points. Choose the minimum over all N_k's as the number of mobile sensors. Deploy the number of mobile sensors, one at each of the partitioning points. Then all mobile sensors start their movement at the same time along their respective tours in the same direction.

Theorem 7.4.5. *The approximation factor of the proposed solution for BSCMC is 5.*

Proof. Let opt be the number of mobile sensor required for the optimal solution of BSCMC. Let opt' be the minimum number of mobile sensor which can guarantee t-sweep coverage of all the vertices of G. Then $opt \geq opt'$.

Since, all the points of each line segments are visited by the mobile sensors in any time period t then $opt \geq \frac{\sum_{i=1}^{n} l_i}{vt}$. Let us consider the movement paths of opt' number of mobile sensors to sweep cover the vertices of G in any time interval $[t_0, t + t_0]$. Let Min_path be total sum of lengths of the paths. Then $Min\_path \leq vt \times opt'$. Again these opt' movement paths form a spanning forest with opt' number of connected components of G. Therefore, we have $|F_{opt'}| \leq Min\_path$ as $F_{opt'}$ is the minimum spanning forest of G with opt' components. Let us consider the iteration of our solution for $k = opt'$. Total number of mobile sensors N in this iteration is given below.

$N = \sum_{i=1}^{k} \left\lceil \frac{|\mathcal{E}^i_{\mathcal{L}}|}{vt} \right\rceil \leq \sum_{i=1}^{k} \frac{|\mathcal{E}^i_{\mathcal{L}}|}{vt} + k = 2\sum_{i=1}^{k} \frac{|G^i_{\mathcal{L}}|}{vt} + k = 2\frac{\sum_{i=1}^{n} l_i}{vt} + 2\frac{|F_k|}{vt} + k \leq 2opt + 2\frac{Min\_path}{vt} + k \leq 2opt + 2opt' + opt' \leq 5opt.$

Therefore, the approximation factor of our proposed solution is 5. $\square$

The two algorithms proposed in this section also work for a set of finite curves as explained below. Let $\mathcal{X}$ be a set of finite curves and S be the set of shortest distance line between every pair of the curves. The complete weighted graph G can be constructed considering each curve as a vertex and the distance between every pair of curves as an edge weight corresponding to the edge. For the MST or any subtree of G, Eulerian tour on the tree or the subtree can be form in the same way by introducing vertices at the end points of each curve and doubling the edges. Mobile sensors also can be deployed after partitioning the tours at each of the partitioning points. The mobile sensors follow same movement strategy as earlier to guarantee sweep coverage of the multiple finite curves.

7.5 Data Gathering by Data Mules

In this section, we consider a data gathering problem by a set of data mules [2, 10, 37, 38, 53], which we formulate as a variation of barrier sweep coverage problem. A set of mobile sensors are moving along finite straight lines in a plane for monitoring or sampling data. Movement of the mobile sensors are arbitrary along their respective paths i.e., a mobile sensor moves in any direction along the straight line with arbitrary speed. A set of data mules are moving with uniform speed v in the same plane for collecting data from the mobile sensors. A data mule can collect data from a mobile sensor whenever it meets the mobile sensor on its path. We assume data transfer can be done instantaneously whenever a data mule meets a mobile sensor. In the data gathering applications, a data mule collects data from all the mobile sensors one by one visiting them and relay the collected data to the sink. The definition of the problem is given below.

Problem 11 (Minimum number of data mule for data gathering (MDMDG))**.** A set of mobile sensors are moving arbitrarily in a plane along line segments. Find minimum number of data mules with uniform speed v such that

1. each mobile sensor is visited by a data mule at least once in every t time period for collecting data,
2. each data mule visits all mobile sensors.

The sweep coverage problem [41] is a special instance of MDMDG when two end points of every line segment are same, *i.e.* the mobile sensors are behaving like a static sensor. Therefore we can state the following theorem.

Theorem 7.5.1. *MDMDG is NP-hard and cannot be approximated within a factor of 2, unless P=NP.*

The proof of the above theorem directly follows from Theorem 7.4.1. Following Lemma 7.5.2 shows that to visit the mobile sensors, each point of all paths must be visited by the data mules.

