

Nonlinear Dynamic Analysis of Pneumatic Artificial Muscle Actuator under Forced and Parametric Excitations

A Thesis Submitted in Partial Fulfilment of the
Requirements for the Degree of

Doctor of Philosophy

by

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(Roll No.-146103047)



**DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
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Certificate

This is to certify that the work contained in this thesis entitled “**Nonlinear Dynamic Analysis of Pneumatic Artificial Muscle Actuator under Forced and Parametric Excitations**” submitted by **Mr. Bhaben Kalita** to the Indian Institute of Technology Guwahati for the award of degree of Doctor of Philosophy has been carried out under my supervision in the Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Assam, India. To the best of my knowledge, no part of the work reported in this thesis has been submitted elsewhere for the award of any other degree or diploma.

Prof. Santosha K. Dwivedy

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Acknowledgements

I am very much grateful to Prof. Santosha K. Dwivedy for his excellent supervision and unwavering faith in me. I will forever be indebted to him for his constant support, encouragement, motivation and guidance in molding me up as a researcher. His patience as an advisor, unlimited energy while teaching, promptness while reviewing all my writings and passion for research are to be commended and worth emulating. I remember how he made him easily accessible at any point of time to resolve my query.

I would also like to thank my Doctoral committee members Prof. D. Chakraborty, Prof. S. Kanagaraj and Prof. S. Talukder for constantly reviewing the progress of my work with valuable suggestions and to improve the quality of the thesis. I am obliged to Prof. Rajiv Tiwari for giving me the permission to use and perform experiments in the Vibration and Acoustic Laboratory, IIT Guwahati. Besides, the incredible support extended by the authority of NIT Meghalaya and Dr. D. K. Sarma, NIT Meghalaya at different phases to complete the research work, is really commendable.

I am thankful to Dr. D. J. Bordoloi for his contribution in the handling the vibration measuring equipment and helping me to conduct the experiments. I would like to express my thanks to my friend Mr. Arunjyoti Borgohain for his continuous support, help in developing the experimental setup and carrying out the experiments. The support received from my senior scholars Dr. Anshul Garg, Mr. Sibananda Mohanty, Dr. Rajesh K. Ojha and Mr. Rajesh Ranjan at different stages of my work is also acknowledged. I would like to thank my friends Kamal, Jyotindra, Avilash and Ronit who made my journey memorable.

Finally, none of this would have been possible without my family's love and support. My deepest gratitude goes to my father, Mr. Biren Kalita for inspiring me to do hard work and keep patience which make it easier to achieve my goal. I don't have words to express thanks to my mother, Mrs. Kanika Kalita for her love and belief without bothering what kind of science I do. And lastly, I thank my partner, Ashamoni Kakati for her love, care and unconditional support and help through the many highs and lows of this beautiful journey.

Finally, I express my sincere thanks to all who helped me in whatever form during the successful completion of this thesis.

Bhaben Kalita



Abstract

In this work, two physical systems with pneumatic artificial muscle (PAM) have been considered for the nonlinear dynamic analyses. In the first physical system, a PAM and a spring are connected to a mass which is subjected to an external harmonic force with a constant pressure inside the muscle. In the second physical system, a PAM is connected to a mass which is subjected to a time varying pneumatic pressure. The muscle force is expressed as a nonlinear function of pressure. The governing equation of motions are derived for five different cases considering different excitations and resonance conditions.

In the first case considering first physical system, the study has been carried out to investigate the dynamics of a harmonically excited system under simple resonance condition. In the second case with the second physical system, a parametrically excited system is studied under the simple and principal parametric resonance conditions. In the third case, the nonlinear dynamics of the second physical system is investigated under principal parametric and simultaneous resonance conditions. In the fourth case, the first physical system is subjected to hard excitation with super- and sub-harmonic resonance conditions. Finally, in the fifth case, the second physical system is studied under parametric and hard excitations, which is subjected to super- and sub-harmonic resonance conditions.

The nonlinear governing equations are solved using the first and second order method of multiple scales. The numerical simulations have been carried out to find the steady state responses along with the stability and critical bifurcations of the fixed point responses for different resonance conditions. The frequency response plots are obtained with the variation in the different sets of physical system parameters such as operating pressure, static and dynamic pressure, damping, various muscle parameters, nonlinear stiffness, spring stiffness and external force. The time response, phase portraits and basin of attraction are also used to analyse the system behaviour for

different resonance conditions. The trivial state instability regions, along with the critical values for different system parameters, are also investigated for the parametrically excited system.

In all the cases, saddle node bifurcation point has been noticed and Hopf bifurcations are observed in the simple, simultaneous and superharmonic resonance conditions. In the case of principal parametric resonance condition, pitchfork bifurcation points have been observed. The trivial state is completely stable for the subharmonic resonance condition. In all the cases, the response amplitude of the system increases with a decrease in the damping parameter, muscle parameters, nonlinearity, stiffness of the external spring, and vice-versa. Specifically, in the fourth case, periodic, multi-harmonic, homoclinic orbits and three periodic orbits leading to chaotic responses are observed, and the butterfly effect for chaos is also noticed by varying the initial conditions.

It is observed that the results obtained by solving the reduced equations from the MMS are in good agreement with that by numerically solving the original nonlinear temporal equation of motion. From the present analyses, the designer or researchers may get to know about the safe operating range along with the effects of various system parameters on the dynamics of the muscle actuation. To achieve the desired amplitude for a specific application, one should choose the system parameters in their safe operating range to avoid the failure of the system.

An experimental setup has been developed to characterize the muscle (to obtain the muscle parameters) by using the experimental force-contraction data. Further, free and forced vibration experiments have been carried out to validate the numerically obtained results from the method of multiple scales. Also, a novel PAM has been developed, which is fabricated with the help of silicon rubber and locally available fabrics in this work. Therefore, the developed numerical model and the reduced equations can be used for further analysis of PAM used in various medical as well as industrial applications, which will take a small amount of memory along with less computational time.

Keywords: Pneumatic Artificial Muscle (PAM), harmonic excitation, parametric excitation, hard excitation, nonlinearity, Method of Multiple Scales (MMS), stability, bifurcations, basin of attraction, parametric instability regions, simple resonance, principal parametric resonance, simultaneous resonance, superharmonic resonance, subharmonic resonance, least square method.

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Nomenclature

English Symbols

c	coefficient of viscous damping
c_1, c_2, c_3	muscle parameter
d_1, d_2	muscle parameter
F	external load
F_{mus}	muscle force
f	nondimensional forcing parameter
f_1, f_2	nondimensional parameter
g	acceleration due to gravity
K	coefficient of the cubic nonlinearity
k	stiffness of the external spring
l_{max}	maximum possible length of the muscle
m	mass attached to the system
P	operating pressure input
P_m	static pressure inside the muscle
P_0	dynamic pressure inside the muscle
p_1, p_2	nondimensional parameter
r	scaling factor
u	displacement
x	nondimensional displacement

Greek Symbols

ω	frequency of the harmonic forcing term/input pressure
ω_0	fundamental natural frequency
τ	nondimensional time
μ	nondimensional damping parameter
ε	book keeping parameter
α	nondimensional nonlinearity
Ω	external excitation frequency
σ	detuning parameter

Acronyms

PAM	Pneumatic Artificial Muscle
MMS	Method of Multiple Scales



Chapter 1

Introduction and Literature Review

1.1 Introduction

The development of robotic systems is becoming increasingly prevalent today, with possible applications including personal or mobile robots working alongside humans along with the exoskeletons worn to restore or improve human abilities. While a variety of possible actuation schemes are present, there is still a need for the development of a soft actuator that is very light-weight, compact, and flexible with high power to weight ratio. To achieve this, the development of the Pneumatic Artificial muscle (PAM) actuator, which is as soft as the human muscle, becomes an interesting topic for many researchers. PAMs are the simple mechanical actuators that consist of an elastomeric bladder within a braided mesh sleeve with two end-fittings to seal both the ends of the muscle. Upon pressurization of the bladder (by air, water, oil, etc.), the actuator either contracts or extends axially, with the direction of motion dependent on the orientation of the braided sleeve fibers. The contractile PAMs are able to produce higher forces than extensile PAMs and are less prone to buckling. So, the contractile PAMs are more commonly utilized, which is also known as the contractile actuator.

There are different types of conventional actuating mechanisms such as electric motors, hydraulic actuators, pneumatic pistons and shape memory alloys (SMA) along with the flexible actuators used in various applications. But the use of artificial muscles is more beneficial in comparison to the other actuation mechanism due to their following advantages [1-4].

- PAMs are lightweight and produce a strong force, which provides high power to weight ratio for the applications.
- PAMs provide flexibility and compliance for the applications in pressurized or unpressurized condition.
- PAMs deliver a safe and compatible interaction with the human environment only by controlling the volume flow rate of operating air into the muscle.
- Simple PAMs can be fabricated from inexpensive materials, which reduce the overall manufacturing cost.
- PAMs provide the linear contractile motion with a monotonically decreasing load-contraction relation like the skeletal muscle, which can control joint compliance.
- PAMs exhibit the nature of essential compliance for installation for providing required power, and the values of speed along with force are in the range of what is needed.
- Because of the direct connection, replacement of a defective PAM with a new one can be done easily and swiftly by uncoupling the muscle along with pneumatic tubing.
- Due to the intrinsic and adjustable compliance, PAMs can be made to have a soft touch and, consequently, well-suited for safe human-robot interaction.

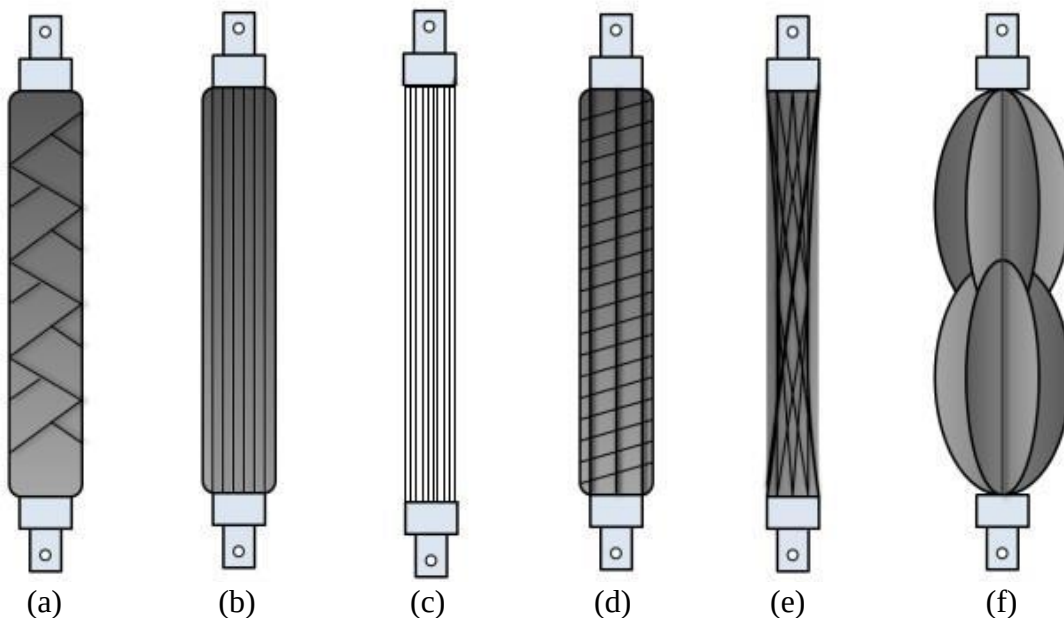


Fig. 1.1 Various types of conventional PAMs [5]: (a) McKibben Muscle/Braided Muscle (b) Pleated Muscle (c) PAM reinforced by Kevlar Fiber (d) Yarlott Netted Muscle (e) Paynter Hyperboloid Muscle and (f) ROMAC Muscle.

With the benefits, a number of groups have investigated the design geometries and fabrication methods of different types of artificial muscle actuators. The physics behind the operation of PAMs is the same as that of human muscle as it converts the pneumatic power to pulling force. The compressibility of the air plays a significant role in the artificial muscles for the unpredictable impact. Various types of conventional PAMs, as shown in Fig. 1.1 have been developed depending on the different operating principles, applications and design concepts [5]. These can be classified in the literature into four major types, such as braided muscles, pleated muscles, netted muscles, and embedded muscles.

For various medical applications, high torques are required at low to moderate speeds. So, the proper selection of the actuator is the crucial part to avoid the undesired phenomena like backlash, friction and extra inertia to the structure. It may lead to a more complicated design with heavy construction, which is not acceptable by the user. Hence, there is a need to make it lightweight and compact to actuate the body's compliant joints in a real environment. For the actuation of these devices, electrical motors like DC motors, AC motors, stepper motors, and linear motors are commonly used through which high acceleration and positional accuracy can be achieved. But for human-robot interaction, the high rigidity of the joints in these robots will be very dangerous for the operator. Weight is the major drawback of electric motors in medical applications. Electric motors have the power to weight ratios in the order of magnitude of 100 W/kg, although peak values are higher, and their torque to weight ratios range more or less within 1–10 Nm/kg [6-8]. So, to achieve high torques at a low speed, some gearing attachment or high performance motors has to be developed. Raab and Isermann [7] mentioned that hydraulic actuators have a very good power to weight ratio of 2000W/kg and high torques can be achieved at low speeds. Though there is a chance of leakage in the energy source due to high operating pressure, it can be directly connected to the robot joints. With the use of the servo-valve control, this kind of actuation can acquire the compliance characteristics.

The most common pneumatic actuators are cylinders with the power to weight ratios of 400W/kg, and these can be operated with moderate speeds [7]. The inherent compliance characteristics are present in the pneumatic actuators due to the compressibility of the air pressure. So, PAM has been used as an alternative pneumatic actuator in the medical applications to achieve better power to weight ratio than the pneumatic cylinder. The muscle actuator is operated with the help of the

pressurized air through a core deformable membrane, and the required motion can be obtained. Due to the use of the viscoelastic materials in most of the available PAM, it is lightweight in nature, and the deformation is based on the material elongation. The power to weight ratio of these kinds of actuators is normally more than the values of 1 kW/kg [9]. Some more and new particular actuators like shape memory alloys (SMA) and polymeric actuators have been used in various applications. But these actuators are very much specific application oriented, and their speeds of operation are also very low [8, 10].

Table 1.1 Comparison of actuator technologies [11].

Actuation Technology	Maximum Strain	Actuation Stress (MPa)	Specific Work (J/kg)
Hydraulic	1	70	35000
Electromechanical	0.5	1	300
Solenoid	0.4	0.1	5
Piezoelectric	0.002	9	1
Magnetostrictive	0.002	200	20
SMA	0.07	700	4500
Pneumatic Cylinder	1	0.9	1200
PAM	0.4	16	4400

Table 1.1 presents a comparison of the maximum strain (stroke divided by resting length), actuation stress (force divided by cross-sectional area) and specific work (work divided by mass) of several types of actuators, which is inspired by the work of Woods [11]. PAMs combine large usable strains with significant actuation stresses to yield a higher ratio of work to mass (i.e., specific action) than several other actuation technologies such as electromechanical actuators, solenoids, and piezoelectric or magnetostrictive materials. Hence, due to the advantages of PAM over the other actuators, these muscle actuators are the most suitable for various medical and industrial applications. In addition to these performance advantages, PAMs boasts a variety of structural advantages as well. PAMs are very durable and have long fatigue life, having demonstrated only small wear and tear signs after 120 million actuation cycles [12].

It is very necessary to have accurate mathematical modeling to understand the dynamics of the PAM for different applications. But, due to the presence of various nonlinear characteristics in PAMs like, the compressibility of air, basic properties of rubber materials along with the geometry, flexible structure and the valve actuation, make these muscles difficult to model and

control precisely. Hence, a proper mathematical model for PAM is desired to improve the control system, which can be easily implementable and computationally efficient model for the better response of the controller. Also, a model describing the dynamics of PAM based on actuator dimensions, braiding characteristics, and material properties, helps to efficiently choose or fabricate a muscle actuator with optimized qualities required for a particular application. Due to static frictional forces and nonlinearity in the material of the bladder, a quasi-static contraction (or elongation) of PAM shows different curves for force versus length and pressure versus length plots for compression as well as the decompression of air. In force modeling methods, this additional frictional force is generally added (or subtracted) from a mean curve for contraction (and elongation) of PAM, which is mostly calculated empirically. Like any other pneumatic system, PAM exhibits hysteresis, which is a significant hindrance in modeling the statics and dynamics of PAM.

In this work, an attempt has been made to derive a significant dynamic model to understand the nonlinear characteristics present in the muscles. Here, a single degree of freedom system is considered containing a spring-mass-damper along with a PAM. While the system is considered to be under harmonic excitation, the PAM is subjected to a harmonically varying external load for some application. The direction of the excitation force and the resulting oscillation takes place in the same direction, and resonance occurs when the frequency of the external excitations is nearly equal to any of the natural frequencies of the system. It may be noted that when the PAM vibrates in a direction perpendicular to the direction of the external forcing, for certain frequencies and amplitude of forcing parameters, the system may reduce to that of a parametrically excited system. The nonlinear governing equation of motion of such a system contains time-dependent excitation terms. In the case of parametrically excited system, resonance may occur when the external excitation frequency is away from the natural frequency of the system. When the external excitation frequency is twice the natural frequency of the system, the resulting resonance is known as principal parametric resonance condition. In other cases, it is known as simultaneous resonance condition where two different kinds of resonance conditions acting concurrently on the system. As a time varying pneumatic pressure inside the PAM is considered in the present work, the resulting nonlinear system is similar to a combination of forced and parametrically excited system.

Hence, the required analysis to obtain the nonlinear response is expected to be very complicated for different resonance conditions.

It may also be noted that in most of the literature, the forcing is considered to be weak, i.e., the magnitude of the forcing term is very less than that of the coefficient of the linear term. Hence, an attempt has been made to study the hard excitation where the forcing amplitude is higher than the coefficient of the linear term. In this case, the system will be subjected to superharmonic and subharmonic resonance conditions. For superharmonic resonance condition, the external excitation frequency is one third of the natural frequency of the system, and in subharmonic resonance condition, the external excitation frequency is three times the natural frequency of the system.

It may be noted that the governing equation of motion of the system can be derived by using force and moment balance methods or energy based models. In force and moment balance methods, one may use Newton's 2nd law or d'Alembert's principle; while in energy methods Lagrange principle or extended Hamilton's principle can be applied. Due to the scalar approach, energy methods are particularly useful for multi-degree of freedom and continuous systems. Besides these methods, one may use the finite element method or lumped parameter models based on multi-body dynamics. As in the present work, a single degree of freedom system has been considered, Newton's 2nd law has been used for finding the governing nonlinear differential equation of motion. Then by using the nondimensional parameters, the governing temporal equation of motion can be obtained.

In the case of linear systems, one may use the superposition theory to predict the response of one system when the response for a similar system is known. But, unlike the linear systems, in a nonlinear system, this superposition theory will not be applicable, and hence, one has to perform the analysis for each case separately. Due to the presence of many nonlinear terms in the governing equation motion of the system, it is very difficult to find the closed form solution of the system. In such cases, one may obtain the approximate solution by using perturbation techniques. In the present work, the method of multiple scales (MMS) which is a popular perturbation technique, is used to analyze the nonlinear temporal equation of motion. In all the cases, the results obtained by using the perturbation technique are verified with those obtained by

numerically solving the temporal equation of motion. Further, these obtained mathematical results are also verified with the experimental findings.

It may be noted that while the parametric instability regions only predict the trivial state instability regions or trivial state bifurcation points, in actual nonlinear systems, the response may be a nontrivial fixed point response or it may be the periodic, quasi-periodic or chaotic response. Further, the response can be a combination of all these types of responses, which may be stable or unstable. Unlike linear systems, while analyzing the nonlinear dynamic system, one may observe many nonlinear phenomena such as multiple solutions, jump up and jump down phenomena, saturations, butterfly effect and chaos, etc. One may observe many bifurcation phenomena such as sub-critical and super-critical pitchfork, trans-critical, saddle-node, and Hopf bifurcations, where either the number or the nature of the existing response curve changes. The detailed analysis of the nonlinear dynamical systems can be found from the books of Nayfeh and Mook [13] and Nayfeh and Balachandran [14]. Hence, to analyze the system, in the present work, the time response, frequency response, phase portraits have been plotted for all resonance conditions. To study the multiple stable states, the basin of attraction have also been plotted. So, for a better understanding of the dynamic and stability of the PAM, in the following sections, an extensive literature review of available research works on PAM is presented and the current work is also outlined.

1.2 Literature Review

1.2.1 Pneumatic Artificial Muscle: Concept and Operation

Pneumatic Artificial Muscle (PAM) is a contractile and linear motion actuator operated by the air pressure. The main two parts of most of these kinds of actuators are the flexible core and the reinforced closed membrane like shell, diaphragm. Both the ends of these muscle actuators are closed with the end fitting (closures, clamps). The core membrane is bulged outward or squeezed due to the supply of the air pressure and through which the mechanical power is transmitted to a load. The core contracts axially apply a pulling force on its load due to the radial expansion or contraction. Therefore, the force (tension, load) and the motion achieved by the PAM actuator are linear and unidirectional. This contractile operation differentiates these actuators from the

bellows, which extend upon inflation. Throughout literature different names are found for PAMs: Pneumatic Muscle Actuator [9], Fluid Actuator [15], Fluid-Driven Tension Actuator [16], Axially Contractible Actuator [17, 18], Tension Actuator [19, 20].

The basic energy source to a PAM is primarily air either forced into it or extracted from it. This creates a pressure difference between the air inside the muscle and its surroundings, through which the PAM can be actuated to obtain the required motion. The more energy can be transported by overpressure than by underpressure. There is a possibility to design an underpressure operating muscle [21, 22], PAMs usually operate at an overpressure with ambient pressure mostly at about 100 kPa. Hence, these muscles generate and supply compressed air to obtain the required motion easily to move a load, discharging it, conversely, makes it yield to a load.

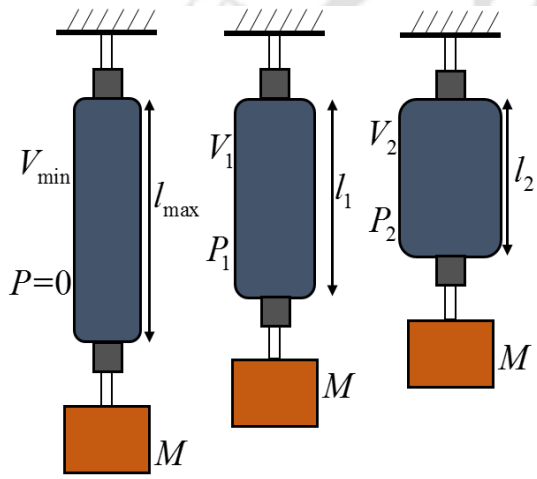


Fig. 1.2 PAM operation at constant load.

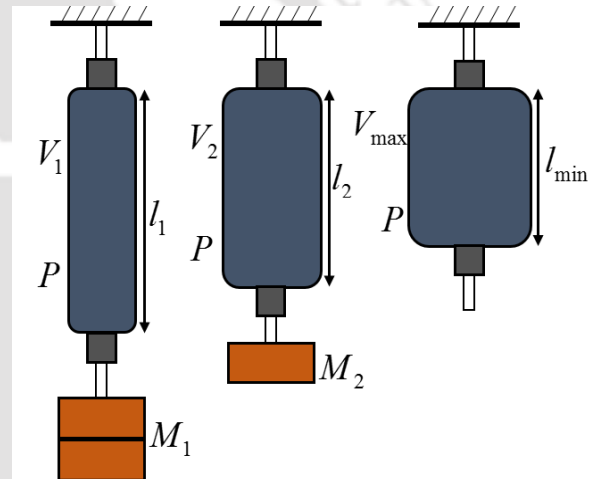


Fig. 1.3 PAM operation at constant pressure.

The basic operations of the PAM can be considered with two examples: (1) under a constant load and with varying gauge pressure, and (2) with a constant gauge pressure and a varying load. In Fig. 1.2 for the first operation shows a PAM is fixed at one end with a constant mass (M) hanging from the other side. The pressure difference across the membrane, i.e., its gauge pressure, can be increased from an initial value of zero. At zero gauge pressure, the volume of the membrane is considered to be minimum $V_{\min}$ and its length maximum $l_{\max}$. Now, the actuator is pressurized to a gauge pressure P_1 which makes the muscle to bulge and at the same time it will develop a pulling force. This force will lift the mass (M) until the equilibrium point where the generated force is equal to the mass weight Mg . In this condition, the volume of the PAM is increased to V_1 and its

length contracted to l_1 . Further increasing the pressure to P_2 , this process will continue until it reaches the maximum allowable gauge pressure $P_{\max}$. From this kind of operation of the PAM, two basic actuator behavior rules can be noticed, where, a PAM shortens by increasing its enclosed volume and it will contract against a constant load if the pneumatic pressure is increased.

The second type operation of PAM under constant gauge pressure is shown in Fig. 1.3 with an example. Here, the gauge pressure is kept at a constant value P with a decrease in the load where the muscle will inflate and shorten. If the load is completely removed as shown in Fig. 1.3, the swelling goes to its full extent with the maximum volume of the PAM, $V_{\max}$. In this situation, the length of the muscle will reach the minimum value $l_{\min}$ and the pulling force will drop to zero. PAM will not be able to contract beyond this point and behaves as a bellows at a shorter length, which generates a pushing instead of pulling force. This means that a PAM will shorten at a constant pressure if its load is decreased and its contraction has an upper limit at which it develops no force along with its enclosed volume is maximal. From these two examples of PAM operations, it can be concluded that PAM has an equilibrium length for each pair of pressure and load. This characteristic is in total contrast to the operation of a pneumatic cylinder where the developed actuation force only depends on the pressure and the piston surface area so that at constant pressure, it will be consistent regardless of the displacement [3]. In the next section, the design of four classifications of PAMs has been explained along with some advanced pneumatic muscle actuators.

1.2.2 Classification of Pneumatic Artificial Muscle

According to Marcinčin and Palko [21], in 1930, various muscle like actuators have been developed by a Russian inventor named S. Garasiev. These actuators can be classified based on the design and operation characteristics, such as, (1) pneumatic or hydraulic operation, (2) overpressure or underpressure operation, (3) braided/netted or embedded membrane and (4) stretching membrane or rearranging membrane. Though the hydraulic operation is considered only for completeness sake, the main feature of the artificial muscles is their inflation and deformation. Due to this feature, the flexibility is needed in the muscles for which the material strength along with the pressure difference should be limited. This makes the PAM as an attractive

and advantageous actuator over the other hydraulic actuator. The third characteristic mentions the tension carrying element of the muscle, which is a structure either embracing the membrane or embedded in the membrane. The last characteristic describes how the membrane inflates where it is able to expand radially; either the membrane material has to stretch, or the radial section has to change by rearranging the membrane's surface. In the case of pure rearranging, the total membrane surface is constant regardless of contraction and volume. This allows for greater tension to be developed as no energy is put into stretching membrane material. Based on these characteristics, further, this review will focus on four types of PAM, namely the braided muscles, which are the most frequently used, the pleated PAM, the netted muscle and the embedded muscle, which was developed as an improvement with regard to the drawbacks of the braided design.

1.2.2.1 Braided Muscle

Braided muscles are composed of a gas-tight elastic tube or bladder surrounded by a braided netting (weave, braid, sleeve) as is shown in Fig. 1.4. The braid fibers run helically about the long axis of the muscle at an angle (pitch angle, braid angle, weave angle). The tube is pressurized laterally against the sleeve and the internal pressure inside the tube is restricted by braid fiber tension due to fiber curvature. The fiber tension is incorporated at the endpoints of the braid to balance the external load. Hence, the pressing contact between tube and sleeve is necessary to lift the load, braided muscles cannot operate at underpressure. The most common braided muscles are the McKibben muscle [1, 2, 23, 24], where both its inner tube and braid connected to fittings at both ends. Another type of muscle is known as sleeved bladder muscle [25], which has only the braid connected to end fittings and the inner tube is an unattached bladder.

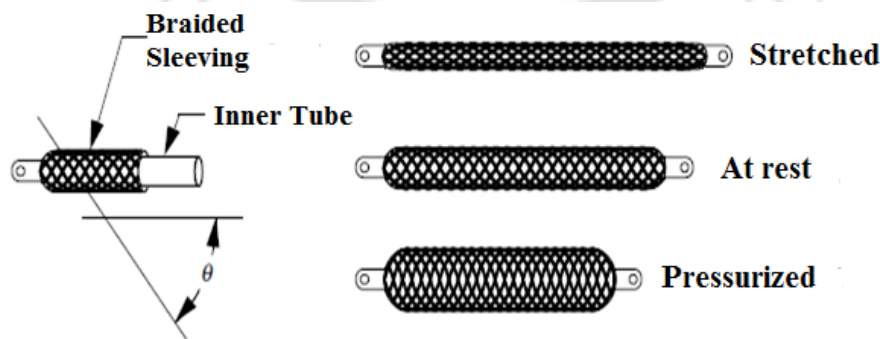


Fig. 1.4 Braided muscle with its working [25].

McKibben muscle, as shown in Fig. 1.5(a) was derived from a patented design by Morin [22], who embedded the fibers into a rubber diaphragm. According to Baldwin [26], it was first introduced by the physician, Joseph L. McKibben, in the 1950s to actuate an orthotic device for polio patients due to the similarity in length-load curves between this artificial muscle and skeletal muscle [27, 28]. The general behavior of these muscles with regard to shape, contraction and tension when inflated, will depend on the geometry of the bladder and surrounded braid at unpressurized condition along with the materials used. Usually, these braided muscles have a cylindrical shape because of a cylindrical bladder and a constant pitch angle throughout the braid.



Fig. 1.5 Different versions of McKibben muscle [23, 24].

The sleeved bladder muscles, patented by Beullens [25], are different from the McKibben type in the design of the inner bladder, where it is not connected to the sleeve, which means that no passive spring force is added to muscle tension. The main advantage of this type of PAM is its extreme ease of assembly.

1.2.2.2 Pleated Muscle

Pleated muscle actuator was developed by Daerden and Lefeber [29, 30], which is shown in Fig. 1.6(a). There is no material strain and friction involved in this type of muscle when it is inflated. This is due to the muscle membrane, which consists of the number of pleats in the axial direction and inflates by unfolding the pleats. Furthermore, membrane stresses in the parallel direction (perpendicular to the axis) are kept negligibly small and decrease with increasing the number of folds. Basically, there is no requirement of any external energy to inflate the membrane and due to the absence of friction, no hysteresis is observed.

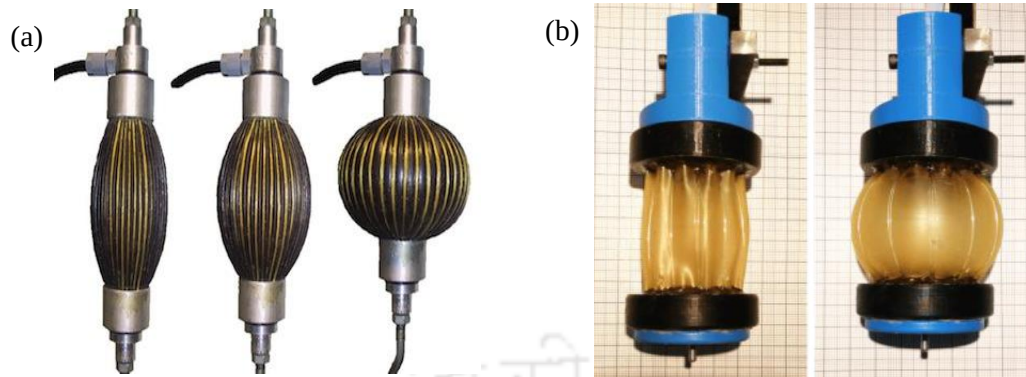


Fig. 1.6 (a) Pleated PAM [29, 30] (b) modified pleated PAM [31].

The characteristics of this type of muscle depend on the ratio of full length to minimum diameter, strain behavior of the membrane's material, contraction rate and applied pressure. A modified version of the pleated muscle actuator by Terryn et al. [31] is shown in Fig. 1.6(b) where the membrane is constructed out of self-healing polymer.

1.2.2.3 Netted Muscle

The main difference between braided and netted muscles is the density of the grid surrounding the membrane, where a mesh with relatively large holes with a braid is tightly woven. Due to this, the membrane is of the stretching kind where the muscle can withstand only low pressures. Therefore this type of muscle actuator will usually have a diaphragm of the rearranging kind.

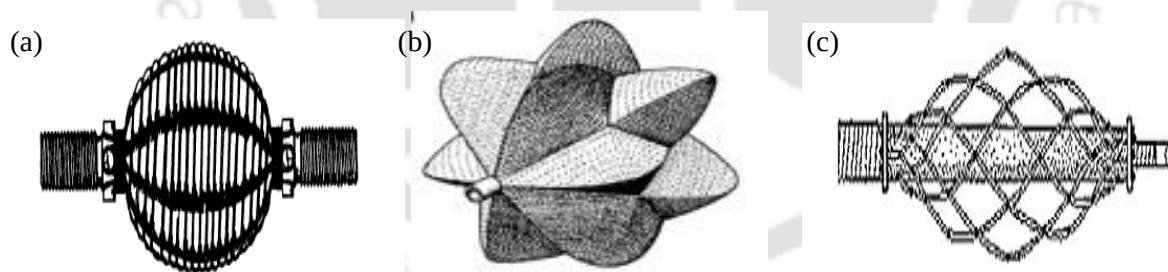


Fig. 1.7 Netted muscle (a) Yarlott muscle [15] (b) ROMAC [17] (c) Kukolj muscle [18].

Figure 1.7(a) shows the Yarlott muscle by J. M. Yarlott [15], which includes an elastomeric bladder of a prolate spheroidal shape netted by a series of cords or strands. This actuator takes the spheroid bladder shape in its inflated state, and when elongated, the axial strands straighten out and push the bladder into a shape characterized by a series of ridges and valleys. Figure 1.7(b) shows another type of netted muscle, Robotic Muscle Actuator (ROMAC), which was designed by G. Immega and M. Kukolj in 1986 [17]. ROMAC consists of an articulating polylobe bladder

harnessed by a wire netting and closed at either end by fittings [32] where the bladder is made of a sheath that is characterized by its high tensile stiffness, flexibility and fluid-tightness. The netting or harness is comprised of non-stretchable flexible tension links joining at nodes to form four-sided diamond-shaped apertures in the network. In Fig. 1.7(c) shows the Kukolj muscle by M. Kukolj [18], which is a modification of the standard McKibben muscle. The main difference between the muscles is the sleeve, where the McKibben muscles have a tightly woven braid, and the Kukolj design uses an open-meshed net. In nonleaded condition, there is a gap between the net and the membrane, which only disappears at a suitably high extending load.

1.2.2.4 Embedded Muscles

In embedded muscles, the load carrying structure of the actuators is embedded in its membrane. The Morin muscle by A. H. Morin [22], consists of a rubber tube embedded by threads of a high tensile stiffness. The threads can be arranged along the actuator's long axis or in a double helix about that axis, and the fiber material may be used like cotton, rayon, asbestos or steel. Depending on the working of operating pressure, three designs of Morin muscle have been shown in Fig. 1.8(a), namely, an overpressure design as shown in a longitudinal cross-section, an underpressure design, and a concentric membranes design. The Baldwin muscle by H. A. Baldwin [26], based on the design of Morin muscle [22], which consists of an elastomeric membrane, a very thin surgical rubber, embedded by glass filaments in the axial direction. The Under Pressure Artificial Muscle (UPAM) developed by K. Nazarczuk in 1964 at the Warsaw Polytechnic [21], exhibits similar design as the Morin muscle [22]. As gas is sucked out of the membrane it collapses in a non-axisymmetric way, i.e., it is squeezed and flattened in the middle.

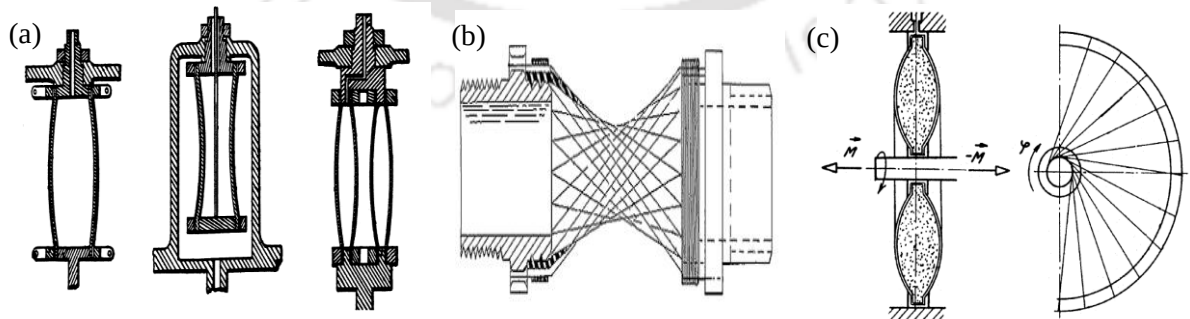


Fig. 1.8 Embedded muscle (a) Morin muscle [22] (b) Paynter Hyperboloid Muscle [16] (c) Kleinwachter torsion device [37].

The Paynter Knitted Muscle developed by H. M. Paynter [20] has a spherical bladder of elastomeric material that is reinforced by a knitted structure of strong, tough and flexible fibers. When fully inflated, the muscle takes on the shape of the original bladder and knitting sphere. In Fig. 1.8(b) shows the Paynter Hyperboloid Muscle, which is an alternative design described by H. M. Paynter [16]. The elastomeric membrane of the muscle is made with the neoprene rubber or polyurethane which takes the shape of a hyperboloid of revolution in its fully elongated state. The membrane is embedded by a sleeve of inextensible, flexible threads of metal wires, cord, polyester fibers or para-aramid fibers attached to the end fittings. Figure 1.8(c) shows the Kleinwachter torsion device by Kleinwachter and Geerk [33] where the inflatable membrane technique can be used in designing a torsional device, which refers to as a torsion muscle. The device has a toroid diaphragm embedded with stiffening filaments and attached at its outer edge to a ring-shaped structure along with its inner edge to a shaft.

Along with these classifications of the traditional PAMs, there are more advanced pneumatic muscles designed and developed in the literature [34-42]. An artificial muscle tube has been developed by Nakamura et al. [34] as shown in Fig. 1.9(a) in which high intensity glass fiber had been built into a natural latex rubber tube (Straight fibre type). Veale et al. [35] developed the Planar Fluidic Muscles, known as Peano muscles or pouch motors as shown in Fig. 1.9(b), which have the potential to provide the high force and compliance of McKibben muscles with the low threshold pressure of pleated PAMs. Fig. 1.9(c) shows a Soft Robotic Muscle (SRM) actuator fabricated by Oguntosin and Akindele [36] with the help of silicone rubber using a molding process. Han et al. [37] developed a High-Contraction Ratio Pneumatic Artificial Muscle (HCRPAM) as shown in Fig. 1.9(g), which consists of three parts viz., an elliptical tube that generates the expansion force, a diamond-shaped band that converts the expansion force of the elliptical tube into the output contraction force, and a two-stage spline that maintains the straight elongation of the elliptical tube. Wirekoh and Park [38] designed the Flat Pneumatic Artificial Muscle (FPAM) as shown in Fig. 1.9(e), which is made of a silicon substrate that contains embedded Kevlar fibers. Further, a modification has been done to the FPAM by integrating with force and position sensors, which results in a sensitized, Flat, Pneumatic Artificial Muscle (sFPAM) by Wirekoh et al. [39] as shown in Fig. 1.9(f).

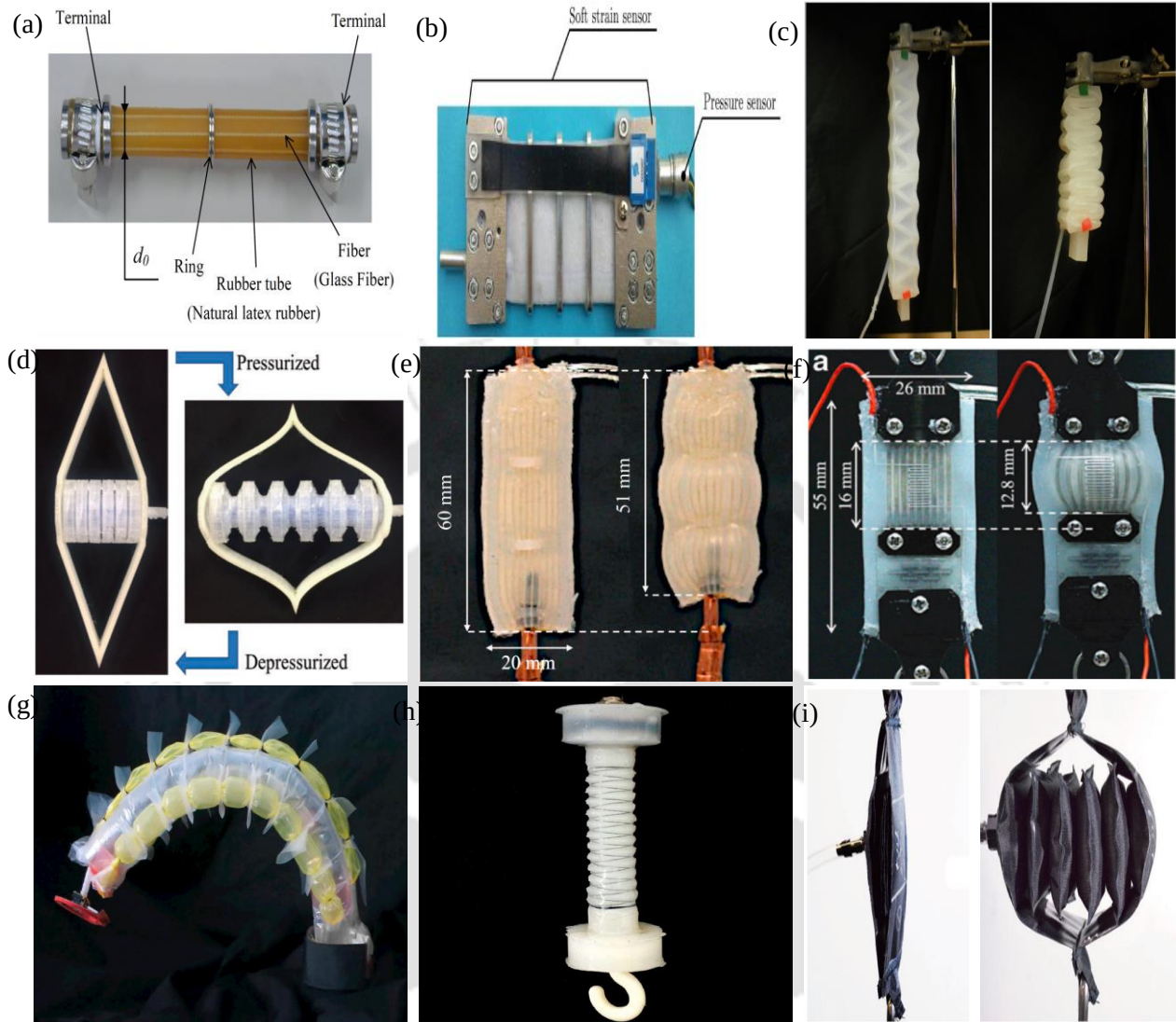


Fig. 1.9 Various advanced PAMs (a) glass fibers reinforced PAM [34] (b) Peano muscle [35] (c) SRM [36] (d) sPAM [37] (e) FPAM [38] (f) sFPAM [39] (g) HCRPAM [40] (h) rPAM [41] (i) high-displacement PAM [42].

Greer et al. [40] designed the series Pneumatic Artificial Muscle (sPAM) as shown in Fig. 1.9(g), which is made from a thin rectangular sheet of polyethylene tube that is sealed (heat bonded) on one end so that it inflates. Figure 1.9(h) shows the reverse Pneumatic Artificial Muscle (rPAM) by Skorina et al. [41], which consists of silicone rubber that is radially constrained by symmetrical double-helix threading. Yang et al. [42] designed a high-displacement pneumatic artificial muscle as shown in Fig. 1.9(i) is made of textiles or plastics that can include integrated electronics to sense its pressure and displacement. Depending on the various design of PAMs, number of applications can be found for the muscle actuators which have been presented in the next section.

1.2.3 Applications of Pneumatic Artificial Muscle

The first commercially available Pneumatic Artificial Muscle (PAM), which was manufactured and marketed by Bridgestone Co. in Japan, known as the Rubbertuators [43] in the late 1980s, was used in various application for some time. Nowadays, Shadow Group of Company from the U.K. [44] and Festo AG & Co. from Germany [45] are manufacturing various kinds of PAM for industrial robotics and medical applications [46]. The PAMs are very much suitable for multiple applications like exoskeletons along with rehabilitation assistance [5, 47, 48], grasping and handling delicate objects with complex geometry [49–51], the robotic mechanism where multiple sophisticated movements required [52–56], industrial and service robotics [5, 57, 58]. PAMs are mainly used as robotic actuators in applications where compliance and low power to weight ratio are essential. Winters [59] mentions sleeved bladder muscles as having potential as a suitable power system in prosthetics.

Caldwell et al. [9] used 18 small McKibben muscles with 90 mm long, 10 mm diameter and 5.5 gm to power a dexterous four-fingered manipulator. Each finger of the manipulator is powered by four muscles and the thumb by six muscles. In another work of Caldwell et al. [60], the authors reported about the McKibben muscle to power the elbow and wrist of an anthropomorphic arm along with full humanoid robot [61]. Shadow Group of Company used the twenty eight PAMs with fourteen on each leg for designing the Shadow Biped Walker [44]. Hannaford et al. [62] manufactured an anthropomorphic arm with the help of fifteen McKibben Muscles, called as Anthroform Biorobotic Arm and which is controlled by simulated spinal neural networks. Grodski and Immea [63] used ROMAC muscles to control a single degree of freedom (DOF) teleoperated arm with the help of myoelectric signals acquired from the biceps and triceps of the operator. Inoue [43] the Soft Arm powered by Rubbertuators, and it was developed by Bridgestone Co. The robot has a shoulder, an upper arm, a lower arm and wrist, 4 to 5 DOFs, and a useful payload of a maximum of 3 kg. Depending on the various properties of the muscle actuators, the applications of PAM can be found in the most three significant areas in the literature [5], namely a) Biorobotic, b) Medical, and c) Industrial applications. In the following subsections, the uses of PAM in these three applications are explained, starting with the various biorobotic applications using PAMs in the next subsection.

1.2.3.1 Biorobotic Applications

PAMs have been used in various biorobotic applications or biomimetic robots that imitate the morphology and physiology of humans and animals. As shown in Fig. 1.10(a), Kawamura et al. [64] designed a humanoid robot, Intelligent Soft Arm Control or ISAC, which consist of two six DOF arms and multiple PAMs that antagonistically actuated each joint of the arms. A six-legged insect-like robot, Airbug developed by Berns et al. [65] using fluidic muscles in the antagonistic arrangement. Daerden et al. [66] designed a hopping robot composed of a lower and upper leg, hip and body that slides along a guide shaft with pleated PAMs for the knee joint movement. Witte et al. [67] developed the four-legged prototype Pneumatically Actuated dyNamicallY sTable quadrupEd Robot (PANTER) using PAM, which has four active DOFs. Ajax, a cockroach, inspired robot with legs as shown in Fig. 1.10(b), was designed by Kingsley et al. [68], which is controlled with the help of several PAMs. The biped robot, Lucy, a two dimensional walking robot has been constructed by Verrelst et al. [69] using 12 pleated PAMs for 6 DOFs in two articulated legs each as shown in Fig. 1.10(c). Tsujita et al. [70] developed a three-legged robot with antagonistic pairs of PAMs, which is controlled by a nonlinear oscillator network.

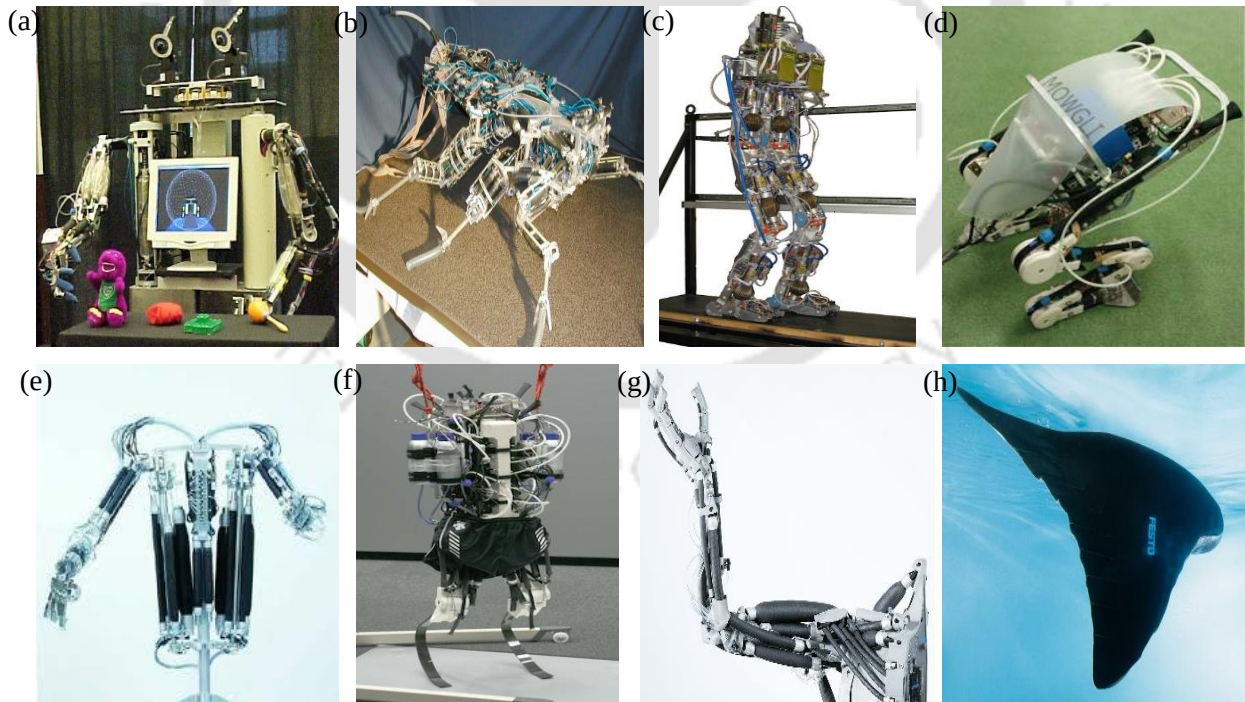


Fig. 1.10 Biorobotic applications of PAMs (a) ISAC [64] (b) Ajax [68] (c) Lucy [69] (d) Mowgli [71] (e) Zwei-Arm-Roboter [74] (f) Pneumatic Athlete Robot [75] (g) Airic's robotic arm [45] (h) Aqua ray robot [45]

Niiyama et al. [71] designed a bipedal jumping and landing robot as shown in Fig. 1.10(d), called Mowgli, which consists of six PAMs. Niiyama and Kuniyoshi [72] developed a bipedal robot with an Artificial Musculoskeletal System based on PAMs. Tsujita et al. [73] constructed and controlled a quadruped robot with PAMs in antagonistic pairs to drive the musculoskeletal system. A fully PAM actuated humanoid muscle robot torso as shown in Fig. 1.10(e), Zwei-Arm-Roboter developed by Boblan and Schulz [74] in human-like proportions and functionality. In Fig. 1.10(f), a Pneumatic Athlete Robot has been presented by Niiyama et al. [75] with musculoskeletal system driven by PAMs and applied human muscle activation patterns for dynamic bipedal running.

Andrikopoulos and Manesis [76] discussed the development and control of a vertical climbing robot, which is actuated with the help of four PAMs. Festo AG & Co. constructed two robots with the help of PAM, namely, Airic's robotic arm and Aqua ray robot [45]. Airic's robotic arm as shown in Fig. 1.10(g), depicts in with artificial bones and muscles where the bone structure is moved via 30 PAMs using very small valves based on piezo technology. In Fig. 1.10(h), the Aqua ray robot is a remotely controlled fish where three antagonistic pairs of PAMs move the two wings and the tail with the help of artificial tendons. Shadow Robot Co. presented the Shadow Robot Leg, which is a human-size PAM actuated robotic leg developed for investigation of myoelectric control of powered prosthetic legs [44]. In the next subsection, the uses of PAMs in the various medical applications have been discussed.

1.2.3.2 Medical Applications

Due to the presence of characteristics like high power to weights ratio, soft and flexible movement, stable motion speed and adaptable compliance in the PAM, it has been widely used in many medical applications. Along with these characteristics, PAM possesses similar properties with those of human muscle and safe human-robot interaction, which is essential for the therapeutic devices, rehabilitation, surgical equipment and neurological injuries [5]. Prior and White [77] discussed about the performance of a PAM actuated wheelchair mounted robot arm. Misuraca and Mavroidis [78] described the design and control of a Human Muscle Enhancer (HME) system, which is used for augmentation of the muscle capabilities of subjects requiring partial lower limb weight-bearing gait support.

Tsagarakis and Caldwell [79] studied the utilization and control of a pair of PAMs in antagonistic arrangement during the construction of dexterous prosthetic hands, which is used for strength augmentation. Figure 1.11(a) shows a PAM actuated prosthetic forearm, which has been developed by Wongsiri and Laksanacharoen [80, 81] with flexor and extensor groups of muscles. An armor type muscle suit actuated through PAM has been developed by Kobayashi and Hiramatsu [82] which provide the muscular support to paralyzed patients without attaching any metal frame. Herr and Kornbluh [83] designed a powered ankle-foot orthosis as shown in Fig. 1.11 (b) with the use of PAM for leg rehabilitation to provide single joint support to the plantar flexion in different phases of gait. RUPERT [84, 85], a therapeutic device as shown in Fig. 1.11(c), which had five DOFs and powered by four PAMs which can be able to provide a supplementing therapy in addition to the clinic treatment of patients suffering from upper extremity impairment. Vimieiro et al. [86] developed an exoskeleton system consisting of a hip orthosis equipped with PAMs to assist the lower limb movements. Ferris et al. [87-89] developed a powered ankle-foot orthosis as shown in Fig. 1.11(d) that utilized PAMs for assisting patients during gait rehabilitation after injury. Gupta and O'Malley [90] designed a five DOFs PAM actuated haptic arm exoskeleton for training and rehabilitation in virtual environments.

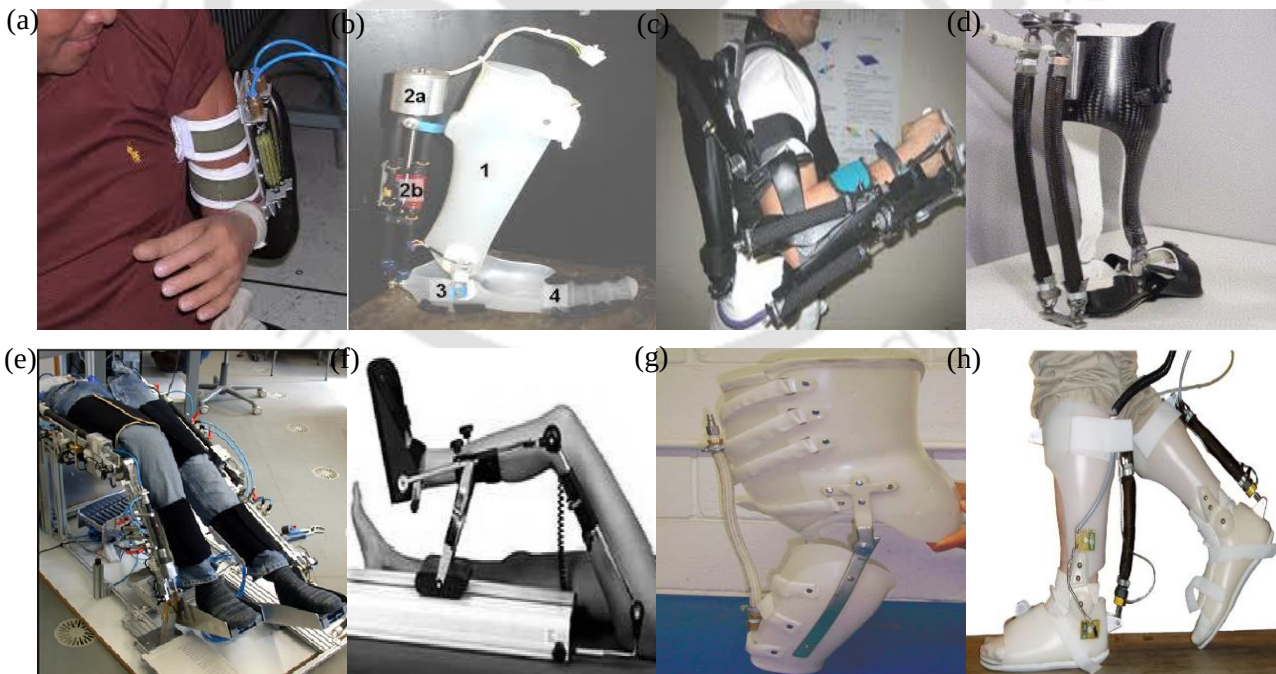


Fig. 1.11 Medical applications of PAMs (a) PAM-actuated forearm [81] (b) Powered ankle-foot orthosis [83] (c) RUPERT [84, 85] (d) Powered ankle-foot orthosis [87-89] (e) Orthosis for home training [95] (f) Isokinetic equipment [98] (g) Hip exoskeleton [101] (h) PAFO [106]

Tsagarakis and Caldwell [91, 92] developed compliant PAM actuated exoskeletons that permit the execution of upper and lower limb physiotherapy. Noritsugu et al. [93] described a wearable power assist glove driven by rubber PAMs which can assist the bending motion and increase the grasping force of the fingers. Thanh and Phuc [94] developed a knee rehabilitation device that is powered by PAMs. In Fig. 1.11(e), a novel motorized orthosis actuated through PAM was presented by Knestel et al. [95] for intensive home-based gait training in patients with neurological disorders. Kadota et al. [96] constructed a power-assisted robot arm that utilized rubber PAMs covered with exoskeleton suits to mimic the motion of biarticular muscles. Wickramatunge and Leephakpreeda [97] developed a robotic rehabilitation system actuated through PAMs with biofeedback arrangement. Deaconescu and Deaconescu designed a PAM actuated isokinetic equipment as shown in Fig. 1.11(f) for recovery exercises of the hip and knee joints [98] and a robotized arm for persons with locomotive disabilities [99]. Yeh et al. [100] developed a lower-limb pneumatic muscles exoskeleton system with the knee f/e and keeping the ankle joints passive.

Figure 1.11(g) shows a hip exoskeleton for gait support, which has been designed by Nascimento et al. [101] with pelvic braces and thigh supports. These are actuated with pneumatic artificial muscle connected to the hip joint of the user. Lewis and Ferris [102] designed an exoskeleton which comprises of a pelvic part and thigh cuffs being attached to the hip joint for f/e motions. It is actuated by a pair of pneumatic muscles placed on the lateral side, which provides passive a/a. A pneumatic actuated knee exoskeleton has been proposed by Kim et al. [103] with two PAMs that mimic the functions of rectus and biceps femoris muscles. Kao et al. [104, 105] developed an ankle-foot orthosis (AFO) for in-lab studies at the University of Michigan using two PAMs and analyzed the ankle d/p motion. Like AFO, a pair of powered ankle-foot orthosis (PAFOs) as shown in Fig. 1.11(h) has been designed with pneumatic muscles for ankle d/p assistance by Norris et al. [106]. Leclair et al. [107] developed an ankle-foot exoskeleton powered by a PAM to obtain the passive ankle d/p motion. Kim et al. [108] designed another AFO to provide ankle plantarflexion motion for elderly people, which comprises of a basic single DOF exoskeleton and an artificial pneumatic actuator. Malcolm et al. [109, 110] developed an AFO with pneumatic muscles to study the effects of an active AFO for human movements. The uses of PAMs in various industrial applications have been discussed in the next subsection.

1.2.3.3 Industrial Applications

Due to the presence of characteristics in the PAM, such as generating high torques at low and moderate speeds, essential compliance for installation, less mechanical wear, and working as the actuator of portable machinery due to their lightweight properties, it is attracted towards the many industrial applications. The PAMs are a suitable solution for the actuation of industrial machinery and particularly industrial robots due to their safe human-robot interaction and shock resistance ability. Pomiers [111] developed a modular robot with a single DOF pneumatic element and a three DOFs robot arm using rubber PAMs. In Fig. 1.12(a), a Robot Arm System designed by Kawashima et al. [112] actuated through rubber PAM, which has six DOFs. Damme et al. [113, 114] presented a two DOFs planar robotic manipulator, as shown in Fig. 1.12(b), which is powered by pleated PAM for assisting in handling heavy loads. Caldwell et al. [115] proposed a prototype design of a teleoperation rig for retrieval of radioactive material, which is a combination of the traditional man-handled manipulation pole and PAMs. PAMs have been used for the identification of the modal parameters of bridges by Deckers et al. [116].

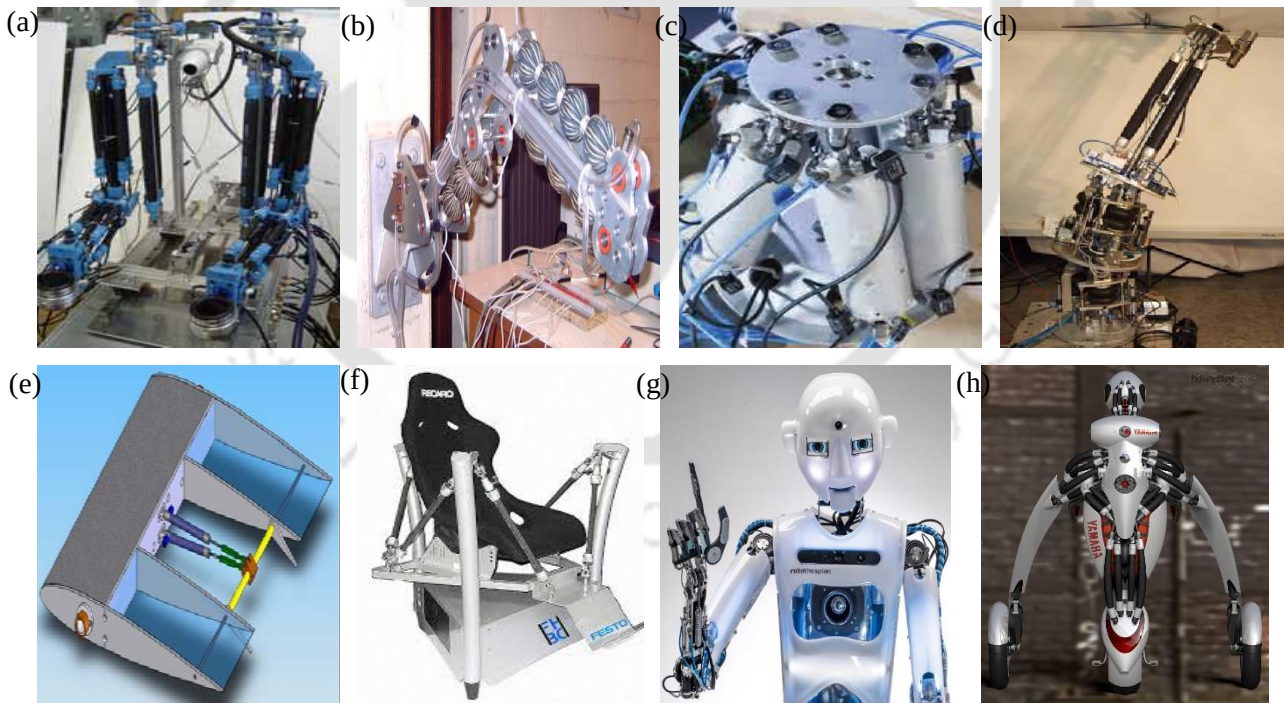


Fig. 1.12 Industrial applications of PAMs (a) PAM-actuated robot arm system [112] (b) 2 DOFs planar robotic manipulator [113] (c) parallel-kinematic hexapod tool [117] (d) hybrid robot [118] (e) PAM-activated trailing edge flap [120] (f) FM Motion Seat [122] (g) RoboThespian [123] (h) Deus Ex Machina [124]

Wedler and Denkena [117] developed a parallel–kinematic hexapod tool, which is driven by six PAMs in antagonistic arrangements with pressure and position sensors, as shown in Fig. 1.12(c). Radojicic et al. [118] developed a prototype of a hybrid robot as shown in Fig. 1.12(d), which included two bellows and one PAM actuated module for safe human-robot interaction. Brown et al. [119] developed the AGAS system, an autonomously controlled deployable airdrop system that consisted of a round parachute and controlled by four PAMs. Yerkes and Wereley [120] described a PAM activated trailing edge flap for flight control as shown in Fig. 1.12(e). In another work, Wereley et al. [121] presented the applications of PAM for the the actuation schemes of a morphing cell for a wing section and trailing edge flaps for wings or rotorcraft blades.

Pohl [122] developed the FM Motion Seat as shown in Fig. 1.12(f), a driving and flight simulator based on a hexapod structure with 6 spatially oriented PAMs moved the seat in all the 6 axes and acted as a passive suspension. In Fig. 1.12(g), Engineered Arts Limited from U.K. [123] developed the RoboThespian; PAM actuated humanoid robot, which is created for education, communication, interaction and entertainment. Jake Loniak from Art Center Pasadena, California [124] designed the Deus Ex Machina project as shown in Fig. 1.12(h), which is an electric, single passenger, vertically parking, and wearable motorcycle. With seven artificial vertebrae behind the helmet, for supporting the rider's head, the control of the Deus Ex Machina is achieved by 36 PAMs and 2 linear actuators combined with the rider's body.

In addition to these applications, many indirect applications of PAMs in medical engineering can be found [55, 56, 125-130]. Li et al. [125] developed a compact and lightweight forceps' manipulator for laparoscopic surgery with PAMs as shown in Fig. 1.13(a), which allows for the perception of manipulation force without the usage of the force sensor. Ashwin and Ghosal [55, 56] designed an endoscopic attachment which can be used to independently position an endoscopic catheter tip at a desired location with the help of three miniaturized pneumatic artificial muscles (MPAMs). Each MPAMs used in the device is of 1.2 mm diameter and 45 mm in length and applied a pressure of 137–827 kPa to one or more muscles. Due to the small size, these MPAMs are mainly found in the various medical applications like wearable hand exoskeletons in Fig. 1.13(b) [126], cardiac compression devices [127], and tool manipulation in surgical devices as shown in Fig. 1.13(c) and Fig. 1.12(d) [128-130]. The different mathematical

modeling of PAMs available in the literature, has been discussed in the next section to better understand the dynamics and control of the PAM used in various applications.

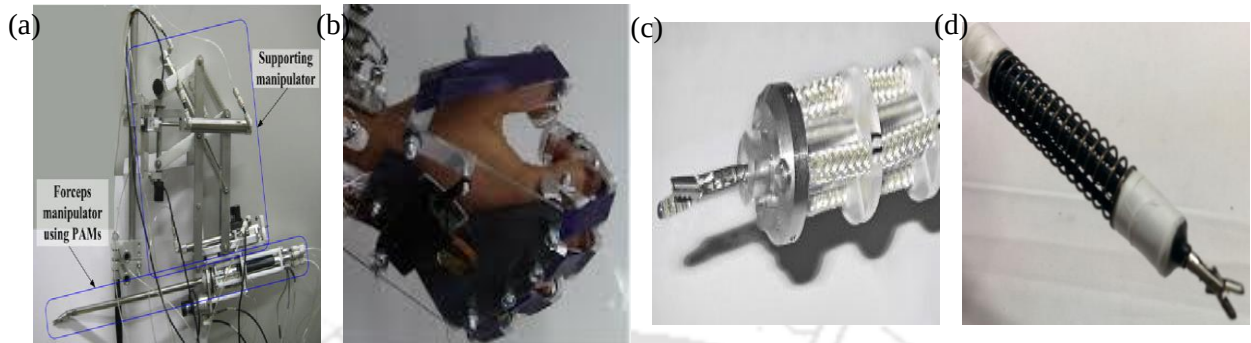


Fig. 1.13 Indirect application of PAMs in medical engineering (a) forceps' manipulator for laparoscopic surgery [125] (b) wearable hand exoskeletons [126] (c) instrument for keyhole surgery [128] (d) end-effector with forceps [130]

1.2.4 Modeling of Pneumatic Artificial Muscle

There are many literature available regarding the mathematical modeling of PAMs. The basic of the modeling is the relation between the pressure and the length of the PAM actuator to the exerted force along its entire axis. The muscle force is primarily considered a function of operating pressure, material properties, and dimensions of the muscle, which play an important role in the PAM dynamical behavior. Therefore, the mathematical modeling of PAM is very important in understanding the relationship between the factors for various applications and controlling the overall functioning of the muscle. But, PAM exhibits strong nonlinear force-length characteristics, making it challenging to model and control along with to obtain the demanded performance features [30, 60]. Kelasidi et al. [131] explained the most common and important mathematical modeling of PAM found in the literature.

1.2.4.1 Geometrical Model of PAM

There are two geometric models of PAM, which are the most common and widely used, namely the Chou and Hannaford model [1] and the Tondu and Lopez model [2]. The Chou and Hannaford model [1] is the simplest geometrical model for static performance of the PAM. With this approach, the PAM has been modeled as a cylinder as shown in Fig. 1.14, with length L , thread length b , diameter D and number of thread turns n . The angle θ is defined as the angle of the

threads with the longitudinal axis. Utilizing the energy conservation principle, the simple geometric force of PAM, F_g is calculated, by multiplying the gauge pressure P with the change in volume with respect to length as follows [132].

$$F_g = \frac{Pb^2 \left(\frac{3L^2}{b^2} - 1 \right)}{4n^2\pi} \quad (1.1)$$

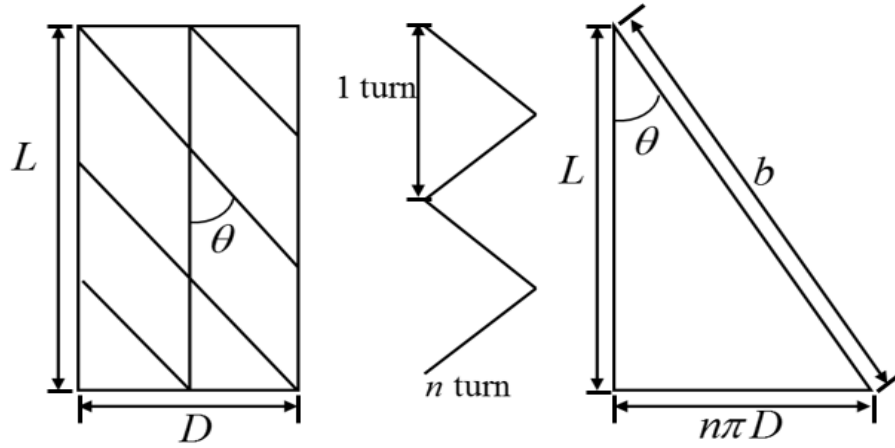


Fig. 1.14 Simplified geometrical model of PAM.

Another widely used geometrical model of PAM is the Tondu and Lopez model [2], which is based on the virtual work theorem. The expression for the contraction force F generated by the muscle which is a function of the control pressure P and the contraction ratio ε , is written as follows [2, 133, 134].

$$F(\varepsilon, P) = \pi r_0^2 \times P \left[\alpha (1 - \varepsilon)^2 - \beta \right] \quad (1.2)$$

where, $\varepsilon = \frac{l_0 - l}{l_0}$, $0 \leq \varepsilon \leq \varepsilon_{\max}$, $\alpha = \frac{3}{\tan^2 \theta_0}$, $\beta = \frac{1}{\sin^2 \theta_0}$ and r_0 is the nominal inner radius, l is the deformed length of the muscle, l_0 is the initial nominal length, P is the pressure and θ_0 is the initial angle between the membrane fibres and the muscle axis. A disadvantage of the model is its design, which based on the hypothesis of a continuously cylindrical shaped muscle. In contrast, it takes a conic shape at both ends when it contracts. Consequently, the more the muscle contracts, the more its active part decreases.

This phenomenon results in the actual maximum contraction theoretically being smaller than that expected from Eq. (1.2) [135]. Hence, to improve Eq. (1.2), Tondu and Lopez [2] considered an empirical factor k in addition to account for the end deformation of the PAM as follows.

$$F(\varepsilon, P) = \pi r_0^2 \times P \left[\alpha (1 - k\varepsilon)^2 - \beta \right] \quad (1.3)$$

where, $0 \leq \varepsilon \leq \varepsilon_{\max}$ and $\varepsilon_{\max} = \left(\frac{1}{k} \right) \left(1 - \sqrt{\beta/\alpha} \right)$. By inserting the parameter k within the considered static model, the value of the maximum force given at zero contraction does not get affected. This is in concordance with the conducted experiment since the PAM has a cylindrical shape only when its contraction ratio is zero.

1.2.4.2 Biomimetic/Biomechanical Model

There is a method of PAM modeling which is popularly known as biomimetic or biomechanical model due to similarities with the biological muscle discussed in the literature [24]. Basically, PAM is a composite of a rubber mold with embedded structural fibers, which creates a force in the longitudinal direction extending the circumferential direction in pressurized condition. A phenomenological approach is considered by Serres et al. [136, 137] to understand the dynamics of PAMs by modeling it with three elements: a spring element, a damping element, and a contractile element in parallel as shown in Fig. 1.15.

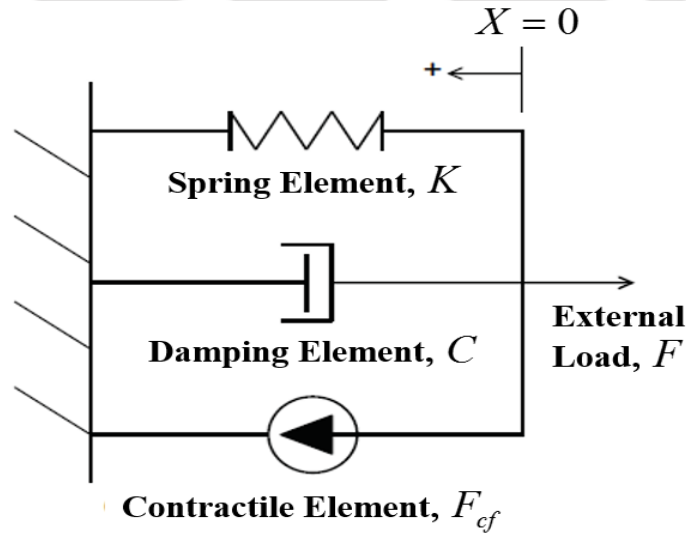


Fig. 1.15 Biomechanical model of PAM.

In Fig. 1.15, X is the displacement of PAM, C is the damping coefficient, F_{cf} is the contractile force coefficient, K is the spring coefficient of PAM and F is the external load. Due to the low mass of the moving parts of the system, it is neglected by the authors in the formulation [136]. Further, by taking the Laplace transform and setting the initial conditions to zero, the transfer function of the phenomenological model of PAM is derived as follows [137] where, X_d is the desired displacement of the PAM and $\tau = (C/K)$.

$$\frac{X(s)}{X_d(s)} = \frac{\left(\frac{1}{\tau}\right)}{s + \left(\frac{1}{\tau}\right)} \quad (1.4)$$

1.2.4.3 Empirical Model of PAM

PAM with different lengths and diameters can be modeled with the help of simplified empirical models. In this approach, The stretched length is zero when the air pressure inside PAM only the atmospheric pressure, and this length increases when the air pressure also increases [97]. The natural mechanical operation of the PAM and the mechanical spring exhibit similar behavior when a tensile force is being applied to them as shown in Fig. 1.16. Usually, the stiffness of the spring is fixed and depends on the material properties and geometry of the spring whereas, the stiffness of PAM is variable and also depends on the above properties in addition to the air pressure inside muscle.

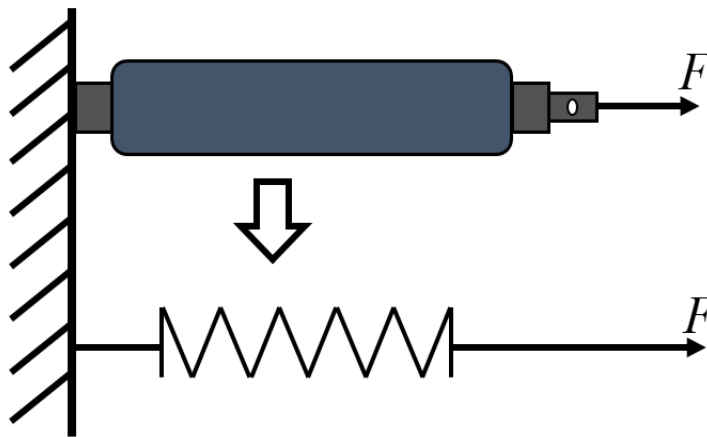


Fig. 1.16 Comparison of PAM and spring system.

Based on the experimental observation by Wickramatunge and Leephakpreeda [138, 139], the pulling force acting on PAM can be modeled as similar to the force acting on a mechanical spring where the stiffness parameter of PAM can also be specified as K . The stiffness parameter K is the function of the operating gauge pressure P and the stretched length of the PAM L_s . The elastic force generated by PAM, $F_{elastic}$ is given below.

$$F_{elastic} = K(P, L_s) L_s \quad (1.5)$$

The stiffness K explicitly has been considered as a second order polynomial of P and L_s which is represented as follows.

$$K = q_1 + q_2 L_s^2 + q_3 P L_s + q_4 P^2 \quad (1.6)$$

where the constant parameters q_1 , q_2 , q_3 and q_4 can be found from experimental data by applying the method of least squares. Further, higher degree polynomials can be applied to obtain the desired precision requirement.

In another experimental work by Li et al. [125], the force-contraction quasi-static characterization of the PAM is considered along with a linear least squares fit at each operating pressure. The muscle force F_{mus} is estimated to vary as a linear function of the contraction ε and operating pressure P as follows.

$$F_{mus}(\varepsilon, P) = (\alpha_1 + \alpha_2 P + \alpha_3 P^2) \varepsilon + \beta_1 + \beta_2 P \quad (1.7)$$

where, α_1 , α_2 , α_3 , β_1 and β_2 are the experimental constant parameters which are obtained from the least squares fit.

Along with these, various other novel methods have been proposed in the literature for the analysis of PAM to establish accurate empirical models. A PAM motion caused by a nonlinear force that mainly depends on parameters like pressure and deformation is obtained analytically by applying the optimal parameter identification method. This method has finally been proved and tested on an experimental rig by Sárosi et al. [140] and Palomares et al. [141]. Cullinan et al. [142] and Cveticanin et al. [143] have investigated the steady state position of the mass of the system and the force influenced by the pressure of the PAM mathematically and experimentally. Colbrunn et al. [144] studied the static and dynamic mathematical model for the braided pneumatic actuator. Tóthová et al. [145] used two PAMs for antagonistic motion connected through the chain gear

and investigated based on modified Hill's basic model. Hildebrandt et al. [146] presented a nonlinear cascaded control strategy to move a trolley actuated through pneumatic muscle actuators to follow a reference path. Carneiro et al. [147] used two servo valves for pneumatic force actuation. Tothova et al. discussed the analytical model of muscle force based on the measured values [148].

In cases of higher positioning requirements and motion in two opposite directions needs, two PAMs in an antagonistic connection highly preferred, just like in real muscles in the living bodies [140, 149-152]. Šitum and Trsljić [152] have discussed a nonlinear mathematical model for a PAM pair in an antagonistic arrangement based on an original and unique ball-on-beam system. Liu et al. [153] and Xie et al. [154] are used the advanced technique like the modified Prandtl–Ishlinskii (MPI) model to control the inherent hysteresis nonlinearity for the accuracy of trajectory tracking control of PAM. Other advanced force models for the PAM have been mentioned, which are discussed in the literature such as, the approximate deducted model using the maximum force of PAM [155], the polynomial function model [156] and the exponential function model [157].

Ashwin and Ghosal [4] presented a discussion on some more different empirical models from the literature and the key equations for the muscle force has been discussed as in Table 1.2. In Table 1.2, l_0 , r_0 and t_0 represent length, outer radius and thickness of the bladder before deformation and after deformation these change to l , r and t respectively. The initial inner radius of the bladder represents by $r_i = r_0 - t_0$ whereas the initial and final winding angles of the braid are denoted as θ_0 and θ respectively. The symbols N , m and b represent the number of turns of the braid along the length, number of strands of braid as well as the length of a single braid strand, respectively. P is the input pressure, F represents the muscle force along with the strain in the bladder along the axial direction is $\varepsilon = \left(l_0 - \frac{l}{l_0} \right)$. All other unknown parameters used in the equations are considered to be constant in general.

Table 1.2 Various models with muscle force equation of PAM.

Author	Muscle Force Equation
Schulte [27]	$F = \frac{b^2}{4\pi N^2} P(3\cos^2 \theta - 1) + \frac{bE}{N} \left[l_0 \sin \theta - \frac{\cos^2 \theta}{\sin \theta} \left(\frac{b}{N} \sin \theta - 2\pi r_0 \right) \right]$ $- \frac{l_0 b}{N} (P - P_u)(\mu_s + \mu_{st}) \sin \theta$
Carlo Ferraresi et al. [158]	$F = \frac{P}{4\pi N} (3l^2 - b^2) + \sigma_1 \frac{V_b}{l} - \frac{\sigma_2 t l^2}{2\pi N^2 r}$
Liu and Rahn [159]	$F = 2\pi E r_0 \left(\frac{2\sigma_2(0)}{\lambda_2(0)} - \frac{PE}{r_0} \right), l_0 = - \int_{\lambda_2(0)}^1 \frac{d\lambda_2}{\lambda_1 \sin \gamma(\lambda_2)}$
Delson et al. [160]	$F = P \frac{dV}{dl} + V_b \frac{dW}{dl}$
Davis and Caldwell [161]	$F = \pi r_0^2 P \left[q_1 (1 - k_\varepsilon \varepsilon)^2 - q_2 \right] \pm \mu_s P \left[\frac{S_{contact}^{(2)}}{S_{scale}^{(2)}} \right]$
Trivedi et al. [162]	$F = \pi E (r_0^2 - r_i^2) (\lambda_1 - 1)$
Doumit [163]	$F = m \cos \beta \cos \theta \left[\left(\frac{P(r - t_0 - 2r_n) - \sigma_1 t_0}{mN \sin \theta} \right) l_{cyl} - \frac{P(r - 2r_n - t_0)^2}{2mr \cos \theta} \frac{l}{N} \right]$ $+ \left[\frac{P(r + r_c - 2t_0 - 2r_n) \cos \beta - 2\sigma_1 t_0}{mN \sin \left(\frac{\theta + \theta_c}{2} \right)} l_{cone} \right] - PS_{contact}^{(2)} \mu_s$
de Volder et al. [128]	$F = \max \left[\left(F_{\min}, \frac{(P - P_{corr})b^2}{4\pi N^2} \right) \left(\frac{3(l - l_{corr})^2}{b} - 1 \right) \right] + \max [0, k_b (l - l_{b0})]$
Chen and Ushijima [164]	$F = \frac{\pi P}{4} \left(\frac{b}{N\pi} \right)^2 (3\cos^2 \theta - 1) \mp \frac{4m^2 N}{b \sin \theta} (M_f \pm M_d \pm M_r \pm M_{tr})$
Wang et al. [165]	$F = F_z - P\pi r_i^2 - \frac{Pr_i l^2 - F_\theta l^2}{2\pi N^2 r_0^2}$
Hocking and Wereley [166]	$F = \frac{P}{4\pi N} (3l^2 - b^2) + \frac{V_b}{l} \sum_{i=1}^n E_i \left(\frac{l}{l_0} - 1 \right)^i - \frac{t l^2}{2\pi N^2 r} \sum_{i=1}^n E_i \left(\frac{r}{r_0} - 1 \right)^i$
Sangian et al. [167]	$F = \pi r_0^2 [q_1 (1 - \varepsilon^2) - q_2] \times \left[P - P_{th} + \frac{Et_0 b}{2\pi N r_0^2} \left(\left(1 - \frac{l^2}{l_0^2} \cos^2 \theta_0 \right)^{\frac{1}{2}} + \sin \theta_0 \right) \right]$
Andrikopoulos et al. [168]	$F = r_0^2 P k_f^{(2)} \left[q_1 (1 - k_\varepsilon^{(2)} (\varepsilon + \alpha_l \Delta T))^2 - q_2 \right]$ $- \left(\frac{P \operatorname{sgn}(\nu)}{S_{scale}^{(1)}} \right) \frac{2\pi r_0 l_0 \mu_s \sin \theta_0}{(1 - k_\varepsilon (\varepsilon + \alpha_l \Delta T)) \sqrt{1 - \cos^2 \theta_0 (1 - k_\varepsilon (\varepsilon + \alpha_l \Delta T))^2}}$
Ball and Garcia [169]	$F = F_{strands} + F_{pressure} + F_{elastic} \pm \mu_f P_{eff}$

A highly nonlinear nature is present in the PAM due to the air compressibility, valve actuation, viscoelastic material properties and load contraction characteristics due to flexible structure. The other factors like the irregular shape of the rubber bladder geometry [169], elastic energy [162, 165, 170, 171] present in the bladder and braid along with threshold pressure [172] also contribute to the nonlinear characteristics of the PAM. So, to include these nonlinearities, some specific models have been presented in the literature. For determining accurate values for constants in nonlinear models often requires precise experimentation. So, by considering the material properties precisely along with an extensive computation, the nonlinear material modeling may provide better accuracy. A nonlinear Mooney-Rivlin material [173, 174] is considered for the elastic properties of the bladder, and the stress components from strain energy density functions are used in the force balance equations. Considering this material Kothera et al. [171] derived the following muscle force expression by using an energy balance method.

$$F = \frac{P}{4N^2\pi}(3l^2 - b^2) - V_b \left[2C_{10} \left(\lambda_1 \frac{d\lambda_1}{dl} + \lambda_2 \frac{d\lambda_2}{dl} + \lambda_3 \frac{d\lambda_3}{dl} \right) + 2C_{01} \left(\lambda_1 (\lambda_2^2 + \lambda_3^2) \frac{d\lambda_1}{dl} + \lambda_2 (\lambda_3^2 + \lambda_1^2) \frac{d\lambda_2}{dl} + \lambda_3 (\lambda_1^2 + \lambda_2^2) \frac{d\lambda_3}{dl} \right) \right] - \frac{P^2 b^3 l}{4\pi^2 m r_n E_b N^4} \quad (1.8)$$

where E_b is Young's modulus and λ_1 , λ_2 and λ_3 are the principal stretch of braiding Mooney-Rivlin material [173, 174]. Later, this model is used by Woods et al. [175] for generating large trailing edge flap in a helicopter with the help of PAM. Wang et al. [165] described a static model of PAM by using Mooney-Rivlin material where Hoop force F_θ and axial force F_z acting on bladder during inflation are found analytically in terms of λ_1 and P . Thus from the static force balance equations from braid, the muscle force can be obtained as below.

$$F = F_z - P\pi r_i^2 - \frac{Pr_i l^2 - F_\theta l^2}{2\pi N^2 r_0^2} \quad (1.9)$$

Some other nonlinear material like neo-Hookean material [176] is also considered for the bladder of PAM in the work of Trivedi et al. [162] modeling and the axial muscle force expression can be written as follows.

$$F = \pi E (r_0^2 - r_i^2) (\lambda_1 - 1) \quad (1.10)$$

In the work of Ball and Garcia [169], the authors considered the thin wall approach in modeling of PAM and the force expression is given as below.

$$F = F_{strands} + F_{pressure} + F_{elastic} \pm \mu_f P_{eff} \quad (1.11)$$

where,

$$F_{strands} + F_{pressure} = \frac{P_0 l^2}{2\pi N^2} - P \left(\frac{b^2 - l^2}{4\pi N^2} - \frac{V_b}{l} \right),$$

$$P_0 = P - \frac{\sigma_2 (r_0 - r_i)}{\lambda_1 \lambda_2 \sqrt{\frac{b^2 - l^2}{n^2 \pi^2} - \frac{4V_b}{\pi l}}}, F_{elastic} = \frac{\sigma_1 V_b}{l} \quad (1.12)$$

with P_0 is the acting pressure at the outer radius by the bladder on the sleeve and σ_1 and σ_2 are the material constants and stretch ratios. In the case of the thick-walled bladder as well as prestrained bladders, the thin wall tube model is applied successively as if the thick cylinder is an array of concentric nested thin tubes. The computational method calculates the pressure P_0 of the innermost layer and works sequentially outward. To understand the various nonlinear characteristics present in the PAMs precisely, in the next section, nonlinear dynamics under different excitations have been discussed with examples from the literature.

1.2.5 Nonlinear Dynamic Analysis

As explained in the previous sections, the governing equation of the system containing artificial muscle can be written as a second order nonlinear differential equation. It may provide harmonic excitation or any other type of periodic or nonperiodic excitation. In the case of harmonic excitation, the amplitude may be small or it may be very large leading to a weak or hard excitation. The frequency of excitation may give rise to many types of resonance conditions. Further, the system may be similar to that of a parametrically excited system where the coefficients of the response contain a time varying terms. It has been observed that though a large number of work are available to study the nonlinear responses for several other dynamical systems, a few works are available in the field of PAM. Hence, in the next four subsections, the related works are briefly reviewed.

1.2.5.1 Harmonic Excitation

Harmonic excitation refers to a sinusoidal external force of a specific frequency applied to a system. The response of a system to harmonic excitation is an important topic because it is encountered very commonly and also covers the concept of resonance. Resonance occurs when the external excitation has the same frequency as the natural frequency of the system, which leads to simple resonance condition. Harmonic excitation is also commonly observed in systems that contain a rotating mass, for example, tires, engines, rotors, etc. Ishida and Inoue [177] studied the detection of a rotor crack based on the nonlinear vibration diagnosis using a harmonic excitation force. Calio and Greco [178] investigated the nonlinear dynamic behavior of a flexible rocking structure subjected to harmonic excitation. It is characterized by a sequence of full contact and rocking phases, which may be halted by the overturning of the system. Li et al. [179] presented a harmonically excited structure coupled with the nonlinear energy sink (NES) and a piezoelectric harvester where the bifurcation study has been explored by varying different system parameters.

The dynamic response of a laminated composite beam subjected to a harmonic excitation has been investigated by Kiral [180] with the help of the Newmark method, which is a numerical time integration method. Ramnarace and Bridge [181] analysed the super elasticity effect of shape memory alloy (SMA) spring system, which is modeled using a piecewise linear force–displacement curve and included a geometric nonlinearity. Chen and Yang [182] studied the nonlinear vibration of a shallow arch with one end attached to an electro-mechanical shaker, which generates a harmonic magnetic force on the central core of the shaker by controlling the electric current. Argüelles and Casanova [183] presented a numerical approach to study the steady-state response amplitudes of a piping system subjected to harmonic excitations and considered dry friction between the pipe and the support surfaces. Dhar and Barrett [184] investigated the method for calculating the control forces, the bearing stiffness and damping coefficients to control the response of multi-mass flexible rotor systems mounted on magnetic bearings which is subjected to unbalance or harmonic excitation forces.

Kiran et al. [185] studied both experimentally and numerically the inelastic dynamic behaviour of a pressurized stainless steel elbow under harmonic base excitation with the emphasis on strain accumulation, which is known as ratcheting. Moghaddam and Ahmadi [186] investigated active

vibration control of functionally graded truncated thin conical shell, which is subjected to the lateral harmonic excitation with the help of a semi-analytical method. Fahsi and Belhaq [187] studied the effect of high-frequency harmonic excitation on the entrainment area of the main resonance in a van der Pol–Mathieu–Duffing oscillator. In another study by Bourkha and Belhaq [188], the authors investigated the effect of horizontal and vertical fast harmonic parametric excitation on Van der Pol pendulum's periodic self-excited motion. In the next subsection, the systems under parametric excitation have been studied.

1.2.5.2 Parametric excitation

Parametric excitation is the increasing character of oscillation caused by the periodic modulation of the system [13, 14]. In this case, the state of equilibrium becomes unstable because of the progressively increasing amplitude, and the system leaves it executing oscillations [189]. It is originated in a narrow excitation frequency band and has a very high response amplitude compared to the excitation amplitude. Because of these features of the parametric resonance, it has been thoroughly examined and is used in various electromechanical systems like in robotic manipulator [190, 191], energy harvester [192-195], MEMS [196, 197]. In all these cases, principal parametric resonance conditions have been studied where the instabilities occur to the trivial equilibrium with the external excitation frequency nearly equal to twice the natural frequency of the system in the absence of any dissipative effects.

Pratihari and Dwivedy [190] investigated the nonlinear response of a single-link flexible Cartesian manipulator with payload subjected to a pulsating axial load. Ju et al. [191] studied the nonlinear modeling and dynamic stability analysis of a flexible Cartesian robotic manipulator with the influence of driving-based disturbance and considering the terminal load. He et al. [192] investigated the nonlinear analysis of piezoelectric nanocomposite energy harvesters under principle parametric resonance condition in prebuckling and postbuckling domains. Dwivedy et al. [193] studied the dynamics of an energy harvester by modeling as a base excited cantilever beam with tip mass having piezoelectric patches on both top and bottom surfaces. Further, Garg and Dwivedy [194, 195] presented both theoretical and experimental investigation of a vertical cantilever beam-based piezoelectric energy harvester, which is a parametrically excited system. The authors attached the piezoelectric patch near the fixed end of the cantilever beam along with

an attached mass positioned at an arbitrary location. Caruntu and Reyes [198, 199] investigated the nonlinear dynamics of the electrostatically actuated micro-electro-mechanical systems (MEMS) cantilever, which undergoes parametric resonance and included the fringe effect for the electrostatic force. The MEMS consists of a cantilever over a parallel ground plate and an AC voltage between them. In another work, Caruntu and Juarez [200] studied the amplitude-frequency response of coaxial parametric resonance of electrostatically actuated double-walled carbon nanotubes (DWCNTs). Belhaq and Sah [201, 202] investigated the interaction effect of fast vertical parametric excitation and time delay on a van der Pol oscillator's self-oscillation.

Along with these applications, in many practical cases due to nonlinear nature present in the system, it may be subjected to simultaneous resonance conditions i.e., simultaneously, the system is subjected to principal parametric excitation along with some additional excitation. Because of these features of simultaneous resonances, it has been discussed thoroughly in the literature for various electromechanical applications. Fang et al. [203] studied the nonlinear dynamics of cantilevered piezoelectric energy harvesters under simultaneous parametric and external excitations. Ostiguy et al. [204] presented the occurrence of simultaneous resonances in parametrically excited rectangular plates where the analysis is based on the dynamic analogy of von Karman's large-deflection theory. Qaderi et al. [205] and Hosseini and Khadem [206] studied the dynamic response of a rotating shaft with geometrical nonlinearity under parametric and external excitations where external excitation is due to shaft unbalance and parametric excitation is due to periodic axial force. To apply the simultaneous resonances in practical applications, a more precise analysis of the motion is required. The systems that undergo hard excitation have been discussed in the next subsection.

1.2.5.3 Hard Excitation

It may also be noted that in most of the literature, the forcing is considered to be weak, i.e., the magnitude of the forcing term is very less than that of the coefficient of the linear term. Hence, an attempt has been made to study the hard excitation where the forcing amplitude is higher than the coefficient of the linear term. Beatriz and Caruntu investigated the amplitude-frequency response of parametric resonance of electrostatically actuated circular plates [207] and MEMS [208] under hard excitations. Abusoua and Daqaq [209, 210] demonstrated the dependence of the

effective nonlinearity of the slow dynamics on the hard excitation magnitude and highlighted the essential design parameters to control the behavior. Botello et al. [211, 212] studied the nonlinear dynamics of MEMS under superharmonic resonance, and the magnitude of the AC voltage is large enough to be considered as hard excitation. But in most of the cases of hard excitations, the system has been mainly subjected to superharmonic and subharmonic resonance conditions.

The response behavior of the various dynamical system at superharmonic and subharmonic resonance conditions have attracted many researchers. The response behavior of these resonance conditions for different applications can be found in the literature [15-34]. Krishnan [213] studied the sub and superharmonic resonance conditions for strongly-excited non-linear systems with the help of the difference equations. Elnaggar and El-Bassyouny [214] studied the super and subharmonic resonance conditions along with the combination resonances of the additive type of self-excited two coupled-second order systems subjected to multi-frequency excitation. Bichri and Belhaq [215] investigated these resonance conditions for the effect of fast harmonic base displacement with periodically time varying stiffness on Vibro impact dynamics of a forced single-sided Hertzian contact oscillator.

Davis and Rajan [216, 217] discussed the random sub and superharmonic resonance conditions extensively with the help of a Duffing oscillator. Cai and Yang [218] and Zhang and Yang [219] studied the bifurcations and chaos in Duffing Equation for the super and subharmonic resonance conditions. An extensive study has been done for Duffing's equation in the literature for these resonance conditions to the bifurcations and stability [220-223]. The responses for super and subharmonic resonance conditions have been presented in the literature with the help of some applications like in spur gear [224], cracked beams [225, 226], piezoelectric sandwich beams [227], elastic cables [228], Cartesian manipulator [229], wind turbine blades [230] and marine applications [231]. To understand the systems under different excitation, the nonlinear differential equations have been solved by using various perturbation techniques, which are discussed in the next section.

1.2.5.4 Solution of Nonlinear Differential Equation using Perturbation Technique

There are several perturbation methods, such as the method of multiple scales (MMS), the averaging method (AM), Lindstedt-Poincare's method (LPM), and method of normal forms (MNF). One may also go for the harmonic balance method (HBM), which is the most commonly used technique for the analysis of nonlinear differential equations. But this method has several limitations. To apply this method, one should have an apriori knowledge about the solution, which will guide the selection of the number of harmonic terms for the analysis. It is also very tedious and time consuming to make the formulation for a multi-degree-of-freedom (MDOF) nonlinear system and when higher harmonic terms have been taken into account. Further, a separate analysis is required to find the stability of the system while using the harmonic balance method. In averaging method (AV), while one needs to know apriori the solution to get a consistent solution, the response and stability can be determined from the same analysis. Unlike HBM and AV, in the method of multiple scales, which is a straight forward method, it is not required to know the solution before the analysis, and it is very efficient for MDOF nonlinear analysis. In this method, the response and stability can be obtained from the same analysis. Like HBM and AV, this method can be used to determine the fixed-point, periodic solution, quasi-periodic and chaotic responses. A few demerits of using MMS include the use of a small book-keeping parameter and an increase in mathematical complexity with an increase in the order of analysis. Besides these methods, there are several other methods viz., intrinsic HBM, incremental HBM, incremental HBM with multiples time scales, multiple-time scales harmonic balance (MHB), and modified Lindstedt-Poincare method used for the analysis of nonlinear systems. However, these methods have been tested only for a few specific nonlinear systems, and their usability has not been fully explored. In the present work, the first and second order methods of multiple scales (MMS) are used to analyze the nonlinear temporal equation of motion.

The method of multiple scales (MMS) comprises techniques used to construct uniformly valid approximations to the solutions of perturbation problems, both for small and large independent variables. The method is done by introducing fast-scale and slow-scale variables for an independent variable, and subsequently, these variables are treated as fast and slow, as if they are independent. In the solution process of the perturbation problem, the resulting additional freedom

is introduced by the new independent variables to remove the secular terms, which may lead to unbounded solutions. The detailed expansion, solution process and responses are well explained and can be found in the books of Nayfeh and Mook [13] and Nayfeh and Balachandran [14] along with the various examples. Janowicz [232] presented a detailed review of the different applications of the method of multiple scales to the quantum-optical problems. Dwivedy and Kar investigated the nonlinear behavior along with the responses using this perturbation technique under different resonance conditions of a slender beam with an attached mass [233, 234] and with a lumped mass [235, 236]. Cartmell et al. [237] addressed a detailed review of the mathematical principles of the perturbation method of multiple scales of weakly nonlinear ordinary differential equations, which can be found in machine and structural dynamics. Jamshidi and Jafari [238] investigated the vibration behavior of a rotor with an asymmetric shaft subjected to unbalanced forces theoretically using the multiple-scales method. Also, the applications in Ref. [182, 187, 188] under harmonic excitation, in Ref. [190, 193-202, 205, 206] under parametric excitation and finally in all applications [207-212] under hard excitation, the MMS and the reconstitution method [239] has been used to solve the nonlinear equation of motion and find various responses of these systems. Further, the response of the applications in Ref. [213-215, 224, 227-231] under super and subharmonic resonance conditions is investigated with the help of the MMS.

