

ONLINE HANDWRITING REPRESENTATION AND SYNTHESIS USING  
SINUSOIDAL MODEL



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**ONLINE HANDWRITING REPRESENTATION AND  
SYNTHESIS USING SINUSOIDAL MODEL**

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*Thesis submitted*

*for the award of the degree of*

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## Certificate

This is to certify that the thesis entitled “**ONLINE HANDWRITING REPRESENTATION AND SYNTHESIS USING SINUSOIDAL MODEL**”, submitted by **Himakshi Choudhury** (146102005), a research scholar in the *Department of Electronics and Electrical Engineering, Indian Institute of Technology Guwahati*, for the award of **Doctor of Philosophy**, is a record of an original research work carried out by her under our supervision and guidance. The thesis has fulfilled all requirements as per the regulations of the institute and in our opinion has reached the standard needed for submission. The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

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To  
**The Memories of My Father**  
Late Nilamoni Choudhury

**My Mother**  
Binita Choudhury

&

**My Brother**  
Janardan Choudhury  
for their love and support



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# Abstract

This thesis aims to model online handwriting using a sinusoidal model and explore its efficacy for handwriting representation and synthesis. The online handwriting data provides the dynamic information of the handwriting along with its two-dimensional (2-D) trace in the form  $(x, y)$  coordinate sequence. Typically, various shape information is derived from the 2-D trace and used for characterization of the handwriting. On the other hand, the sinusoidal model-based approaches can extract movement-based information alternative to the shape-based attributes. Therefore, the sinusoidal modeling of handwriting can be utilized in different areas of handwriting-based applications. In this work, three directions are identified for exploring the sinusoidal model- handwriting representation for recognition, handwriting modification for data augmentation, and personalized handwriting synthesis. The experiments are carried out on three databases: UNIPEN character and UNIPEN-ICROW-03 word databases, IRONOFF character and word databases, and Assamese character and word databases.

In the first part of our work, we propose a multi-component sinusoidal model for online handwriting. In this model, the sinusoidal parameters: amplitude, frequency, and phase are extracted from the horizontal and vertical velocities. These parameters are obtained by modeling the velocity between each successive pair of zero-crossing points with a half period of the sine function. It is observed that a single component is not sufficient to model complex handwriting shapes. Therefore, multiple sinusoidal components are used to model online handwriting trajectories. The parameters of different components are computed iteratively from the residual signals. Two methods are used to evaluate the efficacy of the multi-component sinusoidal model on English and Assamese handwriting. The first is measuring the reconstruction quality in terms of the signal-to-noise ratio (SNR) obtained by computing the difference between the original and reconstructed velocities. The second method to measure the reconstruction quality is the performance of a hidden Markov

model (HMM) based character recognizer. The HMM models are trained using original handwritten data and tested using the reconstructed handwriting. The optimal number of HMM states and mixtures are found to be 11 and 15, respectively, for English and Assamese databases. On the other hand, the number of mixtures in the Gaussian mixture model (GMM) is fixed to 20. The recognition results show that two or three components in the proposed model are sufficient to represent the handwriting data.

In our second work, a feature set is proposed for online handwriting recognition using the sinusoidal model. The sinusoidal parameters help to extract hidden movement-based information in contrast to the conventional point-based feature that captures the geometric shape of the handwriting. Essentially the predominant information of handwriting is modeled in the velocity profiles itself, sufficient for the reconstruction purpose. However, apart from these velocity signals, the acceleration and the position signals are also crucial in improving the discrimination ability of the feature set. Therefore, these signals are also represented by the multi-component sinusoidal model. The parameters from the different signals are analyzed, and an optimal set of features is proposed to develop the online handwriting character and word recognition systems. We also combine the proposed feature representation with the popular point-based features as both these feature sets contain different information. The HMM and support vector machine (SVM) are used to build the recognition systems. The results obtained are promising in comparison with the reported works.

In the third work, we propose a handwriting modification method using the sinusoidal model. A handwriting shape is primarily controlled by the relation between its horizontal and vertical velocities. The multi-component sinusoidal model represents these velocities using sinusoidal oscillations whose parameters— amplitude, frequency, and phase are modulated at different time instants. Therefore, it provides flexibility to vary the model parameters extracted from handwriting to introduce natural variability to the original handwriting. We hypothesize that the modified handwriting obtained by parameter variation within a given limit does not lose the structural and dynamic characteristics of natural handwriting. We demonstrate the effect of varying different model parameters on the handwriting shape and obtain acceptable ranges for variation of the different

sinusoidal parameters. The efficacy of the proposed method is shown by using the modified handwriting for data-augmentation to train a GMM-HMM and DNN (deep neural network)-HMM-based recognition system.

In the last part of our work, we introduce a novel approach for personalized handwriting synthesis using the sinusoidal model. It is observed that the multi-component sinusoidal model could efficiently reconstruct the handwriting (2-D trace) and its velocities. Thus, the model parameters can also be utilized to store the writer-specific handwriting styles efficiently in a compact manner. The proposed method follows the concatenation-based approach, where a generation model produces handwriting at smaller units that are concatenated to synthesize handwriting at a higher level. The generation model in this work is at the character level stored in the form of the sinusoidal parameters. The output of the system is produced at word-level formed by the concatenation of the generated characters. Given an input text in the form of ASCII code, the isolated handwriting for the characters is first generated using the model parameters. These characters are then joined using a sinusoidal model-based method for joining characters. The concatenation method estimates sinusoidal parameters to synthesize the joining ligature based on the ending and beginning region of two neighboring characters. Thus, natural-looking cursive handwriting can be synthesized, which follows the dynamic properties of handwriting. The personalization and naturalness of synthesized handwriting are measured by subjective evaluation methods.