Lemma 7.5.2. *To solve MDMDG each and every point of all line segments must be visited by the set of data mules.*

Proof. We prove the lemma by the method of contradiction. Let l be a line segment for which all points of l are not visited by the data mules. Therefore, there exist one point p on l such that p is not visited by any data mule. One mobile sensor can stop its movement for sometime and which can be allowed for the arbitrary nature of its movements. Now, if the mobile sensor on l remains static at p for more than t time then it will not be visited by any data mule in a time period t, which is a contradiction. Hence, each and every point of all line segments must be visited by the set of data mules. □

Now, we find the minimum path traveled by a single data mule to visit all the mobile sensors. According to the problem definition and by Lemma 7.5.2, within any time interval $[t_0, t_0 + t]$, all points of each line segment are visited by the data mules. Therefore, total length of the tour traversed by all the data mules within the time interval is greater or equals to the optimal tour traversed by a single data mule. Following lemma gives the nature of the optimal tour traversed by a single data mule for visiting all the mobile sensors.

Lemma 7.5.3. *It may not be possible to visit a mobile sensor by a data mule unless the data mule visits the whole line segment from one end to the other end continuously.*

Proof. If a line segment is not visited 'continuously' then in the following scenario the data mule cannot visit a mobile sensor. Let l' and l'' be the two parts of a line segment l. There is a time gap between two continuous visits of l' and l'' by the data mule. It may happen that during the visit of l' the mobile sensor remains on l'' and during the visit of l'' the mobile sensor remains on l'. The mobile sensor changes its position during the time gap. So, in this scenario the data mule cannot visit mobile sensor. □

Let L_{opt} be the optimal tour for visiting all mobile sensors by one data mule. Now, the optimal tour L_{opt} contains two types of movement paths: the movement paths along the line segments and the paths between pair of line segments. According to Lemma 7.5.3, the paths between pair of line segments are the lines which connect end points of the pair of line segments. We construct Euclidian complete graph G_{2n} with $2n$ vertices $a_i, b_i, i = 1, 2, \cdots n$, where a_i and b_i are two end points of the line l_i. The edge set $E(G_{2n})$ of G_{2n} is given by $E(G_{2n}) = \{l_i : i = 1, 2, \cdots, n\} \cup \{(a_i, a_j) : i \neq j\} \cup \{(b_i, b_j) : i \neq j\} \cup \{(a_i, b_j) : i \neq j\} \cup \{(b_i, a_j) : i \neq j\}$. The weight of each edge is equal to the Euclidian distance between two vertices of the edge. Let T_{2n} be a MST of G_{2n} containing all edges $l_i \in E(T_{2n}), i = 1, 2, \cdots, n$. We compute T_{2n} using Kruskal's algorithm after including all edges $l_i, i = 1, 2, \cdots, n$ in the initial edge set of T_{2n}. Until the spanning tree is formed, we apply Kruskal's algorithm on the remaining edges, $E(G_{2n}) \backslash \{l_i : i = 1, 2, \cdots, n\}$ of G_{2n}. An Eulerian graph is formed from T_{2n} as described in Christofides algorithm [13]. We compute an Eulerian tour $\mathcal{E}_{2n}$ from the above Eulerian graph.

We cannot directly apply the movement strategy of the mobile sensors, which is used in Algorithm 7 and Algorithm 8 to give the movement strategy of the data mules. If there exist a line segment with length greater than vt and the mobile sensor moves with a speed v along the line in the same direction as the data mule moves then it may not be possible by the data mule to meet the mobile sensor within time t. Hence, we apply a new strategy as explained below in order solve the MDMDG problem.

Partition the tour $\mathcal{E}_{2n}$ into equal parts of length vt and consider two sets of data mules DM_1 and DM_2, each of which contains $\lceil \frac{\mathcal{E}_{2n}}{vt} \rceil$ number of data mules. Deploy two data mules at each of the partitioning points one from each set. Then each data mule from the set DM_1 moves in a clockwise direction, whereas other set of data mules DM_2 move in the counter clockwise direction. All data mules, irrespective of the sets start their movement at same time. Based on the above discussions, we propose following Algorithm 9 (MDMDG).

Algorithm 9 MDMDG

1: Use Kruskal algorithm to find an MST T_{2n} of G_{2n} with the initial set of edges containing all edges l_i for $i = 1, 2, \cdots, n$.
2: Construct an Eulerian graph from T_{2n} using Christofides algorithm [13].
3: Find an Eulerian tour $\mathcal{E}_{2n}$ from the Eulerian graph. Let $|\mathcal{E}_{2n}|$ be the length of $\mathcal{E}_{2n}$.
4: Partition $\mathcal{E}_{2n}$ into $\left\lceil \frac{|\mathcal{E}_{2n}|}{vt} \right\rceil$ parts of length vt. Deploy two data mules, one from DM_1 and other from DM_2 at each of the partitioning points.
5: All data mules start moving at the same time along $\mathcal{E}_{2n}$ such that the data mules from DM_1 move in clockwise direction and the data mules from DM_2 move in anticlockwise direction.