In the work of Rahman and Burton [240], the authors proposed an improvement to the MMS with different expansion and ordering where the presence of spurious results can be neglected. This second version of the traditional MMS proposed by the authors is known as MMS version II [240]. The comparison between both the versions of MMS and recommended MMS version II is illustrated in the literature for different nonlinear systems [241, 242]. The MMS version II is specifically applied to the harmonically excited Duffing-type equations in most of the cases [240, 241]. Dwivedy and Kar [242] compared the versions I and II to find out the nonlinear response of a parametrically excited system. The results have also been compared with the experimental work of Zavodney and Nayfeh [243]. In most of these literatures, the MMS version II has been used for the primary resonance condition only.

1.3 Summary of the Review and Motivation

Over the years, many researchers and designers have been attracted to the use of PAM in industrial and medical applications due to their advantages, such as high force to mass ratio, soft and flexible structure, and easy installation with safer human interactions. PAMs are suitable for the execution of positive load feedback, where smooth, accurate and fast responses are essential. It also produces a significant force when fully stretched and can be used in applications like exoskeletons and rehabilitation devices due to the lightweight nature of this actuator, where the weight of the equipment is a restriction. Basically, these muscle actuators consist of a thin rubber tube (bladder) that is covered or embedded with a braided mesh shell. Both the ends of the muscle are closed where one end is used as an air inlet, and another is connected to the load. When the compressed air is supplied to the muscle through the inlet port, the internal bladder tends to increase its volume against the braided mesh shell. But the non-extensibility of the braided mesh threads causes the actuator to shorten and produce pulling forces if it is connected with the load.

Many types of PAMs have appeared in the literature, which are different in their mechanical construction and the mathematical model describing the principal operation. Due to the presence of high nonlinearities in the PAM, the operation of the PAM mainly depends on the proper mathematical model for dynamic behavior and control of the muscle. A detailed study can be found in the literature on the modeling and control of PAM, considering various muscle factors. Understanding the hysteresis of PAM in the phenomenological model is still a challenge, so many applications depend only on the empirical formulations. But, there is still a lack of achieving an accurate mathematical description of the nonlinear dynamics behind the actuation of PAMs. These qualities and drawbacks make the PAM as an attractive topic for many researchers and industry to study the behavior in the various environments. In the field of nonlinear dynamics, the response behavior of different types of excitations, like harmonic, parametric and hard excitation has been studied for various applications. These excitations may lead to different resonance conditions, and the study of resonance conditions is also an exciting topic in the nonlinear systems. The responses are studied with the help of perturbation techniques, which provides closed form solutions with less computational time and memory.

The nonlinear characteristics present highly in PAMs due to the compressibility of air, basic properties of rubber materials, flexible structure and valve actuation. The presence of nonlinearities in the muscles makes it difficult to understand the dynamics for modeling and control. Though different models have been presented in the literature to understand the behavior and to control the PAM, there is a lack of the significant dynamic model for the nonlinear characteristics present in the muscles. So, an improved model of the PAM is highly required for various practical applications. The presence of high nonlinearity in the muscle dynamics gives rise to the vibration issue with the PAM. Efforts have been made to improve their dynamic models, provide a better control strategy to attenuate the vibration of the muscle. As it has been clear from the literature review presented in the earlier sections, most of the works on PAMs have been limited to the study of the different control strategies. Therefore, it is necessary to develop a more accurate nonlinear mathematical model for the PAM with different excitation. As the superposition rule is not applicable to nonlinear systems, so the response behavior of PAM under various resonance conditions may be useful for specific applications. The most commonly used PAM, the McKibben Muscle, has some crucial drawbacks, mainly about its control. So, the study of dynamics and control of this kind of PAM is an interesting matter for different applications in the real environment.

1.4 Objective of the Present Work

Taking into consideration the gaps from the current state-of-the-art literature, the primary objective of the present work is to study the nonlinear dynamics of a system attached with pneumatically actuated artificial muscle under forced and parametric excitations with various resonance conditions. Both weak and hard excitations are considered depending on the amplitude of the forcing. To achieve the objective, the following five different cases have been investigated.

- Nonlinear dynamic analysis of harmonically excited PAM with simple resonance condition.
- Dynamic analysis of parametrically excited PAM with simple and principal parametric resonance conditions.
- Nonlinear dynamic analysis of parametrically excited PAM with principal parametric and simultaneous resonance conditions.

- Nonlinear dynamic analysis of harmonically excited PAM under hard excitation with superharmonic and subharmonic resonance conditions.
- Nonlinear dynamic analysis of parametrically excited PAM under hard excitation with superharmonic and subharmonic resonance conditions.

Further, the objective of the present work is also to carry out experimental analysis to validate theoretical findings by developing an experimental setup and using the McKibben type of muscle. Also, it is planned to develop a setup to manufacture a novel low-cost PAM with locally available fabrics and silicon rubber.

1.5 Thesis Organization

The thesis mainly focuses on the study of nonlinear dynamics of a system attached with a Pneumatic Artificial Muscle (PAM) actuator under different excitation and various resonance conditions. So, for better understanding, the thesis has been organized into eight chapters in a systematic and methodical way. In the present chapter, a preliminary introduction to the problems is provided, which includes justification for the research and background information on the motivation behind the study. In Chapter 2, the nonlinear dynamic response of a harmonically excited system with PAM is studied under simple resonance condition. Chapter 3 deals with the dynamic responses of a parametrically excited PAM system under simple and principal parametric resonance conditions. In Chapter 4, nonlinear analyses of the system subjected to principal parametric resonance condition along with simultaneous simple and principal parametric resonance condition have been carried out. In Chapter 5, the nonlinear behavior of the system subjected to hard excitation under the superharmonic and subharmonic resonance conditions has been investigated. Chapter 6 deals with the system under parametric and hard excitation, which is also subjected to the superharmonic and subharmonic resonance conditions. Chapter 7 presents the development of an experimental setup and a new type of PAM. Finally, the general and specific conclusions from the present research are highlighted, and the potential areas for future exploration are proposed in Chapter 8.

Chapter 2

Nonlinear Dynamics of a Harmonically Excited Pneumatic Artificial Muscle

2.1 Introduction

This chapter deals with a single degree of freedom system containing a spring-mass-damper along with a pneumatic artificial muscle (PAM). The system is also subjected to an external harmonic force. The system is modeled as a second order nonlinear differential equation due to the presence of large nonlinear terms in the muscle force. So, to find out the approximate analytical solution, the equation which is subjected to simple resonance condition has been solved by using the second order method of multiple scales (MMS). The safe operating range of various system parameters has been determined using the bifurcation points from frequency response plots. Finally, experiments have been conducted to validate the theoretical findings. In section 2.2, the mathematical modeling of such a system attached to the PAM has been carried out. The approximate solution to the second order nonlinear differential equation has been found out in section 2.3, where simple resonance condition is studied. In section 2.4, time and frequency responses, phase portraits and basin of attraction are plotted to study the response along with the stability and bifurcations of the system. The theoretical results obtained by solving the original equation of motion are compared with the experimental results in section 2.5. Finally, in section 2.6, a brief summary is made.

2.2 Mathematical Modeling

Figure 2.1(a) shows the schematic diagram of the system consisting of mass m , external spring with stiffness k , damper with the coefficient of viscous damping c and a PAM with muscle force F_{mus} . Also an external harmonic force of $F \sin \omega t$ is acting on the system. The various forces acting on the whole system have been shown in the free body diagram in Fig. 2.1(b) and the equation of motion which is obtained using Newton's 2nd law can be written as follows.

$$m\ddot{u} + ku + c\dot{u} + F_{mus} = F \sin \omega t \quad (2.1)$$

Here, the overdot signifies the differentiation with respect to time t . The displacement with respect to the static equilibrium position is u and in this case the pressure P is considered to be constant.

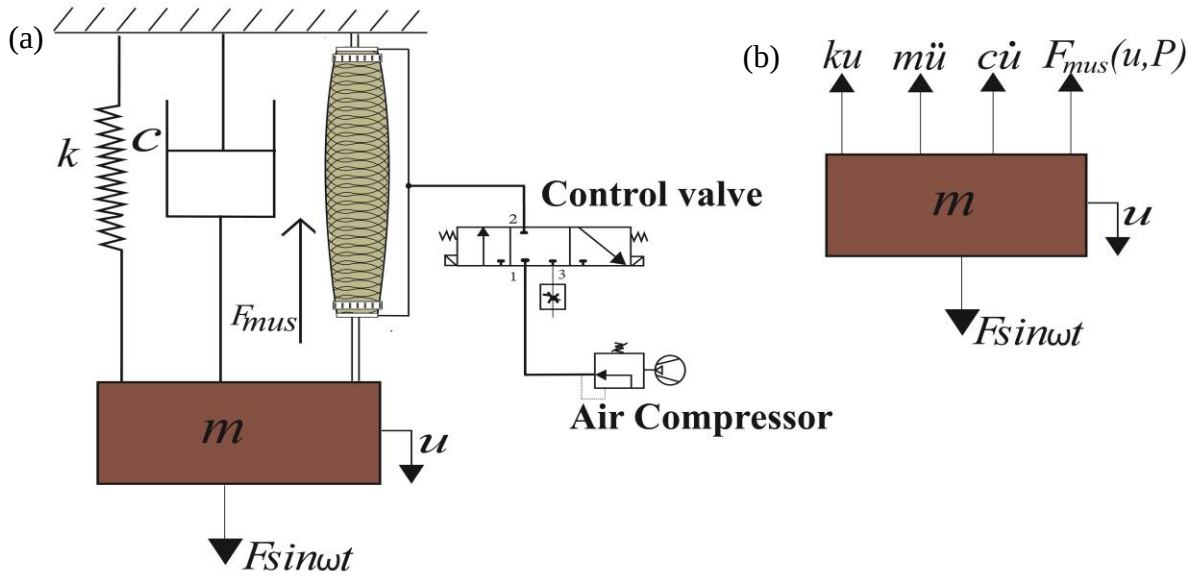


Fig. 2.1 (a) Schematic diagram of a system spring-mass-damper along with a PAM
(b) free-body diagram

The muscle force F_{mus} in this work is considered to be similar to that mentioned by Li et al. [125], which is a fully empirical model dependent on quasi-static characterization. But, instead of considering five parameters as in Ref. [125], here, four parameters (c_1 , c_2 , c_3 and K) are considered for modeling the PAM. It may be noted that several researchers have taken two [244, 245, 246], three [141, 247], four [248] and five [125] experimental parameters in their modeling to derive the muscle equation. Hence, the present modeling is a reasonable one that can be used

for analytical modeling of the PAM. Because of the highly nonlinear nature present in the PAM, an additional cubic nonlinear term, Ku^3 is added to the expression and it can be written as follows.

$$F_{mus}(u, P) = (c_1 + c_2 P + c_3 P^2) \left(\frac{u}{l_{\max}} \right) + Ku^3 \quad (2.2)$$

where c_1 , c_2 , c_3 and K are the constants to be determined from the experiment for a particular application and the maximum length that can be achieved from the muscle is $l_{\max}$. Therefore, substituting Eq. (2.2) in Eq. (2.1), one obtains the following equation.

$$\ddot{u} + \left[\frac{k}{m} + \frac{(c_1 + c_2 P + c_3 P^2)}{ml_{\max}} \right] u + \frac{c}{m} \dot{u} + \frac{K}{m} u^3 = \frac{F}{m} \sin \omega t \quad (2.3)$$

Taking a non-dimensional time, $\tau = \omega_0 t$, the fundamental natural frequency ω_0 of the system is written below.

$$\omega_0 = \sqrt{\frac{k}{m} + \frac{[c_1 + c_2 P + c_3 P^2]}{ml_{\max}}} \quad (2.4)$$

The nondimensional displacement can be written as $u = r x$ by using a scaling factor r .

With the use of various nondimensional parameters as follows,

$$\Omega = \frac{\omega}{\omega_0}, \quad \mu = \frac{c}{2\epsilon m \omega_0}, \quad \alpha = \frac{r^2 K}{\epsilon m \omega_0^2}, \quad f = \frac{F}{r \epsilon m \omega_0^2} \quad (2.5)$$

Eq. (2.3) can be modified to a simplified form as follows.

$$\frac{d^2 x}{d\tau^2} + 2\epsilon\mu \frac{dx}{d\tau} + x + \epsilon\alpha x^3 = \epsilon f \sin \Omega \tau \quad (2.6)$$

Here, the book keeping parameter ϵ is less than 1 and the nondimensional damping parameter is μ . The amplitude of external force F is a constant term which is not a function of air pressure.

It may be noted that the nonlinear stiffness considered for the PAM will yield the same results if one considers that for the external spring. Hence, a separate nonlinear stiffness for the spring is not considered in this work. Also, the equation Eq. (2.6) is similar to that of a Duffing equation, but, here, the coefficients of the response ' x ' and forcing term ' $f \sin \Omega \tau$ ' are the function of muscle parameters and operating pressure. Though the first order solution of Eq. (2.6) is available in the book of Nayfeh [249] and Nayfeh and Mook [13], to obtain more accurate results, one

should go for the higher order solution. The approximate analytical solution procedure of the nonlinear equation Eq. (2.6) is discussed in the next section.

2.3 Approximate Solution

In this section, the analytical approximate solution for the equation of motion in Eq. (2.6) has been obtained by using the second order method of multiple scales (MMS) [13, 14]. With the use of the basic formulation of the MMS, the following expression can be written for the primary variable x .

$$x(\tau, \varepsilon) = x_0(T_0, T_1, T_2) + \varepsilon x_1(T_0, T_1, T_2) + \varepsilon^2 x_2(T_0, T_1, T_2) \quad (2.7)$$

where, $T_n = \varepsilon^n \tau, n = 0, 1, 2$. Now, $D_0 = \frac{d}{dT_0}$, $D_1 = \frac{d}{dT_1}$ and $D_2 = \frac{d}{dT_2}$ substituting Eq. (2.7) into

Eq. (2.6) yields,

$$D_0^2 x_0 + \varepsilon D_0^2 x_1 + \varepsilon^2 D_0^2 x_2 + 2\varepsilon D_0 D_1 x_0 + 2\varepsilon^2 D_0 D_1 x_1 + \varepsilon^2 x_0 (D_1 + 2D_0 D_2) + 2\varepsilon \mu D_0 x_0 + 2\varepsilon^2 \mu D_0 x_1 + 2\varepsilon^2 \mu D_1 x_0 + x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \varepsilon \alpha x_0^3 + \varepsilon^2 3\alpha x_1 x_0^2 = \varepsilon f \sin \Omega \tau. \quad (2.8)$$

Now, the following expression can be obtained by separating the terms with different order of ε from Eq. (2.8).

Order of ε^0

$$D_0^2 x_0 + x_0 = 0 \quad (2.9)$$

Order of ε^1

$$D_0^2 x_1 + x_1 = f \sin \Omega T_0 - 2D_0 D_1 x_0 - 2\mu D_0 x_0 - \alpha x_0^3 \quad (2.10)$$

Order of ε^2

$$D_0^2 x_2 + x_2 = -2D_0 D_1 x_1 - x_0 (D_1 + 2D_0 D_2) - 2\mu D_0 x_1 - 2\mu D_1 x_0 - 3\alpha x_1 x_0^2 \quad (2.11)$$

The common solution of Eq. (2.9) can be found as

$$x_0 = A(T_1, T_2) e^{iT_0} + CC \quad (2.12)$$

where CC represents the complex conjugate of the proceeding terms, and the amplitude of the response is denoted by an unknown complex function A depends on T_1 and T_2 .

The following equation can be written by substituting Eq. (2.12) in Eq. (2.10).

$$D_0^2 x_1 + x_1 = -\frac{if}{2} e^{i\Omega T_0} - 2D_1 i A e^{iT_0} - 2\mu i A e^{iT_0} - \alpha A^3 e^{3iT_0} - \alpha 3A^2 \bar{A} e^{iT_0} + CC \quad (2.13)$$

where the $\bar{A}$ represents the complex conjugate of A . From Eq. (2.13), it can be observed from the first term on the right hand side, the response of the system has a tendency to infinity when $\Omega = 1$. This may give rise to simple resonance condition that may be handled by introducing a detuning parameter. So, by introducing the detuning parameter σ as $\Omega = 1 + \varepsilon\sigma$, Eq. (2.13) can be written as follows.

$$D_0^2 x_1 + x_1 = -\frac{if}{2} e^{i(1+\varepsilon\sigma)T_0} - 2D_1 i A e^{iT_0} - 2\mu i A e^{iT_0} - \alpha A^3 e^{3iT_0} - \alpha 3A^2 \bar{A} e^{iT_0} + CC \quad (2.14)$$

It can be observed from Eq. (2.14) that the particular solution for the terms which contain e^{iT_0} will be unbounded. These terms are known as secular terms [13, 14]. Along with these terms, the particular solution of the term containing $e^{i(1+\varepsilon\sigma)T_0}$ will also be unbounded when $\varepsilon\sigma$ tends to zero. These terms are known as nearly secular terms [13, 14]. But, the solution of the physical system always should have a bounded solution. So, these secular or nearly secular terms should be eliminated to have the bounded solution, and the terms present in Eq. (2.14) are given below.

$$-\frac{if}{2} e^{i\varepsilon\sigma T_0} - 2D_1 i A - 2\mu i A - \alpha 3A^2 \bar{A} = 0 \quad (2.15)$$

$$D_1 A = -\frac{f}{4} e^{i\varepsilon\sigma T_0} - \mu A + \frac{3i}{2} \alpha A^2 \bar{A} \quad (2.16)$$

Now, Eq. (2.14) can be rewritten as,

$$D_0^2 x_1 + x_1 = -\alpha A^3 e^{3iT_0} + CC. \quad (2.17)$$

The solution of Eq. (2.17) which is found to be as below.

$$x_1 = \frac{\alpha A^3 e^{3iT_0}}{8} + CC \quad (2.18)$$

Now, substituting Eq. (2.18) and Eq. (2.12) in Eq. (2.11) with the elimination of secular terms the following equation can be derived.

$$D_2 A = -(1 + 2\mu) \frac{D_1 A}{2i} + \frac{i3\alpha^2 A^3 \bar{A}^2}{16} \quad (2.19)$$

With the use of the method of reconstitution [239] along with Eq. (2.16) and Eq. (2.19); one may write the following form.

$$\frac{dA}{d\tau} = \frac{dA}{dT_1} \frac{dT_1}{d\tau} + \frac{dA}{dT_2} \frac{dT_2}{d\tau} = \varepsilon D_1 A + \varepsilon^2 D_2 A$$

$$\frac{dA}{d\tau} = \left[\varepsilon - \frac{\varepsilon^2(1+2\mu)}{2i} \right] \left[-\frac{f}{4} e^{i\varepsilon\sigma T_0} - \mu A + \frac{3i}{2} \alpha A^2 \bar{A} \right] + \varepsilon^2 \frac{i3\alpha^2 A^3 \bar{A}^2}{16} \quad (2.20)$$

Introducing $A = \frac{1}{2} a e^{i\beta}$ where a is the real modal amplitude and β is the phase. Now, by taking

$\varepsilon\sigma T_0 - \beta = \gamma$ in Eq. (2.20), the following reduced equations can be found by separating the real and imaginary parts.

$$a' = \frac{da}{dt} = \varepsilon \left(-\frac{f}{2} \cos \gamma - \mu a \right) + \frac{\varepsilon^2(1+2\mu)}{2} \left(\frac{f}{2} \sin \gamma - \frac{3}{8} \alpha a^3 \right) \quad (2.21)$$

$$a\gamma' = a \frac{d\gamma}{dt} = a\varepsilon\sigma - \left[\varepsilon \left(-\frac{f}{2} \sin \gamma + \frac{3}{8} \alpha a^3 \right) + \frac{\varepsilon^2(1+2\mu)}{2} \left(-\frac{f}{2} \cos \gamma - \mu a \right) + \varepsilon^2 \frac{3\alpha^2 a^5}{256} \right] \quad (2.22)$$

Further, it may be noted that the first order MMS solution available in the book of Nayfeh [249] for the solution of the Duffing equation, which can be obtained by neglecting the terms with ε^2 order (last term of the Eq. (2.21) and last two term of Eq. (2.22)), and can be written as follows.

$$a' = \varepsilon \left(-\frac{f}{2} \cos \gamma - \mu a \right) \quad (2.23)$$

$$a\gamma' = a\varepsilon\sigma - \varepsilon \left(-\frac{f}{2} \sin \gamma + \frac{3}{8} \alpha a^3 \right) \quad (2.24)$$

One may obtain the following equation after eliminating the secular terms from Eq. (2.14) and further rearranging Eq. (2.18), the following equation can be obtained.

$$x_1 = \frac{\alpha a^3}{32} \cos(6T_0) \quad (2.25)$$

Thus from Eq. (2.7) and Eq. (2.25) the response of the system $x(\tau)$ is presented below.

$$x(\tau) = a \cos(\Omega T_0 - \gamma) + \varepsilon \frac{\alpha a^3}{32} \cos(6T_0) \quad (2.26)$$

Eq. (2.26) gives the expression for the complete time response under harmonic excitation with simple resonance condition of the system in the presence of nonlinearity. In the next section, the various system behaviour have been observed with the help of the time and frequency responses.

2.4 Results and Discussion

In this section, the different system parameters used for the analysis of the system with pneumatically actuated artificial muscle are given in Table 2.1, which are almost same as in the work by Li et al. [125]. The steady state responses of the system have been found out by using the reduced equations Eq. (2.21) and Eq. (2.22), and comparisons have been made by solving numerically the original equation Eq. (2.6). The whole system gives a typical nonlinear behavior due to the presence of various nonlinear parameters in the equation Eq. (2.6). The influence of these parameters for simple resonance condition has been investigated with the help of time response, phase portraits and frequency responses on the nonlinear response of the system.

Table 2.1 System parameters used for numerical simulation.

Parameter	Numerical Value	Parameter	Numerical Value	Parameter	Numerical Value
$l_{\max}$	74 mm	α	150	ε	0.1
c_1	-234.25 N	m	6 N	k	12 N/mm
c_2	1.96 N/kPa	μ	0.01	F	2 kN
c_3	-0.003 N/kPa ²	P	500 kPa	r	1

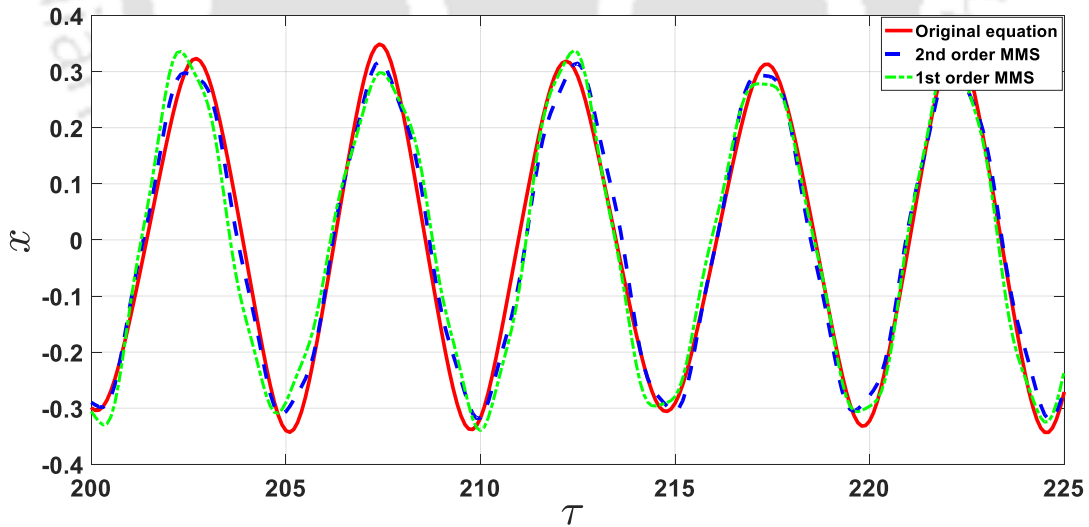


Fig. 2.2 Time response obtained by solving original equation Eq. (2.6), reduced equations Eq. (2.21) and Eq. (2.22) from 2nd order MMS and Eq. (2.23) and Eq. (2.24) from 1st order MMS.

In Fig. 2.2, the comparison has been made among the responses obtained from the reduced equations using first order MMS (Eq. (2.23) and Eq. (2.24)) and second order MMS (Eq. (2.21))

and Eq. (2.22)) with that by solving numerically the original equation Eq. (2.6) using the Runge-Kutta method. Here, the first order MMS solution is found to fluctuate while second order MMS solution has a constant peak value over the cycles. It is observed that the second order MMS solution is closer to that obtained using original equation. Hence, it may clearly be noticed that in the case of simple resonance condition, the solution obtained from the reduced equations using higher order MMS provides more accuracy with the solution obtained from solving the original equation. So, for further analyses, the reduced equations Eq. (2.21) and Eq. (2.22) from second order MMS have been used to study the responses of the system.

Further, in Fig. 2.3, the comparisons have been made with the variation of the different system parameters in the muscle using time response and phase portrait plots. These plots have been found numerically by solving the original equation Eq. (2.6) (marked NS in the figures) with solving the reduced equations Eq. (2.21) and Eq. (2.22) (marked AS in the figures). Considering all the system parameters as per Table 2.1, the comparison has been made in Fig. 2.3(a, b), where it can be observed that both the solutions are giving almost the same response with a mean amplitude approximately equal to zero. The maximum response amplitude in the case of the solution obtained from the reduced equations is around 0.306, and the solution obtained from the original equation is 0.321, which are almost similar in Fig. 2.3(a). Now, with the variation of different system parameters from Table 2.1, the responses have given almost the same amplitude for both the solutions like the previous case. For example, with variation in nonlinearity ($\alpha = 50$) and other system parameters as per Table 2.1, the responses are observed in Fig. 2.3(c, d). The mean amplitude for both the solutions are in good agreement, and the maximum amplitude is increased by 58% from Fig. 2.3(a). In another case, reducing the external force (F) to 1 kN and keeping other parameters the same as Fig. 2.3(a), the mean response amplitude obtained from Eq. (2.6) and those by the second order MMS Eq. (2.21) and Eq. (2.22) are the same in Fig. 2.3(e, f). Here, the maximum amplitude is decreased around 12.5% from that in the first case.

In Fig. 2.3(g, h) reducing the value of the operating pressure (P) to 150 kPa and keeping other parameters the same as in Fig. 2.3(a), the mean responses for both the cases are in good agreement. The maximum response amplitude is almost similar with the response in Fig. 2.3(a, b), which obtained by taking the system parameters as in Table 2.1. In all the cases, from phase portrait plots it can be observed that the results obtained by the second order MMS Eq. (2.21) and Eq.

(2.22) are found to be quasi periodic due to the presence of harmonics, and the numerical solutions of Eq. (2.6) are found to be periodic. But in both cases, the response amplitudes are found to be in good agreement. Hence, the developed reduced equations from second order MMS, Eq. (2.21) and Eq. (2.22) may be used conveniently for further analysis.

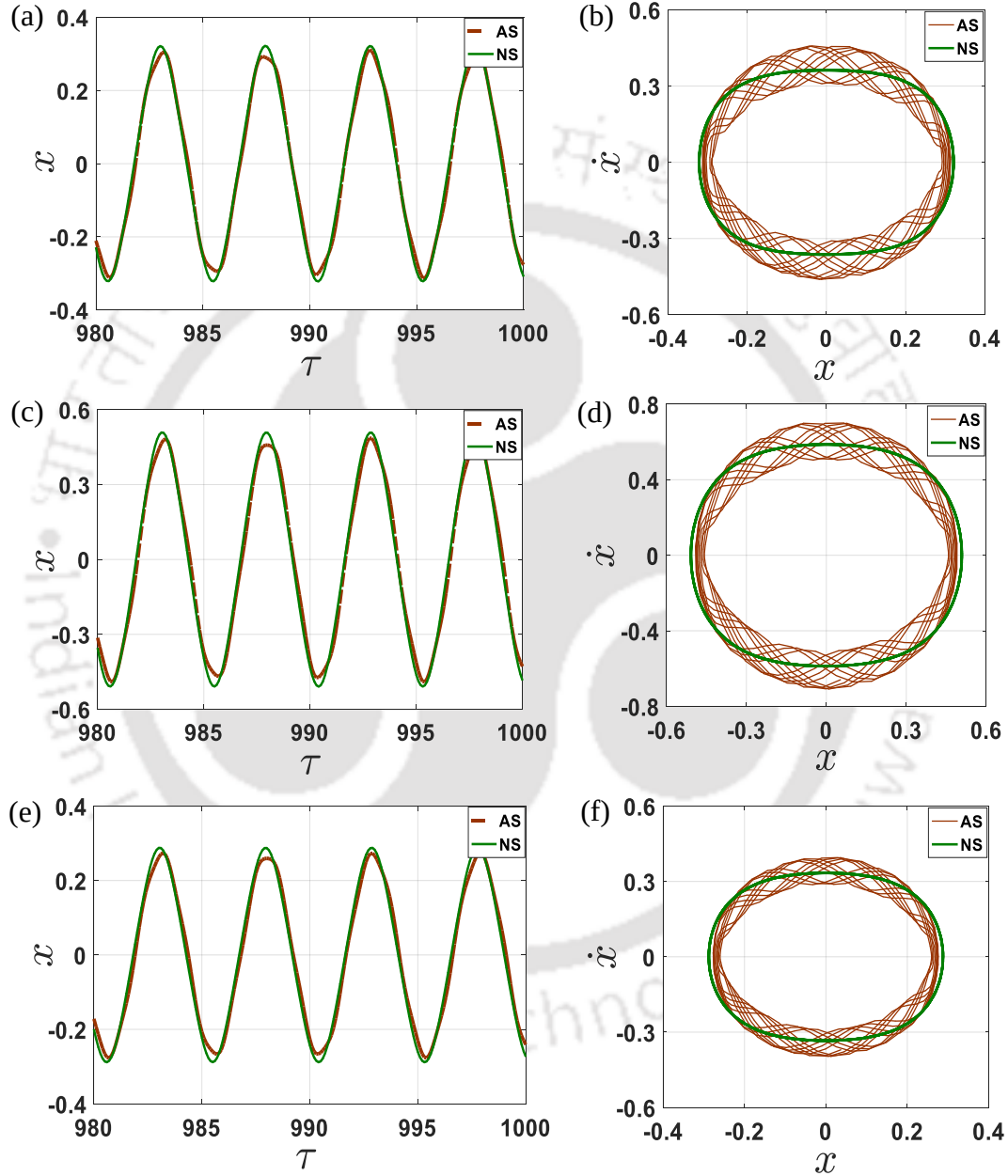


Fig. 2.3 Time response and phase portraits comparison (a, b) parameter values as per Table 2.1 (c, d) variation with α (e, f) variation with F .

2.4.1 Frequency response

In the case of simple resonance condition ($\Omega \approx 1$), the external excitation frequency of the system is nearly equal to the natural frequency of the system. The frequency response plots for the muscle have been found out with the help of Newton's method for solving the set of reduced equations Eq. (2.21) and Eq. (2.22). In Fig. 2.4, the frequency response has been plotted as per the system parameter mentioned in Table 2.1. In all the frequency response plots, the stable solutions are marked as blue color and the unstable solutions are shown in red color.

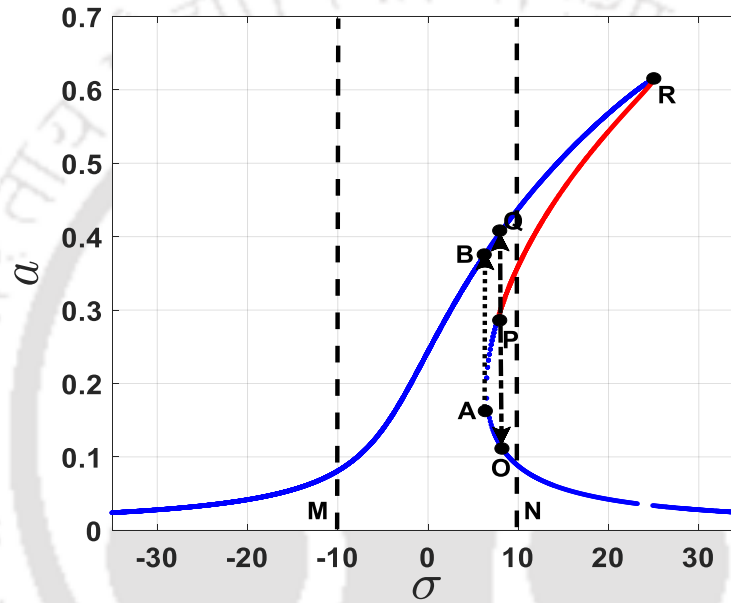


Fig. 2.4 Frequency response of the system with $\Omega \approx 1$ under harmonic excitation as per parameters values in Table 2.1.

The saddle node bifurcation point can be observed at point 'R' in Fig. 2.4, where the maximum amplitude $a = 0.61$ of the system achieved. The point of vertical tangency can be observed at point A, where the detuning parameter, $\sigma = 6.5$. From this point with decrease in σ , as there is only one stable state (point B), so, one may notice a jump-up phenomenon from point A to point B. But, the unstable solution at point P ($\sigma = 7.7$) may have a tendency to jump up to the stable solution at point Q or jump down to the stable solution at point O depending upon the initial conditions. So, from Fig. 2.4, it may clearly be observed that upto point A, the system exhibits single stable state; between point A and P the system displays tri-stable state, between point P and R the system shows bi-stable state, and after point R again has single stable state. Therefore, to reach any of the stable states, the system will depend on the initial conditions. Therefore, the

system can achieve a specific displacement by actively controlling the system parameters of the artificial muscle like air pressure (P), external force (F) and detuning parameter (σ) directly or by changing the passive parameters of the muscle with suitable values of c_1 , c_2 , c_3 , α and the stiffness of the passive spring (k).

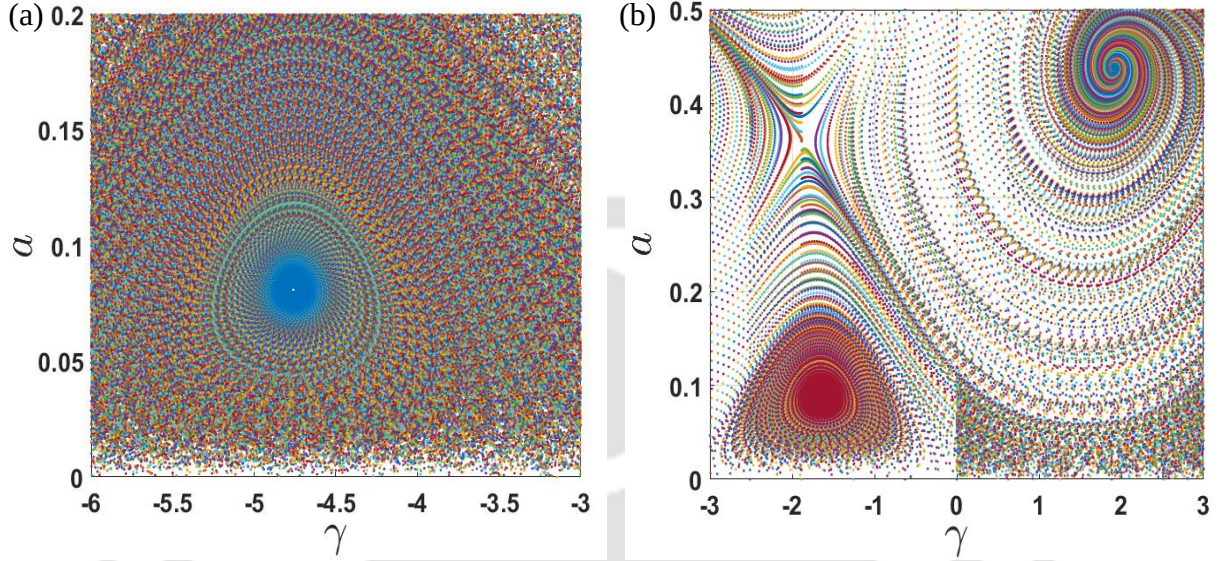


Fig. 2.5 Basin of attraction: (a) $\sigma = -10$ and (b) $\sigma = 10$ from Fig. 2.3.

The basin of attraction for the system have been plotted in the plane $a \sim \gamma$ in Fig. 2.5 to validate the frequency responses at points M and N as per Fig. 2.4. It may be observed from Fig. 2.5(a) that with an amplitude $a = 0.08$, the system has a stable solution which corresponds to the point M in Fig. 2.4 with detuning parameter $\sigma = -10$. From Fig. 2.5(b), the basin of attraction has been plotted and verify the solutions for point N in Fig. 2.4 at $\sigma = 10$. At this point, the system clearly depicts bi-stable non-trivial state with one unstable and two stable solutions. So, it may be verified that the same results can be obtained from Newton's method and the numerical solution.

In Fig. 2.6, the frequency responses have been illustrated for different combinations of the system parameter. From these results, the designer or researchers may know about the effect of various system parameters on the dynamics of the muscle actuation. To achieve the desired amplitude for a specific application, one should choose the system parameters in between their safe operating range to avoid the failure of the system.

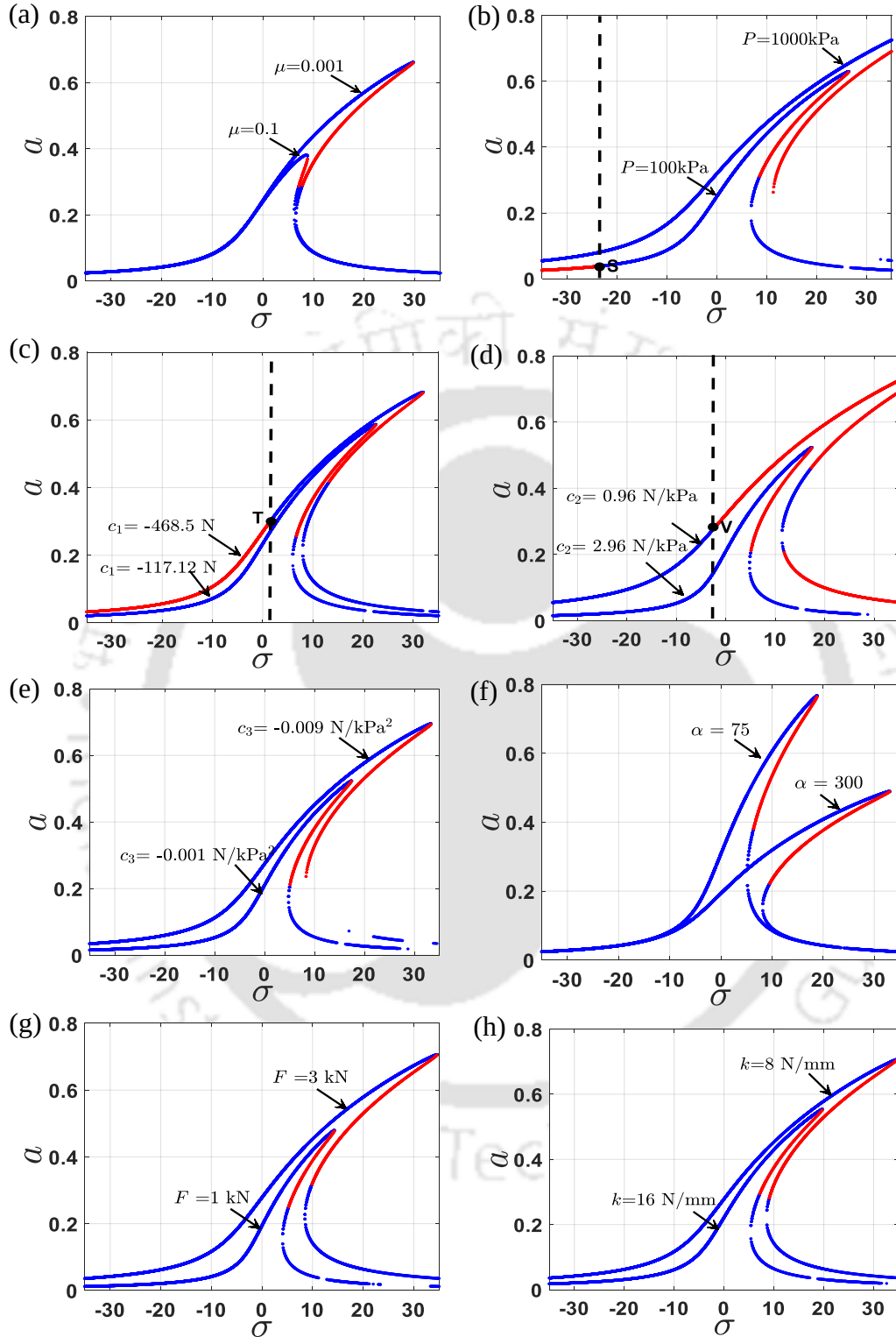


Fig. 2.6 Frequency response curve for excitation close to simple resonance excitation with the variation in (a) μ (b) P (c) c_1 (d) c_2 (e) c_3 (f) α (g) F (h) k .

From Fig. 2.6(a), it may be observed that with the increase in the value of the damping $\mu = 0.1$, decreases the maximum response amplitude of the system, and vice versa. But at saddle node bifurcation point, the system will achieve the maximum response amplitude $a = 0.38$ and $a = 0.66$ for $\mu = 0.1$ and $\mu = 0.001$ respectively. These response amplitude of the system is around 38% lower for $\mu = 0.1$ and 8% higher for $\mu = 0.001$ as compared to that of at $\mu = 0.01$ in Fig. 2.4.

Figure 2.6(b) depicts the frequency response plots of the operating pressure of the muscle (P) with two different values. It can be observed that the maximum response of the system increases with increase and decreases with the decrease in the value of P . From Fig. 2.6(b), the response amplitude of the system is decreased around 5% at detuning parameter $\sigma = 25$ with the decrease in the value of P . For the value of $P = 100$ kPa, the bifurcation point attains maximum amplitude of the system $a = 0.63$ at $\sigma = 26.4$. But in Fig. 2.4 the maximum amplitude is around $a = 0.61$ at $\sigma = 24.9$ for the pressure in the muscle $P = 500$ kPa. Further, the time response and phase portrait have been plotted in Fig. 2.7 to understand the existence of the supercritical Hopf bifurcation point marked as 'S' in Fig. 2.6(b).

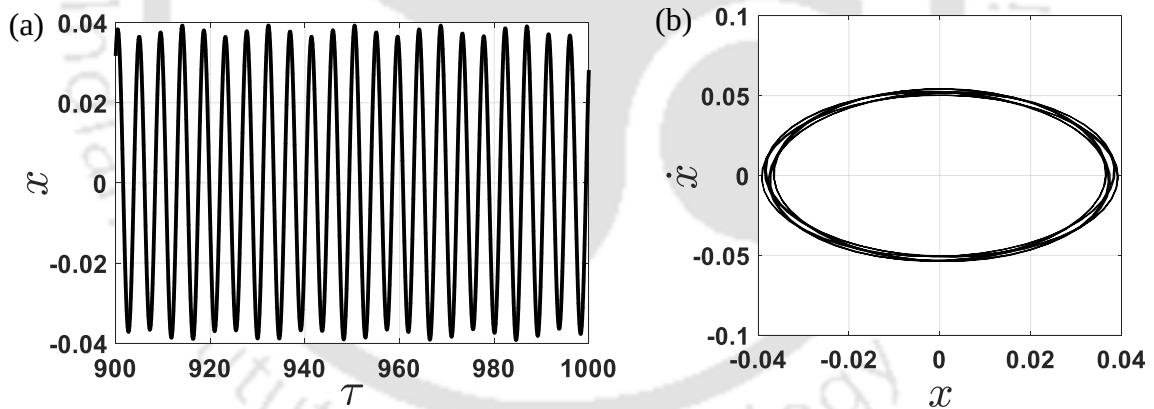


Fig. 2.7 (a) Time response and (b) phase portrait corresponding to the point 'S' in Fig. 2.6(b).

In Fig. 2.6(c), with two different values of muscle parameter (c_1), the frequency responses have been plotted. It is noticed that by doubling the muscle parameter ($c_1 = -117.12$ N) as compared to Fig. 2.4, the maximum response amplitude of the system at saddle node bifurcation point slightly reduced by 3.3%. From Fig. 2.4, where $c_1 = -234.25$ N and reducing to half ($c_1 = -458.5$ N), the maximum response amplitude of the system will increase by 11.5% in Fig. 2.6(c). Like the previous case, the system suffers from Hopf bifurcation due to the lower value of c_1 in Fig. 2.6(c)

with the point 'T' at $\sigma = 1.7$, which is verified by the time response and phase portrait in Fig. 2.8. The response is clearly quasi-periodic which can be observed from Fig. 2.8(b).

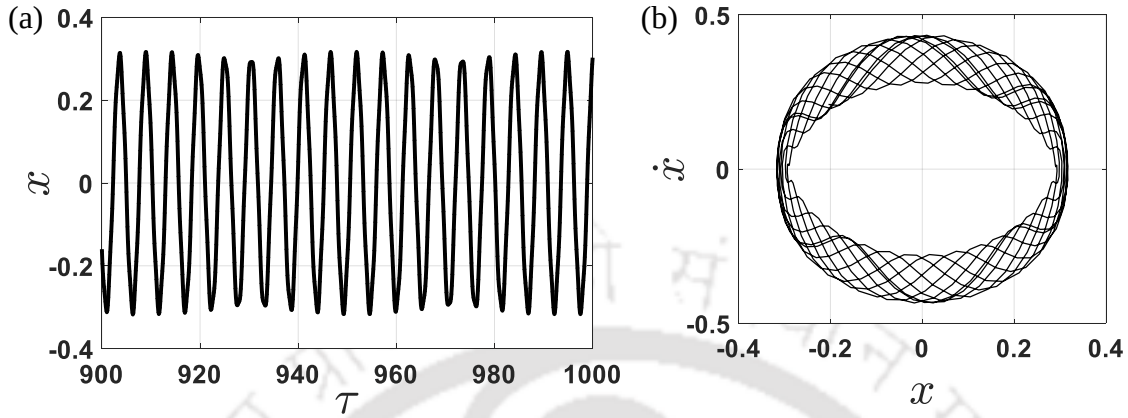


Fig. 2.8 (a) Time response and (b) phase portrait corresponding to the point 'T' in Fig. 2.6(c).

Figure 2.6(d) shows the frequency response plots of the muscle parameter (c_2) for two different values viz., $c_2 = 0.96$ N/kPa and $c_2 = 2.96$ N/kPa. The maximum response of the system increases with decrease in the value of c_2 , and vice-versa like the effect for the muscle parameter c_1 in Fig. 2.6(c). For $c_2 = 2.96$ N/kPa, the maximum response amplitude of the system at saddle node bifurcation point at $\sigma = 17.4$ is $a = 0.52$, which is decreased by 15% as compared to the Fig. 2.4. From point marked as 'V' in Fig. 2.6(d), Hopf bifurcation can be observed at $\sigma = -2.2$ for the muscle parameter value $c_2 = 0.96$ N/kPa. In Fig. 2.9, the time response and phase portrait have been plotted to understand the nature of the supercritical Hopf bifurcation point.

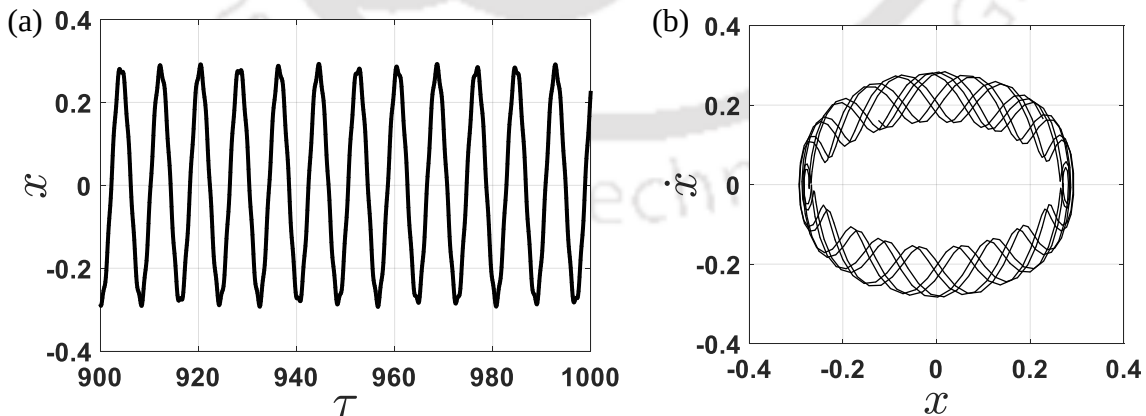


Fig. 2.9 (a) Time response and (b) phase portrait corresponding to the point 'V' in Fig. 2.6(d).

To know the effect of the another muscle parameter (c_3), the frequency responses have been plotted with two different values of c_3 . Like the previous cases for muscle parameter c_1 and c_2 , similar kind of responses are observed for c_3 , where maximum response amplitude increases with decrease in the value of c_3 and vice versa. In Fig. 2.6(e), for higher value of $c_3 = -0.001 \text{ N/kPa}^2$ as compared to the Fig. 2.4, the maximum response amplitude at the saddle node bifurcation point is $a = 0.52$, which is same like the case in Fig. 2.6(d) for $c_2 = 2.96 \text{ N/kPa}$. The maximum response amplitude of the system at saddle node bifurcation point at $\sigma = -33.3$ is increased around 15% as compared to that in Fig. 2.4 with the decrease in the value of $c_3 = -0.009 \text{ N/kPa}^2$.

From Fig. 2.4 and Fig. 2.6(f), it can be observed that the maximum response amplitude $a = 0.48$ is reduced with the increase in the value nonlinearity α . The same observation can be made by decreasing the value of α , where the saddle node bifurcation point can be achieved at $a = 0.76$ as shown in Fig. 2.6(f). Further, a steady attenuation in the response can be observed in comparison with Fig. 2.4 by making the nonlinear parameter α into double or half, the maximum response amplitude is varied by 23%.

To study the effect of the external force F in the system, Fig. 2.6(g) has been plotted for two different values. It may be observed that with the increase in the value of F the maximum response amplitude of the system will be increased, and decreasing F the response will decrease just like the operating pressure (P) shown in Fig. 2.6(b). For $F = 3 \text{ kN}$, at bifurcation point, the response amplitude $a = 0.71$, which is around 16.4% more than that shown in Fig. 2.4 where $F = 2 \text{ kN}$. In the other case, for $F = 1 \text{ kN}$ in Fig. 2.6(g) the maximum response amplitude $a = 0.48$ which is 21.3% less than that in Fig. 2.4.

The frequency response curves have been plotted in Fig. 2.6(h) for two values of the stiffness of the external spring k for the system. The saddle node bifurcation points have been observed which gives the maximum amplitude $a = 0.70$ and $a = 0.55$ for the stiffness value $k = 8 \text{ N/mm}$ and $k = 16 \text{ N/mm}$, respectively. Comparing with Fig. 2.4, where $k = 12 \text{ N/mm}$, by increasing the k to 16 N/mm the response amplitude has reduced by 10%. Also, by decreasing the value of stiffness k to 8 N/mm , the response amplitude increased around 14.8%.

2.4.1.1 Application of the developed results

Figure 2.10 shows the equivalent system representing the human arm, where two artificial muscles are connected in antagonistic direction. In Fig. 2.10(a), both the artificial muscle actuators are in the starting position of the arm at point A. Now, to actuate the forearm through the joint to point B, the antagonistic muscle PAM1 is active and PAM2 acts like a passive spring that needs no control, as shown in Fig 2.10(b). So, the desired movement and position can be achieved by the active PAM1 by controlling the air pressure through its valve. This kind of system provides uniformity and maximum stiffness of the mechanism for specific conditions. The active artificial muscle always depends on the variable air pressure where passive one acts like spring at a constant pressure.

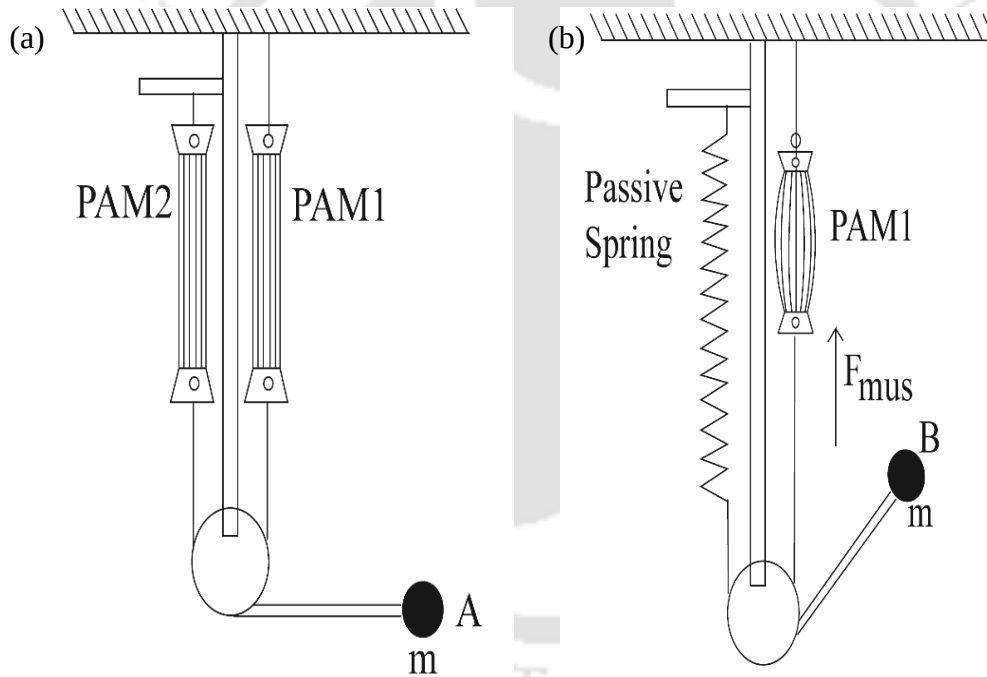


Fig. 2.10 Equivalent system in antagonistic arrangement with (a) two PAMs at rest (b) one active PAM and passive spring.

To apply the above developed results for the movement of the arm, as shown in Fig. 2.10 one may use the following tables (Table 2.2 - 2.4). Here, Table 2.2 gives the list of system parameters the designer may use to achieve a displacement of $\alpha = 0.35$ to use a load of $F = 1, 2, 3$ kN for a wide range of frequency. It may be noted that the same response can be passively achieved by changing the parameter μ , c_1 , c_2 , c_3 , k and α which can be obtained by changing the PAM and connected

spring. Also, the same displacement can be achieved actively by changing the amplitude and frequency of the operating pressure P , forcing parameter F . This will help the designer to choose the system parameters according to the particular application.

From Table 2.2, it may clearly be observed that Case 4 gives the same displacement with the least value of operating pressure (i.e., $P = 100$ kPa), and Case 14 illustrated the condition when the amplitude of the operating load is maximum (i.e., $F = 3$ kN). Further, Case 11 gives the condition when the frequency of the system is minimum. Hence, one may obtain the Pareto optimal condition depending on the exact application of the actuator.

Table 2.2 System parameters to achieve an amplitude of $a = 0.35$.

Case	μ	P	c_1	c_2	c_3	α	F	k	σ	Ref. Figure No.
1	0.01	500	-234.25	1.96	-0.003	150	2	12	5.2	Fig.2.4
2	0.001	500	-234.25	1.96	-0.003	150	2	12	4.9	Fig. 2.6(a)
3	0.1	500	-234.25	1.96	-0.003	150	2	12	5.8	Fig. 2.6(a)
4	0.01	100	-234.25	1.96	-0.003	150	2	12	4.8	Fig. 2.6(b)
5	0.01	1000	-234.25	1.96	-0.003	150	2	12	1.8	Fig. 2.6(b)
6	0.01	500	-468.50	1.96	-0.003	150	2	12	4.1	Fig. 2.6(c)
7	0.01	500	-117.12	1.96	-0.003	150	2	12	5.2	Fig. 2.6(c)
8	0.01	500	-234.25	2.96	-0.003	150	2	12	5.8	Fig. 2.6(d)
9	0.01	500	-234.25	1.96	-0.009	150	2	12	3.9	Fig. 2.6(e)
10	0.01	500	-234.25	1.96	-0.001	150	2	12	5.8	Fig. 2.6(e)
11	0.01	500	-234.25	1.96	-0.003	75	2	12	1.2	Fig. 2.6(f)
12	0.01	500	-234.25	1.96	-0.003	300	2	12	13.2	Fig. 2.6(f)
13	0.01	500	-234.25	1.96	-0.003	150	1	12	6.2	Fig. 2.6(g)
14	0.01	500	-234.25	1.96	-0.003	150	3	12	3.8	Fig. 2.6(g)
15	0.01	500	-234.25	1.96	-0.003	150	2	8	3.8	Fig. 2.6(h)
16	0.01	500	-234.25	1.96	-0.003	150	2	16	5.6	Fig. 2.6(h)

Depending on the weight of the forearm, one may use different muscle parameters and operating pressures to obtain the desired position. For example, to lift a load of 2 kN upto a maximum displacement of $a = 0.61$, one can use a muscle with the system parameter mentioned in Fig. 2.3 with an operating pressure of $P = 500$ kPa. Similarly, to lift a load of $F = 1$ kN and 3 kN upto a response amplitude of $a = 0.35$, the operator should use the system parameters mentioned in the Case 13 and Case 14 of Table 2.2, respectively.

Table 2.3 Variation in the jump up phenomenon with different system parameters while sweeping down the frequency.

Case	Variation of System parameters	σ	a		Jump length	Ref. Figure No.
			Stable point	Unstable point		
1	As per Table 2.1	7.8	0.40	0.29	0.11	Fig.2.4
2	$\mu = 0.001$	7.8	0.40	0.29	0.11	Fig. 2.6(a)
3	$\mu = 0.1$	7.1	0.37	0.29	0.08	Fig. 2.6(a)
4	$P = 100$	8.6	0.42	0.31	0.11	Fig. 2.6(b)
5	$P = 1000$	11.3	0.49	0.26	0.23	Fig. 2.6(b)
6	$c_1 = -458.50$	13.3	0.50	0.42	0.08	Fig. 2.6(c)
7	$c_1 = -117.12$	6.7	0.38	0.26	0.12	Fig. 2.6(c)
8	$c_2 = 2.96$	5.1	0.33	0.21	0.12	Fig. 2.6(d)
9	$c_3 = -0.009$	8.3	0.43	0.24	0.19	Fig. 2.6(e)
10	$c_3 = -0.001$	5.1	0.33	0.21	0.12	Fig. 2.6(e)
11	$\alpha = 75$	6.4	0.52	0.38	0.14	Fig. 2.6(f)
12	$\alpha = 300$	9.6	0.32	0.22	0.10	Fig. 2.6(f)
13	$F = 1$	5.3	0.33	0.25	0.08	Fig. 2.6(g)
14	$F = 3$	9.9	0.45	0.32	0.13	Fig. 2.6(g)
15	$k = 8$	9.1	0.44	0.28	0.16	Fig. 2.6(h)
16	$k = 16$	7.2	0.38	0.30	0.08	Fig. 2.6(h)

Similarly, Table 2.3 gives the variation of saddle node bifurcation point where the system has a tendency to jump up while sweeping down the frequency for a wide range of system parameters. It may be noted that one may achieve the same amplitude by sweeping up the frequency or sweeping down the frequency. But while sweeping down, one must take care of the jump up phenomenon, which may lead to catastrophic failure of the system. In this table, the amplitude (a) for both stable and unstable points is mentioned where jump up phenomenon will take place for different system parameters. In Case 5, the jump length is highest and in Case 3, 6, 13 and 16 the jump length is equal and lowest compared to the other cases.

Table 2.4 gives the idea of the maximum response amplitude observed near the saddle node bifurcation point for the considered system parameters. One should not operate the system beyond this point while sweeping the frequency, which may lead to a jump down phenomenon. For example, in Case 10 the maximum response amplitude is $a = 0.77$ at the saddle node bifurcation

point $\sigma = 18.8$. So, the system may undergo a jump down phenomenon if one increases the operating frequency beyond $\sigma = 18.8$ while sweeping up the frequency. Hence, the designer may take note of this bifurcation points while designing the system for a particular application.

Table 2.4 Saddle node bifurcation point where maximum amplitude achieve for the studied PAM.

Case	Variation of System parameters	a	σ	Ref. Figure No.
1	As per Table 1	0.61	24.9	Fig.2.4
2	$\mu = 0.001$	0.66	29.7	Fig. 2.6(a)
3	$\mu = 0.1$	0.38	8.7	Fig. 2.6(a)
4	$P = 100$	0.63	26.4	Fig. 2.6(b)
5	$c_1 = -458.50$	0.68	31.9	Fig. 2.6(c)
6	$c_1 = -117.12$	0.59	22.5	Fig. 2.6(c)
7	$c_2 = 2.96$	0.52	17.4	Fig. 2.6(d)
8	$c_3 = -0.009$	0.70	33.3	Fig. 2.6(e)
9	$c_3 = -0.001$	0.52	17.4	Fig. 2.6(e)
10	$\alpha = 75$	0.77	18.8	Fig. 2.6(f)
11	$\alpha = 300$	0.49	33	Fig. 2.6(f)
12	$F = 1$	0.48	14.3	Fig. 2.6(g)
13	$F = 3$	0.71	34.6	Fig. 2.6(g)
14	$k = 8$	0.71	34.7	Fig. 2.6(h)
15	$k = 16$	0.55	19.7	Fig. 2.6(h)

To find the response amplitude of the muscle in the dimensional form, an example has been illustrated here. The dimensional value of operating frequency can be calculated using the relation $\Omega = 1 + \varepsilon\sigma$ and $\omega = \Omega\omega_0$ from Eq. (2.5). Using Eq. (2.26), corresponding to nondimensional amplitude (a), nondimensional (x) and dimensional (u) displacement, $u = rx$ can be obtained. Using Fig. 2.4, at detuning parameter $\sigma = 20$ ($\omega = 21$ Hz), the nondimensional amplitude $a = 0.042$ and the corresponding dimensional response amplitude is found to be 4.3 cm. In the next section, the results obtained from the above mathematical work have been validated with the help of an experimental setup.

2.5 Experimental Validation

To validate the above theoretical work, an experimental setup was designed and developed as shown in Fig. 2.11 with different components. The detailed description and features of this setup have been presented in Chapter 7 of the thesis. The setup was attached with a McKibben type PAM and a spring in an antagonistic arrangement as shown in Fig. 2.10(b) through a chain-sprocket mechanism. The McKibben muscle is composed of a gas-tight elastic tube or bladder surrounded by a braided nylon sleeve. Both the ends of the inner tube and braid of this muscle are closed with the end fitting. The upper end of the muscle and the spring were connected to the rigid support of the aluminum slotted frame with the help of snap hooks. The other end of the muscle, along with the spring, were connected to each other chain-sprocket mechanism to achieve the desired motion as shown in Fig. 2.11. The chain sprocket mechanism was designed in the antagonistic arrangement, which is adjustable with the required dimension of the PAM. The load was attached to a shaft which is connected to the chain-sprocket mechanism and rotated to the desired position by supplying air pressure to the muscle.

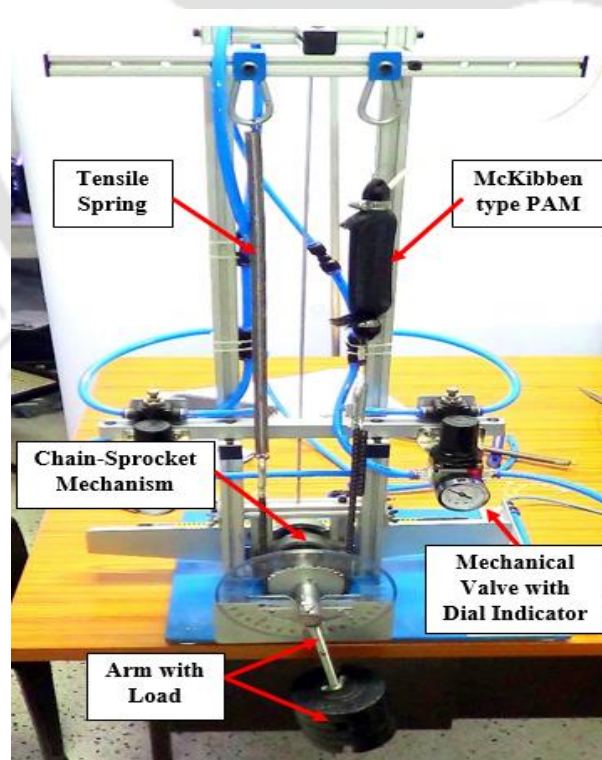


Fig. 2.11 Developed experimental setup.

The compressed air was supplied through the 4mm FESTO pneumatic tube to actuate the muscle mounted on the frame and controlled with a TECHNO mechanical flow control valve with a dial indicator. The spring was providing the necessary support to the system, which does not require any control. The load attached to the setup will act like the weight of the forearm, and sprocket through which the muscle and the spring were connected, will mimic like the elbow of the human arm. With the help of this experimental setup, the changes in the contraction of the muscle (u) were measured while applying a load of 5 N to 30 N with the variation in the pressure from 50 kPa to 300 kPa in the discrete steps of 50 kPa. These experiment data are shown in Table 2.5.

Table 2.5 Experiment Data: Contraction values (u) for different loads with the variation of pressure.

Pressure Muscle Force	u in mm					
	50 kPa	100 kPa	150 kPa	200 kPa	250 kPa	300 kPa
5 N	4.0	6.0	8.0	10.0	12.0	13.0
10 N	3.6	5.6	7.5	9.0	11.0	12.5
15 N	3.3	5.2	7.0	8.4	10.4	12.0
20 N	3.0	4.8	6.6	7.8	9.7	11.6
25 N	2.8	4.5	6.3	7.3	9.2	11.3
30 N	2.6	4.2	6.0	7.0	9.0	11.0

The selected static analytical model for the muscle force in Eq. (2.2), which relates the force exerted by the muscle with its internal pressure and contraction, comprises a set of unknown parameters. With the help of the experimental data in Table 2.5, one may find the values of these parameters. Now, taking $y = u / l_{\max}$, one may use the least square method to find the coefficients of Eq. (2.2). In this method, the error can be written as follows.

$$E(c_1, c_2, c_3, K_n) = \sum_{i=1}^{36} \left[F_{mus_i} - \left\{ (c_1 + c_2 P_i + c_3 P_i^2) y_i + K_n y_i^3 \right\} \right]^2 \quad (2.27)$$

where, $K_n = Kl_{\max}^3$. Hence, the error can be minimized by finding the first derivative with respect to c_1 , c_2 , c_3 and K_n , and equating it to zero as follows.

$$\begin{aligned}\frac{\partial E}{\partial c_1} &= \sum_{i=1}^{36} 2 \left[F_{mus_i} - \left\{ (c_1 + c_2 P_i + c_3 P_i^2) y_i + K_n y_i^3 \right\} \right] y_i = 0 \\ \frac{\partial E}{\partial c_2} &= \sum_{i=1}^{36} 2 \left[F_{mus_i} - \left\{ (c_1 + c_2 P_i + c_3 P_i^2) y_i + K_n y_i^3 \right\} \right] y_i P_i = 0 \\ \frac{\partial E}{\partial c_3} &= \sum_{i=1}^{36} 2 \left[F_{mus_i} - \left\{ (c_1 + c_2 P_i + c_3 P_i^2) y_i + K_n y_i^3 \right\} \right] y_i P_i^2 = 0 \\ \frac{\partial E}{\partial K_n} &= \sum_{i=1}^{36} 2 \left[F_{mus_i} - \left\{ (c_1 + c_2 P_i + c_3 P_i^2) y_i + K_n y_i^3 \right\} \right] y_i^3 = 0\end{aligned}$$

One may rewrite these equations as follows.

$$\begin{aligned}\left(\sum_{i=1}^{36} y_i^2 \right) c_1 + \left(\sum_{i=1}^{36} P_i y_i^2 \right) c_2 + \left(\sum_{i=1}^{36} P_i^2 y_i^2 \right) c_3 - \left(\sum_{i=1}^{36} y_i^4 \right) K_n &= \sum_{i=1}^{36} F_{mus_i} y_i \\ \left(\sum_{i=1}^{36} P_i y_i^2 \right) c_1 + \left(\sum_{i=1}^{36} P_i^2 y_i^2 \right) c_2 + \left(\sum_{i=1}^{36} P_i^3 y_i^2 \right) c_3 - \left(\sum_{i=1}^{36} P_i y_i^4 \right) K_n &= \sum_{i=1}^{36} F_{mus_i} P_i y_i \\ \left(\sum_{i=1}^{36} P_i^2 y_i^2 \right) c_1 + \left(\sum_{i=1}^{36} P_i^3 y_i^2 \right) c_2 + \left(\sum_{i=1}^{36} P_i^4 y_i^2 \right) c_3 - \left(\sum_{i=1}^{36} P_i^2 y_i^4 \right) K_n &= \sum_{i=1}^{36} F_{mus_i} P_i^2 y_i \\ \left(\sum_{i=1}^{36} y_i^4 \right) c_1 + \left(\sum_{i=1}^{36} P_i y_i^4 \right) c_2 + \left(\sum_{i=1}^{36} P_i^2 y_i^4 \right) c_3 - \left(\sum_{i=1}^{36} y_i^6 \right) K_n &= \sum_{i=1}^{36} F_{mus_i} y_i^3\end{aligned}$$

This leads to following matrix equation,

$$\begin{bmatrix} \sum_{i=1}^{36} y_i^2 & \sum_{i=1}^{36} P_i y_i^2 & \sum_{i=1}^{36} P_i^2 y_i^2 & \sum_{i=1}^{36} y_i^4 \\ \sum_{i=1}^{36} P_i y_i^2 & \sum_{i=1}^{36} P_i^2 y_i^2 & \sum_{i=1}^{36} P_i^3 y_i^2 & \sum_{i=1}^{36} P_i y_i^4 \\ \sum_{i=1}^{36} P_i^2 y_i^2 & \sum_{i=1}^{36} P_i^3 y_i^2 & \sum_{i=1}^{36} P_i^4 y_i^2 & \sum_{i=1}^{36} P_i^2 y_i^4 \\ \sum_{i=1}^{36} y_i^4 & \sum_{i=1}^{36} P_i y_i^4 & \sum_{i=1}^{36} P_i^2 y_i^4 & \sum_{i=1}^{36} y_i^6 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ K_n \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{36} F_{mus_i} y_i \\ \sum_{i=1}^{36} F_{mus_i} P_i y_i \\ \sum_{i=1}^{36} F_{mus_i} P_i^2 y_i \\ \sum_{i=1}^{36} F_{mus_i} y_i^3 \end{bmatrix} \quad (2.28)$$

To obtain the required coefficients the matrix equation Eq. (2.28) can be written as follows.

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ K_n \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{36} y_i^2 & \sum_{i=1}^{36} P_i y_i^2 & \sum_{i=1}^{36} P_i^2 y_i^2 & \sum_{i=1}^{36} y_i^4 \\ \sum_{i=1}^{36} P_i y_i^2 & \sum_{i=1}^{36} P_i^2 y_i^2 & \sum_{i=1}^{36} P_i^3 y_i^2 & \sum_{i=1}^{36} P_i y_i^4 \\ \sum_{i=1}^{36} P_i^2 y_i^2 & \sum_{i=1}^{36} P_i^3 y_i^2 & \sum_{i=1}^{36} P_i^4 y_i^2 & \sum_{i=1}^{36} P_i^2 y_i^4 \\ \sum_{i=1}^{36} y_i^4 & \sum_{i=1}^{36} P_i y_i^4 & \sum_{i=1}^{36} P_i^2 y_i^4 & \sum_{i=1}^{36} y_i^6 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{36} F_{mus_i} y_i \\ \sum_{i=1}^{36} F_{mus_i} P_i y_i \\ \sum_{i=1}^{36} F_{mus_i} P_i^2 y_i \\ \sum_{i=1}^{36} F_{mus_i} y_i^3 \end{bmatrix} \quad (2.29)$$

By solving the above equation Eq. (2.29) the values of the coefficients can be found out. The values of the muscle parameters are $c_1 = -7.2 \times 10^3$ N, $c_2 = 85.2$ N/kPa and $c_3 = -0.149$ N/kPa². The value of nonlinear, K_n is 81.657×10^{-3} N/mm³ and the value of the maximum possible length of the particular muscle used in this experiments $l_{\max} = 125$ mm; hence the value of K in Eq. (2.2) is found to be 4.18×10^{-8} N/mm⁶.

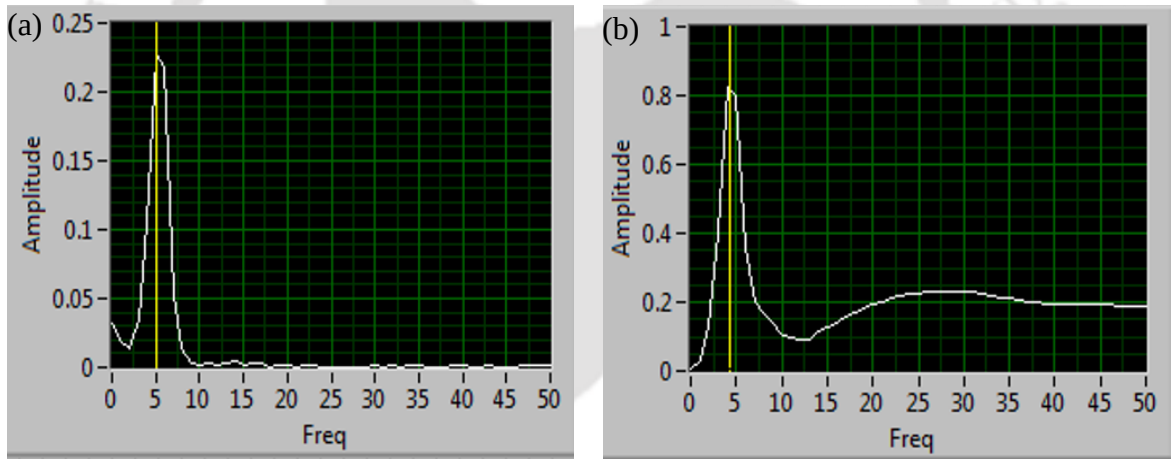


Fig. 2.12 Experimental fundamental frequencies without applying pressure for (a) 1.5 kg (b) 2 kg.

A Bruel and Kjaer made accelerometer (Type: 4396, Serial No.: 2247754) mounted below the attached load in the experimental setup, which is connected to a five-channel dynamic signal acquisition module (NI USB-4432). With the help of this channel, high-accuracy measurements from the accelerometer signals can be obtained, and NI LabVIEW 2009 software is used for post-processing the signals. The fundamental frequencies of the muscle attached with the setup can be attained with an impact on the load to the setup with the help of the response through an accelerometer. The response function is then calculated from the Fast-Fourier transformed force and acceleration data. Initially, the fast Fourier transform (FFT) of the free vibration response of

the muscle is noticed with 1.5 kg and 2 kg load, respectively, to the setup without applying the air pressure inside the muscle. The fundamental frequencies are found to be 5 Hz for 1.5 kg and 4.2 Hz for 2 kg load, which can be observed from Fig. 2.12(a) and Fig. 2.12(b), respectively.

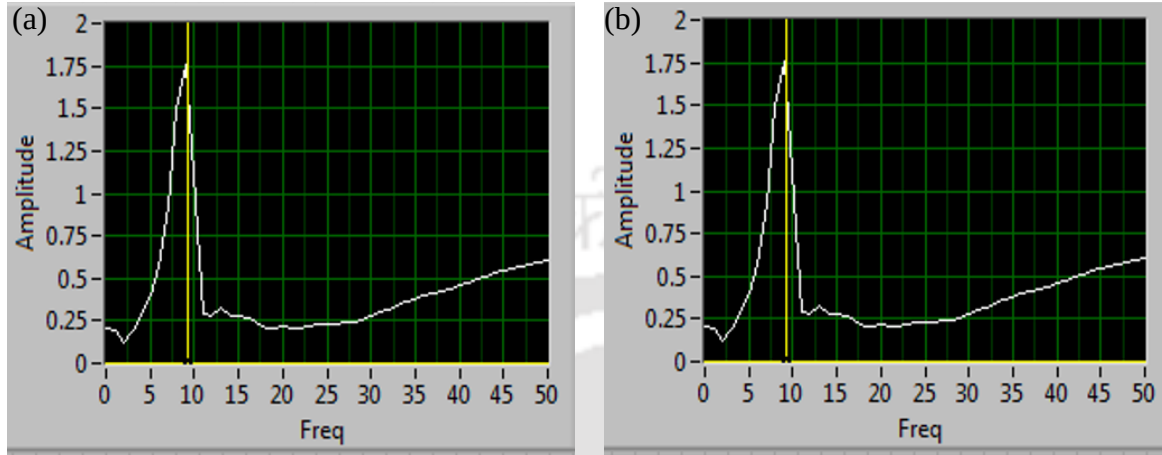


Fig. 2.13 Experimental fundamental frequencies for (a) 1.5 kg at 150 kPa (b) 2 kg at 200 kPa.

Further, the same procedure has been repeated by supplying the air pressure inside the muscle for two cases, and the maximum response amplitude for each case can be found out using the formula, $\text{Response Amplitude} = \text{Acceleration Amplitude} / (2 \times \pi \times \nu)^2$. For the first case in Fig. 2.13(a), to lift the load of 1.5 kg with 150 kPa air pressure inside the muscle attached with setup, the acceleration amplitude is observed 1.83 m/s² for the frequency, $\nu = 9.1$ Hz. Hence, the corresponding maximum response amplitude is found to be 0.56 mm for this case. Similarly, in the second case for 2 kg load with 200 kPa air pressure inside the muscle, the acceleration amplitude is 1.96 m/s² for $\nu = 9.2$ Hz where the maximum response amplitude is obtained 0.59 mm, which can be observed from Fig. 2.13(b). These experimental findings are given in Table 2.6.

The frequency responses have been plotted in Fig. 2.14 for the two above mentioned cases using the reduced equations Eq. (2.21) and Eq. (2.22) along with the above experimentally obtained coefficients. The nature of these plots is also in the good agreement with the plots obtained from the theoretical work (e.g., Fig. 2.4 and Fig. 2.6). For the first case, as mentioned in the above section, to lift a load of $m = 1.5$ kg at a constant pressure of $P = 150$ kPa, the maximum amplitude can be achieved $a = 0.138$, which can be observed from Fig. 2.14(a). Similarly, in the second case from Fig. 2.14(b), by applying a load of $m = 2$ kg at a constant pressure of $P = 200$ kPa, the

maximum amplitude will be $a = 0.141$, which can be obtained at a lower frequency compared to the previous case. In both cases, amplitude $a = 0.1$ can be obtained at frequency $\sigma = 0.264$ for the first case, whereas in the second case frequency will be $\sigma = 0.143$, which is lower to the first case. Hence, the required displacement can be achieved at a lower frequency with higher pressure and load.

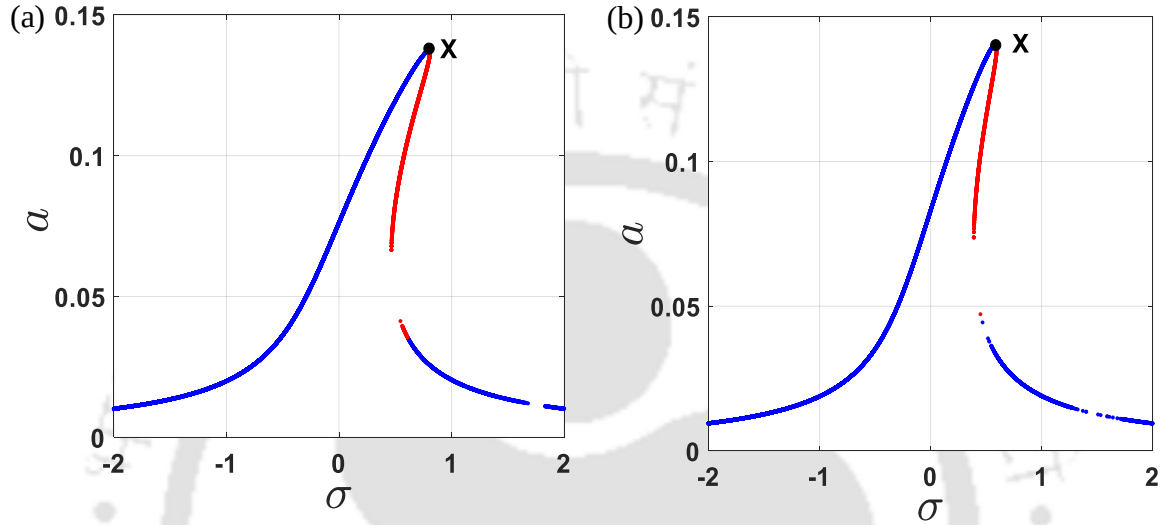


Fig. 2.14 Frequency response using experimentally obtained coefficients for the used McKibben muscle with (a) 1.5 kg at 150 kPa (b) 2 kg at 200 kPa.

Now, from these frequency response plots in Fig. 2.14(a), the maximum response amplitude can be achieved at the saddle node bifurcation point at point X with $\sigma = 0.8$ ($\nu = 8.5$ Hz), where 1.5 kg weight is lifted with 150 kPa pressure. In Fig. 2.15, the comparison among the solutions obtained by numerically solving original equation Eq. (2.6) and the reduced equations from first order MMS (Eq. (2.23) and Eq. (2.24)) and second order MMS (Eq. (2.21) and Eq. (2.22)) have been made. It may be observed from Fig. 2.15(a), the maximum response amplitude obtained from the original equation is 0.93 mm, and the corresponding values for first order MMS and second order MMS are found to be 0.80 mm and 0.85 mm, respectively. Clearly, the result from the second order MMS is found to be closer (8.6% deviation) to that of the original equation. There is a deviation of 14% in the results between the original equation and first order MMS. Similarly, corresponding to point X at $\sigma = 0.6$ ($\nu = 8.7$ Hz) in Fig. 2.14(b), the time response comparison is shown in Fig. 2.15(b) where the response amplitude from the original equation is 0.98 mm, from first order MMS is 0.89 mm and from second order MMS is 0.94 mm. Here also clearly, the results from the second order MMS is found to be more closer with 4% deviation to

that of the original equation and 9% deviation in the results between the original equation and first order MMS. Further, in both the cases, the presence of harmonics is not observed in first order MMS which is clearly visible for the solution of the original equation and second order MMS.

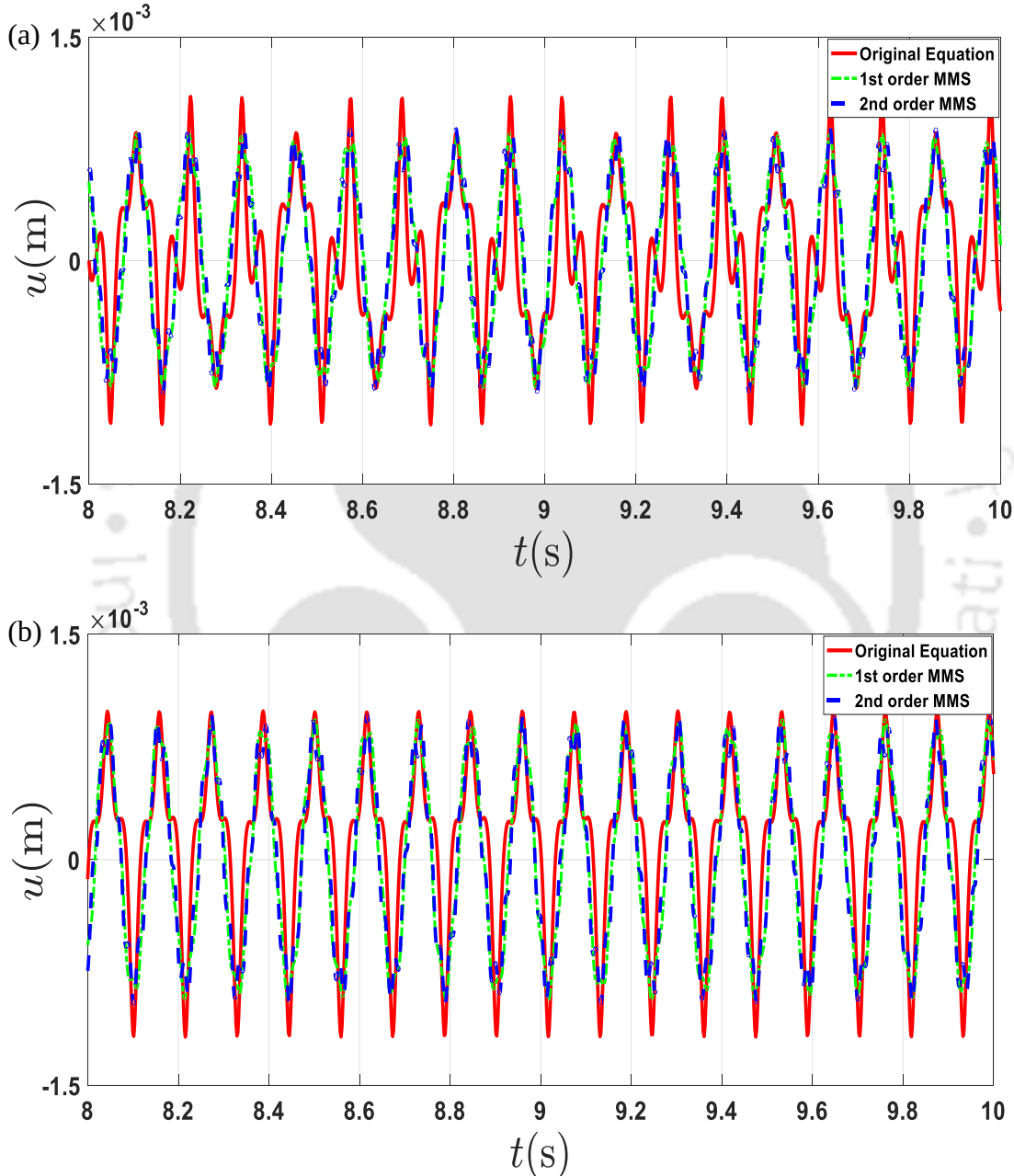


Fig. 2.15 Time response obtained by solving original equation Eq. (2.6), reduced equations Eq. (2.21) and Eq. (2.22) from 2nd order MMS and Eq. (2.23) and Eq. (2.24) from 1st order MMS corresponding to system parameters as in (a) Fig. 2.14(a) with $\sigma = 0.8$ (b) Fig. 2.14(b) with $\sigma = 0.6$.

Now, from Fig. 2.13, Fig. 2.14 and Fig. 2.15, one may compare the experimental and theoretical frequencies and response amplitudes, which are given in Table 2.6. The minor difference in the frequencies between the theoretical and experimental works have been observed, which may be due to the fixing condition of the muscle in the experimental work. Though the theoretical response amplitude with first order MMS is closer to experimental response amplitude, but the second order MMS gives conservative results with higher harmonics, which is closer to the solution of original governing equation of motion. Hence, second order MMS is recommended for studying the response of artificial muscles.

Table 2.6 Comparison of experimental and theoretical results of the used McKibben Muscle.

Case	Load (in kg)	Pressure (in kPa)	Frequency (in Hz)			Response Amplitude (in mm)			
			Experimental	Theoretical		Experimental	Theoretical (At point 'X' in Fig. 2.14)		
				Fundamental frequency	At point 'X' in Fig. 2.14		Solution from Original Equation	Solution from 1 st order MMS	Solution from 2 nd order MMS
I	1.5	150	9.1	8.0	8.5	0.56	0.93	0.80	0.85
II	2	200	9.2	8.2	8.7	0.59	0.98	0.89	0.94

2.6 Summary

In this chapter, the nonlinear analysis of the system with nonlinear PAM have been carried out considering the external excitation frequency to be nearly equal to the natural frequency, which leads to simple resonance condition of the system under harmonic excitation. The governing nonlinear equation of motion of the system has been derived with the help of Newton's 2nd law. Using second order method of multiple scales (MMS), the governing equation of motion has been solved, and a set of reduced equations are obtained. The comparison has been made among the responses obtained by numerically solving the original equation with the reduced equations from the first order MMS and the second order MMS. Here, the results from the second order MMS are found to be more conservative, and hence, it is recommended to be used for analysis purpose.

Further, the effect of different system parameters has been studied with the help of time response, phase portraits and frequency response using the reduced equations from the second order MMS. To attain the desired response of the nonlinear system, the different system parameters values should be chosen suitably to actuate the muscle. So with the help of frequency response plots, the safe operating range of the different system parameters for a particular application can be evaluated. Basin of attraction also have been plotted to verify the actually realized solutions from initial conditions. With the help of examples, the desired position of the forearm can be achieved using various active and passive system parameters, as explained in the tables using different examples. From the experiments conducted with the help of designed experimental setup, conservative results have been found out which validates the theoretical work. The different unknown coefficients present in the muscle force are found by using the least square method. With the use of these experimental coefficients, the frequencies and response amplitudes from the theoretical work are in good agreement that found from the experimental findings.

Chapter 3

Dynamics of a Parametrically Excited Pneumatic Artificial Muscle

3.1 Introduction

In this chapter, a single degree of freedom system consisting of a mass and a pneumatic artificial muscle (PAM) is studied where a time varying pneumatic pressure inside the muscle is considered. The derived resulting governing equation of motion is in the form of a forced and parametrically excited system. The first order and second order method of multiple scales (MMS) have been used to obtain the response of the system. Here, the system is subjected to the simple resonance condition and principal parametric resonance condition. The approximate analytical solution for the developed equation is compared with those obtained by solving numerically the governing equation of motion. The parametric instability regions have also been plotted for several system parameters such as muscle parameters, damping, amplitude and frequency of the time varying muscle pressure. The critical system parameters have been determined by the stability analysis. In section 3.2, the mathematical modeling of the system has been carried out to obtain the governing equation of motion which is solved using the method of multiple scales as explained in section 3.3. In section 3.4 numerical results have been discussed, and section 3.5 experimental validation has been carried out. Finally, a brief summary is presented in section 3.6.

3.2 Mathematical Modeling

The schematic diagram of the system containing a mass and a PAM has been shown in Fig. 3.1(a). Figure 3.1(b) depicts the equivalent single degree of freedom system with a spring-mass and damper. The free body diagram showing the forces acting on the system is shown in Fig. 3.1(c). The equation of motion of the system can be given by the following equation.

$$m\ddot{u} + c\dot{u} + F_{mus} - mg = 0 \quad (3.1)$$

The overdot denotes the differentiation with respect to time t . The displacement with respect to the relaxed position of the PAM is u . The force applied by the PAM is F_{mus} and g signifies the acceleration due to gravity along with the viscous damping co-efficient is c .

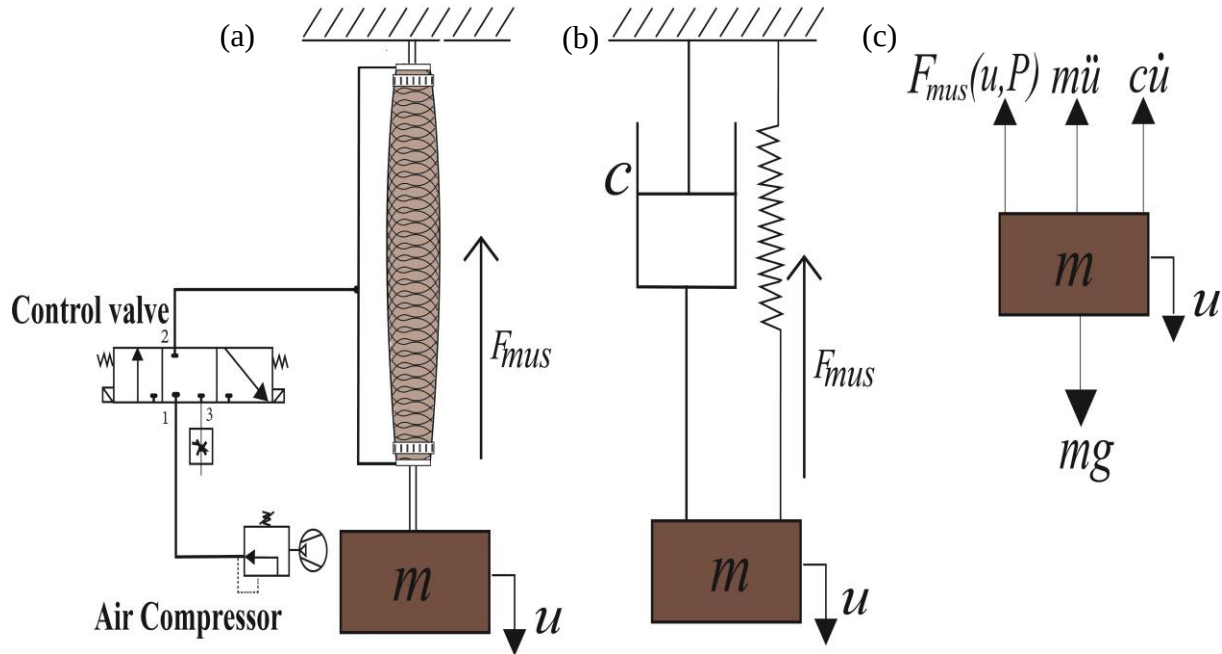


Fig. 3.1 (a) Schematic diagram of a system attached with a PAM (b) equivalent spring-mass-damper system (c) free-body diagram

Thus, F_{mus} represents the muscle model defining a relation between the tensile force exerted by the muscle, the resulting contraction and the internal muscle pressure. The PAM force, F_{mus} is estimated to be comparable with the work of Li et al. [125] which is a quasi-static characterization experimental model.

$$F_{mus}(u, P) = (c_1 + c_2 P + c_3 P^2) \left(\frac{u}{l_{\max}} \right) + (d_1 + d_2 P) \quad (3.2)$$

where c_1 , c_2 , c_3 , d_1 and d_2 are the constants which can be determined with the help of experiments and the possible maximum muscle length is $l_{\max}$. In the present work Eq. (3.2) is used to describe the governing muscle characteristics in its working range due to its linearity in contraction which simplifies further analysis. Thus, the equation of motion of the system can be written by combining Eq. (3.1) and Eq. (3.2) is as follows.

$$\ddot{u} + \left[\frac{(c_1 + c_2 P + c_3 P^2)}{ml_{\max}} \right] u + \frac{c}{m} \dot{u} + \left[\frac{(d_1 + d_2 P)}{m} - g \right] = 0 \quad (3.3)$$

To study the dynamics of the system in a realistic condition, a harmonic internal pressure input is assumed to the muscle [128, 248]. This is due to many constraints such as leakage, incorrect valve opening and closing, elasticity and inhomogeneity of the muscle parameters, it may be very difficult to maintain a constant pressure. Hence, in many practical applications, instead of a constant pressure, a time varying pressure is required to actuate the system. This approximation broadly depicts the case in applications involving position control with an oscillatory motion [125, 250, 251]. A harmonics pressure input, $P = P_m + P_0 \sin \omega t$ inside the muscle is assumed to understand the dynamics of the system in a practical condition where P_0 and P_m represents the dynamic pressure and static pressure inside the muscle, respectively.

This approximation broadly depicts the case similar to the work by Tondl and Nabergoj [252] and Li et al. [125] where the position control with an oscillatory motion is considered. In practice, this may also be applied using a proportional integrator controller and proportional flow control valves [128, 247, 248]. A non-dimensional time $\tau = \omega_0 t$ has been considered with the fundamental natural frequency of the system is ω_0 and the expression is given below.

$$\omega_0 = \sqrt{\frac{c_1 + c_2 P_m + c_3 \left(P_m^2 + \frac{P_0^2}{2} \right)}{ml_{\max}}} \quad (3.4)$$

The nondimensional displacement can be written as $u = r x$ by using a scaling factor r along with the following non-dimensional parameters have been used for formulation.

$$\Omega = \frac{\omega}{\omega_0}, \mu = \frac{c}{2\varepsilon m \omega_0}, p_1 = \frac{c_2 P_0 + 2c_3 P_0 P_m}{\varepsilon m \omega_0^2 l_{\max}}, p_2 = -\frac{c_3 P_0^2}{2\varepsilon m \omega_0^2 l_{\max}},$$

$$f_1 = \frac{d_2 P_m + d_1 - mg}{r \varepsilon m \omega_0^2}, f_2 = \frac{d_2 P_0}{r \varepsilon m \omega_0^2} \quad (3.5)$$

The simplified form of the Eq. (3.3) as follows.

$$\frac{d^2 x}{d\tau^2} + 2\varepsilon \mu \frac{dx}{d\tau} + \varepsilon (1 + p_1 \sin \Omega \tau + p_2 \cos 2\Omega \tau) x = \varepsilon (f_1 + f_2 \sin \Omega \tau) \quad (3.6)$$

Here, a dimensionless book keeping parameter ε is used whose value is less than 1. From Eq. (3.5), it can be observed that the nondimensional parameter p_1 is dependent on the both dynamic pressure P_0 and static pressure P_m inside the muscle. But p_2 and f_2 depend on P_0 , whereas, f_1 depends only on the P_m . From Eq. (3.6), it can be noticed that the coefficients of response x comprises time varying terms with the frequency Ω and 2Ω . Therefore, with multi frequency excitation this is a parametrically excited system. From the right hand side of the governing equation Eq. (3.6) with sinusoidally varying force f_2 and frequency Ω , the system exhibits forced vibration. In the next section 3.3, second order method of multiple scales (MMS) has been used to obtain the approximate solutions of the system.

3.3 Approximate Solution

In this work, the approximate analytical solution for the developed equation of motion in Eq. (3.6) is derived by using the second order MMS [13, 14]. The primary variable is written as follows with the basic formulation of method of multiple scales.

$$x(\tau, \varepsilon) = x_0(T_0, T_1, T_2) + \varepsilon x_1(T_0, T_1, T_2) + \varepsilon^2 x_2(T_0, T_1, T_2) \quad (3.7)$$

where, $T_n = \varepsilon^n \tau, n = 0, 1, 2$

Then substituting Eq. (3.7) in Eq. (3.6) yields

$$D_0^2 x_0 + \varepsilon D_0^2 x_1 + \varepsilon^2 D_0^2 x_2 + 2\varepsilon D_0 D_1 x_0 + 2\varepsilon^2 D_0 D_1 x_1 + \varepsilon^2 x_0 (D_1 + 2D_0 D_2) + 2\varepsilon \mu D_0 x_0 + 2\varepsilon^2 \mu D_0 x_1$$

$$+ 2\varepsilon^2 \mu D_1 x_0 + x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \varepsilon (x_0 + \varepsilon x_1)(p_1 \sin \Omega \tau + p_2 \cos 2\Omega \tau) = \varepsilon (f_1 + f_2 \sin \Omega \tau) \quad (3.8)$$

The following equations have been obtained by extracting the terms from Eq. (3.8) with different order of ε .

$$\text{Order of } \varepsilon^0, D_0^2 x_0 + x_0 = 0 \quad (3.9)$$

Order of ε^1 ,

$$D_0^2 x_1 + x_1 = f_1 + f_2 \sin \Omega T_0 - 2D_0 D_1 x_0 - 2\mu D_0 x_0 - x_0 p_1 \sin \Omega T_0 - x_0 p_2 \cos 2\Omega T \quad (3.10)$$

Order of ε^2 ,

$$D_0^2 x_2 + x_2 = -2D_0 D_1 x_1 - x_0 (D_1 + 2D_0 D_2) - 2\mu D_0 x_1 - 2\mu D_1 x_0 - x_1 (p_1 \sin \Omega T_0 + p_2 \cos 2\Omega T_0) \quad (3.11)$$

Solving Eq. (3.9) gives the 0th order solution:

$$x_0 = A(T_1, T_2) e^{iT_0} + CC \quad (3.12)$$

where the complex conjugate of the preceding terms is represented as CC . A denotes the amplitude of response which is a function of T_1 and T_2 .

Eq. (3.12) is substituted in the Eq. (3.10) which results as follows.

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{if_2}{2} e^{i\Omega T_0} - 2D_1 i A e^{iT_0} - 2\mu i A e^{iT_0} - \frac{ip_1}{2} A e^{i(1-\Omega)T_0} - \frac{ip_1}{2} \bar{A} e^{-i(1+\Omega)T_0} - \frac{p_2}{2} A e^{i(1+2\Omega)T_0} - \frac{p_2}{2} \bar{A} e^{i(2\Omega-1)T_0} + CC \quad (3.13)$$

where the overbar represents the complex conjugate. In the next subsections 3.3.1 and 3.3.2, the simple resonance condition when the nondimensional external excitation frequency $\Omega \approx 1$ and principal parametric resonance condition i.e., $\Omega \approx 2$ have been studied respectively.

3.3.1 Simple resonance condition

In this case, introduce the detuning parameter, σ and one may write $\Omega = 1 + \varepsilon\sigma$. Then Eq. (3.13) becomes as follows.