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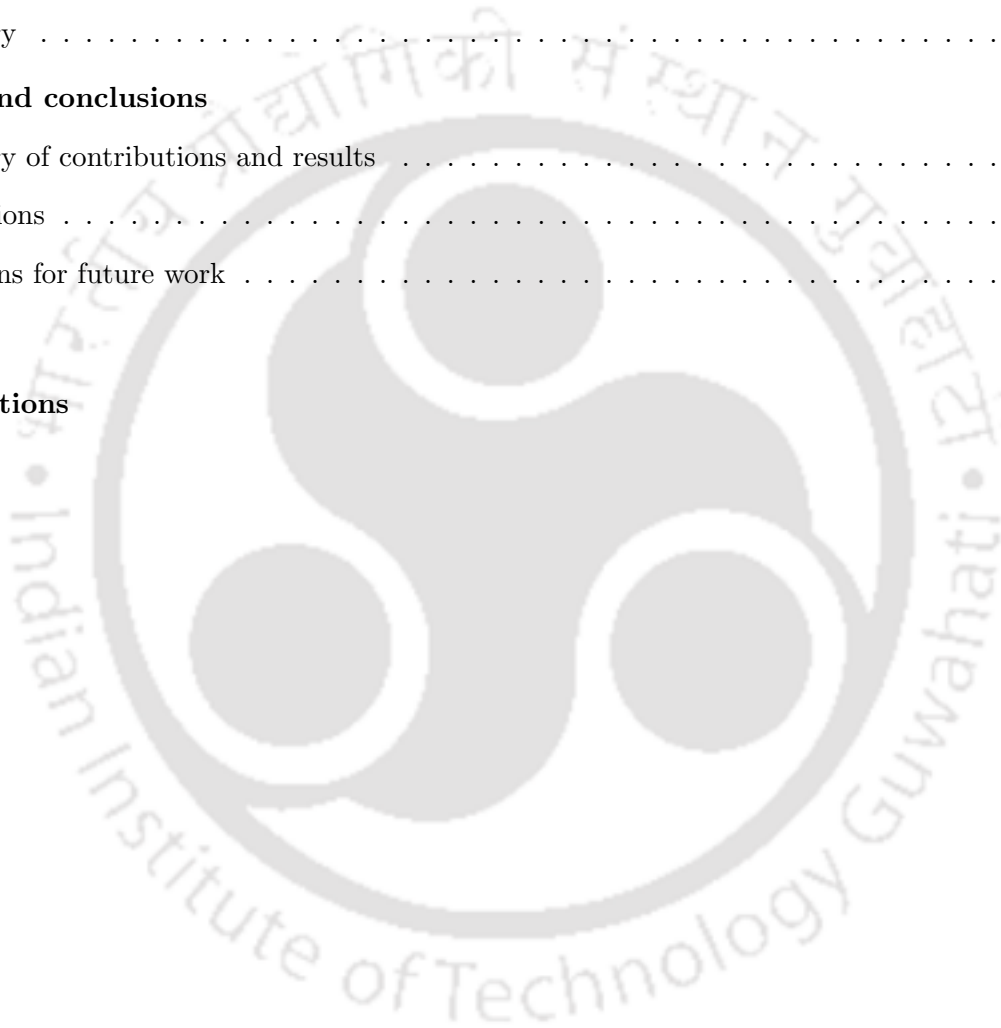
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# List of Acronyms

2D	Two dimensional
CAPTCHA	Completely Automated Public Turing test
CNN	Convolution Neural Network
DFT	Discrete Fourier Transform
DNN	Deep Neural Network
DNN-HMM	Deep Neural Network-Hidden Markov Model
DTW	Dynamic Time Warping
GMM	Gaussian Mixture Model
GMM-HMM	Gaussian Mixture Model-Hidden Markov Model
GP	Gaussian Posterior
HMM	Hidden Markov Model
OCR	Optical Character Recognition
POMH	Parsimonious Oscillatory Model of Handwriting
RBF	Radial Basis Function
SNR	Signal to Noise Ratio
SVM	Support Vector Machine
TDNN	Time Delay Neural Network



# List of Symbols

$a$	Amplitude of horizontal oscillation in Hollerbach model
$a_{v_x}(t)$	Amplitude of horizontal oscillation in POMH model at time instant $t$
$a_{v_y}(t)$	Amplitude of vertical oscillation in POMH model at time instant $t$
$a_{S,i}$	Amplitude of sinusoidal oscillation representing signal $S$ between $z_{S,i}$ and $z_{S,i+1}$
$a_{v_x,i}^m$	The amplitude of $m^{th}$ sinusoidal component of horizontal velocity between $z_{v_x,i}$ and $z_{v_x,i+1}$
$a_{v_y,i}^m$	The amplitude of $m^{th}$ sinusoidal component of vertical velocity between $z_{v_y,i}$ and $z_{v_y,i+1}$
$a_{v_x}^m(t)$	Amplitude of $m^{th}$ sinusoidal component of horizontal velocity at time instant $t$
$a_{v_y}^m(t)$	Amplitude of $m^{th}$ sinusoidal component of vertical velocity at time instant $t$
$a_{v_x}^{(1-K)}$	Amplitude parameter of the $K$ components extracted from horizontal velocity $v_x$
$a_{v_y}^{(1-K)}$	Amplitude parameter of the $K$ components extracted from vertical velocity $v_y$
$a_{r_x}^{(1-K)}$	Amplitude parameter of the $K$ components extracted from horizontal acceleration $r_x$
$a_{r_y}^{(1-K)}$	Amplitude parameter of the $K$ components extracted from vertical acceleration $r_y$
$a_{s_x}^{(1-K)}$	Amplitude parameter of the $K$ components extracted from normalized $x$ -coordinate signal $s_x$
$a_{s_y}^{(1-K)}$	Amplitude parameter of the $K$ components extracted from $y$ -coordinate signal $s_y$
$a_{v_x}^{m,new}$	Modified amplitude of $m^{th}$ sinusoidal component of horizontal velocity
$a_{v_y}^{m,new}$	Modified amplitude of $m^{th}$ sinusoidal component of vertical velocity
$a_{ij}$	State transition probability from state $i$ to $j$
$A$	Transition matrix of HMM
$b$	Amplitude of vertical oscillation in Hollerbach model
$B$	Observation probability matrix of HMM
$\beta$	Slant of handwriting