7.5.1 Analysis

Theorem 7.5.4. *Each mobile sensor is visited by a data mule at least once in every t time period according to Algorithm 9.*

Proof. Let position of a mobile sensor be p when it visited last time by a data mule at time t_0. According the deployment strategy of data mules, two data mules visit p again within time $t_0 + t$, from two different directions. The statement of the theorem follows if the mobile sensor remains static at p till $t_0 + t$. Now we consider the case when the mobile sensor moves clockwise direction from p after time t_0. In this case there exist a data mule, which is moving in counter clockwise direction that visits the mobile sensor within time $t_0 + t$. Similarly, if the mobile sensor moves counter clockwise direction from p after time t_0, then there exist a data mule, which is moving in clockwise direction that visits the mobile sensor within time $t_0 + t$. Hence, irrespective of the nature of movement, a mobile sensor is visited by a data mule at least once in every t time period. □

Theorem 7.5.5. *The approximation factor of Algorithm 9 is 3.*

Proof. According to the Christofides algorithm [13] we can write $|\mathcal{E}_{2n}| \leq \frac{3}{2} L_{opt}$. Let N_{opt} be the number of data mules required for optimal solution.
Then $N_{opt} \times vt \geq L_{opt}$, *i.e.*, $N_{opt} \geq \left\lceil \frac{L_{opt}}{vt} \right\rceil$. The number of data mule (N) calculated by the Algorithm 9 is $\left\lceil \frac{2|\mathcal{E}_{2n}|}{vt} \right\rceil$. Therefore, the approximation factor for Algorithm 9 is equal to $\frac{N}{N_{opt}} \leq \left\lceil \frac{2 \times \frac{3}{2} L_{opt}}{vt} \right\rceil \Big/ \left\lceil \frac{L_{opt}}{vt} \right\rceil \leq 3$. □

7.6 Simulation Results

To the best of our knowledge there is no related work on barrier sweep coverage problem in literature. We compare performance of our proposed Algorithm 8 and the algorithm for BSCMC through simulation. We implement both of the algorithms in C++ language. A set of line segments are randomly generated inside a square region of side 200 meter. The length of each line segment is randomly chosen within 5 meter. The uniform speed of each mobile sensor is taken as 1 meter per second.

No. of line segments	Number of Mobile Sensors	
	Algorithm for BSCMC	Algorithm 8
5	5	12
15	14	28
25	21	35
35	26	43
45	35	54
55	42	64
65	51	72
75	57	77
85	66	90
95	75	97
105	76	100
115	80	104
125	83	107
135	86	110

Table 7.1: Average number of mobile sensors to achieve sweep coverage varying with number of line segments for fixed sweep period

Table 7.1 shows comparison of average number of mobile sensors to achieve sweep coverage for both of the algorithms varying with number of line segments. The average number of mobile sensor is calculated for 100 several executions of the algorithms with fixed sweep period 50 second. A graphical representation of the Table 7.1 is illustrated in Fig. 7.8. Table 7.1 and Fig. 7.8 show that with increasing number of line segments, the algorithm BSCMC performs better than Algorithm 8 with respect to average number of mobile sensors.

Table 7.2 shows comparison of average number of mobile sensors to achieve sweep coverage varying with the sweep periods. The average number of mobile sensor is calculated for 100 several executions of the algorithms with fixed number of line segments,

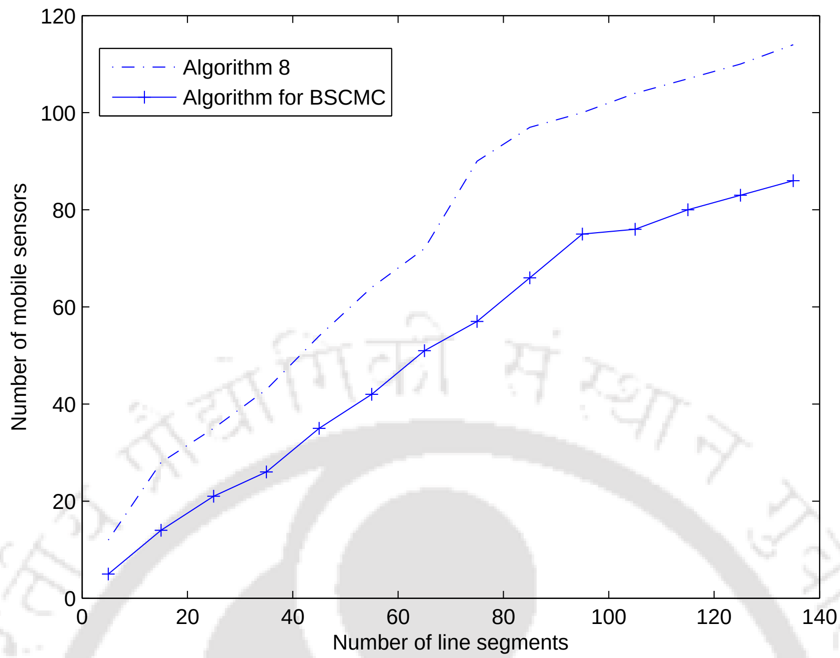


Figure 7.8: Comparison with respect to number of mobile sensors varying with number of line segments