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{if_2}{2} e^{i(1+\varepsilon\sigma)T_0} - 2D_1 i A e^{iT_0} - 2\mu i A e^{iT_0} - \frac{ip_1}{2} A e^{-i\varepsilon\sigma T_0} - \frac{ip_1}{2} \bar{A} e^{-i(2+\varepsilon\sigma)T_0} - \frac{p_2}{2} A e^{i(3+2\varepsilon\sigma)T_0} - \frac{p_2}{2} \bar{A} e^{i(1+2\varepsilon\sigma)T_0} + CC \quad (3.14)$$

The secular terms which contains e^{iT_0} will be leading to singular particular solution which should be removed to have bounded solution of the Eq. (3.14). The secular terms have been neglected from the Eq. (3.14) and are represented as below.

$$-\frac{if_2}{2}e^{i\varepsilon\sigma T_0} - 2D_1iA - 2\mu iA - \frac{p_2}{2}\bar{A}e^{i2\varepsilon\sigma T_0} = 0 \quad (3.15)$$

$$\text{or, } D_1A = -\frac{f_2}{4}e^{i\varepsilon\sigma T_0} - \mu A + \frac{ip_2}{4}\bar{A}e^{i2\varepsilon\sigma T_0} \quad (3.16)$$

Eq. (3.14) can be rewritten as,

$$D_0^2x_1 + x_1 = \frac{f_1}{2} - \frac{ip_1}{2}Ae^{-i\varepsilon\sigma T_0} - \frac{ip_1}{2}\bar{A}e^{-i(2+\varepsilon\sigma)T_0} - \frac{p_2}{2}Ae^{i(3+2\varepsilon\sigma)T_0} + CC. \quad (3.17)$$

Hence,

$$x_1 = \frac{f_1}{2} - \frac{ip_1Ae^{-i\varepsilon\sigma T_0}}{2(1-\varepsilon^2\sigma^2)} - \frac{ip_1\bar{A}e^{-i(2+\varepsilon\sigma)T_0}}{2[1-(2+\varepsilon\sigma)^2]} - \frac{p_2Ae^{i(3+2\varepsilon\sigma)T_0}}{2[1-(3+2\varepsilon\sigma)^2]} + CC \quad (3.18)$$

Now Eq. (3.18) and Eq. (3.12) have been substituted in Eq. (3.11) and the secular terms have been removed to obtain bounded solution.

$$D_0^2x_2 + x_2 = \left[-D_1A - 2iD_2A - 2\mu D_1A - \frac{f_1p_1}{4i}e^{i\varepsilon\sigma T_0} + \frac{p_1^2A}{4(1-\varepsilon^2\sigma^2)} - \frac{p_1^2A}{4(1-(2+\varepsilon\sigma)^2)} + \frac{p_2^2A}{4(1-(3+2\varepsilon\sigma)^2)} \right] e^{iT_0} + \text{other term} \quad (3.19)$$

Therefore,

$$D_2A = -(1+2\mu)\frac{D_1A}{2i} + \frac{f_1p_1}{8}e^{i\varepsilon\sigma T_0} - \frac{ip_1^2A}{8(1-\varepsilon^2\sigma^2)} + \frac{ip_1^2A}{8(1-(2+\varepsilon\sigma)^2)} - \frac{ip_2^2A}{8(1-(3+2\varepsilon\sigma)^2)} \quad (3.20)$$

Now, by using the reconstitution method [239],

$$\begin{aligned} \frac{dA}{d\tau} &= \frac{dA}{dT_1} \frac{dT_1}{d\tau} + \frac{dA}{dT_2} \frac{dT_2}{d\tau} = \varepsilon D_1A + \varepsilon^2 D_2A \\ \frac{dA}{d\tau} &= \left[\varepsilon - \frac{\varepsilon^2(1+2\mu)}{2i} \right] \left(-\frac{f_2}{4}e^{i\varepsilon\sigma T_0} - \mu A + \frac{ip_2}{4}\bar{A}e^{i2\varepsilon\sigma T_0} \right) + \varepsilon^2 \left[\frac{f_1p_1}{8}e^{i\varepsilon\sigma T_0} - \frac{ip_1^2A}{8(1-\varepsilon^2\sigma^2)} \right. \\ &\quad \left. + \frac{ip_1^2A}{8(1-(2+\varepsilon\sigma)^2)} - \frac{ip_2^2A}{8(1-(3+2\varepsilon\sigma)^2)} \right] \end{aligned} \quad (3.21)$$

Introducing $A = \frac{1}{2}ae^{i\beta}$ and $\bar{A} = \frac{1}{2}ae^{-i\beta}$ where a and β are the real numbers and $\varepsilon\sigma T_0 - \beta = \gamma$ in Eq. (3.21) and the following reduced equations can be obtained by the separation of the real and imaginary parts.

$$a' = \frac{da}{dt} = \varepsilon \left(-\frac{f_2}{2} \cos \gamma - \mu a - \frac{p_2 a}{4} \sin 2\gamma \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(\frac{f_2}{2} \sin \gamma - \frac{p_2 a}{4} \cos 2\gamma \right) + \varepsilon^2 \left(\frac{f_1 p_1}{4} \cos \gamma \right) \quad (3.22)$$

$$a\gamma' = a \frac{d\gamma}{dt} = a\varepsilon\sigma - \left[\varepsilon \left(-\frac{f_2}{2} \sin \gamma + \frac{p_2 a}{4} \cos 2\gamma \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\frac{f_2}{2} \cos \gamma - \mu a - \frac{p_2 a}{4} \sin 2\gamma \right) + \varepsilon^2 \left[\frac{f_1 p_1}{4} \sin \gamma - \frac{p_1^2 a}{8(1-\varepsilon^2\sigma^2)} + \frac{p_1^2 a}{8(1-(2+\varepsilon\sigma)^2)} - \frac{p_2^2 a}{8(1-(3+2\varepsilon\sigma)^2)} \right] \right] \quad (3.23)$$

The following equation can be obtained by neglecting the secular terms from Eq. (3.14).

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{ip_1}{2} A e^{i(1-\Omega)T_0} - \frac{ip_1}{2} \bar{A} e^{-i(1+\Omega)T_0} - \frac{p_2}{2} A e^{i(1+2\Omega)T_0} + CC \quad (3.24)$$

From Eq. (3.24), the particular solution of x_1 is as follows.

$$x_1 = \frac{f_1}{2} - \frac{1}{(1-(1-\Omega)^2)} \frac{ip_1}{2} A e^{i(1-\Omega)T_0} - \frac{1}{(1-(1+\Omega)^2)} \frac{ip_1}{2} \bar{A} e^{-i(1+\Omega)T_0} - \frac{1}{(1-(1+2\Omega)^2)} \frac{p_2}{2} A e^{i(1+2\Omega)T_0} + CC \quad (3.25)$$

Thus,

$$x(\tau) = a(T_1) \cos(\Omega T_0 - \gamma(T_1)) + \varepsilon \left[f_1 - \frac{p_1 a(T_1) \sin \gamma(T_1)}{2(1-(1-\Omega)^2)} - \frac{p_1 a(T_1) \sin(2\Omega T_0 - \gamma(T_1))}{2(1-(1+\Omega)^2)} - \frac{p_2 a(T_1) \cos(3\Omega T_0 - \gamma(T_1))}{2(1-(1+2\Omega)^2)} \right] \quad (3.26)$$

Eq. (3.26) gives the expression for the total time response of the parametrically excited system under simple resonance condition.

3.3.1.1 Stability of the steady state response

It can be noticed from the reduced equations Eq. (3.22) and Eq. (3.23) for simple resonance condition, the system exhibits both trivial solution ($a = 0$) and nontrivial solution ($a \neq 0$). Therefore, the stability of the steady state can be studied by examining the nature of the equilibrium points of the system. The trivial state stability solutions may be obtained by changing the Eq. (3.22) and Eq. (3.23) to the Cartesian form of modulation equations and introduce the conversion $p = a \cos \gamma$ and $q = a \sin \gamma$ [13, 14]. Now one may obtain from Eq. (3.22) and Eq. (3.23) in the following form of Cartesian modulation equations.

$$p' = \varepsilon \left(-\frac{f_2}{2} - \mu p - \frac{p_2 q}{4} \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\mu q - \frac{p_2 p}{4} \right) + \varepsilon^2 \left[\frac{f_1 p_1}{4} - \frac{p_1^2 q}{8(1-\varepsilon^2 \sigma^2)} + \frac{p_1^2 q}{8(1-(2+\varepsilon \sigma)^2)} - \frac{p_2^2 q}{8(1-(3+2\varepsilon \sigma)^2)} \right] - \varepsilon \sigma q \quad (3.27)$$

$$q' = \varepsilon \left(-\mu q - \frac{p_2 p}{4} \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(\frac{f_2}{2} + \mu p + \frac{p_2 q}{4} \right) + \varepsilon^2 \left[\frac{p_1^2 p}{8(1-\varepsilon^2 \sigma^2)} - \frac{p_1^2 p}{8(1-(2+\varepsilon \sigma)^2)} + \frac{p_2^2 p}{8(1-(3+2\varepsilon \sigma)^2)} \right] + \varepsilon \sigma p \quad (3.28)$$

Stability of the steady state response (p_0, q_0) can be obtained from the eigenvalues of the following Jacobian matrix (J) which is found by the perturbation of Eq. (3.27) and Eq. (3.28).

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \quad (3.29)$$

$$J_{11} = -\varepsilon \mu - \frac{\varepsilon^2 (1+2\mu)}{2} \frac{p_2}{4}$$

$$J_{12} = -\frac{\varepsilon p_2}{4} - \frac{\mu \varepsilon^2 (1+2\mu)}{2} + \varepsilon^2 \left[-\frac{p_1^2}{8(1-\varepsilon^2 \sigma^2)} + \frac{p_1^2}{8(1-(2+\varepsilon \sigma)^2)} - \frac{p_2^2}{8(1-(3+2\varepsilon \sigma)^2)} \right] - \varepsilon \sigma$$

$$J_{21} = -\frac{\varepsilon p_2}{4} + \frac{\mu \varepsilon^2 (1+2\mu)}{2} + \varepsilon^2 \left[\frac{p_1^2}{8(1-\varepsilon^2 \sigma^2)} - \frac{p_1^2}{8(1-(2+\varepsilon \sigma)^2)} + \frac{p_2^2}{8(1-(3+2\varepsilon \sigma)^2)} \right] + \varepsilon \sigma$$

$$J_{22} = -\varepsilon \mu + \frac{\varepsilon^2 (1+2\mu)}{2} \frac{p_2}{4}$$

Here, the trivial or non-trivial response of the parametrically excited system will be stable for simple resonance condition, if the real parts of all the eigenvalues of the Jacobian matrix (J) in Eq. (3.29) are negative.

3.3.2 Principal parametric resonance condition

Here also the detuning parameter, σ has been introduced by considering $\Omega = 2 + \varepsilon\sigma$ and the Eq. (3.13) can be written as follows.

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{if_2}{2} e^{i(2+\varepsilon\sigma)T_0} - 2D_1 i A e^{iT_0} - 2\mu i A e^{iT_0} - \frac{ip_1}{2} A e^{i(-1-\varepsilon\sigma)T_0} - \frac{ip_1}{2} \bar{A} e^{-i(3+\varepsilon\sigma)T_0} - \frac{p_2}{2} A e^{i(5+2\varepsilon\sigma)T_0} - \frac{p_2}{2} \bar{A} e^{i(3+2\varepsilon\sigma)T_0} + CC \quad (3.30)$$

Similar to the previous case for achieving bounded solution, the secular terms containing e^{iT_0} has to be neglected from Eq. (3.30) and terms have been expressed below.

$$-2D_1 i A - 2\mu i A + \frac{ip_1}{2} \bar{A} e^{i\varepsilon\sigma T_0} = 0 \quad (3.31)$$

$$\text{or, } D_1 A = -\mu A + \frac{p_1}{4} \bar{A} e^{i\varepsilon\sigma T_0} \quad (3.32)$$

Eq. (3.30) can be written as follows.

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{if_2}{2} e^{i(2+\varepsilon\sigma)T_0} - \frac{ip_1}{2} \bar{A} e^{-i(3+\varepsilon\sigma)T_0} - \frac{p_2}{2} A e^{i(5+2\varepsilon\sigma)T_0} - \frac{p_2}{2} \bar{A} e^{i(3+2\varepsilon\sigma)T_0} + CC \quad (3.33)$$

$$x_1 = \frac{f_1}{2} - \frac{if_2}{2(1-(2+\varepsilon\sigma)^2)} e^{i(2+\varepsilon\sigma)T_0} - \frac{ip_1}{2[1-(3+\varepsilon\sigma)^2]} \bar{A} e^{-i(3+\varepsilon\sigma)T_0} - \frac{p_2}{2[1-(5+2\varepsilon\sigma)^2]} A e^{i(5+2\varepsilon\sigma)T_0} - \frac{p_2}{2[1-(3+2\varepsilon\sigma)^2]} \bar{A} e^{i(3+2\varepsilon\sigma)T_0} + CC \quad (3.34)$$

Now, Eq. (3.34) and Eq. (3.12) have been substituted in Eq. (3.11) and the secular terms are neglected.

$$D_0^2 x_2 + x_2 = \left[-D_1 A - 2iD_2 A - 2\mu D_1 A + \frac{p_1^2 A}{4(1-(3+\varepsilon\sigma)^2)} + \frac{ip_1 p_2 \bar{A} e^{i\varepsilon\sigma T_0}}{4(1-(3+2\varepsilon\sigma)^2)} \right. \\ \left. + \frac{ip_1 p_2 \bar{A} e^{i\varepsilon\sigma T_0}}{4(1-(3+\varepsilon\sigma)^2)} + \frac{p_2^2 A}{4(1-(3+2\varepsilon\sigma)^2)} + \frac{p_2^2 A}{4(1-(5+2\varepsilon\sigma)^2)} \right] e^{iT_0} + \text{other term} \quad (3.35)$$

Therefore,

$$D_2 A = -(1+2\mu) \frac{D_1 A}{2i} - \frac{ip_1^2 A}{8(1-(3+\varepsilon\sigma)^2)} + \frac{p_1 p_2 \bar{A} e^{i\varepsilon\sigma T_0}}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{p_1 p_2 \bar{A} e^{i\varepsilon\sigma T_0}}{8(1-(3+\varepsilon\sigma)^2)} \\ - \frac{ip_2^2 A}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{ip_2^2 A}{8(1-(5+2\varepsilon\sigma)^2)} \quad (3.36)$$

Now,

$$\frac{dA}{d\tau} = \left[\varepsilon - \frac{\varepsilon^2(1+2\mu)}{2i} \right] \left(-\mu A + \frac{p_1}{4} \bar{A} e^{i\varepsilon\sigma T_0} \right) + \varepsilon^2 \left[-\frac{ip_1^2 A}{8(1-(3+\varepsilon\sigma)^2)} + \frac{p_1 p_2 \bar{A} e^{i\varepsilon\sigma T_0}}{8(1-(3+2\varepsilon\sigma)^2)} \right. \\ \left. + \frac{p_1 p_2 \bar{A} e^{i\varepsilon\sigma T_0}}{8(1-(3+\varepsilon\sigma)^2)} - \frac{ip_2^2 A}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{ip_2^2 A}{8(1-(5+2\varepsilon\sigma)^2)} \right] \quad (3.37)$$

Again, by introducing $A = \frac{1}{2} a e^{i\beta}$ and $\varepsilon\sigma T_0 - 2\beta = \gamma$ in Eq. (3.37), the following reduced equations can be obtained by the separation of the real and imaginary parts.

$$a' = \frac{da}{dt} = \varepsilon \left(-\mu a + \frac{p_1}{4} a \cos 2\gamma \right) - \frac{\varepsilon^2(1+2\mu)}{2} \left(\frac{p_1 a}{4} \sin 2\gamma \right) + \varepsilon^2 \left[\frac{p_1 p_2 a \cos 2\gamma}{8(1-(3+2\varepsilon\sigma)^2)} \right. \\ \left. + \frac{p_1 p_2 a \cos 2\gamma}{8(1-(3+\varepsilon\sigma)^2)} \right] \quad (3.38)$$

$$a\gamma' = a \frac{d\gamma}{dt} = \frac{a\varepsilon\sigma}{2} - \left[\frac{\varepsilon p_1 a}{4} \sin 2\gamma + \frac{\varepsilon^2(1+2\mu)}{2} \left(-\mu a + \frac{p_1 a}{4} \cos 2\gamma \right) + \varepsilon^2 \left[-\frac{p_1^2 a}{8(1-(3+\varepsilon\sigma)^2)} \right. \right. \\ \left. \left. + \frac{p_1 p_2 a \sin 2\gamma}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{p_1 p_2 a \sin 2\gamma}{8(1-(3+\varepsilon\sigma)^2)} - \frac{p_2^2 a}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{p_2^2 a}{8(1-(5+2\varepsilon\sigma)^2)} \right] \right] \quad (3.39)$$

Now, the following equation can be obtained by neglecting the secular terms from Eq. (3.30).

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{if_2}{2} e^{i\Omega T_0} - \frac{ip_1}{2} \bar{A} e^{-i(1+\Omega)T_0} - \frac{p_2}{2} A e^{i(1+2\Omega)T_0} - \frac{p_2}{2} \bar{A} e^{i(2\Omega-1)T_0} + CC \quad (3.40)$$

The particular solution to x_1 in Eq. (3.40) is thus obtained to be as follows.

$$x_1 = \frac{f_1}{2} - \frac{1}{(1-\Omega^2)} \frac{if_2}{2} e^{i\Omega T_0} - \frac{1}{(1-(1+\Omega)^2)} \frac{ip_1}{2} \bar{A} e^{-i(1+\Omega)T_0} - \frac{1}{(1-(1+2\Omega)^2)} \frac{p_2}{2} A e^{i(1+2\Omega)T_0} - \frac{1}{(1-(2\Omega-1)^2)} \frac{p_2}{2} \bar{A} e^{i(2\Omega-1)T_0} + CC \quad (3.41)$$

Thus,

$$x(\tau) = a(T_1) \cos\left(\frac{\Omega T_0 - 2\gamma(T_1)}{2}\right) + \varepsilon \left[f_1 + \frac{f_2}{(1-\Omega)^2} \sin \Omega T_0 - \frac{p_1 a(T_1)}{2(1-(1+\Omega)^2)} \sin\left(\frac{3\Omega T_0 - 2\gamma(T_1)}{2}\right) - \frac{p_2 a(T_1)}{2(1-(1+2\Omega)^2)} \cos\left(\frac{5\Omega T_0 - 2\gamma(T_1)}{2}\right) - \frac{p_2 a(T_1)}{2(1-(2\Omega-1)^2)} \cos\left(\frac{3\Omega T_0 + 2\gamma(T_1)}{2}\right) \right] \quad (3.42)$$

Eq. (3.42) gives the expression for the total time response of the parametrically excited system under principal parametric resonance condition.

3.3.2.1 Stability of the steady state response

It can be noticed from the reduced equations Eq. (3.38) and Eq. (3.39) for principal resonance condition, the system exhibits both trivial solution ($a = 0$) and nontrivial solution ($a \neq 0$). So, the stability of the steady state response of the system can be studied as mentioned in the section 3.3.1.1. Now one may obtain the following form of Cartesian modulation equations from Eq. (3.38) and Eq. (3.39).

$$p' = \varepsilon \left(-\mu p + \frac{p_1 p}{4} \right) - \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\mu q + \frac{p_1 q}{4} \right) + \varepsilon^2 \left[-\frac{p_1^2 q}{8(1-(3+\varepsilon\sigma)^2)} + \frac{p_1 p_2 p}{8(1-(3+2\varepsilon\sigma)^2)} \right. \\ \left. + \frac{p_1 p_2 p}{8(1-(3+\varepsilon\sigma)^2)} - \frac{p_2^2 q}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{p_2^2 q}{8(1-(5+2\varepsilon\sigma)^2)} \right] - \frac{\varepsilon\sigma q}{2} \quad (3.43)$$

$$q' = \varepsilon \left(-\mu q - \frac{p_1 q}{4} \right) - \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\mu p + \frac{p_1 p}{4} \right) + \varepsilon^2 \left[\frac{p_1^2 p}{8(1-(3+\varepsilon\sigma)^2)} - \frac{p_1 p_2 q}{8(1-(3+2\varepsilon\sigma)^2)} \right. \\ \left. - \frac{p_1 p_2 q}{8(1-(3+\varepsilon\sigma)^2)} + \frac{p_2^2 p}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{p_2^2 p}{8(1-(5+2\varepsilon\sigma)^2)} \right] + \frac{\varepsilon\sigma p}{2} \quad (3.44)$$

The stability of the steady state response (p_0, q_0) can be obtained from the eigenvalues of the following Jacobian matrix (J) which is found by the perturbation of Eq. (3.43) and Eq. (3.44).

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \quad (3.45)$$

$$J_{11} = \varepsilon \left(-\mu + \frac{p_1}{4} \right) + \varepsilon^2 \left[\frac{p_1 p_2}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{p_1 p_2}{8(1-(3+\varepsilon\sigma)^2)} \right] \\ J_{12} = -\frac{\varepsilon^2 (1+2\mu)}{2} \left(-\mu + \frac{p_1}{4} \right) + \varepsilon^2 \left[-\frac{p_1^2}{8(1-(3+\varepsilon\sigma)^2)} - \frac{p_2^2}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{p_2^2}{8(1-(5+2\varepsilon\sigma)^2)} \right] - \frac{\varepsilon\sigma}{2} \\ J_{21} = -\frac{\varepsilon^2 (1+2\mu)}{2} \left(-\mu + \frac{p_1}{4} \right) + \varepsilon^2 \left[\frac{p_1^2}{8(1-(3+\varepsilon\sigma)^2)} + \frac{p_2^2}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{p_2^2}{8(1-(5+2\varepsilon\sigma)^2)} \right] + \frac{\varepsilon\sigma}{2} \\ J_{22} = \varepsilon \left(-\mu - \frac{p_1}{4} \right) + \varepsilon^2 \left[-\frac{p_1 p_2}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{p_1 p_2}{8(1-(3+\varepsilon\sigma)^2)} \right]$$

Here the trivial or non-trivial response of the system will be stable for principal parametric resonance condition, if the real parts of all the eigenvalues of the Jacobian matrix (J) in Eq. (3.45) are negative. In the following section taking physical parameters, the numerical results has been discussed.

3.4 Results and Discussion

For numerical analysis, Table 3.1 shows the values of different muscle parameters along with the other system parameters. The values of the various parameters are similar to the values in the work of Li et al. [125]. The time response behaviors of the system have been found out by using the reduced equations Eq. (3.22) and Eq. (3.23) for simple resonance condition and Eq. (3.38) and Eq. (3.39) for principal parametric resonance condition. The trivial state instability region plots for different system parameters will be discussed in subsection 3.4.1.1 for $\Omega \approx 1$ and subsection 3.4.2.1 for $\Omega \approx 2$. These plots have been marked as ‘U’ to represent the unstable regions which is bounded by curve and outside the curve is stable regions which is marked as ‘S’.

Table 3.1 System parameters numerical values.

Parameter	Numerical Value	Parameter	Numerical Value	Parameter	Numerical Value
$l_{\max}$	74 mm	d_1	-100 N	P_0	58 kPa
c_1	-234.25 N	d_2	1 N/kPa	ε	0.1
c_2	1.96 N/kPa	m	6 N	μ	0.01
c_3	-0.003 N/kPa ²	P_m	300 kPa	r	1

3.4.1 Simple resonance condition

In this section, the response of the system has been studied which is subjected to simple resonance condition where the external excitation frequency is nearly equal to the natural frequency of the system. In Fig. 3.3 taking the parameters value as in Table 3.1, the comparison has been made among responses from the reduced equations from first and second order MMS (marked AS in the figures) with the original equation (marked NS in the figures) using Runge-Kutta method. The time response and phase portrait has been plotted using the reduced equations Eq. (3.22) and Eq. (3.23) in Fig. 3.2(a, b) from the first order MMS and in Fig. 3.2(c, d) from the second order MMS. These responses are compared with those found by solving numerically the original equation Eq. (3.6) for both the cases and are found to be in good agreement. The mean response amplitude of the system is 0.137 in both the cases for first or second order MMS. From Fig. 3.2(a) and Fig. 3.2(c) the maximum response amplitude solving by original equation Eq. (3.6) is 0.208. The maximum response amplitude solving the reduced equations Eq. (3.22) and Eq. (3.23) from the

first order MMS is 0.253 (Fig. 3.3(a)) which increase around 22% and from the second order MMS is 0.235 mm (Fig. 3.3(c)) which increase around 13% from the solution obtained by solving the original equation Eq. (3.6). Hence, it may clearly be noticed that the solution obtained from the reduced equations using higher order MMS gives conservative results that obtained by solving the original equation. So, for further analyses, the reduced equations from second order MMS have been used to study the responses of the system.

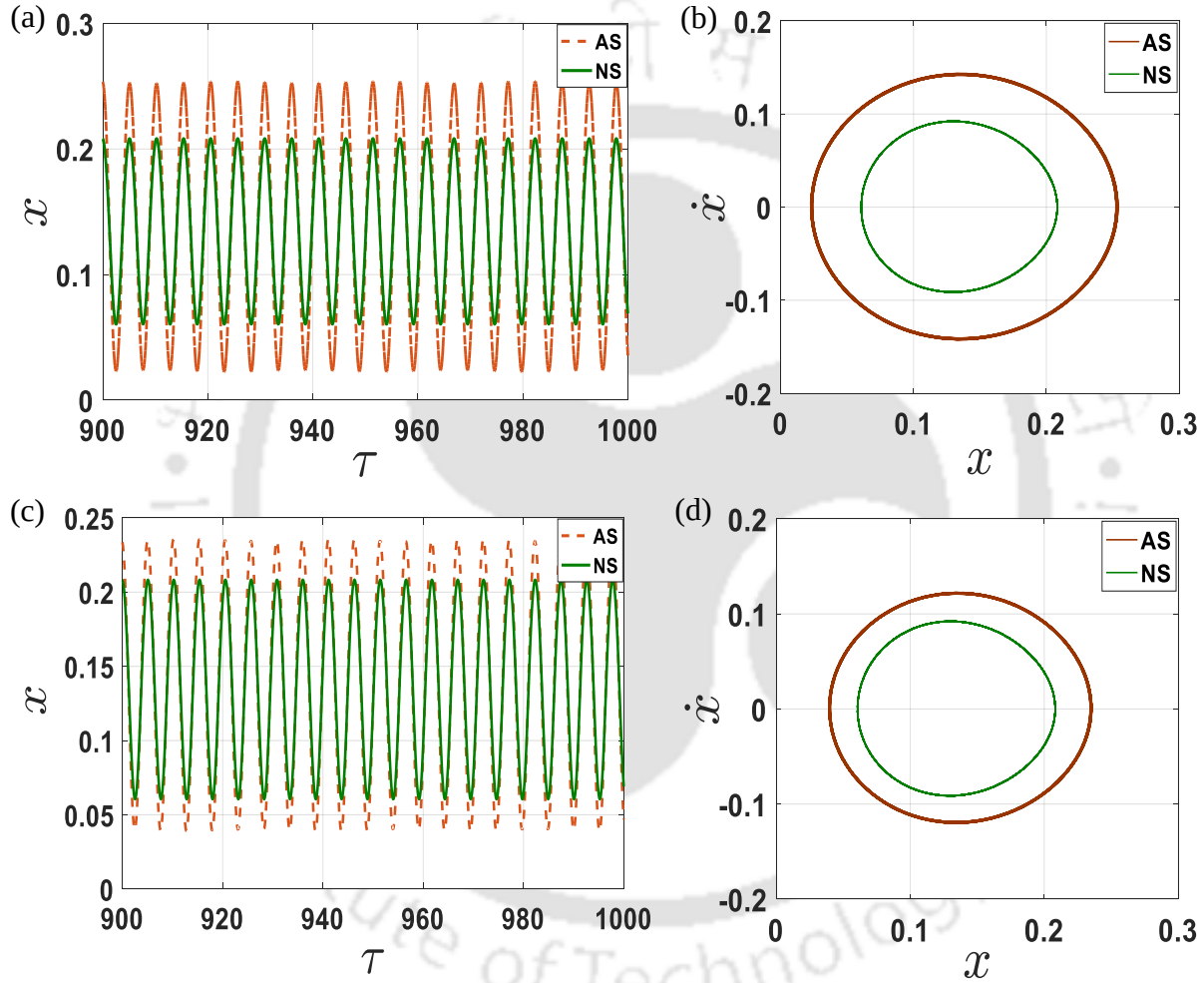


Fig. 3.2 Time response and phase portrait comparison under $\Omega \approx 1$ (a, b) 1st order MMS (c, d) 2nd order MMS.

3.4.1.1 Trivial state instability regions

The trivial state instability regions for different system parameters values have been plotted where the real parts of the eigenvalues of the Jacobian matrix change its sign. The regions are plotted in

Fig. 3.3 and Fig. 3.5 with Jacobian matrix (J) in Eq. (3.29) for variation with different system parameters under simple resonance condition.

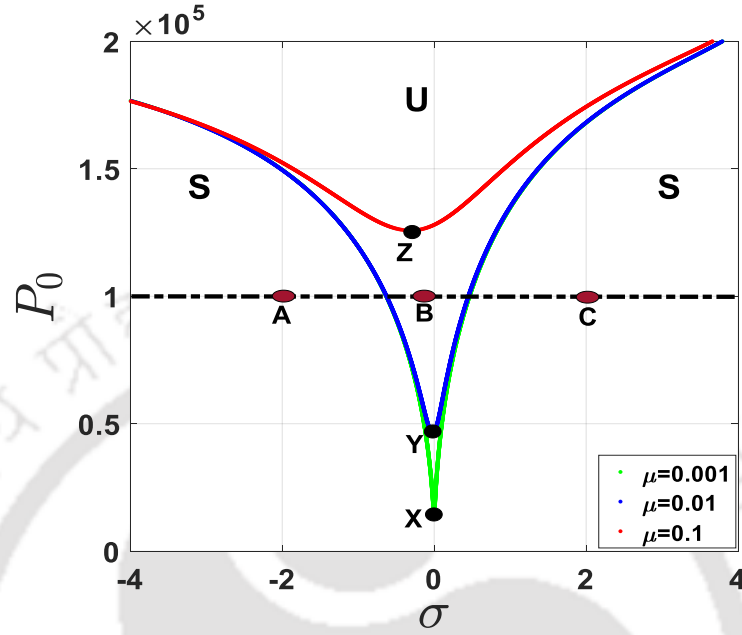


Fig. 3.3 Trivial state instability region in $P_0 \sim \sigma$ plane for three different value of $\mu = 0.001, 0.01$ and 0.1 under $\Omega \approx 1$.

In Fig. 3.3, the instability regions are plotted with the variation of detuning parameter (σ) with dynamic pressure of the muscle (P_0) for three different values of damping (μ). For $P_0 = 100$ kPa and $\mu = 0.01$ in Fig. 3.3, in between the detuning parameter values from $\sigma = -0.627$ and $\sigma = 0.45$ the instability regions exists. So, to verify the results time response and phase portraits have been plotted in Fig. 3.4 for three different points A ($\sigma = -2$), B ($\sigma = -0.2$) and C ($\sigma = 2$) on the instability plot. It can be clearly observed from the time response in Fig. 3.4(a, c) and phase portraits in Fig. 3.4(b, d) that the responses for the points A and C are stable whereas from Fig. 3.4(e, f), it depicts the unstable response for point B. From Fig. 3.3, it can also be noticed that the system will behave as stable below the points X, Y and Z for a certain system parameter value of P_0 for any σ value. This is called as the critical value of P_0 (P_{0cr}), e.g., at point X for $\mu = 0.001$, $P_{0cr} = 15$ kPa; at point Y for $\mu = 0.01$, $P_{0cr} = 46.4$ kPa and finally at point Z for $\mu = 0.1$, $P_{0cr} = 125.9$ kPa in Fig. 3.3. Later, it is also be noted that the unstable region will decrease with the increase in the value of damping (μ) and observed at higher value of the dynamic pressure (P_0).

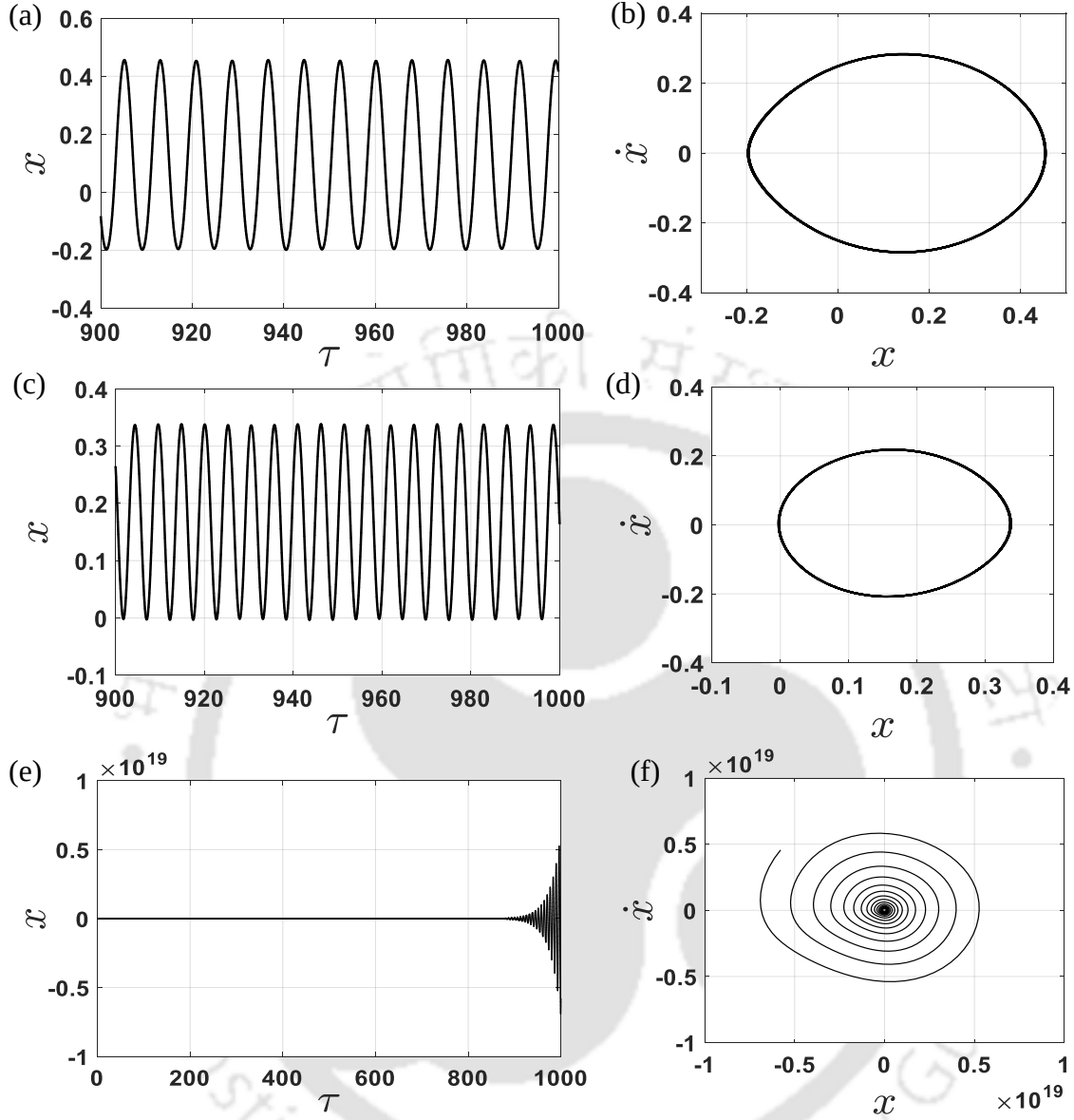


Fig. 3.4 Time responses and phase portraits for (a, b) point A (c, d) point C (e, f) point B from Fig. 3.3 for $\Omega \approx 1$.

The trivial state instability regions have been plotted with three different values for muscle parameters c_1 , c_2 , c_3 , d_1 and d_2 along with the static pressure in the muscle P_m in Fig. 3.5. A large range of unstable region exists for the optimum value of muscle parameter c_1 , c_2 and c_3 which can be observed from Fig. 3.5(a), Fig. 3.5(b) and Fig. 3.5(c) respectively. The variation in the trivial state instability regions is not prominent for another two muscle parameters d_1 and d_2 in Fig. 3.5(d) and Fig. 3.5(e) respectively. The effect of three different values for static pressure

in the muscle P_m in the trivial state instability regions can be noticed in Fig. 3.5(f). In this case also for an optimum value of P_m , a wide range of unstable region can be obtained.

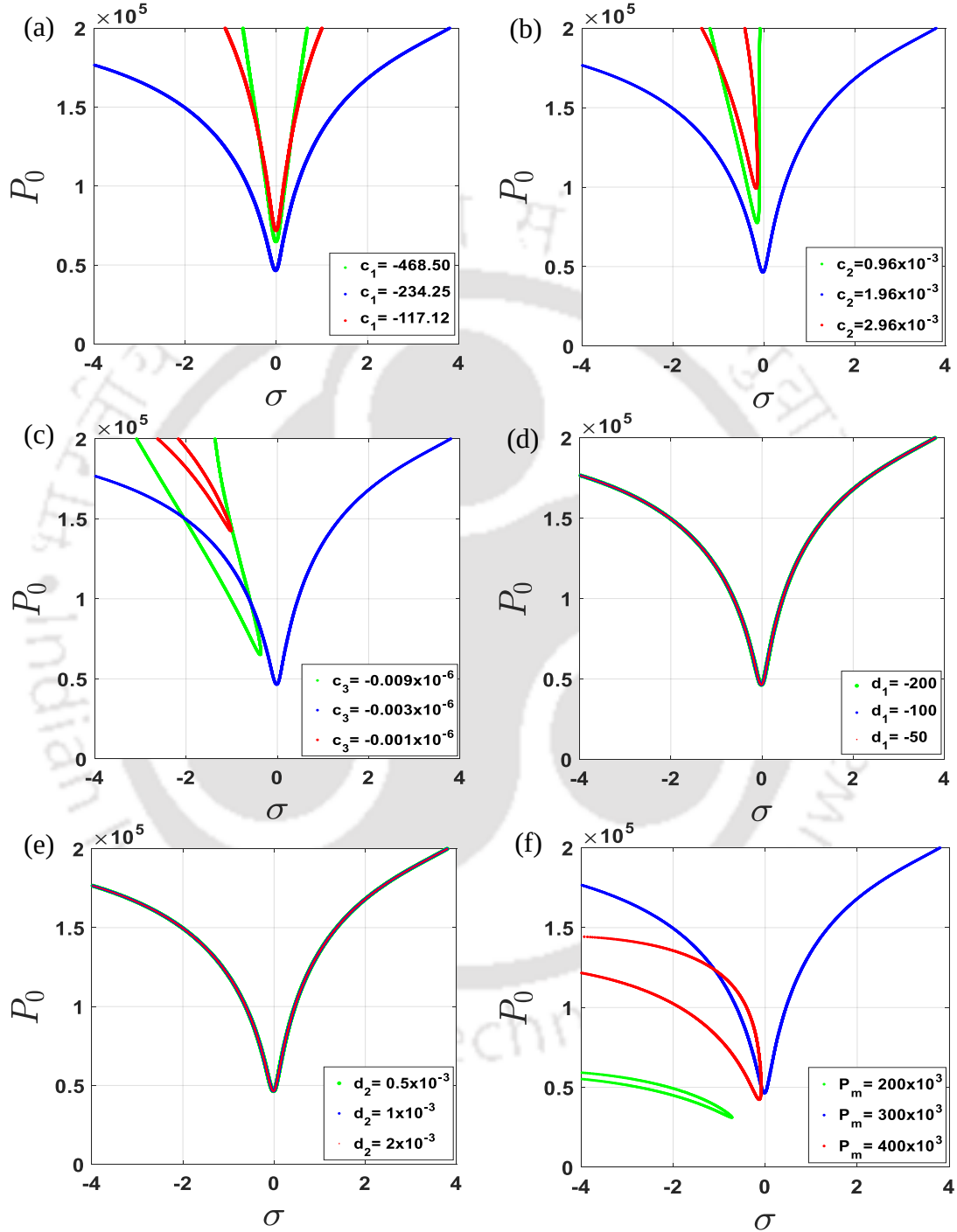


Fig. 3.5 Trivial state instability region in $P_0 \sim \sigma$ plane under $\Omega \approx 1$ with variation in (a) c_1 (b) c_2 (c) c_3 (d) d_1 (e) d_2 (f) P_m .

The critical value of P_0 (P_{0cr}) for different sets system parameters for $\Omega \approx 1$ have been tabulated in Table 3.2 from Fig. 3.5 with corresponding value of detuning parameter σ below which the system will behave as stable. From Table 3.2, it can be observed that Case 7 gives the highest value of $P_{0cr} = 142.4$ kPa with $c_3 = -0.001$ N/kPa² as compared to the other cases. Similarly, in Case 18 with $P_m = 200$ kPa provides the lowest value of $P_{0cr} = 31.1$ kPa. This will help the designer to choose the system parameters properly to actuate the muscle in a wide range operating pressure value for a particular application under simple resonance condition.

Table 3.2 Critical value of P_0 (P_{0cr}) with variation in system parameters from Fig. 3.5.

Case	Variation in system parameter	P_{0cr} (kPa)	σ	Reference Figure No.
1	$c_1 = -117.12$ N	71.8	- 0.01	Fig. 3.5(a)
2	$c_1 = -234.25$ N	46.5	- 0.03	Fig. 3.5(a)
3	$c_1 = -468.50$ N	64.7	- 0.02	Fig. 3.5(a)
4	$c_2 = 2.96$ N/kPa	99.2	- 0.16	Fig. 3.5(b)
5	$c_2 = 1.96$ N/kPa	46.4	- 0.01	Fig. 3.5(b)
6	$c_2 = 0.96$ N/kPa	77.5	- 0.14	Fig. 3.5(b)
7	$c_3 = -0.001$ N/kPa²	142.4	- 1.01	Fig. 3.5(c)
8	$c_3 = -0.003$ N/kPa ²	46.4	- 0.02	Fig. 3.5(c)
9	$c_3 = -0.009$ N/kPa ²	65.0	- 0.35	Fig. 3.5(c)
10	$d_1 = -50$ N	46.4	- 0.01	Fig. 3.5(d)
11	$d_1 = -100$ N	46.4	- 0.01	Fig. 3.5(d)
12	$d_1 = -200$ N	46.4	- 0.01	Fig. 3.5(d)
13	$d_2 = -2$ N/kPa	46.4	- 0.01	Fig. 3.5(e)
14	$d_2 = -1$ N/kPa	46.4	- 0.01	Fig. 3.5(e)
15	$d_2 = -0.5$ N/kPa	46.4	- 0.01	Fig. 3.5(e)
16	$P_m = 400$ kPa	42.3	- 0.12	Fig. 3.5(f)
17	$P_m = 300$ kPa	46.5	- 0.03	Fig. 3.5(f)
18	$P_m = 200$ kPa	31.1	- 0.72	Fig. 3.5(f)

3.4.2 Principal parametric resonance condition

In the case of principal parametric resonance condition where the external excitation frequency is nearly equal to twice the natural frequency of the system has been studied in this section. In this case, the reduced equations Eq. (3.38) and Eq. (3.39) from the second order MMS have been used for the analysis, and the trivial state instability regions have plotted in the next subsection.

3.4.2.1 Trivial state instability regions

The trivial state instability regions for different system parameters values have been plotted in Fig. 3.6 and Fig. 3.8 with Jacobian matrix (J) in Eq. (3.45) for principal parametric resonance condition like in subsection 3.4.1.1. Figure 3.6 shows the variation of detuning parameter (σ) with dynamic pressure of the muscle (P_0) for three different values of damping (μ) like the previous case for $\Omega \approx 1$.

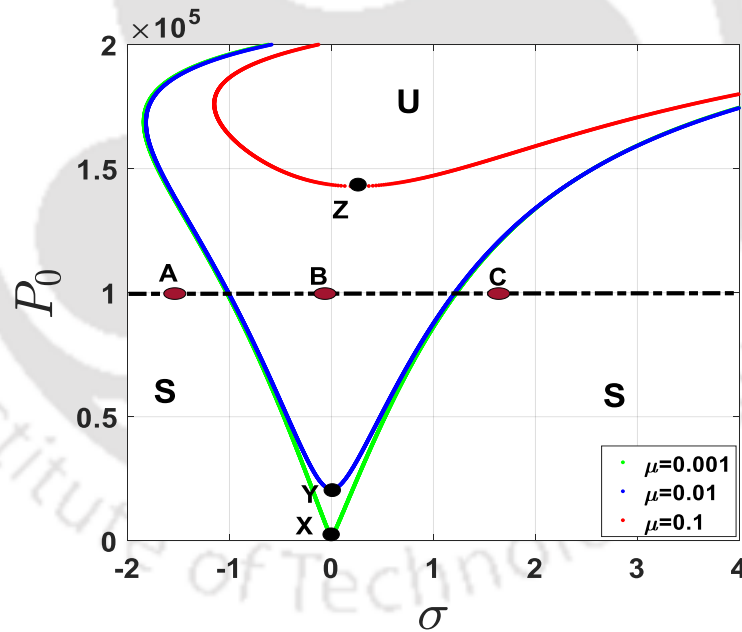


Fig. 3.6 Trivial state instability region in $P_0 \sim \sigma$ plane for three different value of $\mu = 0.001, 0.01$ and 0.1 under $\Omega \approx 2$.

In Fig. 3.6, the instability regions presents in between the detuning parameter values from $\sigma = -1.011$ to $\sigma = 1.209$ for $P_0 = 100$ kPa and $\mu = 0.01$. So, to validate the results time response along with the phase portraits have been shown in Fig. 3.7 for three different points A ($\sigma = -1.5$), B ($\sigma = -0.2$) and C ($\sigma = 1.5$) on the instability plot. It is observed from the time response in Fig.

3.7(a, c) and phase portraits in Fig. 3.7(b, d) that the plots for the points A and C are stable whereas from Fig. 3.7(e, f), it represents the unstable response for point B. The system will always behave as stable below the critical value of P_0 (P_{0cr}), such as at point X for $\mu = 0.001$, $P_{0cr} = 2.1$ kPa; at point Y for $\mu = 0.01$, $P_{0cr} = 20.8$ kPa and finally at point Z for $\mu = 0.1$, $P_{0cr} = 142.9$ kPa in Fig. 3.6. In principal parametric resonance condition also the instability region will decrease with increase in the value of damping (μ) like the simple resonance condition.

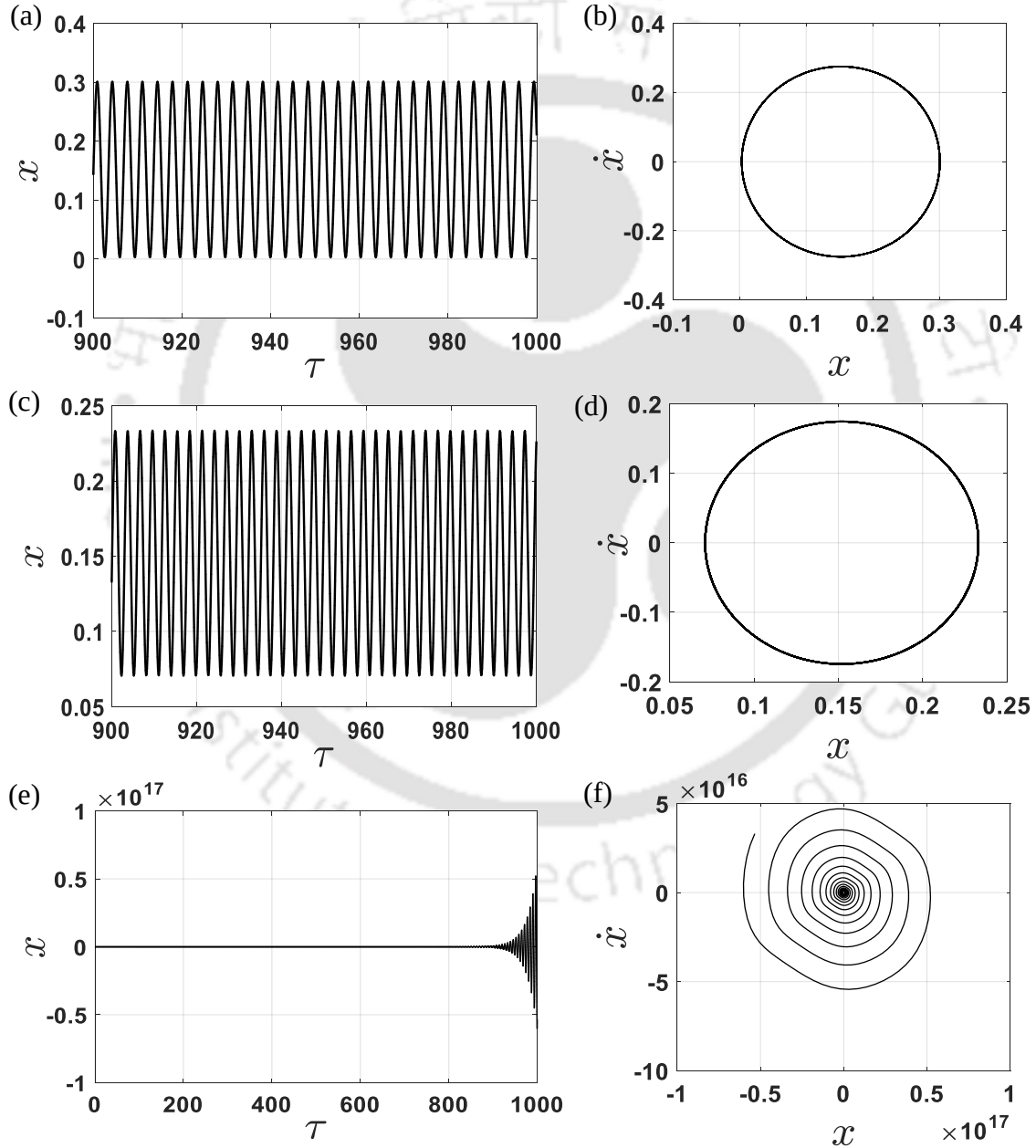
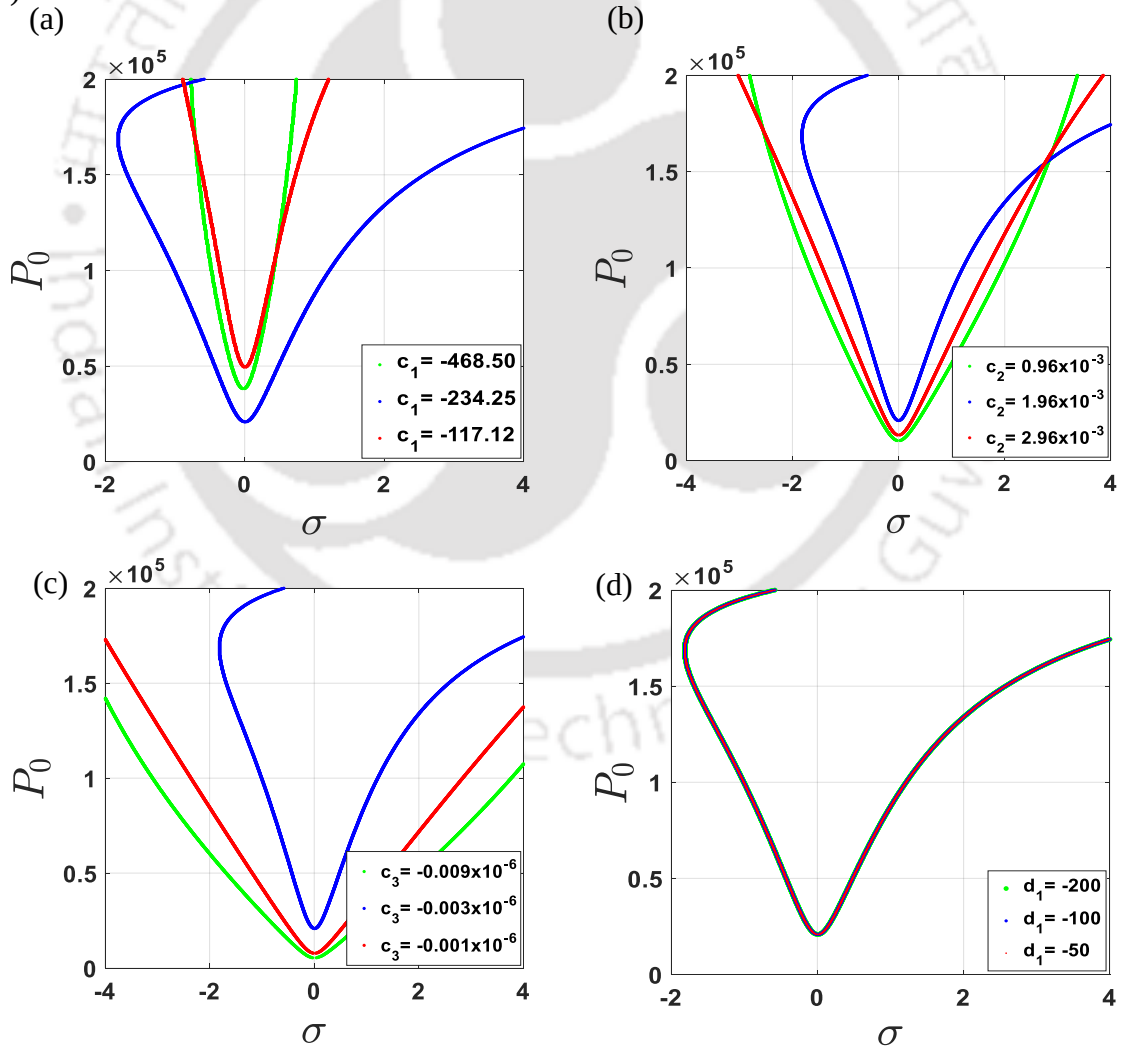


Fig. 3.7 Time responses and phase portraits for (a, b) point A (c, d) point C (e, f) point B from Fig. 3.6 for $\Omega \approx 2$.

The trivial state instability regions have been plotted with three different values for muscle parameters c_1 , c_2 , c_3 , d_1 and d_2 along with the static pressure in the muscle P_m in Fig. 3.8, like in the cases in Fig. 3.5. From Fig. 3.8(a), it can be observed that for an optimum value of muscle parameter c_1 , a wide range of unstable regions can be obtained. With a higher value of muscle parameter c_2 and c_3 , the unstable region increases which can be verified from Fig. 3.8(b) and Fig. 3.8(c), respectively. Like in the case of $\Omega \approx 1$, the trivial state instability regions for $\Omega \approx 2$ also behave almost same way with the variation in another two muscle parameters d_1 and d_2 in Fig. 3.8(d) and Fig. 3.8(e). But with decrease in the value of static pressure in the muscle P_m , the unstable range in the trivial state instability regions increases, which can be noticed from Fig. 3.8(f).



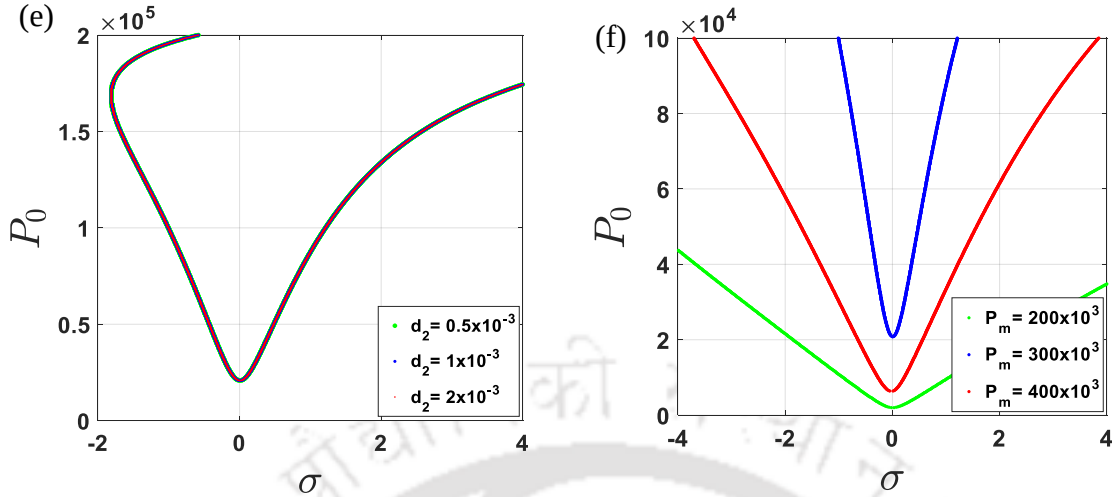


Fig. 3.8 Trivial state instability region in $P_0 \sim \sigma$ plane under $\Omega \approx 2$ with variation in (a) c_1 (b) c_2 (c) c_3 (d) d_1 (e) d_2 (f) P_m .

Table 3.3 Critical value of P_0 (P_{0cr}) with variation in system parameters from Fig. 3.8.

Case	Variation in system parameter	P_{0cr} (kPa)	σ	Reference Figure No.
1	$c_1 = -117.12$ N	49.5	0.02	Fig. 3.8(a)
2	$c_1 = -234.25$ N	20.8	0.01	Fig. 3.8(a)
3	$c_1 = -468.50$ N	38.2	-0.03	Fig. 3.8(a)
4	$c_2 = 2.96$ N/kPa	13.3	-0.02	Fig. 3.8(b)
5	$c_2 = 1.96$ N/kPa	20.8	0.01	Fig. 3.8(b)
6	$c_2 = 0.96$ N/kPa	10.4	0.03	Fig. 3.8(b)
7	$c_3 = -0.001$ N/kPa ²	7.8	-0.02	Fig. 3.8(c)
8	$c_3 = -0.003$ N/kPa ²	20.8	0.01	Fig. 3.8(c)
9	$c_3 = -0.009$ N/kPa ²	5.4	-0.03	Fig. 3.8(c)
10	$d_1 = -50$ N	20.8	0.01	Fig. 3.8(d)
11	$d_1 = -100$ N	20.8	0.01	Fig. 3.8(d)
12	$d_1 = -200$ N	20.8	0.01	Fig. 3.8(d)
13	$d_2 = -2$ N/kPa	20.8	0.01	Fig. 3.8(e)
14	$d_2 = -1$ N/kPa	20.8	0.01	Fig. 3.8(e)
15	$d_2 = -0.5$ N/kPa	20.8	0.01	Fig. 3.8(e)
16	$P_m = 400$ kPa	6.4	0.01	Fig. 3.8(f)
17	$P_m = 300$ kPa	20.8	0.01	Fig. 3.8(f)
18	$P_m = 200$ kPa	2.0	0.03	Fig. 3.8(f)

The critical value of P_0 (P_{0cr}) below which the system will behave as stable with different sets system parameters for $\Omega \approx 2$ have been presented in Table 3.3 from Fig. 3.8 with the corresponding value of detuning parameter σ . From Table 3.3, it can be observed that Case 1 gives the highest value of $P_{0cr} = 49.5$ kPa with $c_1 = -117.12$ N as compared to the other cases. Case 18 with $P_m = 200$ kPa provides the lowest value of $P_{0cr} = 2$ kPa same as the simple resonance condition. With the help of these results, one may actuate the muscle for different applications with wide range operating pressure for principal parametric resonance condition.

3.5 Experimental Validation

For validation of the above theoretical work, the experimental data obtained from the experimental setup as discussed in section 2.6 is used which is again given in Table 3.4. Unlike four muscle parameters determined in Chapter 2 using least square method, here, five parameters based on muscle force characterization has been carried out using a method proposed in the work of Li et al. [125]. Table 3.4 presents the PAM contraction (u) for an external load of 5 N to 30 N with the variation in the pressure from 50 kPa to 300 kPa in the discrete steps of 50 kPa in the experimental setup.

Table 3.4 Experiment Data: Contraction values (u) for different loads with the variation of pressure.

u in mm						
Pressure	50 kPa	100 kPa	150 kPa	200 kPa	250 kPa	300 kPa
Muscle Force						
5 N	4.0	6.0	8.0	10.0	12.0	13.0
10 N	3.6	5.6	7.5	9.0	11.0	12.5
15 N	3.3	5.2	7.0	8.4	10.4	12.0
20 N	3.0	4.8	6.6	7.8	9.7	11.6
25 N	2.8	4.5	6.3	7.3	9.2	11.3
30 N	2.6	4.2	6.0	7.0	9.0	11.0

To find the unknown parameters present in Eq. (3.2), the muscle force F_{mus} can be rewritten using the contraction parameter u and operating pressure P as given in the following linear equation.

$$F_{mus} = c(P)u + d(P) \quad (3.46)$$

Here, $c(P)$ can be estimated from the slope and $d(P)$ can be estimated from the intercept of the $u \sim F_{mus}$ linear curve. Now comparing with the Eq. (3.2) and Eq. (3.46), one can obtain the following expressions.

$$c(P) = \left(\frac{c_1 + c_2 P + c_3 P^2}{l_{\max}} \right) \quad (3.47)$$

$$d(P) = (d_1 + d_2 P) \quad (3.48)$$

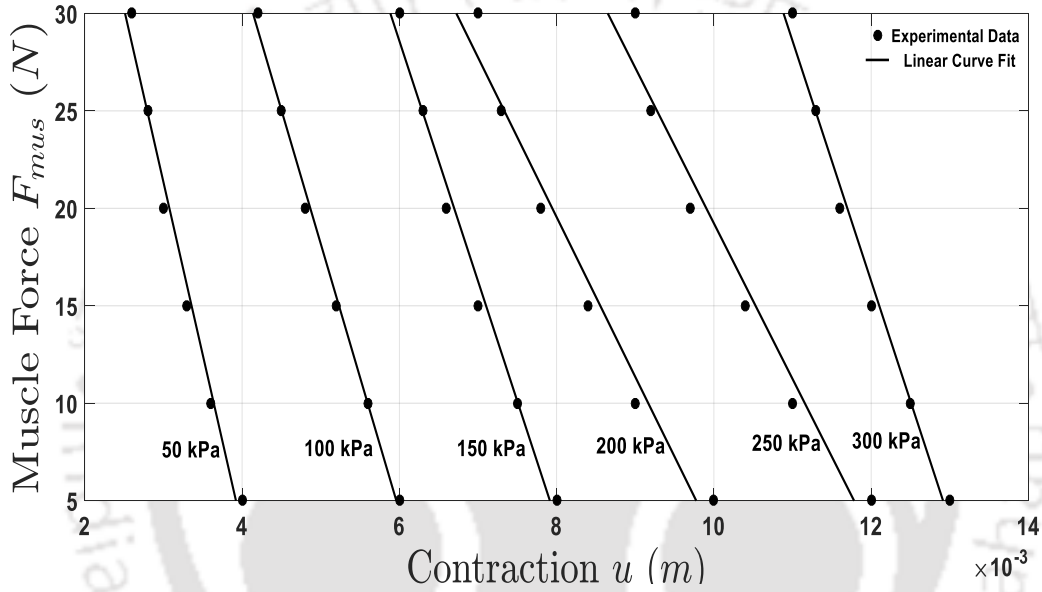


Fig. 3.9 Experimental results as in Table 3.4 and linear least squares fit for set of experiments.

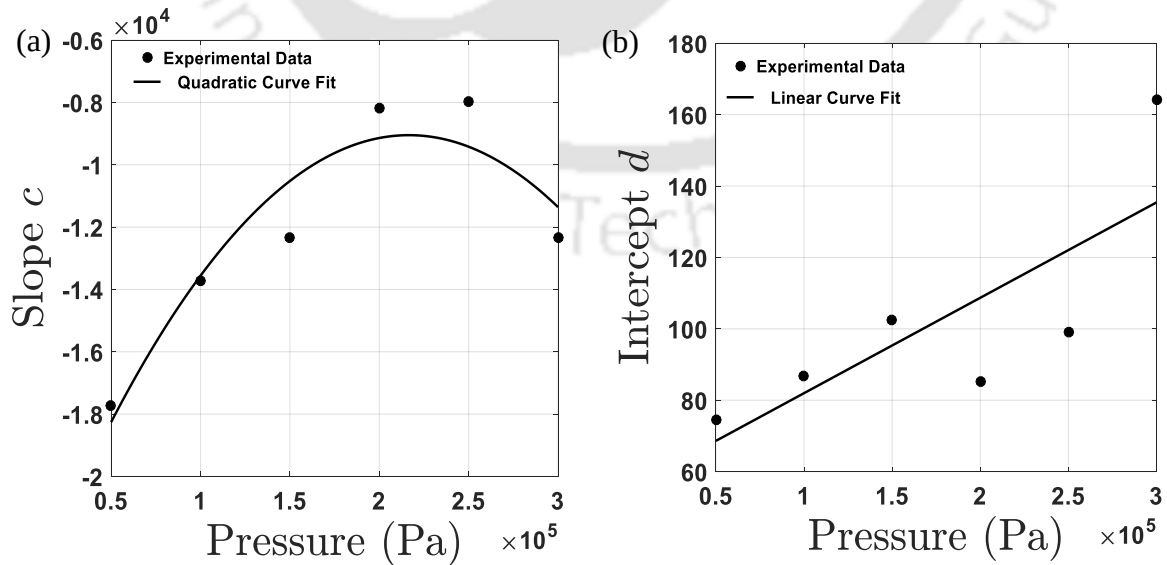


Fig. 3.10 Determination of empirical relation between (a) slope c and (b) intercept d .

Figure 3.9 shows the experimental results of the force contraction characterization of the PAM along with a linear least squares fit at each operating pressure. The control pressures are corresponding to the different applied pressures in the experimental results shown in Fig. 3.9, ranging from 50 to 300 kPa. Subsequently, functions $c(P)$ and $d(P)$ are determined using a least squares fitting with a polynomial basis as shown in Fig. 3.10(a) and Fig. 3.10(b), respectively. It can be observed from Eq. (3.47) and Eq. (3.48) that a second-order relation exists between the slope and the operating pressure while a first-order relation for the intercept respectively. From the least square fit from Fig. 3.10, the values of the muscle parameters are $c_1 = -3.08 \times 10^3$ N, $c_2 = 18$ N/kPa, $c_3 = -0.0415$ N/kPa², $d_1 = 55.26$ N and $d_2 = 0.267$ N/kPa.

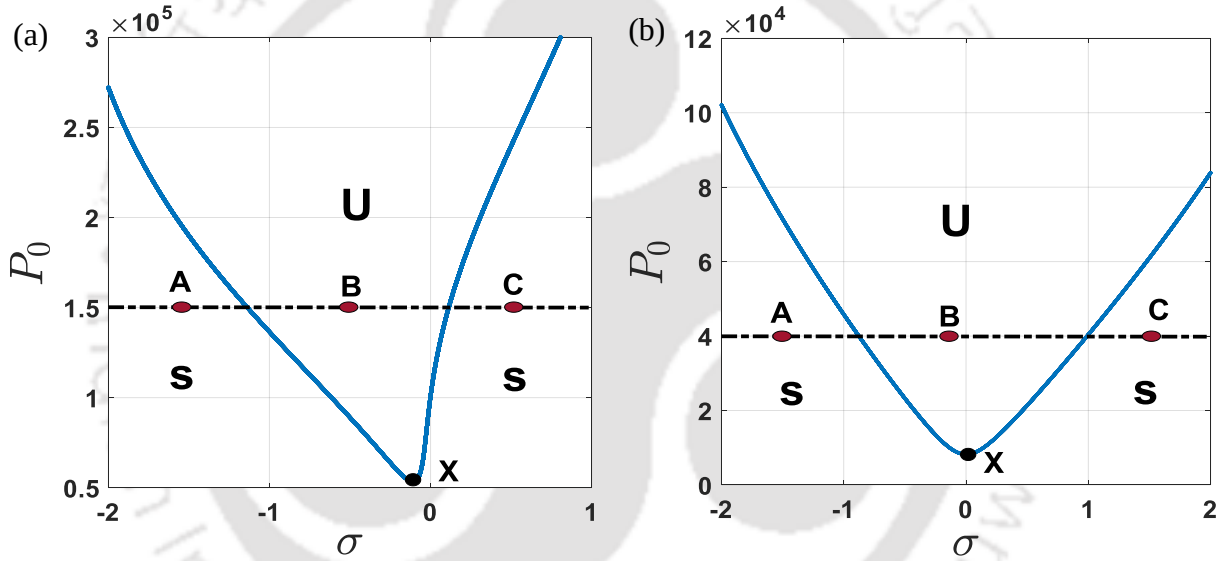


Fig. 3.11 Trivial state instability regions obtained using experimentally obtained coefficients for (a) $\Omega \approx 1$ (b) $\Omega \approx 2$.

Taking the above value of experimentally obtained coefficients c_1 , c_2 , c_3 , d_1 and d_2 with static pressure $P_m = 300$ kPa to lift a load of 20 N the parametric instability regions have been plotted in Fig. 3.11. The instability regions are obtained from the reduced equations Eq. (3.22) and Eq. (3.23) for simple resonance condition and from Eq. (3.38) and Eq. (3.39) for principal parametric resonance condition in Fig. 3.11(a) and Fig. 3.11(b), respectively. The nature of these two plots are also in the good agreement with the plots which are obtained from the theoretical work. The critical value of dynamic pressure P_0 (P_{0cr}) below which the system will always behave as stable at point X for both the cases. In Fig. 3.11(a) for $\Omega \approx 1$ and Fig. 3.11(b) for $\Omega \approx 2$, it can be

observed that $P_{0cr} = 53.4$ kPa at $\sigma = -0.111$ and $P_{0cr} = 8.22$ kPa at $\sigma = 0.013$ respectively. Hence, the system can be actuated for a lower value of dynamic pressure in the muscle along with a large range of instability region under principal parametric resonance condition as compared to simple resonance condition.

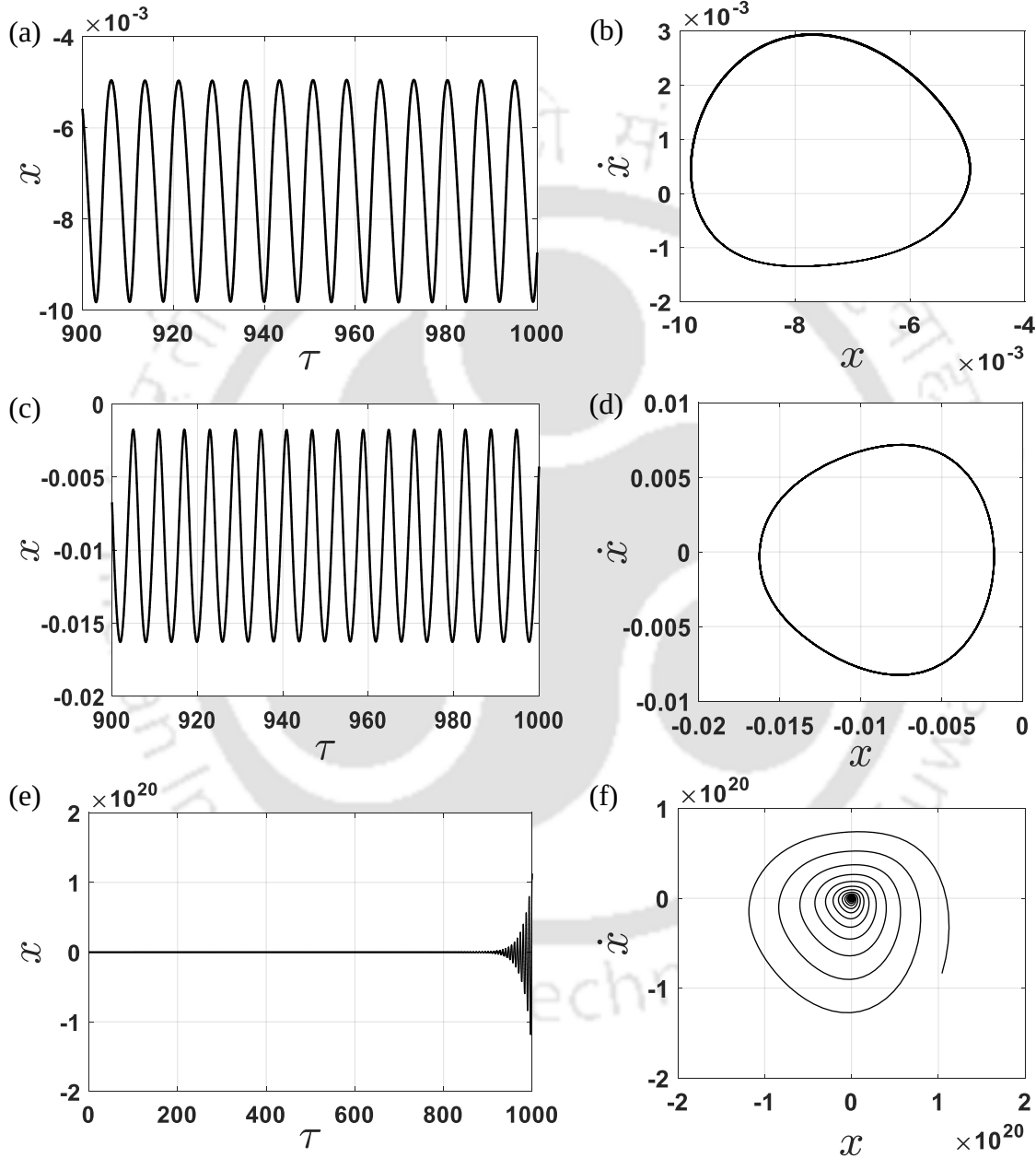


Fig. 3.12 Time responses and phase portraits for (a, b) point A (c, d) point C (e, f) point B from Fig. 3.11(a) for $\Omega \approx 1$.

Taking the value of the different system parameters found from the experiments, for $P_0 = 150$ kPa in Fig. 3.11(a) for simple resonance condition, the instability region exists in between the detuning parameter value from $\sigma = -1.137$ and ends at $\sigma = 0.113$. So, to verify the results like theoretical work, time response and phase portraits have been plotted in Fig. 3.12 for three different points A ($\sigma = -1.5$), B ($\sigma = -0.5$) and C ($\sigma = 0.5$) on the instability plot. It can be clearly observed from the time response in Fig. 3.12(a, c) and phase portraits in Fig. 3.12(b, d) that the responses for the points A and C are stable whereas from Fig. 3.12(e, f), it depicts the unstable response for point B.

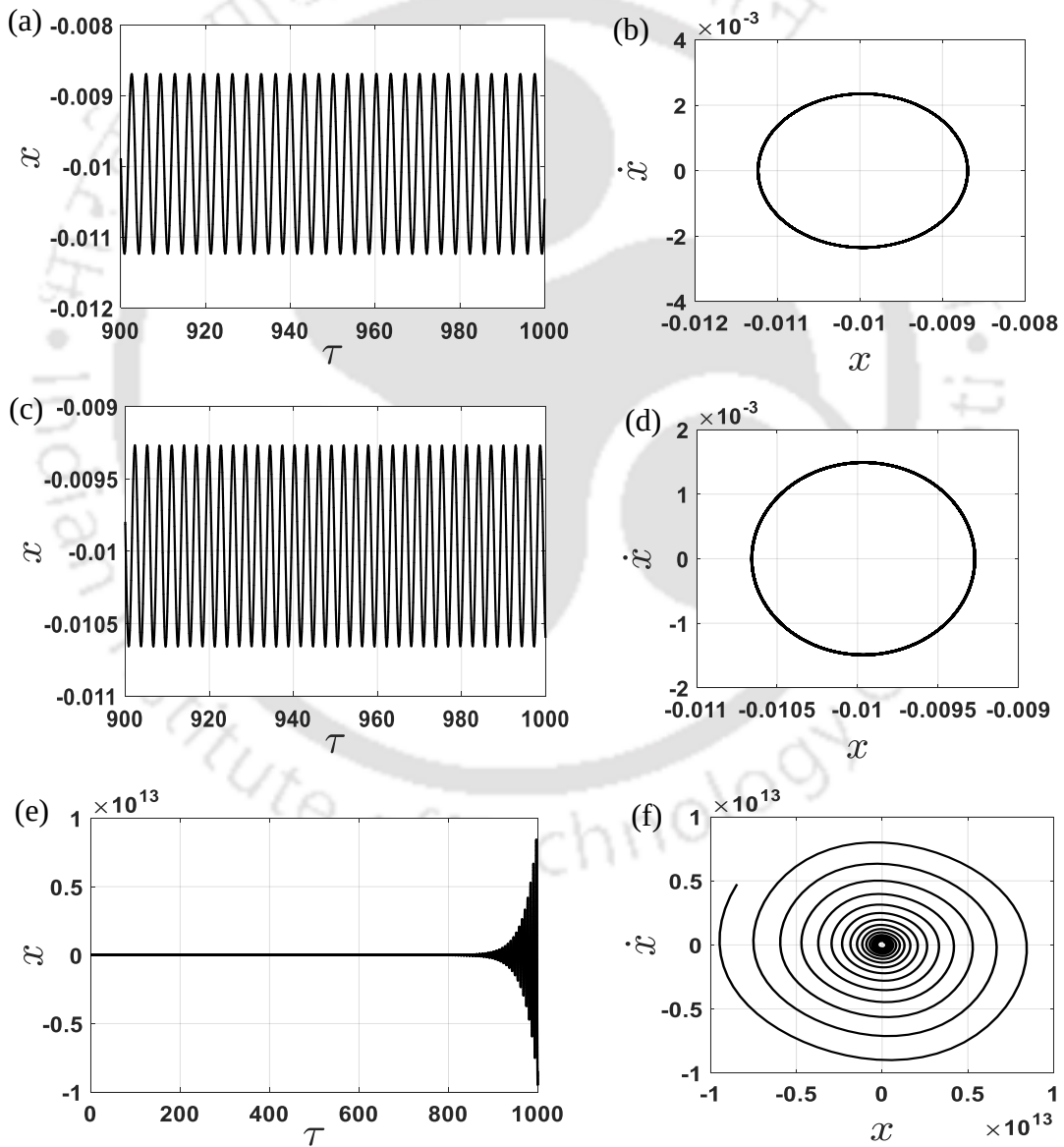


Fig. 3.13 Time responses and phase portraits for (a, b) point A (c, d) point C (e, f) point B from Fig. 3.11(b) for $\Omega \approx 2$.

Similarly, for principal parametric resonance condition in Fig. 3.11(b), the instability region presents in between the detuning parameter value from $\sigma = -0.876$ and ends at $\sigma = 0.99$ for $P_0 = 40$ kPa. So, the results have been validated with the help of the time response along with the phase portraits in Fig. 3.13 for three different points A ($\sigma = -1.5$), B ($\sigma = -0.2$) and C ($\sigma = 1.5$) on the instability plot. It is observed from the time response in Fig. 3.13(a, c) and phase portraits in Fig. 3.13(b, d) that the plots for the points A and C are stable whereas from Fig. 3.13(e, f), it represents the unstable response for point B.

3.6 Summary

In this chapter, dynamic analysis has been carried out for a parametrically excited single degree of freedom of system consisting of a mass and a PAM. The governing forced and parametric equation of motion of the system has been derived with the help of Newton's 2nd law. In the analysis, for two resonance conditions viz., simple resonance condition and principal parametric resonance conditions have been studied with the help of reduced equations from the method of multiple scales (MMS). The maximum response amplitude obtained from the second order MMS is much suitable, as both the numerical and analytical solutions are in good agreement. Hence, the reduced equations from the second order MMS are used for calculating the response of the system for different pressures, which will form the basis for developing the controller for different applications. The trivial state instability regions have been plotted for the different muscle parameters, along with the damping parameter and static pressure inside the muscle. The safe operating range of the system parameters can be observed to achieve a definite response for a particular application with the help of these regions. The critical value of dynamic pressure inside the muscle also has been obtained for different sets of system parameters for actuation. The unknown coefficients present in the muscle force equation have been found with the help of experiments and by using the least squares curve fittings. The nature of the responses using the experimentally obtained coefficients presents the similar responses obtained from the mathematical work. Hence, the reduced equations can be used for further analysis to study the behaviour of the muscle for various applications.

Chapter 4

Nonlinear Dynamics of a Pneumatic Artificial Muscle under Parametric Excitation

4.1 Introduction

In this chapter, a system similar to that studied in the previous chapter is considered by including a nonlinear term in the expression for muscle force of pneumatic artificial muscle (PAM). This nonlinear term may arise due to the presence of the compressibility of air, basic properties of rubber materials, flexible structure and the valve actuation. The mathematical modeling of the system is discussed in section 4.2 and the solution of the governing equation has been carried out in section 4.3 for principal parametric resonance condition and simultaneous simple and principal parametric resonance condition. In section 4.4, the numerical results have been discussed using the parametric instability regions, phase portraits, time and frequency responses for various system parameters. In section 4.5, a brief summary is presented.

4.2 Mathematical Modeling

Figure 4.1(a) shows the schematic diagram of a pneumatically actuated artificial muscle attached to an air compressor through an ON/OFF control valve. This is similar to the use of artificial muscle in the work [145, 253]. When the valve is open, the compressed air will come from the compressor and flows from the inlet 1 to the outlet 2 of the valve. In this condition, the artificial muscle will contract through which the required displacement can be achieved. But when the

valve is closed, air flows from the outlet 2 to the exhaust 3 and the muscle will return back to its original position. The muscle is actuated by adjusting the air pressure by throttle valves. The corresponding equivalent single degree of freedom system with spring-mass and damper is shown in Fig. 4.1(b). The free body diagram showing the forces acting on the system is shown in Fig. 4.1(c).

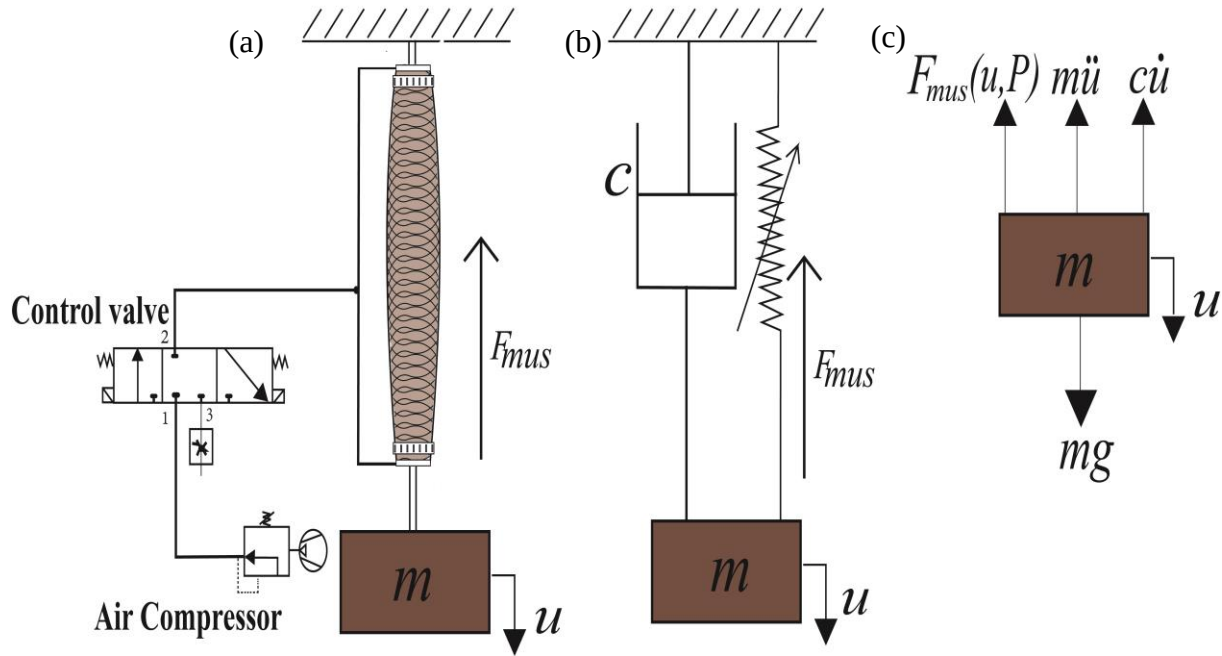


Fig. 4.1 (a) Schematic diagram of a system actuated using PAM (b) equivalent spring-mass-damper system (c) free-body diagram.

From the free body diagram given in Fig. 4.1(c), using Newton's 2nd law the equation of motion of the system can be written as given below.

$$m\ddot{u} + F_{mus} + c\dot{u} - mg = 0 \quad (4.1)$$

where overdot indicates the differentiation with respect to time t and u is the displacement with respect to the unstressed position of the PAM. g denotes the acceleration due to gravity, c is the co-efficient of viscous damping and F_{mus} represents the force exerted by the PAM.

Thus, F_{mus} represents the muscle model defining a relation between the tensile force exerted by the muscle, the resulting contraction and the internal muscle pressure. In this work, the expression for the muscle force F_{mus} is assumed to be similar to that given in the work of Li et al. [125] with an additional cubic nonlinear term.

$$F_{mus}(u, P) = (c_1 + c_2 P + c_3 P^2) \left(\frac{u}{l_{\max}} \right) + (d_1 + d_2 P) + Ku^3 \quad (4.2)$$

where c_1 , c_2 , c_3 , d_1 , d_2 and K are constants, which can be determined from the experiments. $l_{\max}$ is the maximum possible length of the muscle. As in many physical system generally cubic nonlinearity effect is found [13, 14, 197, 249, 254], hence cubic nonlinear term is considered in this muscle force.

In the present work, Eq. (4.2) is used to describe the governing muscle characteristics in its working range due to its linearity in contraction, which simplifies further analysis. Hence, the equation of motion of the system Eq. (4.1) can be rewritten using Eq. (4.2) as follows.

$$\ddot{u} + \left[\frac{(c_1 + c_2 P + c_3 P^2)}{ml_{\max}} \right] u + \frac{K}{m} u^3 + \frac{c}{m} \dot{u} + \left[\frac{(d_1 + d_2 P)}{m} - g \right] = 0 \quad (4.3)$$

Here, the harmonics internal pressure input $P = P_m + P_0 \sin \omega t$ with static pressure P_m and dynamic pressure P_0 is assumed for the muscle. The fundamental natural frequency ω_0 of the system is given by the following expression.

$$\omega_0 = \sqrt{\frac{c_1 + c_2 P_m + c_3 \left(P_m^2 + \frac{P_0^2}{2} \right)}{ml_{\max}}} \quad (4.4)$$

In this work a non-dimensional time $\tau = \omega_0 t$ is introduced. Also a scaling factor r is used to write nondimensional displacement $u = r x$. The following nondimensional parameters are used in the analysis.

$$\begin{aligned} \Omega &= \frac{\omega}{\omega_0}, \quad \mu = \frac{c}{2\epsilon m \omega_0}, \quad \alpha = \frac{r^2 K}{\epsilon m \omega_0^2}, \quad p_1 = \frac{c_2 P_0 + 2c_3 P_0 P_m}{\epsilon m \omega_0^2 l_{\max}}, \\ p_2 &= -\frac{c_3 P_0^2}{2\epsilon m \omega_0^2 l_{\max}}, \quad f_1 = \frac{d_2 P_m + d_1 - mg}{r \epsilon m \omega_0^2}, \quad f_2 = \frac{d_2 P_0}{r \epsilon m \omega_0^2} \end{aligned} \quad (4.5)$$

Using the nondimensional parameters, Eq. (4.3) can be simplified to the following form.

$$\frac{d^2x}{d\tau^2} + 2\varepsilon\mu \frac{dx}{d\tau} + x + \varepsilon(p_1 \sin \Omega\tau + p_2 \cos 2\Omega\tau)x + \varepsilon\alpha x^3 = \varepsilon(f_1 + f_2 \sin \Omega\tau) \quad (4.6)$$

Here, the book keeping ε is a dimensionless parameter which is less than 1 and the non-dimensional damping parameter is μ . The nondimensional parameters p_1 , p_2 and f_2 are the function of dynamic pressure P_0 , p_1 and f_1 are the function of static pressure P_m . It may be observed that the coefficients of response x contains time varying terms with frequency Ω and 2Ω from the fourth term on the left hand side of Eq. (4.6), which results a parametrically excited system with multi frequency excitation. Finally, the last term of the left-hand side contains a cubic nonlinear term with the coefficient α . In addition to this, the system is also subjected to a forced vibration due to the presence of a sinusoidally varying force of amplitude f_2 and frequency Ω . In this chapter, Ω is considered to be twice the natural frequency and hence a principal parametric resonance condition is considered. Also, the system may be subjected to simultaneous (i.e. both simple and principal parametric) resonance conditions which will be evident due to the presence of the term $(p_2 \cos 2\Omega\tau)x$ in Eq. (4.6), which is similar to that of a parametrically excited system. The solution to this equation is studied using the second order method of multiple scales (MMS) in the following sections.

4.3 Approximate Solution

The approximate analytical solution for the developed equation of motion Eq. (4.6) is derived by applying the second order method of multiple scales [13, 14]. Using the standard formulation of the method of multiple scales, the displacement x can be represented in terms of different time scales (T_0, T_1, T_2) and a book keeping parameter ε as follows.

$$x(\tau, \varepsilon) = x_0(T_0, T_1, T_2) + \varepsilon x_1(T_0, T_1, T_2) + \varepsilon^2 x_2(T_0, T_1, T_2) \quad (4.7)$$

where, $T_n = \varepsilon^n \tau, n = 0, 1, 2$

Here, $D_0 = \frac{d}{dT_0}$, $D_1 = \frac{d}{dT_1}$ and $D_2 = \frac{d}{dT_2}$.

Substituting Eq. (4.7) into Eq. (4.6) results in the following form.

$$\begin{aligned} D_0^2 x_0 + \varepsilon D_0^2 x_1 + \varepsilon^2 D_0^2 x_2 + 2\varepsilon D_0 D_1 x_0 + 2\varepsilon^2 D_0 D_1 x_1 + \varepsilon^2 x_0 (D_1 + 2D_0 D_2) + 2\varepsilon \mu D_0 x_0 \\ + 2\varepsilon^2 \mu D_0 x_1 + 2\varepsilon^2 \mu D_1 x_0 + x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \varepsilon (x_0 + \varepsilon x_1)(p_1 \sin \Omega \tau + p_2 \cos 2\Omega \tau) \\ + \varepsilon \alpha x_0^3 + \varepsilon^2 3\alpha x_1 x_0^2 = \varepsilon (f_1 + f_2 \sin \Omega \tau) \end{aligned} \quad (4.8)$$

Now, separating the terms with different order of ε from Eq. (4.8) following equations are obtained.

Order of ε^0

$$D_0^2 x_0 + x_0 = 0 \quad (4.9)$$

Order of ε^1

$$D_0^2 x_1 + x_1 = f_1 + f_2 \sin \Omega T_0 - 2D_0 D_1 x_0 - 2\mu D_0 x_0 - x_0 p_1 \sin \Omega T_0 - x_0 p_2 \cos 2\Omega T_0 - \alpha x_0^3 \quad (4.10)$$

Order of ε^2

$$\begin{aligned} D_0^2 x_2 + x_2 = -2D_0 D_1 x_1 - x_0 (D_1 + 2D_0 D_2) - 2\mu D_0 x_1 - 2\mu D_1 x_0 \\ - x_1 (p_1 \sin \Omega_1 T_0 + p_2 \cos 2\Omega_1 T_0) - 3\alpha x_1 x_0^2 \end{aligned} \quad (4.11)$$

The general solution of Eq. (4.9) can be written as

$$x_0 = A(T_1, T_2) e^{iT_0} + CC \quad (4.12)$$

where CC is the complex conjugate of the proceeding terms.

Substituting Eq. (4.12) in Eq. (4.10) results in the following equation.

$$\begin{aligned} D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{if_2}{2} e^{i\Omega T_0} - 2D_1 i A e^{iT_0} - 2\mu i A e^{iT_0} + \frac{ip_1}{2} A e^{i(\Omega-1)T_0} \\ - \frac{ip_1}{2} \bar{A} e^{-i(1+\Omega)T_0} - \frac{p_2}{2} A e^{i(1+2\Omega)T_0} - \frac{p_2}{2} \bar{A} e^{i(2\Omega-1)T_0} - \alpha A^3 e^{3iT_0} - \alpha 3A^2 \bar{A} e^{iT_0} + CC \end{aligned} \quad (4.13)$$

The 5th term in the right-hand side of the Eq. (4.13) shows that when $\Omega \approx 2$, the response will be unbounded which leads to the condition of principal parametric resonance. Also it is apparent from Eq. (4.13) that the response of the system tends to infinity when $\Omega \approx 1$. This may lead to

simple resonance condition due to the presence of the term $\frac{p_2}{2} \bar{A} e^{i(2\Omega-1)T_0}$ and also may lead to

principal parametric resonance condition due to the presence of the term $\frac{ip_1}{2} A e^{i(\Omega-1)T_0}$. Hence the

system is subjected to simultaneous both simple and principal parametric resonance conditions.

4.3.1 Principal parametric resonance condition

To take into the account of the nearness of the frequency near two, one may introduce the detuning parameter σ as $\Omega = 2 + \varepsilon\sigma$. Then Eq. (4.13) can be written as follows.

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{if_2}{2} e^{i(2+\varepsilon\sigma)T_0} - 2D_1 i A e^{iT_0} - 2\mu i A e^{iT_0} + \frac{ip_1}{2} A e^{i(1+\varepsilon\sigma)T_0} - \frac{ip_1}{2} \bar{A} e^{-i(3+\varepsilon\sigma)T_0} - \frac{p_2}{2} A e^{i(5+2\varepsilon\sigma)T_0} - \frac{p_2}{2} \bar{A} e^{i(3+2\varepsilon\sigma)T_0} - \alpha A^3 e^{3iT_0} - \alpha 3A^2 \bar{A} e^{iT_0} + CC \quad (4.14)$$

Eliminating the secular terms (terms with coefficient of e^{iT_0}) from Eq. (4.14) to have bounded solution one may write the following modulation equation in complex form.

$$-2D_1 i A - 2\mu i A + \frac{ip_1}{2} \bar{A} e^{i\varepsilon\sigma T_0} - \alpha 3A^2 \bar{A} = 0 \quad (4.15)$$

Now, Eq. (4.14) can be rewritten as in the following form.

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{if_2}{2} e^{i(2+\varepsilon\sigma)T_0} - \frac{ip_1}{2} \bar{A} e^{-i(3+\varepsilon\sigma)T_0} - \frac{p_2}{2} A e^{i(5+2\varepsilon\sigma)T_0} - \frac{p_2}{2} \bar{A} e^{i(3+2\varepsilon\sigma)T_0} - \alpha A^3 e^{3iT_0} + CC \quad (4.16)$$

$$x_1 = \frac{f_1}{2} - \frac{if_2}{2(1-(2+\varepsilon\sigma)^2)} e^{i(2+\varepsilon\sigma)T_0} - \frac{ip_1}{2[1-(3+\varepsilon\sigma)^2]} \bar{A} e^{-i(3+\varepsilon\sigma)T_0} - \frac{p_2}{2[1-(5+2\varepsilon\sigma)^2]} A e^{i(5+2\varepsilon\sigma)T_0} - \frac{p_2}{2[1-(3+2\varepsilon\sigma)^2]} \bar{A} e^{i(3+2\varepsilon\sigma)T_0} + \frac{\alpha A^3 e^{3iT_0}}{8} + CC \quad (4.17)$$

Now substituting Eq. (4.17) and Eq. (4.12) in Eq. (4.11) and eliminating the secular terms the following modulation equation in complex form can be found.

$$D_2 A = -(1+2\mu) \frac{D_1 A}{2i} - \frac{ip_1^2 A}{8(1-(3+\varepsilon\sigma)^2)} + \frac{p_1 p_2 \bar{A} e^{i\varepsilon\sigma T_0}}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{\alpha A^3 p_1 e^{-i\varepsilon\sigma T_0}}{32} + \frac{p_1 p_2 \bar{A} e^{i\varepsilon\sigma T_0}}{8(1-(3+\varepsilon\sigma)^2)} - \frac{ip_2^2 A}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{i\alpha \bar{A}^3 p_2 e^{2i\varepsilon\sigma T_0}}{32} - \frac{ip_2^2 A}{8(1-(5+2\varepsilon\sigma)^2)} - \frac{3\alpha p_1 \bar{A}^2 e^{i\varepsilon\sigma T_0}}{4(1-(3+\varepsilon\sigma)^2)} - \frac{3i\alpha p_2 \bar{A}^3 e^{i2\varepsilon\sigma T_0}}{4(1-(3+2\varepsilon\sigma)^2)} + \frac{3i\alpha^2 A^3 \bar{A}^2}{16} \quad (4.18)$$

Now using the method of reconstitution [239], one may write the following form.

$$\begin{aligned} \frac{dA}{d\tau} &= \frac{dA}{dT_1} \frac{dT_1}{d\tau} + \frac{dA}{dT_2} \frac{dT_2}{d\tau} = \varepsilon D_1 A + \varepsilon^2 D_2 A. \\ \frac{dA}{d\tau} &= \left(\varepsilon - \frac{\varepsilon^2(1+2\mu)}{2i} \right) \left(-\mu A + \frac{p_1}{4} \bar{A} e^{i\varepsilon\sigma T_0} + \frac{3i\alpha A^2 \bar{A}}{2} \right) + \varepsilon^2 \left[-\frac{ip_1^2 A}{8(1-(3+\varepsilon\sigma)^2)} \right. \\ &+ \frac{p_1 p_2 \bar{A} e^{i\varepsilon\sigma T_0}}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{\alpha A^3 p_1 e^{-i\varepsilon\sigma T_0}}{32} + \frac{p_1 p_2 \bar{A} e^{i\varepsilon\sigma T_0}}{8(1-(3+\varepsilon\sigma)^2)} - \frac{ip_2^2 A}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{i\alpha \bar{A}^3 p_2 e^{2i\varepsilon\sigma T_0}}{32} \\ &\left. - \frac{ip_2^2 A}{8(1-(5+2\varepsilon\sigma)^2)} - \frac{3\alpha p_1 A \bar{A}^2 e^{i\varepsilon\sigma T_0}}{4(1-(3+\varepsilon\sigma)^2)} - \frac{3i\alpha p_2 \bar{A}^3 e^{i2\varepsilon\sigma T_0}}{4(1-(3+2\varepsilon\sigma)^2)} + \frac{3i\alpha^2 A^3 \bar{A}^2}{16} \right] \end{aligned} \quad (4.19)$$

Here one may introduce $A = \frac{1}{2} a e^{i\beta}$ and $\bar{A} = \frac{1}{2} a e^{-i\beta}$, where a is the real modal amplitude and β is the phase. Again, taking $\varepsilon\sigma T_0 - 2\beta = 2\gamma$ where γ is known as the relative phase and σ is the detuning parameter in Eq. (4.19) and separating the real and imaginary parts one may obtain the following reduced equations.

$$\begin{aligned} a' = \frac{da}{dt} &= \varepsilon \left(-\mu a + \frac{p_1 a}{4} \cos 2\gamma \right) + \frac{\varepsilon^2(1+2\mu)}{2} \left(-\frac{p_1 a}{4} \sin 2\gamma - \frac{3\alpha a^3}{8} \right) \\ &+ \varepsilon^2 \left[\frac{p_1 p_2 a \cos 2\gamma}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{\alpha p_1 a^3}{128} \cos 2\gamma + \frac{p_1 p_2 a \cos 2\gamma}{8(1-(3+\varepsilon\sigma)^2)} - \frac{\alpha p_2 a^3}{128} \sin 4\gamma \right. \\ &\left. - \frac{3\alpha p_1 a^3 \cos 2\gamma}{16(1-(3+\varepsilon\sigma)^2)} + \frac{3\alpha p_2 a^3 \sin 4\gamma}{16(1-(3+2\varepsilon\sigma)^2)} \right] \end{aligned} \quad (4.20)$$

$$\begin{aligned} a\gamma' = a \frac{d\gamma}{dt} &= \frac{a\varepsilon\sigma}{2} - \varepsilon \left(\frac{p_1 a}{4} \sin 2\gamma + \frac{3\alpha a^3}{8} \right) - \frac{\varepsilon^2(1+2\mu)}{2} \left(-\mu a + \frac{p_1 a}{4} \cos 2\gamma \right) \\ &- \varepsilon^2 \left[-\frac{p_1^2 a}{8(1-(3+\varepsilon\sigma)^2)} + \frac{p_1 p_2 a \sin 2\gamma}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{\alpha p_1 a^3}{128} \sin 2\gamma + \frac{p_1 p_2 a \sin 2\gamma}{8(1-(3+\varepsilon\sigma)^2)} \right. \\ &- \frac{p_2^2 a}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{\alpha p_2 a^3}{128} \cos 4\gamma - \frac{p_2^2 a}{8(1-(5+2\varepsilon\sigma)^2)} - \frac{3\alpha p_1 a^3 \sin 2\gamma}{16(1-(3+\varepsilon\sigma)^2)} \\ &\left. - \frac{3\alpha p_2 a^3 \cos 4\gamma}{16(1-(3+2\varepsilon\sigma)^2)} + \frac{3\alpha^2 a^5}{256} \right] \end{aligned} \quad (4.21)$$

For steady state $a' = 0$, $\gamma' = 0$ and from Eq. (4.20) and Eq. (4.21), one may obtain the both trivial and nontrivial responses. To find the stability of the steady state responses, perturbing Eq. (4.20) and Eq. (4.21) by substituting $a = a_0 + a_1$ and $\gamma = \gamma_0 + \gamma_1$ where a_0 , γ_0 are the singular points and then exploring the eigenvalues of the resulting Jacobian matrix J . The system is stable if the real parts of all the eigenvalues of the following Jacobian matrix J in Eq. (4.22) are negative.