## List of Symbols

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$c$	The magnitude of horizontal sweep in Hollerbach model
$\hat{c}$	Recognized character class.
$d_x$	Variation parameter for $x$ -coordinate of horizontal velocity zero-crossing points
$d_y$	Variation parameter for $y$ -coordinate of vertical velocity zero-crossing points
$d_\phi$	Variation parameter for phase modification
$d_\theta^g$	Variation parameter for global slant modification
$d_\theta^l$	Variation parameter for local slant modification
$k_1$	Horizontal velocity constraint at the top corner in Hollerbach model
$k_2$	Horizontal velocity constraint at the bottom corner in Hollerbach model
$k_3$	Slant constraint in Hollerbach model
$K$	Number of components in the multi-component sinusoidal model.
$\lambda_j$	Learned HMM for the class $j$ .
$N_S$	Number of zero-crossings in a signal $S$
$N_{state}$	Number of states in HMM
$N_{samples}$	Number of real samples present in the training set of a database
$N_{train}$	Number of real samples used to train the system
$N_x$	Number of zero-crossings in $v_x$
$N_y$	Number of zero-crossings in $v_y$
$N_x^m$	Number of zero-crossings in the signal $v_x^m$
$N_y^m$	Number of zero-crossings in the signal $v_y^m$
$\mathbf{o}_i$	Feature vector corresponding to the $i^{th}$ point of the online handwriting coordinate sequence
$p_{real}$	Percentage of $N_{samples}$ used to train the system
$P$	$(x, y)$ co-ordinate sequence of online handwriting
$\phi_x$	The phase of horizontal oscillation in Hollerbach model
$\phi_y$	The phase of vertical oscillation in Hollerbach model
$\phi$	Phase difference between the horizontal and vertical oscillations in Hollerbach model
$\phi_{v_x}(t)$	Phase of horizontal oscillation in POMH model at time instant $t$
$\phi_{v_y}(t)$	Phase of vertical oscillation in POMH model at time instant $t$
$\phi_{S,i}$	Phase of sinusoidal oscillation representing signal $S$ between $z_{S,i}$ and $z_{S,i+1}$

$\phi_{v_x}^m(t)$	Phase of $m^{th}$ sinusoidal component of horizontal velocity at time instant $t$
$\phi_{v_y}^m(t)$	Phase of $m^{th}$ sinusoidal component of vertical velocity at time instant $t$
$\phi_{v_x,i}^m$	The phase of $m^{th}$ sinusoidal component of horizontal velocity between $z_{v_x,i}$ and $z_{v_x,i+1}$
$\phi_{v_y,i}^m$	The phase of $m^{th}$ sinusoidal component of vertical velocity between $z_{v_y,i}$ and $z_{v_y,i+1}$
$\phi_{v_x}^{(1-K)}$	Frequency parameter of the $K$ components extracted from horizontal velocity $v_x$
$\phi_{v_y}^{(1-K)}$	Frequency parameter of the $K$ components extracted from vertical velocity $v_y$
$\phi_{r_x}^{(1-K)}$	Frequency parameter of the $K$ components extracted from horizontal acceleration $r_x$
$\phi_{r_y}^{(1-K)}$	Frequency parameter of the $K$ components extracted from vertical acceleration $r_y$
$\phi_{s_x}^{(1-K)}$	Frequency parameter of the $K$ components extracted from normalized $x$ -coordinate signal $s_x$
$\phi_{s_y}^{(1-K)}$	Frequency parameter of the $K$ components extracted from $y$ -coordinate signal $s_y$
$\phi_{v_x}^{m,new}$	Modified amplitude of $m^{th}$ sinusoidal component of horizontal velocity
$\Pi$	Initial probabilities of HMM states
$Q$	Number of sample points in the sinusoidal feature vector
$r_x$	Horizontal acceleration
$r_y$	Vertical acceleration
$\hat{r}_x$	Sinusoidal feature vector obtained for $r_x$
$\hat{r}_y$	Sinusoidal feature vector obtained for $r_y$
$R_{v_x}^m$	Residual signal obtained for the horizontal velocity after $m^{th}$ iteration
$R_{v_y}^m$	Residual signal obtained for the vertical velocity after $m^{th}$ iteration
$s_x$	normalized $x$ -coordinate signal
$s_y$	normalized $y$ -coordinate signal
$\hat{s}_x$	Sinusoidal feature vector obtained from the signal $s_x$
$\hat{s}_y$	Sinusoidal feature vector obtained from the signal $s_y$
$\text{SNR}_{v_x,\text{dB}}$	The SNR computed between the reconstructed and the original horizontal velocity in dB
$\text{SNR}_{v_y,\text{dB}}$	The SNR computed between the reconstructed and the original vertical velocity in dB
$t$	Time/sample point number of the handwriting sequence
$t_0$	Initial time in Hollerbach model
$\tau_x^{p,i}$	Absolute difference between $x$ -coordinates of $(i-1)^{th}$ and $(i)^{th}$ zero-crossing points