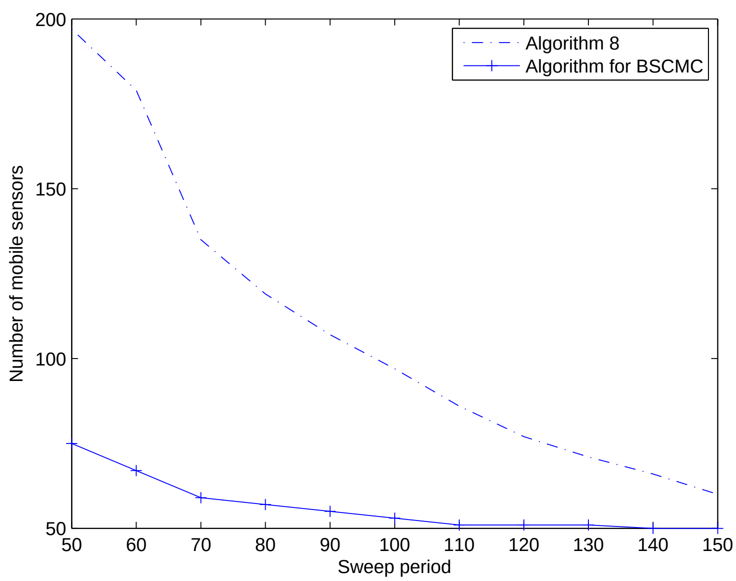


Figure 7.9: Comparison with respect to number of mobile sensors varying with sweep period (in second)

Sweep period	Number of Mobile Sensors	
	Algorithm for BSCMC	Algorithm 8
50	75	197
60	67	179
70	62	135
80	58	119
90	56	107
100	54	97
110	52	86
120	51	77
130	51	71
140	50	66
150	50	60

Table 7.2: Comparison of the number of mobile sensors to achieve sweep coverage varying sweep period for fixed number of line segments

which is equal to 50. A graphical representation of the Table 7.2 is illustrated in Fig. 7.9. Table 7.2 and Fig. 7.9 show that with increasing number of line segments, the difference between the average number of mobile sensors decreases. In general the algorithm for BSCMC performs better than Algorithm 8.

7.7 Conclusion

In this chapter, we have introduced sweep coverage concept for barriers. In barrier sweep coverage, mobile sensors periodically visit all points of a set of finite curves. For a single curve, we have solved the problem optimally. Energy utilization for the mobile sensors is restricted as they are battery-powered. Considering the limitation, an energy restricted barrier sweep coverage problem is defined for a finite curve. A $\frac{13}{3}$-approximation algorithm is proposed to solve the problem. We have proved that finding minimum number of mobile sensors to sweep cover a set of finite curves is NP-hard and cannot be approximated within a factor of 2, unless P=NP. We have proposed a 5-approximation algorithm to solve the problem. A 2-approximation algorithm is proposed for a special case of the problem. As an application of barrier sweep coverage, we have defined a data gathering problem with data mules. A 3-approximation algorithm is proposed to solve the problem.

Chapter 8

Distributed Sweep Coverage

8.1 Introduction

Li et al. [41] comments on the impossibility of designing distributed sweep coverage algorithm for a given set of PoIs. An individual mobile sensor cannot have knowledge of the movement paths of all other mobile sensors without global information. Therefore, a mobile sensor cannot locally say “yes” or “no” to the question of whether a given set of PoIs is globally sweep covered. In this chapter, we propose a distributed sweep coverage algorithm in a different context. We assume static sensors are placed at the location of PoIs, one for each. The set of static sensors form a connected network. A set of mobile sensors visits the static sensors once in a given time period to sweep cover the static sensors. In this setting topology of the underlying network may change due to failure or addition of the static sensors. But it is not possible to apply sequential Algorithm 2 (chapter 3) for every change in network topology. In this scenario distributed solution of the sweep coverage plays an important role. The static sensors themselves decide the positions and movement strategy of the mobile sensors by some distributed coordinations. There are several applications in data gathering, where a set of mobile sensors periodically visits the static sensors to collect and/or exchange data. This periodic visit is important because the static sensors may not have sufficient memory for frequent sampling and storing the data in its memory. Another reason for periodic visit might be for updating sensing instructions.