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \quad (4.22)$$

where,

$$\begin{aligned} J_{11} &= \varepsilon \left(-\mu + \frac{p_1}{4} \cos 2\gamma_0 \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\frac{p_1}{4} \sin 2\gamma_0 - \frac{9\alpha a_0^2}{8} \right) + \varepsilon^2 \left[\frac{p_1 p_2 \cos 2\gamma_0}{8(1-(3+2\varepsilon\sigma)^2)} \right. \\ &\quad \left. - \frac{3\alpha p_1 a_0^2}{128} \cos 2\gamma_0 + \frac{p_1 p_2 \cos 2\gamma_0}{8(1-(3+\varepsilon\sigma)^2)} - \frac{3\alpha p_2 a_0^2}{128} \sin 4\gamma_0 - \frac{9\alpha p_1 a_0^2 \cos 2\gamma_0}{16(1-(3+\varepsilon\sigma)^2)} + \frac{9\alpha p_2 a_0^2 \sin 4\gamma_0}{16(1-(3+2\varepsilon\sigma)^2)} \right] \\ J_{12} &= \varepsilon \left(-\frac{p_1 a_0}{2} \sin 2\gamma_0 \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\frac{p_1 a_0}{2} \cos 2\gamma_0 \right) + \varepsilon^2 \left[-\frac{p_1 p_2 a_0 \sin 2\gamma_0}{4(1-(3+2\varepsilon\sigma)^2)} \right. \\ &\quad \left. + \frac{\alpha p_1 a_0^3}{64} \sin 2\gamma_0 - \frac{p_1 p_2 a_0 \sin 2\gamma_0}{4(1-(3+\varepsilon\sigma)^2)} - \frac{\alpha p_2 a_0^3}{32} \cos 4\gamma_0 + \frac{3\alpha p_1 a_0^3 \sin 2\gamma_0}{8(1-(3+\varepsilon\sigma)^2)} + \frac{3\alpha p_2 a_0^3 \cos 4\gamma_0}{4(1-(3+2\varepsilon\sigma)^2)} \right] \\ J_{21} &= -\varepsilon \left(\frac{3\alpha a_0}{4} \right) - \varepsilon^2 \left[\frac{\alpha p_1 a_0}{64} \sin 2\gamma + \frac{\alpha p_2 a_0}{64} \cos 4\gamma - \frac{3\alpha p_1 a_0 \sin 2\gamma}{8(1-(3+\varepsilon\sigma)^2)} - \frac{3\alpha p_2 a_0 \cos 4\gamma}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{3\alpha^2 a_0^3}{64} \right] \\ J_{22} &= -\varepsilon \left(\frac{p_1}{2} \cos 2\gamma_0 \right) - \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\frac{p_1}{2} \sin 2\gamma_0 \right) - \varepsilon^2 \left[\frac{p_1 p_2 \cos 2\gamma_0}{4(1-(3+2\varepsilon\sigma)^2)} + \frac{\alpha p_1 a_0^2}{64} \cos 2\gamma_0 \right. \\ &\quad \left. + \frac{p_1 p_2 \cos 2\gamma_0}{4(1-(3+\varepsilon\sigma)^2)} - \frac{\alpha p_2 a_0^2}{32} \sin 4\gamma_0 - \frac{3\alpha p_1 a_0^2 \cos 2\gamma_0}{8(1-(3+\varepsilon\sigma)^2)} + \frac{3\alpha p_2 a_0^2 \sin 4\gamma}{4(1-(3+2\varepsilon\sigma)^2)} \right] \end{aligned}$$

After elimination of the secular terms from Eq. (4.14) one may obtain the following expression.

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{if_2}{2} e^{i\Omega T_0} - \frac{ip_1}{2} \bar{A} e^{-i(1+\Omega)T_0} - \frac{p_2}{2} A e^{i(2\Omega+1)T_0} - \frac{p_2}{2} \bar{A} e^{i(2\Omega-1)T_0} - \alpha A^3 e^{3iT_0} + CC \quad (4.23)$$

The particular solution of x_1 in Eq. (4.23) can be written as follows.

$$x_1 = f_1 + \frac{\alpha a^3}{32} \cos(6T_0) + \frac{f_2}{(1-\Omega)^2} \sin \Omega T_0 - \frac{p_1 a}{2(1-(1+\Omega)^2)} \sin\left(\frac{3\Omega T_0 - 2\gamma}{2}\right) - \frac{p_2 a}{2(1-(1+2\Omega)^2)} \cos\left(\frac{5\Omega T_0 - 2\gamma}{2}\right) - \frac{p_2 a}{2(1-(2\Omega-1)^2)} \cos\left(\frac{3\Omega T_0 + 2\gamma}{2}\right) \quad (4.24)$$

Thus from Eq. (4.7), Eq. (4.12) and Eq. (4.24) the following expression can be achieved.

$$x = x_0 + \varepsilon x_1 = a \cos\left(\frac{\Omega T_0 - 2\gamma}{2}\right) + \varepsilon \left[f_1 + \frac{f_2}{(1-\Omega)^2} \sin \Omega T_0 - \frac{p_1 a}{2(1-(1+\Omega)^2)} \sin\left(\frac{3\Omega T_0 - 2\gamma}{2}\right) - \frac{p_2 a}{2(1-(1+2\Omega)^2)} \cos\left(\frac{5\Omega T_0 - 2\gamma}{2}\right) - \frac{p_2 a}{2(1-(2\Omega-1)^2)} \cos\left(\frac{3\Omega T_0 + 2\gamma}{2}\right) + \frac{\alpha a^3}{32} \cos(6T_0) \right] \quad (4.25)$$

Eq. (4.25) gives the complete time response of the parametrically excited system attached with a nonlinear PAM under principal parametric resonance condition.

4.3.1.1 Stability of the steady state response

From the reduced equations Eq. (4.20) and Eq. (4.21) for principal parametric resonance condition with cubic nonlinearity, it is observed that the system has both trivial solution ($a = 0$) and nontrivial solution ($a \neq 0$). So, the stability of the steady state response of the system can be studied by investigating the nature of the equilibrium points. The stability of the trivial solutions may be obtained by converting the Eq. (4.20) and Eq. (4.21) to their Cartesian form of modulation equations by introducing the transformation $p = a \cos \gamma$ and $q = a \sin \gamma$ [13, 14]. Now one may obtain from Eq. (4.20) and Eq. (4.21), the resulting Cartesian modulation equations are as follows.

$$\begin{aligned}
 p' = & \varepsilon \left(-\mu p + \frac{p_1 p}{4} + \frac{3\alpha q (p^2 + q^2)}{8} \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\mu q - \frac{p_1 q}{4} - \frac{3\alpha p (p^2 + q^2)}{8} \right) \\
 & + \varepsilon^2 \left[-\frac{p_1^2 q}{8(1-(3+\varepsilon\sigma)^2)} + \frac{p_1 p_2 p}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{4\alpha p_1 p^3}{128} + \frac{3\alpha p_1 p (p^2 + q^2)}{128} + \frac{p_1 p_2 p}{8(1-(3+\varepsilon\sigma)^2)} \right. \\
 & - \frac{p_2^2 q}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{3\alpha p_2 q (p^2 + q^2)}{128} + \frac{4\alpha p_2 q^3}{128} - \frac{p_2^2 q}{8(1-(5+2\varepsilon\sigma)^2)} - \frac{3\alpha p_1 p (p^2 + q^2)}{16(1-(3+\varepsilon\sigma)^2)} \\
 & \left. + \frac{9\alpha p_2 q (p^2 + q^2)}{16(1-(3+2\varepsilon\sigma)^2)} - \frac{12\alpha p_2 q^3}{16(1-(3+2\varepsilon\sigma)^2)} + \frac{3\alpha^2 q (p^2 + q^2)^2}{256} \right] - \frac{\varepsilon\sigma q}{2}
 \end{aligned} \tag{4.26}$$

$$\begin{aligned}
 q' = & \varepsilon \left(-\mu q - \frac{p_1 q}{4} - \frac{3\alpha p (p^2 + q^2)}{8} \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(\mu p - \frac{p_1 p}{4} - \frac{3\alpha q (p^2 + q^2)}{8} \right) \\
 & + \varepsilon^2 \left[\frac{p_1^2 p}{8(1-(3+\varepsilon\sigma)^2)} - \frac{p_1 p_2 q}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{3\alpha p_1 q (p^2 + q^2)}{128} + \frac{4\alpha p_1 q^3}{128} - \frac{p_1 p_2 q}{8(1-(3+\varepsilon\sigma)^2)} \right. \\
 & + \frac{p_2^2 p}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{4\alpha p_2 p^3}{128} + \frac{3\alpha p_2 p (p^2 + q^2)}{128} + \frac{p_2^2 p}{8(1-(5+2\varepsilon\sigma)^2)} + \frac{3\alpha p_1 q (p^2 + q^2)}{16(1-(3+\varepsilon\sigma)^2)} \\
 & \left. - \frac{12\alpha p_2 p^3}{16(1-(3+2\varepsilon\sigma)^2)} + \frac{9\alpha p_2 p (p^2 + q^2)}{16(1-(3+2\varepsilon\sigma)^2)} - \frac{3\alpha^2 p (p^2 + q^2)^2}{256} \right] + \frac{\varepsilon\sigma p}{2}
 \end{aligned} \tag{4.27}$$

Stability of the steady state response (p_0, q_0) for this case can be achieved by investigating the eigenvalues of the Jacobian matrix (J) obtained by perturbing Eq. (4.26) and Eq. (4.27). Here the trivial or non-trivial response of the system for principal parametric resonance condition will be stable, if the real parts of all the eigenvalues of the Jacobian matrix (J) in Eq. (4.28) as given below are negative.

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \tag{4.28}$$

where,

$$\begin{aligned}
 J_{11} = & \varepsilon \left(-\mu + \frac{p_1}{4} + \frac{6\alpha p_0 q_0}{8} \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\frac{9\alpha p_0^2}{8} - \frac{3\alpha q_0^2}{8} \right) + \varepsilon^2 \left[\frac{p_1 p_2}{8(1-(3+2\varepsilon\sigma)^2)} \right. \\
 & - \frac{3\alpha p_1 p_0^2}{128} + \frac{3\alpha p_1 q_0^2}{128} + \frac{p_1 p_2}{8(1-(3+\varepsilon\sigma)^2)} - \frac{6\alpha p_2 p_0 q_0}{128} - \frac{9\alpha p_1 p_0^2}{16(1-(3+\varepsilon\sigma)^2)} - \frac{3\alpha p_1 q_0^2}{16(1-(3+\varepsilon\sigma)^2)} \\
 & \left. + \frac{18\alpha p_2 p_0 q_0}{16(1-(3+2\varepsilon\sigma)^2)} + \frac{12\alpha^2 p_0^3 q_0}{256} + \frac{12\alpha^2 p_0 q_0^3}{256} \right] \\
 J_{12} = & \varepsilon \left(\frac{3\alpha p_0^2}{8} + \frac{9\alpha q_0^2}{8} \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\mu - \frac{p_1}{4} - \frac{6\alpha p_0 q_0}{8} \right) + \varepsilon^2 \left[-\frac{p_1^2}{8(1-(3+\varepsilon\sigma)^2)} \right. \\
 & + \frac{6\alpha p_1 p_0 q_0}{128} - \frac{p_2^2}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{3\alpha p_2 p_0^2}{128} + \frac{3\alpha p_2 q_0^2}{128} - \frac{p_2^2}{8(1-(5+2\varepsilon\sigma)^2)} - \frac{6\alpha p_1 p_0 q_0}{16(1-(3+\varepsilon\sigma)^2)} \\
 & \left. + \frac{9\alpha p_2 p_0^2}{16(1-(3+2\varepsilon\sigma)^2)} - \frac{9\alpha p_2 q_0^2}{16(1-(3+2\varepsilon\sigma)^2)} + \frac{3\alpha^2 p_0^4}{256} + \frac{18\alpha^2 p_0^2 q_0^2}{256} + \frac{15\alpha^2 q_0^4}{256} \right] - \frac{\varepsilon\sigma}{2} \\
 J_{21} = & \varepsilon \left(-\frac{9\alpha p_0^2}{8} - \frac{3\alpha q_0^2}{8} \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(\mu - \frac{p_1}{4} - \frac{6\alpha p_0 q_0}{8} \right) + \varepsilon^2 \left[\frac{p_1^2}{8(1-(3+\varepsilon\sigma)^2)} \right. \\
 & - \frac{6\alpha p_1 p_0 q_0}{128} + \frac{p_2^2}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{3\alpha p_2 p_0^2}{128} + \frac{3\alpha p_2 q_0^2}{128} + \frac{p_2^2}{8(1-(5+2\varepsilon\sigma)^2)} + \frac{6\alpha p_1 p_0 q_0}{16(1-(3+\varepsilon\sigma)^2)} \\
 & \left. - \frac{9\alpha p_2 p_0^2}{16(1-(3+2\varepsilon\sigma)^2)} + \frac{9\alpha p_2 q_0^2}{16(1-(3+2\varepsilon\sigma)^2)} - \frac{15\alpha^2 p_0^4}{256} - \frac{18\alpha^2 p_0^2 q_0^2}{256} - \frac{3\alpha^2 q_0^4}{256} \right] + \frac{\varepsilon\sigma}{2} \\
 J_{22} = & \varepsilon \left(-\mu - \frac{p_1}{4} - \frac{6\alpha p_0 q_0}{8} \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\frac{3\alpha p_0^2}{8} - \frac{9\alpha q_0^2}{8} \right) + \varepsilon^2 \left[-\frac{p_1 p_2}{8(1-(3+2\varepsilon\sigma)^2)} \right. \\
 & - \frac{3\alpha p_1 p_0^2}{128} + \frac{3\alpha p_1 q_0^2}{128} + \frac{p_1 p_2}{8(1-(3+\varepsilon\sigma)^2)} + \frac{6\alpha p_2 p_0 q_0}{128} + \frac{3\alpha p_1 p_0^2}{16(1-(3+\varepsilon\sigma)^2)} + \frac{9\alpha p_1 q_0^2}{16(1-(3+\varepsilon\sigma)^2)} \\
 & \left. + \frac{18\alpha p_2 p_0 q_0}{16(1-(3+2\varepsilon\sigma)^2)} - \frac{12\alpha^2 p_0^3 q_0}{256} - \frac{12\alpha^2 p_0 q_0^3}{256} \right]
 \end{aligned}$$

4.3.2 Simultaneous resonance condition

Again, introducing the detuning parameter σ one may write $\Omega = 1 + \varepsilon\sigma$. Then Eq. (4.13) becomes,

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{if_2}{2} e^{i(1+\varepsilon\sigma)T_0} - 2D_1 i A e^{iT_0} - 2\mu i A e^{iT_0} - \frac{ip_1}{2} A e^{-i\varepsilon\sigma T_0} - \frac{ip_1}{2} \bar{A} e^{-i(2+\varepsilon\sigma)T_0} - \frac{p_2}{2} A e^{i(3+2\varepsilon\sigma)T_0} - \frac{p_2}{2} \bar{A} e^{i(1+2\varepsilon\sigma)T_0} - \alpha A^3 e^{3iT_0} - \alpha 3A^2 \bar{A} e^{iT_0} + CC. \quad (4.29)$$

The secular terms containing e^{iT_0} shall lead to a singular particular solution. These terms should be eliminated from Eq. (4.29) to obtain the bounded solution. The secular terms in Eq. (4.29) which should be eliminated are given below.

$$-\frac{if_2}{2} e^{i\varepsilon\sigma T_0} - 2D_1 i A - 2\mu i A - \frac{p_2}{2} \bar{A} e^{i2\varepsilon\sigma T_0} - \alpha 3A^2 \bar{A} = 0 \quad (4.30)$$

$$D_1 A = -\frac{f_2}{4} e^{i\varepsilon\sigma T_0} - \mu A + \frac{ip_2}{4} \bar{A} e^{i2\varepsilon\sigma T_0} + \frac{3i}{2} \alpha A^2 \bar{A} \quad (4.31)$$

Now, Eq. (4.29) can be rewritten as below.

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{ip_1}{2} A e^{-i\varepsilon\sigma T_0} - \frac{ip_1}{2} \bar{A} e^{-i(2+\varepsilon\sigma)T_0} - \frac{p_2}{2} A e^{i(3+2\varepsilon\sigma)T_0} - \alpha A^3 e^{3iT_0} + CC \quad (4.32)$$

$$x_1 = \frac{f_1}{2} - \frac{ip_1}{2(1-\varepsilon^2\sigma_1^2)} A e^{-i\varepsilon\sigma T_0} - \frac{ip_1}{2[1-(2+\varepsilon\sigma_1)^2]} \bar{A} e^{-i(2+\varepsilon\sigma)T_0} - \frac{p_2}{2[1-(3+2\varepsilon\sigma_1)^2]} A e^{i(3+2\varepsilon\sigma)T_0} - \frac{\alpha A^3 e^{3iT_0}}{8} + CC. \quad (4.33)$$

Now substituting Eq. (4.33) and Eq. (4.12) in Eq. (4.11) and eliminating the secular terms yields,

$$D_2 A = -(1+2\mu) \frac{D_1 A}{2i} + \frac{f_1 p_1}{8} e^{i\varepsilon\sigma T_0} - \frac{ip_1^2 A}{8(1-\varepsilon^2\sigma^2)} + \frac{ip_1^2 A}{8(1-(2+\varepsilon\sigma)^2)} - \frac{ip_2^2 A}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{i\alpha A^3 p_2}{32} e^{-2i\varepsilon\sigma T_0} - \frac{i3\alpha p_2 A \bar{A}^2 e^{i2\varepsilon\sigma T_0}}{4(1-(3+2\varepsilon\sigma)^2)} + \frac{i3\alpha^2 A^3 \bar{A}^2}{16}. \quad (4.34)$$

Again using the method of reconstitution [239],

$$\frac{dA}{d\tau} = \frac{dA}{dT_1} \frac{dT_1}{d\tau} + \frac{dA}{dT_2} \frac{dT_2}{d\tau} = \varepsilon D_1 A + \varepsilon^2 D_2 A$$

$$\begin{aligned} \frac{dA}{d\tau} = & \left[\varepsilon - \frac{\varepsilon^2(1+2\mu)}{2i} \right] \left[-\frac{f_2}{4} e^{i\varepsilon\sigma T_0} - \mu A + \frac{ip_2}{4} \bar{A} e^{i2\varepsilon\sigma T_0} + \frac{3i}{2} \alpha A^2 \bar{A} \right] + \varepsilon^2 \left[\frac{f_1 p_1}{8} e^{i\varepsilon\sigma T_0} \right. \\ & - \frac{ip_1^2 A}{8(1-\varepsilon^2\sigma^2)} + \frac{ip_1^2 A}{8(1-(2+\varepsilon\sigma)^2)} - \frac{ip_2^2 A}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{i\alpha A^3 p_2}{32} e^{-2i\varepsilon\sigma T_0} - \frac{i3\alpha p_2 A \bar{A}^2 e^{i2\varepsilon\sigma T_0}}{4(1-(3+2\varepsilon\sigma)^2)} \\ & \left. + \frac{i3\alpha^2 A^3 \bar{A}^2}{16} \right] \end{aligned} \quad (4.35)$$

Introducing $A = \frac{1}{2} a e^{i\beta}$ and taking $\varepsilon\sigma T_0 - \beta = \gamma$ in Eq. (4.35), the following reduced equations can be obtained by separating the real and imaginary parts.

$$\begin{aligned} a' = \frac{da}{dt} = & \varepsilon \left(-\frac{f_2}{2} \cos \gamma - \mu a - \frac{p_2 a}{4} \sin 2\gamma \right) + \frac{\varepsilon^2(1+2\mu)}{2} \left(\frac{f_2}{2} \sin \gamma - \frac{p_2 a}{4} \cos 2\gamma - \frac{3}{8} \alpha a^3 \right) \\ & + \varepsilon^2 \left[\frac{f_1 p_1}{4} \cos \gamma + \frac{\alpha a^3 p_2}{128} \sin 2\gamma + \frac{3\alpha p_2 a^3}{16(1-(3+2\varepsilon\sigma)^2)} \sin 2\gamma \right] \end{aligned} \quad (4.36)$$

$$\begin{aligned} a\gamma' = a \frac{d\gamma}{dt} = & a\varepsilon\sigma - \varepsilon \left(-\frac{f_2}{2} \sin \gamma + \frac{p_2 a}{4} \cos 2\gamma + \frac{3}{8} \alpha a^3 \right) - \frac{\varepsilon^2(1+2\mu)}{2} \left(-\frac{f_2}{2} \cos \gamma - \mu a \right. \\ & \left. - \frac{p_2 a}{4} \sin 2\gamma \right) - \varepsilon^2 \left[\frac{f_1 p_1}{4} \sin \gamma - \frac{p_1^2 a}{8(1-\varepsilon^2\sigma^2)} + \frac{p_1^2 a}{8(1-(2+\varepsilon\sigma)^2)} - \frac{p_2^2 a}{8(1-(3+2\varepsilon\sigma)^2)} \right. \\ & \left. + \frac{\alpha a^3 p_2}{128} \cos 2\gamma - \frac{3\alpha p_2 a^3 \cos 2\gamma}{16(1-(3+2\varepsilon\sigma)^2)} + \frac{3\alpha^2 a^5}{256} \right] \end{aligned} \quad (4.37)$$

Like in the previous case for principal parametric resonance condition, the stability of the steady state responses for the simultaneous resonance condition can be obtained by perturbing Eq. (4.36) and Eq. (4.37). The system is stable if the real parts of all the eigenvalues of the following Jacobian matrix J in Eq. (4.38) are negative.

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \quad (4.38)$$

where,

$$\begin{aligned}
 J_{11} &= \varepsilon \left(-\mu - \frac{p_2}{4} \sin 2\gamma_0 \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(-\frac{p_2}{4} \cos 2\gamma_0 - \frac{9}{8} \alpha a_0^2 \right) + \varepsilon^2 \left[\frac{3\alpha a_0^2 p_2}{128} \sin 2\gamma \right. \\
 &\quad \left. + \frac{9\alpha p_2 a_0^2 \sin 2\gamma}{16(1-(3+2\varepsilon\sigma)^2)} \right] \\
 J_{12} &= \varepsilon \left(\frac{f_2}{2} \sin \gamma_0 - \frac{p_2 a_0}{2} \cos 2\gamma_0 \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(\frac{f_2}{2} \cos \gamma_0 + \frac{p_2 a_0}{2} \sin 2\gamma_0 \right) + \varepsilon^2 \left[-\frac{f_1 p_1}{4} \sin \gamma \right. \\
 &\quad \left. + \frac{\alpha a_0^3 p_2}{64} \cos 2\gamma_0 + \frac{3\alpha p_2 a_0^3 \cos 2\gamma_0}{8(1-(3+2\varepsilon\sigma)^2)} \right] \\
 J_{21} &= -\varepsilon \left(\frac{f_2}{2a_0^2} \sin \gamma_0 + \frac{3}{4} \alpha a_0 \right) - \frac{\varepsilon^2 (1+2\mu)}{2} \left(\frac{f_2}{2a_0^2} \cos \gamma_0 \right) - \varepsilon^2 \left[-\frac{f_1 p_1}{4a_0^2} \sin \gamma_0 + \frac{\alpha a_0 p_2}{64} \cos 2\gamma_0 \right. \\
 &\quad \left. - \frac{3\alpha p_2 a_0 \cos 2\gamma_0}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{3\alpha^2 a_0^3}{64} \right] \\
 J_{22} &= -\varepsilon \left(-\frac{f_2}{2a_0} \cos \gamma_0 - \frac{p_2}{2} \sin 2\gamma_0 \right) - \frac{\varepsilon^2 (1+2\mu)}{2} \left(\frac{f_2}{2a_0} \sin \gamma_0 - \frac{p_2}{2} \cos 2\gamma_0 \right) - \varepsilon^2 \left[\frac{f_1 p_1}{4a_0} \cos \gamma_0 \right. \\
 &\quad \left. - \frac{\alpha a_0^2 p_2}{64} \sin 2\gamma_0 + \frac{3\alpha p_2 a_0^2 \sin 2\gamma_0}{8(1-(3+2\varepsilon\sigma)^2)} \right]
 \end{aligned}$$

After elimination of the secular terms from Eq. (4.29) one may obtain the following equation.

$$D_0^2 x_1 + x_1 = \frac{f_1}{2} - \frac{ip_1}{2} A e^{i(1-\Omega)T_0} - \frac{ip_1}{2} \bar{A} e^{-i(1+\Omega)T_0} - \frac{p_2}{2} A e^{i(2\Omega+1)T_0} - \alpha A^3 e^{3iT_0} + CC \quad (4.39)$$

Thus the particular solution of x_1 from Eq. (4.39) can be written as follows.

$$\begin{aligned}
 x_1 &= \frac{f_1}{2} + \frac{\alpha A^3 e^{3iT_0}}{8} - \frac{1}{(1-(1-\Omega)^2)} \frac{ip_1}{2} A e^{i(1-\Omega)T_0} - \frac{1}{(1-(1+\Omega)^2)} \frac{ip_1}{2} \bar{A} e^{-i(1+\Omega)T_0} \\
 &\quad - \frac{1}{(1-(2\Omega+1)^2)} \frac{p_2}{2} A e^{i(2\Omega+1)T_0} + CC
 \end{aligned} \quad (4.40)$$

Reorganizing Eq. (4.40) results in the following equation.

$$x_1 = f_1 + \frac{\alpha a^3}{32} \cos(6T_0) - \frac{p_1 a}{2(1-(1-\Omega)^2)} \sin \gamma - \frac{p_1 a}{2(1-(1+\Omega)^2)} \sin(2\Omega T_0 - \gamma) - \frac{p_2 a}{2(1-(2\Omega+1)^2)} \cos(3\Omega T_0 - \gamma) \quad (4.41)$$

Thus from Eq. (4.7), Eq. (4.12) and Eq. (4.41) the response of the system $x(\tau)$ is given below.

$$x = x_0 + \varepsilon x_1 = a \cos(\Omega T_0 - \gamma) + \varepsilon \left[f_1 - \frac{p_1 a}{2(1-(1-\Omega)^2)} \sin \gamma - \frac{p_1 a}{2(1-(1+\Omega)^2)} \sin(2\Omega T_0 - \gamma) - \frac{p_2 a}{2(1-(2\Omega+1)^2)} \cos(3\Omega T_0 - \gamma) + \frac{\alpha a^3}{32} \cos(6T_0) \right] \quad (4.42)$$

Eq. (4.42) provides the expression for the complete time response of the parametrically excited system attached with a nonlinear PAM under simultaneous resonance condition.

4.3.2.1 Stability of the steady state response

It can be noticed from the reduced equations Eq. (4.36) and Eq. (4.37) for simultaneous resonance condition, the system shows both trivial solution ($a = 0$) and nontrivial solution ($a \neq 0$). As same as discussed in the section 4.3.1.1, the stability of the steady state response of the system can be studied and following form of Cartesian modulation equations from Eq. (4.36) and Eq. (4.37) can be obtained.

$$p' = \varepsilon \left(-\frac{f_2}{2} - \mu p - \frac{p_2 q}{4} + \frac{3\alpha q(p^2 + q^2)}{8} \right) + \frac{\varepsilon^2(1+2\mu)}{2} \left(-\mu q - \frac{p_2 p}{4} - \frac{3\alpha p(p^2 + q^2)}{8} \right) + \varepsilon^2 \left[\frac{f_1 p_1}{4} + \frac{3\alpha p_2 q(p^2 + q^2)}{128} - \frac{4\alpha p_2 q^3}{128} + \frac{3\alpha p_2 q(p^2 + q^2)}{16(1-(3+2\varepsilon\sigma)^2)} - \frac{p_1^2 q}{8(1-\varepsilon^2\sigma^2)} + \frac{p_1^2 q}{8(1-(2+\varepsilon\sigma)^2)} - \frac{p_2^2 q}{8(1-(3+2\varepsilon\sigma)^2)} + \frac{3\alpha^2 q(p^2 + q^2)^2}{256} \right] - \varepsilon \sigma q \quad (4.43)$$

$$\begin{aligned}
 q' = & \varepsilon \left(-\mu q - \frac{p_2 p}{4} - \frac{3\alpha p(p^2 + q^2)}{8} \right) + \frac{\varepsilon^2(1+2\mu)}{2} \left(\frac{f_2}{2} + \mu p + \frac{p_2 q}{4} - \frac{3\alpha q(p^2 + q^2)}{8} \right) \\
 & + \varepsilon^2 \left[-\frac{4\alpha p_2 p^3}{128} + \frac{3\alpha p_2 p(p^2 + q^2)}{128} + \frac{3\alpha p_2 p(p^2 + q^2)}{16(1-(3+2\varepsilon\sigma)^2)} + \frac{p_1^2 p}{8(1-\varepsilon^2\sigma^2)} - \frac{p_1^2 p}{8(1-(2+\varepsilon\sigma)^2)} \right. \\
 & \left. + \frac{p_2^2 p}{8(1-(3+2\varepsilon\sigma)^2)} - \frac{3\alpha^2 p(p^2 + q^2)^2}{256} \right] + \varepsilon\sigma p
 \end{aligned} \quad (4.44)$$

Similarly, the stability of the steady state response (p_0, q_0) can be achieved by examining the eigenvalues of the following Jacobian matrix (J) found by perturbing Eq. (4.43) and Eq. (4.44). Here, the system will be stable for both trivial and non-trivial responses for simultaneous resonance condition when all the real parts the eigenvalues of the Jacobian matrix (J) in Eq. (4.45) are negative.

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} \quad (4.45)$$

where,

$$\begin{aligned}
 J_{11} = & \varepsilon \left(-\mu + \frac{6\alpha p_0 q_0}{8} \right) + \frac{\varepsilon^2(1+2\mu)}{2} \left(-\frac{p_2}{4} - \frac{9\alpha p_0^2}{8} - \frac{3\alpha q_0^2}{8} \right) + \varepsilon^2 \left[\frac{6\alpha p_2 p_0 q_0}{128} + \frac{6\alpha p_2 p_0 q_0}{16(1-(3+2\varepsilon\sigma)^2)} \right. \\
 & \left. + \frac{12\alpha^2 p_0^3 q_0}{256} + \frac{12\alpha^2 p_0 q_0^3}{256} \right] \\
 J_{12} = & \varepsilon \left(-\frac{p_2}{4} + \frac{3\alpha p_0^2}{8} + \frac{9\alpha q_0^2}{8} \right) + \frac{\varepsilon^2(1+2\mu)}{2} \left(-\mu - \frac{3\alpha p_0 q_0}{4} \right) + \varepsilon^2 \left[\frac{3\alpha p_2 p_0^2}{128} - \frac{3\alpha p_2 q_0^2}{128} \right. \\
 & + \frac{3\alpha p_2 p_0^2}{16(1-(3+2\varepsilon\sigma)^2)} + \frac{9\alpha p_2 q_0^2}{16(1-(3+2\varepsilon\sigma)^2)} - \frac{p_1^2}{8(1-\varepsilon^2\sigma^2)} + \frac{p_1^2}{8(1-(2+\varepsilon\sigma)^2)} - \frac{p_2^2}{8(1-(3+2\varepsilon\sigma)^2)} \\
 & \left. + \frac{3\alpha^2 p_0^4}{256} + \frac{18\alpha^2 p_0^2 q_0^2}{256} + \frac{15\alpha^2 q_0^4}{256} \right] - \varepsilon\sigma
 \end{aligned}$$

$$\begin{aligned}
J_{21} = & \varepsilon \left(-\frac{p_2}{4} - \frac{9\alpha p_0^2}{8} - \frac{3\alpha q_0^2}{8} \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(\mu - \frac{3\alpha p_0 q_0}{4} \right) + \varepsilon^2 \left[-\frac{3\alpha p_2 p_0^2}{128} + \frac{3\alpha p_2 q_0^2}{128} \right. \\
& + \frac{9\alpha p_2 p_0^2}{16(1-(3+2\varepsilon\sigma)^2)} + \frac{3\alpha p_2 q_0^2}{16(1-(3+2\varepsilon\sigma)^2)} + \frac{p_1^2}{8(1-\varepsilon^2\sigma^2)} - \frac{p_1^2}{8(1-(2+\varepsilon\sigma)^2)} + \frac{p_2^2}{8(1-(3+2\varepsilon\sigma)^2)} \\
& \left. - \frac{15\alpha^2 p_0^4}{256} - \frac{18\alpha^2 p_0^2 q_0^2}{256} - \frac{3\alpha^2 q_0^4}{256} \right] + \varepsilon\sigma \\
J_{22} = & \varepsilon \left(-\mu - \frac{6\alpha p_0 q_0}{8} \right) + \frac{\varepsilon^2 (1+2\mu)}{2} \left(\frac{p_2}{4} - \frac{3\alpha p_0^2}{8} - \frac{9\alpha q_0^2}{8} \right) + \varepsilon^2 \left[\frac{6\alpha p_2 p_0 q_0}{128} + \frac{6\alpha p_2 p_0 q_0}{16(1-(3+2\varepsilon\sigma)^2)} \right. \\
& \left. - \frac{12\alpha^2 p_0^3 q_0}{256} - \frac{12\alpha^2 p_0 q_0^3}{256} \right]
\end{aligned}$$

4.4 Results and Discussion

The system parameters for the PAM used in this work for numerical analyses are mentioned in Table 4.1, which are almost similar to the parameters reported in the work of Li et al. [125]. The steady state responses of the system have been obtained by solving the reduced equations Eq. (4.20) and Eq. (4.21) for principal parametric resonance condition and Eq. (4.36) and Eq. (4.37) for simultaneous simple and principal parametric resonance condition by using Newton's method. The obtained approximate solutions from the method of multiple scale have been compared with those found by numerically solving the original equation Eq. (4.6). The system shows a typical nonlinear behaviour because of the presence of various nonlinear parameters in the original equations of motion Eq. (4.6). In this section, the effects of various system parameters like operating pressure, muscle parameter and damping on the nonlinear response of the system have been studied for the principal parametric resonance and simultaneous resonance conditions using time response, phase portraits, instability regions and frequency responses. For comparison, the time response and phase portrait curves have been plotted where the green lines are the numerical solution (NS) from original governing equation and brown lines are the analytical solution (AS) from the reduced equations using the method of multiple scales. It may be noted that the regions bounded by the curve is unstable (marked by U) and regions outside the curve is stable (marked by S) in the case of trivial state instability regions in subsections 4.4.1.1 and 4.4.2.1 for principal parametric and simultaneous resonance condition respectively. For the frequency response plots,

the blue line signifies the stable solution and the red line shows the unstable solution in subsections 4.4.1.2 and 4.4.2.2.

Table 4.1 System parameters used for numerical simulation.

Parameter	Numerical Value	Parameter	Numerical Value	Parameter	Numerical Value
$l_{\max}$	74 mm	d_1	-100 N	P_0	58 kPa
c_1	-234.25 N	d_2	1 N/kPa	ε	0.1
c_2	1.96 N/kPa	m	6 N	μ	0.01
c_3	-0.003 N/kPa ²	P_m	300 kPa	α	150

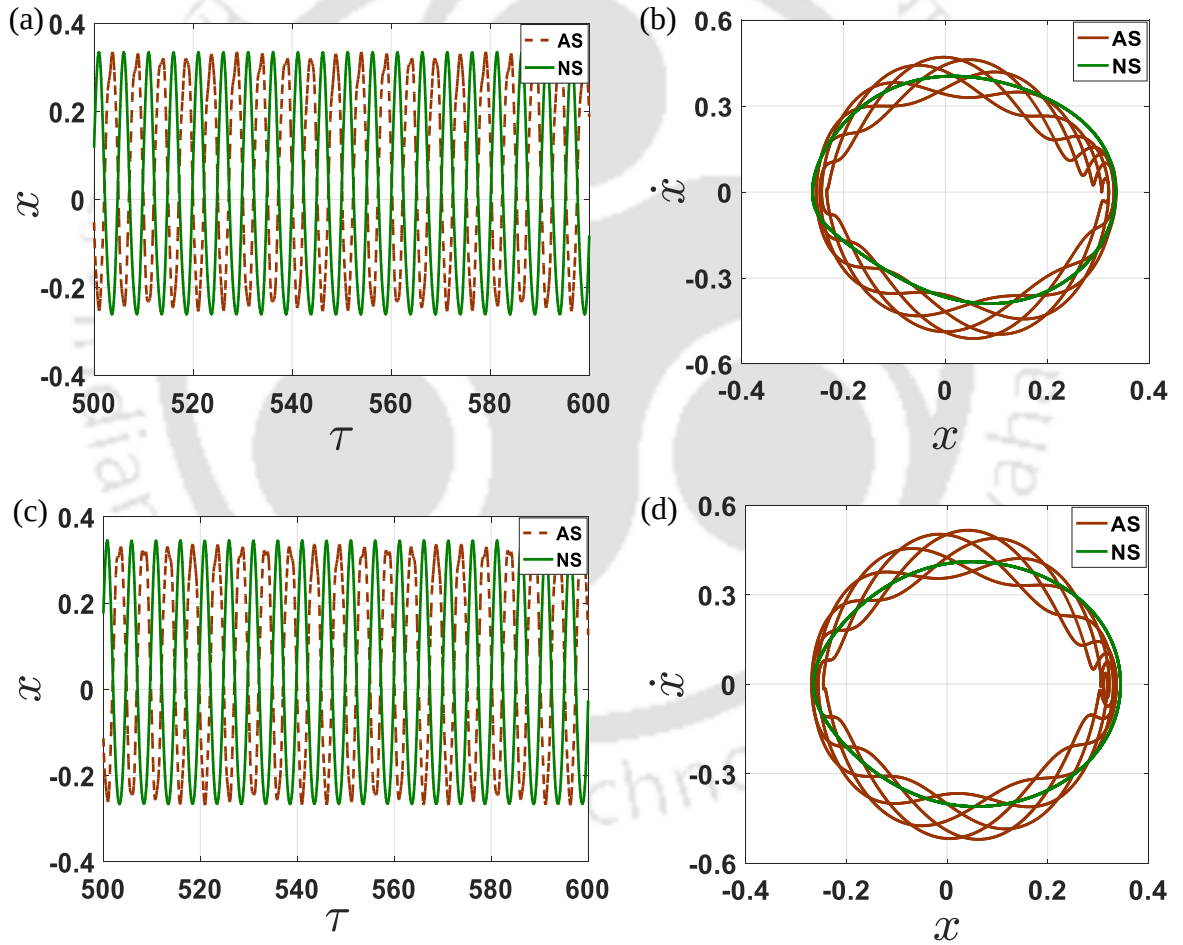
4.4.1 Principal parametric resonance condition

In the case of principal parametric resonance condition, the frequency is nearly equal to twice the natural frequency of the system. Figure 4.2 shows time responses and phase portraits comparison between the solutions obtained by solving the original equation Eq. (4.6) and solving the reduced equations Eq. (4.20) and Eq. (4.21) numerically using the Runge-Kutta method by changing the different system parameters of the muscles.

Taking the static pressure of the muscle (P_m) = 100 kPa and damping parameter (μ) = 0.1 and other parameters from Table 4.1, the time response and phase portrait are shown in Fig. 4.2(a) and 4.2(b), respectively. It may clearly be observed that the solution obtained by solving the original equation (4.6) and those obtained by the method of multiple scales (4.20) and (4.21) are found to be in good agreement with mean response amplitude equal to 0.037 in both the cases. Now, reducing the damping parameter (μ) to 0.01 and keeping other system parameters the same as before, the time response and phase portrait are shown in Fig. 4.2(c) and 4.2(d), respectively. Here also it is observed that both the responses have the same mean response amplitude of 0.044 which is 18.9% more than the previous case. Further changing only the dynamic pressure (P_0) to 70 kPa and keeping other parameters same as in Fig. 4.2(a), the responses obtained from Eq. (4.6) and those from the method of multiple scales have the mean amplitude 0.038 (Fig. 4.2(e) and Fig. 4.2(f)) which is almost the same as in Fig. 4.2(a). Now finally reducing the nonlinear parameter (α) to 50 and keeping other system parameter is the same as in Fig. 4.2(a), the responses are

observed in Fig. 4.2(g) and 4.2(h). Here also both the solutions are in good agreement with mean amplitude 0.037 which is exactly similar to Fig. 4.2(a).

In all of the above cases, the results obtained by solving Eq. (4.6) are found to be periodic and those obtained by the second order method of multiple scales are found to be quasi-periodic due to the presence of harmonics. Similarly by changing the other different parameters of the PAM, one can obtain the time responses and phase portraits in both cases. As the results obtained by using the method of multiple scales are very close to those obtained by solving the original equation Eq. (4.6), so the developed reduced equations Eq. (4.20) and Eq. (4.21) may be used conveniently for further analysis.



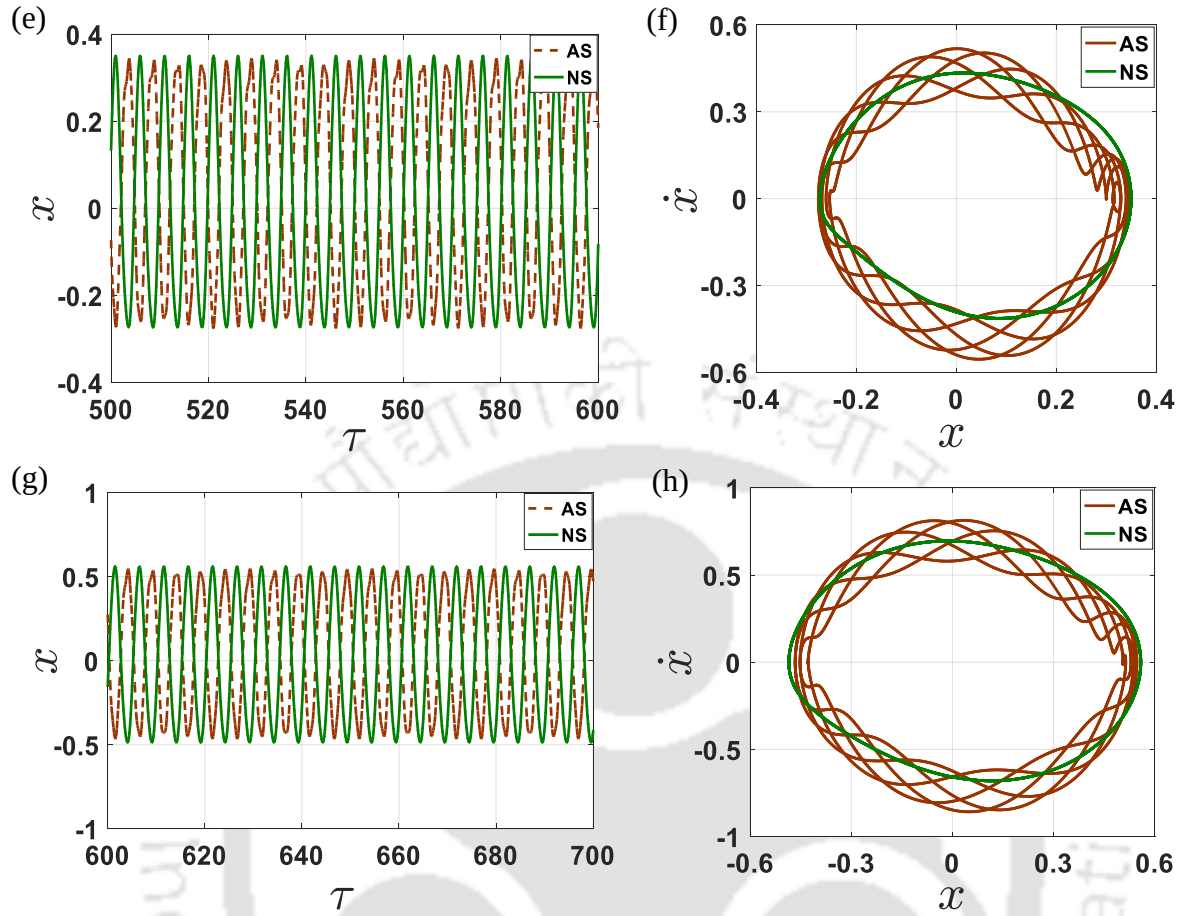


Fig. 4.2 Time response and phase portrait comparison under principal parametric resonance condition with (a, b) $\mu = 0.1, P_0 = 58\text{kPa}$ and $\alpha=150$ (c, d) $\mu=0.01, P_0 = 58\text{kPa}$ and $\alpha=150$ (e, f) $\mu = 0.1, P_0 = 70\text{kPa}$ and $\alpha=150$ (g, h) $\mu = 0.1, P_0 = 58\text{kPa}$ and $\alpha=50$.

4.4.1.1 Trivial state instability regions

Trivial state instability regions are plotted by finding the values of system parameters for which the real part of the eigenvalues of the Jacobean matrix (J) Eq. (4.28) for principal parametric resonance condition change its sign. These regions are shown in Fig. 4.3 plotted in terms of detuning parameter(σ) with dynamic pressure of the muscle(P_0), muscle parameter(c_1) and static pressure of the muscle(P_m) for different values of damping(μ).

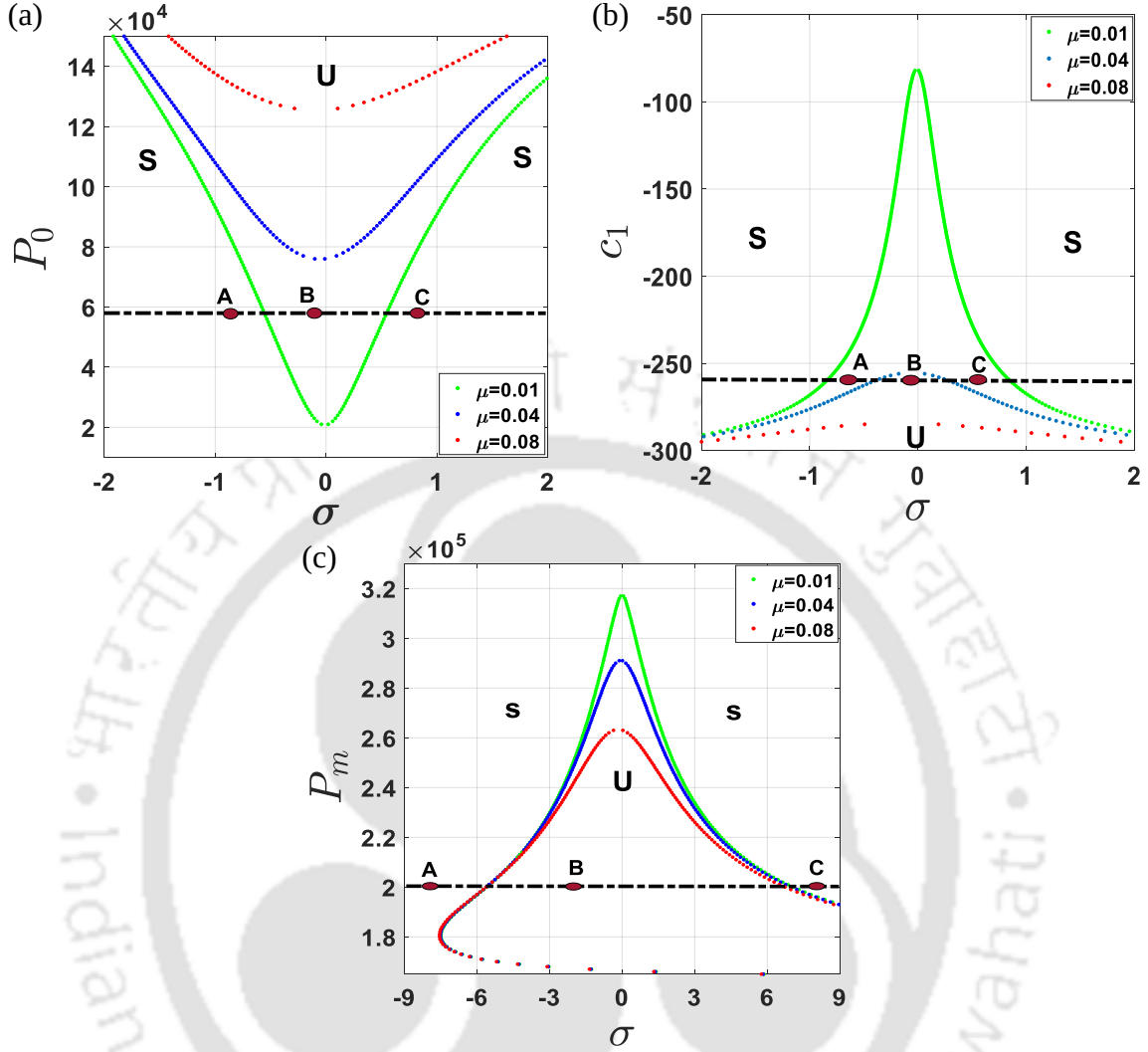


Fig. 4.3 Trivial state instability region for $\mu = 0.01, 0.04$ and 0.08 under principal parametric resonance condition with (a) $P_0 \sim \sigma$ (b) $c_1 \sim \sigma$ (c) $P_m \sim \sigma$.

For $P_0 = 58$ kPa and $\mu = 0.01$ in Fig. 4.3(a), the instability region starts for detuning parameter $\sigma = -0.556$ and ends at $\sigma = 0.544$. This results have been verified by taking three points on the instability plot A ($\sigma = -0.8$), B ($\sigma = -0.2$) and C ($\sigma = 0.8$) and plotting the time response and phase portraits in Fig. 4.4. Clearly, it can be noticed that the response is stable for point A and C as shown in time responses Fig. 4.4(a, c) and phase portraits Fig. 4.4(b, d). For point B taking $\alpha = 0$, the time response and phase portraits are shown in Fig. 4.4(e, f) which clearly depicts the response to be unstable. But for the same point taking $\alpha = 150$ one may get the nontrivial response which is bounded and stable with two periodic response as shown in Fig. 4.4(g, h). It may be observed from these figures that though $a = 0$ and $\gamma = 0$, $x \neq 0$. This is due to the drifting of

amplitude which is evident from Eq. (4.25). It may be noted that there is a critical value of $P_0(P_{0cr})$ below which the system always remains stable for any value of σ . For example, for $\mu = 0.01$, $P_{0cr} = 20$ kPa; $\mu = 0.04$, $P_{0cr} = 76$ kPa and $\mu = 0.08$, $P_{0cr} = 126$ kPa. Hence, the unstable region decreases with increase in μ and occurs at a higher value of P_0 .

Figure 4.3(b) shows the instability curve in the $c_1 \sim \sigma$ plane. Unlike the case of $P_0 \sim \sigma$, here the instability plot is inverted. Similar to the previous case the time responses and phase portraits are plotted by taking three points A ($\sigma = -0.5$), B ($\sigma = -0.1$) and C ($\sigma = 0.5$) which are found to be in good agreement as point A (Fig. 4.5(a, b)) and C (Fig. 4.5 (c, d)) are stable and point B (Fig. 4.5(e, f)) is unstable. But, point B with $\alpha = 150$ depicts stable response in Fig. 4.5(g, f) where one may obtain the solution in the nontrivial state. For example, to operate the PAM with $c_1 = -260$ N and $\mu = 0.04$, σ should be in between supercritical (-0.398) and subcritical (0.280) pitchfork bifurcation points which are observed from Fig. 4.3(b). Here, one may find the critical value of $c_1(c_{1cr})$ for the different value of damping (μ) which is for $\mu = 0.01$, $c_{1cr} = -82$ N; $\mu = 0.04$, $c_{1cr} = -256$ N and $\mu = 0.08$, $c_{1cr} = -285$ N, above which the system remains always stable. With an increase in μ the value of c_1 decreases and the unstable region increases.

In Fig. 4.3(c) trivial state instability regions have been plotted in $P_m \sim \sigma$ plane for three values of damping parameters. The time responses and phase portraits have been plotted for $P_m = 200$ kPa and $\mu = 0.08$ at three different points A ($\sigma = -8$), B ($\sigma = -2$) and C ($\sigma = 8$) while point A (Fig. 4.6(a, b)) and C (Fig. 4.6(c, d)) are stable, point B (Fig. 4.6(e, f)) is found to be unstable. This validates the instability region in Fig. 4.3(c). As an example while designing the muscle, one should operate the system within the detuning parameter $\sigma = -5.56$ and $\sigma = 6.7$ where the system has an unstable trivial state (Fig. 4.6(e, f)) and nontrivial state stable corresponding to $\alpha = 150$ (Fig. 4.6(g, h)). Similar to the previous two cases the critical value of $P_m(P_{mcr})$ above which the system will always be stable and increase in the value of damping (μ), the value of P_m decreases and the unstable region increases. The critical values of $P_m(P_{mcr})$ for different damping (μ) are, for $\mu = 0.01$, $P_{mcr} = 317$ kPa; $\mu = 0.04$, $P_{mcr} = 291$ kPa and $\mu = 0.08$, $P_{mcr} = 263$ kPa.

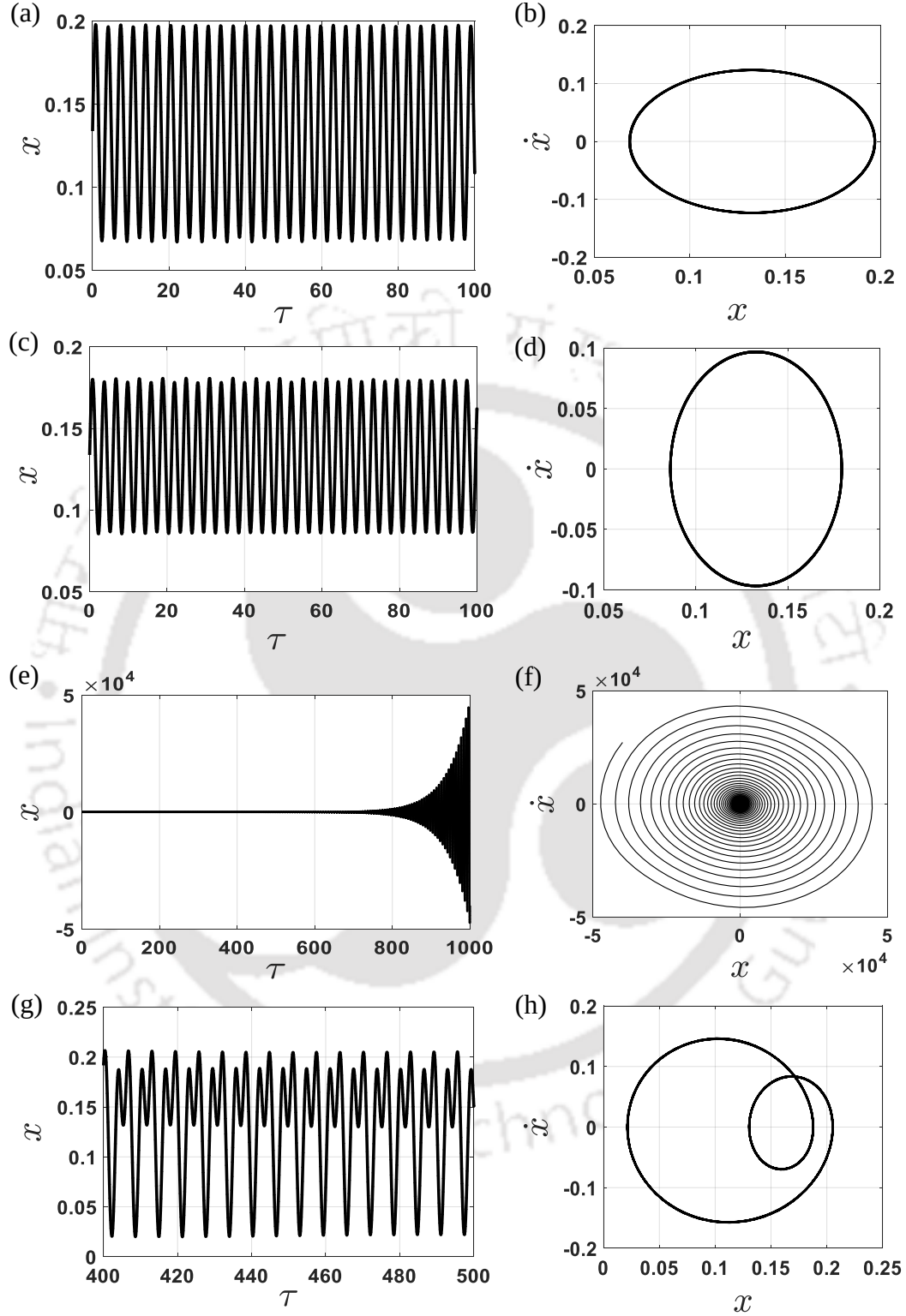


Fig. 4.4 Time responses and phase portraits for (a, b) point A (c, d) point C (e, f) point B with $\alpha = 0$ (g, h) point B with $\alpha = 150$ from Fig. 4.3(a) under principal parametric resonance condition.

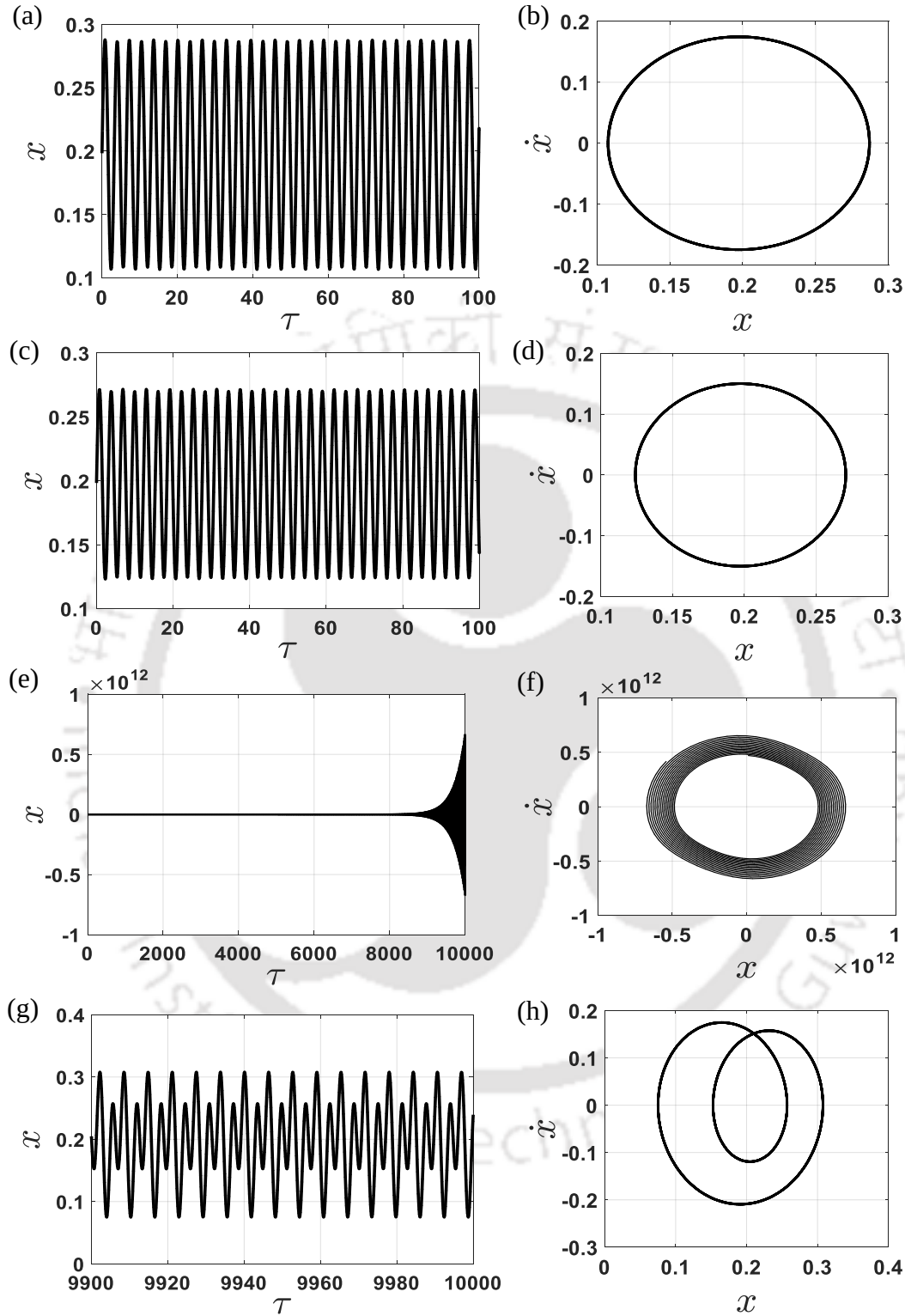


Fig. 4.5 Time responses and phase portraits for (a, b) point A (c, d) point C (e, f) point B with $\alpha = 0$ (g, h) point B with $\alpha = 150$ from Fig. 4.3(b) under principal parametric resonance condition.

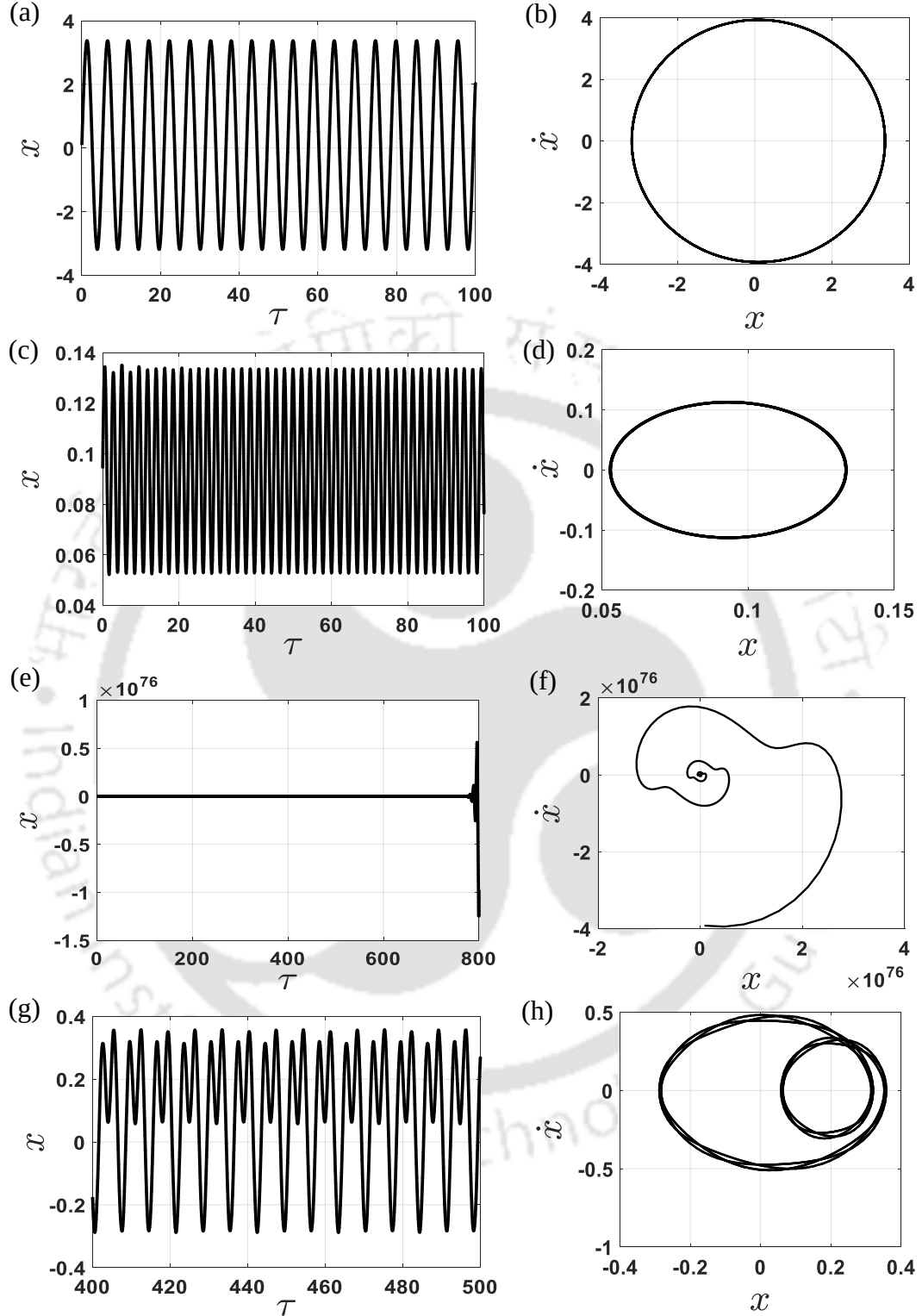


Fig. 4.6 Time responses and phase portraits for (a, b) point A (c, d) point C (e, f) point B with $\alpha = 0$ (g, h) point B with $\alpha = 150$ from Fig. 4.3(c) under principal parametric resonance condition.

4.4.1.2 Frequency response

The frequency responses (Fig. 4.7-4.17) of the muscle is plotted by solving the set of equations Eq. (4.20) and Eq. (4.21) using Newton's method. As observed in Fig. 4.3, in these frequency responses the trivial state becomes unstable between the supercritical and subcritical pitchfork bifurcation points. Figure 4.7 shows the frequency response plot considering the system parameters as given in Table 4.1. As mentioned above the supercritical and subcritical pitchfork bifurcation points are marked as point A and B, respectively. It may clearly be observed that the muscle will not contract below point A. With the increase in frequency beyond this point, there will be more contraction in the muscle and bulging effect will take place. Also as the system has a trivial state beyond point B, by reducing the frequency of the system will experience a jump-up phenomenon at the point B where the unstable solution P has a tendency to jump up to the stable solution Q. So depending on the requirement of displacement of the muscle one should choose proper system parameters to actively operate the system by controlling the air pressure (P_0 and P_m) and detuning parameter (σ). Passively one may achieve the same effect by changing the muscle with suitable value of c_1 , c_2 , c_3 , d_1 , d_2 and α . To verify the frequency response plot, the basin of attraction are plotted.

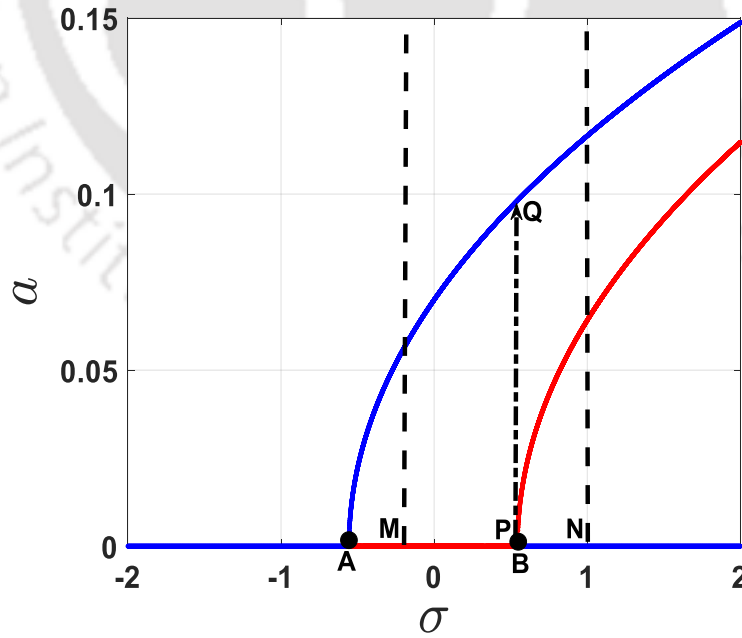


Fig. 4.7 Frequency response of the parametrically excited system with principal parametric resonance condition as the parameter values in Table 5.2.

In Fig. 4.8(a), the basin of attraction has been plotted in the $a \sim \gamma$ plane for the system with detuning parameter, $\sigma = -0.2$ which corresponds to point M as marked in Fig. 4.7. Clearly, it shows that the system has a stable solution with amplitude $a = 0.057$. Corresponding to point N marked in Fig. 4.7 the basin of attraction is shown in Fig. 4.8(b) which clearly shows that the system has a bistable state with two stable and one unstable solution. So the results obtained by using Newton's method and the numerical solution yields the same result.

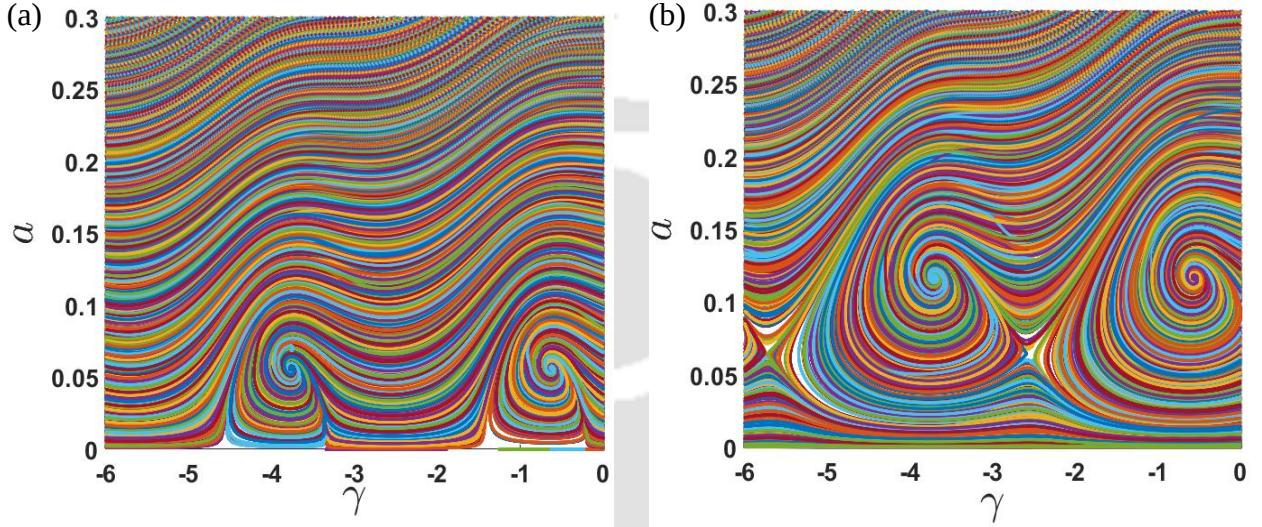


Fig. 4.8 Basin of attraction: (a) $\sigma = -0.2$ and (b) $\sigma = 1$ from Fig. 4.7.

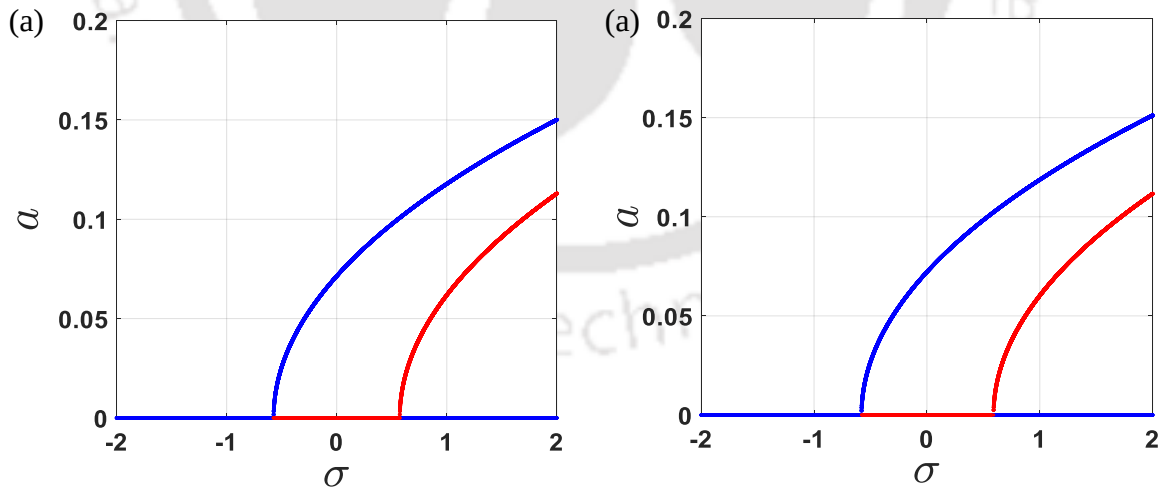


Fig. 4.9 Effect of damping (μ) on the frequency response curve of the muscle with (a) $\mu = 0.006$ and (b) $\mu = 0.001$.

With the decrease in the damping parameter (μ), the frequency plots are given in Fig. 4.9(a, b). One may observe that the response amplitude of the system slightly increases and the range of unstable trivial state between the supercritical and subcritical bifurcation points also increases.

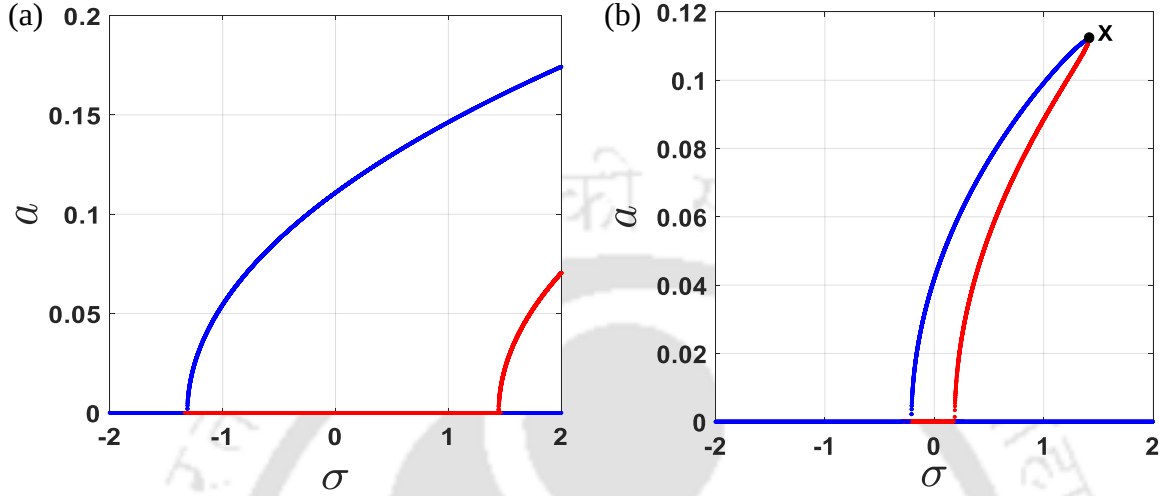


Fig. 4.10 Effect of dynamic pressure (P_0) of the muscle on the frequency response curve with
(a) $P_0 = 116$ kPa (b) $P_0 = 29$ kPa.

Figure 4.10 shows the frequency response curves for the two different values of dynamic pressure of the muscle (P_0). It is observed that with the increase in P_0 , the maximum response amplitude of the system increases and the trivial state instability region between the super- and sub-critical pitchfork bifurcation points also increases. But with the decrease in the value P_0 (Fig. 4.10(b)), the maximum response amplitude and the unstable trivial range are decreased. From Fig. 4.10(a, b) the response amplitude of the system is decreased around 32% at detuning parameter $\sigma = 1$ with decreased in the value of P_0 . In Fig. 4.10(b), one may observe the saddle-node bifurcation point at 'X' beyond which the system has a single stable trivial state. Clearly, in this case, the operating range of the muscle is in between the supercritical pitchfork bifurcation and saddle-node bifurcation point.

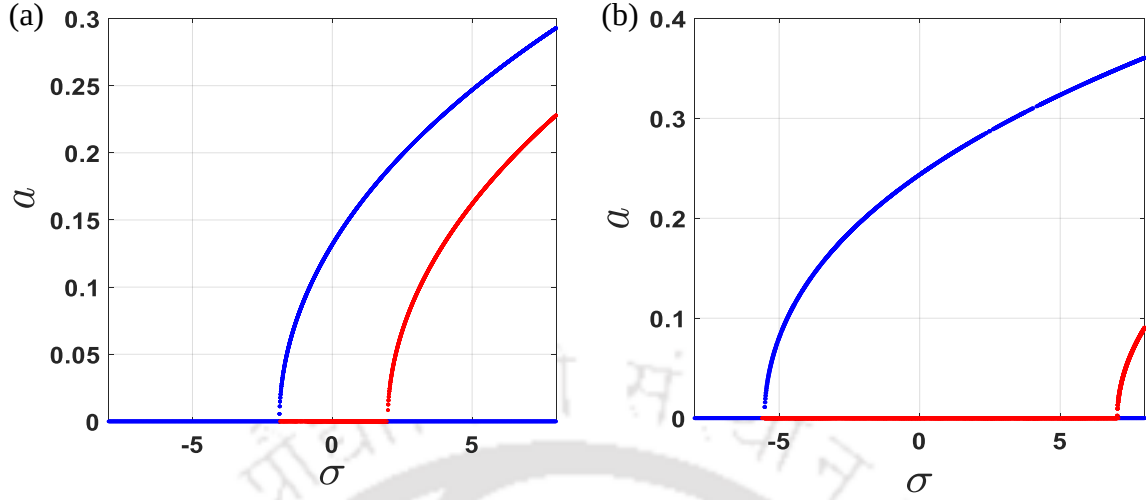


Fig. 4.11 Effect of static pressure (P_m) of the muscle on the frequency response curve with
(a) $P_m = 400$ kPa (b) $P_m = 200$ kPa.

In Fig. 4.11, the frequency response curves for the two values of static muscle pressure P_m are shown. Here it can clearly be noted from Fig. 4.7 and Fig. 4.11 that with the increase or decrease in P_m , the response amplitude of the system increases. But the trivial state instability region increases with decrease in P_m . For the same muscle displacement one may use the lower value of P_m with higher value of σ . Here the response amplitude of the system is increased by around 19% at detuning parameter $\sigma = 8$ with the decrease in the value of P_m .

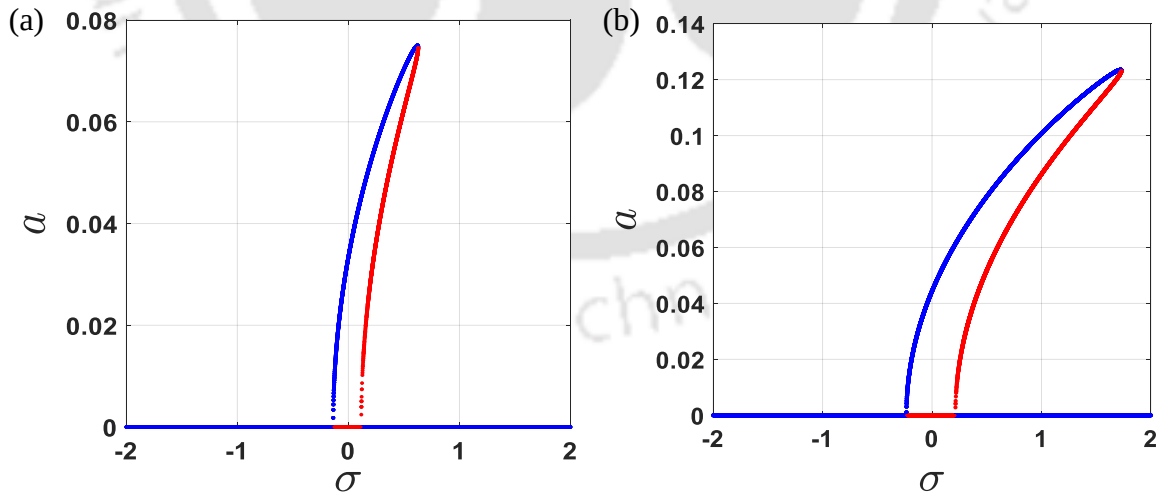


Fig. 4.12 Effect of muscle parameter (c_1) on the frequency response curve with
(a) $c_1 = -117.12$ N (b) $c_1 = -468.50$ N.

Figure 4.12 shows the frequency response curves for the two different values muscle parameter (c_1). It is observed that with either increase or decrease in c_1 , the response amplitude of the system decreases and the unstable trivial range between the super- and sub-critical pitchfork bifurcation points also decreases. The maximum response amplitude for these cases increase by around 40% with the decrease in the muscle parameter c_1 . The muscle should be operated between the supercritical pitchfork bifurcation and saddle-node bifurcation point for getting the desired displacement.

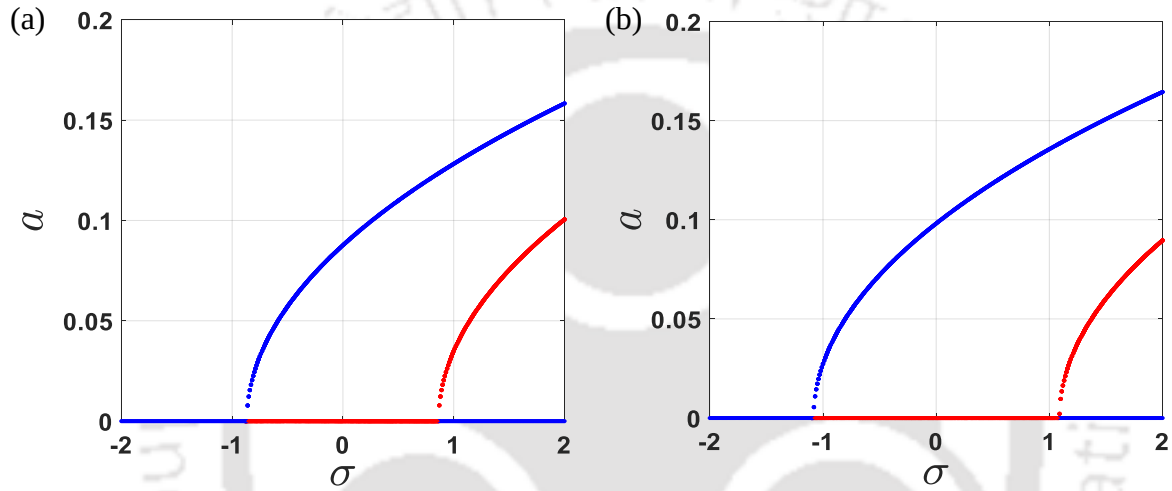


Fig. 4.13 Effect of muscle parameter (c_2) on the frequency response curve with
(a) $c_2 = 2.96$ N/kPa (b) $c_2 = 0.96$ N/kPa.

In Fig. 4.13, the frequency response curves for the two values of muscle parameter c_2 are shown. Here it can clearly be noted from Fig. 4.7 and Fig. 4.13 that with the increase or decrease in the value of c_2 , the response amplitude increases. But in the case of the trivial state instability region is increased with increase or decrease in c_2 . The response amplitude is increased by around 7% and 11% with increase in the value of $c_2 = 2.96$ N/kPa and decrease in the value of $c_2 = 0.96$ N/kPa as compared to the Fig. 4.7 respectively. Similarly the unstable trivial range is increased around 56% and 96% by increasing and decreasing the value of c_2 respectively as compared to the Fig. 4.7.

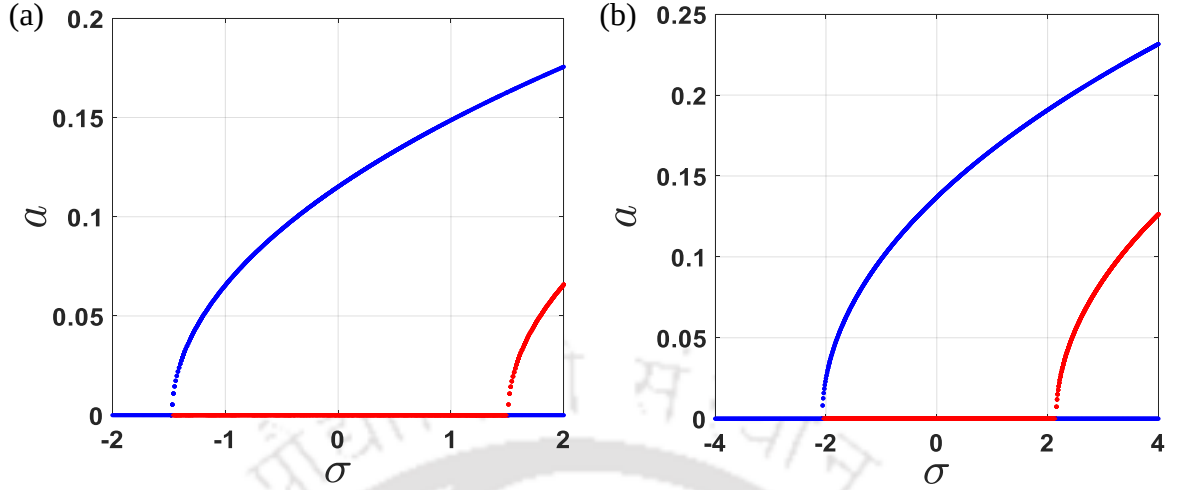


Fig. 4.14 Effect of muscle parameter (c_3) on the frequency response curve with
 (a) $c_3 = -0.001 \text{ N/kPa}^2$ (b) $c_3 = -0.009 \text{ N/kPa}^2$.

Like in the previous case for muscle parameter c_2 , the response amplitude and the unstable trivial range is increased with increase or decrease in the muscle parameter value c_3 which can be observed from Fig. 4.14 and Fig. 4.7. The response amplitude and the unstable trivial range are increased by around 18% and 170% respectively as compared to Fig. 4.7 with increase in the value of $c_3 = -0.001 \text{ N/kPa}^2$. Similarly the response amplitude and unstable trivial range are increased around 28% and 283% respectively by decreasing the value of $c_3 = -0.009 \text{ N/kPa}^2$ as compared to the Fig. 4.7. Hence for design and particular application purpose to achieve a definite response amplitude one may find the optimum value of c_1 , c_2 and c_3 .

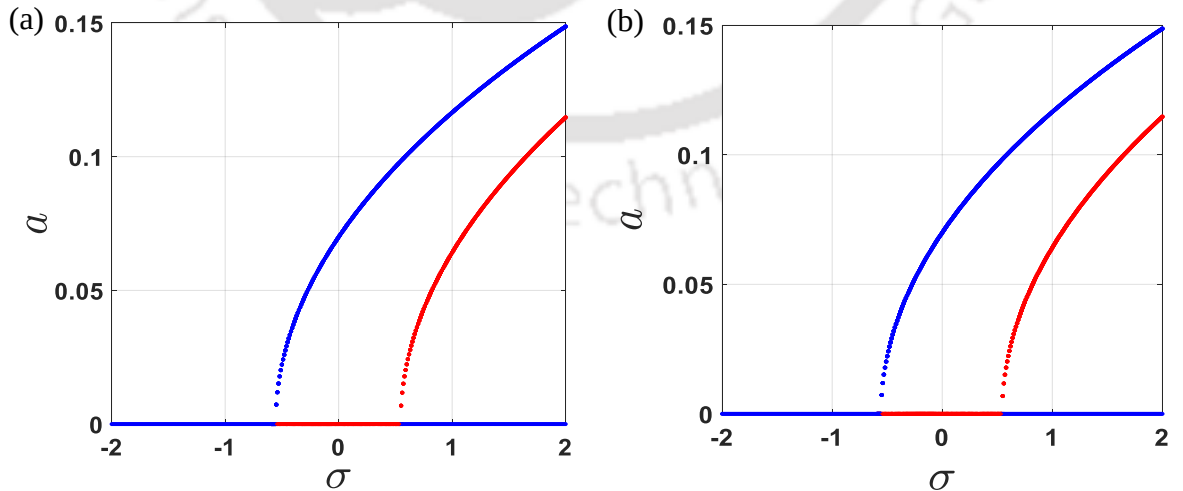


Fig. 4.15 Effect of muscle parameter (d_1) on the frequency response curve with
 (a) $d_1 = -50 \text{ N}$ (b) $d_1 = -200 \text{ N}$.

From Fig. 4.15 and Fig. 4.7, it can be observed that the effect of the muscle parameter d_1 is not prominent. By making half the value of $d_1 = -50$ N in Fig. 4.15(a) and double in Fig. 4.15(b) where $d_1 = -200$ N as compared to Fig. 4.7, there is no significant change in the response amplitude as well as in the trivial state instability region.

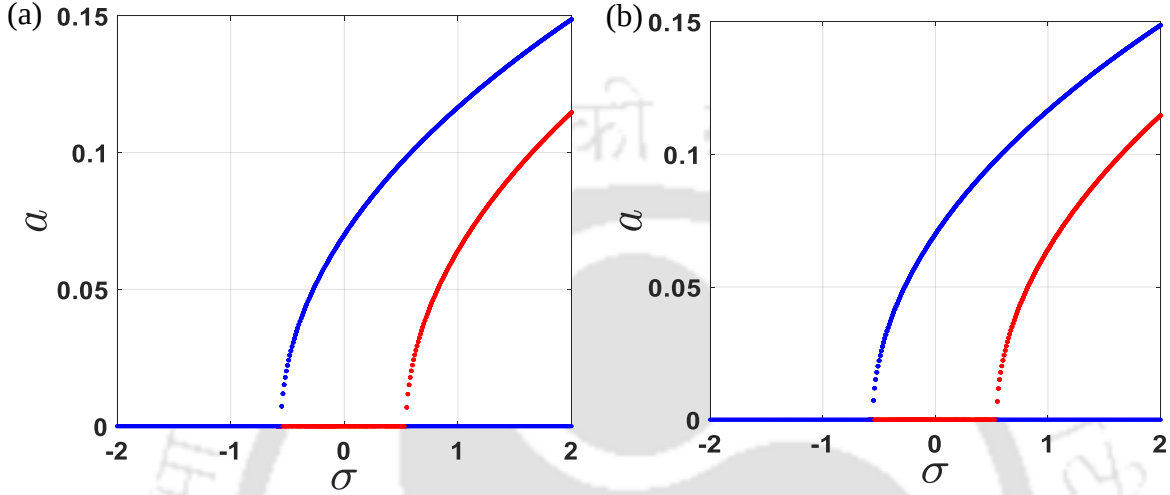


Fig. 4.16 Effect of muscle parameter (d_2) on the frequency response curve with
(a) $d_2 = 2$ N/kPa (b) $d_2 = 0.5$ N/kPa.

The effect of the muscle parameter d_2 also not significant like the previous case for the muscle parameter d_1 . There is no change in the response amplitude and the trivial state instability region by making half or doubling the value of d_2 which can be observed from Fig. 4.16 and Fig. 4.7. So, the effects of d_1 and d_2 are negligible which can be observed from the responses in Fig. 4.7, Fig. 4.15 and Fig. 4.16 which are similar.

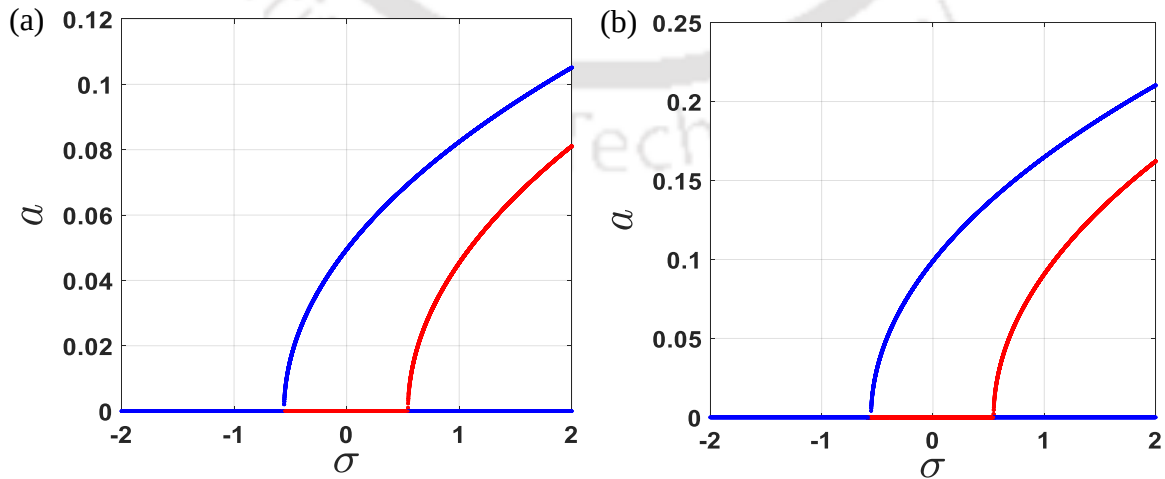


Fig. 4.17 Effect of nonlinearity (α) on the frequency response curve with (a) $\alpha = 300$ (b) $\alpha = 75$.

From Fig. 4.17(a), it is observed that by increasing the value nonlinearity α the response amplitude is reduced. A similar observation has been made by decreasing the value of α as shown in Fig. 4.17(b). Clearly, the trivial unstable range remains unchanged. From Fig. 4.7 and Fig. 4.17(a) it can be observed that by making double the nonlinearity α , the response amplitude is decreased around 29% and in Fig. 4.17(b) reducing the nonlinearity α to half, the response amplitude is increased by 29% as compared to Fig. 4.7.

4.4.1.2.1 Application of the developed results

To illustrate the application of the above frequency response plots for principal parametric resonance condition, Table 4.2 shows how for different system parameters one can obtain the same response amplitude. Some of the cases are passive where one may change the artificial muscle (change of parameter c_1 , c_2 , c_3 , α as shown in Table 4.2) itself and in other cases, one may achieve the same response actively by changing the amplitude and frequency of the air pressure.

Table 4.2 System parameters to achieve an amplitude of $a = 0.05$.

Case	μ	P_0	P_m	c_1	c_2	c_3	α	σ	Ref. Figure No.
1	0.01	58	300	-254.25	1.96	-0.003	150	-0.272	Fig. 4.7
2	0.006	58	300	-254.25	1.96	-0.003	150	-0.293	Fig. 4.9(a)
3	0.001	58	300	-254.25	1.96	-0.003	150	-0.302	Fig. 4.9(b)
4	0.01	116	300	-254.25	1.96	-0.003	150	-1.049	Fig. 4.10(a)
5	0.01	29	300	-254.25	1.96	-0.003	150	0.090	Fig. 4.10(b)
6	0.01	58	400	-254.25	1.96	-0.003	150	-1.630	Fig. 4.11(a)
7	0.01	58	200	-254.25	1.96	-0.003	150	-5.320	Fig. 4.11(b)
8	0.01	58	300	-117.12	1.96	-0.003	150	0.174	Fig. 4.12(a)
9	0.01	58	300	-468.50	1.96	-0.003	150	0.063	Fig. 4.12(b)
10	0.01	58	300	-254.25	2.96	-0.003	150	-0.580	Fig. 4.13(a)
11	0.01	58	300	-254.25	0.96	-0.003	150	-0.800	Fig. 4.13(b)
12	0.01	58	300	-254.25	1.96	-0.001	150	-1.190	Fig. 4.14(a)
13	0.01	58	300	-254.25	1.96	-0.009	150	-1.790	Fig. 4.14(b)
14	0.01	58	300	-254.25	1.96	-0.003	300	0.012	Fig. 4.17(a)
15	0.01	58	300	-254.25	1.96	-0.003	75	-0.414	Fig. 4.17(b)

For example to obtain response amplitude $a = 0.05$, one may change the damping (μ) like in cases 1, 2 and 3 in Table 4.2 and frequency or the detuning parameter σ . To achieve the same value of a , one may change the air pressure P_0 (Cases 1, 4 and 5) or P_m (Cases 1, 6 and 7) actively from Table 4.2. It has been observed with numerical simulations in Fig. 4.7, Fig. 4.15 and Fig. 4.16 that the effect of d_1 and d_2 are not prominent and hence they have not been reported in this section. Hence, using these parameters for a particular system, one may use inverse methods or soft computing techniques to arrive the system control parameters to effectively move the artificial muscle to the required position.

Table 4.3 Typical response amplitude with system parameters while sweeping down the frequency.

Case	a	μ	P_0	P_m	c_1	c_2	c_3	α	σ	Ref. Figure No.
1	0.12	0.01	58	300	-254.25	1.96	-0.003	150	1.096	Fig. 4.7
2	0.12	0.006	58	300	-254.25	1.96	-0.003	150	1.059	Fig. 4.9(a)
3	0.12	0.001	58	300	-254.25	1.96	-0.003	150	1.035	Fig. 4.9(b)
4	0.17	0.01	116	300	-254.25	1.96	-0.003	150	1.834	Fig. 4.10(a)
5	0.1	0.01	29	300	-254.25	1.96	-0.003	150	1.023	Fig. 4.10(b)
6	0.23	0.01	58	400	-254.25	1.96	-0.003	150	4.060	Fig. 4.11(a)
7	0.36	0.01	58	200	-254.25	1.96	-0.003	150	7.950	Fig. 4.11(b)
8	0.07	0.01	58	300	-117.12	1.96	-0.003	150	0.493	Fig. 4.12(a)
9	0.12	0.01	58	300	-468.50	1.96	-0.003	150	1.566	Fig. 4.12(b)
10	0.15	0.01	58	300	-254.25	2.96	-0.003	150	1.700	Fig. 4.13(a)
11	0.15	0.01	58	300	-254.25	0.96	-0.003	150	1.480	Fig. 4.13(b)
12	0.17	0.01	58	300	-254.25	1.96	-0.001	150	1.780	Fig. 4.14(a)
13	0.22	0.01	58	300	-254.25	1.96	-0.009	150	3.390	Fig. 4.14(b)
14	0.1	0.01	58	300	-254.25	1.96	-0.003	300	1.751	Fig. 4.17(a)
15	0.2	0.01	58	300	-254.25	1.96	-0.003	75	1.751	Fig. 4.17(b)

It may be noted from Figs. 4.7 to 4.17 that, to achieve the required response amplitude of the PAM one may either sweep up the frequency through the supercritical pitchfork bifurcation point or undergo a jump up phenomena though the subcritical pitchfork bifurcation point while sweeping down the frequency. Table 4.3 presents one such example where to achieve an amplitude of

$a = 0.12$, one may use 4 different system parameters as mentioned in the cases 1, 2, 3 and 9. Among these cases, the required amplitude can be obtained at lower value of detuning parameter $\sigma = 1.04$ considering the system parameters as in case 3 of the Table 4.3. Several other typical amplitudes with systems parameters are also mentioned in Table 4.3 which will be useful for the designer.

Table 4.4 Super- and sub-critical bifurcation points for the studied PAM.

Case	σ Super Critical PFB	σ Sub Critical PFB	Length of the unstable trivial state	Ref. Figure No.
1	-0.557	0.546	1.103	Fig. 4.7
2	-0.581	0.592	1.173	Fig. 4.9(a)
3	-0.575	0.574	1.149	Fig. 4.9(b)
4	-1.339	1.468	2.807	Fig. 4.10(a)
5	-0.208	0.186	0.394	Fig. 4.10(b)
6	-1.900	1.990	3.800	Fig. 4.11(a)
7	-5.600	7.060	12.660	Fig. 4.11(b)
8	-0.232	0.211	0.443	Fig. 4.12(a)
9	-0.137	0.115	0.252	Fig. 4.12(b)
10	-0.860	0.860	1.720	Fig. 4.13(a)
11	-1.080	1.080	2.160	Fig. 4.13(b)
12	-1.470	1.500	2.970	Fig. 4.14(a)
13	-2.060	2.150	4.210	Fig. 4.14(b)
14	-0.557	0.546	1.103	Fig. 4.17(a)
15	-0.557	0.546	1.103	Fig. 4.17(b)

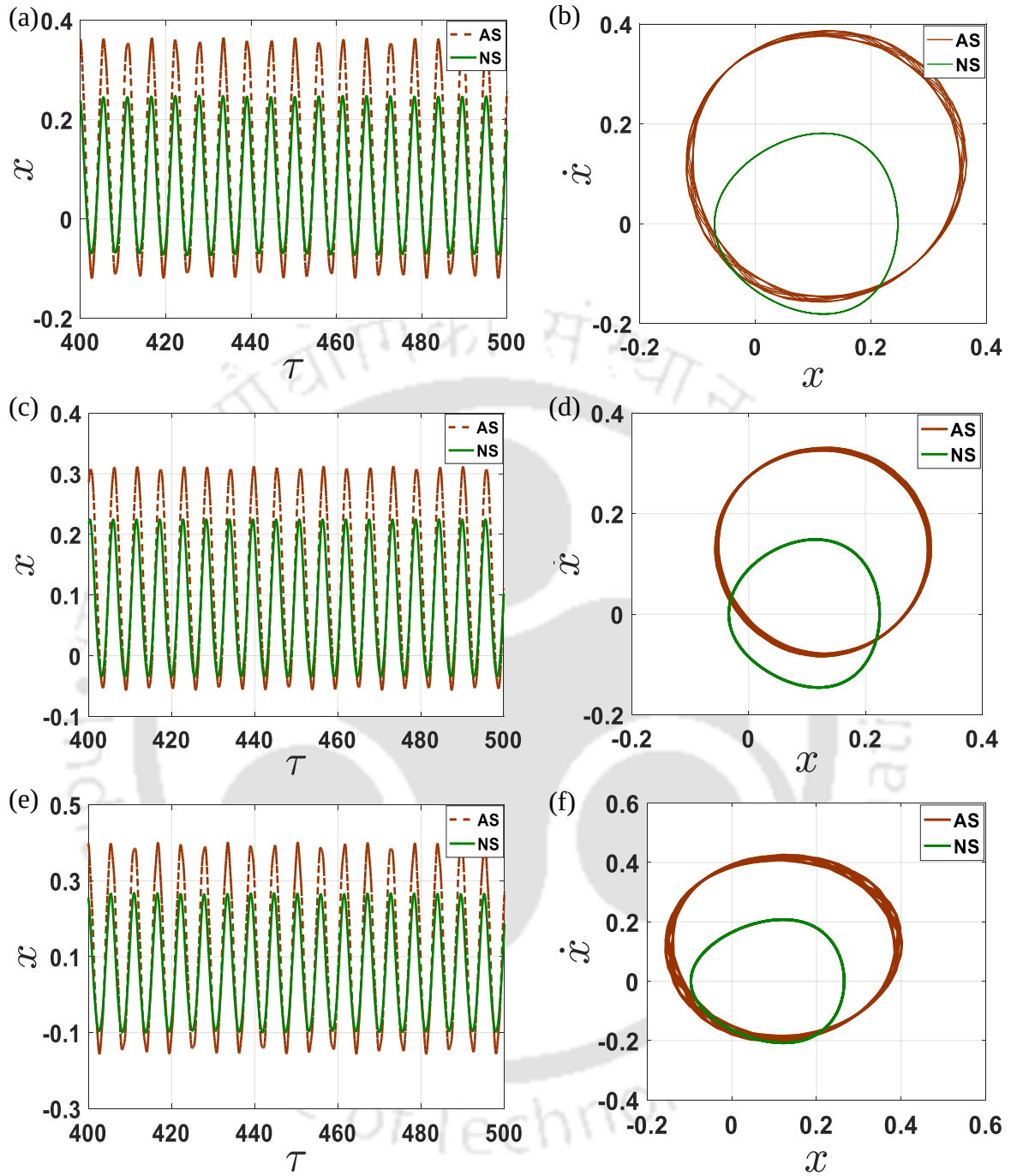
Table 4.4 gives all the supercritical and subcritical pitchfork bifurcation points (PFB) where the case 7 exhibits the longest and case 9 shows the smallest unstable trivial state. These PFBs will be useful for the designer to choose the system parameters for rehabilitation purpose while deciding the appropriate amplitude for the artificial muscle.