## List of Symbols

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	of horizontal velocities
$\tau_x^{n,i}$	Absolute difference between $x$ -coordinates of $(i)^{th}$ and $(i+1)^{th}$ zero-crossing points of horizontal velocities
$\tau_y^{p,i}$	Absolute difference between $y$ -coordinates of $(i-1)^{th}$ and $(i)^{th}$ zero-crossing points of vertical velocities
$\tau_y^{n,i}$	Absolute difference between $y$ -coordinates of $(i)^{th}$ and $(i+1)^{th}$ zero-crossing points of vertical velocities
$T$	Number of points in a velocity signal
$v_x$	Horizontal velocity of handwriting
$v_y$	Vertical velocity of handwriting
$v_x^m$	Input signal to the $m^{th}$ iteration of parameter estimation method for modeling $v_x$
$v_y^m$	Input signal to the $m^{th}$ iteration of parameter estimation method for modeling $v_y$
$v_x^{rm}$	Reconstructed $v_x^m$ using the $m^{th}$ component
$v_y^{rm}$	Reconstructed $v_y^m$ using the $m^{th}$ component
$v_x^r$	Final reconstructed horizontal velocity $v_x$
$v_y^r$	Final Reconstructed vertical velocity $v_y$
$\hat{v}_x$	Sinusoidal feature vector obtained for horizontal velocity $v_x$
$\hat{v}_y$	Sinusoidal feature vector obtained for vertical velocity $v_y$
$\omega_x$	The frequency of horizontal oscillation in Hollerbach model
$\omega_y$	The frequency of vertical oscillation in Hollerbach model
$\omega_{S,i}$	Frequency of sinusoidal oscillation representing signal $S$ between $z_{S,i}$ and $z_{S,i+1}$
$\omega_{v_x}(t)$	Frequency of horizontal oscillation in POMH model at time instant $t$
$\omega_{v_y}(t)$	Frequency of vertical oscillation in POMH model at time instant $t$
$\omega_{v_x}^m(t)$	Frequency of $m^{th}$ sinusoidal component of horizontal velocity at time instant $t$
$\omega_{v_y}^m(t)$	Frequency of $m^{th}$ sinusoidal component of vertical velocity at time instant $t$
$\omega_{v_x,i}^m$	The frequency of $m^{th}$ sinusoidal component of horizontal velocity between $z_{v_x,i}$ and $z_{v_x,i+1}$
$\omega_{v_y,i}^m$	The frequency of $m^{th}$ sinusoidal component of vertical velocity between $z_{v_y,i}$ and $z_{v_y,i+1}$
$\omega_{v_x}^{(1-K)}$	Frequency parameter of the $K$ components extracted from horizontal velocity $v_x$

$\omega_{v_y}^{(1-K)}$	Frequency parameter of the $K$ components extracted from vertical velocity $v_y$
$\omega_{r_x}^{(1-K)}$	Frequency parameter of the $K$ components extracted from horizontal acceleration $r_x$
$\omega_{r_y}^{(1-K)}$	Frequency parameter of the $K$ components extracted from vertical acceleration $r_y$
$\omega_{s_x}^{(1-K)}$	Frequency parameter of the $K$ components extracted from normalized $x$ -coordinate signal $s_x$
$\omega_{s_y}^{(1-K)}$	Frequency parameter of the $K$ components extracted from $y$ -coordinate signal $s_y$
$\hat{\omega}$	Recognized word label.
$x$	$x$ co-ordinate of the online handwriting
$y$	$y$ co-ordinate of the online handwriting
$z_{S,i}$	The $i^{th}$ zero-crossing instant of a signal $S$
$z_{v_x,i}^m$	The $i^{th}$ zero-crossing instant of a signal $v_x^m$
$z_{v_y,i}^m$	The $i^{th}$ zero-crossing instant of a signal $v_y^m$





# 1

## Introduction

### Contents

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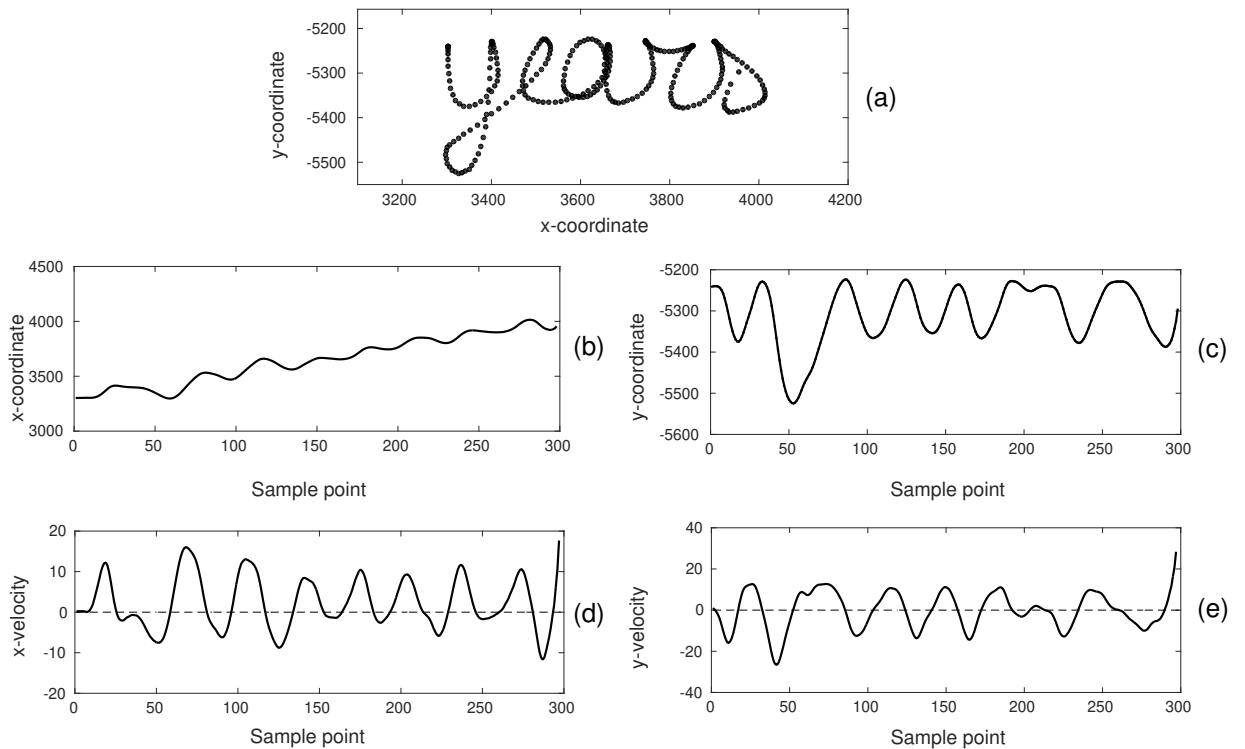
### Objectives of the thesis