Definition 8.1.1 (Distributed sweep coverage problem)**.** Let $U = \{u_1, u_2, \cdots, u_n\}$ be the set of PoIs in a two dimensional plane and $S = \{s_1, s_2, \cdots, s_n\}$ be the set of static sensors such that s_i is located at u_i for all $i = 1$ to n. Let $M = \{m_1, m_2, \cdots, m_n\}$ be the set of mobile sensors with uniform speed v. For a given $t > 0$, S finds minimum number of mobile sensors with their positions such that each PoI in U is t-sweep covered.

8.1.1 Contribution

- We introduce a distributed sweep coverage problem and propose a 4-approximation algorithm as a solution.
- A 2-approximation algorithm is also proposed for a special case.

8.2 Approximation for a Special Case of Distributed Sweep Coverage

We first propose a distributed algorithm for a special case of the problem where each mobile sensor visits all static sensors. Then solution for the distributed sweep coverage problem is proposed, where each mobile sensor does not necessarily visit all static sensors.

8.2.1 System Model and Algorithm

Let n static sensors with unique $id \in \{0, 1, 2, \cdots, n-1\}$ and communication range r_c are placed at all PoIs. Two static sensors are neighbor of each other if they are within a distance r_c. Let $G_c = (U, E_c, w)$ be the connected communication graph, where each static sensor i is a vertex and (i, j) is the edge between two neighbors i and j. Edge weight $w(i, j)$ for the edge (i, j) is the Euclidian distance between i and j. Let $G = (U, E, w)$ be the corresponding complete graph of G_c. For any subgraph H of G, we write $w(H)$ for the sum of the edge weights of H. Let t be the sweep period of every static sensor. All mobile sensors can move with uniform speed v. Every static sensor uses *control message* for finding Euler tour by marking the edges and the positions of the mobile sensors by marking the point on the edges. There is an initiator, static sensor

s for initiating the control message. Every static sensor maintains a variable *status* for each edge incident on it, which can be either *used* or *unused*. An edge *status:= used* means one control message for finding *Euler tour* has passed through the edge, otherwise *status:= unused*. Based on the above discussion we propose Algorithm 10 (DistributedSweepCoverage).

Algorithm 10 DistributedSweepCoverage

1: Find a MST $\mathcal{T}_c$ of G_c by the distributed MST algorithm [24].
2: Doubling every edge $(i, j) \in \mathcal{T}_c$ with same weight, one edge is (i, j) and other is (j, i).
3: Initial status of each edge $(i, j) \in \mathcal{T}_c$ is *unused*
4: Initiator s selects an *unused* edge $(s, j) \in \mathcal{T}_c$, marks the edge for *Euler tour* and assigns $d = w(s, j)$. If $\lfloor \frac{d}{vt} \rfloor \geq 1$ then s calculates $\lfloor \frac{d}{vt} \rfloor$ number of points on (s, j) and marks them as positions of mobile sensors and sends $< m, d - vt \times \lfloor \frac{d}{vt} \rfloor >$ to j. Otherwise s sends a message $< m, d >$ to j. The status of the edge (s, j) become *used*.
5: j executes following after receiving a message $< m, d >$ from i.
Choose an *unused* edge $(j, k) \in \mathcal{T}_c$ other than (j, i) and marks the edge for *Euler tour*. Now $d = d + w(j, k)$. If $\lfloor \frac{d}{vt} \rfloor \geq 1$, calculates $\lfloor \frac{d}{vt} \rfloor$ number of points on (j, k) and marks them as positions of mobile sensors and sends $< m, d - vt \times \lfloor \frac{d}{vt} \rfloor >$ to k. Otherwise j sends a message $< m, d >$ to k. The status of the edge (j, k) become *used*. If no such *unused* edge $(j, k) \in \mathcal{T}_c$ other than (j, i) exists then j executes step 5 for the edge (j, i).
6: When the message $< m, d >$ returns back to the initiator s and every edge in $\mathcal{T}_c$ incident to s is *used*, and if $d > 0$, the s marks its position for a mobile sensor.

The Algorithm 10 returns marked edges for Euler tour and corresponding marked points for placement of mobile sensors. The mobile sensors start moving along the edges of the Euler tour at the same time once they all are placed. The movement of all mobile sensors guarantee sweep coverage for all the PoIs. Following theorems give correctness and approximation factor of the algorithm.