4.4.2 Simultaneous resonance condition

Here, simultaneous simple and principal parametric resonance condition is studied when the external excitation frequency nearly equal to the natural frequency of the system. Figure 4.18 shows the time responses found by solving the reduced equations Eq. (4.36) and Eq. (4.37) and the original equation Eq. (4.6) for various system parameters by using Runge-Kutta method. It is clearly observed that for the system with low value of nonlinearity Fig. 4.18(g, h), the results using both the methods are almost matching. In other cases the results found by using method of multiple scales gives a conservative response which can be clearly observed from Table 4.5. It may be noted that the simulated results of maximum and minimum amplitudes given in Table 4.5 are obtained from the figure mentioned in last column of the Table 4.5. Hence one may use the reduced equations Eq. (4.36) and Eq. (4.37) for the analysis of artificial muscle to obtain the time response and phase portraits. From Eq. (4.6) and Eq. (4.42), it may be noted that the mean value of the response is εf_1 which is independent of μ , P_0 and α . Hence the mean value remain almost same $x = 0.126$ in Fig. 4.18(a, c, e) and with a minor variation $x = 0.118$ in Fig. 4.18(g). From Fig. 4.18 it may be noted that with increase in μ the response amplitude decreases and with increase in the value of the dynamic pressure P_0 and nonlinear term α the response amplitude increases.

Table 4.5 Comparison of the amplitude response under simultaneous resonance condition from Fig.4.18.

Observation	System parameters			Response obtained from reduced equations (Eq. (4.36) and Eq. (4.37))		Response obtained from governing equations (Eq. (4.6))		Ref. Figure No.
	μ	P_0	α					
				Max Amp.	Min Amp.	Max Amp.	Min Amp.	
Case1	0.01	58	150	0.363	-0.111	0.245	-0.068	Fig. 4.18(a, b)
Case2	0.1	58	150	0.312	-0.053	0.224	-0.034	Fig. 4.18(c, d)
Case3	0.01	70	150	0.392	-0.146	0.266	-0.098	Fig. 4.18(e, f)
Case4	0.01	58	50	0.493	-0.265	0.389	-0.225	Fig. 4.18(g, h)



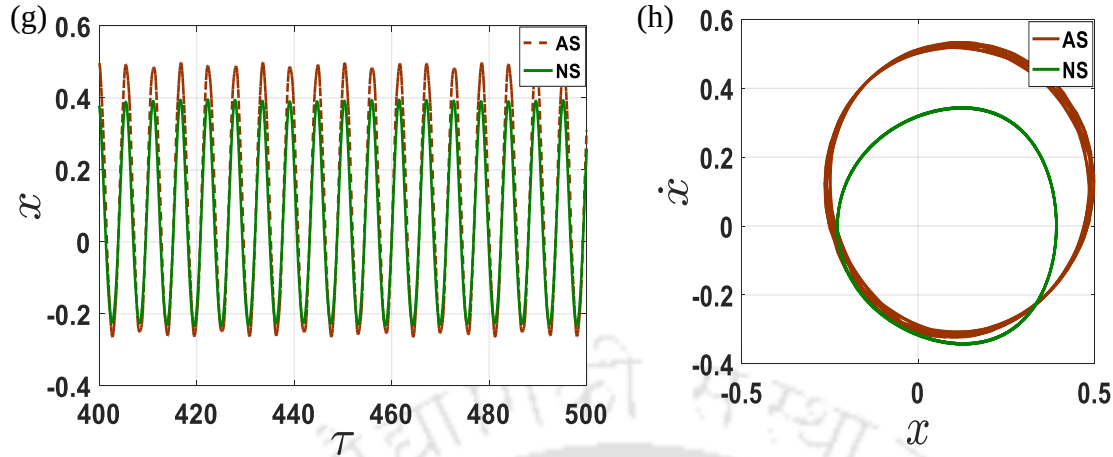
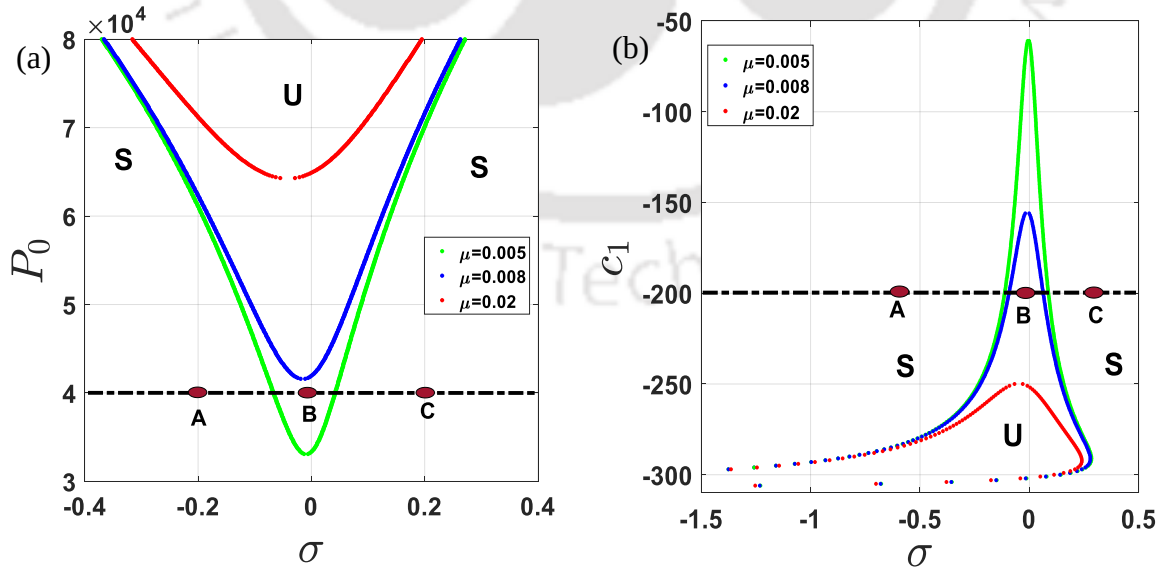


Fig. 4.18 Time response and phase portrait comparison under simultaneous resonance condition with (a, b) $\mu = 0.01$, $P_0 = 58$ kPa and $\alpha = 150$ (c, d) $\mu = 0.1$, $P_0 = 58$ kPa and $\alpha = 150$ (e, f) $\mu = 0.01$, $P_0 = 70$ kPa and $\alpha = 150$ (g, h) $\mu = 0.01$, $P_0 = 58$ kPa and $\alpha = 50$.

4.4.2.1 Trivial state instability regions

The trivial state instability regions are plotted by finding the values of system parameter for which the real part of the eigenvalues of the Jacobian matrix Eq. (4.45) for simultaneous resonance condition changes its sign. These regions have been plotted in Fig. 4.19 in terms of detuning parameter (σ) with dynamic pressure of the muscle (P_0), muscle parameter (c_1) and static pressure of the muscle (P_m) for different values of damping (μ) like in the subsection 4.4.1.1.



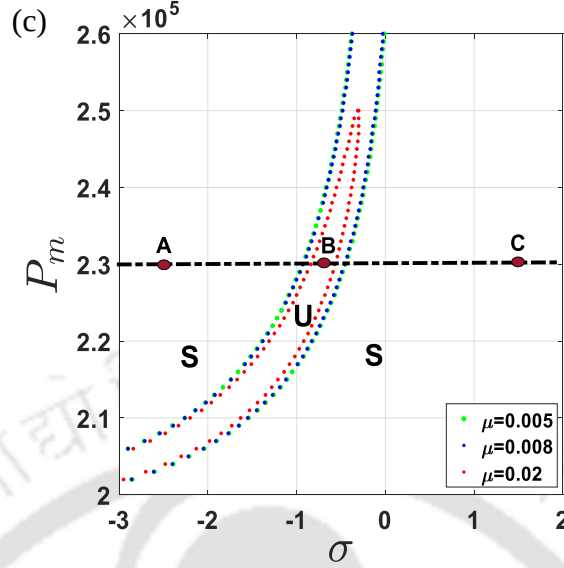


Fig. 4.19 Trivial state instability region for $\mu = 0.005, 0.008$ and 0.02 under simultaneous resonance condition with (a) $P_0 \sim \sigma$ (b) $c_1 \sim \sigma$ (c) $P_m \sim \sigma$.

As shown in Fig. 4.19(a), for $P_0 = 40$ kPa and $\mu = 0.005$, the instability region starts for detuning parameter $\sigma = -0.067$ and ends at $\sigma = 0.042$. These results have been verified by taking three points on the instability plot A ($\sigma = -0.2$), B ($\sigma = -0.02$) and C ($\sigma = 0.2$) and plotting the time responses and phase portraits in Fig. 4.20. Clearly it has been shown that the response is stable for point A and C as shown in time response Fig. 4.20(a, c) and phase portraits Fig. 4.20(b, d). For the point B taking $\alpha = 0$, the response clearly depicts to be unstable as shown in Fig. 4.20(e, f). But for the same point B, taking $\alpha = 150$, the time response and phase portraits are shown in Fig. 4.20(e, f) where one may get the nontrivial response which is bounded and stable periodic response. Like in the case of principal parametric resonance condition here also drifting of amplitude can be observed from Eq. (4.42). It may be clearly noted that there is a critical value of P_0 (P_{0cr}) below which the system always remains stable for any value of σ . For example, for $\mu = 0.005$, $P_{0cr} = 33.1$ kPa; for $\mu = 0.008$, $P_{0cr} = 41.6$ kPa and for $\mu = 0.02$, $P_{0cr} = 64.3$ kPa. Further, it may be noted that with increase in damping (μ), the unstable region decreases and occurs at a higher value of P_0 . For the artificial muscle, the system parameter should be chosen in such a way that the required deformation can be obtained with minimum computational and control effort.

Figure 4.19(b) shows the instability curve in the $c_1 \sim \sigma$ plane. Unlike the case of $P_0 \sim \sigma$, here the instability plot is inverted. Similar to the previous case the time responses and phase portraits are plotted by taking three points A ($\sigma = -0.6$), B ($\sigma = -0.02$) and C ($\sigma = 0.3$) and they are found to be in good agreement as point A (Fig. 4.21(a, b)) and C (Fig. 4.21(c, d)) are found to be stable and point B (Fig. 4.21(e, f)) is found to be unstable. But, point B with $\alpha = 150$ depicts stable response in Fig. 4.21(g, f) where one may obtain the solution in the nontrivial state. As an example, to operate the artificial muscle with $c_1 = -200$ N and $\mu = 0.008$, the detuning parameter σ should be in between $\sigma = -0.093$ and $\sigma = 0.063$ which is observed from Fig. 4.19(b). Here one may find the critical value of c_1 (c_{1cr}) for different value of damping (μ) above which the system remains always stable. For example, for $\mu = 0.005$, $c_{1cr} = -61$ N; for $\mu = 0.008$, $c_{1cr} = -156$ N and for $\mu = 0.02$, $c_{1cr} = -250$ N. With increase in damping (μ), the value of c_1 decreases and the unstable region increases.

In Fig. 4.19(c) trivial state instability regions have been plotted in $P_m \sim \sigma$ plane for three values of damping parameters. The time responses and phase portraits have been plotted for $P_m = 230$ kPa and $\mu = 0.02$ at three different points A ($\sigma = -2.5$), B ($\sigma = -0.75$) and C ($\sigma = 1.5$). While point A and C are stable, point B is found to be unstable when $\alpha = 0$. This validates the instability region in Fig. 4.19(c). As an example while designing the muscle, one should operate the system within the detuning parameter $\sigma = -0.84$ and $\sigma = -0.57$ where the system has a nontrivial state (Fig. 4.22(g, h)) and unstable trivial state corresponding to $\alpha = 0$ (Fig. 4.22(e, f)). Similar to the previous two cases, the critical value of P_m (P_{mcr}) above which the system will always be stable and increase in the value of damping (μ), the value of P_m decreases and the unstable region increases. For example critical value of P_m (P_{mcr}) is 250 kPa for damping (μ) equal to 0.02.

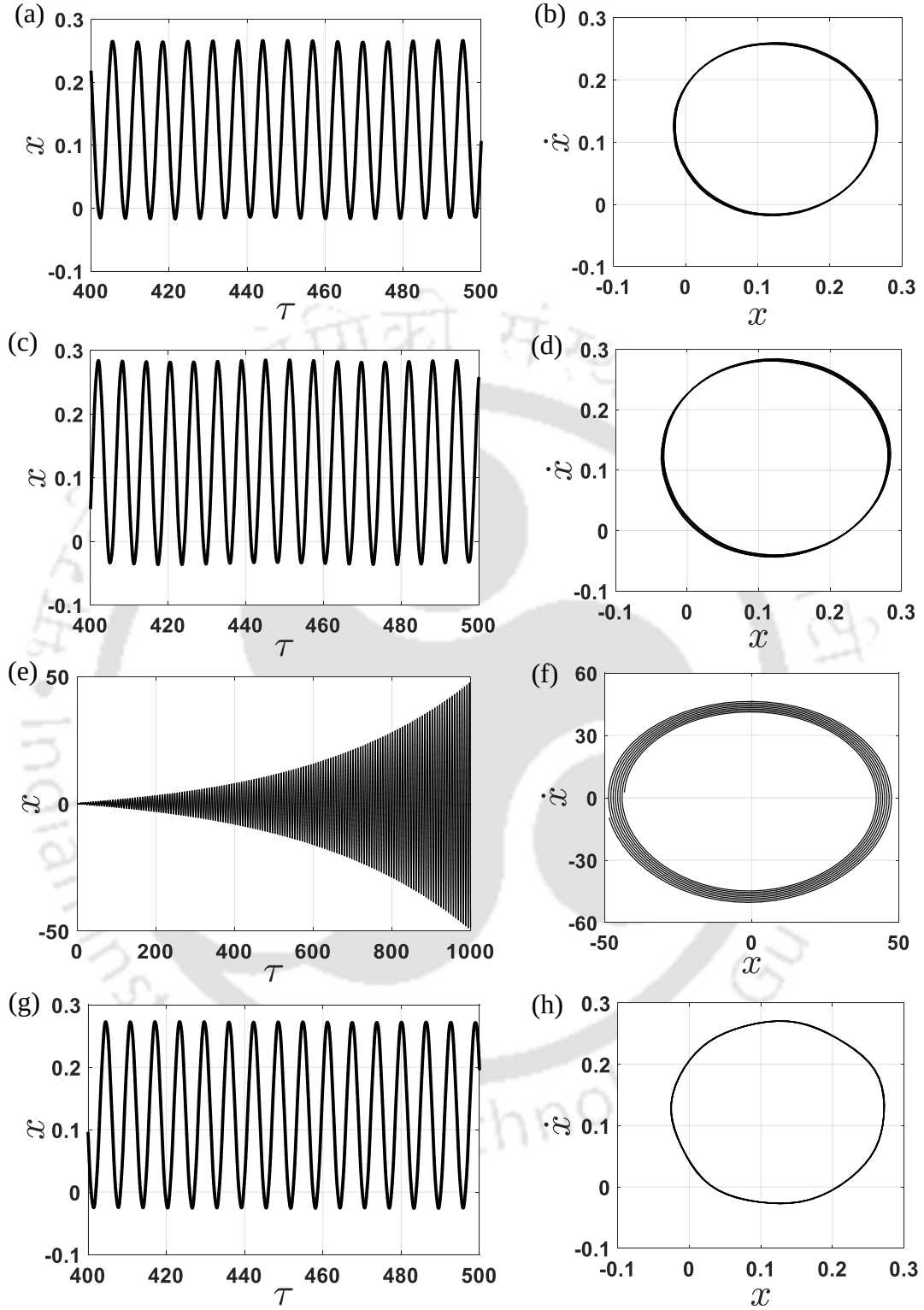


Fig. 4.20 Time responses and phase portraits for (a, b) point A (c, d) point C (e, f) point B with $\alpha = 0$ (g, h) point B with $\alpha = 150$ from Fig. 4.19(a) under simultaneous resonance condition.

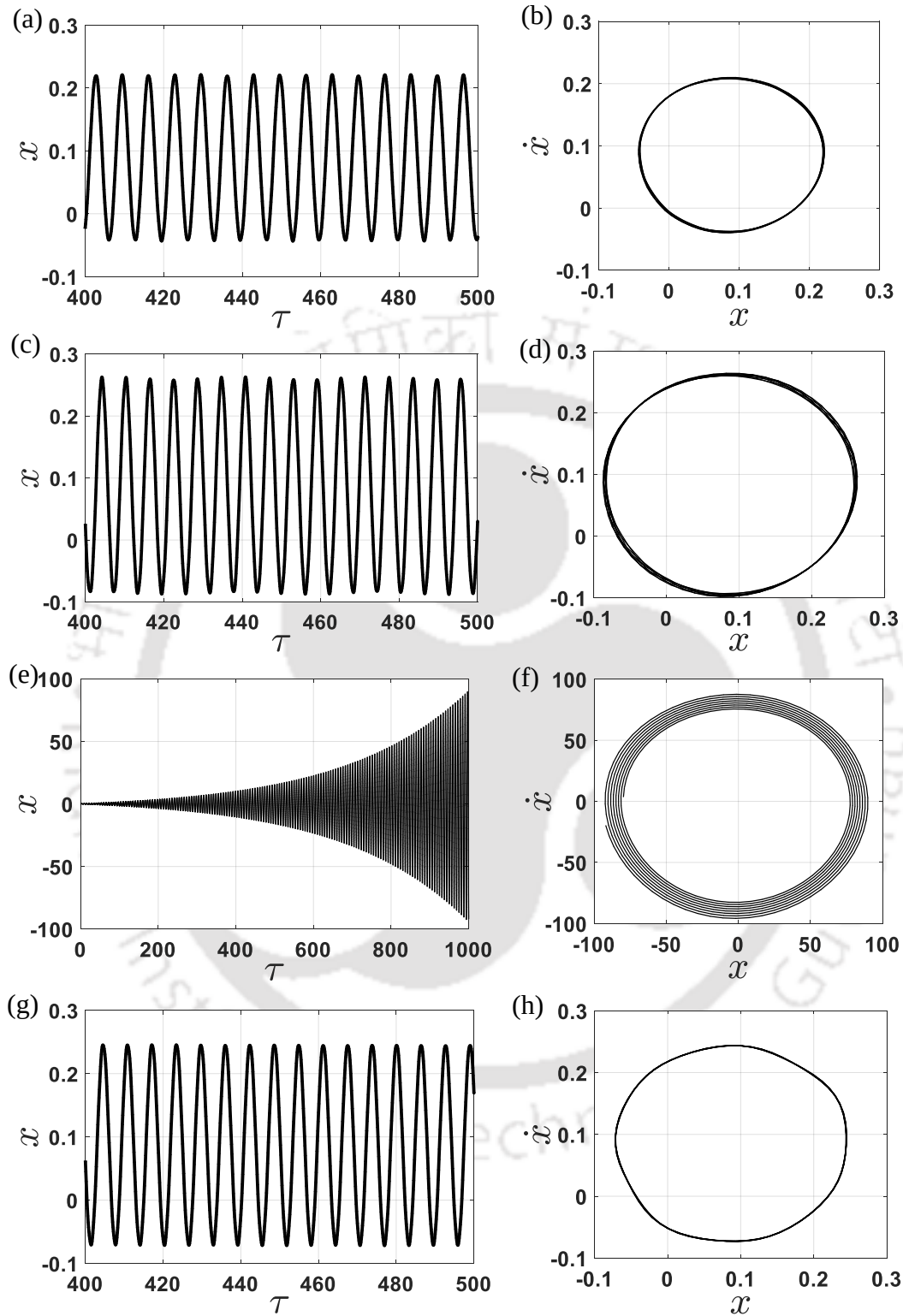


Fig. 4.21 Time responses and phase portraits for (a, b) point A (c, d) point C (e, f) point B with $\alpha = 0$ (g, h) point B with $\alpha = 150$ from Fig. 4.19(b) under simultaneous resonance condition.

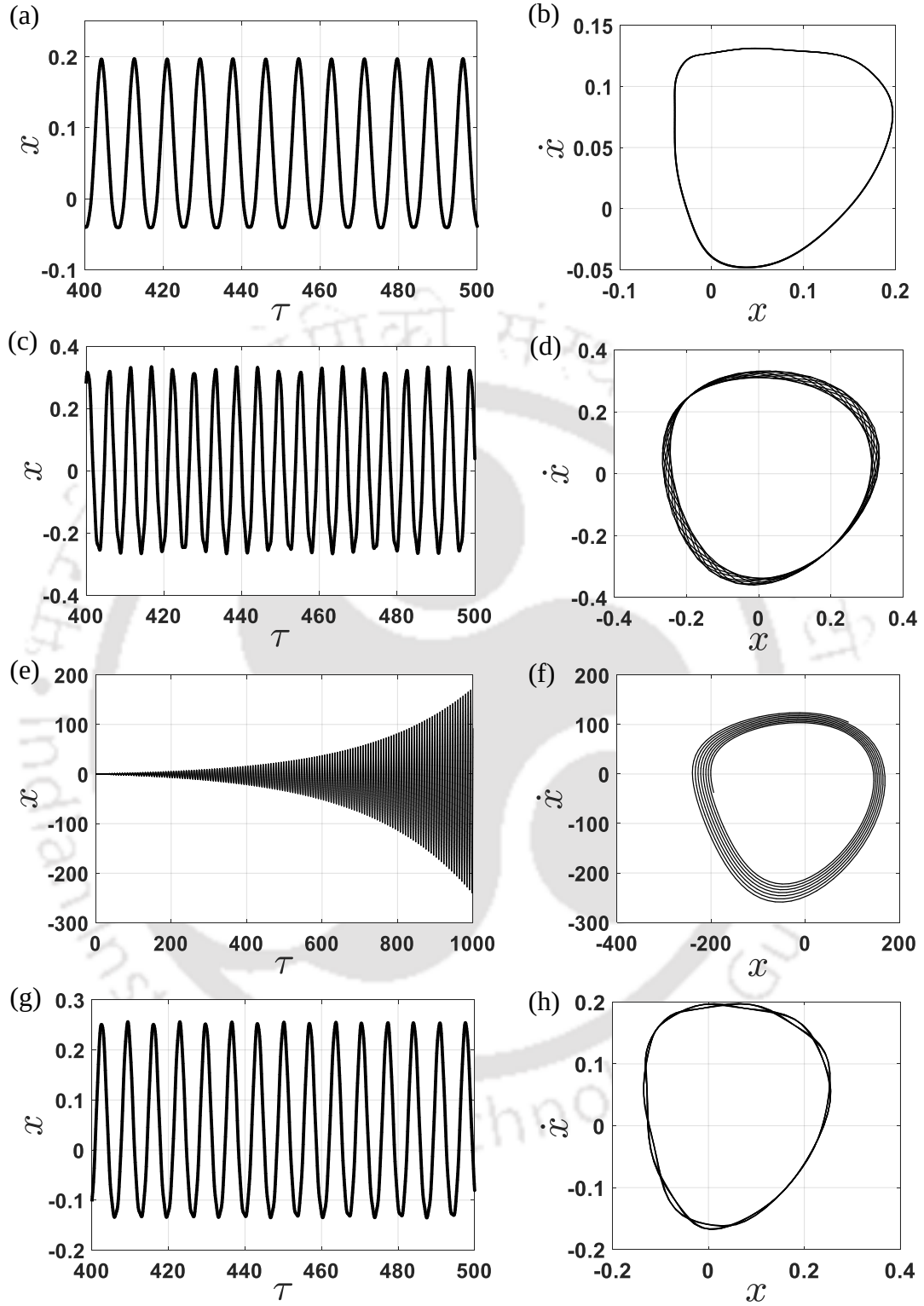


Fig. 4.22 Time responses and phase portraits for (a, b) point A (c, d) point C (e, f) point B with $\alpha = 0$ (g, h) point B with $\alpha = 150$ from Fig. 4.19(c) under simultaneous resonance condition.

4.4.2.2 Frequency response

In the case of simultaneous resonance condition, the system undergoes both principal parametric resonance and simple resonance conditions simultaneously when the external excitation frequency is nearly equal to the natural frequency of the system. The frequency responses (Fig. 4.23 - 4.35) of the muscle is plotted by solving the set of equations Eq. (4.36) and Eq. (4.37) using Newton's method. For these frequency response plots, the system parameters are considered as given in Table 4.1. The trivial state becomes unstable between the supercritical (marked point A) and subcritical (marked point B) Hopf bifurcation points as shown in Fig. 4.23.

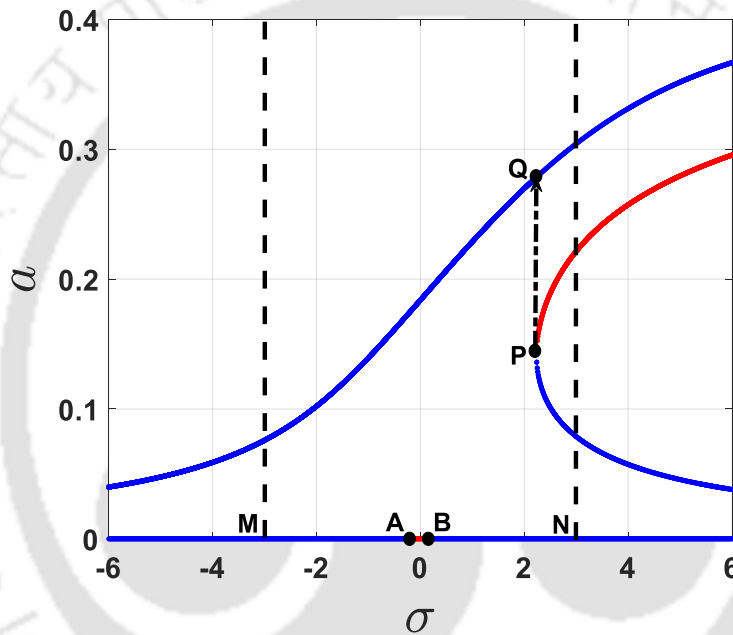


Fig. 4.23 Frequency response of the parametrically excited system with simultaneous resonance condition as the parameter values in Table 5.2.

It may clearly be observed that between the points A and B as the trivial state is unstable and there is stable nontrivial state, the muscle will contract and a bulging effect will take place. It may be noted that upto point A and between point B and P the system has bi-stable state, between point A and B the system has single stable state and after point P it has tri-stable state. So, depending on the initial conditions the system will achieve any of the stable state. For example, by swipping down the frequency at point P the system will experience a jump-up phenomenon where the unstable solution point P ($\sigma = 2.24$) has a tendency to jump up to the stable solution point Q as shown in Fig. 4.23. Hence, depending on the requirement of displacement of the muscle, one should chose proper system parameters to actively operate the system by controlling the air

pressure (P_0 and P_m) and detuning parameter (σ). Passively one may achieve the same effect by changing the muscle with suitable value of c_1 , c_2 , c_3 , d_1 , d_2 and α . To verify the frequency response plot, the basin of attraction are plotted in Fig. 4.24.

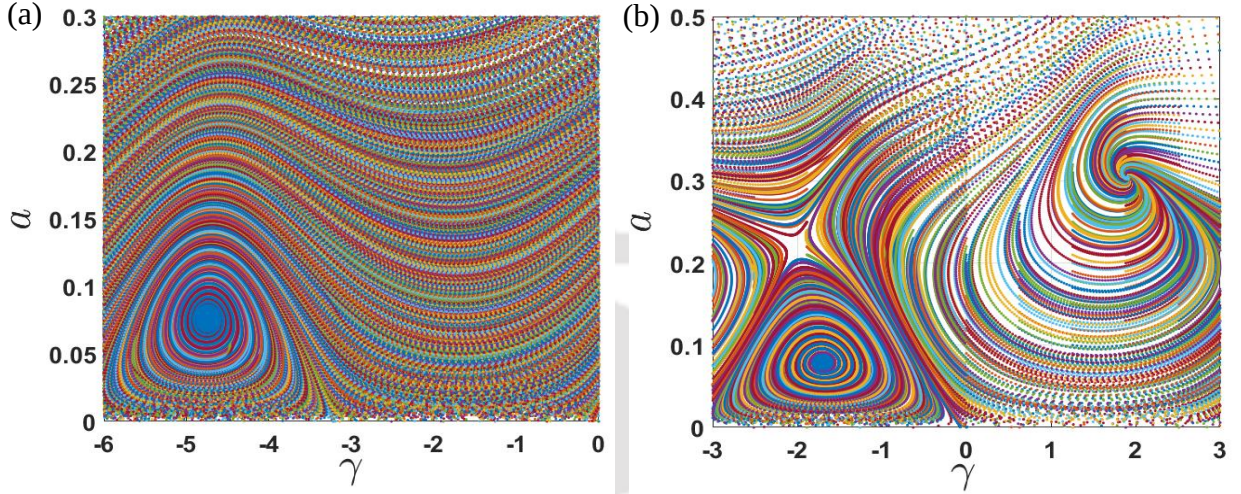


Fig. 4.24 Basin of attraction: (a) $\sigma = -3$ and (b) $\sigma = 3$ from Fig. 4.23.

In Fig. 4.24(a), the basin of attraction has been plotted in the $a \sim \gamma$ plane for the system with detuning parameter, $\sigma = -3$ which corresponds to the point M as marked in Fig. 4.23. Clearly it depicts that the system has a stable solution with amplitude $a = 0.076$. For the point N marked in Fig. 4.23, the basin of attraction is shown in Fig. 4.24(b) which clearly shows that the system has bi-stable nontrivial state with two stable and one unstable solution. So the results obtained by using the Newton's method and the numerical solution yields the same result.

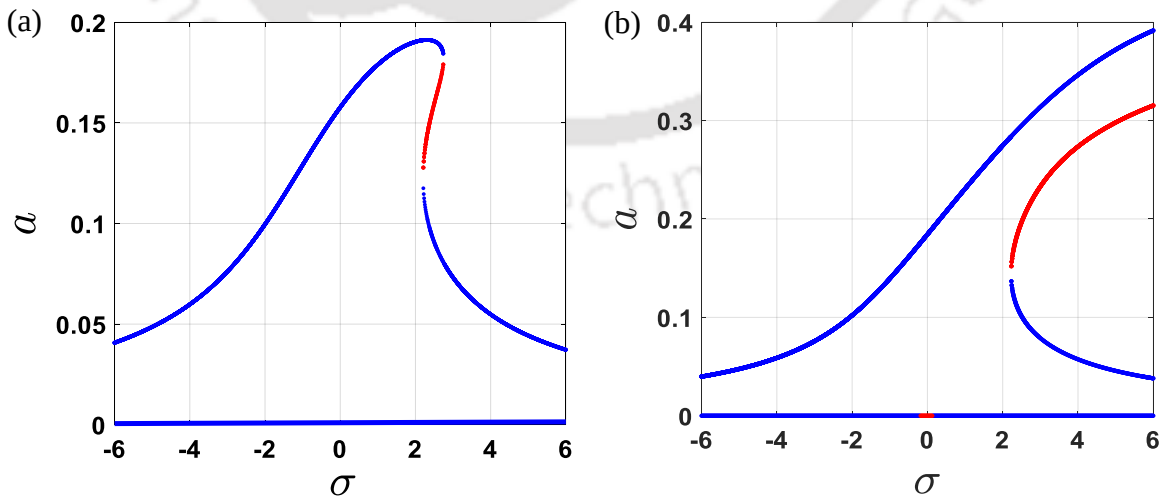


Fig. 4.25 Effect of damping (μ) on the frequency response curve of the muscle with (a) $\mu = 0.1$ and (b) $\mu = 0.001$.

From Fig. 4.25(a), with increase in the damping parameter (μ) the maximum response amplitude of the system decreases and the trivial state is completely stable. In Fig. 4.25(b), with decrease in the damping parameter (μ) the maximum response amplitude of the system has been slightly increased and the range of unstable trivial state between the supercritical and subcritical bifurcation points also increases. The trivial state results can also be validated from the trivial state instability region Fig. 4.23.

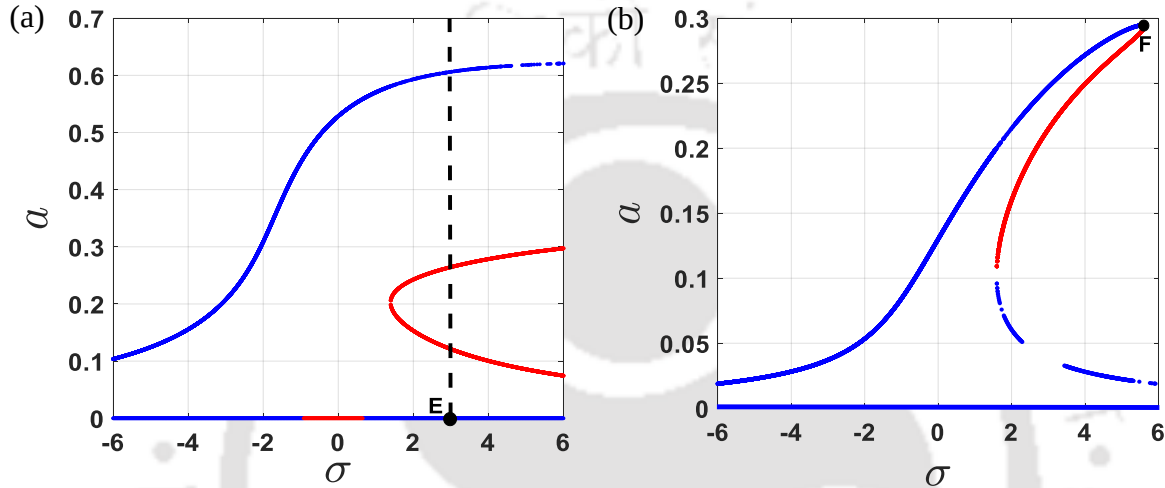


Fig. 4.26 Effect of dynamic pressure (P_0) of the muscle on the frequency response curve with
(a) $P_0 = 116$ kPa (b) $P_0 = 29$ kPa.

Figure 4.26 shows the frequency response curves for the two different values of dynamic pressure of the muscle (P_0). It is noticed that the maximum response amplitude of the system increases with increase in P_0 and the trivial state instability region between the super- and sub-critical Hopf bifurcation points also increases. But with the decrease in the value P_0 (Fig. 4.26(b)), the maximum response amplitude is decreased and the trivial state is stable. From Fig. 4.23 and Fig. 4.26 it is observed that the response amplitude of the system is increased around 85% with doubling in the value of P_0 (i.e. increasing P_0 from 58 kPa to 116 kPa) and decreased around 18% with decrease in the value of P_0 from 58 kPa to 29 kPa for detuning parameter $\sigma = 4$. In Fig. 4.26(b), one may observe the saddle node bifurcation point at 'F' where maximum response amplitude $a = 0.294$, beyond which the system has a single stable solution in the trivial and nontrivial state each. Hence, the muscle cannot be operated at a frequency beyond this saddle node bifurcation point 'F'. Corresponding to the point 'E' as marked in Fig. 4.26(a), the time response and phase portrait are shown in Fig. 4.27 and it clearly depicts the response is aperiodic.

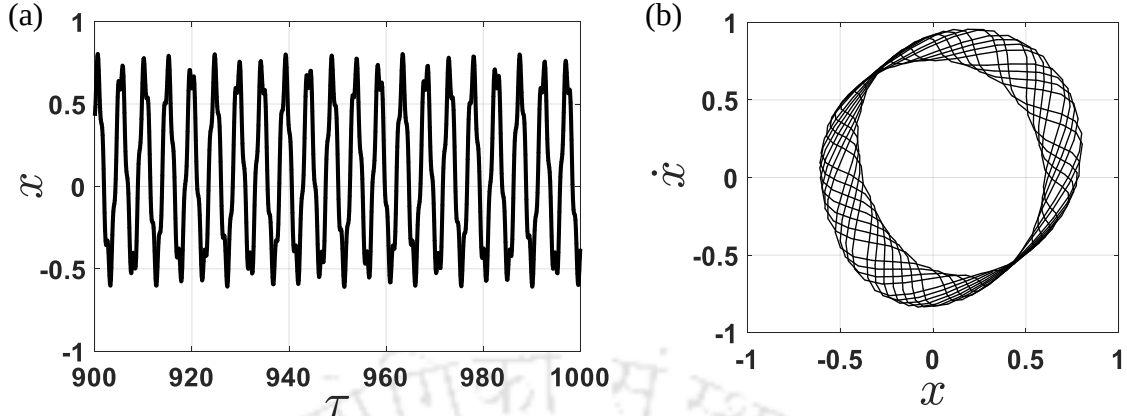


Fig. 4.27 (a) Time response and (b) phase portrait corresponding to the point 'E' in Fig. 4.26(a).

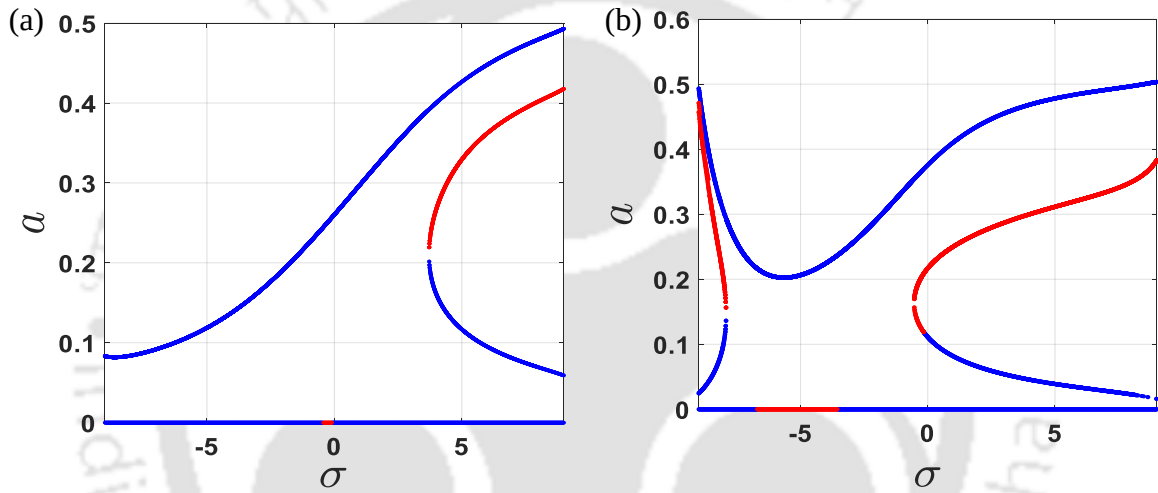


Fig. 4.28 Effect of static pressure (P_m) of the muscle on the frequency response curve with (a) $P_m = 400$ kPa (b) $P_m = 200$ kPa.

In Fig. 4.28, the frequency response curves for the two values of static pressure of the muscle P_m are shown. Here it can clearly be noted from Fig. 4.23 and Fig. 4.28 that with either increase or decrease in the value of P_m , the response amplitude increases and also the trivial state instability region between the super- and sub-critical Hopf bifurcation increases. For the same muscle displacement one may use the lower value of P_m with higher value of σ . Here the response amplitude of the system is increased by 21% in Fig. 4.28(a) and 41% in Fig. 4.28(b) at detuning parameter $\sigma = 4$ in comparison to the Fig. 4.23. For lower value of P_m (e.g., $P_m = 200$ kPa, Fig. 4.28(b)), with increase in detuning parameter σ initially the response amplitude decreases and then it increases with increase in σ . Hence one may achieve the same response amplitude for two values of σ .

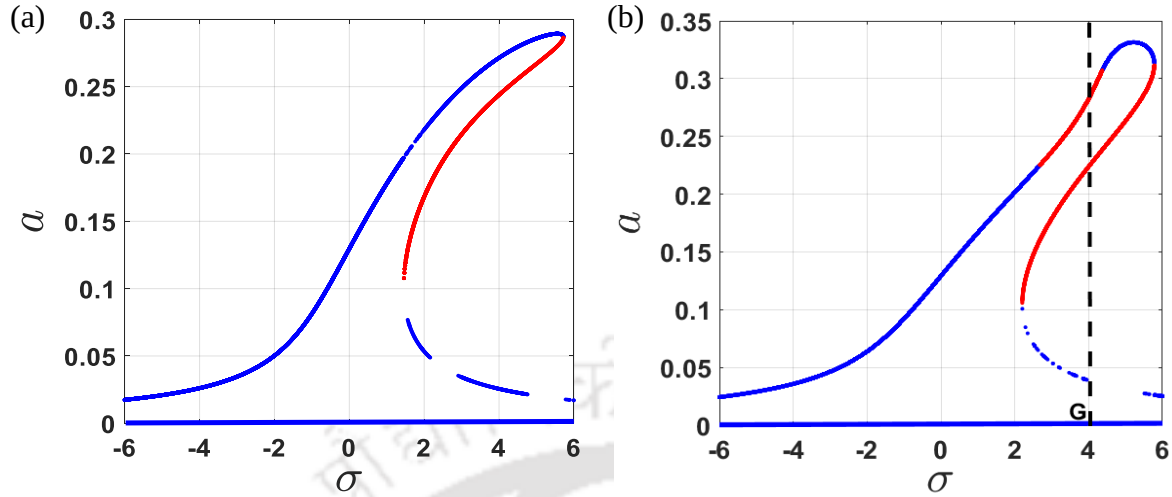


Fig. 4.29 Effect of muscle parameter (c_1) on the frequency response curve with

(a) $c_1 = -117.12$ N (b) $c_1 = -468.5$ N.

Figure 4.29 depicts the frequency response curves for the two different values muscle parameter (c_1). It is noticed that with either increase or decrease in c_1 , the maximum response amplitude of the system decreases and the trivial range remain stable. The maximum response amplitude increase by 13% by reducing the value of the parameter c_1 from -117.12 N to -468.5 N. For lower value of c_1 , the system suffers both hopf bifurcation and saddle node bifurcation in the nontrivial state. The time response and phase portrait shown in Fig. 4.30 corresponding to the point 'G' in Fig. 4.29(b) clearly depicts the presence of the supercritical Hopf bifurcation point.

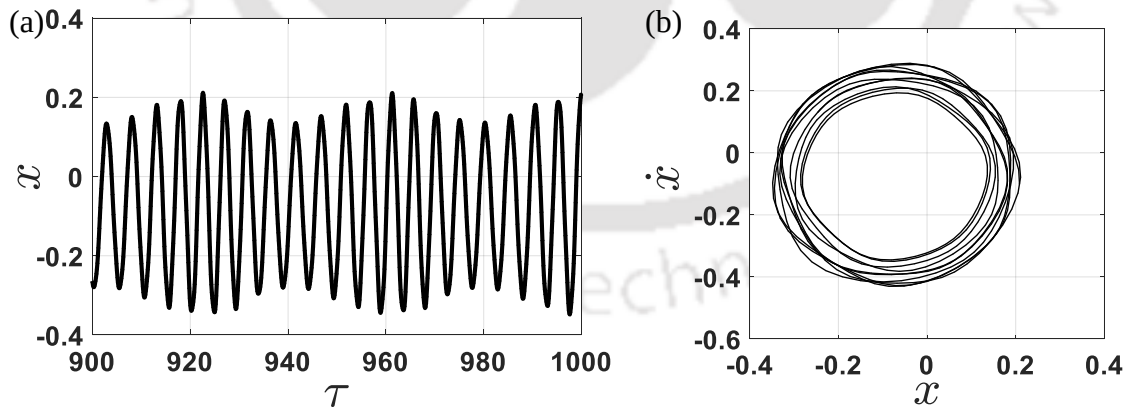


Fig. 4.30 (a) Time response and (b) phase portrait corresponding to the point 'G' in Fig. 4.29(b).

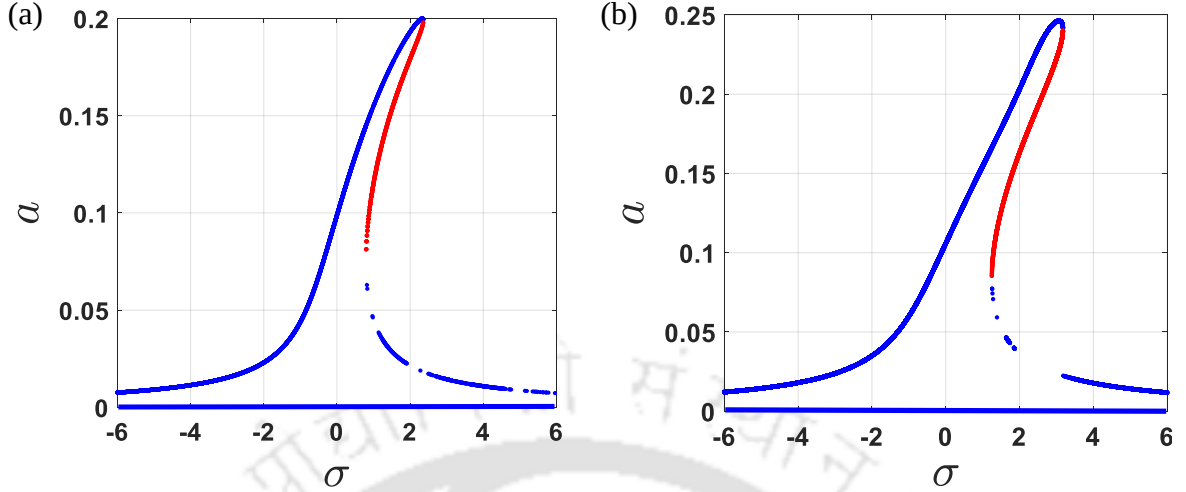


Fig. 4.31 Effect of muscle parameter (c_2) on the frequency response curve with
(a) $c_2 = 2.96$ N/kPa (b) $c_2 = 0.96$ N/kPa.

In Fig. 4.31, the frequency response curves have been plotted for the two different values muscle parameter (c_2). In comparison with Fig. 4.23, it can be observed that with either increase or decrease in c_2 , the maximum response amplitude of the system decreases and the trivial range remain stable like the previous case for muscle parameter c_1 in Fig. 4.29. The response amplitudes are decreased around 28% in Fig. 4.31(a) and 24% in Fig. 4.31(b) by increasing and decreasing the value of c_2 respectively as compared to the Fig. 4.23.

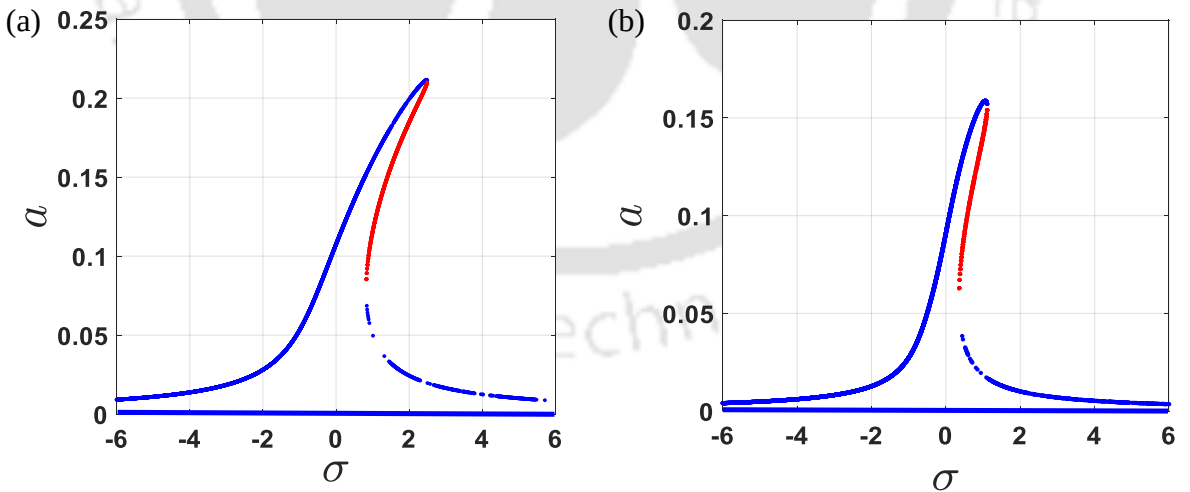


Fig. 4.32 Effect of muscle parameter (c_3) on the frequency response curve with
(a) $c_3 = -0.001$ N/kPa² (b) $c_3 = -0.009$ N/kPa².

Like the previous cases for the muscle parameters c_1 and c_2 , the response amplitude decreases in with increase or decrease in the value of another muscle parameter c_3 comparison with Fig. 4.23 and the trivial range is completely stable which can be observed from Fig. 4.32. The maximum response amplitude can be achieved $a = 0.211$ at $\sigma = 2.49$ in Fig. 4.32(a) and $a = 0.159$ at $\sigma = 1.07$ in Fig. 4.32(b) by increasing $c_3 = -0.001 \text{ N/kPa}^2$ and decreasing $c_3 = -0.009 \text{ N/kPa}^2$ respectively. Hence to design a system for a particular application to achieve a definite response amplitude one may find the optimum value of the muscle parameters c_1 , c_2 and c_3 .

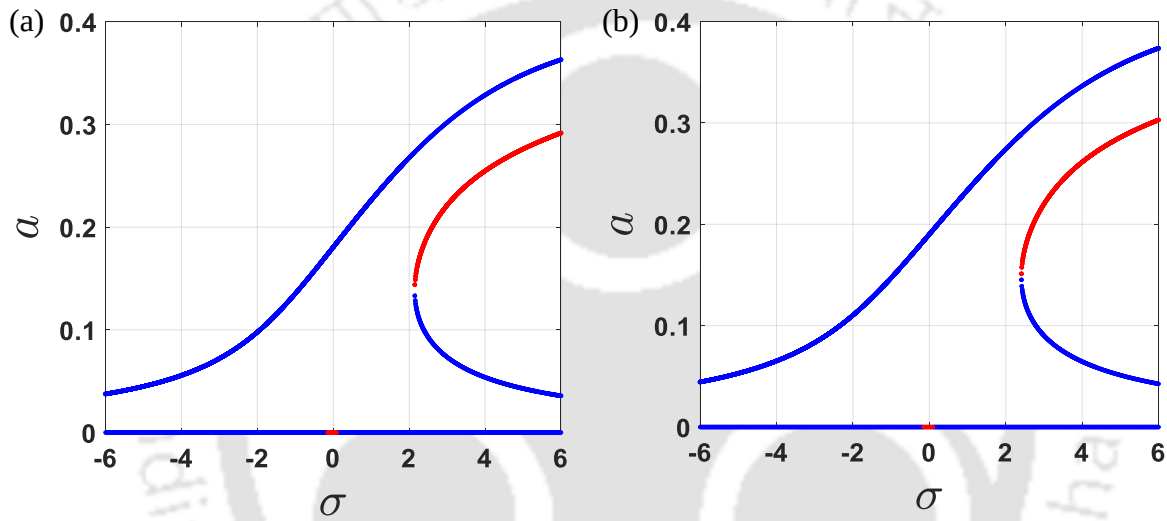


Fig. 4.33 Effect of muscle parameter (d_1) on the frequency response curve with
(a) $d_1 = -50 \text{ N}$ (b) $d_1 = -200 \text{ N}$.

In Fig. 4.33, the frequency response curves have been plotted for two different value of muscle parameter d_1 . By increasing the value of $d_1 = -50 \text{ N}$, the response amplitude decreases by around 1% and with decrease in $d_1 = -200 \text{ N}$, the response amplitude increases by around 2% as compared to the Fig. 4.23. There is no change in the trivial state for both the cases in Fig. 4.33 with Fig. 4.23.

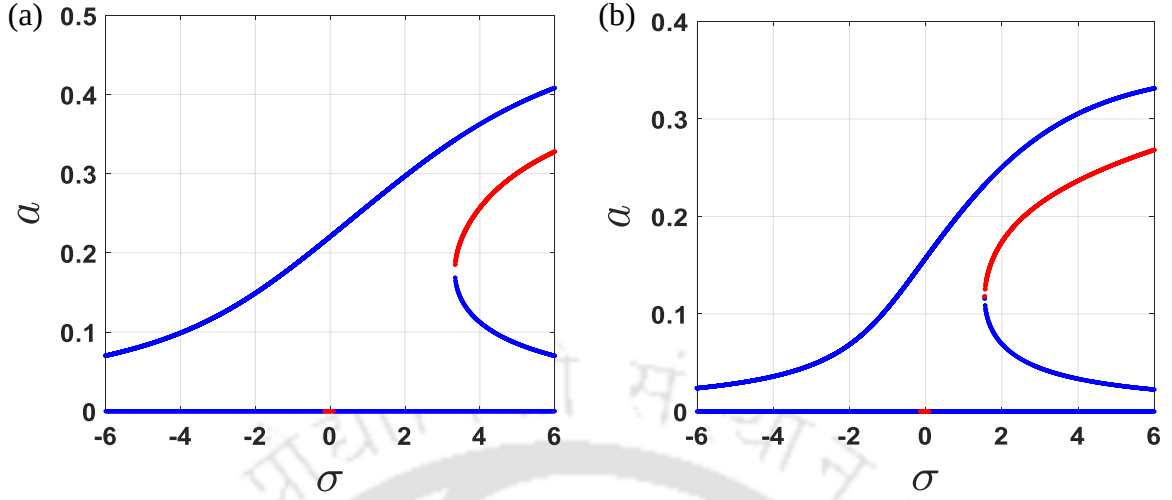


Fig. 4.34 Effect of muscle parameter (d_2) on the frequency response curve with
(a) $d_2 = 2$ N/kPa (b) $d_2 = 0.5$ N/kPa.

The frequency responses have been plotted by making double and half in Fig. 4.34(a) and Fig. 4.34(b) respectively for the muscle parameter d_2 value in Fig. 4.23. It can be observed that the response amplitude increases around 11% by doubling the value of $d_2 = 2$ N/kPa and decreases around 10% by making half of the value of $d_2 = 0.5$ N/kPa as compared to the Fig. 4.23 where $d_2 = 1$ N/kPa. Like the previous case for the muscle parameter d_1 , in the case of d_2 also the trivial state is remain unchanged. Hence, the effect of the muscle parameter d_1 and d_2 is not prominent in the trivial state which can be observed from the Fig. 4.23, Fig. 4.33 and Fig. 4.34.

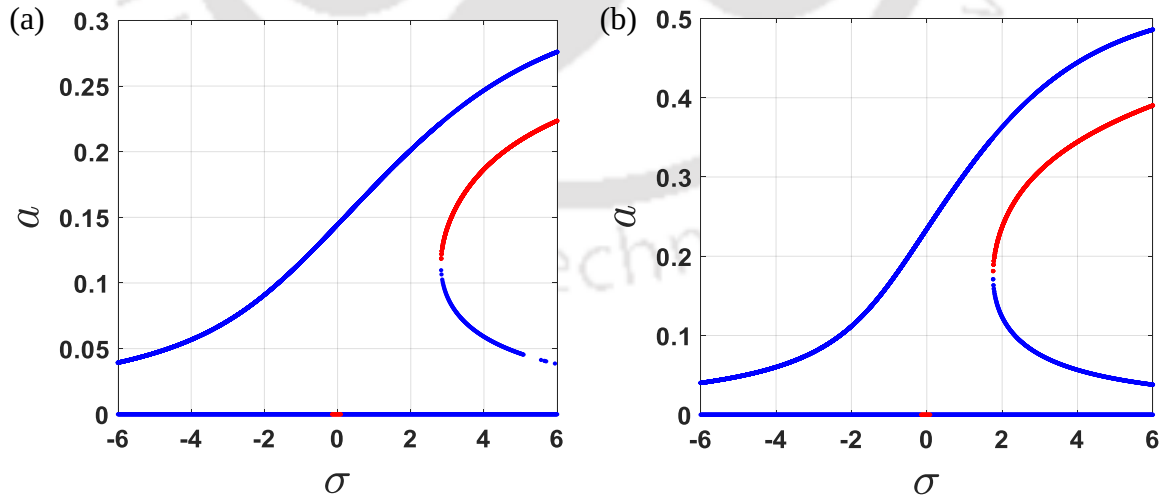


Fig. 4.35 Effect of nonlinearity (α) on the frequency response curve with (a) $\alpha = 300$ (b) $\alpha = 75$.

From Fig. 4.23 and Fig. 4.35(a), it is observed that by increasing the value nonlinearity α the response amplitude is reduced. Similar observation has been made by decreasing the value of α as shown in Fig. 4.35(b). Clearly the trivial unstable range remains unchanged. Further it is observed that by doubling the nonlinear parameter α , the response amplitude is decreased by 26% and by reducing the nonlinear parameter α to its half value, the response amplitude is increased by 34%.

4.4.2.2.1 Application of the developed results

The application of the above frequency response plots has been demonstrated in Table 4.6 to show how for different system parameters one can obtain the same response amplitude. Here is illustrated the set of system parameters viz. μ , P_0 , P_m , c_1 , c_2 , c_3 , d_1 , d_2 , α and σ to achieve a response amplitude $a = 0.15$.

Table 4.6 System parameters to achieve an amplitude of $a = 0.15$.

Case	μ	P_0	P_m	c_1	c_2	c_3	d_1	d_2	α	σ	Ref. Figure No.
1	0.01	58	300	-234.25	1.96	-0.003	-100	1	150	-0.75	Fig. 4.23
2	0.1	58	300	-234.25	1.96	-0.003	-100	1	150	-0.29	Fig. 4.25(a)
3	0.001	58	300	-234.25	1.96	-0.003	-100	1	150	-0.75	Fig. 4.25(b)
4	0.01	116	300	-234.25	1.96	-0.003	-100	1	150	-4.15	Fig. 4.26(a)
5	0.01	29	300	-234.25	1.96	-0.003	-100	1	150	0.42	Fig. 4.26(b)
6	0.01	58	400	-234.25	1.96	-0.003	-100	1	150	-3.49, 4.16	Fig. 4.28(a)
7	0.01	58	300	-117.12	1.96	-0.003	-100	1	150	0.40	Fig. 4.29(a)
8	0.01	58	300	-468.50	1.96	-0.003	-100	1	150	0.548	Fig. 4.29(b)
9	0.01	58	300	-234.25	2.96	-0.003	-100	1	150	0.90	Fig. 4.31(a)
10	0.01	58	300	-234.25	0.96	-0.003	-100	1	150	0.90	Fig. 4.31(b)
11	0.01	58	300	-234.25	1.96	-0.001	-100	1	150	0.78	Fig. 4.32(a)
12	0.01	58	300	-234.25	1.96	-0.009	-100	1	150	0.78	Fig. 4.32(b)
13	0.01	58	300	-234.25	1.96	-0.003	-50	1	150	-0.66	Fig. 4.33(a)
14	0.01	58	300	-234.25	1.96	-0.003	-200	1	150	-0.93	Fig. 4.33(b)
15	0.01	58	300	-234.25	1.96	-0.003	-100	2	150	-1.97	Fig. 4.34(a)
16	0.01	58	300	-234.25	1.96	-0.003	-100	0.5	150	-0.14	Fig. 4.34(b)
17	0.01	58	300	-234.25	1.96	-0.003	-100	1	300	0.19	Fig. 4.35(a)
18	0.01	58	300	-234.25	1.96	-0.003	-100	1	75	-1.24, 1.81	Fig. 4.35(b)

One may achieve this response either actively by changing P_0 , P_m and σ or passively by the changing the parameter μ , c_1 , c_2 , c_3 , d_1 , d_2 and α . In this example, Case 5 with low value of P_0 and P_m will be the best option for achieving the response. Hence, using these parameters for a particular application, inverse methods or soft computing techniques can be used to obtain the system parameters to effectively move the artificial muscle to the proper position.

Table 4.7 Typical response amplitude with system parameters while sweeping down the frequency.

Case	a	μ	P_0	P_m	c_1	c_2	c_3	d_1	d_2	α	σ	Ref. Figure No.
1	0.30	0.01	58	300	-234.25	1.96	-0.003	-100	1	150	2.86	Fig. 4.23
2	0.19	0.1	58	300	-234.25	1.96	-0.003	-100	1	150	2.57	Fig. 4.25(a)
3	0.30	0.001	58	300	-234.25	1.96	-0.003	-100	1	150	2.63	Fig. 4.25(b)
4	0.60	0.01	116	300	-234.25	1.96	-0.003	-100	1	150	2.46	Fig. 4.26(a)
5	0.20	0.01	29	300	-234.25	1.96	-0.003	-100	1	150	1.59	Fig. 4.26(b)
6	0.40	0.01	58	400	-234.25	1.96	-0.003	-100	1	150	3.99	Fig. 4.28(a)
7	0.50	0.01	58	200	-234.25	1.96	-0.003	-100	1	150	8.50	Fig. 4.28(b)
8	0.20	0.01	58	300	-117.12	1.96	-0.003	-100	1	150	1.52	Fig. 4.29(a)
9	0.20	0.01	58	300	-468.50	1.96	-0.003	-100	1	150	1.95	Fig. 4.29(b)
10	0.19	0.01	58	300	-234.25	2.96	-0.003	-100	1	150	1.90	Fig. 4.31(a)
11	0.20	0.01	58	300	-234.25	0.96	-0.003	-100	1	150	1.94	Fig. 4.31(b)
12	0.20	0.01	58	300	-234.25	1.96	-0.001	-100	1	150	2.02	Fig. 4.32(a)
13	0.15	0.01	58	300	-234.25	1.96	-0.009	-100	1	150	0.78	Fig. 4.32(b)
14	0.30	0.01	58	300	-234.25	1.96	-0.003	-50	1	150	3.00	Fig. 4.33(a)
15	0.30	0.01	58	300	-234.25	1.96	-0.003	-200	1	150	2.73	Fig. 4.33(b)
16	0.40	0.01	58	300	-234.25	1.96	-0.003	-100	2	150	5.56	Fig. 4.34(a)
17	0.30	0.01	58	300	-234.25	1.96	-0.003	-100	0.5	150	3.73	Fig. 4.34(b)
18	0.25	0.01	58	300	-234.25	1.96	-0.003	-100	1	300	4.18	Fig. 4.35(a)
19	0.40	0.01	58	300	-234.25	1.96	-0.003	-100	1	75	2.76	Fig. 4.35(b)

Table 4.7 gives the value of detuning parameter σ corresponding to saddle node bifurcation point. To achieve an amplitude one may sweep down the frequency till the saddle node bifurcation point where it may jump up to the stable nontrivial branch. For example as shown in Fig. 4.23 to achieve an amplitude $a = 0.3$ one may use the system parameters mentioned in case 1. Along with the

case 1, amplitude $a = 0.3$ can be achieved with cases 3, 14, 15 and 17 but in case 3 shows the value of system parameters corresponding to lower value of frequency. Similarly, amplitude $a = 0.2$ can be obtained with system parameters values in cases 5, 8, 9, 11 and 12 with lower frequency in case 8. Hence one should judiciously choose the system parameters to achieve particular response of the system for a particular application.

4.5 Summary

In this chapter, the study of the nonlinear dynamics of a parametrically excited system attached with a nonlinear PAM and a mass under principal resonance condition and simultaneous simple and principal parametric resonance condition has been carried out. The governing nonlinear equation of motion of the system has been derived and solved using second order method of multiple scales. It is shown that the method of multiple scale gives a conservative result by comparing the results obtained by method of multiple scale and by solving numerically the original nonlinear equation of motion. The instability regions have been plotted in different planes which will be useful for choosing the appropriate system parameters. The nonlinear response of the system is very complex and one should use the appropriate system parameters to achieve the required response which have been achieved by plotting frequency responses. The frequency response diagrams with different bifurcation points will also give the idea about the amplitude and frequency to avoid catastrophic failure for a wide range of system parameters. This will help in open loop control without going for a closed loop feedback control which is expensive. Basin of attraction have been plotted to observe the physically realized solutions from initial conditions. One may also achieve the desired system response through passive or active use of system parameters under both the resonance conditions as demonstrated using different examples.

Chapter 5

Nonlinear Dynamics of a Pneumatic Artificial Muscle under Hard Excitation

5.1 Introduction

In this chapter, an attempt has been made to find the nonlinear response of a system consisting of a similar system as discussed in chapter 2, which is subjected to a hard excitation. Here, the pneumatic artificial muscle (PAM) and an external spring are attached to a mass which is subjected to a harmonic force with high amplitude. The nonlinear governing equation of motion is derived and then it is solved by using second order method of multiple scales (MMS) version II as discussed in section 5.2 and 5.3 respectively. Unlike in chapter 2 where only the simple resonance condition is considered, in the present chapter, the superharmonic and subharmonic resonance conditions are considered. In section 5.4, numerical results have been discussed using the time and frequency responses, phase portraits and basin of attraction. In this section, the stability and bifurcations analyses have been carried out for different system parameters. Finally, in section 5.5, a summary of this work is presented.

5.2 Mathematical Modeling

Figure 5.1 shows the schematic diagram of the system consisting of a PAM in parallel with a spring and damper. These parts are attached to a mass (m) which is subjected to an external force $F \sin \omega t$. Taking F_{mus} as the muscle force exerted by the PAM, k as the stiffness of the spring

and c as the coefficient of viscous damping, the following governing equation of motion has been derived using Newton's 2nd law.

$$m\ddot{u} + c\dot{u} + ku + F_{mus} = F \sin \omega t \quad (5.1)$$

Here, overdot denotes the differentiation with respect to time t and u is the displacement with respect to the static equilibrium position of the system.

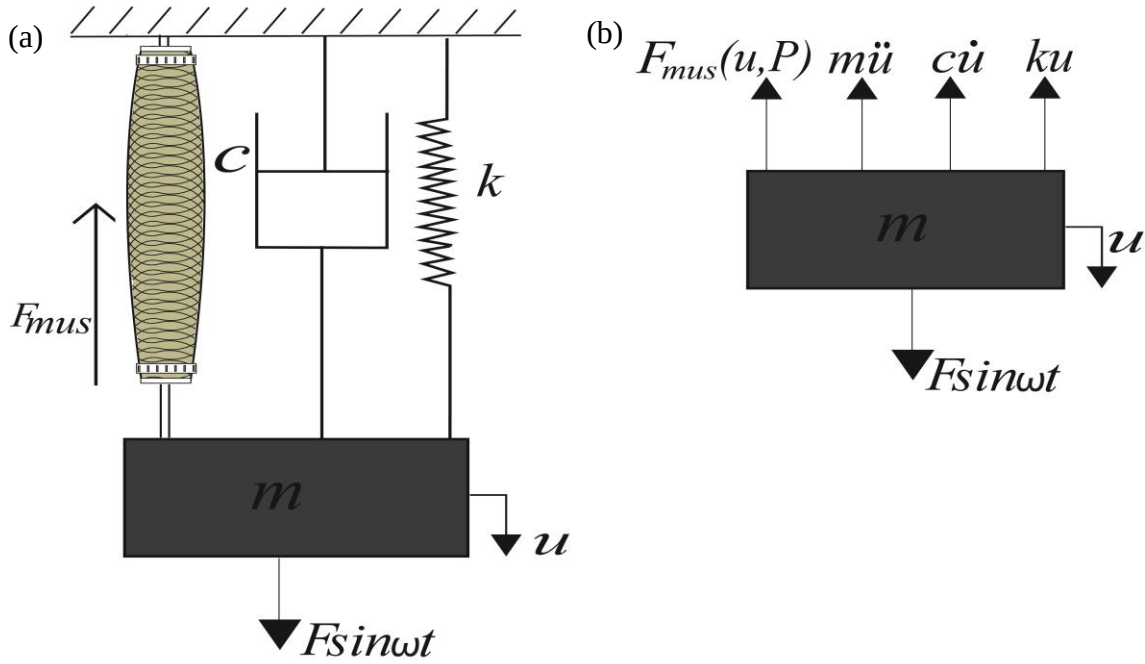


Fig. 5.1 System consisting PAM (a) equivalent spring-mass-damper system, (b) free-body diagram.

The considered muscle force F_{mus} is given by the following equation.

$$F_{mus}(u, P) = (c_1 + c_2 P + c_3 P^2) \left(\frac{u}{l_{max}} \right) + Ku^3 \quad (5.2)$$

where c_1 , c_2 , c_3 and K are the experimental constants and l_{max} is the maximum possible length that can be attained by the muscle. P is the operating pressure in the muscle to actuate. In this work, the muscle force F_{mus} is similar to that considered by the experimental work of Li et al. [125] where the muscle force was considered to be linear. But in the present work considering large nonlinear displacement a cubic term (Ku^3) is included. Substituting Eq. (5.2) in Eq. (5.1), one may find the following equation.

$$\ddot{u} + \frac{c}{m} \dot{u} + \left[\frac{k}{m} + \frac{(c_1 + c_2 P + c_3 P^2)}{ml_{\max}} \right] u + \frac{K}{m} u^3 = \frac{F}{m} \sin \omega t \quad (5.3)$$

For further analysis, a non-dimensional time $\tau = \omega_0 t$ and displacement $u = r x$ is considered. Here, ω_0 is the fundamental natural frequency and r is the scaling factor. Also, the following nondimensional terms are considered in this analysis.

$$\omega_0 = \sqrt{\frac{k}{m} + \frac{(c_1 + c_2 P + c_3 P^2)}{ml_{\max}}} \quad (5.4)$$

$$\Omega = \frac{\omega}{\omega_0}, \quad \mu = \frac{c}{2\varepsilon m \omega_0}, \quad \alpha = \frac{r^2 K}{\varepsilon m \omega_0^2}, \quad f = \frac{F}{mr \omega_0^2} \quad (5.5)$$

Now, from Eq. (5.3), Eq. (5.4) and Eq. (5.5) the following nonlinear temporal equation of motion is obtained.

$$\frac{d^2 x}{d\tau^2} + 2\varepsilon\mu \frac{dx}{d\tau} + x + \varepsilon\alpha x^3 = f \sin \Omega \tau \quad (5.6)$$

Here the book keeping parameter ε is less than 1 and μ is the non-dimensional damping parameter. The forcing parameter f is independent of operating pressure P and can be changed independently. This equation is similar to that of a Duffing equation with cubic nonlinearity and hard excitation. In most of the literature the forcing is considered to be weak, i.e., the magnitude of the forcing term is very less than that of the coefficient of the linear term. Here, an attempt has been made to study the hard excitation where the forcing amplitude is higher than the coefficient of the linear term. Though the first order solutions of similar systems are available in the book of Nayfeh and Mook [13] for other applications, here an attempt has been made to find the higher order solution using the MMS version II proposed by Rahman and Burtan [240]. In the next section, the approximate solution of Eq. (5.6) is discussed.

5.3 Approximate Solution

Following Rahman and Burton [240] and Dwivedy and Kar [242], the approximate solution is derived by using the MMS version II. The authors proposed an improvement to the MMS with different expansion and ordering where the presence of spurious results can be neglected. Here,

the displacement x can be written using a book keeping parameter ε in terms of different time scales $T_n = \varepsilon^n \tau$, $n = 0, 1, 2$ as follows.

$$x(\tau; \varepsilon) = x_0(T_0, T_1, T_2) + \varepsilon x_1(T_0, T_1, T_2) + \varepsilon^2 x_2(T_0, T_1, T_2) \quad (5.7)$$

$$\text{Taking, } \frac{d}{d\tau} = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 \text{ and } \frac{d^2}{d\tau^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 (D_1^2 + 2D_0 D_2) \quad (5.8)$$

where $D_0 = \frac{\partial}{\partial T_0}$, $D_1 = \frac{\partial}{\partial T_1}$ and $D_2 = \frac{\partial}{\partial T_2}$ and substituting Eq. (5.7) and Eq. (5.8) into Eq. (5.6),

the following expression can be obtained.

$$\begin{aligned} D_0^2 x_0 + \varepsilon D_0^2 x_1 + \varepsilon^2 D_0^2 x_2 + 2\varepsilon D_0 D_1 x_0 + 2\varepsilon^2 D_0 D_1 x_1 + \varepsilon^2 x_0 (D_1^2 + 2D_0 D_2) \\ + 2\varepsilon \mu D_0 x_0 + 2\varepsilon^2 \mu D_0 x_1 + 2\varepsilon^2 \mu D_1 x_0 + x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \varepsilon \alpha x_0^3 + \varepsilon^2 3\alpha x_1 x_0^2 = f \sin \Omega \tau \end{aligned} \quad (5.9)$$

Now, equating the coefficient of ε^0 , ε^1 and ε^2 from Eq. (5.9), the following equations are found respectively.

$$D_0^2 x_0 + x_0 = f \sin \Omega T_0 \quad (5.10)$$

$$D_0^2 x_1 + x_1 = -2D_0 D_1 x_0 - 2\mu D_0 x_0 - \alpha x_0^3 \quad (5.11)$$

$$D_0^2 x_2 + x_2 = -2D_0 D_1 x_1 - x_0 (D_1^2 + 2D_0 D_2) - 2\mu D_0 x_1 - 2\mu D_1 x_0 - 3\alpha x_1 x_0^2 \quad (5.12)$$

The general solution of Eq. (5.10) can be expressed as follows.

$$x_0 = A e^{iT_0} + \Lambda e^{i\Omega T_0} + \bar{A} e^{-iT_0} + \Lambda e^{-i\Omega T_0} \quad (5.13)$$

$$\text{where, } \Lambda = \frac{f}{2(1 - \Omega^2)} \quad (5.14)$$

A is a complex number and $\bar{A}$ is the complex conjugate of A . Substituting the value of x_0 from Eq. (5.13) in Eq. (5.11) leads to the following equation.

$$\begin{aligned} D_0^2 x_1 + x_1 = -2iD_1 A e^{iT_0} - 2i\Omega D_1 \Lambda e^{i\Omega T_0} - 2i\mu A e^{iT_0} - 2i\Omega \mu \Lambda e^{i\Omega T_0} \\ - \alpha \left[A^3 e^{i3T_0} + 3\Lambda^3 e^{i\Omega T_0} + \Lambda^3 e^{i3\Omega T_0} + 3A^2 \bar{A} e^{iT_0} + 6A\Lambda^2 e^{iT_0} + 6A\bar{A}\Lambda e^{i\Omega T_0} \right. \\ \left. + 3A\Lambda^2 e^{i(1-2\Omega)T_0} + 3A\Lambda^2 e^{i(1+2\Omega)T_0} + 3A^2 \Lambda e^{i(2+\Omega)T_0} + 3A^2 \Lambda e^{i(2-\Omega)T_0} \right] + CC \end{aligned} \quad (5.15)$$

where CC stands for the complex conjugate of the preceding terms. It can be observed from the Eq. (5.15) that it contains secular or nearly secular terms when $\Omega \approx 1$, $\Omega \approx \frac{1}{3}$ or $\Omega \approx 3$. The particular solution of these terms will be unbounded and therefore should be eliminated. From Eq. (5.14) and (5.15) it may be noted that when $\Omega \approx 1$, the simple resonance condition occur and the system exhibits a response similar to that of a linear system. Hence, this resonance condition is not considered in this work. The other two conditions $\Omega \approx \frac{1}{3}$ and $\Omega \approx 3$ are considered in this work which are known as superharmonic and subharmonic resonance condition. It may be noted that the nonlinear term (i.e. $\alpha\Lambda^3 e^{3i\Omega T_0}$) will contribute to superharmonic resonance condition and the complex conjugate of the nonlinear term (i.e. $3\alpha A^2 \bar{\Lambda} e^{i(2-\Omega)T_0}$) will lead to the subharmonic resonance condition. These two resonance cases are studied in the following subsections.

5.3.1 Superharmonic resonance condition

In this case, the external excitation frequency Ω is nearly equal to one third of the natural frequency of the system. Here, one may introduce the detuning parameter σ as

$$\Omega = \frac{1}{3} + \varepsilon\sigma \quad \sigma = O(1). \quad (5.16)$$

Now, substituting Eq. (5.16) in Eq. (5.15), one may obtain the following equation.

$$\begin{aligned} D_0^2 x_1 + x_1 = & -2iD_1 A e^{iT_0} - 2i\Omega D_1 \Lambda e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} - 2i\mu A e^{iT_0} - 2i\Omega \mu \Lambda e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} \\ & - \alpha \left[A^3 e^{i3T_0} + 3\Lambda^3 e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + \Lambda^3 e^{3i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + 3A^2 \bar{A} e^{iT_0} + 6A\Lambda^2 e^{iT_0} + 6A\bar{A}\Lambda e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} \right. \\ & \left. + 3\bar{A}\Lambda^2 e^{i\left(2\left(\frac{1}{3} + \varepsilon\sigma\right) - 1\right)T_0} + 3A\Lambda^2 e^{i\left(1 + 2\left(\frac{1}{3} + \varepsilon\sigma\right)\right)T_0} + 3A^2 \bar{\Lambda} e^{i\left(2 + \left(\frac{1}{3} + \varepsilon\sigma\right)\right)T_0} + 3A^2 \Lambda e^{i\left(2 - \left(\frac{1}{3} + \varepsilon\sigma\right)\right)T_0} \right] + CC \end{aligned} \quad (5.17)$$

The secular and nearly secular terms in Eq. (5.17) which should be eliminated to have bounded solution are given below.

$$-2iD_1 A - 2i\mu A - \alpha\Lambda^3 e^{i3\varepsilon\sigma T_0} - 3\alpha A^2 \bar{A} - 6\alpha A\Lambda^2 = 0 \quad (5.18)$$

$$D_1 A = -\mu A + \frac{i}{2} \alpha \Lambda^3 e^{i3\varepsilon\sigma T_0} + \frac{3i}{2} \alpha A^2 \bar{A} + 3i\alpha A\Lambda^2 \quad (5.19)$$

Now, Eq. (5.15) can be rewritten as in the following form.

$$D_0^2 x_1 + x_1 = -2i\Omega\mu\Lambda e^{i\Omega T_0} - \alpha \left[A^3 e^{i3T_0} + 3\Lambda^3 e^{i\Omega T_0} + 6A\bar{A}\Lambda e^{i\Omega T_0} + 3A\Lambda^2 e^{i(1-2\Omega)T_0} + 3A\Lambda^2 e^{i(1+2\Omega)T_0} + 3A^2\Lambda e^{i(2+\Omega)T_0} + 3A^2\Lambda e^{i(2-\Omega)T_0} \right] + cc \quad (5.20)$$

$$x_1 = -\frac{2i\Omega\mu\Lambda e^{i\Omega T_0}}{(1-\Omega^2)} - \alpha \left[-\frac{A^3 e^{i3T_0}}{8} + \frac{3\Lambda^3 e^{i\Omega T_0}}{(1-\Omega^2)} + \frac{6A\bar{A}\Lambda e^{i\Omega T_0}}{(1-\Omega^2)} + \frac{3A\Lambda^2 e^{i(1-2\Omega)T_0}}{(1-(1-2\Omega)^2)} + \frac{3A\Lambda^2 e^{i(1+2\Omega)T_0}}{(1-(1+2\Omega)^2)} + \frac{3A^2\Lambda e^{i(2+\Omega)T_0}}{(1-(2+\Omega)^2)} + \frac{3A^2\Lambda e^{i(2-\Omega)T_0}}{(1-(2-\Omega)^2)} \right] + cc \quad (5.21)$$

Now substituting Eq. (5.21) and Eq. (5.13) in Eq. (5.12) and eliminating the secular terms the following modulation equation in complex form can be found.

$$D_2 A = 3\alpha \left[\frac{\Omega\mu\Lambda^3 e^{i3\varepsilon\sigma T_0}}{(1-\Omega^2)} - \frac{i\alpha}{2} \left\{ -\frac{A^3 \bar{A}^2}{8} + \frac{3\Lambda^5 e^{i3\varepsilon\sigma T_0}}{(1-\Omega^2)} + \frac{12A\Lambda^4}{(1-\Omega^2)} + \frac{6A\bar{A}\Lambda^3 e^{i3\varepsilon\sigma T_0}}{(1-\Omega^2)} + \frac{24A^2 \bar{A}\Lambda^2}{(1-\Omega^2)} + \frac{3A\Lambda^4}{(1-(1-2\Omega)^2)} + \frac{6A^2 \Lambda^3 e^{-i3\varepsilon\sigma T_0}}{(1-(1-2\Omega)^2)} + \frac{6A\bar{A}\Lambda^3 e^{i3\varepsilon\sigma T_0}}{(1-(1-2\Omega)^2)} + \frac{3A\Lambda^4}{(1-(1+2\Omega)^2)} + \frac{6A\bar{A}\Lambda^3 e^{i3\varepsilon\sigma T_0}}{(1-(1+2\Omega)^2)} + \frac{6A^2 \bar{A}\Lambda^2}{(1-(2+\Omega)^2)} + \frac{3A^2 \Lambda^3 e^{-i3\varepsilon\sigma T_0}}{(1-(2-\Omega)^2)} + \frac{6A^2 \bar{A}\Lambda^2}{(1-(2-\Omega)^2)} \right\} \right] \quad (5.22)$$

It may be noted that following the method of multiple scales version II the terms $D_1 A$ which appeared in the expression for $D_2 A$ are not considered. Using the method of reconstitution [239], one may write the following form.

$$A' = \varepsilon D_1 A + \varepsilon^2 D_2 A \quad (5.23)$$

$$\begin{aligned}
A' = & \varepsilon \left(-\mu A + \frac{i}{2} \alpha \Lambda^3 e^{i3\varepsilon\sigma T_0} + \frac{3i}{2} \alpha A^2 \bar{A} + 3i\alpha A \Lambda^2 \right) + 3\varepsilon^2 \alpha \left[\frac{\Omega \mu \Lambda^3 e^{i3\varepsilon\sigma T_0}}{(1-\Omega^2)} - \frac{i\alpha}{2} \left\{ -\frac{A^3 \bar{A}^2}{8} \right. \right. \\
& + \frac{3\Lambda^5 e^{i3\varepsilon\sigma T_0}}{(1-\Omega^2)} + \frac{12A\Lambda^4}{(1-\Omega^2)} + \frac{6A\bar{A}\Lambda^3 e^{i3\varepsilon\sigma T_0}}{(1-\Omega^2)} + \frac{24A^2 \bar{A}\Lambda^2}{(1-\Omega^2)} + \frac{3A\Lambda^4}{(1-(1-2\Omega)^2)} + \frac{6A^2 \Lambda^3 e^{-i3\varepsilon\sigma T_0}}{(1-(1-2\Omega)^2)} \\
& + \frac{6A\bar{A}\Lambda^3 e^{i3\varepsilon\sigma T_0}}{(1-(1-2\Omega)^2)} + \frac{3A\Lambda^4}{(1-(1+2\Omega)^2)} + \frac{6A\bar{A}\Lambda^3 e^{i3\varepsilon\sigma T_0}}{(1-(1+2\Omega)^2)} + \frac{6A^2 \bar{A}\Lambda^2}{(1-(2+\Omega)^2)} + \frac{3A^2 \Lambda^3 e^{-i3\varepsilon\sigma T_0}}{(1-(2-\Omega)^2)} \\
& \left. \left. + \frac{6A^2 \bar{A}\Lambda^2}{(1-(2-\Omega)^2)} \right\} \right] \quad (5.24)
\end{aligned}$$

Here, considering the polar form i.e., $A = \frac{1}{2} a e^{i\beta}$ where a and β represents the amplitude and phase of the response. Now, by taking $\gamma = 3\varepsilon\sigma T_0 - \beta$ in Eq. (5.24) and separating the real and imaginary parts, one may find a set of following reduced equations.

$$\begin{aligned}
a' = & \varepsilon \left(-\mu a - \alpha \Lambda^3 \sin \gamma \right) + \varepsilon^2 \left[\frac{6\alpha \Omega \mu \Lambda^3}{(1-\Omega^2)} \cos \gamma + 3\alpha^2 \left\{ \frac{3\Lambda^5}{(1-\Omega^2)} \sin \gamma + \frac{3a^2 \Lambda^3}{2(1-\Omega^2)} \sin \gamma \right. \right. \\
& \left. \left. + \frac{3a^2 \Lambda^3}{2(1-(1+2\Omega)^2)} \sin \gamma - \frac{3a^2 \Lambda^3}{4(1-(2-\Omega)^2)} \sin \gamma \right\} \right] \quad (5.25)
\end{aligned}$$

$$\begin{aligned}
a\gamma' = & 3a\varepsilon\sigma - \varepsilon \left(\alpha \Lambda^3 \cos \gamma + \frac{3}{8} \alpha a^3 + 3\alpha a \Lambda^2 \right) - \varepsilon^2 \left[\frac{6\alpha \Omega \mu \Lambda^3}{(1-\Omega^2)} \sin \gamma - 3\alpha^2 \left\{ -\frac{a^5}{256} \right. \right. \\
& + \frac{3\Lambda^5}{(1-\Omega^2)} \cos \gamma + \frac{6a\Lambda^4}{(1-\Omega^2)} + \frac{3a^2 \Lambda^3}{2(1-\Omega^2)} \cos \gamma + \frac{3a^3 \Lambda^2}{(1-\Omega^2)} + \frac{3a\Lambda^4}{2(1-(1-2\Omega)^2)} \\
& + \frac{3a^2 \Lambda^3}{(1-(1-2\Omega)^2)} \cos \gamma + \frac{3a\Lambda^4}{2(1-(1+2\Omega)^2)} + \frac{3a^2 \Lambda^3}{2(1-(1+2\Omega)^2)} \cos \gamma + \frac{3a^3 \Lambda^2}{4(1-(2+\Omega)^2)} \\
& \left. \left. + \frac{3a^2 \Lambda^3}{4(1-(2-\Omega)^2)} \cos \gamma + \frac{3a^3 \Lambda^2}{4(1-(2-\Omega)^2)} \right\} \right] \quad (5.26)
\end{aligned}$$

Thus the particular solution of x_1 from Eq. (5.20) can be written as

$$x_1 = \frac{4\Omega\mu\Lambda}{(1-\Omega^2)} \sin(\Omega T_0) - \alpha \left[-\frac{a^3}{32} \cos(9\Omega T_0 - 3\gamma) + \frac{6\Lambda^3}{(1-\Omega^2)} \cos(\Omega T_0) + \frac{3a^2\Lambda}{(1-\Omega^2)} \cos(\Omega T_0) \right. \\ \left. + \frac{3a\Lambda^2}{(1-(1-2\Omega)^2)} \cos(\Omega T_0 - \gamma) + \frac{3a\Lambda^2}{(1-(1+2\Omega)^2)} \cos(5\Omega T_0 - \gamma) \right. \\ \left. + \frac{3a^2\Lambda}{2(1-(2+\Omega)^2)} \cos(7\Omega T_0 - 2\gamma) + \frac{3a^2\Lambda}{2(1-(2-\Omega)^2)} \cos(5\Omega T_0 - 2\gamma) \right] \quad (5.27)$$

And the particular solution of x_0 from Eq. (5.13) can be written as

$$x_0 = a \cos(3\Omega T_0 - \gamma) + 2\Lambda \cos(\Omega T_0) \quad (5.28)$$

Thus substituting Eq. (5.27) and Eq. (5.28) in Eq. (5.7), the response of the system $x(\tau)$ is given below.

$$x = a \cos(3\Omega T_0 - \gamma) + 2\Lambda \cos(\Omega T_0) + \frac{4\varepsilon\Omega\mu\Lambda}{(1-\Omega^2)} \sin(\Omega T_0) - \alpha\varepsilon \left[-\frac{a^3}{32} \cos(9\Omega T_0 - 3\gamma) \right. \\ \left. + \frac{6\Lambda^3}{(1-\Omega^2)} \cos(\Omega T_0) + \frac{3a\Lambda^2}{(1-(1-2\Omega)^2)} \cos(\Omega T_0 - \gamma) + \frac{3a\Lambda^2}{(1-(1+2\Omega)^2)} \cos(5\Omega T_0 - \gamma) \right. \\ \left. + \frac{3a^2\Lambda}{(1-\Omega^2)} \cos(\Omega T_0) + \frac{3a^2\Lambda}{2(1-(2+\Omega)^2)} \cos(7\Omega T_0 - 2\gamma) + \frac{3a^2\Lambda}{2(1-(2-\Omega)^2)} \cos(5\Omega T_0 - 2\gamma) \right] \quad (5.29)$$

Eq. (5.29) provides the expression for the complete time response of the harmonically excited system attached with a nonlinear PAM under hard excitation for superharmonic resonance condition.

5.3.2 Subharmonic resonance condition

In this case, the external excitation frequency Ω is nearly equal to the thrice of the natural frequency of the system which is known to be subharmonic resonance condition. Here, one may introduce the detuning parameter σ as

$$\Omega = 3 + \varepsilon\sigma \quad \sigma = O(1). \quad (5.30)$$

Now, substituting Eq. (5.30) in Eq. (5.15), one may lead the following equation.

$$D_0^2 x_1 + x_1 = -2iD_1 A e^{iT_0} - 2i\Omega D_1 \Lambda e^{i(3+\varepsilon\sigma)T_0} - 2i\mu A e^{iT_0} - 2i\Omega\mu\Lambda e^{i(3+\varepsilon\sigma)T_0} \\ - \alpha \left[A^3 e^{i3T_0} + 3\Lambda^3 e^{i(3+\varepsilon\sigma)T_0} + \Lambda^3 e^{i3(3+\varepsilon\sigma)T_0} + 3A^2 \bar{A} e^{iT_0} + 6A\Lambda^2 e^{iT_0} + 6A\bar{A}\Lambda e^{i(3+\varepsilon\sigma)T_0} \right. \\ \left. + 3\bar{A}\Lambda^2 e^{i(2(3+\varepsilon\sigma)-1)T_0} + 3A\Lambda^2 e^{i(1+2(3+\varepsilon\sigma))T_0} + 3A^2 \Lambda e^{i(2+(3+\varepsilon\sigma))T_0} + 3A^2 \Lambda e^{i(2-(3+\varepsilon\sigma))T_0} \right] + cc \quad (5.31)$$

Again, the secular and nearly secular terms in Eq. (5.31) which should be eliminated to have bounded solution are given below.

$$-2iD_1 A - 2i\mu A - 3\alpha A^2 \bar{A} - 6\alpha A\Lambda^2 - 3\alpha \bar{A}^2 \Lambda e^{i\varepsilon\sigma T_0} = 0 \quad (5.32)$$

$$D_1 A = -\mu A + \frac{3i}{2} \alpha A^2 \bar{A} + 3i\alpha A\Lambda^2 + \frac{3i}{2} \alpha \bar{A}^2 \Lambda e^{i\varepsilon\sigma T_0} \quad (5.33)$$

Now, Eq. (5.15) can be rewritten as in the following form.

$$D_0^2 x_1 + x_1 = -2i\Omega\mu\Lambda e^{i\Omega T_0} - \alpha \left[A^3 e^{i3T_0} + 3\Lambda^3 e^{i\Omega T_0} + \Lambda^3 e^{i3\Omega T_0} + 6A\bar{A}\Lambda e^{i\Omega T_0} + 3A\Lambda^2 e^{i(1-2\Omega)T_0} \right. \\ \left. + 3A\Lambda^2 e^{i(1+2\Omega)T_0} + 3A^2 \Lambda e^{i(2+\Omega)T_0} \right] + cc \quad (5.34)$$

$$x_1 = -\frac{2i\Omega\mu\Lambda e^{i\Omega T_0}}{(1-\Omega^2)} - \alpha \left[-\frac{A^3 e^{i3T_0}}{8} + \frac{3\Lambda^3 e^{i\Omega T_0}}{(1-\Omega^2)} + \frac{\Lambda^3 e^{i3\Omega T_0}}{(1-9\Omega^2)} + \frac{6A\bar{A}\Lambda e^{i\Omega T_0}}{(1-\Omega^2)} + \frac{3A\Lambda^2 e^{i(1-2\Omega)T_0}}{(1-(1-2\Omega)^2)} \right. \\ \left. + \frac{3A\Lambda^2 e^{i(1+2\Omega)T_0}}{(1-(1+2\Omega)^2)} + \frac{3A^2 \Lambda e^{i(2+\Omega)T_0}}{(1-(2+\Omega)^2)} \right] + cc \quad (5.35)$$

Now substituting Eq. (5.35) and Eq. (5.13) in Eq. (5.12) and eliminating the secular terms the following modulation equation in complex form can be found.

$$D_2 A = 3\alpha \left[\frac{\Omega\mu\bar{A}^2 \Lambda e^{i\varepsilon\sigma T_0}}{(1-\Omega^2)} - \frac{i\alpha}{2} \left\{ -\frac{A^3 \bar{A}^2}{8} - \frac{A^4 \Lambda e^{-i\varepsilon\sigma T_0}}{4} - \frac{2\bar{A}^3 A \Lambda e^{i\varepsilon\sigma T_0}}{(1-\Omega^2)} + \frac{3\bar{A}^2 \Lambda^3 e^{i\varepsilon\sigma T_0}}{(1-\Omega^2)} \right. \right. \\ \left. \left. + \frac{12A\Lambda^4}{(1-\Omega^2)} + \frac{6A\bar{A}^3 \Lambda e^{i\varepsilon\sigma T_0}}{(1-\Omega^2)} + \frac{24A^2 \bar{A}\Lambda^2}{(1-\Omega^2)} + \frac{3A\Lambda^4}{(1-(1-2\Omega)^2)} + \frac{6\bar{A}^2 \Lambda^3 e^{i\varepsilon\sigma T_0}}{(1-(1-2\Omega)^2)} + \frac{3A\Lambda^4}{(1-(1+2\Omega)^2)} \right. \right. \\ \left. \left. + \frac{6A^2 \bar{A}\Lambda^2}{(1-(2+\Omega)^2)} + \frac{3\bar{A}^2 \Lambda^3 e^{i\varepsilon\sigma T_0}}{(1-(2+\Omega)^2)} \right\} \right] \quad (5.36)$$

Here,

$$A' = \varepsilon D_1 A + \varepsilon^2 D_2 A \quad (5.37)$$

$$\begin{aligned} A' = \varepsilon \left(-\mu A + \frac{3i}{2} \alpha A^2 \bar{A} + 3i\alpha A \Lambda^2 + \frac{3i}{2} \alpha \bar{A}^2 \Lambda e^{i\varepsilon\sigma T_0} \right) + 3\varepsilon^2 \alpha \left[\frac{\Omega \mu \bar{A}^2 \Lambda e^{i\varepsilon\sigma T_0}}{(1-\Omega^2)} - \frac{i\alpha}{2} \left\{ -\frac{A^3 \bar{A}^2}{8} \right. \right. \\ \left. \left. - \frac{A^4 \Lambda e^{-i\varepsilon\sigma T_0}}{4} - \frac{2\bar{A}^3 A \Lambda e^{i\varepsilon\sigma T_0}}{(1-\Omega^2)} + \frac{3\bar{A}^2 \Lambda^3 e^{i\varepsilon\sigma T_0}}{(1-\Omega^2)} + \frac{12A\Lambda^4}{(1-\Omega^2)} + \frac{6A\bar{A}^3 \Lambda e^{i\varepsilon\sigma T_0}}{(1-\Omega^2)} + \frac{24A^2 \bar{A} \Lambda^2}{(1-\Omega^2)} \right. \right. \\ \left. \left. + \frac{3A\Lambda^4}{(1-(1-2\Omega)^2)} + \frac{6\bar{A}^2 \Lambda^3 e^{i\varepsilon\sigma T_0}}{(1-(1-2\Omega)^2)} + \frac{3A\Lambda^4}{(1-(1+2\Omega)^2)} + \frac{6A^2 \bar{A} \Lambda^2}{(1-(2+\Omega)^2)} + \frac{3\bar{A}^2 \Lambda^3 e^{i\varepsilon\sigma T_0}}{(1-(2+\Omega)^2)} \right\} \right] \end{aligned} \quad (5.38)$$

Again, also substituting $A = \frac{1}{2} a e^{i\beta}$ and $\gamma = \varepsilon\sigma T_0 - 3\beta$ in Eq. (5.38) and separating the real and imaginary parts, one may get the following reduced equations.

$$\begin{aligned} a' = \varepsilon \left(-\mu a - \frac{3}{4} \alpha a^2 \Lambda \sin \gamma \right) + \varepsilon^2 \left[\frac{3\alpha \Omega \mu a^2 \Lambda}{2(1-\Omega^2)} \cos \gamma + 3\alpha^2 \left\{ \frac{a^4 \Lambda}{64} \sin \gamma - \frac{a^4 \Lambda}{8(1-\Omega^2)} \sin \gamma \right. \right. \\ \left. \left. + \frac{3a^2 \Lambda^3}{4(1-\Omega^2)} \sin \gamma + \frac{3a^4 \Lambda}{8(1-\Omega^2)} \sin \gamma + \frac{3a^2 \Lambda^3}{2(1-(1-2\Omega)^2)} \sin \gamma + \frac{3a^2 \Lambda^3}{4(1-(2+\Omega)^2)} \sin \gamma \right\} \right] \end{aligned} \quad (5.39)$$

$$\begin{aligned} a\gamma' = a\varepsilon\sigma - 3\varepsilon \left(\frac{3}{8} \alpha a^3 + 3\alpha a \Lambda^2 + \frac{3}{4} \alpha a^2 \Lambda \cos \gamma \right) - 3\varepsilon^2 \left[\frac{3\alpha \Omega \mu a^2 \Lambda}{2(1-\Omega^2)} \sin \gamma - 3\alpha^2 \left\{ -\frac{a^5}{256} \right. \right. \\ \left. \left. - \frac{a^4 \Lambda}{64} \cos \gamma - \frac{a^4 \Lambda}{8(1-\Omega^2)} \cos \gamma + \frac{3a^2 \Lambda^3}{4(1-\Omega^2)} \cos \gamma + \frac{6a\Lambda^4}{(1-\Omega^2)} + \frac{3a^3 \Lambda}{8(1-\Omega^2)} \cos \gamma + \frac{3a^3 \Lambda^2}{(1-\Omega^2)} \right. \right. \\ \left. \left. + \frac{3a\Lambda^4}{2(1-(1-2\Omega)^2)} + \frac{3a^2 \Lambda^3}{2(1-(1-2\Omega)^2)} \cos \gamma + \frac{3a\Lambda^4}{2(1-(1+2\Omega)^2)} + \frac{3a^3 \Lambda^2}{4(1-(2+\Omega)^2)} \right. \right. \\ \left. \left. + \frac{3a^2 \Lambda^3}{4(1-(2+\Omega)^2)} \cos \gamma \right\} \right] \end{aligned} \quad (5.40)$$

Thus the particular solution of x_1 from Eq. (5.34) can be written as

$$\begin{aligned}
 x_1 = & \frac{4\Omega\mu\Lambda}{(1-\Omega^2)} \sin(\Omega T_0) - \alpha \left[-\frac{a^3}{32} \cos(-\Omega T_0 + 4T_0 + \gamma) + \frac{6\Lambda^3}{(1-\Omega^2)} \cos(\Omega T_0) \right. \\
 & + \frac{2\Lambda^3}{(1-9\Omega^2)} \cos(3\Omega T_0) + \frac{3a^2\Lambda}{(1-\Omega^2)} \cos(\Omega T_0) + \frac{3a\Lambda^2}{(1-(1-2\Omega)^2)} \cos(-3\Omega T_0 + 4T_0 + \gamma) \\
 & \left. + \frac{3a\Lambda^2}{(1-(1+2\Omega)^2)} \cos(\Omega T_0 + 4T_0 + \gamma) + \frac{3a^2\Lambda}{2(1-(2+\Omega)^2)} \cos(-\Omega T_0 + 8T_0 + 2\gamma) \right]
 \end{aligned} \quad (5.41)$$

And the particular solution of x_0 from Eq. (5.12) can be written as

$$x_0 = a \cos(-\Omega T_0 + 4T_0 + \gamma) + 2\Lambda \cos(\Omega T_0) \quad (5.42)$$

Thus substituting Eq. (5.41) and Eq. (5.42) in Eq. (5.7), the response of the system $x(\tau)$ is given below.

$$\begin{aligned}
 x_0 = & a \cos(-\Omega T_0 + 4T_0 + \gamma) + 2\Lambda \cos(\Omega T_0) + \frac{4\varepsilon\Omega\mu\Lambda}{(1-\Omega^2)} \sin(\Omega T_0) \\
 & - \alpha \varepsilon \left[-\frac{a^3}{32} \cos(-\Omega T_0 + 4T_0 + \gamma) + \frac{6\Lambda^3}{(1-\Omega^2)} \cos(\Omega T_0) + \frac{2\Lambda^3}{(1-9\Omega^2)} \cos(3\Omega T_0) \right. \\
 & + \frac{3a^2\Lambda}{(1-\Omega^2)} \cos(\Omega T_0) + \frac{3a\Lambda^2}{(1-(1-2\Omega)^2)} \cos(-3\Omega T_0 + 4T_0 + \gamma) \\
 & \left. + \frac{3a\Lambda^2}{(1-(1+2\Omega)^2)} \cos(\Omega T_0 + 4T_0 + \gamma) + \frac{3a^2\Lambda}{2(1-(2+\Omega)^2)} \cos(-\Omega T_0 + 8T_0 + 2\gamma) \right]
 \end{aligned} \quad (5.43)$$

Eq. (5.43) provides the expression for the complete time response of the harmonically excited system attached with a nonlinear PAM under hard excitation for subharmonic resonance condition.