*Despite the advances made in handwriting research, the analysis and representation of handwriting still remain a challenging task. Various approaches have been employed to study the shape and dynamic characteristics of handwriting. In this context, models based on handmovement theory have also been explored to describe the handwriting. These models can be beneficial for handwriting analysis and define effective generation based features for the designing of various handwriting systems. In this thesis, a multi-component sinusoidal model is proposed to represent online handwriting based on the oscillatory theory of handmovement. The proposed model is further explored in three different directions: feature representation for online handwriting recognition, handwriting modification for data augmentation, and personalized handwriting synthesis. In the first direction, the sinusoidal model parameters extracted from online handwriting are utilized as features to develop character and word recognition systems. In the second direction, the sinusoidal parameters extracted from the online handwriting are varied to modify the handwriting. The modified samples are used for data augmentation to improve the performance of a recognition system. Lastly, the sinusoidal model is employed to synthesize personalized handwriting for any given input text. A concatenation method is also presented to synthesize natural-like joining ligatures between neighboring characters.*

#### 1.1 Introduction to online handwriting

Handwriting is one of the unique capabilities of human, which helps to communicate without speaking. It enables the transfer and storing of information by writing a predefined set of symbols on surfaces, such as paper or a touchscreen device. These symbols are associated with the predefined letters of the writing script. Currently, the printed computer fonts are used for most of the applications, such as storing information and sending emails. However, handwriting still remains an essential skill, as it can be connected with a personal identity by its uniqueness. Therefore, research in handwriting has been one of the exciting areas of signal processing. Several tools and systems have been developed by researchers in the last few decades, such as handwriting recognition, writer identification, and handwriting synthesis [1–3].

Handwriting can be broadly categorized into offline and online handwriting based on the mode of capturing the data [4]. In offline handwriting, the data is captured in the form of an image by scanning a handwritten document. The entire message is written, and the scanner generates a



**Figure 1.1:** The subplot (a) illustrates an online handwritten word ‘years’ from UNIPEN-ICROW-03 word database. The subplot (b)–(e) show the  $x$ -coordinates,  $y$ -coordinates,  $x$ -velocity (horizontal velocity) and  $y$ -velocity (vertical velocity), respectively.

bitmap image of the handwriting. The handwriting is analyzed and represented using various image processing techniques [5, 6]. On the other hand, online handwriting is obtained with the help of an electronic transducer that captures the movement of the pen-tip as a sequence of  $(x, y)$  coordinate, sampled uniformly over time. These devices also detect the pen-up/pen-down state of handwriting. In particular, a pen-down state occurs when the pen touches the writing pad, and when the pen is lifted off, a pen-up state is sensed. The set of points captured between successive pen-down to pen-up states is called a stroke. It may be noted that the online handwriting records the coordinates of successive points as a function of time. This enables us to derive more meaningful dynamic information, such as the velocity or acceleration of  $x$ - and  $y$ -coordinate sequences. An online handwritten word <sup>1</sup> is shown in Figure 1.1(a). The subplots (b)–(e) show the  $x$ -coordinate,  $y$ -coordinate,  $x$ -velocity, and  $y$ -velocity, respectively.

<sup>1</sup>The sample is taken from UNIPEN-ICROW-03 word database, and details of the databases used in this thesis are described in Section 3.6 of Chapter 3.

## 1. Introduction

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The online handwriting allows us to analyze several “hidden” variables that inform us about the movement generating the handwriting. The most common attributes directly conveyed by the acquisition device are position in terms of  $(x, y)$  coordinates, timestamp, button status, pressure, azimuth, and altitude. Handwriting dynamics have been analyzed for different applications such as handwriting recognition, signature verification, and synthesis. It has also helped researchers in the direction of a computer-aided diagnosis system. The following part describes various tasks related to online handwriting and its challenges.

### 1.1.1 Various tasks related to online handwriting

Online handwriting has been a very active area of research explorations. Some of the important online handwriting applications studied in the literature are handwriting recognition, signature verification, writer identification, handwriting synthesis, and medical diagnosis systems. A brief discussion of these applications is as follows.

- **Handwriting recognition**

Online handwriting recognition is a technique for transcribing handwritten messages by a machine [1, 7, 8]. The handwritten message comprises of various trajectory shapes forming the letters depending on its writing script. A recognition system learns to classify or describe these shapes (e.g., character classes) based on different handwriting attributes. The input to the system is online handwriting, and the output is the recognized text. This development has a vast potential in the improvement of human-computer interface by enabling a system to be operated with a stylus using handwriting replacing the keyboard interface.

- **Signature verification**

A signature refers to the handwritten form of a person’s name or a unique mark that someone writes on the documents for identity proof. As every person has their own writing style, the signature is considered an essential behavioral trait of a person. The signature verification system detects if an individual’s signature is genuine or forged [9, 10]. An efficient online signature verification system can substitute a password for accessing a device or application.

- **Writer identification**

The task of a writer identification system is to find the author of a questioned document from a set of writers. It can be a text-dependent or text-independent system [11]. In the text-dependent system, a writer needs to write a predefined text which has been used to train the system. On

the other hand, in the text-independent system, a writer has the freedom to write any text. In this case, the text used for training and testing may not be the same. This text can be a sentence, a paragraph, or page level. Like the signature verification system, the uniqueness of a person's writing style is used to distinguish between different writers and identify a document's actual writer.