Theorem 8.2.1. *Let T be a minimum spanning tree of G and $\mathcal{T}_c$ be the minimum spanning tree of G_c. Then $w(T) = w(\mathcal{T}_c)$.*

Proof. Clearly, $w(T) \leq w(\mathcal{T}_c)$. For all the edges $e \in E(G_c)$ implies $w(e) \leq r_c$, since r_c is the communication range. Therefore, $e \in E(\mathcal{T}_c)$ implies $w(e) \leq r_c$. Now let us consider the following two cases:

Case 1: All edges of T are with weight $\leq r_c$. T is a spanning tree of G_c and therefore

$w(\mathcal{T}_c) = w(T)$.

Case 2: There exist an edge e in T such that $w(e) > r_c$. Let us consider the graph $\mathcal{T}_c \cup \{e\}$. This graph must contains a cycle containing e and $w(e)$ is greater than the weights of all other edges of the cycle. So, by the cycle property of minimum spanning tree, e cannot be a part of any MST of G, contradicting the fact that e is a part of T. Hence $w(e) \leq r_c$ and therefore, by case 1, $w(\mathcal{T}_c) = w(T)$. □

Theorem 8.2.2. *Algorithm 10 produces approximation ratio 2 for finding minimum number of mobile sensors.*

Proof. Let L_{opt} be the optimal TSP tour on the corresponding complete graph G. If T is an MST on G with cost $w(T)$, then one can easily prove $w(T) \leq w(L_{opt})$. Algorithm 10 computes an MST $\mathcal{T}_c$ on the graph G_c and by Theorem 8.2.1, $w(\mathcal{T}_c) = w(T)$. Therefore, $w(\mathcal{T}_c) \leq w(L_{opt})$. Let $\mathcal{E}$ be the Eulerian graph formed after doubling each edge of $\mathcal{T}_c$. Then $w(\mathcal{E}) \leq 2w(L_{opt})$. By Theorem 3.2.2, the approximation ratio of the Algorithm 10 is 2. □

Theorem 8.2.3. *Message complexity of the Algorithm 10 is $O(n \log n + |E_c|)$, where $|E_c|$ is the number of edges in E_c.*

Proof. Total number of messages required for finding number of mobile sensors and their initial positions depends on the distributed construction of MST and finding Euler tour by doubling each edge of the MST. The messages required to construct an MST by the distributed algorithm [24] is $O(n \log n + |E_c|)$. Total $2(n-1)$ messages are required for finding Euler tour by Algorithm 10, since in step 4 each static sensor sends one message through all the tree edges which are incident on it. Therefore, messages complexity of the algorithm is $O(n \log n + |E_c|)$. □

8.3 4-approximation Algorithm for Distributed Sweep Coverage

In this section, we generalize the above Algorithm 10 to solve distributed sweep coverage problem, where each mobile sensor does not necessarily visit all static sensors.

Let $\mathcal{T}_c$ be the minimum spanning tree computed in step 1 of the Algorithm 10. Now, every static sensor removes all incident edges of $\mathcal{T}_c$ from its adjacency list which are of length more than vt. This may split $\mathcal{T}_c$ into several connected components. Run the remaining steps of Algorithm 10 for each of the components to find number of mobile sensors and their initial positions.

To analyze the approximation factor of the proposed algorithm, we construct a complete graph $G' = (U, E, w')$ from the complete graph $G = (U, E, w)$, where weight function w' of G' is defined as follows. For all $e \in E$, if $w(e) \leq vt$ then assign $w'(e) = \frac{w(e)}{vt}$, otherwise assign $w'(e) = 1$. Let *opt* be the number of mobile sensor required in the optimal solution on the communication graph G_c and opt' be the number of mobile sensor required in the optimal solution on G . Following lemma gives a lower bound of *opt*.

Lemma 8.3.1. $\lceil w'(\mathcal{T}_c) \rceil \leq 2opt$.

