



INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

This thesis mainly focuses on the numerical solutions for the various types of fractional order differential and integro-differential equations. We have discussed the ordinary, partial, and integro-partial differential equations involving RLC-type and Caputo-type fractional derivatives. The numerical methods we have proposed in this thesis are mainly the shooting method, the cubic spline, and the discontinuous Galerkin finite element method. Besides finding the numerical solutions, we have discussed the well-posedness of some of the problems.

The thesis begins with the introduction providing the basic definitions, detailed literature survey and describing the outline of the rest of the chapters. First, we have studied the shooting technique for the steady-state linear and semi-linear fractional BVPs of RLC-type. Then, we have considered a class of one-dimensional linear and semi-linear time-fractional diffusion IBVPs. Here, we used the Sumudu decomposition approach and the maximum-minimum principle to demonstrate the existence and uniqueness of the analytical solution of the linear IBVP, respectively. Also, using L1-discretization for the fractional temporal derivative and the cubic spline approximation for the spatial variables, we provide a numerical scheme for both the linear and semi-linear diffusion IBVP. The same analytic and numerical solution approach is applied to a class of linear fractional integro-differential equation (FIDE).

Then, for the linear and semi-linear time-fractional diffusion problem, we obtained the numerical solution by applying the non-symmetric interior penalty Galerkin (NIPG) method for the spatial direction on a uniform mesh and the L1-discretization for the time derivative on a graded mesh. Also, we have studied the superconvergence of error estimates in the discrete energy-norm. After that, we discuss an efficient numerical method for solving nonlinear FIDE, where we first use the Newton's linearization process and then apply the NIPG method for discretizing the spatial variable and both the L1-discretization and L2-discretization for the time-fractional derivative and the trapezoidal rule for the integral term. Next, we suggest a fully discrete numerical solution approach for the time-fractional Burgers' equation, by considering the L2-discretization formula in the temporal direction and the NIPG method for the spatial variable. Finally, we proposed an alternating direction implicit (ADI) type operator splitting discontinuous Galerkin finite element method to solve a class of two-dimensional time-fractional diffusion equations, numerically. For solving both the sub-problems obtained by splitting technique, we use the L2-discretization in time discretization and the NIPG approach for spatial variable.

The thesis concludes with a summary and suggestions for further research.