5.3.2.1 Stability of the trivial steady state response

As Eq. (5.40) contains term $a\gamma'$, while finding the stability of the trivial state the linearized equation will not contain the perturbation γ' . So, the stability of the trivial solutions cannot be found in a normal way and hence one may use the transformation similar to the work of Dwivedy

and Kar [242], i.e., by taking $p = a \cos \gamma$ and $q = a \sin \gamma$. The resulting Cartesian modulation equations are given below.

$$\begin{aligned}
 p' = & \varepsilon \left(-\mu p + \frac{3}{2} \alpha \Lambda p q + \frac{9}{8} \alpha q (p^2 + q^2) + 9 \alpha \Lambda^2 q \right) + \varepsilon^2 \left[\frac{3 \alpha \Omega \mu \Lambda}{2(1-\Omega^2)} (p^2 + 3q^2) \right. \\
 & + 3 \alpha^2 \left\{ \left(\frac{3q}{256} (p^2 + q^2) + \frac{\Lambda p q}{16} - \frac{\Lambda p q}{4(1-\Omega^2)} - \frac{9 \Lambda^2 q}{(1-\Omega^2)} - \frac{9 \Lambda^2 q}{4(1-(2+\Omega)^2)} \right) (p^2 + q^2) \right. \\
 & - \frac{3 \Lambda^3 p q}{2(1-\Omega^2)} - \frac{18 \Lambda^4 q}{(1-\Omega^2)} - \frac{9 \Lambda^4 q}{2(1-(1-2\Omega)^2)} - \frac{3 \Lambda^3 p q}{2(1-(1-2\Omega)^2)} - \frac{9 \Lambda^4 q}{2(1-(1+2\Omega)^2)} \\
 & \left. \left. - \frac{3 \Lambda^3 p q}{2(1-(2+\Omega)^2)} \right) \right] - \varepsilon \sigma q
 \end{aligned} \tag{5.44}$$

$$\begin{aligned}
 q' = & \varepsilon \left(-\mu q - 9 \alpha \Lambda^2 p - \frac{3}{4} \alpha \Lambda (3p^2 + q^2) - \frac{9}{8} \alpha p (p^2 + q^2) \right) + \varepsilon^2 \left[-\frac{6 \alpha \Omega \mu \Lambda p q}{2(1-\Omega^2)} \right. \\
 & + 3 \alpha^2 \left\{ \left(-\frac{3p}{256} (p^2 + q^2) + \left(\frac{\Lambda}{64} + \frac{\Lambda}{8(1-\Omega^2)} \right) (q^2 - 3p^2) + \frac{9 \Lambda^2 p}{(1-\Omega^2)} \right. \right. \\
 & + \frac{9 \Lambda^2 p}{4(1-(2+\Omega)^2)} \left. \right) (p^2 + q^2) + \left(\frac{\Lambda^3}{4(1-\Omega^2)} + \frac{\Lambda(p^2 + q^2)}{8(1-\Omega^2)} + \frac{\Lambda^3}{2(1-(1-2\Omega)^2)} \right. \\
 & \left. \left. + \frac{\Lambda^3}{4(1-(2+\Omega)^2)} \right) (9p^2 + 3q^2) + \frac{18 \Lambda^4 p}{(1-\Omega^2)} + \frac{9 \Lambda^4 p}{2(1-(1-2\Omega)^2)} + \frac{9 \Lambda^4 p}{2(1-(1+2\Omega)^2)} \right\} \right] + \varepsilon \sigma p
 \end{aligned} \tag{5.45}$$

The stability of the steady state response (p_0, q_0) for both trivial and nontrivial state can be determined by perturbing Eq. (5.44) and Eq. (5.45) and finding the eigenvalues of the Jacobian matrix (J) . Here, the system will be stable for trivial state responses when all the real parts the eigenvalues of the Jacobian matrix (J) are less than zero.

5.4 Results and Discussion

In this section the numerical results for system shown in Fig. 5.1 have been found out by taking the system parameters as given in Table 5.1 for superharmonic and subharmonic resonance

conditions. Here, by putting $a' = \gamma' = 0$ in the reduced equations from the MMS version II Eq. (5.25) and Eq. (5.26) for superharmonic resonance condition and Eq. (5.39) and Eq. (5.40) for subharmonic resonance condition the steady state responses have been obtained by using Newton's Method. Further these results have been compared by solving the original equation Eq. (5.6) numerically using Runge-Kutta method. In the following subsections results for superharmonic and subharmonic resonance conditions have been discussed with the variation of system parameters. As minimal effect is observed for variation of the muscle parameter c_1 and c_2 , they have not been discussed in this work. In all the frequency response plots red color depicts the unstable solution and blue color shows the stable solutions.

Table 5.1 System parameters numerical values for simulation.

Parameter	Numerical Value	Parameter	Numerical Value	Parameter	Numerical Value
$l_{\max}$	74 mm	m	6 N	k	12 N/mm
c_1	-234.25 N	P	500 kPa	α	400
c_2	1.96 N/kPa	μ	0.01	ε	0.1
c_3	0.4 N/kPa ²	F	8 kN	r	1

5.4.1 Superharmonic resonance condition

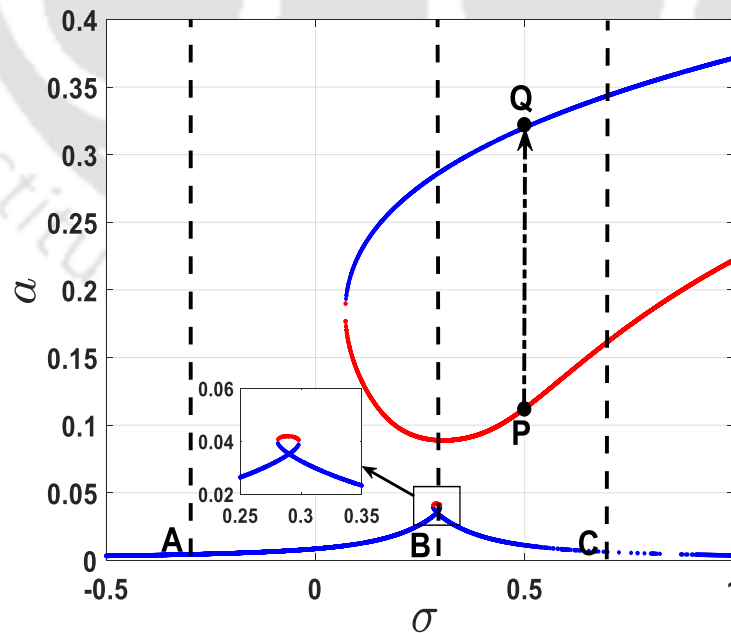


Fig. 5.2 Frequency response of the harmonically excited system under hard excitation with superharmonic resonance condition as the parameter values in Table 5.1.

In this section, the responses are plotted for the superharmonic resonance condition where the external excitation frequency is one third of the natural frequency of the system. A typical frequency response plot is shown Fig. 5.2 with the help of the reduced equations Eq. (5.25) and Eq. (5.26) from the MMS version II by taking the system parameters as given in Table 5.1. Here, one may observe two distinct zones in the frequency response plot viz., regions below $a = 0.042$ (Zone 1) and above $a = 0.089$ (Zone 2). In zone 1, except at detuning parameters $\sigma = 0.281$ and $\sigma = 0.298$ where the saddle node bifurcation occurs, the nontrivial steady state response is stable at other values of σ . Near the mentioned saddle node bifurcation points the time response and phase portraits are shown in Fig. 5.3 which are found to be periodic containing multi frequency components. In these two values of σ , the phase portraits clearly shows how the response contains three periods which may lead to chaos [255].

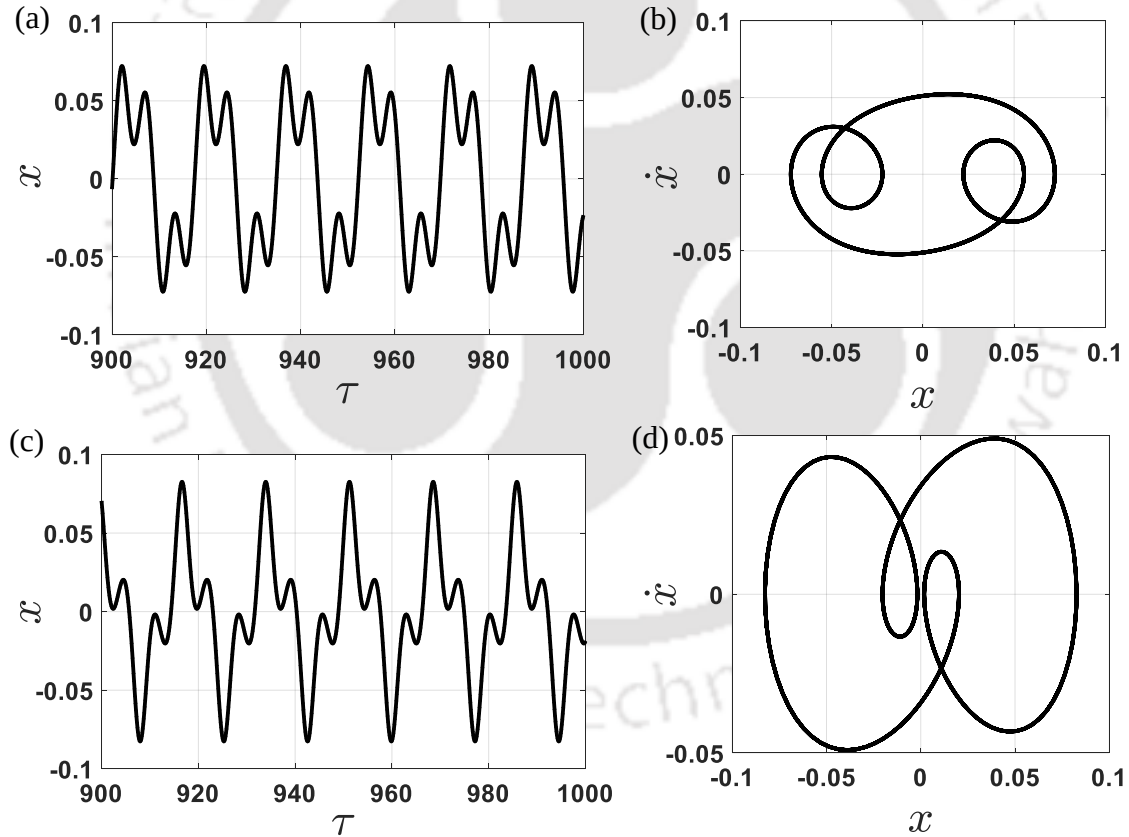


Fig. 5.3 Time response and phase portraits corresponding to the Zone1 in Fig. 5.2 at (a, b) $\sigma = 0.281$ (c, d) $\sigma = 0.298$.

In zone 2, saddle node bifurcation can be observe at $\sigma = 0.072$ where the upper branch of the nontrivial steady state response is stable and lower branch is unstable. The time response and

phase portraits have been plotted in Fig. 5.4 near the saddle node point ($\sigma = 0.072$) which is also found to be periodic containing multi frequency components. In this zone, one may observe the jump up phenomenon in Fig. 5.2 at $\sigma = 0.5$. Here, the unstable solution with amplitude $a = 0.1124$ at point P has always a tendency to reach the stable solution with $a = 0.32$ at point Q in the stable branch of the nontrivial steady state response. It may clearly be observed that the first order solution described in the book of Nayfeh and Mook [13] are completely different from the present work.

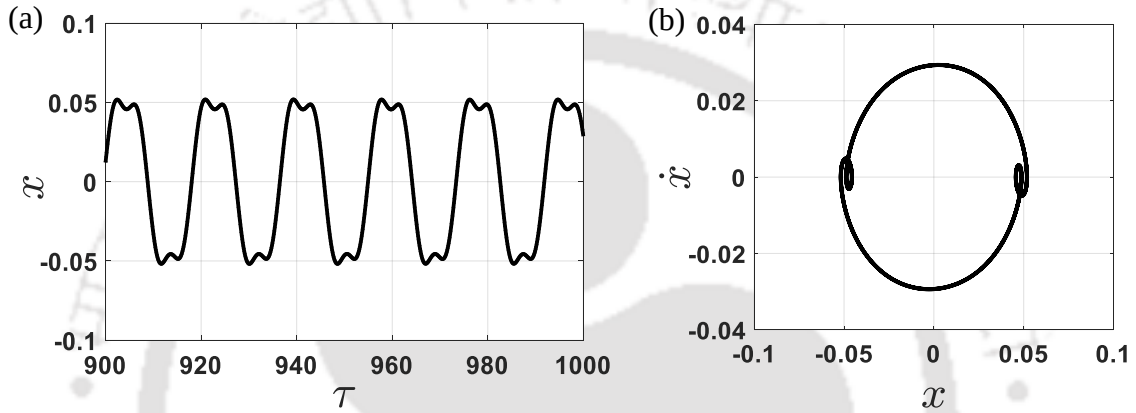


Fig. 5.4 Time response and phase portraits corresponding to the Zone 2 in Fig. 5.2 at (a, b) $\sigma = 0.072$.

In Fig. 5.5, the validation has been done for the frequency response plot in Fig. 5.2 by plotting the basin of attraction in the $a \sim \gamma$ plane with three different values of σ . At point A ($\sigma = -0.3$) in Fig. 5.2, the system has only a stable solution with amplitude $a = 0.004$ which can be observed from the basin of attraction in Fig. 5.5(a). For the point B ($\sigma = 0.293$) marked in Fig. 5.2, the basin of attraction is shown in Fig. 5.5(b) where three stable solutions can be obtained at amplitude $a = 0.034$, $a = 0.036$ and $a = 0.286$ along with two unstable solutions at $a = 0.042$ and $a = 0.089$. Fig. 5.5(c) validates the another point at C ($\sigma = 0.7$) in Fig. 5.2 where the system exhibits bi-stable nontrivial state with two stable solutions at $a = 0.006$, $a = 0.344$ and one unstable solution at $a = 0.162$. From these frequency and basin of attraction plots, the system can be noticed that upto $\sigma = 0.072$ the system has single stable state, between $\sigma = 0.072$ and $\sigma = 0.281$ bi-stable state, between $\sigma = 0.281$ and $\sigma = 0.298$ tri-stable state and finally after $\sigma = 0.298$ the system exists bistable state.

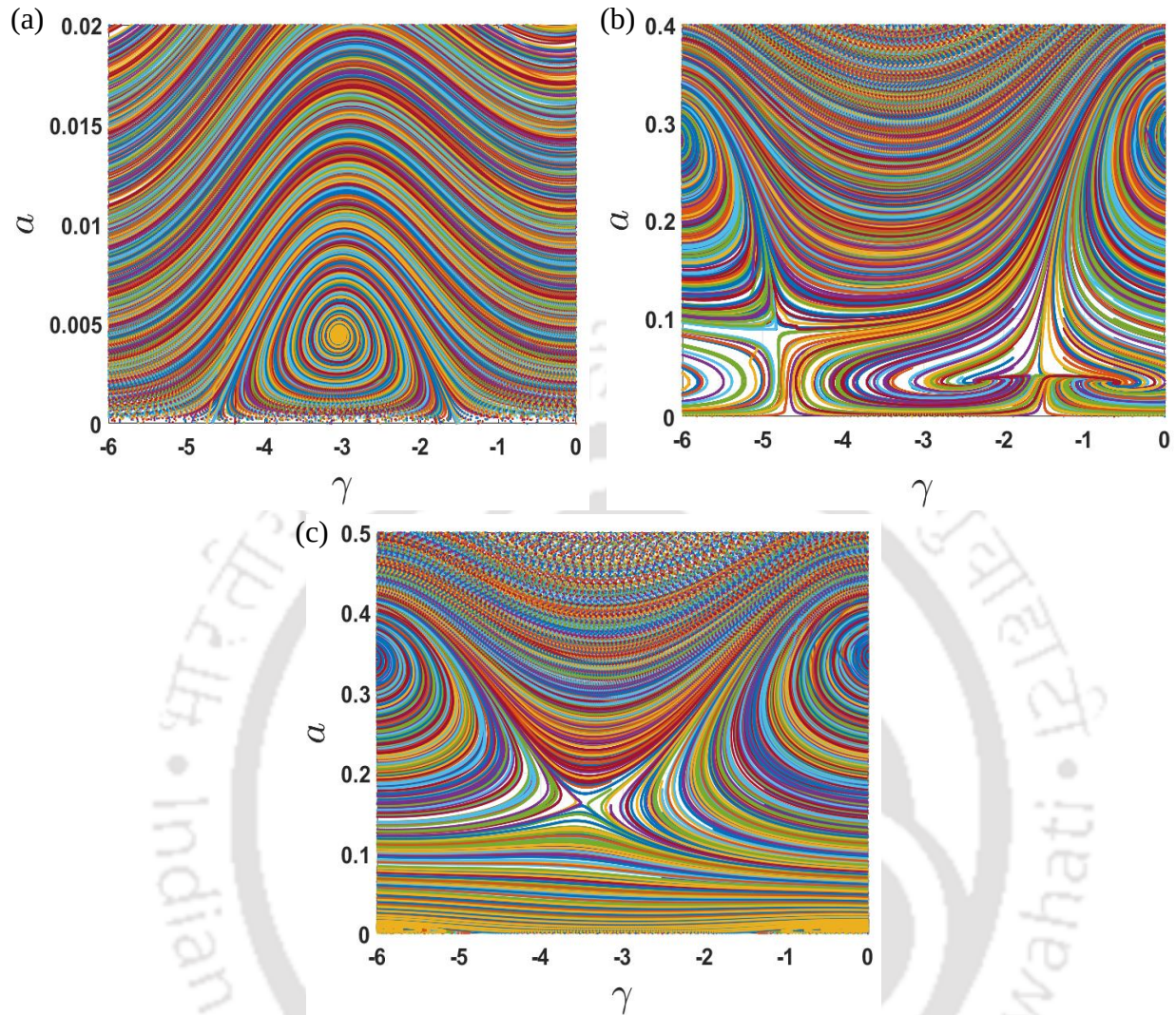
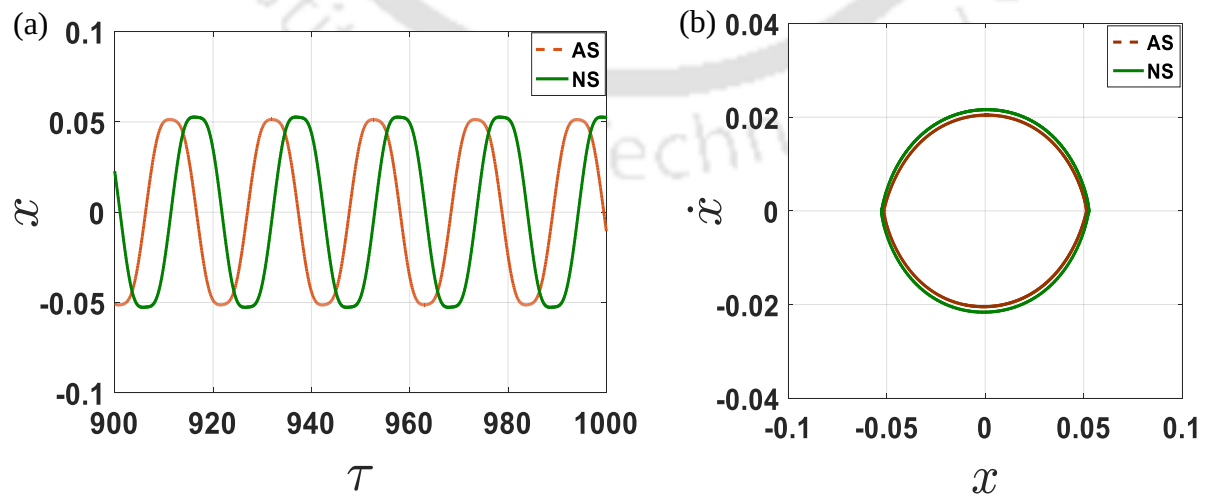


Fig. 5.5 Basin of attraction at (a) point A ($\sigma = -0.3$) (b) point B ($\sigma = 0.293$) and (c) point C ($\sigma = 0.7$) in Fig. 5.2.



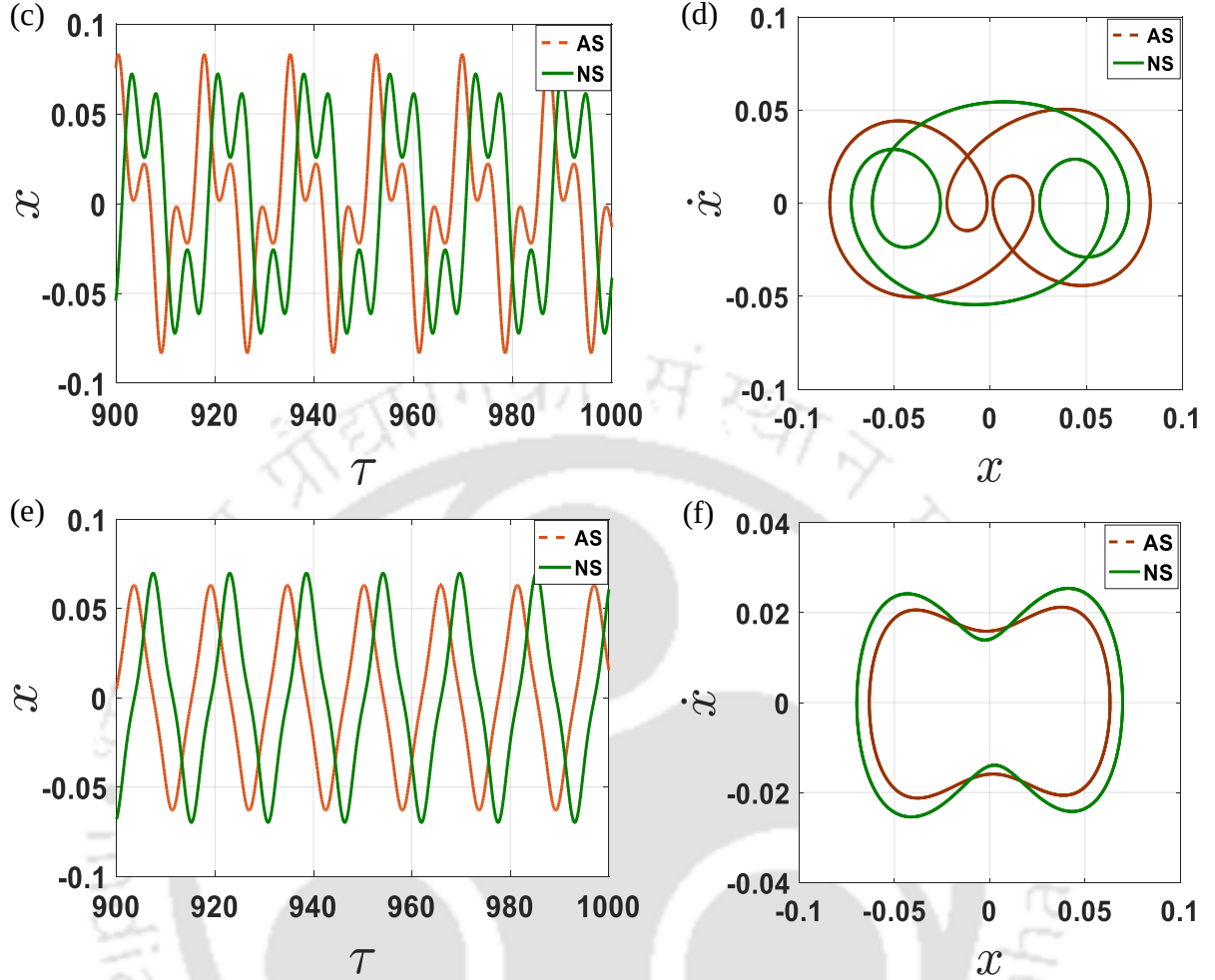


Fig. 5.6 Time response and phase portraits comparison at (a, b) point A ($\sigma = -0.3$) (c, d) point B ($\sigma = 0.293$) and (e, f) point C ($\sigma = 0.7$) in Fig. 5.2.

In Fig. 5.6 the comparison has been made with the help of time responses and phase portraits by solving the reduced equations Eq. (5.25) and Eq. (5.26) from the MMS version II for superharmonic resonance condition (marked 'AS') and the original Eq. (5.6) (marked 'NS'). These comparisons have been made at three different points at point A ($\sigma = -0.3$), B ($\sigma = 0.293$) and C ($\sigma = 0.7$) in Fig. 5.2. It can be observed from Fig. 5.6 that the responses show similar behavior in both the cases. From Fig. 5.6(a) at $\sigma = -0.3$ the maximum response obtained from the reduced equation Eq. (5.25) and Eq. (5.26) from the MMS version II is less than 0.13% from that obtained by solving the original equation Eq. (5.6). But at $\sigma = 0.293$ in Fig. 5.6(c), the maximum response obtained from the reduced equation is increased by 1.12% from the response obtained from the original governing equation. Again from Fig. 5.6(e) at $\sigma = 0.7$, the maximum response from the MMS version II is decreased by 0.66% from the response

obtained from the original equation Eq. (5.6). So, the results obtained by solving the reduced equations from the MMS version II and those from the original equation are in good agreement. Hence, to reduce computational time and memory one may use the reduced equations for further analyses. The phase portraits for the three different points show typical behavior in Fig. 5.6(b, d, f) where Fig. 5.6(b) and Fig. 5.6(f) depicts single periodic but Fig. 5.6(d) shows the three periods which may leads to the chaos. So, the study on the effect of the various system parameters on the system at detuning parameter $\sigma = 0.293$ along with the stability of the system have been carried out in the following sections.

5.4.1.1 Effect of the operating pressure

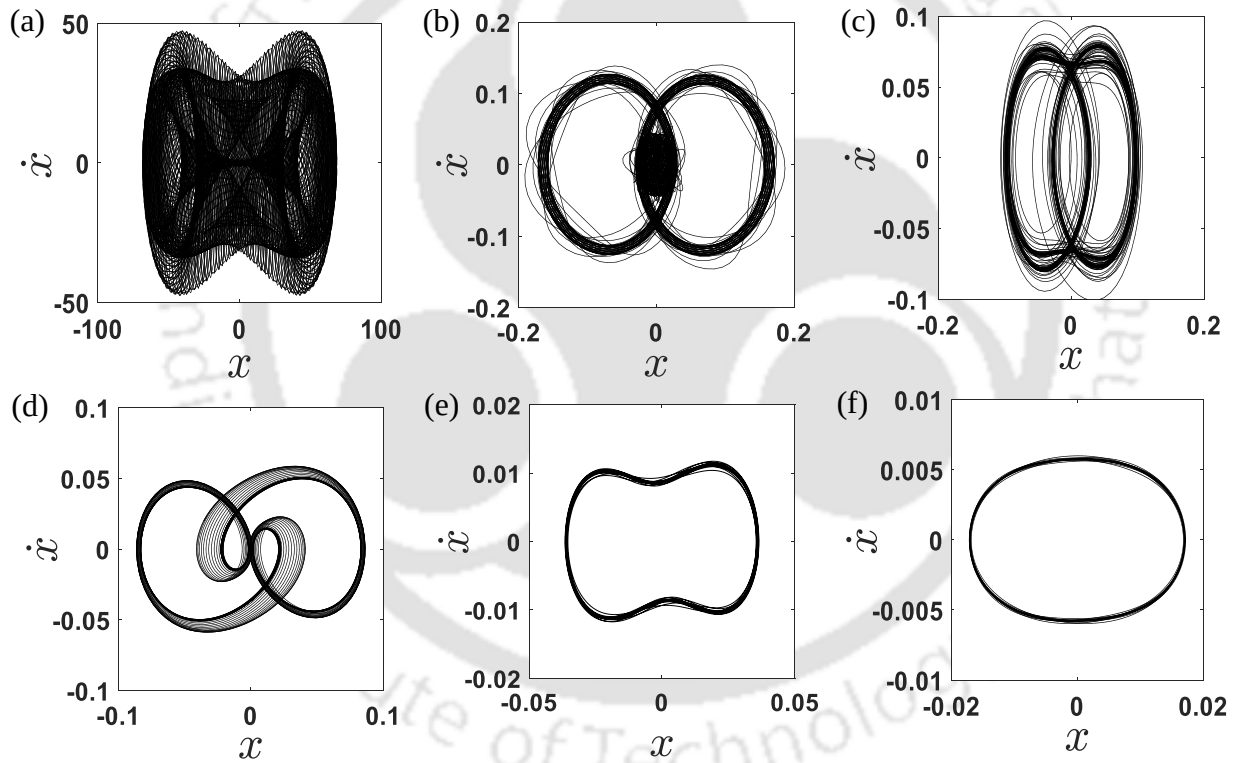


Fig. 5.7 Phase portraits with variation of P with (a) 180 kPa (b) 280 kPa (c) 350 kPa (d) 500 kPa (e) 700 kPa (f) 1000 kPa.

To study the effect of operating pressure (P) on the response of the system at detuning parameter $\sigma = 0.293$, the phase portraits are plotted in Fig. 5.7 taking other parameters same as in Table 5.1. It may clearly be observed from Fig. 5.7(a) that the response is chaotic for pressure $P = 180$ kPa. With increase in the pressure, the number of harmonics is decreases and at $P = 500$ kPa the response is found to be three periodic. For $P = 700$ kPa, the phase portraits is similar to that of a

homoclinic orbit. With further increase in pressure the response is found to be periodic at $\sigma = 0.293$ as shown in Fig. 5.7(f).

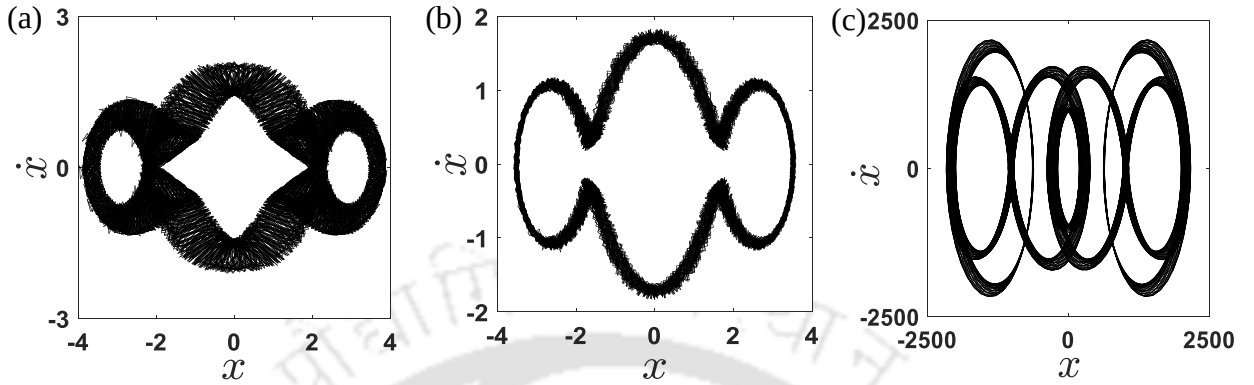


Fig. 5.8 Variation of initial conditions with respect to Fig. 5.7(a), (a) $a = -0.001$, $\gamma = 0.001$
(b) $a = 0.001$, $\gamma = 0.001$ (c) $a = 1.5$, $\gamma = -0.5$.

To study the dependency of the response on initial conditions or to study the butterfly effect, the initial conditions has been changed from $a = 0.001$, $\gamma = -0.001$ (Fig. 5.7(a)) to $a = -0.001$, $\gamma = 0.001$ (Fig. 5.8(a)), $a = 0.001$, $\gamma = 0.001$ (Fig. 5.8(b)) and $a = 1.5$, $\gamma = -0.5$ (Fig. 5.8(c)). The resulting chaotic responses are shown in Fig. 5.8 with the help of phase portraits. Clearly all these chaotic responses are very different. Hence, the operator of the PAM should take care to change the initial condition to avoid the sudden failure of the muscle.

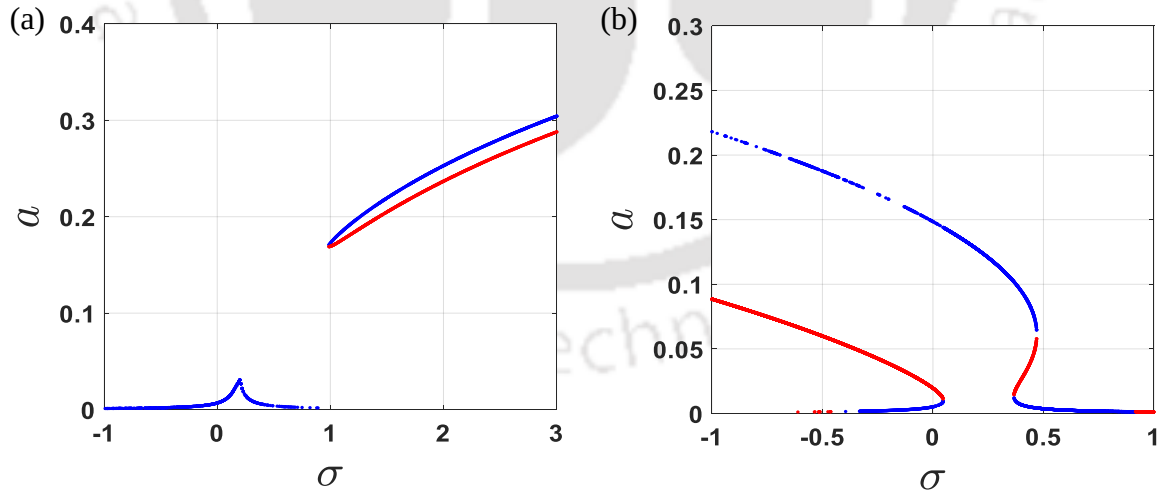


Fig. 5.9 Frequency responses with variation of P with (a) 600 kPa (b) 400 kPa.

The frequency response for operating pressure $P = 600$ kPa is shown in Fig. 5.9(a) taking other parameter same as in Table 5.1. Comparing Fig. 5.2 and Fig. 5.9(a), in Zone 1 the response is completely stable and is similar to that of a linear system and the nontrivial response in Zone 2 is

started at $\sigma = 0.99$. But for $P = 500$ kPa in Fig. 5.2, the nontrivial response in Zone 2 begins at $\sigma = 0.07$. In this case, the maximum response amplitude $a = 0.1727$ at $\sigma = 1$ which is decreased by 53.5% as compare to Fig. 5.2 that can observed from the Zone 2 in both the figures. Now decreasing $P = 400$ kPa, the system response is shown in Fig. 5.9(b). Clearly the bifurcation points can be observed at five different values of σ where three saddle node bifurcation points can be observed at $\sigma = 0.046$, $\sigma = 0.367$ and $\sigma = 0.469$ with two Hopf bifurcation points at $\sigma = -0.462$ and $\sigma = 0.916$. In this case, Zone 2 is absent and the maximum response amplitude can be achieved at Zone 1 with $a = 0.218$ in $\sigma = -1$. The stability in the frequency response plot in Fig. 5.9(b) is distinct in comparison with the Fig. 5.2 and the maximum response amplitude can be obtained at lower value of σ .

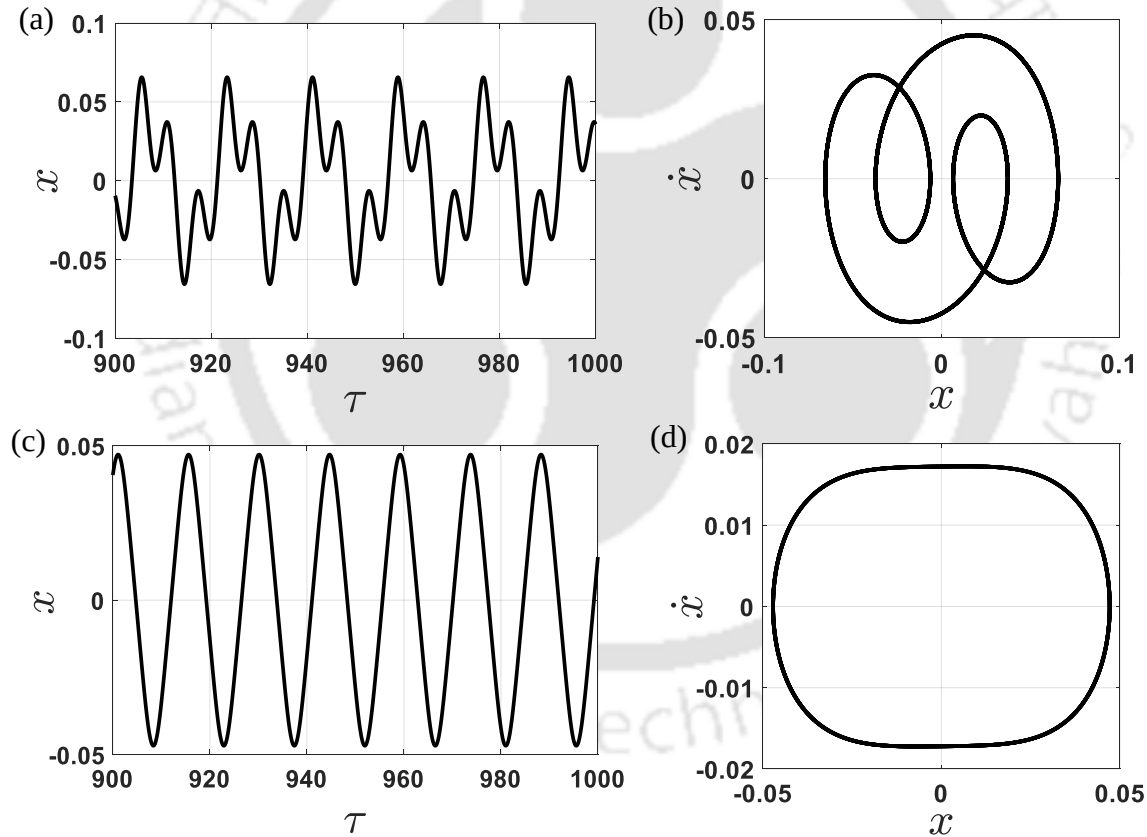
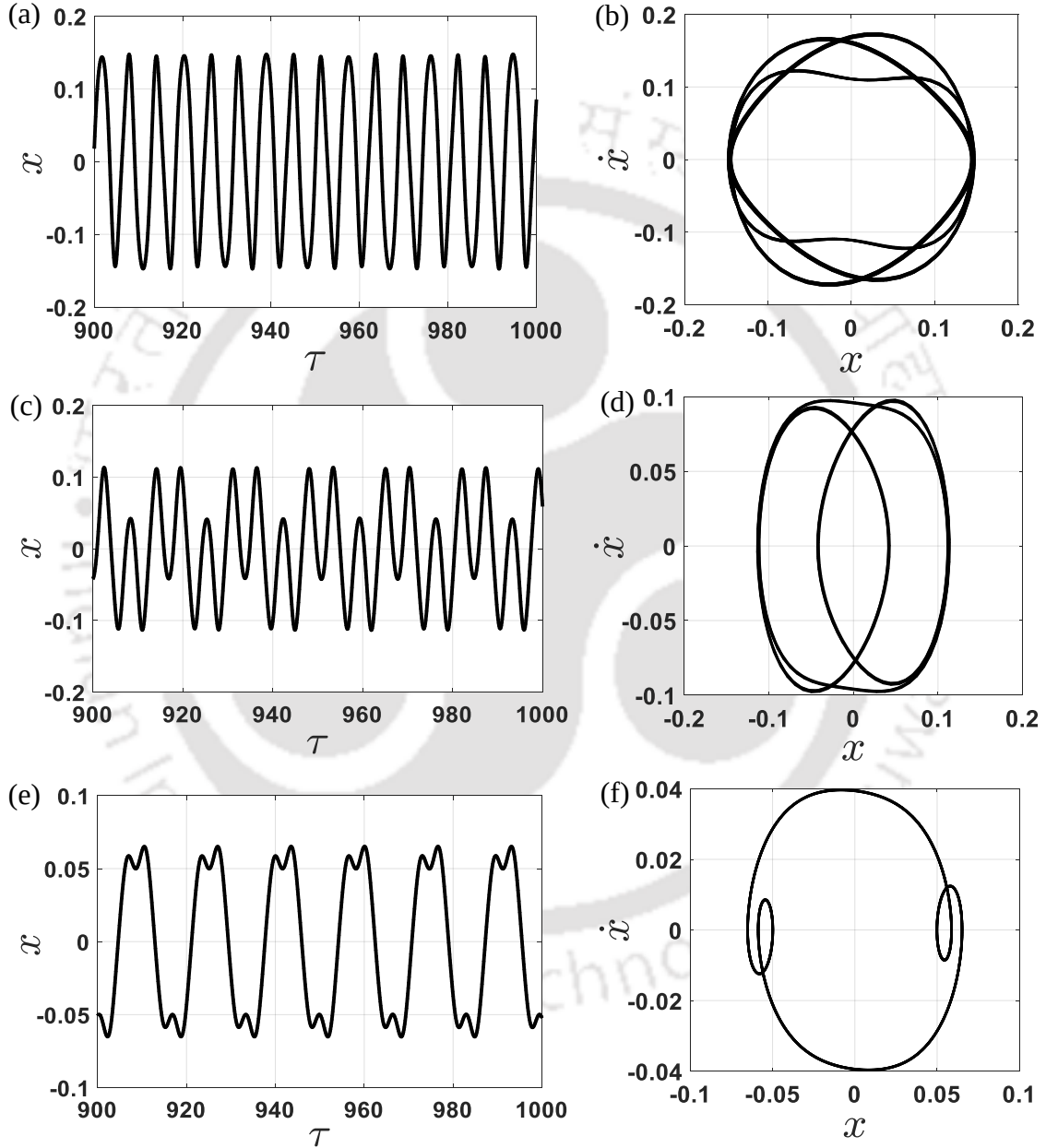


Fig. 5.10 Time response and phase portraits at (a, b) $\sigma = 0.2$ in Zone 1
(c, d) $\sigma = 0.99$ in Zone 2 corresponding to Fig. 5.9(a).

The time response and phase portraits for bifurcation point at $\sigma = 0.2$ in Zone 1 and $\sigma = 0.99$ in Zone 2 in Fig. 5.9(a) are shown in Fig. 5.10(a, b) and Fig. 5.10(c, d) respectively. The response at $\sigma = 0.2$ in Fig. 5.10(a, b) is found to be three periodic which may lead to chaos but at $\sigma = 0.99$ in Fig. 5.10(c, d) is a single periodic response.



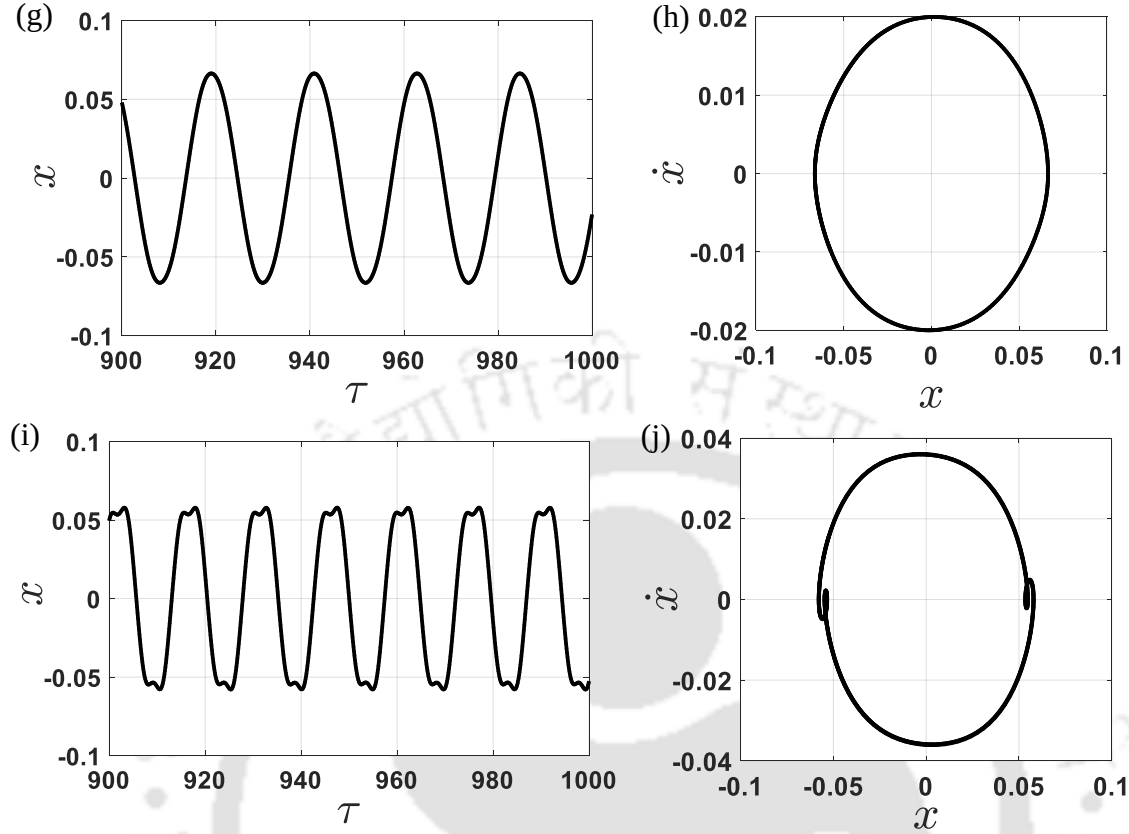


Fig. 5.11 Time response and phase portraits at (a, b) $\sigma = 0.046$ (c, d) $\sigma = 0.367$ (e, f) $\sigma = 0.469$ (g, h) $\sigma = -0.462$ (i, j) $\sigma = 0.916$ corresponding to Fig. 5.9(b).

The five different bifurcation points in Fig. 5.9(b) have been studied with the help of time response and phase portraits in Fig. 5.11. For the saddle node bifurcation points at $\sigma = 0.046$ and $\sigma = 0.367$ in Fig. 5.9(b), the time response and phase portraits have been plotted in Fig. 5.11(a, b) and Fig. 5.11(c, d), respectively. The response is observed as periodic but due to the presence of three or more periods, the system may lead to the chaos. The time response and phase portraits have been shown in Fig. 5.11(e, f) for the saddle node bifurcation point in Fig. 5.9(b) at $\sigma = 0.469$ which is also depicts three periods. Again for the subcritical Hopf bifurcation point at $\sigma = -0.462$ and supercritical Hopf bifurcation point at $\sigma = 0.916$ in Fig. 5.9(b), the time response and phase portraits have been plotted in Fig. 5.11(g, h) and Fig. 5.11(i, j) respectively. The response is periodic in both the cases but at $\sigma = -0.462$ the system has a single period whereas at $\sigma = 0.916$ occurs three periods which can be observed from the phase portraits in Fig. 5.11(h) and Fig. 5.11(j).

5.4.1.2 Effect of the damping parameter

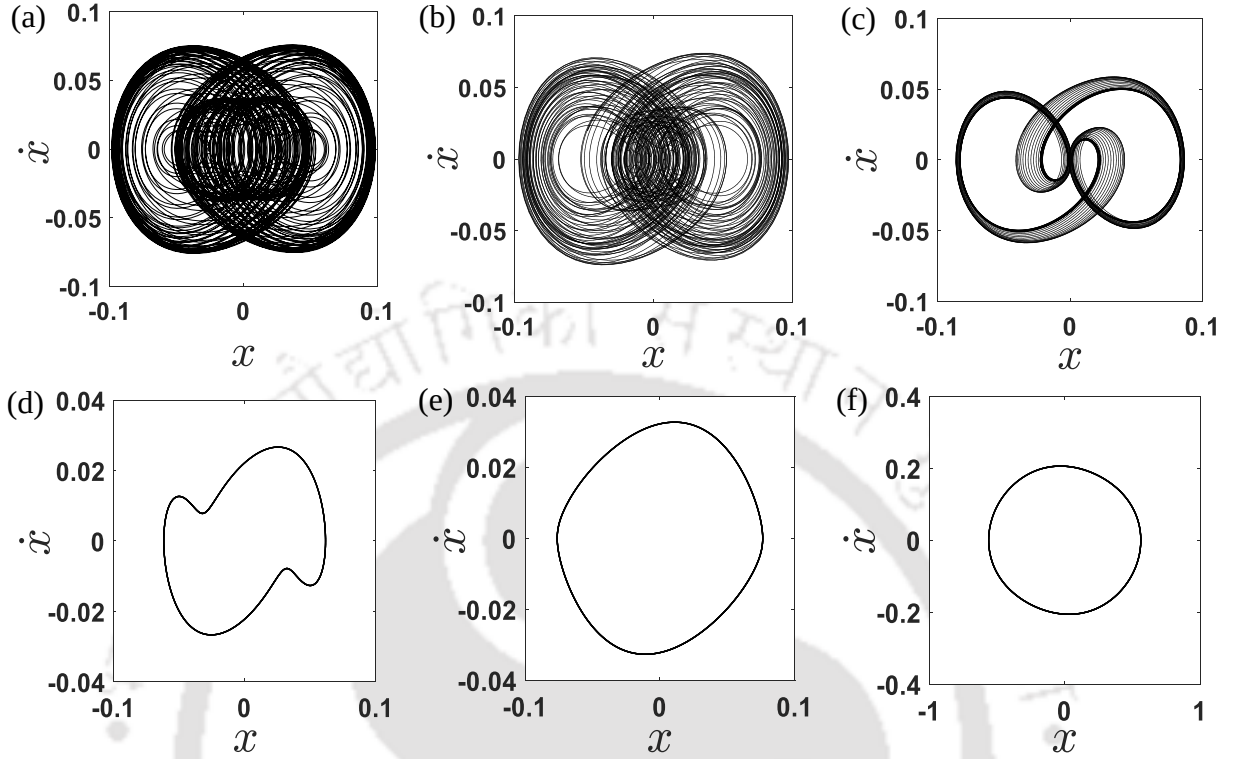


Fig. 5.12 Phase portraits with variation of μ with (a) 0.0001 (b) 0.001 (c) 0.01 (d) 0.1 (e) 1 (f) 10.

At detuning parameter $\sigma = 0.293$, the effect of damping parameter (μ) has been studied with the help of phase portraits in Fig. 5.12 keeping other system parameters same as in Table 5.1. From Fig. 5.12(a) with very less value of μ (e.g., $\mu = 0.0001$), the system is showing chaotic response and with increase in the value of $\mu = 0.01$ the system depicts three periods in Fig. 5.12(c). Further, with increase in the value of μ , the number of harmonics decreases and finally the response is leading to a single periodic response which can be observed from Fig. 5.12(d, e, f).

The frequency responses have plotted in Fig. 5.13 to study the stability of the system for two different values of μ with 0.05 (Fig. 5.13(a)) and 0.005 (Fig. 5.13(b)). With increase in the value of $\mu = 0.05$ in Fig. 5.13(a) as compared to the Fig. 5.2, the Zone 1 will behave as a linear system with a stable response but in Zone 2, the saddle node bifurcation point is observed at $\sigma = 0.55$. The maximum response amplitude $a = 0.3547$ in Zone 2 at $\sigma = 1$ which is decreased by 4.5% as compared to that of Zone 2 in Fig. 5.2. But with decrease in the value of $\mu = 0.005$ in Fig. 5.13(b), the system will behave as same as the response in Fig. 5.2 with two distinct saddle node

bifurcation point in Zone 1 at $\sigma = 0.27$ and $\sigma = 0.32$. In Zone 2, the saddle node bifurcation point is observed at $\sigma = 0.06$ and the maximum response amplitude $a = 0.3719$ at $\sigma = 1$ which is increased by a small amount 0.13% as compared to Fig. 5.2.

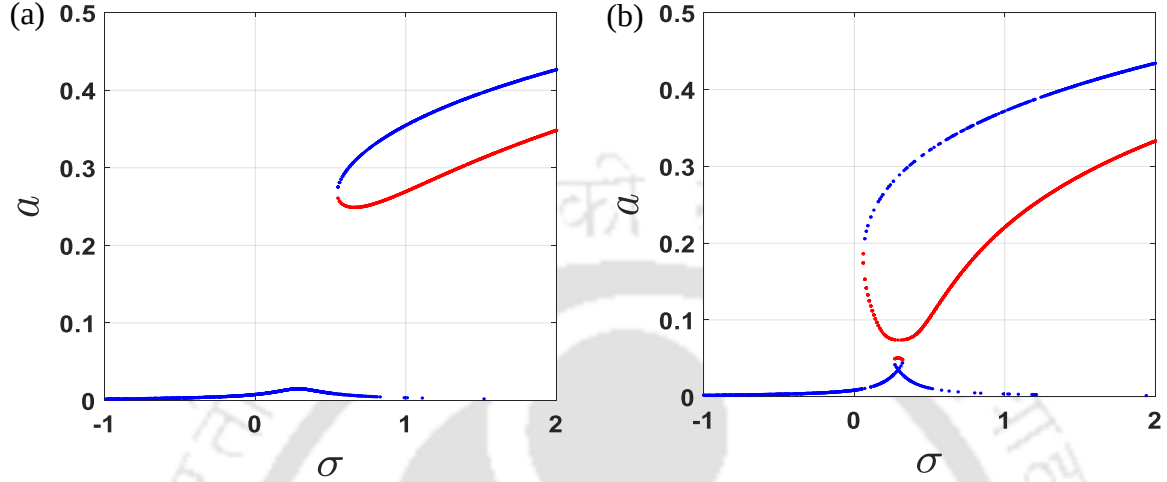


Fig. 5.13 Frequency responses with variation of μ with (a) 0.05 (b) 0.005.

5.4.1.3 Effect of the external load

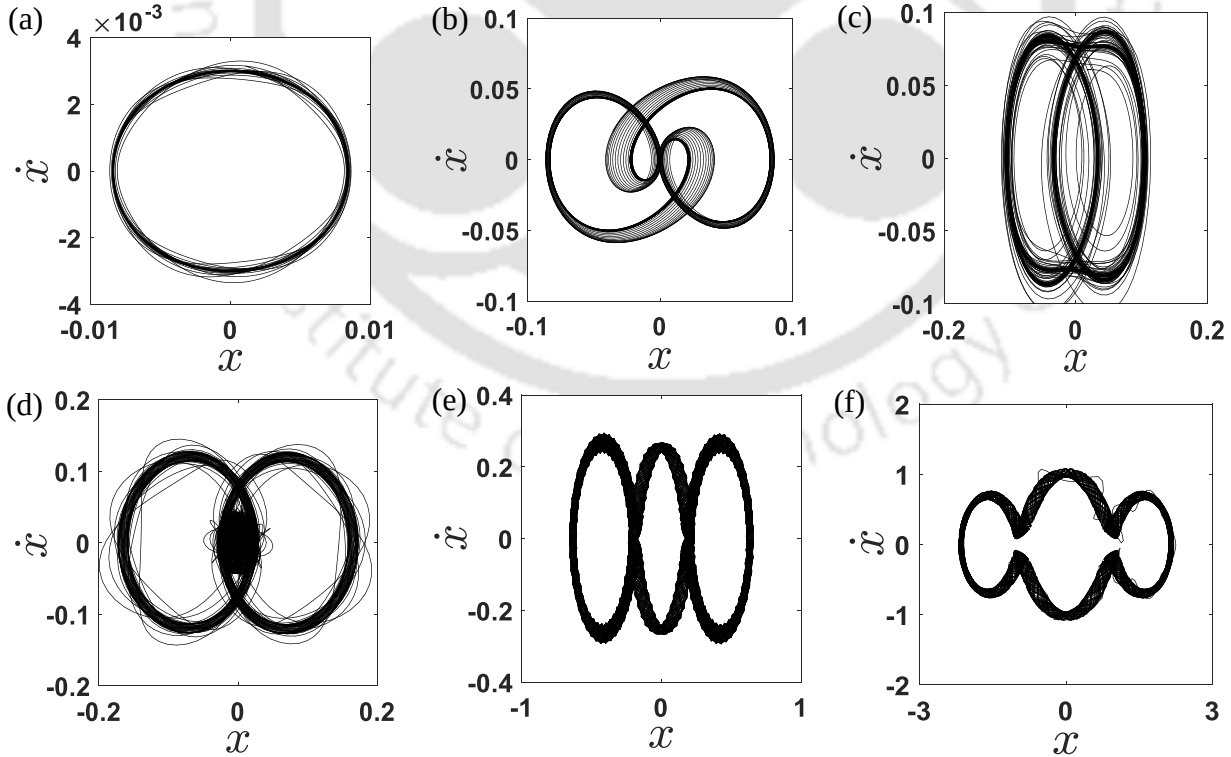


Fig. 5.14 Phase portraits with variation of F with (a) 1 kN (b) 8 kN (c) 15 kN (d) 25 kN (e) 35 kN (f) 50 kN.

The effect of the external load (F) on the system has been studied in Fig. 5.14 by plotting the phase portraits with six different value of F at detuning parameter $\sigma = 0.293$. The system exhibits a single periodic response for smaller value of $F = 1$ kN in Fig. 5.14(a) and with increase in the value $F = 8$ kN in Fig. 5.14(b), the number of harmonics of the system increases with three periods. Further increase in the value of F with 15 kN (Fig. 5.14(c)), 25 kN (Fig. 5.14(d)), 35 kN (Fig. 5.14(e)) and 50 kN (Fig. 5.14(f)), the phase portraits depict that the system may increase the number of harmonics and lead to a chaotic response.

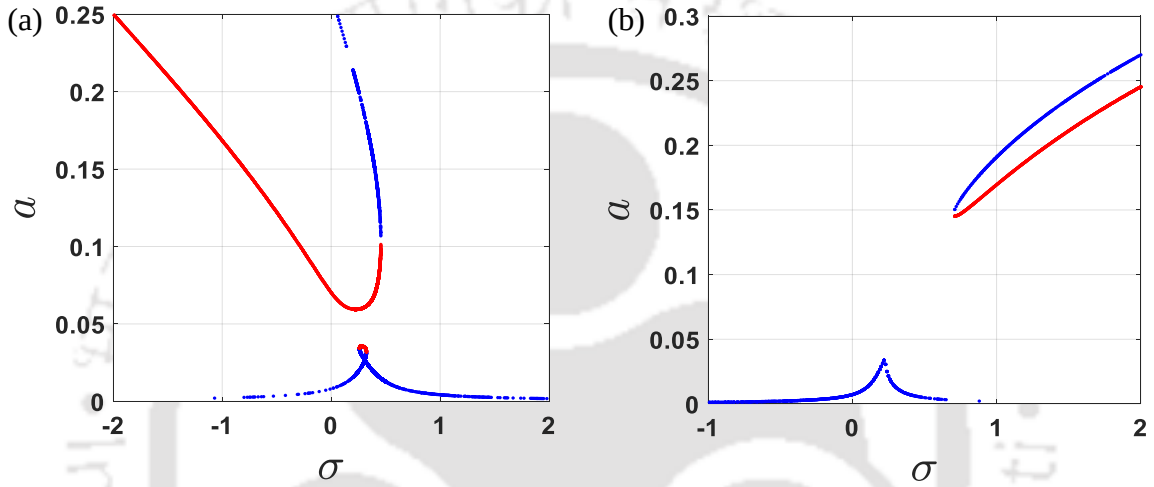


Fig. 5.15 Frequency responses with variation of F with (a) 10 kN (b) 6 kN.

To study the stability of the system with the variation of F , the frequency responses have been plotted with two different values in Fig. 5.15. With increase in the value of $F = 10$ kN as compared to the Fig. 5.2, the response curve in Zone 2 in Fig. 5.15(a) inclined towards left and the subcritical Hopf bifurcation point is observed at $\sigma = 0.456$. The Zone 1 in Fig. 5.15(a) shows two supercritical Hopf bifurcation points at $\sigma = 0.259$ and $\sigma = 0.325$. In this case, the maximum response amplitude a can be attained with the lower value of the σ . In Fig. 5.15(b) with decrease in the value of $F = 6$ kN as compared to Fig. 5.2 shows the similar response behaviour with the increase in the operating pressure (P) in Fig. 5.9(a) and damping parameter (μ) in Fig. 5.13(a). Here, in Zone 1, the bifurcation point is observed at $\sigma = 0.22$ and the system will behave as a linear system like the previous case mentioned in Fig. 5.9(a) and Fig. 5.13(a). The saddle node bifurcation point at Zone 2 in Fig. 5.15(b) can be noticed at $\sigma = 0.71$ and the maximum response amplitude $a = 0.1909$ at $\sigma = 1$ which is decreased by 48.6% as compared to that in Fig. 5.2.

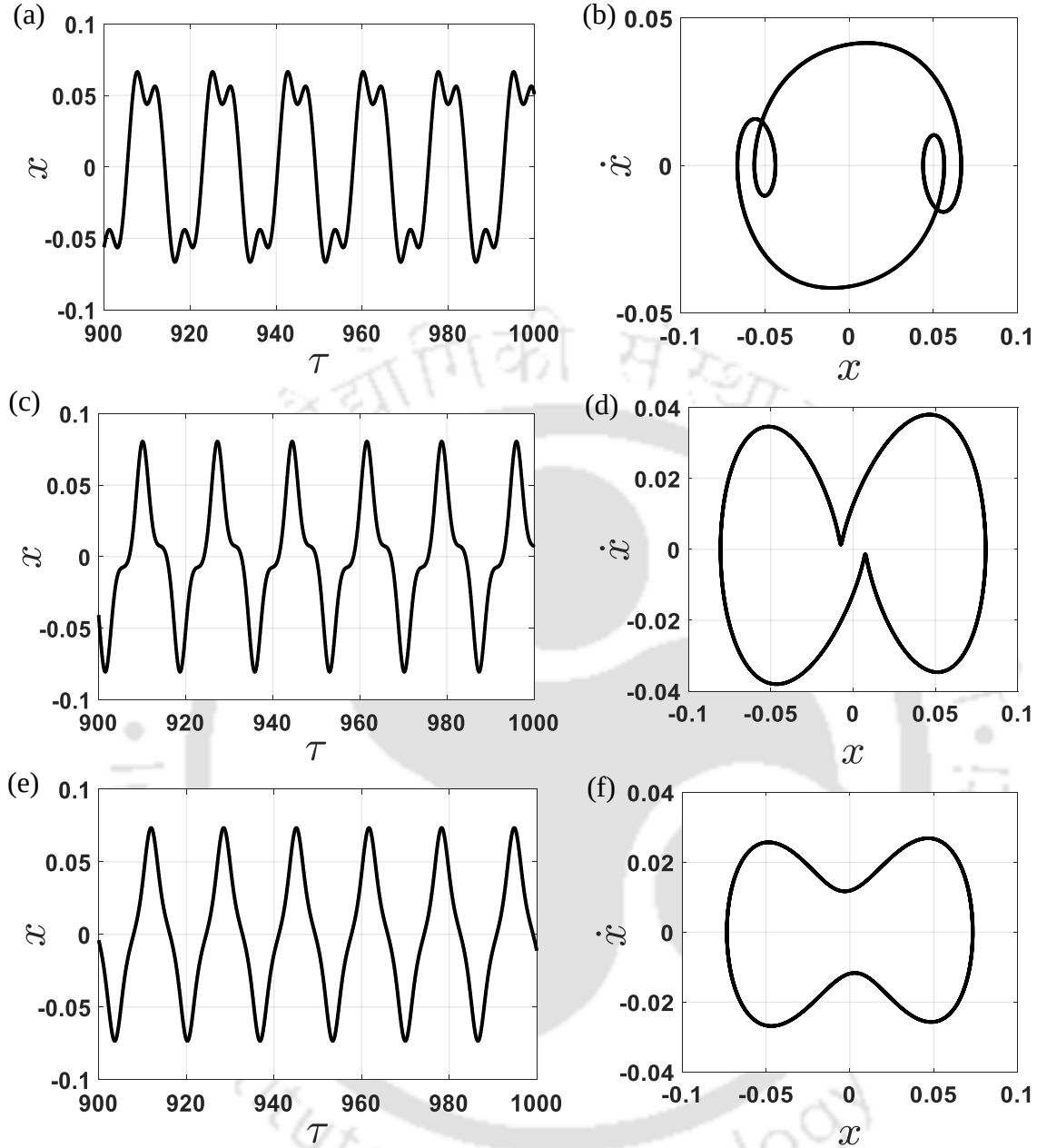


Fig. 5.16 Time response and phase portraits at (a, b) $\sigma = 0.259$ (c, d) $\sigma = 0.325$ (e, f) $\sigma = 0.456$ corresponding to Fig. 5.14(a).

The time response and phase portraits have been plotted in Fig. 5.16 to study the response behavior of the three Hopf bifurcation points at $\sigma = 0.259$ (Fig. 5.16(a, b)), $\sigma = 0.325$ (Fig. 5.16(c, d)) and $\sigma = 0.456$ (Fig. 5.16(e, f)) from Fig. 5.15(a). The responses are periodic in all the three cases which can be observed from the time responses. At $\sigma = 0.259$, the phase portraits in Fig. 5.16(b) shows a triple periodic response whereas at $\sigma = 0.325$ and $\sigma = 0.456$ in Fig. 5.16(d) and Fig. 5.16(f) the phase portraits are similar to that of homoclinic orbit.

5.4.1.4 Effect of the stiffness of external spring

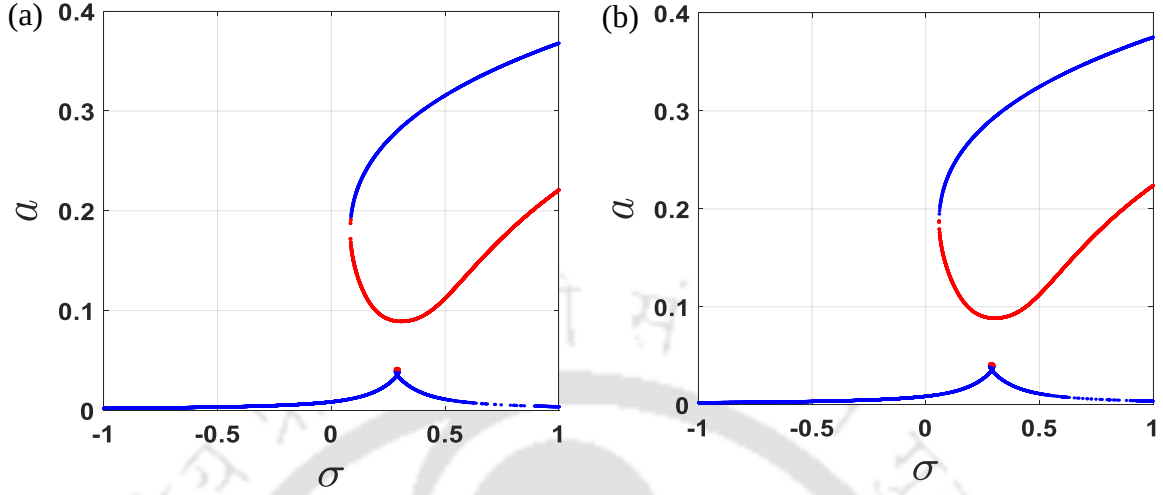


Fig. 5.17 Frequency responses with variation of k with (a) 16 N/mm (b) 8 N/mm.

It is observed that the effect of the stiffness of the external spring (k) on the system response is not changing with different values of k . Hence, the stability of the system has been studied for two different values of k by plotting frequency responses in Fig. 5.17. With increase or decrease in the value of k as compared to Fig. 5.2, the system will exhibit same response in both the zones which can be observed from Fig. 5.17(a) and Fig. 5.17(b). In Fig. 5.17(a), with increase in the value of the $k = 16$ N/mm as compared to Fig. 5.2, the maximum response amplitude $a = 0.3679$ which is decreased by 0.94%. Similarly with decrease in the value of $k = 8$ N/mm, the maximum response amplitude $a = 0.3752$ which is increased by 1% as compared to Fig. 5.2. The saddle node bifurcation points at Zone 2 in both the cases in Fig. 5.17(a) and Fig. 5.17(b) are occurred at detuning parameter $\sigma = 0.09$ and $\sigma = 0.06$ respectively. But the response in Zone 1, for the two different values of k is similar with that of Fig. 5.2 with two saddle node bifurcation points. These points occur at the same value of σ with Fig. 5.2 in both the cases. It may be noted that as nondimensional frequency has been taken for finding the response, the effect of k is not found. But in dimensional form, the effect will be prominent.

5.4.1.5 Effect of the nonlinearity

The effect of nonlinearity (α) has been observed with the help of phase portraits for different values of α in Fig. 5.18 at detuning parameter $\sigma = 0.293$. The system shows a periodic response in Fig. 5.18(a) for $\alpha = 10$ and in Fig. 5.18(b) with $\alpha = 100$, the response is similar to that of

homoclinic orbit. Again by increasing the value of the $\alpha = 400$ in Fig. 5.18(c), the system will depict three periodic response which can lead to chaos. This can be observed from the subsequent phase portraits plots in Fig. 5.18(d), Fig. 5.18(e) and Fig. 5.18(f) for the value of $\alpha = 1500$, $\alpha = 4500$ and $\alpha = 10000$, respectively.

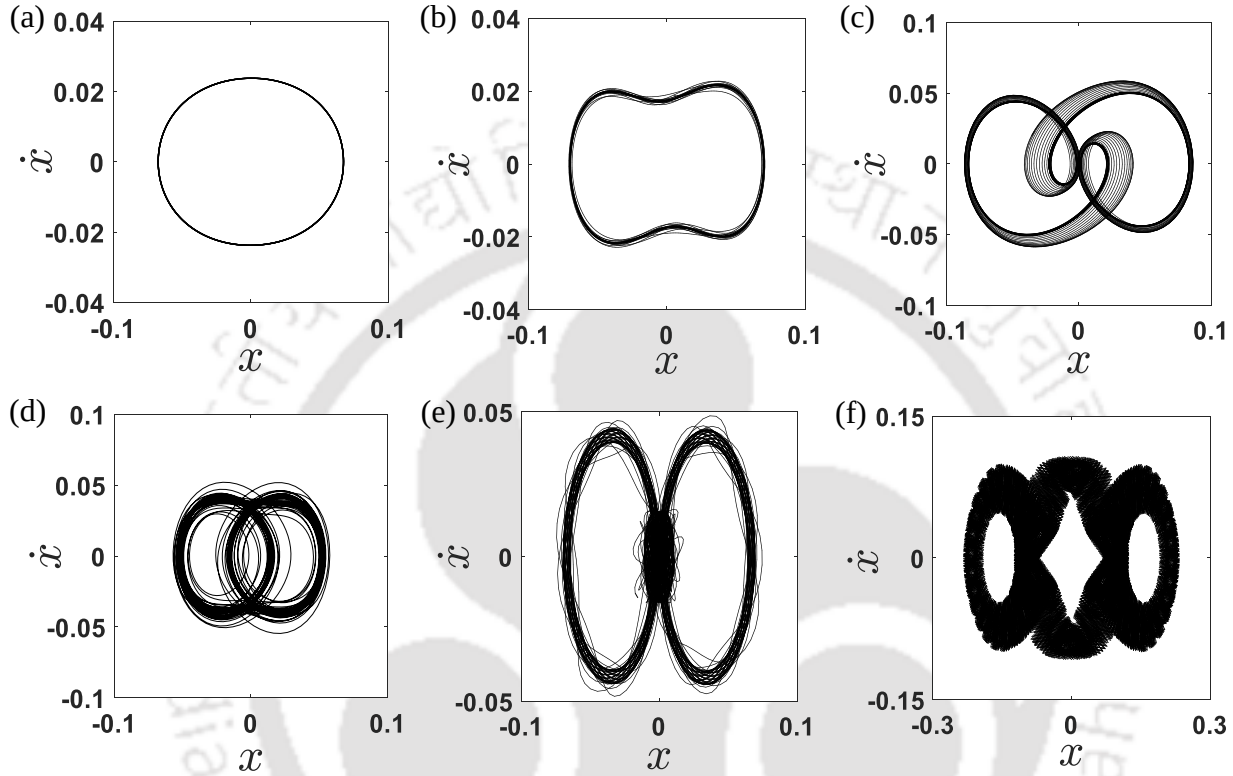


Fig. 5.18 Phase portraits with variation of α with (a) 10 (b) 100 (c) 400 (d) 1500 (e) 4500 (f) 10000.

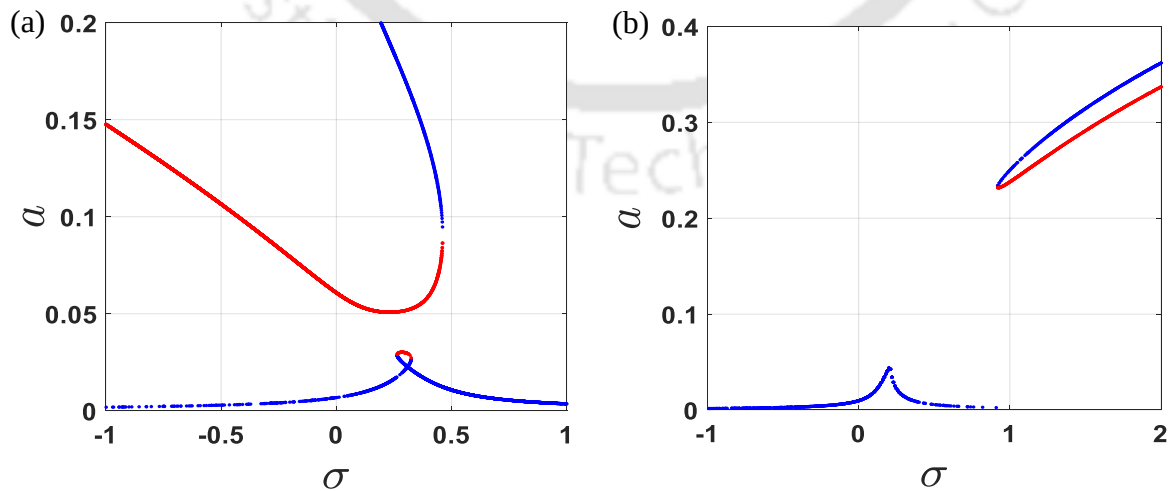


Fig. 5.19 Frequency responses with variation of α with (a) 600 (b) 200.

In Fig. 5.19, the stability of the system has been studied for two different values of α with the help of the frequency response plots. By increasing the value of α from 400 (Fig. 5.2) to 600, the response in Fig. 5.19(a) is showing a similar response as in Fig. 5.15(a). But in this case there is two saddle node bifurcation points in Zone 1 at $\sigma = 0.264$ and $\sigma = 0.325$ and only one Hopf bifurcation point may be observe at $\sigma = 0.461$ in Zone 2. Like the case in Fig. 5.15(a), in this case also maximum response amplitude a can be achieved at lower value of σ . Now, by decreasing the value of α to 200, the system shows the typical response as in Fig. 5.15(b) where all the solutions in the Zone 1 are stable. Here, the response is linear with bifurcation point at $\sigma = 0.2$ which is similar to the Zone 1 of the case in Fig. 5.9(a). In Zone 2 of the Fig. 5.19(b), the saddle node bifurcation point can be observed at $\sigma = 0.92$ and the maximum response amplitude $a = 0.2501$ at $\sigma = 1$ which is decreased by 32.7% as compared to Fig. 5.2.

5.4.1.6 Effect of the muscle parameter

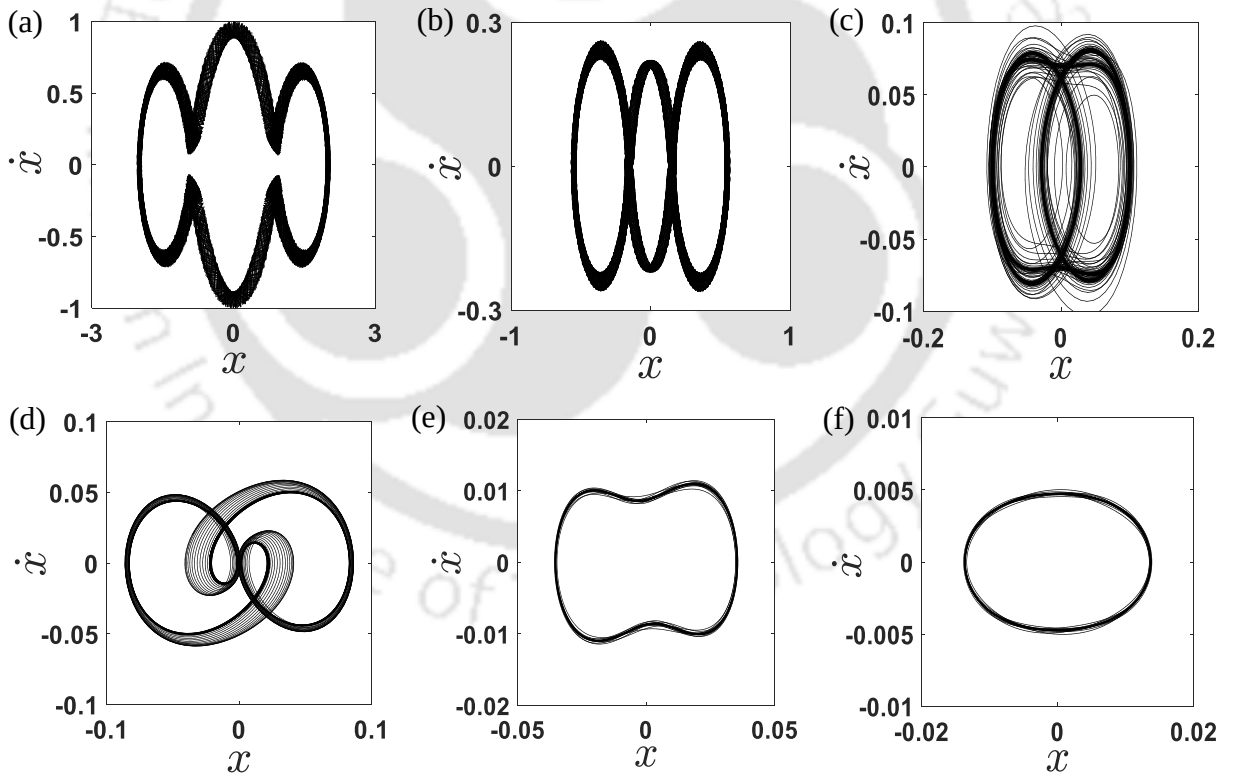


Fig. 5.20 Phase portraits with variation of c_3 with (a) 0.06 N/kPa² (b) 0.09 N/kPa² (c) 0.2 N/kPa² (d) 0.4 N/kPa² (e) 0.8 N/kPa² (f) 2 N/kPa².

In Fig. 5.20, the effect of the muscle parameter (c_3) has been studied at detuning parameter $\sigma = 0.293$ with the help of phase portraits for six different values of c_3 . The system is subjected to chaotic response with the lower value of c_3 which can be observed from the Fig. 5.20(a) with $c_3 = 0.06 \text{ N/kPa}^2$, Fig. 5.20(b) with $c_3 = 0.09 \text{ N/kPa}^2$ and Fig. 5.20(c) with $c_3 = 0.2 \text{ N/kPa}^2$. In Fig. 5.20(d), the system exhibits three periodic response with the value of $c_3 = 0.4 \text{ N/kPa}^2$ which is also a chaotic response. But with further increase in the value of $c_3 = 0.8 \text{ N/kPa}^2$ in Fig. 5.20(e), the response is similar to that of homoclinic orbit and finally when $c_3 = 2 \text{ N/kPa}^2$, the system will exhibit single periodic response which is observed in Fig. 5.20(f).

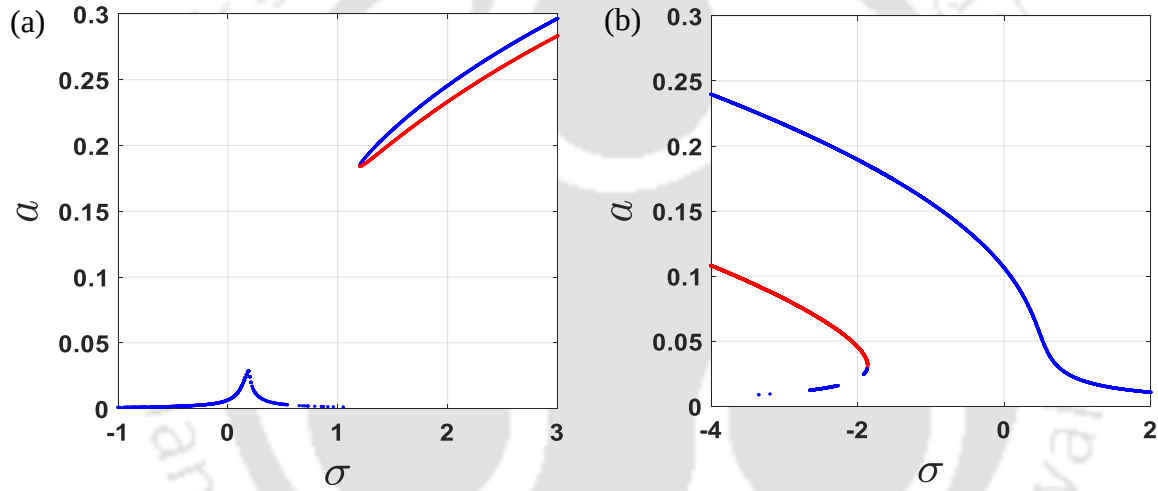


Fig. 5.21 Frequency responses with variation of c_3 with (a) 0.6 N/kPa^2 (b) 0.2 N/kPa^2 .

The stability of the system with the variation of c_3 is shown in Fig. 5.21 by plotting frequency responses. In Fig. 5.21(a) with increase in the value of $c_3 = 0.6 \text{ N/kPa}^2$ as compared to the Fig. 5.2, the system will behave as a linear system in Zone 1 with bifurcation point at $\sigma = 0.19$. In Zone 2 of the Fig. 5.21 (a), the saddle node bifurcation point is observed at $\sigma = 1.2$ and hence, the maximum response amplitude a cannot be achieved at $\sigma = 1$ like the previous cases. With decrease in the value of $c_3 = 0.2 \text{ N/kPa}^2$ in Fig. 5.21(b), the response will be similar to the case in Fig. 5.9(b) where the Zone 2 is not present. In this case, the saddle node bifurcation point can be noticed at $\sigma = -1.9$ and the maximum response amplitude a can be found with the lower value of σ . In the next subsection, subharmonic resonance condition has been studied.

5.4.2 Subharmonic resonance condition

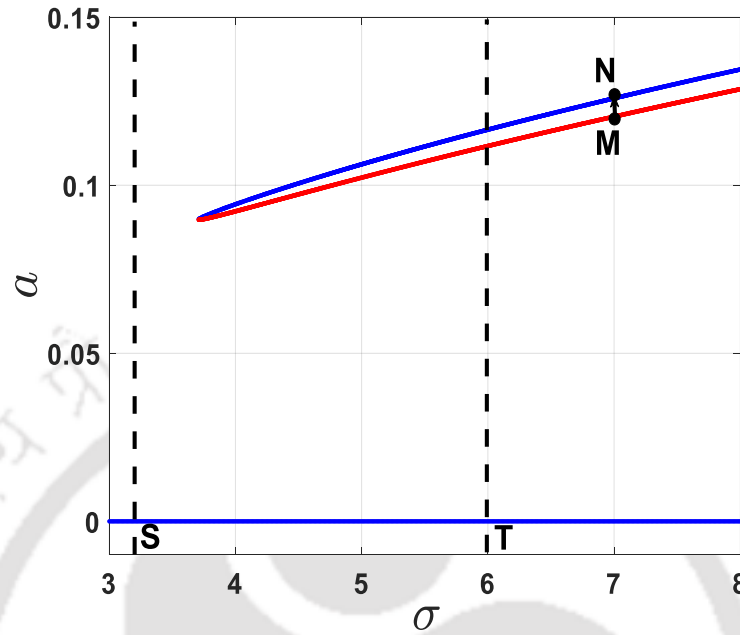


Fig. 5.22 Frequency response of the harmonically excited system under hard excitation with subharmonic resonance condition as the parameter values in Table 5.1.

A typical frequency response have been plotted in Fig. 5.22 for the system at subharmonic resonance condition where the external excitation frequency is nearly equal to three times of the natural frequency by using the system parameters as given in Table 5.1. In this resonance condition the system exhibits both trivial and nontrivial branches in the frequency plots. This can be observed from the reduced equations Eq. (5.39) and Eq. (5.40) using the MMS version II where $a = 0$ is a solution. Hence, to study the stability on the nontrivial branch the reduced equations Eq. (5.39) and Eq. (5.40) are used where the saddle node bifurcation point can be observed at detuning parameter $\sigma = 3.71$. For the trivial branch, the stability can be studied with the help of the Cartesian modulation equations Eq. (5.48) and Eq. (5.49). This trivial steady state response is always stable for a wide range of σ which can be noticed from Fig. 5.22. In the nontrivial branch of the frequency plot, the jump up phenomenon is observed at $\sigma = 7$ where the unstable solution with amplitude $a = 0.1204$ at point M has always a tendency to jump up to the point N with $a = 0.1259$ where the stable solution exists. The system exhibits a single stable solution on the trivial branch upto the bifurcation point at $\sigma = 3.71$. After that value of σ the nontrivial branch arises with a stable and an unstable solution i.e., bistable state can be found after $\sigma = 3.71$. This can be verified by plotting the basin of attraction in Fig. 5.23.

The basin of attraction have been plotted in Fig. 5.23 for two values of $\sigma = 3.2$ and $\sigma = 6$ which are marked as point S and point T respectively in Fig. 5.22. From Fig. 5.23(a) it can be verified that the system has only one stable solution i.e., $a = 0$ in the trivial branch of the frequency response of the system at $\sigma = 3.2$. But from Fig. 5.23(b), the system has bistable solution with $a = 0$ and $a = 0.117$ at $\sigma = 6$ along with an unstable solution $a = 0.112$. From the basin of attraction in Fig. 5.23(b), it can be noticed that the unstable solution $a = 0.112$ has a tendency to go the nearest stable solution with $a = 0.117$ which verifies the frequency response plot in Fig. 5.22.

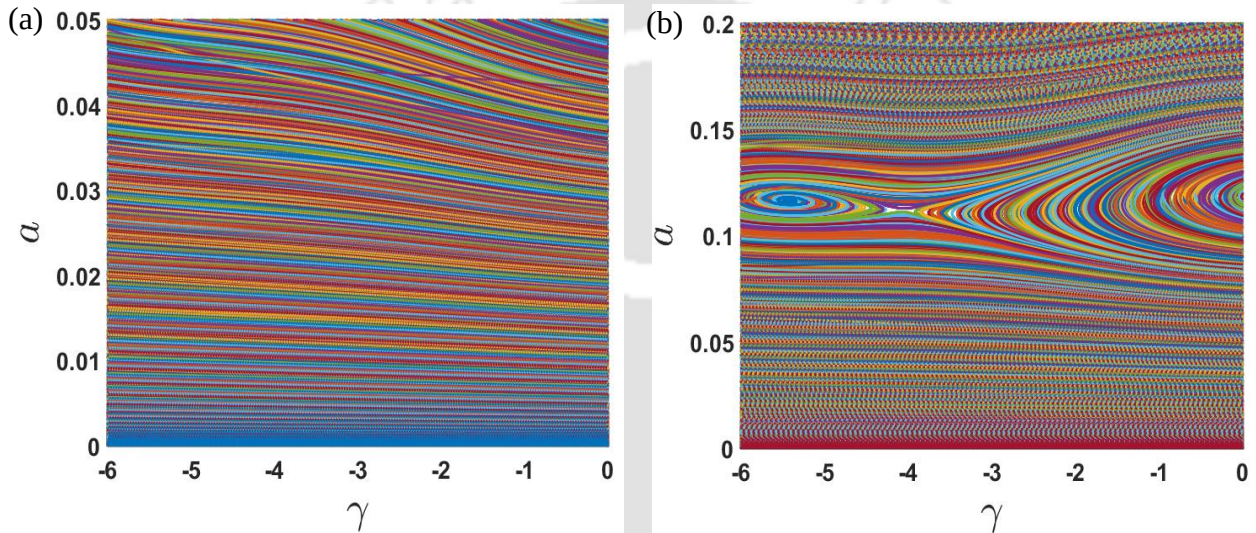


Fig. 5.23 Basin of attraction at (a) point S ($\sigma = 3.2$) (b) point T ($\sigma = 6$) in Fig. 5.22.

The comparison has been done in Fig. 5.24 between the solution obtained by solving the reduced equations Eq. (5.39) and Eq. (5.40) from the MMS version II for subharmonic resonance condition (marked 'AS') and the original Eq. (5.6) (marked 'NS'). The time response and phase portraits have been plotted at $\sigma = 6$ in Fig. 5.23(a, b) which are found to be periodic. The maximum responses are around 0.005 in the both the cases and also are in good agreement for any value of σ . Hence, the reduced equations Eq. (5.39) and Eq. (5.40) from the MMS version II are used to study the effect of the various system parameters on the system with the help of frequency response plots to reduce computational time and memory. In the following subsections the frequency responses have been plotted for different set of system parameters and the trivial branch is found to be stable in all the cases for a wide range of σ . Hence, the nature of the nontrivial branch of the frequency response plots has been discussed extensively.

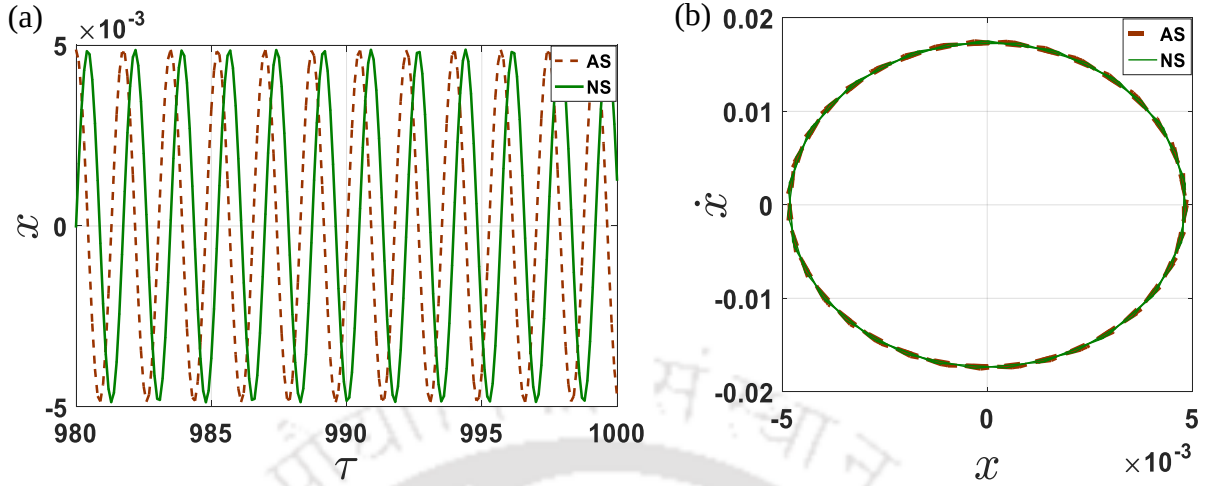


Fig. 5.24 Time response and phase portraits comparison at (a, b) point T ($\sigma = 6$) in Fig. 5.21.

5.4.2.1 Effect of the operating pressure

The effect of the operating pressure (P) on the stability of the nontrivial branch has been studied in Fig. 5.25 with two different values of P . In Fig. 5.25(a) with increase in the value of $P = 600$ kPa as compared to Fig. 5.22, the saddle node bifurcation point is observed at detuning parameter $\sigma = 7.35$. The maximum response amplitude $a = 0.1325$ at $\sigma = 8$ which is decreased by 1.4% from Fig. 5.22. With decrease in the value of $P = 400$ kPa in Fig. 5.25(b), the saddle node bifurcation point occurs at $\sigma = 1.65$ and the maximum response amplitude $a = 0.1085$ at $\sigma = 5$ which is increased by 2.2% in comparison to Fig. 5.22. From Fig. 5.25, it is observed that with lower value of P , the response amplitude can be obtained at lower value of σ .

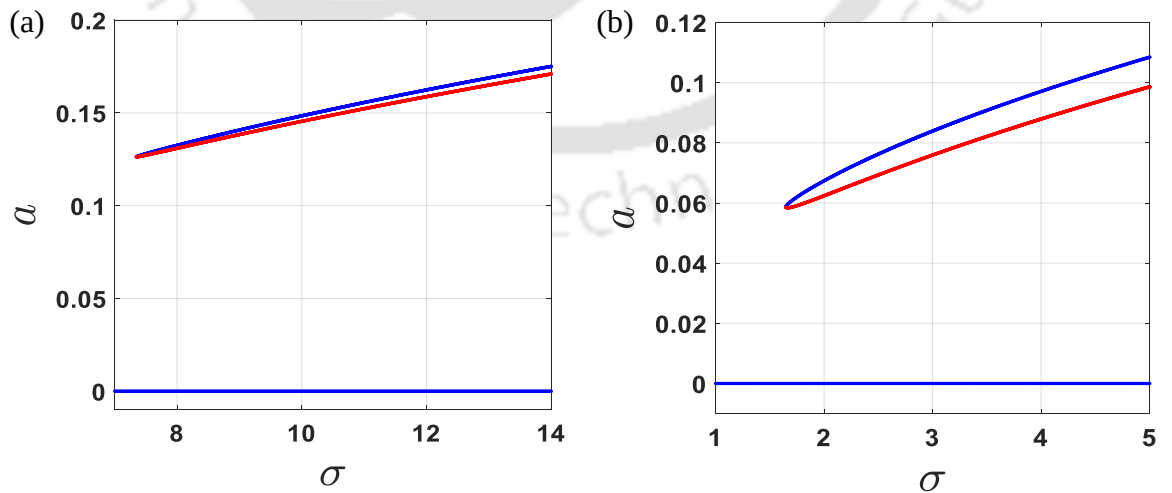


Fig. 5.25 Frequency responses with variation of P with (a) 600 kPa (b) 400 kPa.

5.4.2.2 Effect of the damping parameter

To study the stability on the nontrivial branch with the variation of the damping parameter (μ), the frequency responses have been plotted with two different values in Fig. 5.26. With increase in the value of $\mu = 0.05$ in Fig. 5.26(a) as compared to Fig. 5.21, the saddle node bifurcation point is occurred at very high value of detuning parameter $\sigma = 63.73$. The maximum response amplitude is $a = 0.3503$ at this bifurcation point. With decrease in the value of $\mu = 0.005$ in Fig. 5.26(b), at $\sigma = 0.98$ the bifurcation point occurs and the maximum response amplitude $a = 0.1077$ at $\sigma = 5$ which is increased by 1.4% as that in the case of Fig. 5.22. Hence, with lower value of μ , the response amplitude can be achieved at lower value of σ .

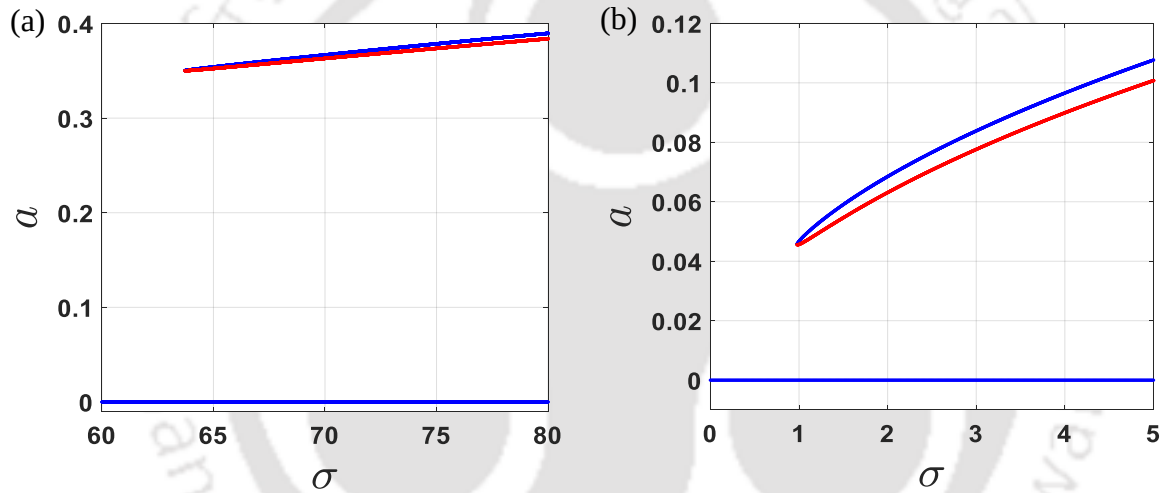


Fig. 5.26 Frequency responses with variation of μ with (a) 0.05 (b) 0.005.

5.4.2.3 Effect of the external load

The frequency responses have plotted in Fig. 5.27 to study the stability of the nontrivial branch of the system for two different values of external load (F). For $F = 10$ kN (Fig. 5.27(a)) and $F = 6$ kN (Fig. 5.27(b)), the maximum response amplitude $a = 0.1354$ and $a = 0.1331$ respectively at detuning parameter $\sigma = 8$. Hence, by comparing with Fig. 5.22, the maximum response amplitude will increased by 0.7% with the increase in the value of $F = 10$ kN and decreased by 1% with decrease in the value of $F = 6$ kN. The saddle node bifurcation points occur at $\sigma = 2.44$ and $\sigma = 6.4$ in Fig. 5.27(a) and Fig. 5.27(b), respectively. Hence, with higher value of F , the response amplitude can be observed at lower value of σ .

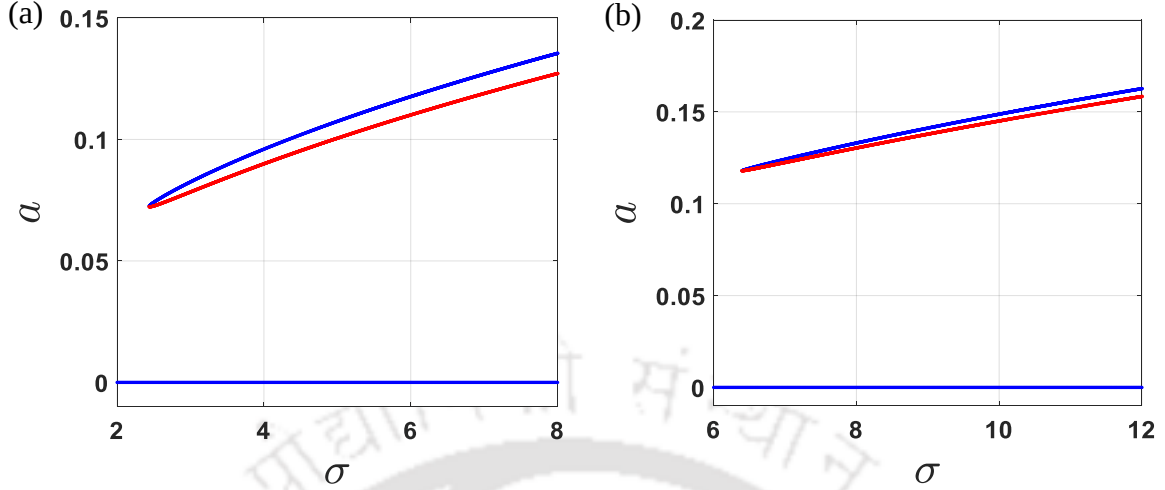


Fig. 5.27 Frequency responses with variation of F with (a) 10 kN (b) 6 kN.