Proof. It is easy to observe that $opt' \leq opt$. Let us consider the movement of the mobile sensors in the optimal solution on G. Let $P_1, P_2, \cdots, P_{opt'}$ be the movement paths of the opt' number of mobile sensors in a time interval $[t_0, t+t_0]$. Since a mobile sensor can move at most vt distance during the time interval, no P_i contains any edge with weight more that vt. Therefore, in G', $w'(P_i) = \frac{w(P_i)}{vt} \leq 1$ and thus $\sum_{i=1}^{opt'} w'(P_i) \leq opt'$. Consider a vertex $v_i \in P_i$ for all $i = 1$ to opt'. Let H' be the subgraph of G' such that $V(H') = U$ and $E(H') = \bigcup_{i=1}^{opt'} E(P_i) \bigcup \{(v_j, v_{j+1})|\ j = 1 \ \ to \ \ opt' - 1\}$. Note that H' is a spanning tree of G' and $w'(H')= \sum_{i=1}^{opt'} w'(P_i) + \sum_{i=1}^{opt'-1} w'(v_i, v_{i+1}) \leq opt' + (opt' - 1) \leq 2opt' - 1$. Since T is the minimum spanning tree of G, $w'(T) \leq w'(H') \leq 2opt' - 1$. Therefore, $\lceil w'(T) \rceil \leq 2opt'$. By Theorem 8.2.1, $w'(\mathcal{T}_c) = w'(T)$, and hence $\lceil w'(\mathcal{T}_c) \rceil \leq 2opt' \leq 2opt$. ☐

Theorem 8.3.2. *The above algorithm is a 4-approximation algorithm.*

Proof. Let N be the number of mobile sensors computed by the algorithm. In step 1 of the proposed algorithm, MST $\mathcal{T}_c$ is computed. After computing the tree $\mathcal{T}_c$, all the edges of weight more than vt are deleted from $\mathcal{T}_c$. Let m be the number of deleted edges. There are following two cases.

Case 1: At least one edge is deleted ($m \neq 0$). The tree $\mathcal{T}_c$ splits into $m + 1$ connected components $C_1, C_2, \cdots, C_{m+1}$ in G_c. The corresponding tours $T_1, T_2, \cdots, T_{m+1}$ are computed after doubling the edges of each component. Total number of mobile sensors needed is $\sum_{i=1}^{m+1}\left\lceil\frac{w(T_i)}{vt}\right\rceil \leq \sum_{i=1}^{m+1}\frac{2w(C_i)}{vt}+(m+1)$, where $w(T_i)=2w(C_i)$. Also, $w'(\mathcal{T}_c)=\sum_{i=1}^{m+1}\frac{w(C_i)}{vt}+m$. Therefore,

$$\begin{aligned} N &\leq \sum_{i=1}^{m+1}\frac{2w(C_i)}{vt}+(m+1) \\ &\leq \sum_{i=1}^{m+1}\frac{2w(C_i)}{vt}+2m \\ &\leq 2w'(\mathcal{T}_c) \\ &\leq 4opt \text{ by Lemma 8.3.1} \end{aligned}$$

Case 2: No edge is deleted ($m=0$). Total number of mobile sensors in this case is given by
$N \leq 2\left\lceil\frac{w(\mathcal{T}_c)}{vt}\right\rceil = 2\lceil w'(\mathcal{T}_c)\rceil \leq 4opt.$

Therefore, from the above two cases, the algorithm is a 4-approximation algorithm. □

8.4 Conclusion

In this chapter we have proposed a distributed sweep coverage algorithm, where static sensors are placed at every point of interest. The mobile sensors move on the plane according to their schedule and designated paths to sweep cover all static sensors. In the distributed algorithm all static sensors communicate with their immediate neighbors and collectively compute number of mobile sensors to guarantee sweep coverage of all static sensors. The approximation factor of the algorithm is 4. A 2-approximation algorithm is also proposed for a special case.