- **Handwriting synthesis**

Handwriting synthesis aims at the automatic generation of handwriting that resembles natural handwriting. It has applications in generating personalized fonts, document retrieval, and CAPTCHA generation [12–14]. Synthesized handwriting has also contributed to the performance improvement of handwriting recognition systems and other biometric security systems [15]. These systems can be online [12] or offline [16]. The final output of an offline handwriting synthesis system is in the form of handwriting images. Whereas, in an online handwriting synthesis system, a  $(x, y)$  coordinate sequence is generated. Different approaches have been used for synthesis, such as perturbation of real samples, template-based methods, and model-based generation systems.

- **Medical diagnosis related systems**

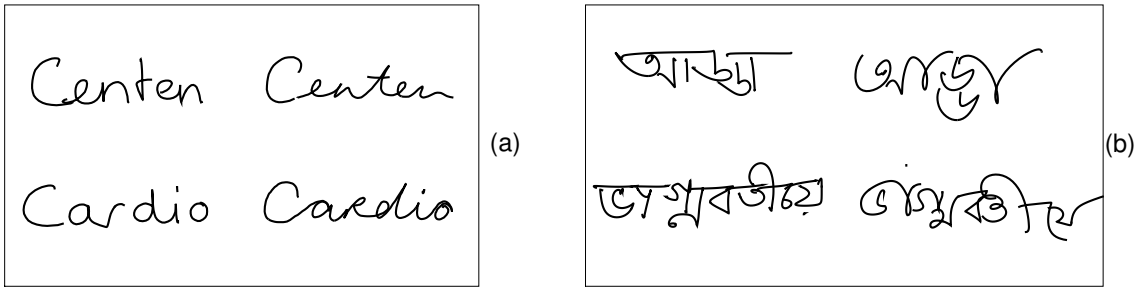
Handwriting analysis has also been used to develop systems for early diagnosis of neurodegenerative diseases such as Alzheimer's disease and Parkinson's disease [17, 18]. These diseases affect the brain's functioning and result in declining motor control, disturbing/changing the person's handwriting significantly. The dynamic information from online handwriting allows investigating the patient's handmovement information. Therefore, several works [17, 18] have studied the effectiveness of online handwriting in diagnosing and assessing the progression of such diseases.

### 1.1.2 Challenges in online handwriting systems

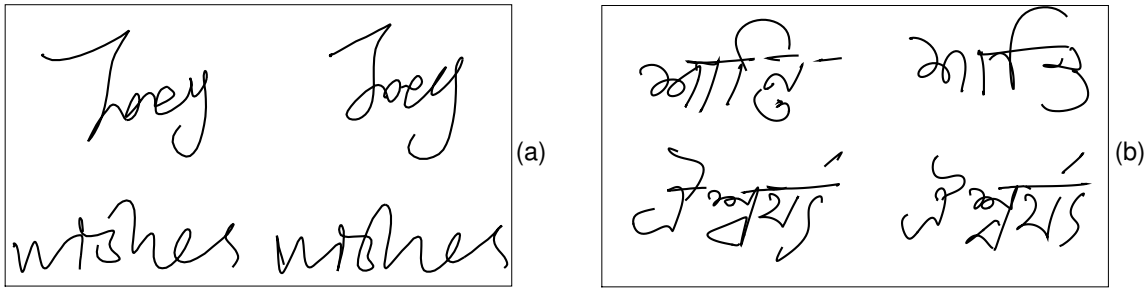
The main challenges of automatic systems trained with online handwriting data are the writing variability that occurs both within and across different writers. Although the letter shapes are fixed for a particular script, every writer develops a unique writing style in the process of learning. As a result, diverse writing styles are present in the handwriting data, such as the discrete style, cursive style, or mixed styles of writing [19]. The shape, slant, and speed of writing also vary significantly in the handwriting. This individuality or the personalized writing style is beneficial to identify a person in case of a signature verification or writer identification system by using it as a behavioral

## 1. Introduction

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**Figure 1.2:** The samples (in each row) depict the inter-writer variability for English and Assamese words in (a) and (b), respectively.