5.4.2.4 Effect of the stiffness of the external spring

The effect of the stiffness of the external spring (k) on the system is not prominent for different values of k . The stability of the nontrivial response has been discussed for two different values of k by plotting frequency responses in Fig. 5.28. With increase or decrease in the value of k as compared to Fig. 5.22, the system will exhibit same maximum response amplitude $a = 0.1344$ at detuning parameter $\sigma = 8$ which can be observed from Fig. 5.28(a) ($k = 16$ N/mm) and Fig. 5.28(b) ($k = 8$ N/mm). The saddle node bifurcation points are slightly shift in both the cases as compared to Fig. 22, i.e., at $\sigma = 3.73$ in Fig. 5.28(a) and at $\sigma = 3.69$ in Fig. 5.28(b).

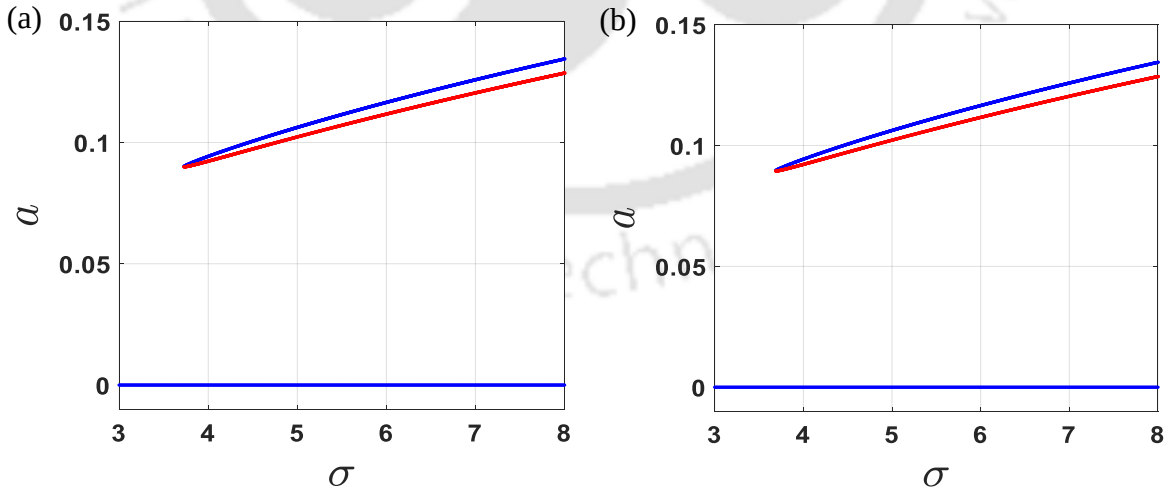


Fig. 5.28 Frequency responses with variation of k with (a) 16 N/mm (b) 8 N/mm.

5.4.2.5 Effect of the nonlinearity

The stability of the nontrivial branch with the variation of the nonlinearity (α) is shown in Fig. 5.28 by plotting frequency responses. In Fig. 5.29(a) with increase in the value of $\alpha = 600$ as compared to Fig. 5.22, the saddle node bifurcation point occurs at detuning parameter $\sigma = 2.53$. Similarly by decreasing the value of $\alpha = 200$ in Fig. 5.29(b), the saddle node bifurcation point can be obtained at $\sigma = 7.16$. The maximum response amplitude $a = 0.1105$ and $a = 0.1876$ at $\sigma = 8$ in Fig. 5.29(a) and Fig. 5.29(b) respectively can be obtained. The response amplitude as compared to Fig. 5.22 is decreased by 17.8% with increase in the value of α in Fig. 5.29(a) and increased by 39.6% with decrease in the value of α . Therefore, the response amplitude can be achieved at lower value of σ with higher value of α , but to obtain maximum response amplitude the value of α should be low.

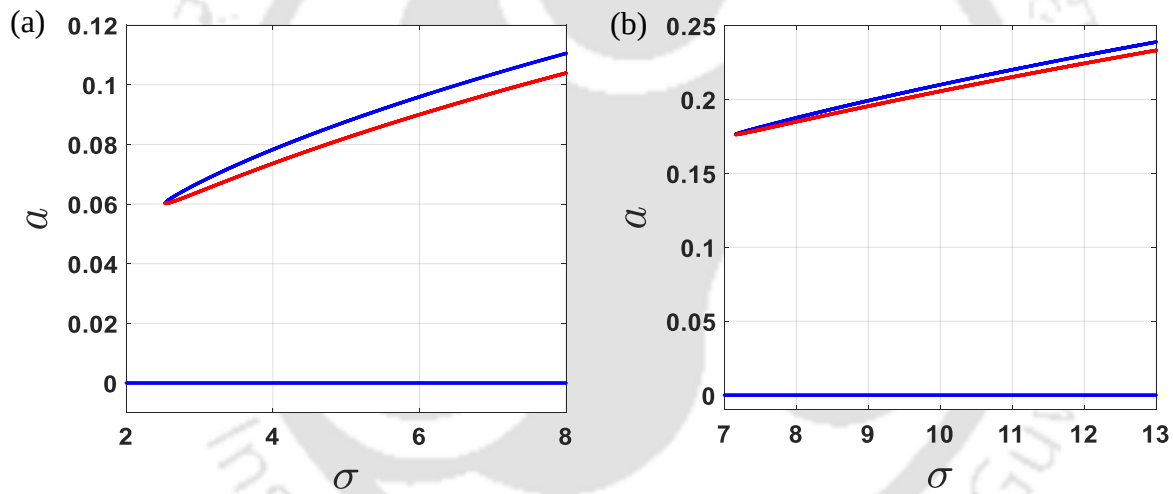


Fig. 5.29 Frequency responses with variation of α with (a) 600 (b) 200.

5.4.2.6 Effect of the muscle parameter

In Fig. 5.30, the stability of the nontrivial branch has been studied for two different values of muscle parameter (c_3) with the help of the frequency response plots. With increase in the value of $c_3 = 0.6 \text{ N/kPa}^2$ in Fig. 5.30(a), the saddle node bifurcation point is observed at detuning parameter $\sigma = 7.91$ and the maximum response amplitude $a = 0.132$ at $\sigma = 8$ which is decreased 0.7% as compared to Fig. 5.22. In Fig. 5.30(b), with decrease in the value of $c_3 = 0.2 \text{ N/kPa}^2$, the saddle node bifurcation point occurs at $\sigma = 1.14$ with the maximum response amplitude

$a = 0.1096$ at $\sigma = 5$ which is increased 3.2% in comparison with Fig. 5.22. Hence, with lower value of c_3 the response amplitude can be obtained at lower value of σ .

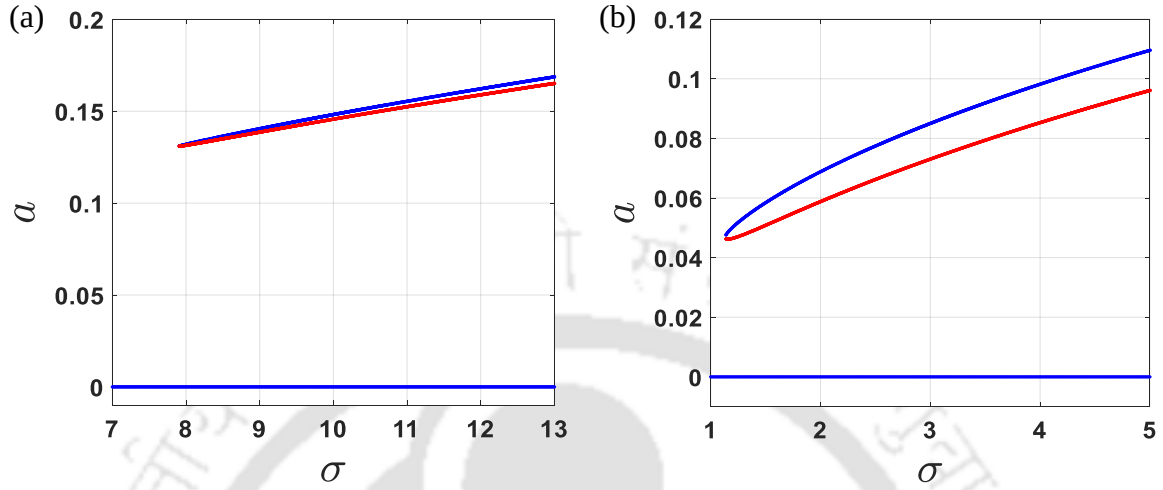


Fig. 5.30 Frequency responses with variation of c_3 with (a) 0.6 N/kPa^2 (b) 0.2 N/kPa^2 .

5.5 Summary

In this chapter, the nonlinear dynamics of a harmonically excited system attached with a nonlinear PAM is studied under hard excitation for superharmonic and subharmonic resonance conditions. Unlike the use of conventional method of multiple scales, here MMS version II is used. The results obtained from the reduced equations from the MMS version II using Newton's method are found to be in good agreement with the results obtained by numerically solving the original governing differential equation. The effects of different system parameters on the time and frequency response of the system have been extensively studied. The system shows fixed point, periodic, homoclinic orbits, three periodic and chaotic responses for different system parameters.

In the frequency response of superharmonic resonance condition, clearly two Zones have been observed. While in Zone 1 the system response is similar to that of a linear system in some cases, and in most of the cases two saddle node bifurcations have been found out. In Zone 2, both hardening and softening effects have been observed along with jump up and jump down phenomena. Here, the system exhibits saddle node, subcritical and supercritical Hopf bifurcations. In this resonance condition periodic, multi-harmonic, three periodic, homoclinic orbits and chaotic responses have been determined using the time responses and phase portraits for different system parameters. The butterfly effect generating chaotic responses has been shown with the

variations of the initial conditions. In case of subharmonic resonance condition, both trivial and nontrivial responses have been found from the frequency response plots. While the trivial state is always stable, the nontrivial state undergoes transition from stable to unstable region by saddle node bifurcation. This study will help the designer or the operator to identify the safe operating zone and required operating conditions such operating pressure, dimension and material properties of the muscle, stiffness of the external spring and load to have required moment of the artificial muscle.



Chapter 6

Nonlinear Dynamics of a Pneumatic Artificial Muscle under Parametric and Hard Excitation

6.1 Introduction

In this chapter, an attempt has been made to study the response of a single degree of freedom system with a PAM having periodically time varying pressure leading to a forced and parametrically excited system. Here, the forcing amplitude is higher than the coefficient of the linear term which leads system under hard excitation. It may be noted that in chapter 4 a similar system is considered with weak excitation force. The second order nonlinear governing equation of motion with hard excitation is obtained which is solved by using the method of multiple scales to obtain the response of the system. The mathematical modeling of such a system is discussed in section 6.2 and the approximate solution is obtained in section 6.3. In section 6.4 numerical results have been discussed and section 6.5 experimental validation has been carried out. Finally, a brief summary is presented in section 6.6.

6.2 Mathematical Modeling

Figure 6.1(a) shows a single degree of freedom system with mass m connected to a PAM. The PAM is modelled as a nonlinear spring and a damper subjected to a muscle force F_{mus} . The

expression for F_{mus} is given by the following equation which is similar to the work of Li et al. [125].

$$F_{mus}(u, P) = (c_1 + c_2 P + c_3 P^2) \left(\frac{u}{l_{\max}} \right) + (d_1 + d_2 P) + Ku^3 \quad (6.1)$$

where c_1 , c_2 , c_3 , d_1 , d_2 and K are the constants which can be obtained from the experiments. P is the operating pressure in the muscle to actuate u is the displacement with respect to the unstressed position of the system and $l_{\max}$ is the maximum possible length that can be attained by the muscle.

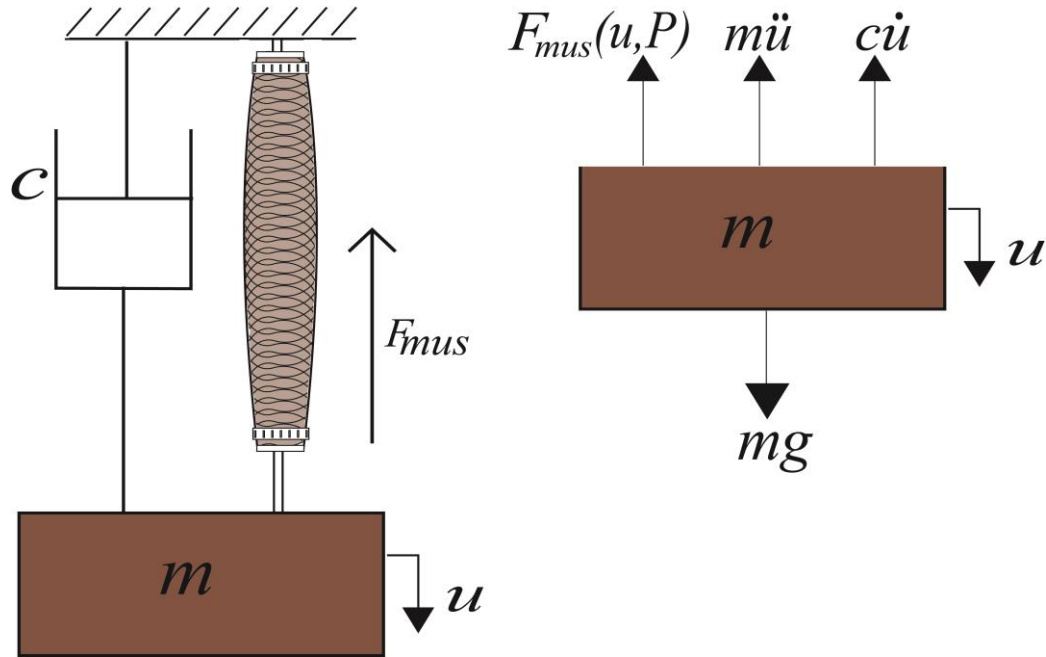


Fig. 6.1 (a) Equivalent spring-mass-damper system, (b) free-body diagram.

In this case, $P = P_m + P_0 \sin \omega t$ is considered where P_m is the static pressure and P_0 is the dynamic pressure in the muscle and ω is the frequency of the dynamic pressure. Figure 6.1(b) shows the free-body diagram to derive the equation of motion which can be given by the following equation.

$$m\ddot{u} + c\dot{u} + (c_1 + c_2 P + c_3 P^2) \left(\frac{u}{l_{\max}} \right) + (d_1 + d_2 P) + Ku^3 - mg = 0 \quad (6.2)$$

The overdot signifies the differentiation with respect to time t . Eq. (6.2) can be rearranged as follows.

$$\ddot{u} + \frac{c}{m} \dot{u} + \left[\frac{(c_1 + c_2 P + c_3 P^2)}{m l_{\max}} \right] u + \frac{K}{m} u^3 = \left[g - \frac{(d_1 + d_2 P)}{m} \right] \quad (6.3)$$

Now, considering a non-dimensional time $\tau = \omega_0 t$ where the fundamental natural frequency ω_0 of the system can be written as below.

$$\omega_0 = \sqrt{\frac{c_1 + c_2 P_m + c_3 \left(P_m^2 + \frac{P_0^2}{2} \right)}{m l_{\max}}} \quad (6.4)$$

The nondimensional displacement can be written as $u = r x$ by using a scaling factor r along with the following non-dimensional parameters have been used for formulation.

$$\Omega = \frac{\omega}{\omega_0}, \quad \mu = \frac{c}{2\varepsilon m \omega_0}, \quad \alpha = \frac{r^2 K}{\varepsilon m \omega_0^2}, \quad p_1 = \frac{c_2 P_0 + 2c_3 P_0 P_m}{\varepsilon m \omega_0^2 l_{\max}}, \quad p_2 = -\frac{c_3 P_0^2}{2\varepsilon m \omega_0^2 l_{\max}},$$

$$f_1 = \frac{d_2 P_m + d_1 - mg}{mr \omega_0^2}, \quad f_2 = \frac{d_2 P_0}{mr \omega_0^2} \quad (6.5)$$

Now, Eq. (6.3) can be simplified to the temporal equation of motion as follows.

$$\frac{d^2 x}{d\tau^2} + 2\varepsilon \mu \frac{dx}{d\tau} + x + \varepsilon (p_1 \sin \Omega \tau + p_2 \cos 2\Omega \tau) x + \varepsilon \alpha x^3 = (f_1 + f_2 \sin \Omega \tau) \quad (6.6)$$

In Eq. (6.6), the book keeping parameter ε is less than 1 and μ is the non-dimensional damping parameter. It may be noted that the nondimensional parameters p_1 , p_2 and f_2 are the function of P_0 ; p_1 and f_1 are the function of P_m . From the fourth term on the left-hand side of Eq. (6.6), it may clearly be observed that the coefficients of the response x contain time-varying terms with frequency Ω and 2Ω . Hence, this is a parametrically excited system with multi frequency excitation. Finally, the last term of the left hand side contains a cubic nonlinear term with the coefficient α . In addition to this, the system is also a forced vibration system having a sinusoidally varying force of amplitude f_2 and frequency Ω . So, in this work, the governing equation Eq. (6.6) with cubic nonlinearity is subjected to hard excitation where the magnitude of the forcing term is large than that of the coefficient of the linear term. In this work, the superharmonic $\left(\Omega \approx \frac{1}{3} \right)$ and

subharmonic ($\Omega \approx 3$) resonance conditions have been considered. The perturbation technique, method of multiple scales is discussed in the next section to find out the approximate solution of the nonlinear equation Eq. (6.6).

6.3 Approximate Solution

Due to the presence of various nonlinear terms in Eq. (6.6), it is very difficult to find the exact solution. So, there is a need to use the perturbation technique to find the approximate solution. Hence, the method of multiple scales is used to find out the solution of the equation of motion Eq. (6.6) where the displacement u can be written as $u(\tau; \varepsilon) = u_0(T_0, T_1) + \varepsilon u_1(T_0, T_1) + O(\varepsilon^2)$ in terms of different time scales $(T_0, T_1, \dots)$ where $T_n = \varepsilon^n \tau$; $n = 0, 1$ with a book keeping parameter ε . Following standard procedure of method of multiple scales [13, 14] the governing equation Eq. (6.6) can now be written as follows.

$$D_0^2 u_0 + \varepsilon D_0^2 u_1 + 2\varepsilon D_0 D_1 u_0 + 2\varepsilon \mu D_0 u_0 + u_0 + \varepsilon u_1 + \varepsilon u_0 (p_1 \sin \Omega \tau + p_2 \cos 2\Omega \tau) + \varepsilon \alpha u_0^3 = f_1 + f_2 \sin \Omega \tau \quad (6.7)$$

Equating the coefficient of ε^0 and ε^1 from Eq. (6.7), the following equations are found respectively.

$$D_0^2 u_0 + u_0 = f_1 + f_2 \sin \Omega \tau \quad (6.8)$$

$$D_0^2 u_1 + u_1 = -2D_0 D_1 u_0 - 2\mu D_0 u_0 - u_0 p_1 \sin \Omega T_0 - u_0 p_2 \cos 2\Omega T_0 - \alpha u_0^3 \quad (6.9)$$

The general solution of Eq. (6.8) can be expressed as follows.

$$u_0 = \frac{f_1}{2} + A e^{iT_0} + \Lambda e^{i\Omega T_0} + \frac{f_1}{2} + \bar{A} e^{-iT_0} + \Lambda e^{-i\Omega T_0} \quad (6.10)$$

where, $\Lambda = \frac{f_2}{2(1-\Omega^2)}$. A is a complex number and $\bar{A}$ is the complex conjugate of A . Substituting

the value of u_0 from Eq. (6.10) in Eq. (6.9) leads to the following equation.

$$\begin{aligned}
 D_0^2 u_1 + u_1 = & -2iD_1 A e^{iT_0} - 2i\Omega D_1 \Lambda e^{i\Omega T_0} - 2i\mu A e^{iT_0} - 2i\Omega \mu \Lambda e^{i\Omega T_0} + \frac{ip_1}{2} \left[\frac{f_1}{2} e^{i\Omega T_0} - \frac{f_1}{2} e^{-i\Omega T_0} \right. \\
 & + A e^{i(1+\Omega)T_0} - A e^{i(1-\Omega)T_0} + \Lambda e^{i2\Omega T_0} - \Lambda \left. \right] - \frac{p_2}{2} \left[\frac{f_1}{2} e^{i2\Omega T_0} + \frac{f_1}{2} e^{-i2\Omega T_0} + A e^{i(1+2\Omega)T_0} + A e^{i(1-2\Omega)T_0} \right. \\
 & + \Lambda e^{i3\Omega T_0} + \Lambda e^{-i3\Omega T_0} \left. \right] - \alpha \left[3\Lambda^3 e^{i\Omega T_0} + \Lambda^3 e^{i3\Omega T_0} + 3A^3 e^{i3T_0} + \frac{1}{2} f_1^3 + 3f_1 \Lambda^2 + 3A\bar{A}f_1 + 3f_1^2 \Lambda e^{i\Omega T_0} \right. \\
 & + 3f_1 \Lambda^2 e^{i2\Omega T_0} + 3A^2 \bar{A} e^{iT_0} + 3A f_1^2 e^{iT_0} + 3A^2 f_1 e^{i2T_0} + 6A\Lambda^2 e^{iT_0} + 6A\bar{A}\Lambda e^{i\Omega T_0} + 3A\Lambda^2 e^{iT_0} e^{-i2\Omega T_0} \\
 & + 3A\Lambda^2 e^{iT_0} e^{i2\Omega T_0} + 3A^2 \Lambda e^{i2T_0} e^{i\Omega T_0} + 3A^2 \Lambda e^{i2T_0} e^{-i\Omega T_0} + 6A f_1 \Lambda e^{iT_0} e^{i\Omega T_0} + 6A f_1 \Lambda e^{iT_0} e^{-i\Omega T_0} \left. \right] + CC
 \end{aligned} \quad (6.11)$$

where CC stands for the complex conjugate of the proceeding terms. It can be observed from the Eq. (6.11) that it will contain secular or nearly secular terms when $\Omega \approx 1$, $\Omega \approx \frac{1}{3}$ or $\Omega \approx 3$. The particular solution of these terms will be unbounded and therefore should be eliminated. From Eq. (6.11) it may be noted that when $\Omega \approx 1$, the simple resonance condition may occur and but as $\Lambda \sim \infty$, the system will exhibit a response similar to that of a linear system. Hence, it is not explored in this work. The other two resonance conditions $\Omega \approx \frac{1}{3}$ and $\Omega \approx 3$ are considered in this work which are known as superharmonic and subharmonic resonance condition. It may be noted that, for $\Omega \approx \frac{1}{3}$ the terms, i.e., $\frac{p_2}{2} \Lambda e^{i3\Omega T_0}$ and $\alpha \Lambda^3 e^{i3\Omega T_0}$ will contribute to superharmonic resonance condition and for $\Omega \approx 3$, the complex conjugate of the term $3\alpha A^2 \Lambda e^{i(2-\Omega)T_0}$ will lead to the subharmonic resonance condition. These two resonance cases have been studied in the following subsections.

6.3.1 Superharmonic resonance condition

In this case, the external excitation frequency Ω is nearly equal to the one third of the natural frequency of the system which is known to be superharmonic resonance condition. Here, one may introduce the detuning parameter σ as $\Omega = \frac{1}{3} + \varepsilon\sigma$ and substituting in Eq. (6.11), the following equation can be obtained. .

$$\begin{aligned}
 D_0^2 u_1 + u_1 = & -2iD_1 A e^{iT_0} - 2i\Omega D_1 \Lambda e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} - 2i\mu A e^{iT_0} - 2i\Omega\mu\Lambda e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + \frac{ip_1}{2} \left[\frac{f_1}{2} e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} \right. \\
 & - \frac{f_1}{2} e^{-i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + A e^{i\left(1 + \left(\frac{1}{3} + \varepsilon\sigma\right)\right)T_0} - A e^{i\left(1 - \left(\frac{1}{3} + \varepsilon\sigma\right)\right)T_0} + \Lambda e^{i2\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} - \Lambda \left. \right] - \frac{p_2}{2} \left[\frac{f_1}{2} e^{i2\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} \right. \\
 & + \frac{f_1}{2} e^{-i2\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + A e^{i\left(1 + 2\left(\frac{1}{3} + \varepsilon\sigma\right)\right)T_0} + A e^{i\left(1 - 2\left(\frac{1}{3} + \varepsilon\sigma\right)\right)T_0} + \Lambda e^{i3\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + \Lambda e^{-i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} \left. \right] \\
 & - \alpha \left[3\Lambda^3 e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + \Lambda^3 e^{i3\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + 3A^3 e^{i3T_0} + \frac{1}{2} f_1^3 + 3f_1 \Lambda^2 + 3A\bar{A}f_1 + 3f_1^2 \Lambda e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} \right. \\
 & + 3f_1 \Lambda^2 e^{i2\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + 3A^2 \bar{A} e^{iT_0} + 3A f_1^2 e^{iT_0} + 3A^2 f_1 e^{i2T_0} + 6A\Lambda^2 e^{iT_0} + 6A\bar{A}\Lambda e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} \\
 & + 3A\Lambda^2 e^{iT_0} e^{-i2\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + 3A\Lambda^2 e^{iT_0} e^{i2\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + 3A^2 \Lambda e^{i2T_0} e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + 3A^2 \Lambda e^{i2T_0} e^{-i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} \\
 & \left. + 6A f_1 \Lambda e^{iT_0} e^{i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} + 6A f_1 \Lambda e^{iT_0} e^{-i\left(\frac{1}{3} + \varepsilon\sigma\right)T_0} \right] + CC
 \end{aligned} \tag{6.12}$$

The secular and nearly secular terms in Eq. (6.12) which should be eliminated to have bounded solution are given below.

$$-2iD_1 A - 2i\mu A - \Lambda e^{3i\varepsilon\sigma T_0} - \alpha\Lambda^3 e^{3i\varepsilon\sigma T_0} - 3\alpha A^2 \bar{A} - 3\alpha A f_1^2 - 6\alpha A\Lambda^2 = 0 \tag{6.13}$$

$$D_1 A = -\mu A + \frac{i}{2} \Lambda e^{3i\varepsilon\sigma T_0} + \frac{i}{2} \alpha\Lambda^3 e^{3i\varepsilon\sigma T_0} + \frac{3i}{2} \alpha A^2 \bar{A} + \frac{3i}{2} \alpha A f_1^2 + 3i\alpha A\Lambda^2 \tag{6.14}$$

Again,

$$A' = \varepsilon D_1 A = \varepsilon \left(-\mu A + \frac{i}{2} \Lambda e^{3i\varepsilon\sigma T_0} + \frac{i}{2} \alpha\Lambda^3 e^{3i\varepsilon\sigma T_0} + \frac{3i}{2} \alpha A^2 \bar{A} + \frac{3i}{2} \alpha A f_1^2 + 3i\alpha A\Lambda^2 \right) \tag{6.15}$$

Here, A is function of T_1 and considering the polar form i.e., $A = \frac{1}{2} a e^{i\beta}$ where a and β represents the amplitude and phase of the response. Now, by taking $\gamma = 3\varepsilon\sigma T_0 - \beta$ in Eq. (6.15) and separating the real and imaginary parts, one may find a set of following reduced equations.

$$a' = \varepsilon \left(-\mu a - \Lambda \sin \gamma - \alpha\Lambda^3 \sin \gamma \right) \tag{6.16}$$

$$a\gamma' = 3a\varepsilon\sigma - \varepsilon \left(\Lambda \cos \gamma + \alpha\Lambda^3 \cos \gamma + \frac{3}{8} \alpha a^3 + \frac{3}{2} \alpha a f_1^2 + 3\alpha a\Lambda^2 \right) \tag{6.17}$$

Now, for steady state response (a_0, γ_0) , a set of two nonlinear equation can be achieved by putting $a' = 0$ and $\gamma' = 0$ in Eq. (6.16) and Eq. (6.17) as follows.

$$(\Lambda + \alpha\Lambda^3)\sin\gamma = -\mu a \quad (6.18)$$

$$(\Lambda + \alpha\Lambda^3)\cos\gamma = 3a\sigma - \frac{3}{8}\alpha a^3 - \frac{3}{2}\alpha a f_1^2 - 3\alpha a\Lambda^2 \quad (6.19)$$

Squaring and adding these equations Eq. (6.18) and Eq. (6.19) leads to the following frequency response equation of the parametrically excited system attached with a nonlinear PAM under hard excitation.

$$(\Lambda + \alpha\Lambda^3)^2 = \left[\mu^2 + \left(3\sigma - \frac{3}{8}\alpha a^2 - \frac{3}{2}\alpha f_1^2 - 3\alpha\Lambda^2 \right)^2 \right] a^2 \quad (6.20)$$

From Eq. (6.20), one may noticed that $a = 0$ is not a solution. Hence, there is no trivial state response exist in the case of superharmonic resonance condition. But one may get the nontrivial state response for the system by solving the Eq. (6.16) and Eq. (6.17) simultaneously. Hence, substituting $a = a_0 + a_1$ and $\gamma = \gamma_0 + \gamma_1$ where a_0 and γ_0 are the equilibrium points in Eq. (6.16) and Eq. (6.17) the stability of the steady state response can be found by determining the eigenvalues of the Jacobian matrix (J). The Jacobian matrix (J) is as follows.

$$J = \varepsilon \begin{pmatrix} -\mu & -3a_0\sigma + \frac{3}{8}\alpha a_0^3 + \frac{3}{2}\alpha a_0 f_1^2 + 3\alpha a_0\Lambda^2 \\ \frac{3\sigma}{a_0} - \frac{9}{8}\alpha a_0 - \frac{3}{2a_0}\alpha f_1^2 - \frac{3\alpha\Lambda^2}{a_0} & -\mu \end{pmatrix} \quad (6.21)$$

The system will be stable for the superharmonic resonance condition if the real parts of all the eigenvalues of the Jacobian matrix (J) in Eq. (6.21) are negative.

6.3.2 Subharmonic resonance condition

In this subharmonic resonance condition, the external excitation frequency Ω is nearly equal to the thrice of the natural frequency of the system. Here, considering $\Omega = 3 + \varepsilon\sigma$, Eq. (6.11) can be rewritten as follows.

$$\begin{aligned}
 D_0^2 u_1 + u_1 = & -2iD_1 A e^{iT_0} - 2i\Omega D_1 \Lambda e^{i(3+\varepsilon\sigma)T_0} - 2i\mu A e^{iT_0} - 2i\Omega\mu\Lambda e^{i(3+\varepsilon\sigma)T_0} + \frac{ip_1}{2} \left[\frac{f_1}{2} e^{i(3+\varepsilon\sigma)T_0} \right. \\
 & - \frac{f_1}{2} e^{-i(3+\varepsilon\sigma)T_0} + A e^{i(1+(3+\varepsilon\sigma))T_0} - A e^{i(1-(3+\varepsilon\sigma))T_0} + \Lambda e^{i2(3+\varepsilon\sigma)T_0} - \Lambda \left. \right] - \frac{p_2}{2} \left[\frac{f_1}{2} e^{i2(3+\varepsilon\sigma)T_0} \right. \\
 & + \frac{f_1}{2} e^{-i2(3+\varepsilon\sigma)T_0} + A e^{i(1+2(3+\varepsilon\sigma))T_0} + A e^{i(1-2(3+\varepsilon\sigma))T_0} + \Lambda e^{i3(3+\varepsilon\sigma)T_0} + \Lambda e^{-i(3+\varepsilon\sigma)T_0} \left. \right] - \alpha \left[3\Lambda^3 e^{i(3+\varepsilon\sigma)T_0} \right. \\
 & + \Lambda^3 e^{i3(3+\varepsilon\sigma)T_0} + 3A^3 e^{i3T_0} + \frac{1}{2} f_1^3 + 3f_1 \Lambda^2 + 3A\bar{A}f_1 + 3f_1^2 \Lambda e^{i(3+\varepsilon\sigma)T_0} + 3f_1 \Lambda^2 e^{i2(3+\varepsilon\sigma)T_0} + 3A^2 \bar{A} e^{iT_0} \\
 & + 3Af_1^2 e^{iT_0} + 3A^2 f_1 e^{i2T_0} + 6A\Lambda^2 e^{iT_0} + 6A\bar{A}\Lambda e^{i(3+\varepsilon\sigma)T_0} + 3A\Lambda^2 e^{iT_0} e^{-i2(3+\varepsilon\sigma)T_0} + 3A\Lambda^2 e^{iT_0} e^{i2(3+\varepsilon\sigma)T_0} \\
 & \left. + 3A^2 \Lambda e^{i2T_0} e^{i(3+\varepsilon\sigma)T_0} + 3A^2 \Lambda e^{i2T_0} e^{-i(3+\varepsilon\sigma)T_0} + 6Af_1 \Lambda e^{iT_0} e^{i(3+\varepsilon\sigma)T_0} + 6Af_1 \Lambda e^{iT_0} e^{-i(3+\varepsilon\sigma)T_0} \right] + CC
 \end{aligned} \quad (6.22)$$

Now by eliminating the secular and nearly secular terms in Eq. (6.22), one may obtain the following expression.

$$-2iD_1 A - 2i\mu A - 3\alpha A^2 \bar{A} - 3\alpha A f_1^2 - 6\alpha A \Lambda^2 - 3\alpha \bar{A}^2 \Lambda e^{i\varepsilon\sigma T_0} = 0 \quad (6.23)$$

$$D_1 A = -\mu A + \frac{3i}{2} \alpha A^2 \bar{A} + \frac{3i}{2} \alpha A f_1^2 + 3i\alpha A \Lambda^2 + \frac{3i}{2} \alpha \bar{A}^2 \Lambda e^{i\varepsilon\sigma T_0} \quad (6.24)$$

Here,

$$A' = \varepsilon D_1 A = \varepsilon \left(-\mu A + \frac{3i}{2} \alpha A^2 \bar{A} + \frac{3i}{2} \alpha A f_1^2 + 3i\alpha A \Lambda^2 + \frac{3i}{2} \alpha \bar{A}^2 \Lambda e^{i\varepsilon\sigma T_0} \right) \quad (6.25)$$

Now, substituting $A = \frac{1}{2} a e^{i\beta}$ and $\gamma = \varepsilon\sigma T_0 - 3\beta$ in Eq. (6.25) and separating the real and imaginary parts, one may get the following reduced equations.

$$a' = \varepsilon \left(-\mu a - \frac{3}{4} \alpha a^2 \Lambda \sin \gamma \right) \quad (6.26)$$

$$a\gamma' = a\varepsilon\sigma - 3\varepsilon \left(\frac{3}{8} \alpha a^3 + \frac{3}{2} \alpha a f_1^2 + 3\alpha a \Lambda^2 + \frac{3}{4} \alpha a^2 \Lambda \cos \gamma \right) \quad (6.27)$$

For steady state response (a_0, γ_0) , following similar procedure as that in superharmonic resonance condition one can obtain the following frequency response equation of the parametrically excited system attached with a nonlinear PAM under hard excitation.

$$\frac{9}{16} \alpha^2 \Lambda^2 a^4 = \left(\mu^2 + \left(\frac{\sigma}{3} - \frac{3}{8} \alpha a^2 - \frac{3}{2} \alpha f_1^2 - 3\alpha \Lambda^2 \right)^2 \right) a^2 \quad (6.28)$$

From Eq. (6.28), it may be noted that for subharmonic resonance condition, one may obtain both trivial state (i.e., $a = 0$) and nontrivial state (i.e., $a \neq 0$) responses. Hence, to obtain the nontrivial state response for the system consider $a = a_0 + a_1$ and $\gamma = \gamma_0 + \gamma_1$ where a_0 and γ_0 are the equilibrium points in Eq. (6.26) and Eq. (6.27) and the stability of the steady state response can be found by determining the eigenvalues of the Jacobian matrix (J). The Jacobian matrix (J) is as follows.

$$J = \varepsilon \begin{pmatrix} \mu & -\frac{a_0 \sigma}{3} + \frac{3}{8} \alpha a_0^3 + \frac{3}{2} \alpha a_0 f_1^2 + 3 \alpha a_0 \Lambda^2 \\ -\frac{9}{4} \alpha a_0 - \frac{\sigma}{a_0} + \frac{9}{8} \alpha a_0 + \frac{9}{2 a_0} \alpha f_1^2 + \frac{9 \alpha \Lambda^2}{a_0} & -3 \mu \end{pmatrix} \quad (6.29)$$

The system will be stable for the subharmonic resonance condition if the real parts of all the eigenvalues of the Jacobian matrix (J) in Eq. (6.29) are negative.

3.2.1 Stability of the trivial steady state response

The polar form of modulation Eq. (6.27) contains term like $a\gamma'$ but for finding the stability of the trivial state the linearized equation will not contain the perturbation γ'_1 . So, the stability of the trivial solutions may be found by converting the Eq. (6.26) and Eq. (6.27) to their Cartesian form of modulations by introducing the transformation $p = a \cos \gamma$ and $q = a \sin \gamma$ [13, 14]. Now one may obtain from Eq. (6.26) and Eq. (6.27), the resulting Cartesian modulation equations are as follows.

$$p' = \varepsilon \left(-\mu p + \frac{3}{2} \alpha \Lambda p q + \frac{9}{8} \alpha q (p^2 + q^2) + \frac{9}{2} \alpha f_1^2 q + 9 \alpha \Lambda^2 q \right) - \varepsilon \sigma q \quad (6.30)$$

$$q' = \varepsilon \left(-\mu q - \frac{3}{4} \alpha \Lambda (3p^2 + q^2) - \frac{9}{8} \alpha p (p^2 + q^2) - \frac{9}{2} \alpha f_1^2 p - 9 \alpha \Lambda^2 p \right) + \varepsilon \sigma p \quad (6.31)$$

Similarly, the stability of the steady state response (p_0, q_0) for this case can be achieved by examining the eigenvalues of the Jacobian matrix (J) found by perturbing Eq. (6.30) and Eq. (6.31). The Jacobian matrix can be written as follows.

$$J = \varepsilon \begin{pmatrix} \begin{pmatrix} -\mu + \frac{3}{2}\alpha\Lambda q_0 + \frac{9}{4}\alpha p_0 q_0 \end{pmatrix} & \begin{pmatrix} \frac{3}{2}\alpha\Lambda p_0 + \frac{9}{8}\alpha p_0^2 + \frac{27}{8}\alpha q_0^2 + \frac{9}{2}\alpha f_1^2 \\ +9\alpha\Lambda^2 \end{pmatrix} - \sigma \\ \begin{pmatrix} -\frac{9}{2}\alpha\Lambda p_0 - \frac{27}{8}\alpha p_0^2 - \frac{9}{8}\alpha q_0^2 - \frac{9}{2}\alpha f_1^2 \\ -9\alpha\Lambda^2 \end{pmatrix} + \sigma & \begin{pmatrix} -\mu - \frac{3}{2}\alpha\Lambda q_0 - \frac{9}{4}\alpha p_0 q_0 \end{pmatrix} \end{pmatrix} \quad (6.32)$$

Here, the system will be stable for trivial state responses when all the real parts the eigenvalues of the Jacobian matrix (J) in Eq. (6.32) are negative.

6.4 Results and Discussion

In this section, the time and frequency responses for the system as shown in Fig. 6.1 have been plotted by taking different set of system parameters values for superharmonic and subharmonic resonance conditions. The value of the different system parameters are given in Table 6.1 which is similar to the work of Li et al. [125]. Here, the steady state responses can be obtained from Eq. (6.21) for superharmonic resonance condition and Eq. (6.29) for subharmonic resonance condition from the method of multiple scales. In the following subsections results for superharmonic and subharmonic resonance conditions have been discussed with the variation of system parameters. In all the frequency response plots red color depicts the unstable solution and blue color shows the stable solutions.

Table 6.1 System parameter values used for the numerical simulation.

Parameter	Numerical Value	Parameter	Numerical Value	Parameter	Numerical Value
$l_{\max}$	74 mm	d_1	-100 N	P_0	58 kPa
c_1	-234.25 N	d_2	1 N/kPa	ε	0.1
c_2	1.96 N/kPa	m	6 N	μ	0.01
c_3	-0.003 N/kPa ²	P_m	300 kPa	α	150

6.4.1 Superharmonic resonance condition

In this section, the responses are plotted for the superharmonic resonance condition where the external excitation frequency is one third of the natural frequency of the system. A typical

frequency response plot is shown Fig. 6.2. The stability of the steady state response of the system is examined with the help of the Jacobian matrix Eq. (6.21).

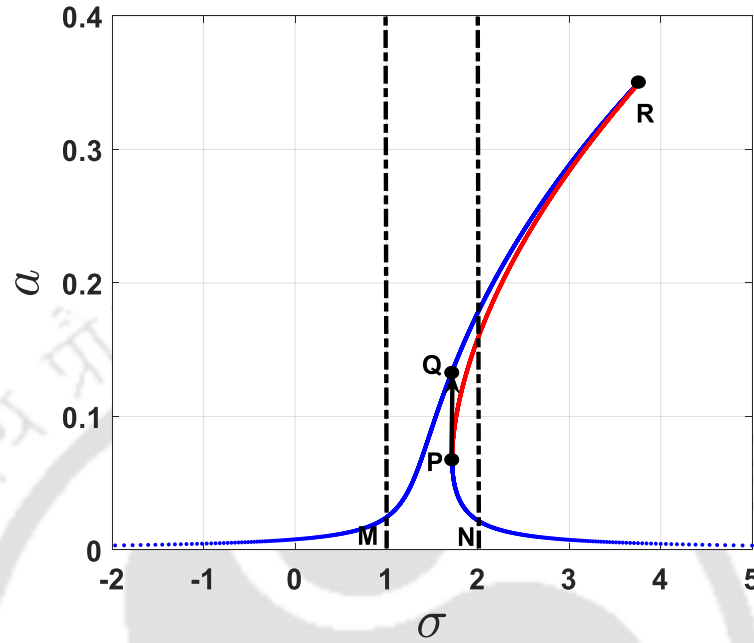


Fig. 6.2 Frequency response of the system under parametric and hard excitation with superharmonic resonance condition as the parameter values in Table 6.1.

Here, one may observe two distinct saddle node bifurcation points at $\sigma=1.718$ (point P) and $\sigma=3.758$ (point R) in the frequency response plot. The jump up phenomenon can be noticed at $\sigma=1.718$ where the unstable solution with amplitude $a=0.0683$ at point P has always a tendency to reach the stable solution with $a=0.1337$ at point Q in the stable branch of the nontrivial steady state response. The maximum response amplitude $a=0.3499$ for the system can be achieved at point R. In Fig. 6.3, the time response and phase portraits have been plotted with the help of the reduced equations Eq. (6.16) and Eq. (6.17) near the mentioned saddle node bifurcation points. The responses are found to be periodic containing multi frequency components.

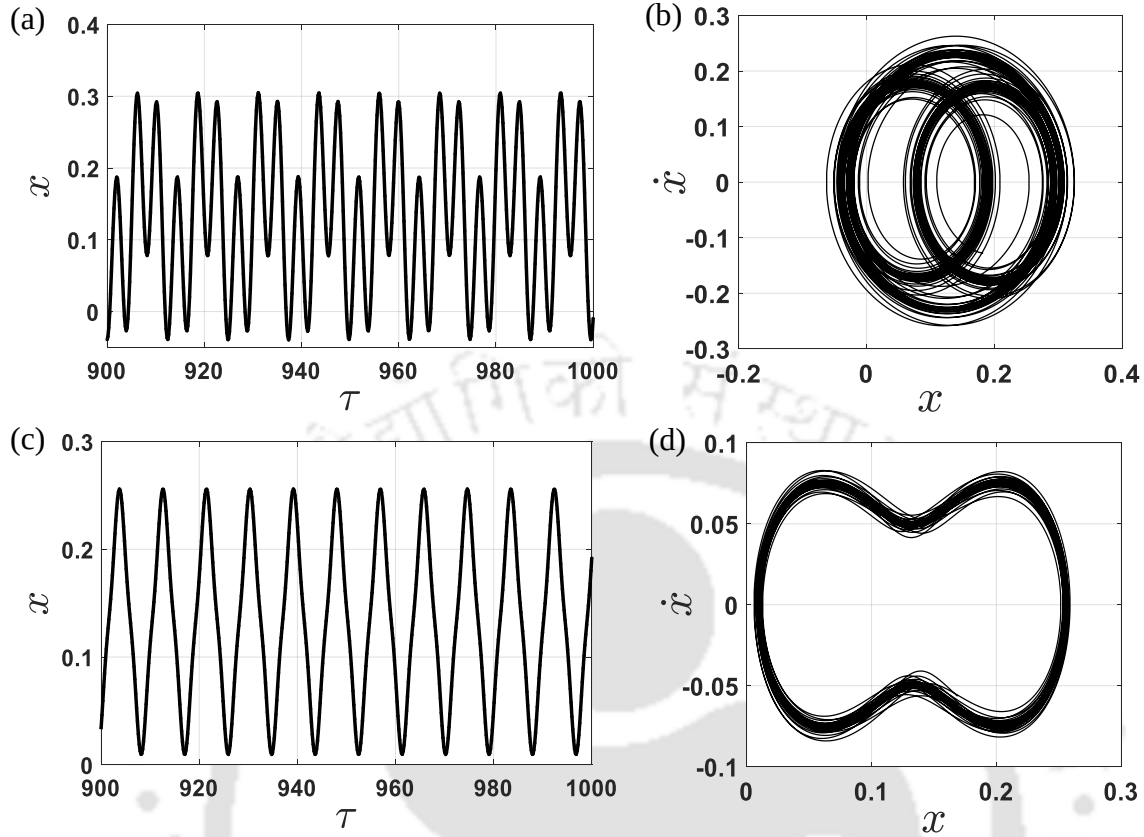


Fig. 6.3 Time response and phase portraits corresponding to Fig. 6.2 at (a, b) point P (c, d) point R.

In Fig. 6.4, the validation has been done for the frequency response plot in Fig. 6.2 by plotting the basin of attraction in the $a \sim \gamma$ plane with two different values of σ . At point M ($\sigma = 1$) in Fig. 6.2, the system has only a stable solution with amplitude $a = 0.0246$ which can be observed from the basin of attraction in Fig. 6.4(a). For the point N ($\sigma = 2$) marked in Fig. 6.2, the basin of attraction is shown in Fig. 6.4(b) where the system exhibits bi-stable nontrivial state where two stable solutions can be obtained at amplitude $a = 0.022$ and $a = 0.1779$ along with a unstable solution at $a = 0.1586$. From these frequency and basin of attraction plots, the system can be noticed that upto the point P ($\sigma = 1.718$) the system has single stable state, between both the saddle node bifurcation points viz., point P and point R the system exhibits bi-stable state and finally after point R ($\sigma = 3.758$) the system exists again single stable state.

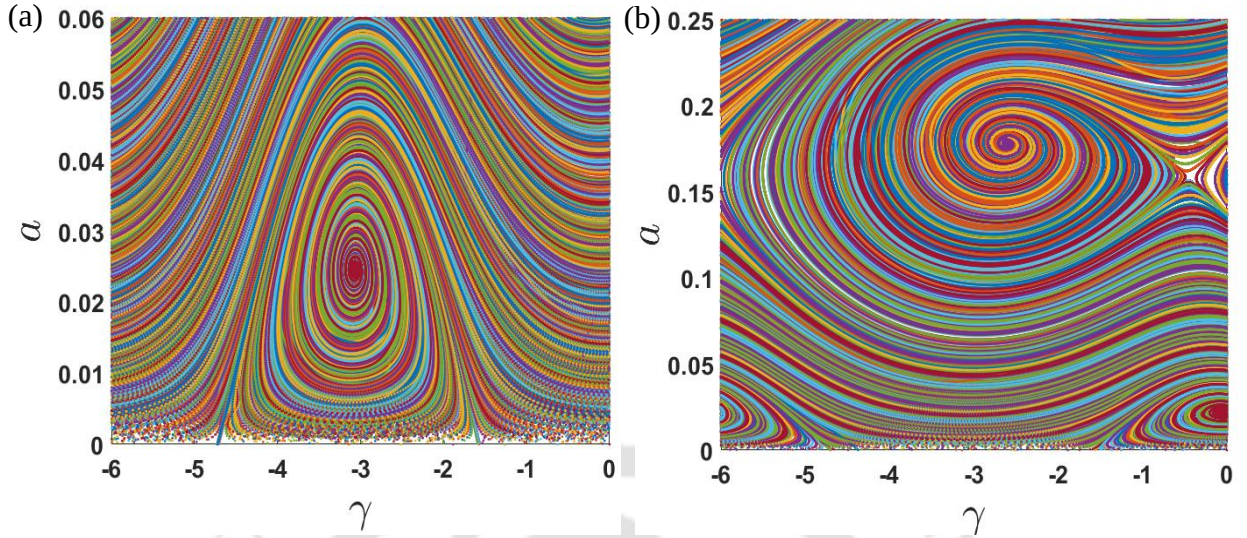


Fig. 6.4 Basin of attraction at (a) point M ($\sigma=1$) (b) point N ($\sigma=2$) in Fig. 6.2.

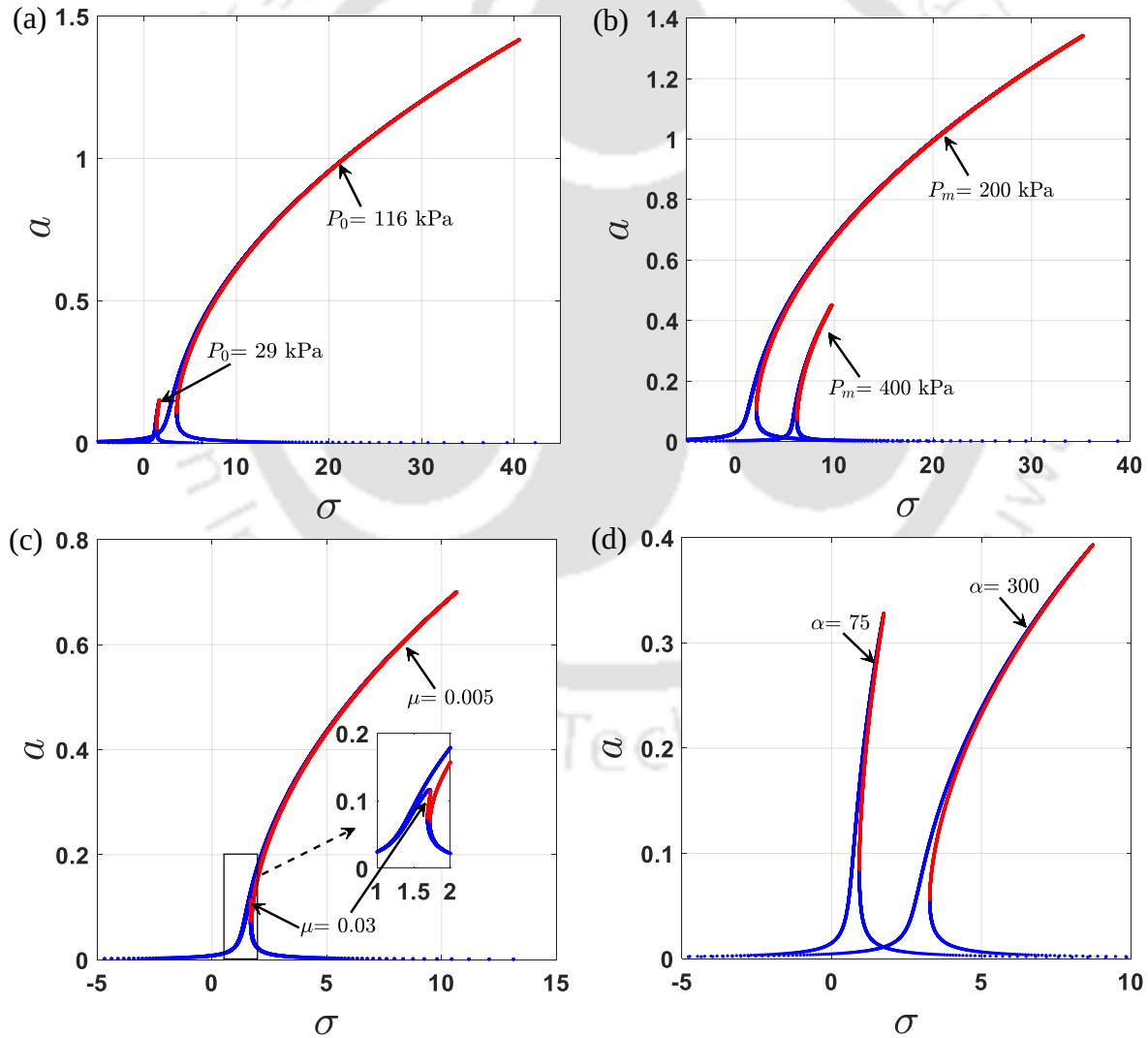
In Fig. 6.5(a) the maximum response amplitude of the system increases with increase in the value of the dynamic pressure in the muscle (P_0) and decreases with decrease in the value of P_0 . It can be observed that by making half in the value of P_0 ($P_0 = 29$ kPa), the maximum response amplitude $a = 0.1509$ of the system can be achieved at detuning parameter $\sigma = 1.664$ which is 56.9% decrease as compared to Fig. 6.2 where $P_0 = 58$ kPa. But making double in the value of $P_0 = 116$ kPa in Fig. 6.5(a), the maximum response amplitude $a = 1.417$ at higher value of $\sigma = 40.54$ which is increased by around three times as that in Fig. 6.2.

Figure 6.5(b) shows the frequency response plots for two different values of the static pressure in the muscle (P_m). Here, one may observed that either increase or decrease in the value of P_m , the maximum response amplitude is increased. The maximum response amplitude $a = 0.4509$ can be attained at $\sigma = 9.727$ with increase in the value of $P_m = 400$ kPa which is 28.9% increase in comparison with $P_m = 300$ kPa in Fig. 6.2. With decrease in the value of $P_m = 200$ kPa, the maximum response amplitude $a = 1.341$ which is also increased by 283% and can be achieved at higher value of $\sigma = 35.21$.

From Fig. 6.5(c) with increase in the value of damping parameter (μ), the maximum response amplitude is decreased and vice-versa. At $\mu = 0.03$ which is increased as compared to Fig. 6.2 where $\mu = 0.01$, the maximum response amplitude $a = 0.1165$ at $\sigma = 1.723$ is decreased by 66.7%.

But with decrease in the value of $\mu = 0.005$, the maximum response amplitude $a = 0.6999$ at $\sigma = 10.65$ which is increased by double as compared to Fig. 6.2.

From Fig. 6.2 and Fig. 6.5(d), it can be observed that by increasing in the value of nonlinearity (α), the maximum response amplitude can be achieved at higher value of detuning parameter σ . A similar observation can be noticed by decreasing the value of α in Fig. 6.5(d). The maximum response amplitude can be obtained $a = 0.3932$ at $\sigma = 8.725$ for $\alpha = 300$ and $a = 0.3282$ at $\sigma = 1.743$ for $\alpha = 75$. By making the nonlinearity α double ($\alpha = 300$), the maximum response amplitude in Fig. 6.5(d) is increased by 12.4% and making half ($\alpha = 75$) the maximum response amplitude decreased by 6.2% as compared to the saddle node point R in Fig. 6.2 where $\alpha = 150$.



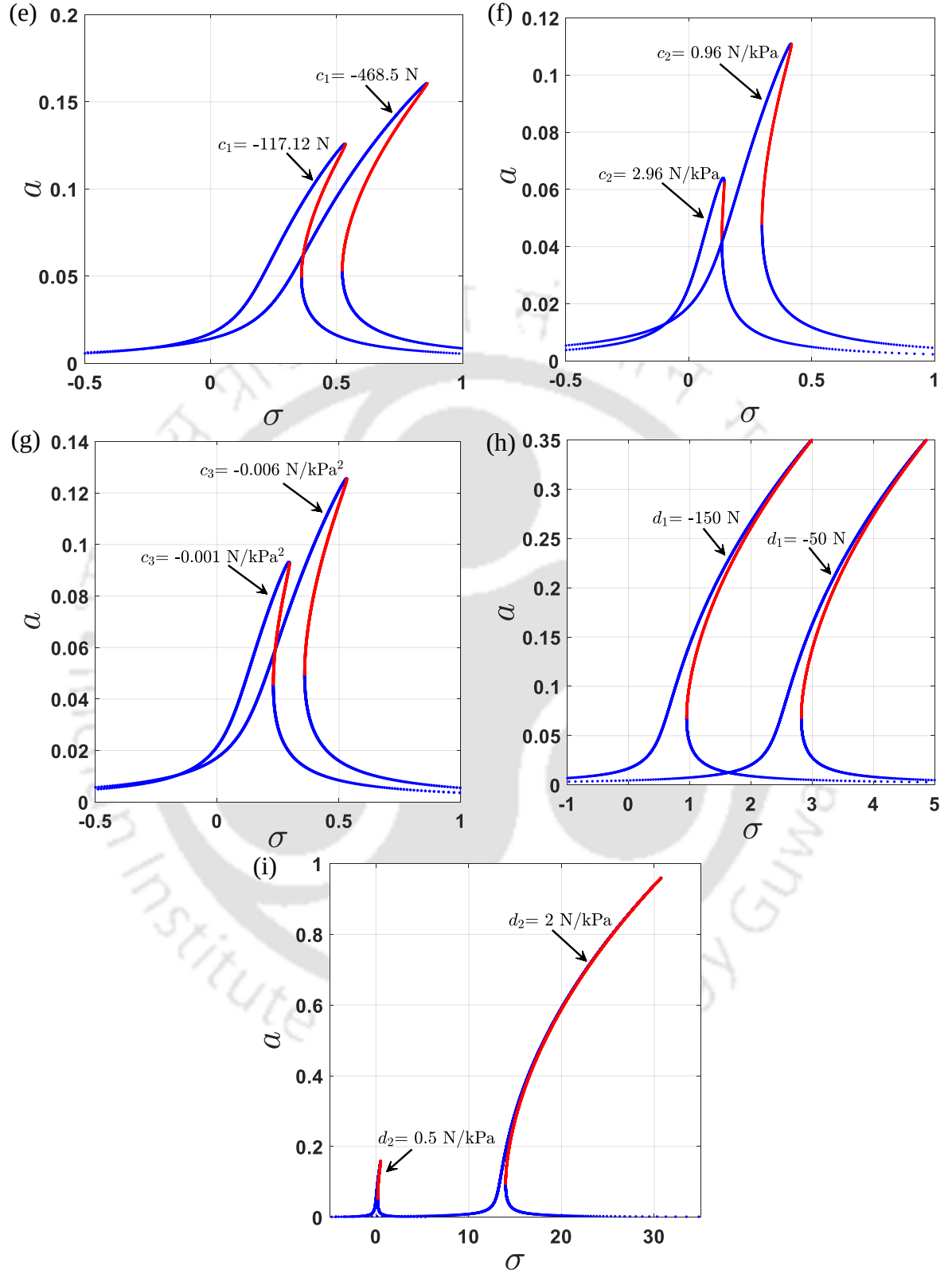


Fig. 6.5 Frequency responses under superharmonic resonance condition with variation in the system parameters (a) P_0 (b) P_m (c) μ (d) α (e) c_1 (f) c_2 (g) c_3 (h) d_1 (i) d_2 .

Figure 6.5(e) depicts the frequency response plots for the muscle parameter (c_1), where the maximum response amplitude decreases with increase or decrease in the value of c_1 as compared to Fig. 6.2. With decrease in the value of $c_1 = -468.5$ N as compared to the $c_1 = -234.25$ N in Fig. 6.2, the maximum response amplitude $a = 0.1607$ at $\sigma = 0.858$ which is decreased by 54%. Also, with increase in the value of $c_1 = -117.12$ N, the maximum response amplitude $a = 0.1259$ which is decreased by 64% as compared to Fig. 6.2 and can be achieved at saddle node bifurcation point at $\sigma = 0.535$ in Fig. 6.5(e). But from Fig. 6.5(e), it can be noticed that the maximum response amplitude at saddle node bifurcation points occurs at lower value of σ as compared to point R in Fig. 6.2.

In Fig. 6.5(f), the frequency response curves have been plotted for the two different values of muscle parameter (c_2). Here, for muscle parameter $c_2 = 2.96$ N/kPa and $c_2 = 0.96$ N/kPa, the maximum response amplitude of the system can be achieved with $a = 0.064$ at $\sigma = 0.142$ and $a = 0.111$ at $\sigma = 0.417$ respectively. Like in the previous case for muscle parameter c_1 , the maximum response amplitude is decreased by 81.7% and 68.3% with increase and decrease in the value of c_2 respectively as compared to the Fig. 6.2 where $c_2 = 1.96$ N/kPa.

Like the previous cases for the muscle parameters c_1 and c_2 , the maximum response amplitude decreases with either increase or decrease in the value of another muscle parameter c_3 . The maximum response amplitude can be achieved $a = 0.0931$ at $\sigma = 0.297$ in Fig. 6.5(g) with increase in the value of $c_3 = -0.001$ N/kPa² which is 73.4% decrease in comparison with Fig. 6.2. With decrease in the value of $c_3 = -0.006$ N/kPa², the maximum response amplitude $a = 0.1256$ which is decreased by 64% and can be attained at $\sigma = 0.533$. In this case also like in Fig. 6.5(e) and Fig. 6.5(f), the saddle node bifurcation point where the maximum response amplitude occurs, can be noticed at lower value of σ as compared to Fig. 6.2.

From Fig. 6.5(h) and Fig. 6.2, it can be observed that the effect of the muscle parameter (d_1) is not prominent. The maximum response amplitude in Fig. 6.5(h) for both the values of $d_1 = -50$ N and $d_1 = -150$ N is same with Fig. 6.2. But the value of detuning parameter σ is slightly different

for the two values of $d_1 = -50$ N and $d_1 = -150$ N which is $\sigma = 4.862$ and $\sigma = 2.989$ respectively.

In Fig. 6.5(i), the maximum response amplitude increases with increase in the value of another muscle parameter (d_2) and decreases with decrease in the value of d_2 same as the cases for P_0 in Fig. 6.5(a) and α in Fig. 6.5(d). For $d_2 = 0.5$ N/kPa which is half of that value in Fig. 6.2, the maximum response amplitude $a = 0.1586$ at $\sigma = 0.514$ is decreased by 54.7%. The maximum response amplitude $a = 0.9597$ which is increased by 174% at $\sigma = 0.514$ for $d_2 = 2$ N/kPa in Fig. 6.5(i) which is double the value of d_2 in Fig. 6.2. Hence to design a system for a particular application to achieve a definite response amplitude one may find the optimum value of the muscle parameters c_1, c_2, c_3, d_1 and d_2 . In all these cases, different PAM has to be used.

To illustrate the application of the above developed results one may consider Table 6.2 where to achieve an amplitude $a = 0.1$; 17 different cases have been presented with different set of system parameters. It may be noted that the same response can be obtained by changing the parameter P_0, P_m and σ actively or the other parameters like $\mu, \alpha, c_1, c_2, c_3, d_1$ and d_2 passively. This will help the designer to choose the system parameters according to the particular application. From Table 6.2, it can be noticed that in Case 3 with low value of P_0 and P_m will give the same displacement as compared to the other cases. Also, Case 17 provides the desired amplitude at a low frequency of the system. So, one may use the different set of system parameters to effectively move the PAM for the exact application.

6.4.1.1 Application of the developed results

Table 6.3 provides the variation of saddle node bifurcation point where the system has a tendency to jump up while sweeping down the frequency for a wide range of system parameters. It may be noted that one may achieve the same amplitude either by sweeping up the frequency or by sweeping down the frequency. But while sweeping down one must take care of the jump up phenomenon which may lead to catastrophic failure of the system. In this table, the amplitude (a) for both stable and unstable point are mentioned where jump up phenomenon will take place

for different system parameters. It may be noted that in Case 12, the jump length is lowest and in Case 2 the jump length is highest as compared to the other cases.

Table 6.2 System parameters to attain an amplitude of $a = 0.1$.

Case	P_0	P_m	μ	α	c_1	c_2	c_3	d_1	d_2	σ	Ref. Figure No.
1	58	300	0.01	150	-234.25	1.96	-0.003	-100	1	1.539	Fig. 6.2
2	116	300	0.01	150	-234.25	1.96	-0.003	-100	1	2.608, 3.551	Fig. 6.5(a)
3	29	300	0.01	150	-234.25	1.96	-0.003	-100	1	1.385	Fig. 6.5(a)
4	58	400	0.01	150	-234.25	1.96	-0.003	-100	1	5.957	Fig. 6.5(b)
5	58	200	0.01	150	-234.25	1.96	-0.003	-100	1	1.211, 2.103	Fig. 6.5(b)
6	58	300	0.03	150	-234.25	1.96	-0.003	-100	1	1.590	Fig. 6.5(c)
7	58	300	0.005	150	-234.25	1.96	-0.003	-100	1	1.535	Fig. 6.5(c)
8	58	300	0.01	300	-234.25	1.96	-0.003	-100	1	3.174	Fig. 6.5(d)
9	58	300	0.01	75	-234.25	1.96	-0.003	-100	1	0.721	Fig. 6.5(d)
10	58	300	0.01	150	-117.12	1.96	-0.003	-100	1	0.398	Fig. 6.5(e)
11	58	300	0.01	150	-468.50	1.96	-0.003	-100	1	0.520	Fig. 6.5(e)
12	58	300	0.01	150	-234.25	0.96	-0.003	-100	1	0.357	Fig. 6.5(f)
13	58	300	0.01	150	-234.25	1.96	-0.006	-100	1	0.397	Fig. 6.5(g)
14	58	300	0.01	150	-234.25	1.96	-0.003	-50	1	2.641	Fig. 6.5(h)
15	58	300	0.01	150	-234.25	1.96	-0.003	-150	1	0.768	Fig. 6.5(h)
16	58	300	0.01	150	-234.25	1.96	-0.003	-100	2	13.34	Fig. 6.5(i)
17	58	300	0.01	150	-234.25	1.96	-0.003	-100	0.5	0.187	Fig. 6.5(i)

Table 6.3 Variation in the jump up phenomenon with different system parameters while sweeping down the frequency.

Case	Ref. Figure No.	Variation of System parameters	σ	a		Jump length
				Unstable point	Stable point	
1	Fig. 6.2	As per Table 1	1.718	0.0683	0.1337	0.0654
2	Fig. 6.5(a)	$P_0 = 116$	3.547	0.1082	0.2156	0.1074
3	Fig. 6.5(a)	$P_0 = 29$	1.377	0.0524	0.0982	0.0458
4	Fig. 6.5(b)	$P_m = 400$	6.219	0.0741	0.1461	0.0720
5	Fig. 6.5(b)	$P_m = 200$	2.100	0.1062	0.2116	0.1054
6	Fig. 6.5(c)	$\mu = 0.03$	1.688	0.0738	0.1144	0.0406
7	Fig. 6.5(c)	$\mu = 0.005$	1.721	0.0679	0.1352	0.0673
8	Fig. 6.5(d)	$\alpha = 300$	3.275	0.0562	0.1110	0.0548
9	Fig. 6.5(d)	$\alpha = 75$	0.924	0.0846	0.1634	0.0788
10	Fig. 6.5(e)	$c_1 = -117.12$	0.360	0.0496	0.0912	0.0416
11	Fig. 6.5(e)	$c_1 = -468.50$	0.523	0.0534	0.1006	0.0472
12	Fig. 6.5(f)	$c_2 = 2.96$	0.135	0.0424	0.0636	0.0212
13	Fig. 6.5(f)	$c_2 = 0.96$	0.298	0.0479	0.0863	0.0384
14	Fig. 6.5(g)	$c_3 = -0.001$	0.230	0.0457	0.0796	0.0339
15	Fig. 6.5(g)	$c_3 = -0.006$	0.359	0.0496	0.0911	0.0415
16	Fig. 6.5(h)	$d_1 = -50$	2.820	0.0683	0.1337	0.0654
17	Fig. 6.5(h)	$d_1 = -150$	0.948	0.0683	0.1337	0.0654
18	Fig. 6.5(i)	$d_2 = 2$	13.980	0.0951	0.1898	0.0947
19	Fig. 6.5(i)	$d_2 = 0.5$	0.187	0.0532	0.1001	0.0469

6.4.2 Subharmonic resonance condition

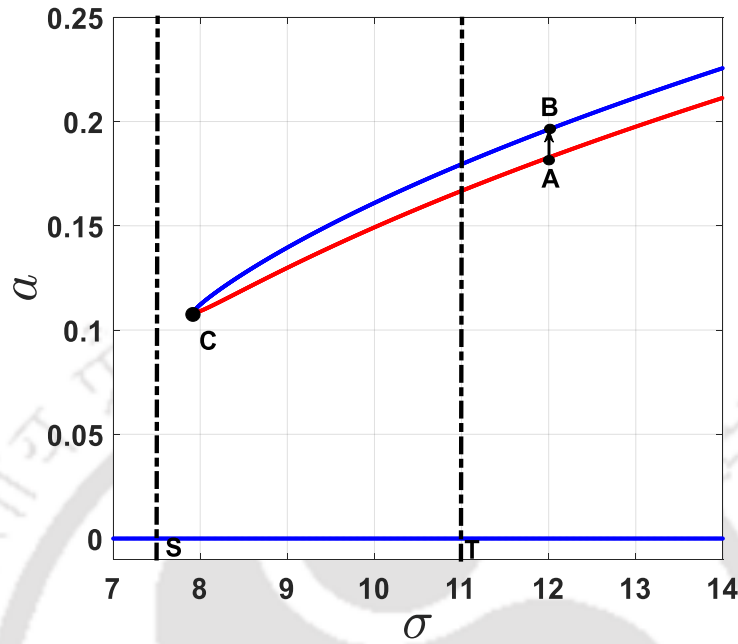


Fig. 6.6 Frequency response of the system under parametric and hard excitation with subharmonic resonance condition as the parameter values in Table 6.1.

In this section, the numerical results for subharmonic resonance condition where the external excitation frequency is three times of the natural frequency of the system have been explored. The frequency response have been plotted in Fig. 6.6 by taking the system parameters values as given in Table 1 except the value of static pressure is considered as $P_m = 200$ kPa. The system exhibits both trivial and nontrivial branches in the frequency plots for this resonance condition which can be observed from Eq. (6.36) where $a = 0$ is a solution. Here, the stability of the steady state response on the nontrivial and trivial branch have been studied with the help Jacobian Matrix Eq. (6.37) and Eq. (6.40). In the nontrivial branch in Fig. 6.6, the jump up phenomenon can be noticed at $\sigma = 12$ where the unstable solution with amplitude $a = 0.1827$ at point A has always a tendency to jump to the solution in the stable branch with amplitude $a = 0.1962$ at point B. The saddle node bifurcation point C can be observed in the nontrivial branch at detuning parameter $\sigma = 7.928$ with amplitude $a = 0.1087$ and the trivial branch in the frequency response plot is stable for a wide range of σ . The time response and phase portraits have been plotted in Fig. 6.7 with the help of the reduced equations Eq. (6.32) and Eq. (6.33) near the mentioned saddle node bifurcation point C ($\sigma = 7.928$) and the point A ($\sigma = 12$) where the responses are found to be periodic.

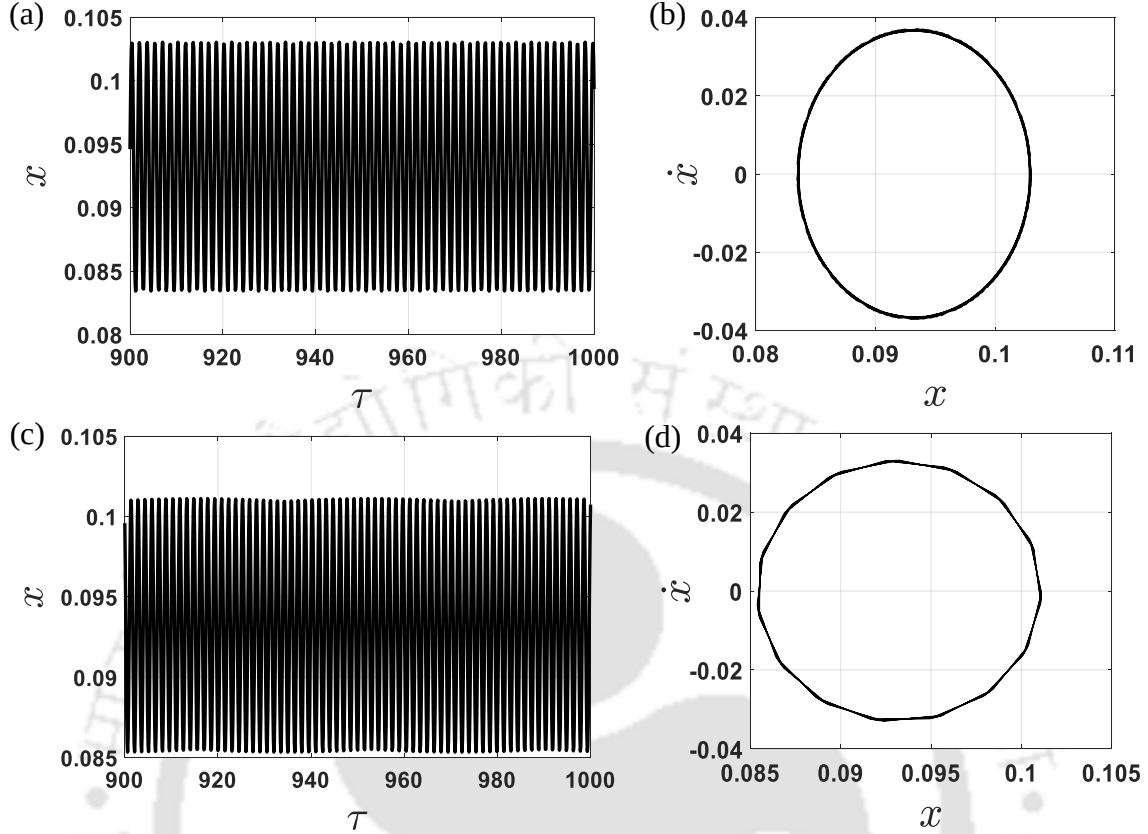


Fig. 6.7 Time response and phase portraits corresponding to Fig. 6.6 at (a, b) point C (c, d) point A.

From Fig. 6.6, it can be observed that the system exhibits a single stable in the trivial branch upto the saddle node bifurcation point C ($\sigma = 7.928$) and after that the nontrivial branch arises with a stable and an unstable solution, i.e., the system shows bistable state. So, to validate the frequency response curve in Fig. 6.6, the basin of attraction have been plotted with the $a \sim \gamma$ plane in Fig. 6.8 for two different values of σ . For the point S ($\sigma = 7.5$) in Fig. 6.6, the system has only one stable solution i.e., $a = 0$ in the trivial branch of the frequency response curve which can be verified from the Fig. 6.8(a). From Fig. 6.8(b), it can be observed that the system has two stable solution with $a = 0$ and $a = 0.1795$ in the trivial and nontrivial branch of the frequency response curve in Fig. 6.6 for $\sigma = 11$. The system has also an unstable solution with $a = 0.1668$ at $\sigma = 11$ in the nontrivial branch of the frequency response which always shows an affinity to go the nearest stable solution with $a = 0.1795$. The frequency responses have been plotted for different set of system parameters and the trivial branch is found to be stable in all the cases for a wide range of σ . Hence, the nature of the nontrivial branch of the frequency response plots has been discussed extensively.

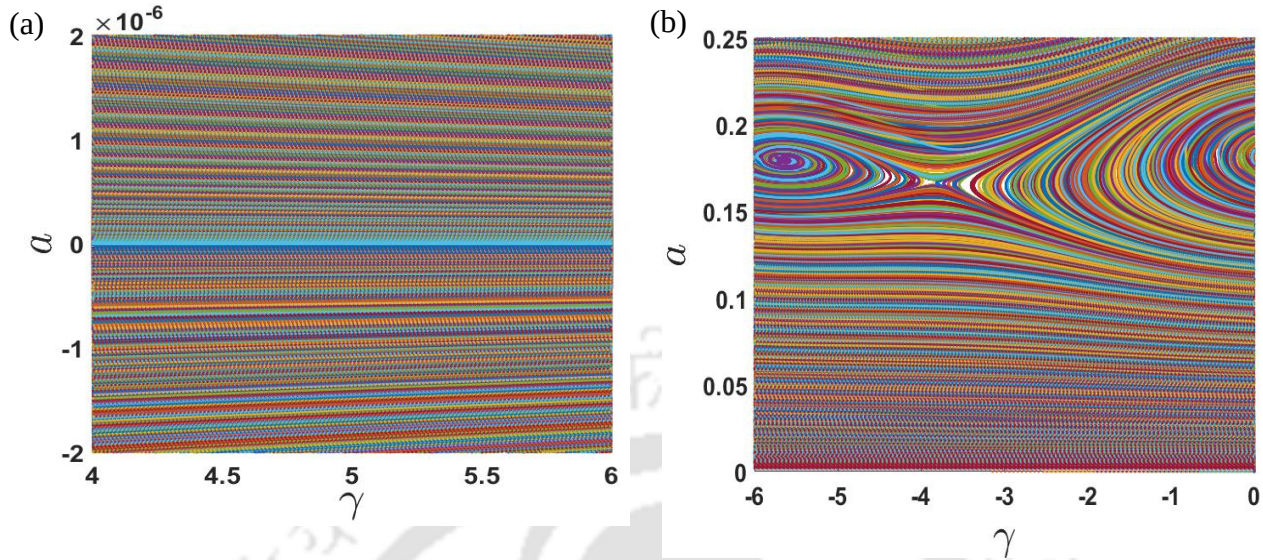
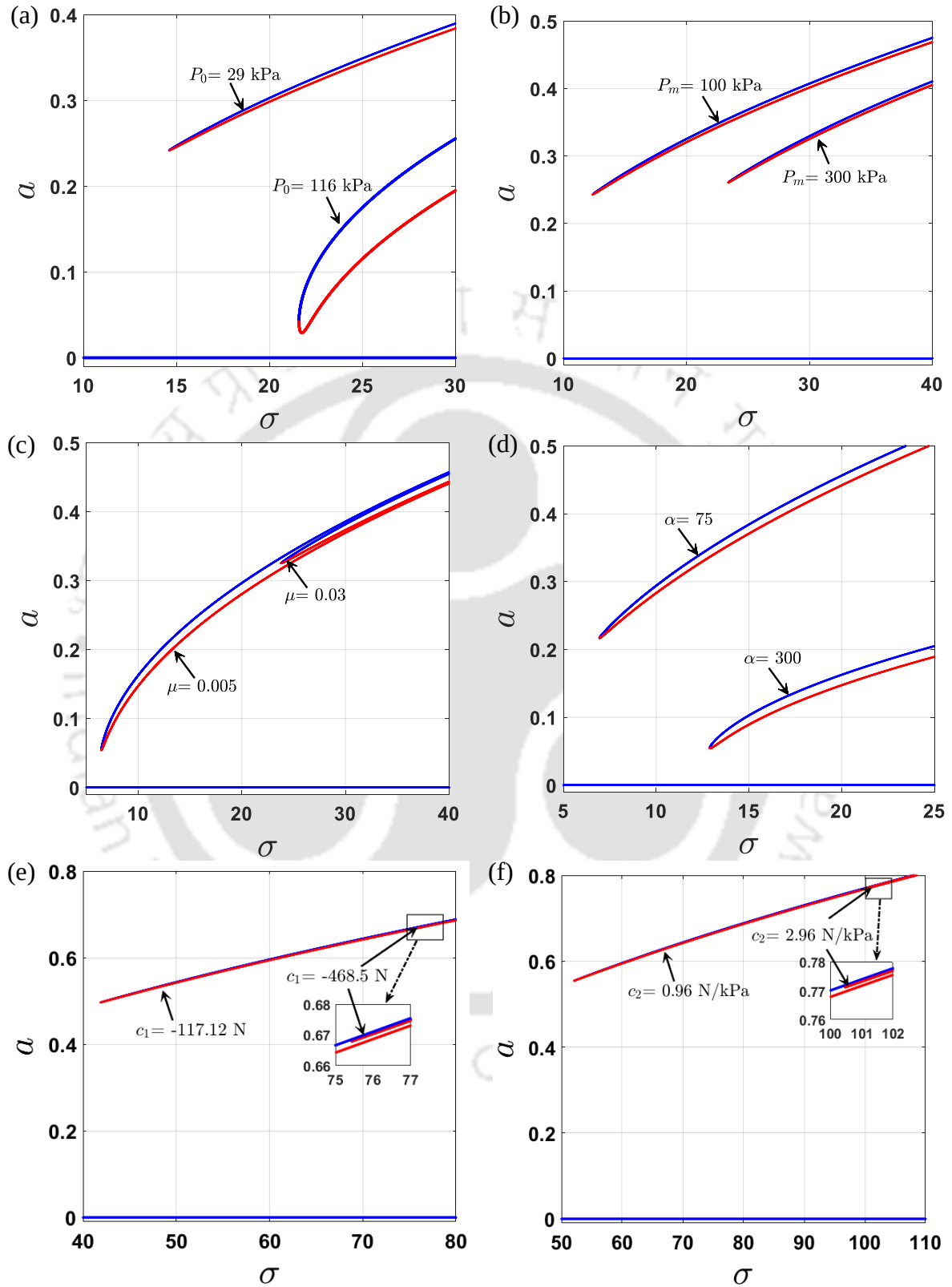


Fig. 6.8 Basin of attraction at (a) point S ($\sigma = 7.5$) (b) point T ($\sigma = 11$) in Fig. 6.6.

In Fig. 6.9(a) the effect of the dynamic pressure in the muscle (P_0) on the stability of the nontrivial branch has been studied with two different values of P_0 . With decrease in the value of $P_0 = 29$ kPa in Fig. 6.9(a) which is half to that in Fig. 6.6 ($P_0 = 58$ kPa), the saddle node bifurcation point occurs at detuning parameter $\sigma = 14.6$ with an amplitude $a = 0.2419$. But by making double in the value of $P_0 = 116$ kPa, the saddle node bifurcation point is observed at higher value of detuning parameter $\sigma = 21.58$ with $a = 0.0422$. From Fig. 6.6 and Fig. 6.9(a), it can be noticed that the saddle node bifurcation point occurs at higher value of σ with either increase or decrease in the value of P_0 .

Figure 6.9(b) shows the frequency response plots for two different values of the static pressure in the muscle (P_m). Here, one may observed that with increase in the value of $P_m = 300$ kPa as compared to Fig. 6.6, the saddle node bifurcation point in the nontrivial branch can be observed at $\sigma = 23.39$ with $a = 0.2608$. With decrease in the value of $P_m = 100$ kPa, the saddle node bifurcation point is at $\sigma = 12.35$ with $a = 0.2429$. Hence, from Fig. 6.6, Fig. 6.9(a) and Fig. 6.9(b), it can be observed that an optimum value of P_0 and P_m should be chosen to operate the system at lower value of σ .



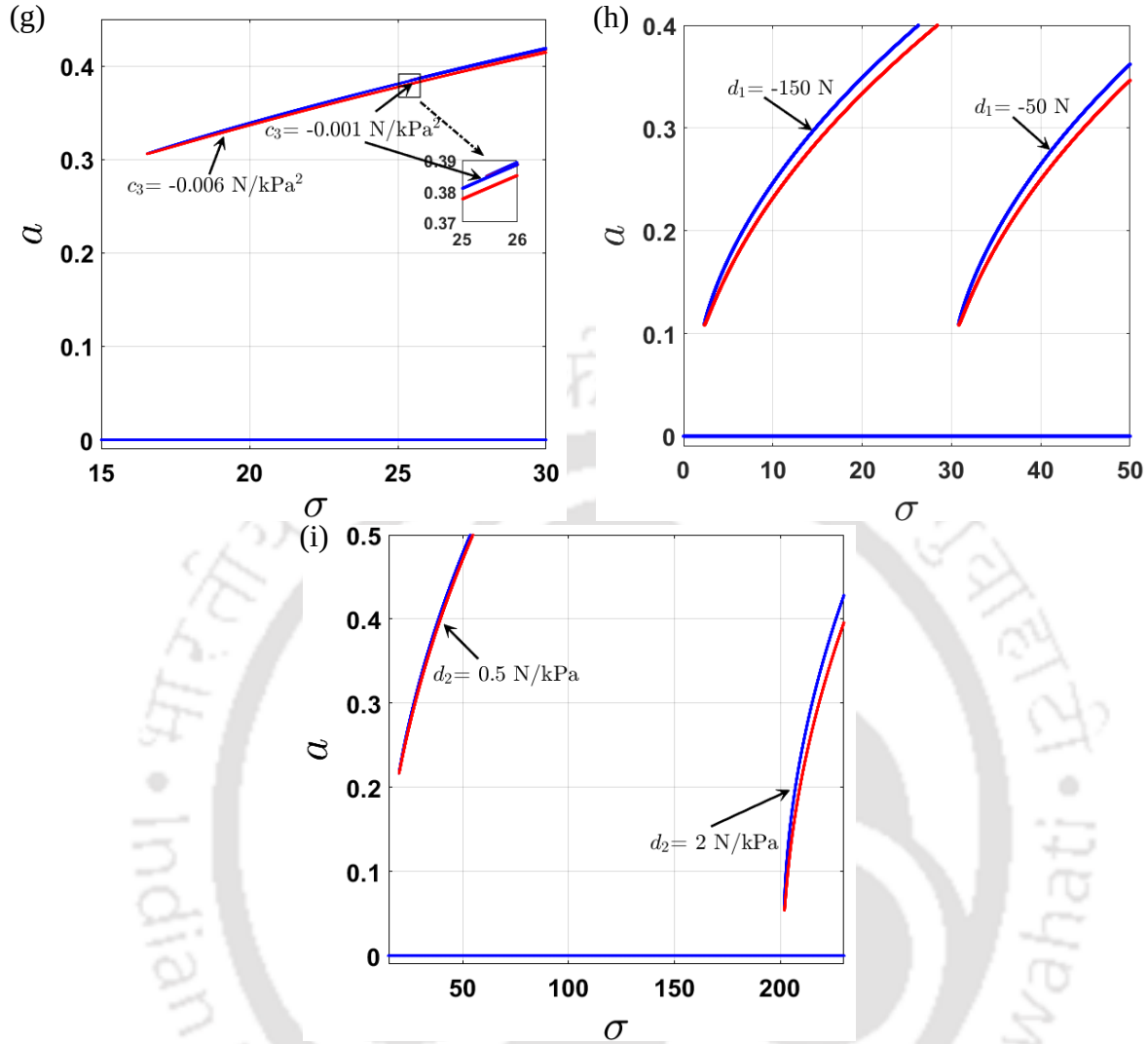


Fig. 6.9 Frequency responses under subharmonic resonance condition with variation in the system parameters (a) P_0 (b) P_m (c) μ (d) α (e) c_1 (f) c_2 (g) c_3 (h) d_1 (i) d_2 .

From Fig. 6.9(c) with decrease in the value of damping parameter $\mu = 0.005$ as compared to Fig. 6, the saddle node bifurcation point occurs at $\sigma = 6.449$ with $a = 0.0542$. Here, the response amplitude is $a = 0.1628$ at $\sigma = 10$ which is increased by 1% from Fig. 6. With increase in the value of $\mu = 0.03$, the saddle node bifurcation point can be noticed at higher value $\sigma = 23.78$ with $a = 0.3252$ in the nontrivial branch of the frequency response curve.

From Fig. 6.6 and Fig. 6.9(d), it can be observed that by increasing in the value of nonlinearity (α), the saddle node bifurcation point can be observed at higher value of σ and decreasing α the same can be attained at lower value of σ . By making double the nonlinearity $\alpha = 300$ and reducing to half $\alpha = 75$ as compared to Fig. 6.6, the saddle node bifurcation point occurs at

$\sigma = 12.88$ with $a = 0.0545$ and $\sigma = 6.936$ with $a = 0.2169$ respectively. Hence, the response amplitude increases with decrease in the value of α and vice versa.

Figure 6.9(e) depicts that the saddle node bifurcation point occurs at higher value of $\sigma = 75.44$ with $a = 0.6679$ by decreasing the value of muscle parameter $c_1 = -468.5$ N as compared to the $c_1 = -234.25$ N in Fig. 6.6. But with increase in the value of $c_1 = -117.12$ N, the saddle node bifurcation point in the nontrivial branch can be noticed at $\sigma = 41.88$ with $a = 0.4965$. From Fig. 6.6 and Fig. 6.9(e), the saddle node bifurcation point can be observed at higher value of σ and the response amplitude increases with either increase or decrease in the value c_1 .

From Fig. 6.9(f), the saddle node bifurcation point is observed at higher value of $\sigma = 52.1$ with $a = 0.5544$ by decreasing the value of muscle parameter $c_2 = 0.96$ N/kPa as compared to Fig. 6.6. With increase in the value of $c_2 = 2.96$ N/kPa, the saddle node bifurcation point occurs at $\sigma = 100.5$ with $a = 0.7712$ in the nontrivial branch. Like the previous case in Fig. 6.9(e), the response amplitude in Fig. 6.9(f) is increased with increase or decrease in the value of c_2 in comparison to Fig. 6.6 where $c_2 = 1.96$ N/kPa.

In Fig. 6.9(g), the frequency response curves have been plotted for the two different values of another muscle parameter (c_3). For muscle parameter $c_3 = -0.001$ N/kPa² which is increased to Fig. 6.6 ($c_3 = -0.003$ N/kPa²), the saddle node bifurcation is noticed at higher value of $\sigma = 25.45$ with $a = 0.3847$. By decreasing the value of $c_3 = -0.006$ N/kPa², the saddle node bifurcation point occurs at $\sigma = 16.55$ with $a = 0.306$ in the nontrivial branch of the frequency response curve. In this case also the response amplitude is increased at a higher value of σ in comparison to Fig. 6.6 whether the value of c_3 is increased or decreased.

Like the case in Fig. 6.5(h) for superharmonic resonance condition, from Fig. 6.9(h) and Fig. 6.6, it can be observed that the effect of the muscle parameter (d_1) is not distinct in case of subharmonic resonance condition. The response amplitude in Fig. 6.9(h) at the saddle node bifurcation point for both the values of $d_1 = -50$ N and $d_1 = -150$ N is same with Fig. 6.7. But the value of detuning parameter σ is different for the two values of $d_1 = -50$ N and $d_1 = -150$ N

which is $\sigma = 30.81$ and $\sigma = 2.329$ respectively. Hence, it can be noted from Fig. 6.9(h) that with decrease in the value of d_1 as compared to Fig. 6.6 ($d_1 = -100$ N), the saddle node bifurcation point can be observed at lower value of σ and vice versa.

In Fig. 6.9(i), the saddle node bifurcation point occurs at a very high value $\sigma = 201.9$ with low response amplitude $a = 0.0542$ making the value of the muscle parameter $d_2 = 2$ N/kPa double in comparison with Fig. 6.6 where $d_2 = 1$ N/kPa. But decreasing the value of $d_2 = 0.5$ N/kPa which is half to that in Fig. 6.6, the saddle node bifurcation point is observed at $\sigma = 19.91$ with $a = 0.2168$. Hence, like the superharmonic resonance condition, the operator or designer should choose the muscle parameters c_1, c_2, c_3, d_1 and d_2 value optimally to obtain the required amplitude at lower frequency for subharmonic resonance condition.

6.5 Experimental Validation

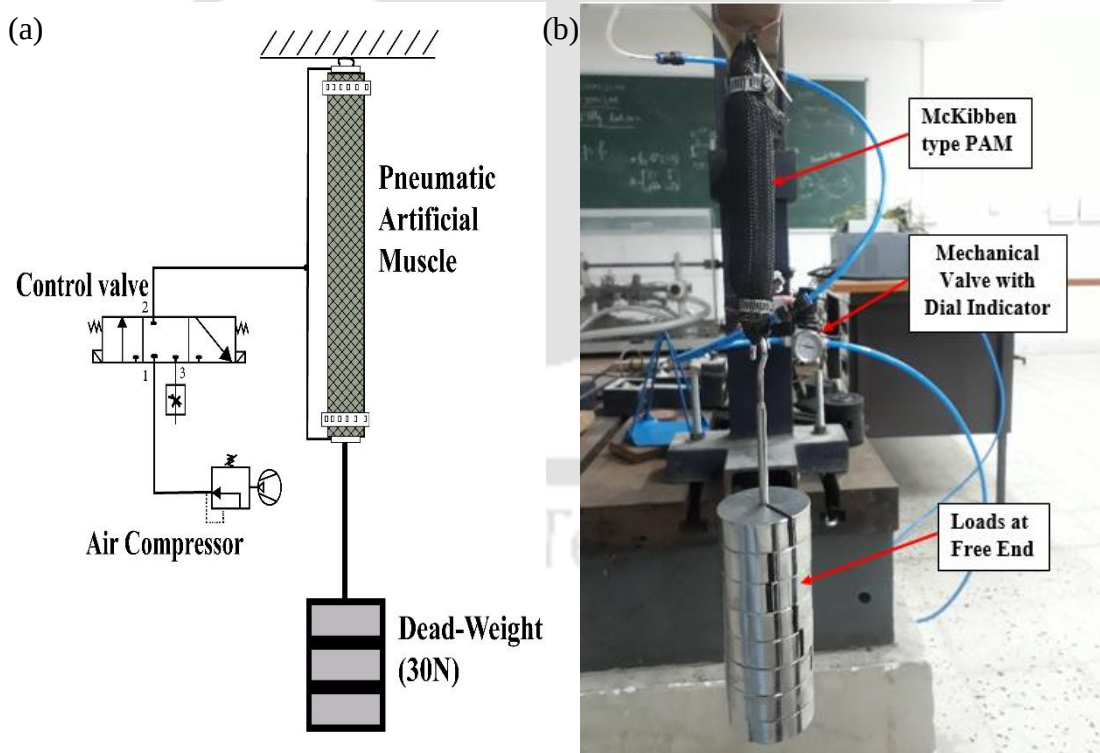


Fig. 6.10 Quasi-static PAM model identification (a) schematic diagram (b) actual experiment setup.

Figure 6.10 (a) shows the schematic diagram and Fig. 6.10(b) shows the actual experimental setup used to conduct free and forced vibration analysis. Further, it is used to determine the different

muscle parameters in Eq. (6.2) in the quasi-static empirical PAM model. The most commonly used PAM, i.e., a McKibben Muscle has been considered which is composed of a gas-tight elastic tube or bladder surrounded by a nylon braided sleeve. Both the ends of the inner tube and braid of this muscle is closed with the end fitting. The McKibben Muscle was hung freely from a rigid support with a dead weight of 3 kg at its free end. This ensures that the rubber tube along with the outer braid of the muscle was at its maximum possible length and the characteristics of PAM (McKibben type) were studied from this position. The changes in the contraction of the muscle were measured by varying the internal pressure inside the muscle with the help of a mechanical control valve. The pressure inside the muscle was increased from 50 kPa to the safe pressure limit of 200 kPa in discrete steps of 50 kPa. At each pressure step, the PAM contraction was measured and noted as given in Table 4. The above steps were repeated for different loads in the range 0–3 kg (0–30 N).

Table 6.4 Variation of contraction of the muscle in mm (u) with muscle force (applied load) and air pressure.

u in mm				
Pressure	50 kPa	100 kPa	150 kPa	200 kPa
Muscle Force				
5 N	7.0	12.0	19.0	22.0
10 N	5.8	10.6	17.0	19.8
15 N	4.2	8.8	15.2	17.8
20 N	2.8	7.2	13.8	15.8
25 N	1.8	5.7	12.2	14.2
30 N	1.0	4.5	11.0	12.0

Now, taking $y = u / l_{\max}$ and the data from Table 6.4 one may use the least square method to find the coefficients of Eq. (6.2). In this method the error can be written as follows.

$$E(c_1, c_2, c_3, d_1, d_2, K_n) = \sum_{i=1}^{24} \left[F_{mus_i} - \left\{ (c_1 + c_2 P_i + c_3 P_i^2) y_i + (d_1 + d_2 P) + K_n y_i^3 \right\} \right]^2 \quad (6.33)$$

where, $K_n = Kl_{\max}^3$. Hence, the error can be minimized by finding the first derivative with respect to c_1, c_2, c_3, d_1, d_2 and K_n and equating it to zero as follows.

$$\begin{bmatrix} \sum_{i=1}^{24} y_i^2 & \sum_{i=1}^{24} P_i y_i^2 & \sum_{i=1}^{24} P_i^2 y_i^2 & \sum_{i=1}^{24} y_i & \sum_{i=1}^{24} P_i y_i & \sum_{i=1}^{24} y_i^4 \\ \sum_{i=1}^{24} P_i y_i^2 & \sum_{i=1}^{24} P_i^2 y_i^2 & \sum_{i=1}^{24} P_i^3 y_i^2 & \sum_{i=1}^{24} P_i y_i & \sum_{i=1}^{24} P_i^2 y_i & \sum_{i=1}^{24} P_i y_i^4 \\ \sum_{i=1}^{24} P_i^2 y_i^2 & \sum_{i=1}^{24} P_i^3 y_i^2 & \sum_{i=1}^{24} P_i^4 y_i^2 & \sum_{i=1}^{24} P_i^2 y_i & \sum_{i=1}^{24} P_i^3 y_i & \sum_{i=1}^{24} P_i^2 y_i^4 \\ \sum_{i=1}^{24} y_i & \sum_{i=1}^{24} P_i y_i & \sum_{i=1}^{24} P_i^2 y_i & 1 & \sum_{i=1}^{24} P_i & \sum_{i=1}^{24} y_i^3 \\ \sum_{i=1}^{24} P_i y_i & \sum_{i=1}^{24} P_i^2 y_i & \sum_{i=1}^{24} P_i^3 y_i & \sum_{i=1}^{24} P_i & \sum_{i=1}^{24} P_i^2 & \sum_{i=1}^{24} P_i y_i^3 \\ \sum_{i=1}^{24} y_i^4 & \sum_{i=1}^{24} P_i y_i^4 & \sum_{i=1}^{24} P_i^2 y_i^4 & \sum_{i=1}^{24} y_i^3 & \sum_{i=1}^{24} P_i y_i^3 & \sum_{i=1}^{24} y_i^6 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ d_1 \\ d_2 \\ K_n \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{24} F_{mus_i} y_i \\ \sum_{i=1}^{24} F_{mus_i} P_i y_i \\ \sum_{i=1}^{24} F_{mus_i} P_i^2 y_i \\ \sum_{i=1}^{24} F_{mus_i} \\ \sum_{i=1}^{24} F_{mus_i} P_i \\ \sum_{i=1}^{24} F_{mus_i} y_i^3 \end{bmatrix} \quad (6.34)$$

To obtain the required coefficients the matrix equation Eq. (6.34) can be written as follows.

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ d_1 \\ d_2 \\ K_n \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{24} y_i^2 & \sum_{i=1}^{24} P_i y_i^2 & \sum_{i=1}^{24} P_i^2 y_i^2 & \sum_{i=1}^{24} y_i & \sum_{i=1}^{24} P_i y_i & \sum_{i=1}^{24} y_i^4 \\ \sum_{i=1}^{24} P_i y_i^2 & \sum_{i=1}^{24} P_i^2 y_i^2 & \sum_{i=1}^{24} P_i^3 y_i^2 & \sum_{i=1}^{24} P_i y_i & \sum_{i=1}^{24} P_i^2 y_i & \sum_{i=1}^{24} P_i y_i^4 \\ \sum_{i=1}^{24} P_i^2 y_i^2 & \sum_{i=1}^{24} P_i^3 y_i^2 & \sum_{i=1}^{24} P_i^4 y_i^2 & \sum_{i=1}^{24} P_i^2 y_i & \sum_{i=1}^{24} P_i^3 y_i & \sum_{i=1}^{24} P_i^2 y_i^4 \\ \sum_{i=1}^{24} y_i & \sum_{i=1}^{24} P_i y_i & \sum_{i=1}^{24} P_i^2 y_i & 1 & \sum_{i=1}^{24} P_i & \sum_{i=1}^{24} y_i^3 \\ \sum_{i=1}^{24} P_i y_i & \sum_{i=1}^{24} P_i^2 y_i & \sum_{i=1}^{24} P_i^3 y_i & \sum_{i=1}^{24} P_i & \sum_{i=1}^{24} P_i^2 & \sum_{i=1}^{24} P_i y_i^3 \\ \sum_{i=1}^{24} y_i^4 & \sum_{i=1}^{24} P_i y_i^4 & \sum_{i=1}^{24} P_i^2 y_i^4 & \sum_{i=1}^{24} y_i^3 & \sum_{i=1}^{24} P_i y_i^3 & \sum_{i=1}^{24} y_i^6 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{i=1}^{24} F_{mus_i} y_i \\ \sum_{i=1}^{24} F_{mus_i} P_i y_i \\ \sum_{i=1}^{24} F_{mus_i} P_i^2 y_i \\ \sum_{i=1}^{24} F_{mus_i} \\ \sum_{i=1}^{24} F_{mus_i} P_i \\ \sum_{i=1}^{24} F_{mus_i} y_i^3 \end{bmatrix} \quad (6.35)$$

By solving the above equation Eq. (6.35) the values of the coefficients can be found out. The values of the muscle parameters in the present case are found to be $c_1 = -6.871 \times 10^3$ N, $c_2 = 34.6$ N/kPa, $c_3 = 0.028$ N/kPa², $d_1 = -79.83$ N and $d_2 = 0.579$ N/kPa. The value of nonlinear parameter, K_n is 3.291×10^{-9} N/mm³ and the value of the maximum possible length of the particular muscle used in this experiments $l_{\max} = 136$ mm; hence the value of K in Eq. (6.2) is found to be 1.308×10^{-15} N/mm⁶.