Chapter 9

Conclusion

Sweep coverage concept is recently introduced in the literature where periodic patrol inspections are sufficient for a given set of points of interest by a set of mobile sensors. In this thesis, we have investigated various types of sweep coverage problem such as point sweep coverage, area sweep coverage, barrier sweep coverage, energy efficient sweep coverage, energy restricted sweep coverage and distributed sweep coverage from the theoretical point of view. In our solution of the sweep coverage problem we have overcome a flaw of the previous solution [41] and proposed a 3-approximation algorithm to solve this NP-hard problem where points are the vertices of a weighted graph. A 2-approximation algorithm is proposed for a special case of the problem, which is the best possible approximation factor of the problem. We have investigated the sweep coverage problem in two different contexts of heterogeneity. In the first context, sweep periods of the PoIs are different, and in the second context speeds of all mobile sensors are different. There is a flaw of the previous solution [41] for the problem corresponding to the first context. We have remarked on the flaw and proposed an $O(\log \rho)$-approximation algorithm for this generalized problem, where $\rho = \frac{t_{max}}{t_{min}}$, t_{min} and t_{max} are the minimum and maximum sweep periods of the PoIs such that points are the vertices of a weighted graph. We have proved that it is impossible to design any constant factor approximation algorithm to solve the problem corresponding to the second context, unless P=NP. We have introduced two variations of sweep coverage problem handling different types of power issues of WSNs. Objective of the first problem is to guarantee sweep coverage for a given set of PoIs by a set of static and/or mobile sensors with minimum energy consumption per unit time. An 8-approximation algorithm is proposed to solve the problem. Another solution for a special case is also proposed, which achieves the best possible approximation factor 2. Objective of the second problem is to find minimum number of mobile sensors to guarantee sweep coverage where the maximum energy utilization by a mobile sensor is bounded. for that, we have proposed a $(5+\frac{2}{\alpha})$-approximation algorithm to solve the problem. We have introduced area sweep coverage problem and proved that the problem is NP-hard. Then a $2\sqrt{2}$-approximation algorithm is proposed for a square area. For arbitrary bounded region the approximation factor is a function of perimeter and area of the bounded region. We have introduced sweep coverage concept for barriers, where the objective is to sweep cover finite curves in a plane. We have solved the problem optimally for a single curve. To resolve the issue of limited battery power of the mobile sensors, we have proposed a solution by introducing an energy source in the plane. To solve the problem we have proposed a $\frac{13}{3}$-approximation algorithm. We have proved that finding minimum number of mobile sensors to sweep cover a set of finite length curves is NP-hard and cannot be approximated within a factor of 2, unless P=NP. Then we have proposed 5-approximation algorithm to solve it. For a special case of this problem we have proposed a 2-approximation algorithm. As an application of barrier sweep coverage, we have defined a data gathering problem with data mules, where the concept of barrier sweep coverage is applied for gathering data by utilizing minimum number of data mules. For that a 3-approximation algorithm is proposed to solve the problem. We have introduced a distributed sweep coverage problem where static sensors are placed at the location of the PoIs, one for each and for that a 4-approximation algorithm is proposed to solve the problem.

Above algorithms can be applied for the class of coverage problems where coverage requirement is time variant. We have designed several constant factor approximation algorithms for various sweep coverage problems. There is a scope of further improvement on the approximation factors of the above problems. In future, we want to investigate some other issues such as fault tolerance, scalability, self-stabilization, etc. in the context of sweep coverage. However designing online algorithm for sweep coverage might be a nice extension for further study.

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Publication from the Contents of the Thesis

Papers published/submitted in International Journals:

[J1] Barun Gorain and Partha Sarathi Mandal, "Approximation Algorithm for Sweep Coverage on Graph", *Information Processing Letters (Elsevier)*, Vol. 115, Issue 9, pp. 712-718, 2015.

[J2] Barun Gorain and Partha Sarathi Mandal, "Approximation Algorithms for Sweep Coverage in Wireless Sensor Networks", *Journal of Parallel and Distributed Computing (Elsevier)*, Vol. 74, Issue 8, pp. 2699-2707, August 2014.

[J3] Barun Gorain and Partha Sarathi Mandal, "Approximation Algorithms for Barrier Sweep Coverage in Wireless Sensor Networks", *Journal of Parallel and Distributed Computing (Elsevier)* (Submitted).

[J4] Barun Gorain and Partha Sarathi Mandal, "Solving Energy Issues for Sweep Coverage in Wireless Sensor Networks", *Discrete Applied Mathematics (Elsevier)*, (Submitted).

Papers Published in International Conference Proceedings:

[C1] Barun Gorain and Partha Sarathi Mandal, "Energy Efficient Sweep Coverage with Mobile and Static Sensors" in *Proc. of International Conference on Algorithms and Discrete Applied Mathematics (CALDAM 2015)*, Lecture Notes in Computer Science (LNCS-8959) (Springer-Verlag), IIT Kanpur, India, pp. 275-285, Feb 8-10, 2015.

[C2] Barun Gorain and Partha Sarathi Mandal, "Brief Announcement: Sweep Coverage with Mobile and Static Sensors" in *Proc. of 16th International Symposium on Stabilization, Safety, and Security of Distributed Systems (SSS 2014)*, Lecture Notes in Computer Science (LNCS-8756) (Springer-Verlag), Paderborn, Germany, pp. 346-348, Sep 28 - Oct 1, 2014.

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Other Publications

[C5] Barun Gorain, Partha Sarathi Mandal and Sandip Das, " Poster Abstract: Approximation Algorithm for Minimizing the Size of a Coverage Hole in Wireless Sensor Networks" in *in Proc. of 14th International Conference on Distributed Computing and Networking (ICDCN'13)*, Lecture Notes in Computer Science (LNCS-7730), (Springer-Verlag), TIFR, Mumbai, India, pp. 463-464, Jan 3-6, 2013.

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