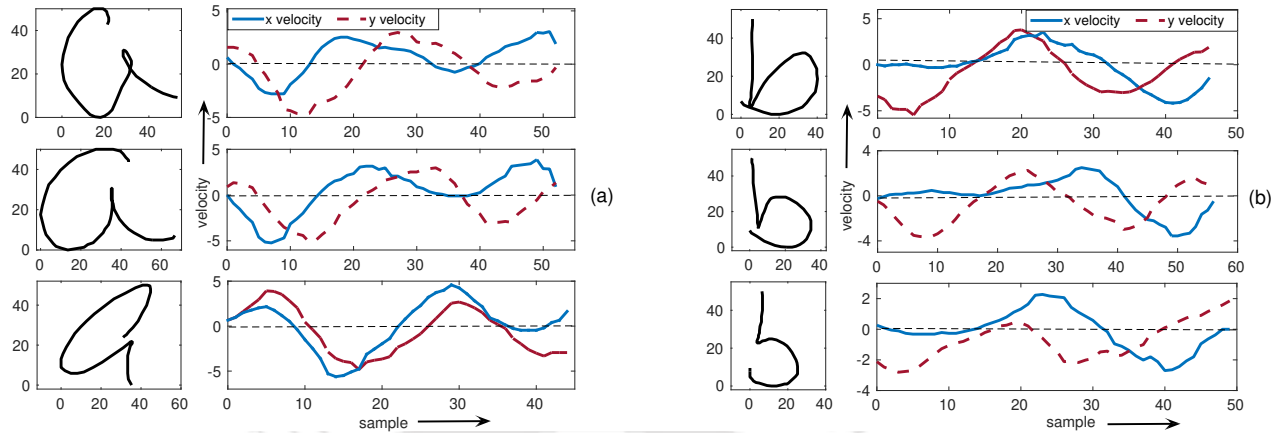


**Figure 1.3:** The samples (in each row) depict intra-writer variability for (a) English words and (b) Assamese words.

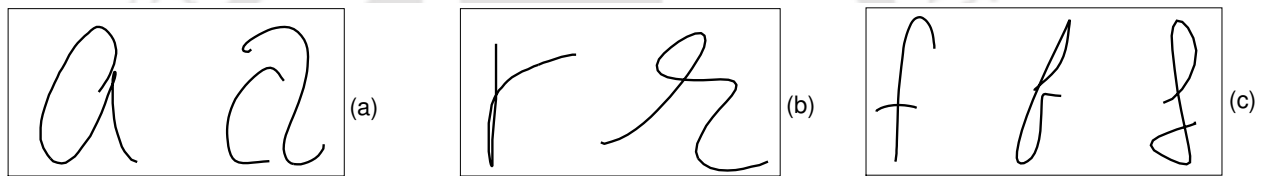
trait. However, it becomes a longstanding challenge for writer-independent systems such as online handwriting recognition. A few samples showing inter-writer variability are given in Figure 1.2 for two English and Assamese words <sup>2</sup>.

Further, it is to be noted that the handwriting of the same person also shows natural variations with time due to physical or emotional factors. Typically, a person's handwriting is based on the mental image of the letters taught and trained from childhood. However, as handwriting involves fine motor skills (controlling small hand muscles for a smooth and coordinated movement), slight variations are always observed in the written samples of the same person due to change in the motor controls. The other kind of variation in handwriting is observed with the underlying effort given in the writing. For instance, speeding up writing indicates that more force should be produced to maintain the required curvature in less time. If this fails, the handwriting becomes sloppier in a peculiar manner. This intra-class or intra-writer variation is an important issue for the verification systems. A few samples showing the intra-writer variability are given in Figure 1.3 for two English and Assamese words.

<sup>2</sup>The details of the English and Assamese databases are given section 3.6 of Chapter 3



**Figure 1.4:** (a) Samples of letter ‘a’ written by different writers (b) Samples of letter ‘b’ written by same writer along with the velocity profiles from the UNIPEN character database.



**Figure 1.5:** Allographs of letter ‘a’, ‘r’, and ‘f’ written by different writers from UNIPEN character database.

Next, we visualize the shape and dynamics (velocities) of different handwriting samples in Figure 1.4. The subplot (a) shows three samples of the letter ‘a’ written by three different writers along with the velocity profiles. The subplot (b) shows three samples of the letter ‘b’ written by the same writer. Along with the letter shapes, the  $x$ - and  $y$ - velocities are also shown for each sample. It can be visualized that a small change in the relation between the two velocities contributes to the shape variations in the handwriting.

Another important variation in handwriting is allographs present for the same character. Allographs are large shape differences between samples of the same character produced by different writers, especially when they are of different nationalities or different generations or taught different writing style methods. This variation is illustrated in Figure 1.5. Moreover, a character can be written using multiple strokes, where a stroke refers to handwriting trajectory between pen-down to pen-up instant. The sequence of writing these strokes does not change the shape or meaning of the letter. However, it may become a different pattern when processed sequentially by a machine learning algorithm. Therefore, it is an important variability in the online handwriting data. The number of pen-ups for the same letter may also vary with writers.

All these variabilities together make recognition and identification-related tasks challenging. On the other hand, in a synthesis system, the large variabilities result in a lack of naturalness in the synthesized handwriting. A synthesis system needs to store all the variations present in the data, or it should have the ability to create different variations to synthesize natural-looking handwriting, which is a challenging task. Thus, various approaches have been employed in different areas of handwriting to deal with these issues. We discuss some of the important research directions in the following.

### 1.1.3 Research explorations in handwriting systems

A considerable amount of work has been reported in the literature for the development and improvement of various online handwriting applications. These explorations can be categorized as follows:

- Handwriting representation: It is the most important aspect that decides the success of any handwriting system. A good representation can significantly minimize the impact of intra-class variations in the feature space, where a class may signify a ‘character’ in recognition/synthesis system or a ‘writer’ in verification/identification system [20]. The selection of suitable attributes/features in the representation can also help in increasing the inter-class variations. Various approaches have been presented in the literature for enhancing the representation of handwriting [21, 22]. The most utilized representation of online handwriting is  $(x, y)$  coordinates of the pen trajectory, which is directly available from the transducer devices. Several features are then derived in the literature by utilizing the  $(x, y)$  coordinates and can be grouped as the following:

- point-based features: the features computed at each point of the trajectory such as writing direction, curvature, and hat features [23],
- global shape feature: the features that describe the overall geometric characteristic of the entire character such as the number of strokes and length of the trajectory [1],
- frequency domain features: these features are obtained by applying different signal processing techniques such as Fourier transform, and Wavelet transform on the  $(x, y)$  coordinates of the online handwriting. The coefficients values are considered as features [24],
- posterior features: these are probabilistic features obtained from statistical classifiers [7],
- offline features: these features are extracted by first converting the original input to an image and then computing various parameters from the image [25].

Depending on the task, a set of suitable features is chosen to describe the handwriting.

- Handwriting synthesis: A quality database should contain sufficient training samples for all kinds of possible variations that can arise at any instant. However, it is often observed that such databases rarely exist and mostly have skewed distribution in terms of the number of variations in character shape, contextual information, and character frequency. The collection of data for all possible writing styles from different writers requires extensive time and effort. Moreover, it is not always possible to find writers, especially for low resource languages and scripts. Therefore, the availability of a large labeled database may not always be possible. This could result in sub-optimal performance of a system. One approach to solving this data shortage problem is to augment the dataset with synthetic handwriting or synthetically modified handwriting. Therefore, an extensive amount of works are reported in the literature to study the handwriting variabilities and add artificial variations in the training samples [15] to enrich the database. The generation of synthetic data has found applications in different areas such as recognition performance improvement and database creation for forensics systems [26–29]. A synthetic database also solves privacy issues for forensic systems.
- Advanced modeling approaches: One of the core modules of handwriting systems is appropriate modeling techniques, based on which actual decision for the input test sample is taken. Thus, the search for better modeling techniques is always considered as one of the main research interests in this field. The early explorations in online handwriting relied primarily on structural and rule-based approaches [30]. A character was described in an abstract fashion, without paying much attention to the irrelevant shape variations of the pattern. In current times, the majority of the explorations in the literature rely on data-driven modeling approaches. In the data-driven approach, the character classes are described by a statistical profile, where the associated parameters are estimated from the feature vectors of the training data. The modeling techniques such as the hidden Markov model (HMM), Gaussian Mixture Model (GMM), support vector machine (SVM), neural networks (e.g., multi-layer perceptron, time-delay neural network, recurrent neural network) are typical examples that follow this paradigm [31–34]. Apart from these statistical model-based approaches, the distance-based dynamic time warping (DTW) approach and its variants have also been used in various online handwriting systems [35].