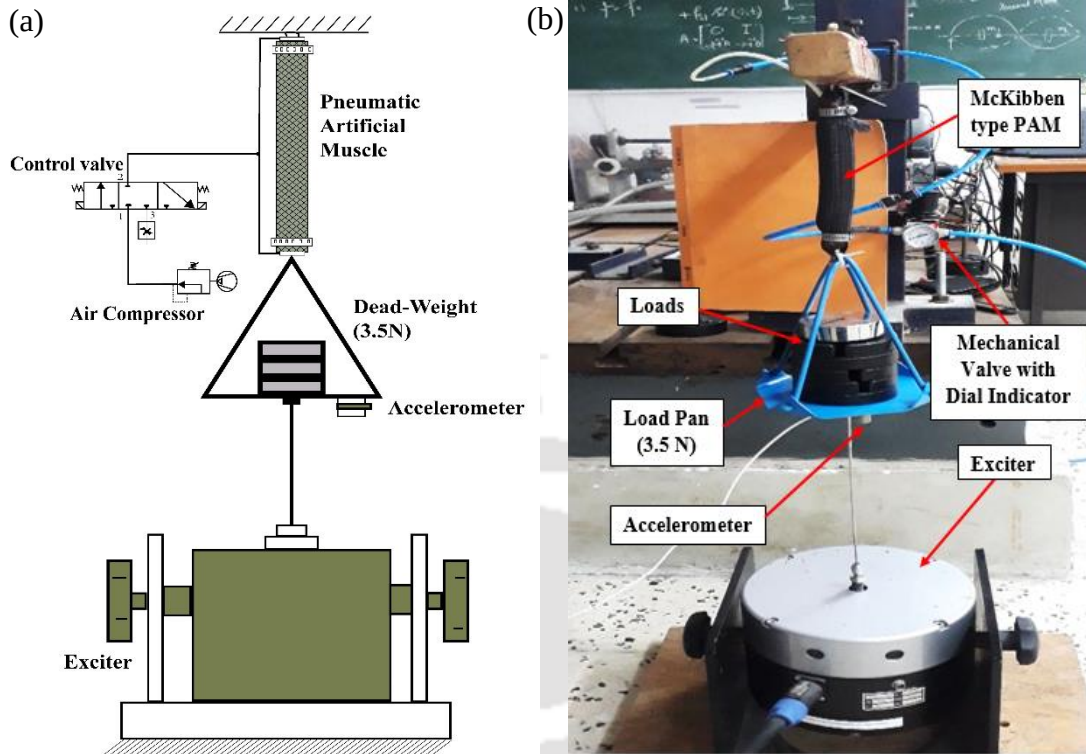


Fig. 6.11 Dynamic PAM model identification using forced excitation (a) schematic diagram (b) actual experiment setup.

The dynamic characteristics of the PAM have been analyzing with time response over a range of external forced excitation. Figure 6.11(a) and Fig. 6.11(b) show the schematic diagram and actual experimental setup, respectively. The load pan of weight 3.5 N was used which considered as dead-weight. The pan was attached to the free end of the muscle first to measure the dynamic characteristics. The system was then excited vertically as shown in Fig. 6.11 with the help of a shaker (Make: B&K, Model: 4824) with a sweep sinusoidal input force for three different external excitation frequencies, namely $f = 8 \text{ Hz}$ ($\Omega = 50.27 \text{ rad/s}$), $f = 10 \text{ Hz}$ ($\Omega = 62.83 \text{ rad/s}$) and $f = 12 \text{ Hz}$ ($\Omega = 75.40 \text{ rad/s}$). The acceleration-time history of the vibrating muscle was recorded using an Bruel and Kjaer made accelerometer (Type: 4396, Serial No.: 2247754) mounted on the load pan. A five-channel dynamic signal acquisition module (NI USB-4432) was connected to the accelerometer for making high-accuracy measurements from the accelerometer signals and NI LabVIEW 2009 software was used for post-processing the signals. The response function was then calculated from the Fast-Fourier transformed force and acceleration data. This experiment was repeated by varying the loads in the load pan with different input pressure inside the muscle. In Fig. 6.12 the time responses (Acceleration data) are shown for the muscle for lifting a load of

2.5 kg with 200 kPa input pressure into the muscle for above mentioned three external excitation frequencies.

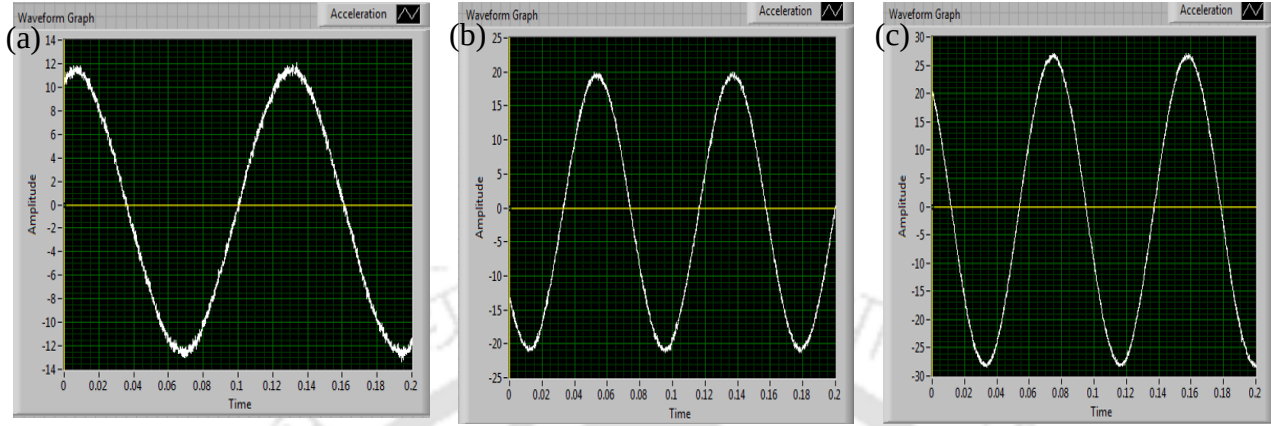


Fig. 6.12 Time response for the system with a load of 2.5 kg at 200 kPa

(a) $f = 8$ Hz (b) $f = 10$ Hz (c) $f = 12$ Hz.

The maximum displacement from experiments can be found out from Fig. 6.12 using the formula,

Displacement = $\frac{\text{Acceleration Amplitude}}{(2 \times \pi \times f)^2}$. In Fig. 6.12(a), the acceleration amplitude is observed

13.5 m/s^2 for $f = 8$ Hz and the corresponding maximum displacement is found to be 5.3 mm. Similarly, the acceleration amplitude is 21 m/s^2 in Fig. 6.12(b) for $f = 10$ Hz where the maximum displacement is obtained 5.3 mm. Finally, for $f = 12$ Hz in Fig. 6.12(c), the acceleration amplitude is 29 m/s^2 and the corresponding maximum displacement is noticed 5.1 mm.

By considering the value of the system parameters obtained from the experiments, the time response comparisons have been for the case studied in Fig. 6.12 (2.5 kg with 200 kPa). These comparisons are shown in Fig. 6.13 and Fig. 6.14 between the solution obtained by solving the reduced equations from the method of multiple scales (marked 'AS') and the original governing equation (marked 'NS') for super and sub harmonic resonance condition respectively. The reduced equations Eq. (6.16) and Eq. (6.17) for superharmonic resonance condition and Eq. (6.26) and Eq. (6.27) for subharmonic resonance condition are used to find the responses. Further these results have been compared by solving the original equation Eq. (6.6) numerically using Runge-Kutta method.

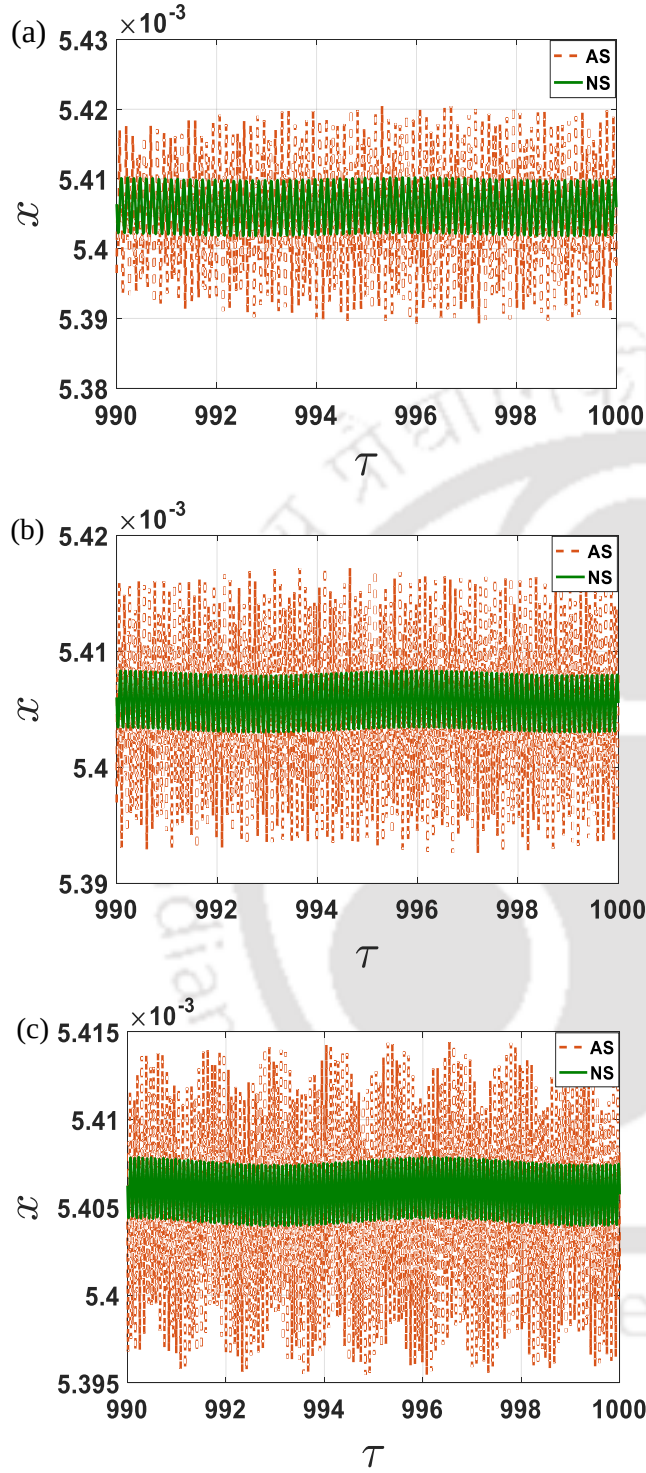


Fig. 6.13 Time response comparison under superharmonic resonance condition at (a) $f = 8$ Hz (b) $f = 10$ Hz (c) $f = 12$ Hz.

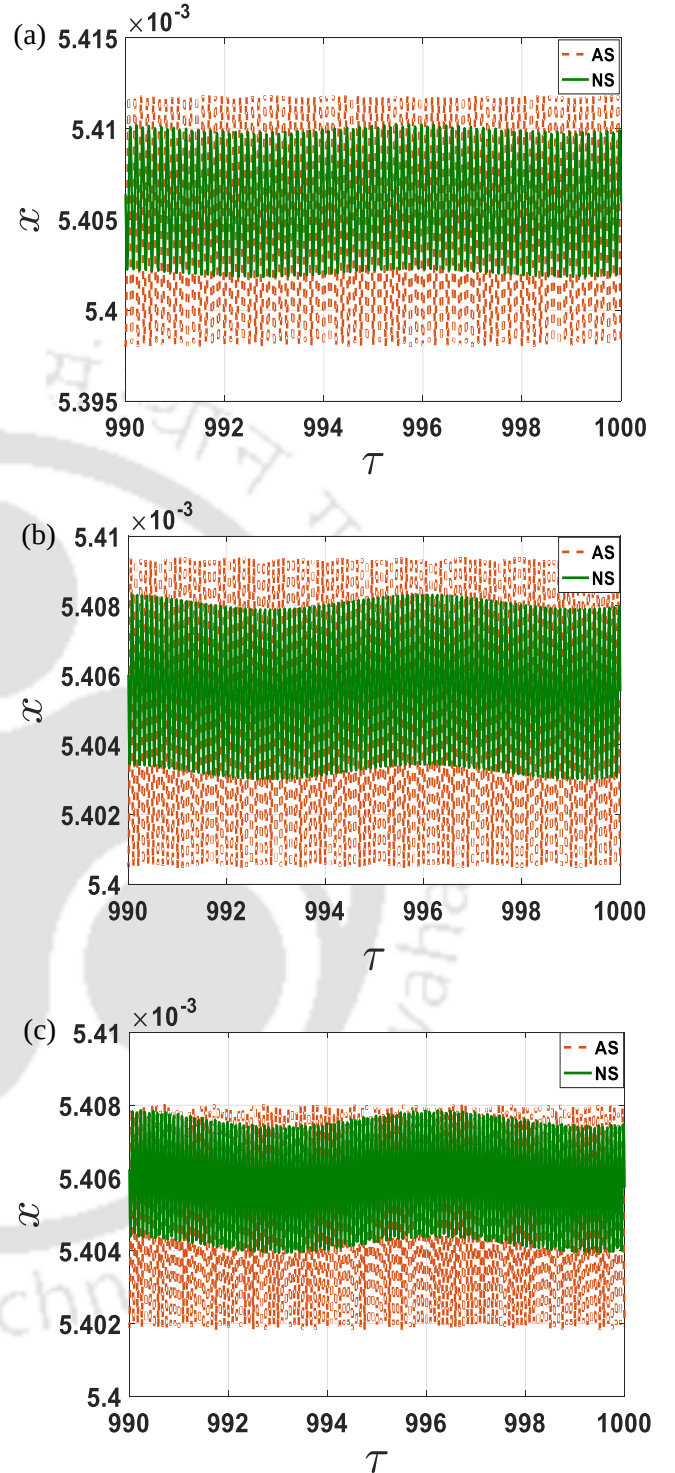


Fig. 6.14 Time response comparison under subharmonic resonance condition at (a) $f = 8$ Hz (b) $f = 10$ Hz (c) $f = 12$ Hz.

For superharmonic resonance condition with $f = 8$ Hz in Fig. 6.13(a), the maximum response of the solution obtained by solving the reduced equations is increased by 0.2% that from the original equation. From Fig. 6.13(b) with $f = 10$ Hz it can be noticed that the response of the solution found from the reduced equations is increased around 0.17% that from the original equation. Similarly, it can be observed from Fig. 6.13(c) for $f = 12$ Hz that the maximum response of the solution obtained from reduced equation also increased around 0.11% to that of the original equation. Hence, it can be noticed that the responses found by using method of multiple scales gives a conservative result in all the cases with same mean response amplitude.

Again, in the case of subharmonic resonance condition in Fig. 6.14, the maximum response of the solution obtained by solving the reduced equations is in good agreement with that from the original equation. The responses have been plotted for $f = 8$ Hz in Fig. 6.14(a), for $f = 10$ Hz in Fig. 6.14(b) and for $f = 12$ Hz in Fig. 6.14(c).

In Table 6.5 for the above mentioned three external excitation frequencies, the experimental displacements of the McKibben Muscle from Fig. 6.12 are presented. Further, these results have been compared with the theoretical displacement from the solution obtained by solving the reduced equations and original equation for super and sub harmonic resonance condition which can be observed from Fig. 6.13 and Fig. 6.14. Hence, in both the resonance conditions, the responses are found to be in good agreement with the experimental results and the reduced equations from the method of multiple scales can be used to study system behaviour which will reduce the computational time and memory.

Table 6.5 Comparison of experimental and theoretical response of the used McKibben Muscle.

External Excitation frequency	Experimental Displacement	Theoretical Displacement (Superharmonic Resonance condition)		Theoretical Displacement (Subharmonic Resonance condition)	
		Using governing equations Eq. (6.6)	Using reduced equations Eq. (6.16) and Eq. (6.17)	Using governing equations Eq. (6.6)	Using reduced equations Eq. (6.26) and Eq. (6.27)
$f = 8$ Hz	5.3 mm	5.410 mm	5.421 mm	5.410 mm	5.412 mm
$f = 10$ Hz	5.3 mm	5.408 mm	5.417 mm	5.408 mm	5.409 mm
$f = 12$ Hz	5.1 mm	5.408 mm	5.414 mm	5.408 mm	5.408 mm

6.6 Summary

In this chapter, the nonlinear analysis of a parametrically excited system attached with a PAM is studied under hard excitation for superharmonic and subharmonic resonance conditions. Using the method of multiple scales, the governing equation of motion has been solved and a set of reduced equations are obtained for each resonance condition. So with the help of frequency response plots, the safe operating range of the different system parameters can be determined. Basin of attraction also have been plotted to verify the actually realized solutions from the initial conditions. The frequency responses show the jump-up and jump-down phenomenon along with the saddle node bifurcations. The periodic responses are observed from the time responses and phase portraits in both the resonance conditions. But in the case of subharmonic resonance condition, both trivial and nontrivial responses have been found from the frequency response plots. While the trivial state is always stable, the nontrivial state undergoes transition from stable to unstable region by the saddle node bifurcation. An experimental setup is developed to conduct a series of experiments which gives conservative results to validate the mathematical work. The different unknown coefficients present in the muscle force are found by using the least square method. The maximum response amplitudes of the system obtained from the time response plots are in good agreement with that from the experimental findings. Hence, the developed mathematical work and the reduced equations can be used for further analysis to know the behavior of various kind of PAM.



Chapter 7

Development of an Experimental Setup and Novel Pneumatic Artificial Muscle

7.1 Introduction

In most of the day to day robotic applications, a bidirectional linear or rotational motion is required. So, to obtain the required motion, there is a need for a setup with an antagonistic arrangement using Pneumatic Artificial Muscle (PAM) actuators similar to the biceps-triceps muscle arrangement of the human arm. In this chapter, the fabrication of an experimental setup is carried out. Further, the manufacturing process of novel white Pat and golden Muga silk embedded PAMs have been discussed. In section 7.2, the design and development of the experimental setup is described. In section 7.3, the mold development and preparation of the novel PAM are presented. The summary of this chapter is briefly given in section 7.4.

7.2 Experimental Setup

The dynamics of the human arm movement to generate a bidirectional motion is mainly depend on the antagonistic arrangement of the biceps and triceps muscle of the human arm as shown in Fig. 7.1. These biceps and triceps muscle can be classified as flexor and extensor muscles to move the human arm. Biceps included in the flexor group, and it will bend the arm by decreasing the angle between the forearm and upper arm whereas, triceps muscle included in the extensor muscle. In flexion condition, the biceps muscle will contract while the triceps muscle will relax.

While in extension condition, the biceps will relax and triceps will contract. Finally, the tensile force acting on the two muscles gives rise to the required rotational motions through the antagonistic pair of the human elbow in the arm.

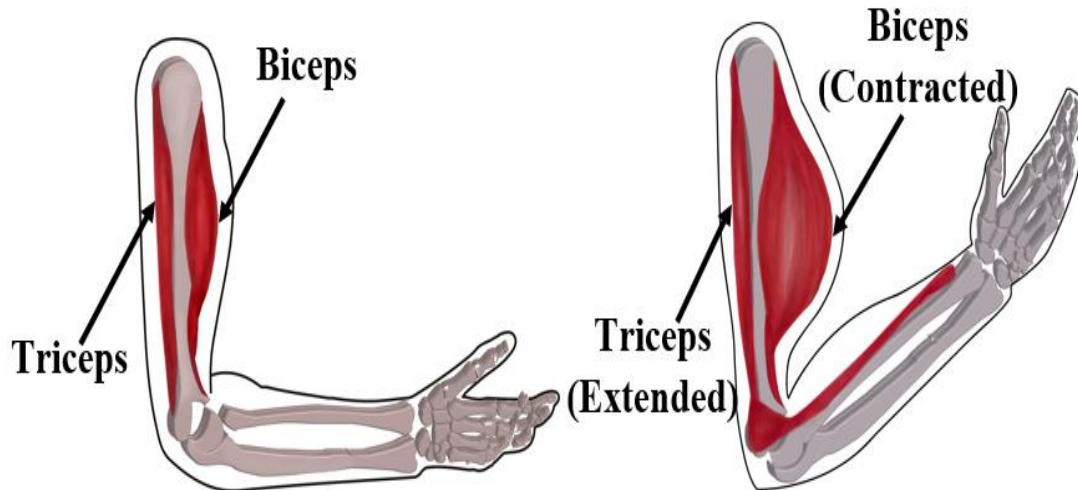


Fig. 7.1 Dynamics of the muscle in a human arm.

This concept of the antagonistic arrangement is mostly employed with the conventional actuators like hydraulic or pneumatic cylinders along with the newer smart materials, such as shape memory alloys [256] and electrostrictive materials [257]. But, due to the advantages of PAM over the other actuators, in various applications like aerospace [12, 121, 175], 3D printing [258], rehabilitation [259] where the artificial muscle is used to obtain the antagonistic actuation. Balara and Tóthová [260] described the static and dynamic properties of the PAM with the help of antagonistic arrangement in an experimental rig. Vocke et al. [151] designed a mechanism with the help of the PAM to achieve the bidirectional motion. But, due to the presence of the highly nonlinear force-contraction characteristic and compliance in PAM, achieving the antagonistic actuation is a challenge.

Many researches have studied various arrangements attached to the PAM to achieve antagonistic actuation with a wide range of actuator motion along with the control of the stiffness of the actuator. These arrangements involve simple pulleys [261, 262], complex variable radius pulleys [263], and offset lever mechanisms [175, 264] to modify the system kinematics. Due to the advantage of this antagonistic arrangement to achieve a wide range motion of the muscle actuator, a setup has been developed in this present work and the details of which are given in the next subsections.

7.2.1 Design of the setup

Figure 7.2(a) shows the schematic diagram of a link with attached mass m , which is rigidly connected to a pulley that is actuated by two artificial muscles PAM1 and PAM2. As explained before in Fig. 7.1 regarding the antagonistic motion of a human arm, here in a similar way, PAM1 and PAM2 will actuate to rotate the link. As the arm has to rotate by 180° , one may replace PAM2 by a spring to achieve a similar motion as shown Fig. 7.2(b). So, the desired movement and position can be achieved by the active PAM1 by controlling the air pressure through its valve. The active artificial muscle always depends on the variable air pressure where passive one acts like a spring at a constant pressure.

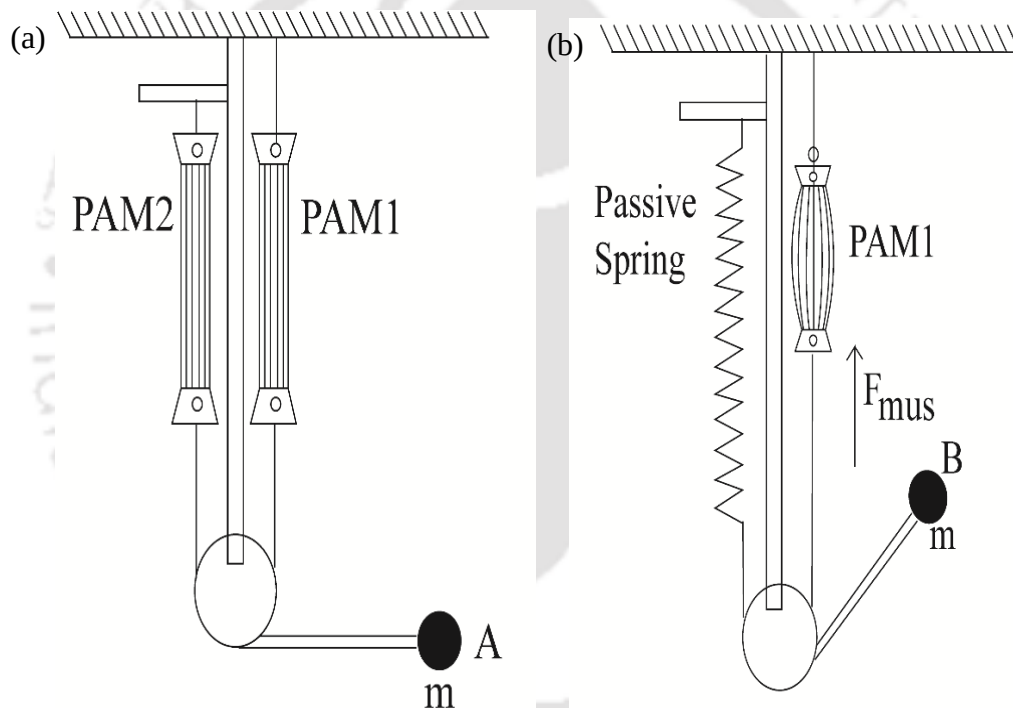


Fig. 7.2 Schematic antagonistic arrangement with (a) two PAMs at unpressurized condition (b) one active PAM and passive spring at pressurized condition.

Based on this principle, an experimental setup is designed and developed. The CAD model of the setup corresponding to Fig. 7.2(a) and Fig. 7.2(b) are given in Fig. 7.3(a) and Fig. 7.3(b), respectively, which are made using SOLIDWORKS software. In Fig. 7.3(a), two PAMs are in the antagonistic arrangement, and in Fig. 7.3(b) one of the PAMs has been replaced by a nonlinear tension spring.

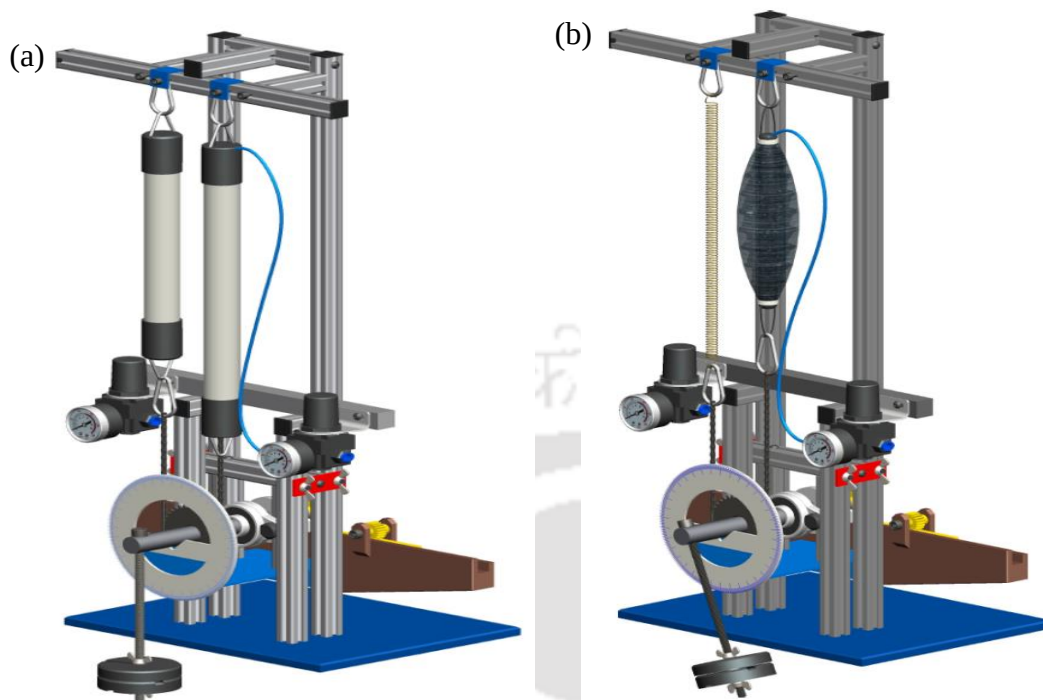


Fig. 7.3 CAD model of the setup in antagonistic arrangement with (a) two PAMs (b) one PAM and a spring.

7.2.2 Development of the setup

Figure 7.4 shows the schematic diagram of the setup with different parts. Figure 7.5 shows the chain-sprocket mechanism and rack and pinion mechanism for the rotational motion of the link. The setup is a bench-top design with size Length x Breadth x Height (L x B x H) = 17 x 15 x 35 inch with a base of mild steel plate (1) of size L x B = 14 x 11 inch and 8 mm thick. The frame is built with aluminum T-slot 2020 extrusions (2) fixed with the help of M5 bolts and T-nuts, which is used primarily to build a lightweight and stable frame. The frame also has two manual TECHNO mechanical flow control valves with dial indicators (3, 4), which enable controlled air supply to the individual muscles. With the help of two M5 Allen bolts (5), two snap hook (6) of steel material is mounted on the top portion of the frame. One end of an artificial muscle (7) and a spring (8) is attached in the setup with the help of the snap hooks, and the other ends are tied to two ends of a chain (16).

The frame also has an adjustable platform (11) with extrusion links on which a shaft (13) of 20 mm diameter is installed with the help of two pillow block bearings (12) as shown in Fig. 7.5.

This helps to fit the PAM of different types, design and length for performing the experiments. The adjustable platform (11) can be fixed to the frame with the help of a small aluminum extrusions slot (20) and four wing nuts (21). The shaft (13) has a through hole of 8mm diameter at one end for the assembling of the arm (14). The arm is a threaded rod of 8mm diameter where the loads (18) are mounted, and the position can be easily adjusted by two M8 wing nuts (19). Now, by supplying air pressure with 4 mm FESTO pneumatic tube (9) to actuate the muscle (7), the angular deflection can be recorded with the help of a protractor (15). The chain (16) roles over a sprocket (17) of teeth with 36 nos., which is mounted on the shaft (13) and fixed with grub screw. On the other end of the shaft (13), a hole with M6 thread is provided to fix a spur gear (26) of the module with one which is driving a rack (25) sliding on a slotted body (24) made of acrylic sheet. An idle gear (27) is also mounted for smooth movement of the rack. The rear end of the frame has a scale with which the linear movement of the rack (25) can be observed and measured simultaneously. This linear displacement corresponds to the change in muscle length, while extension/contraction can be achieved.

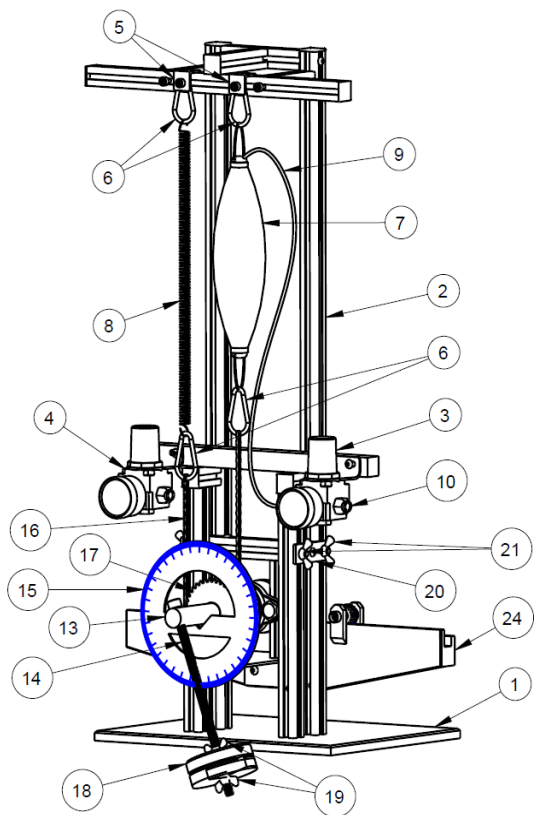


Fig. 7.4 Schematic of the developed setup.

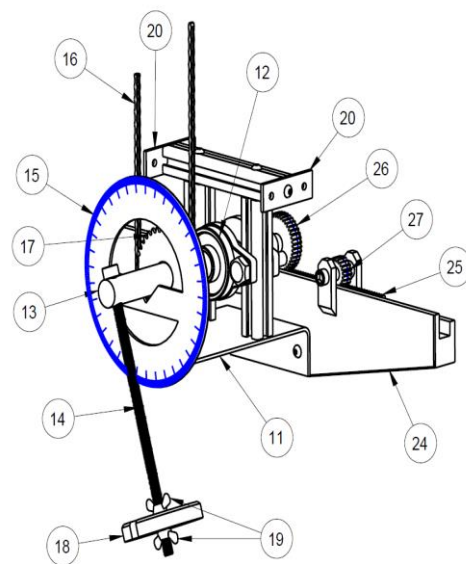


Fig. 7.5 Chain-sprocket mechanism attached to the setup.

7.2.3 Working principle of the setup

Figure 7.6(a) shows the setup when the attached PAM in unpressurized condition. The muscle of length L_1 is mounted between the hook of the frame and the chain. The other end of the chain is attached to one end of a nonlinear spring of length, S_1 whose other end is fixed to the frame with a hook. The movable platform (24) can be adjusted by a distance, H_x so that the artificial muscle achieve its maximum length. A mass of weight, W is loaded to the arm and placed at fixed distance, R with the help of wing nuts. Now, by supplying the compressed air to the muscle with length L_1 , it will make the muscle inflate and contract to a length L_2 . This results in the rotation of the sprocket by an angle and increases the length of spring to a length, S_2 as shown in Fig. 7.6(b). The new tension, T_2 on the chain will depend on the value of the weight, W , radius, R and the deflection angle, The pressure on the muscle while deflecting can be recorded from the dial indicator corresponding to the muscle. The motion in both directions can be achieved by using two PAMs attached to the setup, as shown in the CAD model in Fig. 7.3(a) for the required application.

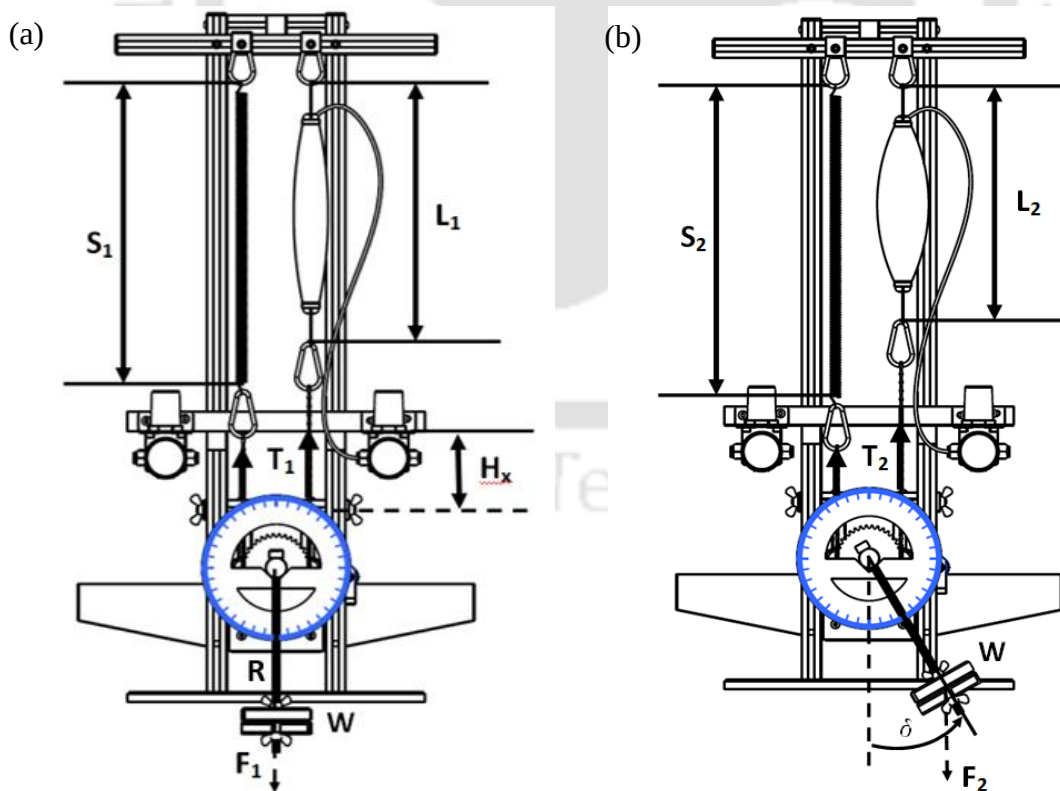


Fig. 7.6 Developed setup as shown in Fig. 7.4 at (a) unpressurized condition (b) pressurized condition.

7.2.4 Final developed setup

The front and rear views of the fabricated setup are shown in Fig. 7.7(a) and Fig. 7.7(b) respectively. This can be used to conduct various experiments for understanding the behavior of the different artificial muscles. Here, one can perform experiments with two muscles arranged in an antagonistic manner as well as single muscle with a nonlinear spring to understand the working of the PAM and its effect on the application. The design concentrates on the use to test efficiently PAMs of different types, materials and dimensions.

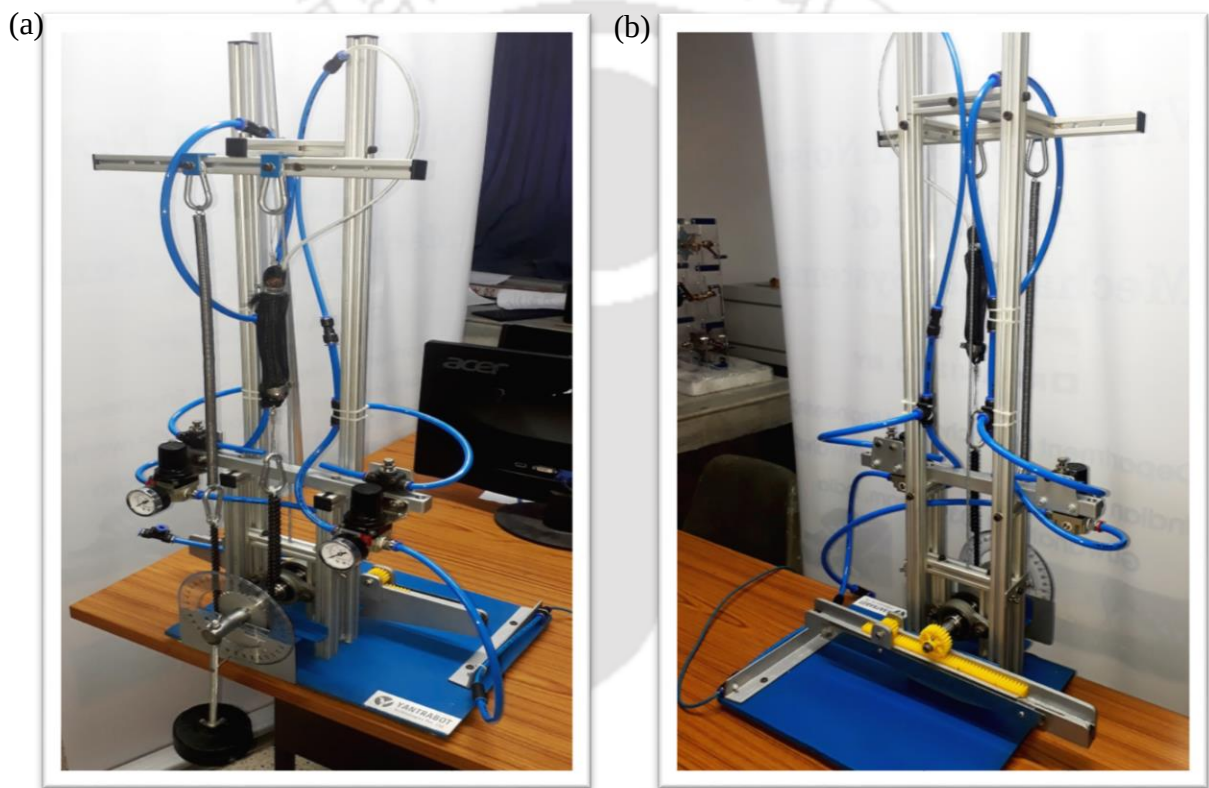


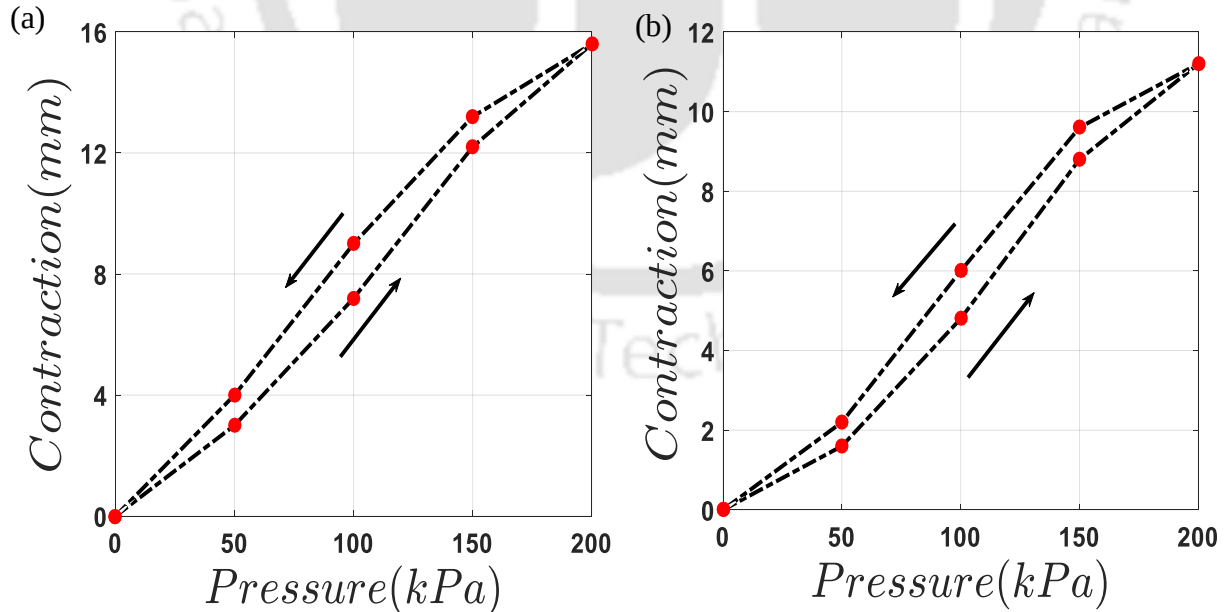
Fig. 7.7 Actual developed experimental setup with (a) front view and (b) rear view.

The setup has been connected with a McKibben type muscle and a spring in an antagonistic arrangement through the chain-sprocket mechanism. The spring will provide the necessary support to the system, which does not require any control. With the help of this experimental setup, the pressure inside the muscle was increased from 50 kPa to the safe pressure limit of 200 kPa in discrete steps of 50 kPa with the help of the flow control valve. At each pressure step, the PAM contraction was measured and noted as given in Table 7.1. The above steps were repeated for two loads of 1.5 kg (15 N) and 3 kg (30 N) attached to the setup.

Table 7.1 Variation of contraction of the muscle with air pressure for 15 N and 30 N loads.

Contraction in mm				
Pressure (kPa)	Load = 15 N		Load = 30 N	
	Loading	Unloading	Loading	Unloading
0	0.0	0.0	0.0	0.0
50	3.0	4.0	1.6	2.2
100	7.2	9.0	4.8	6.0
150	12.2	13.2	8.8	9.6
200	15.6	15.6	11.2	11.2

Taking the data from Table 7.1, Fig. 7.8(a) is plotted to show the effect of variation of pressure on the contraction of the muscle during increasing the pressure from 0 kPa to 200 kPa (loading) and decreasing the pressure from 200 kPa to 0 kPa (unloading) with a load of 15 N. Similarly, in Fig. 7.8(b), the loading and unloading conditions have been shown for a load of 30 N. Clearly, these figures show the existence of hysteresis during the operation of the muscle. These hysteresis effects are similar to observation found in the work of Li et al. [125] and Ganguly et al. [134] for the McKibben type of muscle. As the area enclosed by these curves are not significant, the hysteresis effect has not been considered in the present theoretical work. It has been kept as a scope for future work. With the help of this experimental setup, different muscle parameters can be determined as illustrated in chapter 2, chapter 3 and chapter 6.


Fig. 7.8 Typical hysteresis present in the used McKibben type muscle for (a) 15 N and (b) 30 N.

Though in the present work, McKibben type of muscle has been considered for the theoretical and experimental analyses purpose, it is felt to develop a new kind of artificial muscle which will have the same functionality but with lower cost and use of locally available fabrics. Hence, an attempt has been made to develop a new artificial muscle that is presented in the next section. It may be noted that this developed muscle is not used for analysis purposes in this work, which can be studied in a similar way in the future.

7.3 Novel Pneumatic Artificial Muscle

A novel PAM is developed in this work, in which the high-intensity locally available natural fabrics like Golden *Muga* and White *Pat* silk are embedded in the silicon rubber. These PAMs are one such class of actuators in which fabrics are arranged longitudinally to make it frictionless so that the product life cycle can be extended and hysteresis of the actuator becomes small. These will be capable of delivering a high power-to-weight ratio, just like the widely available McKibben Muscle and significant similarities with our body muscles.

7.3.1 Material selection

Silicon rubber [265] of a premium grade with high performance for long lasting flexible molds called MoldSil 15 as shown in Fig. 7.9 is used with its catalyst for curing. The Catalyst CAT-16 which is a medium speed catalyst with moderate work life for fast demoulding and takes about 16 hours at room temperature for complete curing. The color of the used rubber is white, and the catalyst is transparent. This type of silicon rubber offers a good combination of good flow, high mechanical properties, and excellent detail reproduction with good service life in mold making applications. The main features of MoldSil 15 silicon rubber are, easy to mold, fast curing at room temperature, high tear strength, low shrinkage, long mold life, soft and flexible in nature, low shrinkage (0.1%-0.3%), superior chemical and high heat resistance. It has excellent release properties for easy demolding due to the presence of high elastic nature. The viscosity of the silicon rubber, MoldSil 15 is 23000 mPa.s, and the catalyst, CAT-16 is mPa.s as per the datasheet comes along with the product. Normally, the shelf life of rubber, MoldSil 15, and the catalyst, CAT-16 is 6 months from the date of manufacturing if stored below 27°C in original unopened containers.



Fig. 7.9 Silicon rubber, MoldSil 15 [265].

White Pat silk fiber [266] differs in composition, structure and properties depending on their specific source and function. This kind of silk is produced by *Bombyx textor* silkworms, which feed on mulberry leaves and usually brilliant white or off-white in color. This *Pat* silk or Mulberry silk of Assam is a variety of domestic silk in Assam, India. Silk is a continuous protein fiber (a fibrous biopolymer) produced by the silkworm so as to form the cocoon for their shelter as shown in Fig. 7.10(a). This cocoon has been utilized as textile fiber, and the physical properties of this kind silk are different from all other natural fiber available to date. This silk is unique for the combined extensibility along with high tensile strength, and the toughness is also higher than that of Kevlar fibers. This mechanical superiority is thought to be dependent on the molecular structure, thread substructure (supra-molecular structure) as well as the formation process.

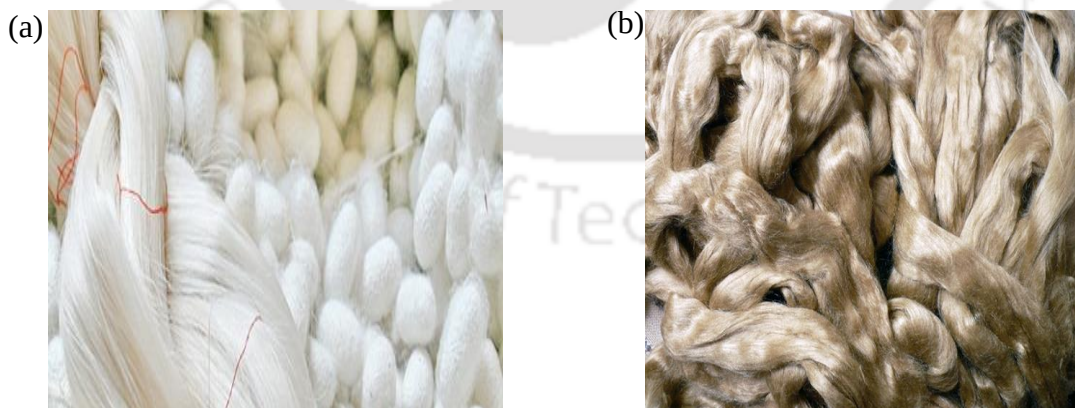


Fig 7.10 Natural fabrics (a) White *Pat* silk cocoon and fiber [266] (b) Golden *Muga* silk fiber [267].

Golden Muga silk fiber [267] as shown in Fig. 7.10(b) is the product of the silkworm *Antheraea assamensis* endemic to Assam, India. The larvae of these moths feed on som (*Machilus*

bombycina), and sualu (*Litsaea polyantha*) leaves. The silk produced is known for its glossy fine texture and durability. Like the *White Pat* silk, it is also produced by the silkworm so as to form the cocoon and, after that, exploited as a textile fiber. The silkworm secretes two continuous filaments made of a core protein “fibroin” producing by its two salivary glands, which are cemented together by another gummy protein “sericin” and finally become the natural silk by hardening the filaments. Due to some outstanding mechanical and aesthetic properties like unique luster, strength, excellent softness, easy handling etc. *Muga* silk is known as the Queen of fiber, and it possesses the highest tenacity among all natural fibers. The higher tenacity of *Muga* imparts a wide range of its use in various purposes in textiles and as non-weave material.

7.3.2 Manufacturing process

A casting setup or die as shown in Fig. 7.11 is prepared for the fabrication of the artificial muscle with the help of the silicon rubber reinforced by the fabrics *Pat* or *Muga* Silk. Here, the casting setup is designed as per the desired shape, length, thickness, etc. for the artificial muscle. The casting setup consists of mainly a hollow CPVC tube, a SS center rod and a pair of brass caps.

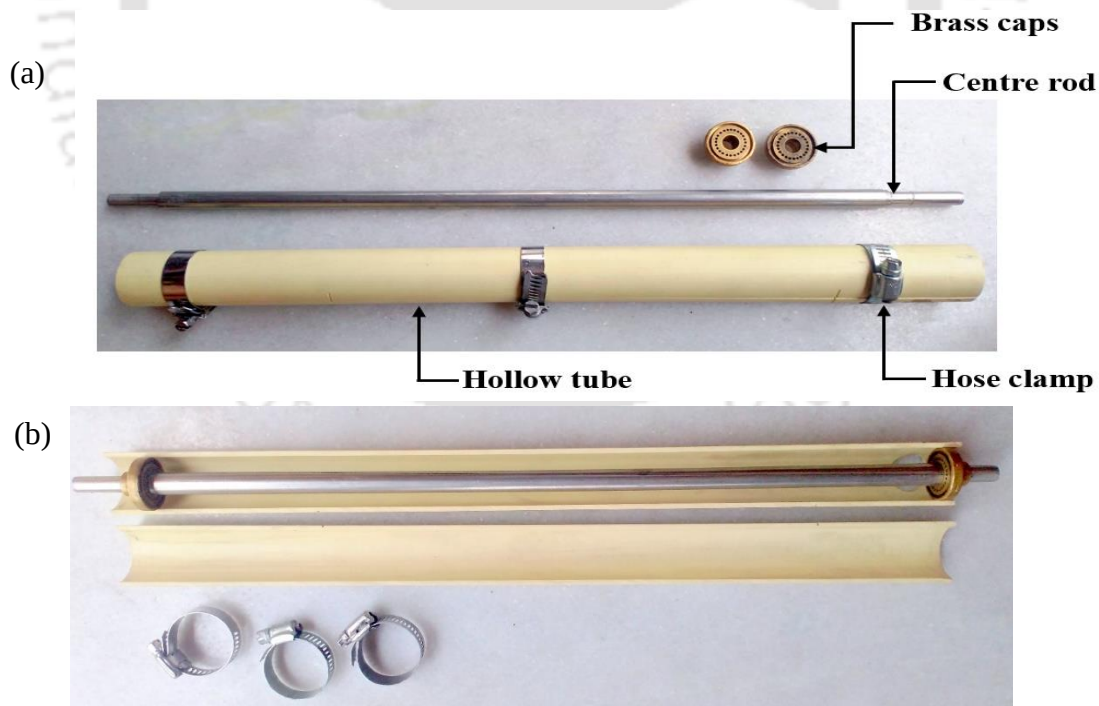


Fig. 7.11 Casting setup for the proposed PAM.

Initially, the threads of Pat or Muga silk fiber, as shown in Fig. 7.12(a) has been made to align in the axial direction with the help of the center rod and a pair of brass caps. The diameter of the center rod is kept 12mm to fix the pneumatic attachments on the final proposed PAM. The brass caps are shown in Fig. 7.12(b) where a number of threads of the fiber can be woven with a maximum of 24 nos. It may be noted that the number of threads, the thickness of the rubber, length, and radius of the PAM can be changed in the casting setup to achieve the required property for a particular application. The number of threads and the thickness of the PAM can be changed with the help of the brass caps and the diameter of the hollow tube. In Fig. 7.12(b), two pairs of brass caps have been shown with different thickness of 6 mm and 10 mm. So, for the proposed PAM, 12 nos. of white Pat silk fiber threads have been woven align in the axial direction with the help of the center rod and the pair of brass caps with 10 mm thickness as shown in Fig. 7.12(c) and the cross-sectional view is shown in Fig. 7.12(d). Then, this center rod with the woven threads of fiber has been placed in the hollow tube, which will behave as the master for the mold as shown in Fig. 7.11(b).

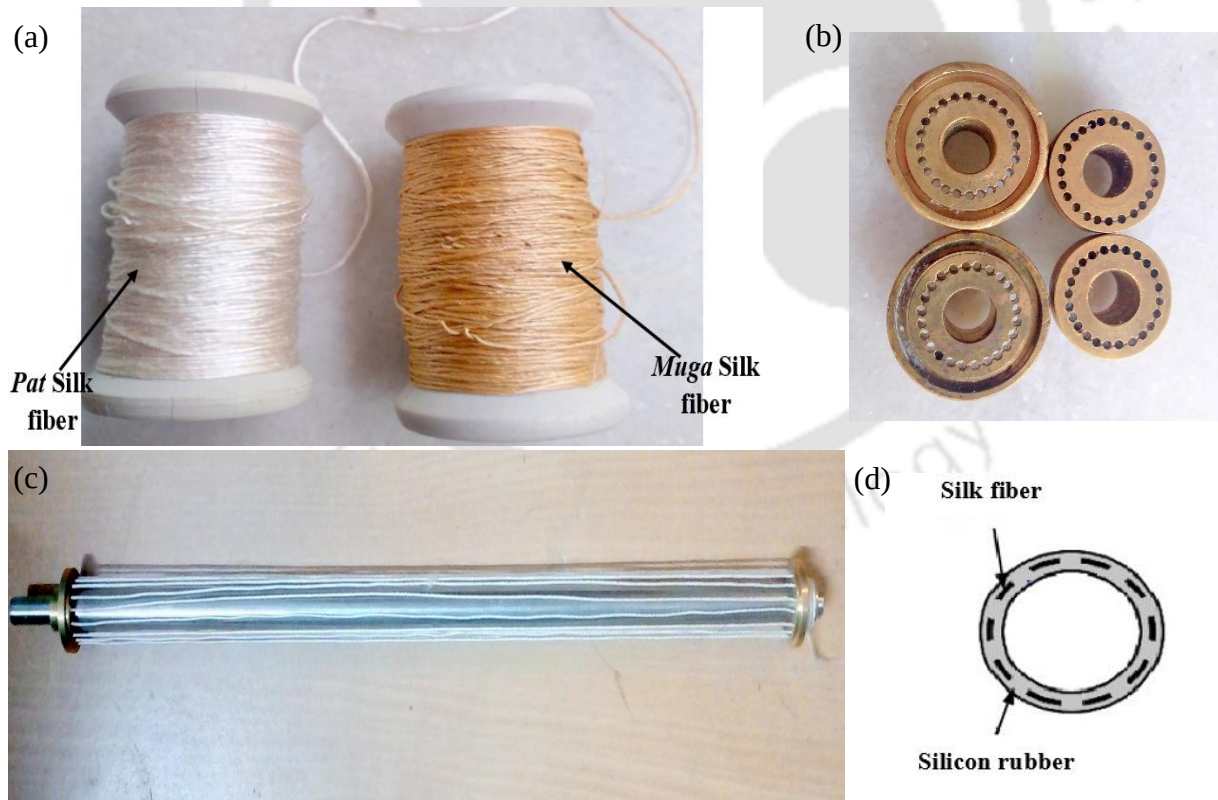


Fig. 7.12 (a) Threads of silk fiber (b) brass caps (c) center rod with woven fiber threads (d) cross-sectional view.

For fabricating the proposed PAM, the surface of the CPVC hollow tube was cleaned and made free of loose materials and dust particles. A suitable release agent such as petroleum jelly or soap solution was used with porous substrates. The silicon rubber, MoldSil 15 was stirred thoroughly in a container before the addition of a catalyst to avoid filler separation, which might have occurred during prolonged storage. This was a crucial step to get the desired performance. After that, the catalyst, CAT-16 was added around 5% to the total volume of silicon rubber to be used. Again, the stirring was done vigorously for several minutes by scraping the sides which produce a homogeneous mix at the bottom of the container. Hand or mechanical mixing can be used but should not mix for an extended period of time to avoid entrapping large amounts of air or causing overheating resulting in shorter work life. It is recommended that entrapped air should be removed under vacuum to eliminate voids in the final product. This process will make the mixture to expand and then collapse. A volume increase of about 4-5 times will occur during the deaeration process. Therefore, a large container which is 7-8 times larger than the total material to be mixed, should be used to accommodate this volume change. It should also be noted that prolonged application of vacuum will remove the volatiles from the mixture that can result in a poor cure. This mixture is sensitive to temperature and humidity, so it can influence the cure speed.

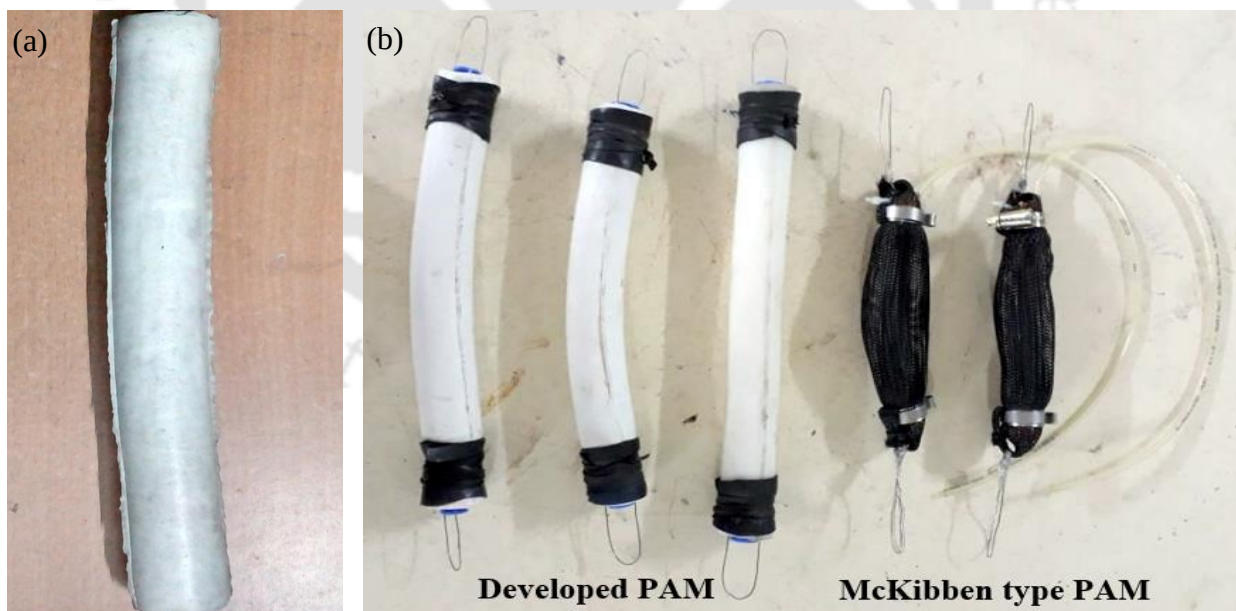


Fig. 7.13 Developed PAM (a) after taking out from mould (b) Final novel PAMs with different dimensions and traditional McKibben type muscle.

Now, the mixture of the silicon rubber and the catalyst was poured as soon as possible on to the original master mold with the center rod containing the fiber threads to avoid air entrapment. This

was cured to a flexible rubber within 24 hours at room temperature, and the mold was separated from the master. The material was cured at speed, depending on the selection and the amount of the catalyst. Finally, the outcome of this manufacturing process has been shown in Fig. 7.13(a), which was the developed PAM. In Fig. 7.13(b), the developed PAMs of different dimensions have been shown with the pneumatic attachments. The traditionally used McKibben type of muscles is also shown there for comparison purposes.

7.3.3 Experimental observation of the Novel PAM

Static and dynamic force analyses had been carried out by fixing the muscle to rigid support at one end and attaching dead loads to the other end. The compressed air was supplied using a compressor through the mechanical valve. Due to the presence of fabrics, the length of the muscle was constant, and hence, the longitudinal and lateral expansion of the newly developed muscles were controlled by applying pneumatic pressure. In Fig. 7.14(a, b) loads of 4 kg (40 N) and 6 kg (60 N) were applied to the developed muscle. So, it was observed that without air pressure, it can take up a load comparable to that of a McKibben muscle.

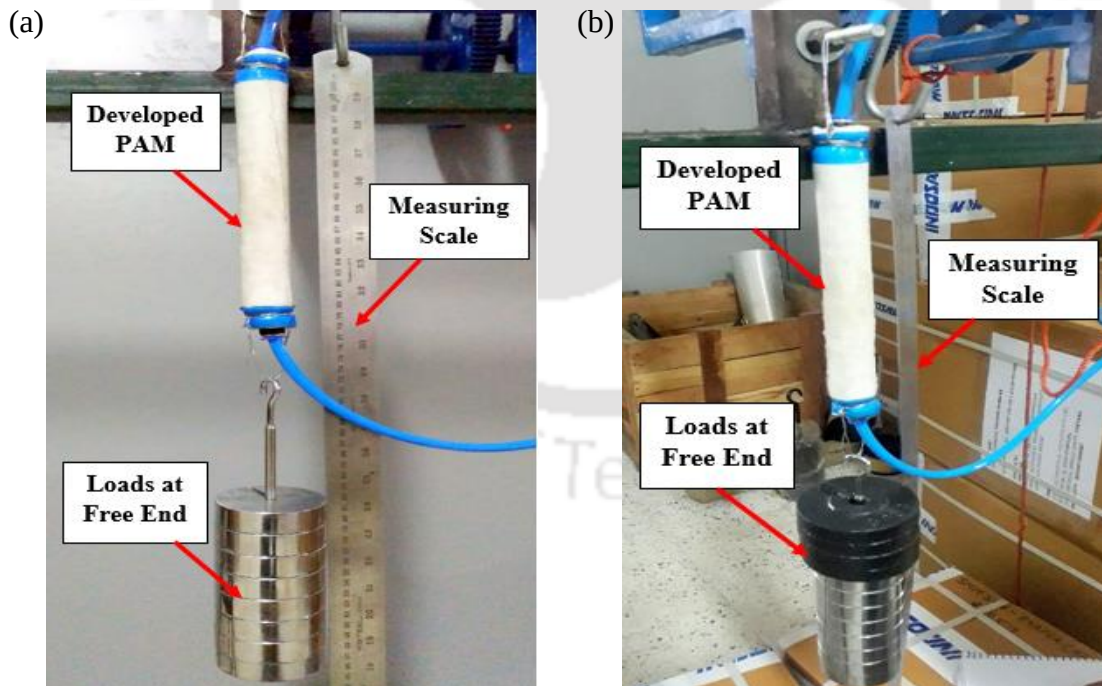


Fig. 7.14 Static loading on the developed PAM with (a) 4 kg (40 N) (b) 6 kg (60 N).

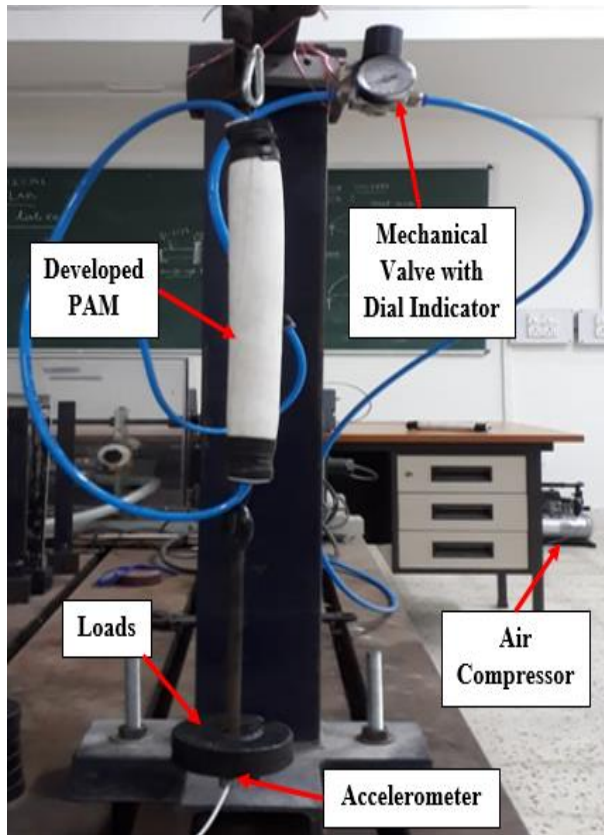


Fig. 7.15 Dynamic loading on the developed muscle.



Fig. 7.16 Novel muscle attached with the developed experimental setup.

Now, air pressure was applied to the muscle, which was attached with a load of 1 kg (10 N) as shown in Fig. 7.15. For the pressure of 100 kPa and a contraction of 20 mm was observed. It may be noted that for a similar situation, the corresponding McKibben muscle provides only 10.6 mm (Table 6.4) contraction. At a load 15 N with a muscle pressure of 200 kPa, the McKibben muscle has a contraction of a maximum 15.6 mm (Table 7.1). So, hence, the developed low cost PAM is sufficiently better in terms of strength and contraction characteristics. Further, free vibration analysis of the developed muscle is carried out by using an accelerometer and NI LabVIEW 2009 software. The fundamental frequency of the developed muscle at a pressure of 100 kPa and 1 kg load was found to be 13.9 Hz (Fig. 7.17(a)). It may be noted that the corresponding McKibben muscle has a fundamental frequency of 16.3 Hz (Fig. 7.17(b)). Figure 7.16 shows the novel muscle is attached with the developed experimental setup for antagonistic motion. So, the developed muscle can be used for various actuator applications like exoskeletons for rehabilitation purposes.

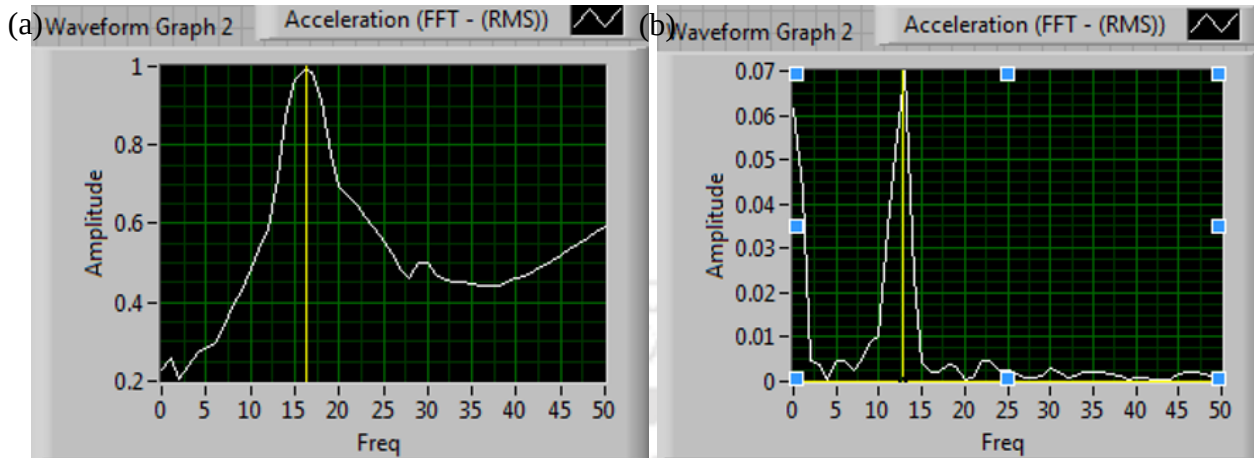


Fig. 7.17 Experimental fundamental frequency (a) developed PAM (b) McKibben muscle

7.4 Summary

In this chapter, an experimental setup has been designed and developed, which can be used for testing a wide range of muscles available in the market before use in different applications. The system has been designed, keeping in mind some major factors like flexibility, robustness, weight and uses. The principle of the antagonistic arrangement is clearly illustrated with the help of this setup. The various nonlinear behavior present in the muscle can be studied by performing experiments on this system. The setup provides both the angular motion and the linear motion simultaneously by attaching a rack and pinion arrangement along with the chain-sprocket mechanism. Using the setup and McKibben muscle, the hysteresis effect is illustrated, and it is found to be small. As the most commonly used McKibben type of PAMs are costly and not easily available in the local market, for a wide use of PAM, it is felt to develop a novel artificial muscle using locally available material. In this work, mold has been prepared, and white Pat silk and golden Muga silk based artificial muscles have been successfully fabricated which can take a load upto 60 N. It is shown that the developed muscle can provide more contraction with a less amount of air pressure than that of the McKibben muscle to lift the same amount of load. Further, the fundamental frequency of the developed muscle is found to be comparable with that of the McKibben muscle. Hence, the experimental setup, along with developed PAM will be very useful for various applications related to PAM, which may find many uses in the field of healthcare and other related industrial applications.

Chapter 8

Conclusions and Scope for Future Work

8.1 General Conclusions

The present work deals with the nonlinear dynamics of a system attached to the pneumatic artificial muscle (PAM) under forced and parametric excitations. Sometimes the forcing is may be weak, and sometimes it may be hard depending on the amplitude of the external forcing. Here, the McKibben type of artificial muscle is used for the analysis purpose. Due to these excitations, the system may undergo various resonance conditions, such as simple, principal parametric, simultaneous, superharmonic and subharmonic resonance conditions. The governing nonlinear equation of motion of the system has been derived with the help of Newton's 2nd law. Using the method of multiple scales (MMS), the governing equation of motion has been solved, and a set of reduced equations are obtained for each case. It is shown that the results obtained by using the method of multiple scales are in good agreement with the results obtained by solving the original nonlinear equation of motion numerically.

To attain the desired response of the nonlinear system, the different system parameters values should be chosen suitably to actuate the muscle. So with the help of the frequency response plots, the safe operating range of the different system parameters for a particular application can be evaluated. The basin of attraction have been determined to illustrate the importance of the initial conditions to obtain an accurate system response, which validate the frequency response plots. The reduced equations are used to obtain periodic, quasi-periodic and chaotic responses along with the saddle-node, Hopf and pitchfork bifurcations. Due to the bifurcation points, jump-up and

jump-down phenomena may be observed while sweeping up and down the frequency. In the case of subcritical bifurcations, the system may undergo catastrophic failure. The time response, phase portraits and frequency response have been plotted to study the behaviour of the nonlinear responses of the system with different set of parameters such as, damping, various muscle parameters, nonlinear muscle force, stiffness of the external spring; amplitude and frequency of external force, and operating pressure.

To study the nonlinear dynamics of the system containing pneumatically actuated muscle, two different single degree of freedom systems have been considered in this work. The first physical system consists of a spring-mass-damper with a PAM at a constant pressure supplied to muscle with an external harmonic force and supported by an external spring. The second physical system consists of a mass and a PAM subjected to a time varying pneumatic pressure, which results in a forced and parametrically excited system.

Taking these two physical systems, five different cases have been investigated. In the first case, the nonlinear dynamics have been studied for the first system by considering nonlinearity in the muscle force under simple resonance condition where the external excitation frequency is nearly equal to the natural frequency of the system. For the second case, the dynamic response has been investigated for the second physical system under the simple and principal parametric resonance conditions. In principal parametric resonance condition, the external excitation frequency is nearly equalled twice the natural frequency of the system. Here, the trivial state instability regions, along with the critical values for different system parameters, have been found out. In the third case, the nonlinear dynamics of the second physical system has been studied under principal parametric and simultaneous resonance condition. For simultaneous resonance condition, both simple and principal parametric resonance occurs simultaneously when the external excitation frequency of the system is nearly equal to the natural frequency of the system.

Unlike in the first three cases, where the forcing is considered to be weak, in the remaining two cases, hard excitation, i.e., very high amplitude of forcing amplitude, is considered. In the fourth case, the first physical system is considered, which is subjected to super- and sub-harmonic resonance conditions. The external excitation frequency is nearly equal to the one third and three times to the natural frequency of the system for the super- and sub-harmonic resonance conditions,

respectively. For the final fifth case, the second physical system is studied under hard excitation and subjected to super- and sub-harmonic resonance conditions.

For the first, second and fifth cases, the theoretical results have been verified by conducting free and forced vibration experiments using the McKibben type of muscle. The experimental results are found to be in good agreement with the numerical work. Finally, an experimental setup has been developed to illustrate the working of the artificial muscle based on the antagonistic arrangement of the biceps and triceps muscles of the human arm. Further, new types of PAM have been fabricated with the help of silicon rubber and locally available fabrics like white *Pat* silk and golden *Muga* silk. Using experimentally obtained force-contraction behavior with different pressure and the least square method, two systematic procedures for determining the system parameters of a PAM are presented in this work. With the use of these experimentally obtained muscle parameters, the responses are obtained theoretically, which are found to be in good agreement with the experimental results.

From the above mentioned five theoretical works, the designers and researchers may get to know about the effect of various system parameters on the dynamics of the muscle actuation. To achieve the desired amplitude for a specific application, one should choose the system parameters in between their safe operating range to avoid the failure of the system. The system can achieve a particular displacement actively by controlling the amplitude and frequency of the air pressure along with the forcing parameter. Also, it may be noted that the same response can be passively achieved by changing the various muscle parameters and damping, which can be obtained by changing the PAM along with the stiffness of the external spring. Hence, the developed closed form solutions and the reduced equations from theoretical work for various resonance conditions can be effectively used for various medical as well as industrial applications, which will take a small amount of memory along with less computational time.

8.2 Specific Conclusions

8.2.1 Nonlinear dynamics of a harmonically excited PAM

- In this case for simple resonance condition, the system does not possess any trivial state response and always vibrates with amplitude equal to that of the nontrivial state.
- The frequency response shows both single and bi-stable states along with supercritical Hopf and saddle node bifurcation points.
- The response amplitude of the system is decreased by 8% while damping parameter (μ) is increased from 0.001 to 0.01 and it decreased by 38% while one increase in the value of μ from 0.01 to 0.1. Hence, one should not simply interpolate to predict the value of the response which will give erroneous results.
- With lower value of operating air pressure P , the system undergoes supercritical Hopf bifurcation at lower value of frequency resulting in oscillatory motion. With increase in the value of P , this bifurcation point disappears and the system has only stable nontrivial response. So, one can actively control the response of the system by suitably changing the air pressure.
- With lower value of muscle parameter c_1 , the system experiences supercritical Hopf bifurcation and the system oscillates for a wide range of frequency of excitation. With increase in the value of c_1 , this bifurcation point disappears. Hence, one has to change the PAM to have a stable nontrivial response of the system.
- With lower value of muscle parameter c_2 , the system undergoes supercritical Hopf bifurcation at higher value of frequency resulting in oscillatory motion. With 100% increase in the value of c_2 , one can achieve a reduction 15% in maximum response amplitude. Similar effect on response amplitude can be observed in the variation of muscle parameter c_3 . But in this case, the variation in the value of c_3 , Hopf bifurcation is not observed.
- A steady attenuation of 23% in the maximum response amplitude can be observed by doubling the coefficient of cubic nonlinear term α .

- Either by increasing the external spring stiffness k or by decreasing the amplitude of external load F , the response amplitude decreases. But it may be noted that this decrease is not linear. Hence, one should use the developed closed form equation to accurately predict the response amplitude for various system parameters.

8.2.2 Dynamics of a parametrically excited PAM

- For the simple resonance condition, using 1st order MMS, the maximum response amplitude is found to be 22% more than that found by solving the original equation. For the same case, using 2nd order MMS the solution is found to be 13% more than that obtained by solving the original equation. Hence, the 2nd order MMS is more suitable which provides a conservative results for the system.
- From the trivial state instability regions, it is observed that the unstable region in the instability plot will decrease with the increase in the value of damping and observed at higher value of the dynamic pressure (P_0) for both the resonance conditions.
- For simple resonance condition, the critical value of P_0 (P_{0cr}) increases 3 times (i.e., 46.4 kPa) by increasing damping 0.001 to 0.01. However, the value of P_{0cr} for $\mu=0.1$ is 8 times (i.e., 125.9 kPa) more than that for $\mu=0.001$ (i.e., 15 kPa).
- For principal parametric resonance condition, the value of P_{0cr} for $\mu=0.01$ is found to be 10 times (i.e., 20.8 kPa) and for $\mu=0.1$, it is 70 times (i.e., 142.2 kPa) more than that for $\mu=0.001$ (i.e., 2.1 kPa).
- The variation of P_{0cr} with damping is not linear and hence, one should use the closed form equations for predicting the instability regions and the corresponding critical values.
- The value of P_{0cr} for principal parametric resonance condition is found to be very less than that of the simple resonance condition for lower value of damping. But for higher value of damping principal parametric resonance condition has slightly more P_{0cr} than that of the simple resonance condition.
- For a particular application, one can find the optimum value of muscle parameters by finding the variation of P_{0cr} with muscle parameters c_1 , c_2 and c_3 . For example, the values

of P_{0cr} are 46.4 kPa and 20.8 kPa for simple and principal parametric resonance conditions, respectively with the values of muscle parameters $c_1 = -234.25$ N, $c_2 = 1.96$ N/kPa and $c_3 = -0.003$ N/ kPa².

- It is observed that the trivial state instability regions is not affected by the variation of muscle parameters d_1 and d_2 for both simple and principal parametric resonance conditions.
- For simple resonance condition, with lower value of static pressure P_m (e.g., $P_m = 200$ kPa), a narrow instability region is obtained, where for the higher value of frequency of the system is found stable. Hence, to have a contraction in the muscle, P_m and frequency should be chosen appropriately. But, in case of principal parametric resonance condition, as the instability region is very wide even for small value of P_m , this resonance condition can be effectively used to have the required contraction.

8.2.3 Nonlinear dynamics of a PAM under parametric excitation

- For both principal parametric and simultaneous resonance conditions, unlike the case for $P_0 \sim \sigma$, the instability regions in case of $c_1 \sim \sigma$ and $P_m \sim \sigma$ are inverted in nature. Hence, there exists a minimum value of P_0 and maximum value of c_1 and P_m for which the trivial state of the system is always stable.
- For principal parametric resonance, the critical values of P_0 and P_m are found to be 6 times and 4.5 times more, respectively whereas the critical value of c_1 is 3.5 times less for the variation of μ from 0.01 to 0.08.
- For simultaneous resonance condition, the critical values of P_0 and c_1 are found to be 2 times more and 4 times less, respectively for the variation of μ from 0.005 to 0.02.
- For both the resonance conditions, the variation in the critical values of the system parameters P_0 , P_m and muscle parameter c_1 with damping is not linear and hence, the closed form equations should be used to plot the instability regions and find the critical values.

- For principal parametric resonance condition, the PAM can operate and a bulging effect will take place between the super- and sub-critical pitchfork bifurcation points.
- For simultaneous resonance condition, the trivial state becomes unstable between the super- and sub-critical Hopf bifurcation points. One may achieve single, bi-stable and tri-stable state depending on the initial conditions.
- With decrease in the value of damping parameter, the response amplitude and the trivial state instability regions increase for both the resonance conditions. But, a stable trivial state is observed for simultaneous resonance condition for higher value of μ (i.e., $\mu = 0.1$).
- With the increase in the value of P_0 , the maximum response amplitude and the trivial state instability regions increase in both the resonance conditions. However, for principal parametric resonance condition, the response amplitude is almost more than 2.5 times that of the case for simultaneous resonance condition. For example, $P_0 = 29$ kPa, the maximum response amplitude is 0.294 and 0.116 for principal parametric and simultaneous conditions, respectively.
- For both the resonance condition, the response amplitude decreases with increase in the value of P_m upto a critical value and after that again the response amplitude increases with increase in value of P_m . In the case of simultaneous resonance condition, the system has additional saddle node and Hopf bifurcation points for lower value of P_m (i.e., $P_m = 200$ kPa). The trivial state instability regions is found to be increased by 12 times more with decrease in the value of P_m from 300 kPa to 200 kPa for principal parametric resonance condition.
- There exist an optimum value of muscle parameter c_1 for which the response amplitude of the system is maximum for both the resonances. The maximum response amplitude increase by 40% for principal parametric resonance condition and 13% for simultaneous resonance condition by reducing the value of the muscle parameter c_1 from -117.12 N to -468.5 N. But, in the case of simultaneous resonance condition, additional Hopf bifurcation points giving rise to periodic responses are observed in the nontrivial branch with lower value of muscle parameter c_1 ($c_1 = -468.5$ N).

- For both the resonance conditions, it is observed that for an optimum value of the muscle parameters c_1 , c_2 and c_3 , the maximum response amplitude of the system is observed. So, these values should be chosen optimally to obtain the required maximum response amplitude for a particular application. With the variation in these muscle parameters, a stable trivial state is noticed for simultaneous resonance condition.
- There is no significant change in the response amplitude with variation in muscle parameters d_1 and d_2 for principal parametric resonance condition. For simultaneous resonance condition, the effect of d_1 is not prominent on response amplitude, but a steady amplification of around 10% can be observed in the response amplitude with increase in the value of d_2 . Further it is observed muscle parameters d_1 and d_2 has no effect on the trivial state unstable region for both the resonance conditions.
- A steady attenuation of 29% in the response amplitude is observed with doubling α for principal parametric resonance condition. But, for simultaneous resonance condition, the interpolation cannot be made to understand the response of the system with the variation in the α . There is no change in the trivial unstable range of the system for both the resonance conditions.

8.2.4 Nonlinear dynamics of a PAM under hard excitation

- For superharmonic resonance condition, the system exhibits two distinct zones in nontrivial frequency response plots. Here, in addition to fixed point responses with saddle node, super- and sub-critical Hopf bifurcations, periodic, multi-harmonic, homoclinic orbits and three periodic orbits leading to chaotic responses are observed. The butterfly effect for chaos is also noticed by varying the initial conditions.
- In the case of subharmonic resonance condition, trivial state is found to always stable and the nontrivial state has a saddle node bifurcation. Also, periodic and quasi-periodic response are observed in this case.
- For superharmonic resonance condition, the frequency response in Zone 1 is almost similar to that of a linear spring-mass-damper system for higher value of P (e.g., $P = 600$ kPa), μ (e.g., $\mu = 0.05$) and c_3 (e.g., $c_3 = 0.6$ N/kPa²) along with the lower

value of F (e.g., $F = 6$ kN) and α (e.g., $\alpha = 200$). But in these cases, the Zone 2 has many types of bifurcations such as, saddle node and super- and sub- critical Hopf bifurcations.

- For the lower value P (e.g., $P = 400$ kPa) and c_3 (e.g., $c_3 = 0.2$ N/kPa²), the system has only a single zone where the softening effect is observed. Also, for the higher value of F (e.g., $F = 10$ kN) and α (e.g., $\alpha = 600$) in Zone 2, this effect is noticed for superharmonic resonance condition.
- For subharmonic resonance condition, it is observed that with lower value of P , μ and c_3 along with of higher value F and α , the nontrivial state starts with a saddle node bifurcation at lower value of σ and the system has a bi-stable after this bifurcation point. As the trivial state is always stable, it is not advisable to operate the PAM in this resonance condition.
- The effect of the stiffness of the external spring (k) on the system is not prominent for both the resonance conditions.

8.2.5 Nonlinear dynamics of a PAM under parametric and hard excitation

- For superharmonic resonance condition, the system exhibits nontrivial fixed point responses with two saddle node bifurcation points. Also, the periodic, multi-harmonic, and homoclinic orbits are observed for this case.
- For subharmonic resonance condition, the trivial state is found to be stable and nontrivial state has a saddle node bifurcation point along with periodic and quasi-periodic responses.
- For superharmonic resonance condition, doubling P_0 from 29 kPa to 58 kPa, the maximum response amplitude of the system increases by 2 times and with further doubling P_0 from 58 kPa to 116 kPa, the maximum response amplitude increases by 3 times. Hence, the variation of response amplitude with P_0 is not linear.
- For a particular value of P_0 , with increase in P_m , reduces the maximum response amplitude and with further increase in P_m , results in increase of the maximum response amplitude for superharmonic resonance condition. For example, for $P_m = 200$ kPa, 300

kPa and 400 kPa, the corresponding response amplitude $a = 1.35, 0.35$ and 0.45 , respectively. Hence, the variation of maximum response amplitude with P_m is not linear.

- In superharmonic resonance condition, by increasing α from 75 to 150, the maximum response amplitude is found to be increased by 6.6% and with further increase in α from 150 to 300, the maximum response amplitude is increased by 12.4%. So, here also, the variation is not linear.
- For both the resonance conditions, it is observed that there is an optimum value of the muscle parameters c_1 , c_2 and c_3 for which the system has a maximum response amplitude. So, these values should be chosen optimally to obtain the required maximum response amplitude for a particular application.
- The effect of the muscle parameter d_1 is not distinct in both the resonance conditions.
- For superharmonic resonance condition, the maximum response amplitude increases by 2 times with increase in the muscle parameter d_2 from 0.5 N/kPa to 1 N/kPa and with further increase in the value of d_2 from 1 N/kPa to 2 N/kPa, the maximum response amplitude increases by 3 times.
- As the variation of maximum response amplitude with most of the system parameters are not linear for both the resonance conditions, hence, one should use the developed closed form solution instead of applying superposition rule.

8.2.6 Development of an experimental setup and novel PAM

- An experimental setup has been developed to enable the antagonistic arrangement of PAM which can mimic the biceps and triceps muscles of the human arm. The design concentrates on the use to test the PAM efficiently with different types, design and length.
- In the developed system, one can perform experiments with two muscles arranged in antagonistic manner as well as single muscle with a nonlinear spring to understand the working of the PAM and its effect on the application.
- The typical hysteresis effect has been observed in the McKibben muscle while increasing and releasing the pressure in the artificial muscle. The energy loss due to hysteresis is found to be small and hence, in the present work this effect has not been considered.

- A novel PAM is developed in this work, in which the high-intensity locally available natural fabrics like Golden *Muga* and White *Pat* silk are embedded in the silicon rubber.
- It may be noted that the number of threads, thickness, length and radius of the PAM can be changed with the help of the casting setup to achieve the required property of the proposed muscle for a particular application.
- The developed low cost PAM is sufficiently better in terms of strength and contraction characteristics compare to the McKibben muscle.

8.3 Scope for Future Work

In the present work, the nonlinear dynamic analyses have been carried out for a system attached with the pneumatic artificial muscle (PAM) under forced and parametric excitations. The present work may be extended in multiple directions and some of them are given below.

- In the present work, the system is modeled as a single degree of freedom system. It can be extended to multi degrees of freedom and continuous systems.
- The analysis has been limited to single frequency of the system. Multi frequency excitation with many resonance conditions may be explored for various practical applications.
- The present work may be extended by considering the hysteresis present in the PAM in the phenomenological model to understand statics and dynamics of the muscle accurately.
- The system response of present work can be compared by using the other perturbation techniques, like the Straight-Forward expansion, the method of Harmonic Balance, methods of Averaging and many other newly developed numerical and analytical techniques.
- In all the above mentioned cases, more experiments may be performed to validate the theoretical findings.
- The present theoretical analysis can be extended by incorporating the developed artificial muscles.
- The developed muscle may be used for various practical applications like, actuator in the lower and upper body exoskeleton systems for paraplegic and elderly persons, in various surgical robot in medical applications, industrial applications with soft actuation, etc.



Appendix

A.1 Derivation of the nonlinear temporal equation of motion for the first physical system.

The muscle force, F_{mus} expression is as follows.

$$F_{mus}(u, P) = (c_1 + c_2 P + c_3 P^2) \left(\frac{u}{l_{\max}} \right) + Ku^3 \quad (\text{A.1})$$

Putting Eq. (A.1) in the equation of motion which is obtained using Newton's 2nd law.

$$m \frac{d^2 u}{dt^2} + c \frac{du}{dt} + ku + (c_1 + c_2 P + c_3 P^2) \left(\frac{u}{l_{\max}} \right) + Ku^3 = F \sin \omega t \quad (\text{A.2})$$

$$\frac{d^2 u}{dt^2} + \frac{c}{m} \frac{du}{dt} + \left(\frac{k}{m} + \frac{c_1 + c_2 P + c_3 P^2}{ml_{\max}} \right) u + \frac{Ku^3}{m} = \frac{F}{m} \sin \omega t \quad (\text{A.3})$$

Now by taking, $\tau = \omega_0 t$ and $\frac{du}{dt} = \frac{du}{d\tau} \left(\frac{d\tau}{dt} \right) = \omega_0 \frac{du}{d\tau} \Rightarrow \frac{d^2 u}{dt^2} = \omega_0^2 \frac{d^2 u}{d\tau^2}$, where,

$$\omega_0 = \sqrt{\frac{k}{m} + \frac{c_1 + c_2 P + c_3 P^2}{ml_{\max}}} \quad (\text{A.4})$$

Eq. (A.3) can be written as follows.

$$\omega_0^2 \frac{d^2 u}{d\tau^2} + \frac{c}{m} \omega_0 \frac{du}{d\tau} + \omega_0^2 u + \frac{Ku^3}{m} = \frac{F}{m} \sin \omega t \quad (\text{A.5})$$

Here, the nondimensional displacement can be written as $u = r x$ by using a scaling factor r .

$$\frac{d^2 x}{d\tau^2} + \frac{c}{m\omega_0} \frac{dx}{d\tau} + x + \frac{r^2 Kx^3}{m\omega_0^2} = \frac{F}{m\omega_0^2} \sin \omega t \quad (\text{A.6})$$

Now by using the following nondimensional parameters in Eq. (A.6)

$$\Omega = \frac{\omega}{\omega_0}, \quad \mu = \frac{c}{2\varepsilon m \omega_0}, \quad \alpha = \frac{r^2 K}{\varepsilon m \omega_0^2}, \quad f = \frac{F}{r \varepsilon m \omega_0^2} \quad (\text{A.7})$$

The governing nonlinear temporal equation of motion can be written as below.

$$\frac{d^2 x}{d\tau^2} + 2\varepsilon\mu \frac{dx}{d\tau} + x + \varepsilon\alpha x^3 = \varepsilon f \sin \Omega \tau \quad (\text{A.8})$$

A.2 Derivation of the nonlinear temporal equation of motion for the second physical system.

The muscle force, F_{mus} expression is as follows.

$$F_{mus}(u, P) = (c_1 + c_2 P + c_3 P^2) \left(\frac{u}{l_{\max}} \right) + (d_1 + d_2 P) + Ku^3 \quad (\text{A.9})$$

By taking the harmonics internal pressure input to the muscle as $P = P_m + P_0 \sin \omega t$, Eq. (A.9) can be written as follows.

$$F_{mus} = (c_1 + c_2 P_m + c_3 P_m^2 + \frac{c_3 P_0^2}{2}) \left(\frac{u}{l_{\max}} \right) + (c_2 P_0 + 2c_3 P_m P_0) \left(\frac{u \sin \omega t}{l_{\max}} \right) - \frac{c_3 P_0^2}{2} \left(\frac{u \cos 2\omega t}{l_{\max}} \right) + (d_1 + d_2 P_m + d_2 P_0 \sin \omega t) + Ku^3 \quad (\text{A.10})$$

Putting Eq. (A.10) in the equation of motion which is obtained using Newton's 2nd law.

$$\begin{aligned} \frac{d^2 u}{dt^2} + \frac{c}{m} \frac{du}{dt} + \left(\frac{c_1 + c_2 P_m + c_3 P_m^2 + c_3 \frac{P_0^2}{2}}{ml_{\max}} \right) u + \left[\left(\frac{c_2 P_0 + 2c_3 P_m P_0}{ml_{\max}} \right) \sin \omega t - \frac{c_3 P_0^2}{2ml_{\max}} \cos 2\omega t \right] u \\ + \left(\frac{d_1 + d_2 P_m - mg}{m} \right) + \left(\frac{d_2 P_0}{m} \right) \sin \omega t + \frac{Ku^3}{m} = 0 \end{aligned} \quad (\text{A.11})$$

Now by taking, $\tau = \omega_0 t$ and $\frac{du}{dt} = \frac{du}{d\tau} \left(\frac{d\tau}{dt} \right) = \omega_0 \frac{du}{d\tau} \Rightarrow \frac{d^2 u}{dt^2} = \omega_0^2 \frac{d^2 u}{d\tau^2}$, where,

$$\omega_0 = \sqrt{\frac{c_1 + c_2 P_m + c_3 \left(P_m^2 + \frac{P_0^2}{2} \right)}{ml_{\max}}} \quad (\text{A.12})$$

Eq. (A.11) can be written as follows.

$$\begin{aligned} \omega_0^2 \frac{d^2 u}{d\tau^2} + \frac{c}{m} \omega_0 \frac{du}{d\tau} + \omega_0^2 u + \left[\left(\frac{c_2 P_0 + 2c_3 P_m P_0}{m l_{\max}} \right) \sin \omega t - \frac{c_3 P_0^2}{2m l_{\max}} \cos 2\omega t \right] u \\ + \left(\frac{d_1 + d_2 P_m - mg}{m} \right) + \left(\frac{d_2 P_0}{m} \right) \sin \omega t + \frac{K u^3}{m} = 0 \end{aligned} \quad (\text{A.13})$$

Here, the nondimensional displacement can be written as $u = r x$ by using a scaling factor r .

$$\begin{aligned} \frac{d^2 x}{d\tau^2} + \frac{c}{m \omega_0} \frac{dx}{d\tau} + \left[1 + \left(\frac{c_2 P_0 + 2c_3 P_m P_0}{m \omega_0^2 l_{\max}} \right) \sin \omega t - \frac{c_3 P_0^2}{2m \omega_0^2 l_{\max}} \cos 2\omega t \right] x \\ + \left(\frac{d_1 + d_2 P_m - mg}{r m \omega_0^2} \right) + \left(\frac{d_2 P_0}{r m \omega_0^2} \right) \sin \omega t + \frac{r^2 K x^3}{m \omega_0^2} = 0 \end{aligned} \quad (\text{A.14})$$

Now by using the following nondimensional parameters in Eq. (A.14)

$$\begin{aligned} \Omega = \frac{\omega}{\omega_0}, \quad \mu = \frac{c}{2\varepsilon m \omega_0}, \quad p_1 = \frac{c_2 P_0 + 2c_3 P_m P_0}{\varepsilon m \omega_0^2 l_{\max}}, \quad p_2 = -\frac{c_3 P_0^2}{2\varepsilon m \omega_0^2 l_{\max}}, \quad f_1 = \frac{d_2 P_m + d_1 - mg}{r \varepsilon m \omega_0^2}, \\ f_2 = \frac{d_2 P_0}{r \varepsilon m \omega_0^2}, \quad \alpha = \frac{r^2 K}{\varepsilon m \omega_0^2} \end{aligned} \quad (\text{A.15})$$

The governing nonlinear temporal equation of motion can be written as below.

$$\frac{d^2 x}{d\tau^2} + 2\varepsilon \mu \frac{dx}{d\tau} + \varepsilon (1 + p_1 \sin \Omega \tau + p_2 \cos 2\Omega \tau) x + \varepsilon \alpha x^3 = \varepsilon (f_1 + f_2 \sin \Omega \tau) \quad (\text{A.16})$$



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List of Publications

Journals publications:

1. Bhaben Kalita and S. K. Dwivedy, **Nonlinear Dynamics of a Parametrically Excited Pneumatic Artificial Muscle (PAM) Actuator with Simultaneous Resonance Condition**, *Mechanism and Machine Theory*, 135, 281-297, 2019. <https://doi.org/10.1016/j.mechmachtheory.2019.01.031>
2. Bhaben Kalita and S. K. Dwivedy, **Dynamic Analysis of Pneumatic Artificial Muscle (PAM) Actuator for Rehabilitation with Principal Parametric Resonance Condition**, *Nonlinear Dynamics*, 97(4), 2271-2289, 2019. <https://doi.org/10.1007/s11071-019-05122-2>
3. Bhaben Kalita and S. K. Dwivedy, **Numerical Investigation of Nonlinear Dynamics of a Pneumatic Artificial Muscle With Hard Excitation**, *ASME-Journal of Computational and Nonlinear Dynamics*, 15(4), p. 041003, 2020. <https://doi.org/10.1115/1.4046246>
4. Bhaben Kalita and S. K. Dwivedy, **Nonlinear Dynamic Response of Pneumatic Artificial Muscle: A Theoretical and Experimental Study**, *International Journal of Non-Linear Mechanics*, 125, p. 103544, 2020. <https://doi.org/10.1016/j.ijnonlinmec.2020.103544>
5. Bhaben Kalita, Jyotindra Narayan and S. K. Dwivedy, **Development of Active Lower Limb Robotic-Based Orthosis and Exoskeleton Devices: A Systematic Review**, *International Journal of Social Robotics*. (Accepted, In Press) <https://doi.org/10.1007/s12369-020-00662-9>
6. Jyotindra Narayan, Bhaben Kalita and S. K. Dwivedy, **Development of Robotic-based Upper Limb Devices for Rehabilitation Purposes: A Systematic Review**, *Augmented Human Research*. (Accepted, In Press)
7. Bhaben Kalita and S. K. Dwivedy, **Study of Parametric Instability Regions of Pneumatic Artificial Muscle under Forced and Parametric Excitation**, *Mechanics Based Design of Structures and Machines*. (Revised Manuscript Under Review)
8. Bhaben Kalita and S. K. Dwivedy, **Pneumatic Artificial Muscle Actuator under Parametric and Hard Excitations: A Theoretical and Experimental Investigation**, *Mechanical Systems and Signal Processing*. (Manuscript Under Review)

Conferences publications:

1. Bhaben Kalita and S. K. Dwivedy, **Dynamic Analysis of a Parametrically Excited Golden Muga Silk Embedded Pneumatic Artificial**, *The 14th International Conference on Vibration Engineering and Technology of Machinery (VETOMAC XIV)*, Lisbon, Portugal, September 10-13, 2018.
2. Bhaben Kalita and S. K. Dwivedy, **Forced Vibration Analysis of a Silk Fibre Embedded Pneumatic Artificial Muscle**, *The 6th International Conference on Robot Intelligence Technology and Applications (RITA 2018)*, Putrajaya, Malaysia, December 16-18, 2018.
3. Bhaben Kalita and S. K. Dwivedy, **Design and Analysis of a Silk embedded Pneumatic Artificial Muscle (PAM) Actuator for Rehabilitation Purpose**, *National Conference on Disability and Social Inclusion-The Role of Technology*, Guwahati, India, January 10-11, 2019.
4. Bhaben Kalita and S. K. Dwivedy, **Nonlinear Study of a Pneumatic Artificial Muscle (PAM) under Superharmonic Resonance Condition using Method of Multiple Scales**, *15th International Conference Dynamical System Theory and Applications (DSTA 2019)*, Łódź, Poland, December 2-5, 2019.
5. Bhaben Kalita, Arunjyoti Borgohain and S. K. Dwivedy, **Antagonistic Actuation of Pneumatic Artificial Muscle (PAM) with Chain-Sprocket Mechanism**, *4th International and 19th National Conference on Machines and Mechanisms (iNaCoMM 2019)*, IIT Mandi, India, December 5-7, 2019.

Patents:

1. Bhaben Kalita, Arunjyoti Borgohain and S. K. Dwivedy, **Testing Machine for Pneumatic Artificial Muscle Actuator in Antagonistic Arrangement**. (Under Preparation)
2. Bhaben Kalita, Arunjyoti Borgohain and S. K. Dwivedy, **Novel Pneumatic Artificial Muscle Actuator using natural fiber reinforcement**. (Under Preparation)