As discussed above, there have been many proposals in the literature to improve the performance of a handwriting system. However, the basic pipeline of these approaches follows a standard way

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of developing any pattern recognition system that is fundamentally common for any signals such as handwriting, speech, or image. However, one approach has evolved only in the past few years in the literature that aims to analyze the handwriting signals from a different perspective - how a handwriting signal is generated and the different generation-based information that can augment a standard handwriting system to enhance its performance. In this context, research in handmovement based models have gained much popularity and has been explored for many applications [36–38]. These models represent the process of handmovement for handwriting and are discussed in detail in the next section.

### 1.2 Handmovement theory based handwriting models

In this section, we give an overview of the handwriting production mechanism and a summary of various handmovement theory based models.

#### 1.2.1 Overview of handwriting production

Handwriting is the representation of a written word of a language by alphabets and symbols. It helps humans to store and transfer information without speaking. However, unlike speech and other abilities of humans like seeing, handwriting is not a natural process. It is learned by constant practice of handmovements required for writing. The handwriting production process starts when a person intends to write a message. Depending on the language, the writer selects the appropriate letters to form a word. A cognitive image of each letter shapes, which are stored in the brain at the time of learning, is fetched and transformed into movement patterns. It requires a coordinated movement of various joints of the hand from fingers to the forearms [39, 40]. The nervous system triggers the appropriate muscles for producing the required movements, and thus the pen trajectory of handwriting is created. This complex movement would have been impossible if we had to think for each of the movements individually. However, after training and practice for many years, handwriting becomes a spontaneous task.

It is worth mentioning that, although there are many degrees of freedom associated with the generation of handwriting, one can consider two main degrees of freedom for analysis: one along the vertical direction of letter height and the other in horizontal writing direction. In a right-handed person, the first movement corresponds to the motion of the finger joints, and the second motion corresponds to the motion of the wrist and vice-versa for a left-handed person. Translational motion

of the elbow and shoulder joints separate the letters in the handwriting. While imitating the standard shapes or fonts of the letters, every individual develops their writing style. The writing speed, pressure, and way of holding a pen are some factors that vary from person to person, leading to a unique style of writing.

### 1.2.2 Various handmovement theory-based models

For many decades, human movement theories, especially the handwriting production process, have been studied, analyzing the planning and execution of various motor tasks [41,42]. These models can be subdivided into two main categories: cognitive and computational models [43]. The cognitive models focus on the brain processes that generate handwriting and address issues such as learning and planning the writing process. These models have been used to analyze and understand the process of learning handwriting [44,45]. The computational models use mathematics or physics tools and aim to reproduce the outcome of handwriting movements such as velocity profiles or the relations between different aspects of the handwriting dynamics [42,46,47]. The computational models provide an analytical representation of the handwriting and are described in the following.

Numerous studies have been conducted to model the handwriting generation process. The kinematic theory proposed by Plamondon et al. [42] considers handwriting production as a neuromuscular system, which has a lognormal impulse response. This theory has been very successful in reproducing the invariant properties observed in rapid handwriting strokes and is referred to as the delta lognormal model. According to the delta-lognormal model, a velocity profile of a rapid movement is the output of two neuro-muscular systems, one agonist and one antagonist acting in opposite directions [48]. Another variation of the lognormal model, the sigma-lognormal model, considers a complex handwriting movement as a vectorial summation of lognormal segments between target points [49]. Apart from lognormal models, various beta function-based models have also been proposed for describing the handwriting generation process. The beta-elliptic model described handwriting as a superposition of elliptic strokes [46]. This model was proposed by Beziere et al., which represents the handwriting trajectory by a combination of kinematic and geometric features. The curvilinear velocity of the handwriting is modeled by overlapping beta function [50], and the geometry of the 2D trajectory is represented by parameters of an ellipse.

Another velocity related model is the oscillatory theory of handwriting, proposed by Hollerbach [47]. According to this modeling approach, handwriting production results from two superimposed

orthogonal oscillators along with a horizontal drift. The orthogonal oscillations are responsible for producing character shape, while the horizontal sweep, which is a constant rightward velocity, results in spatially separated characters. The lognormal and beta-elliptic models are discrete models that consider handwriting to be formed by the concatenation of strokes. On the other hand, the oscillatory theory considers writing as a continuous process. It is based on the idea that handwriting is produced continuously by two orthogonal oscillators whose parameters are modulated after every small interval. In this thesis, we propose a sinusoidal model for online handwriting, which is based on the oscillatory theory of handwriting. In the next subsection, we give an introduction to the oscillatory theory of handwriting.

### 1.2.3 Oscillatory theory of online handwriting

The oscillatory theory of handwriting proposed by Hollerbach considers handwriting to be a continuous process produced through two motion generators: vertical oscillations of finger joints, horizontal oscillations of the wrist joint, and an added drift motion in the writing direction associated with the elbow and shoulder joints. The relation between the two orthogonal oscillations corresponding to the finger joints and wrist joint controls the character shape. On the other hand, the drift motion separates the characters in the writing direction. The oscillation parameters are modulated at zero-crossings of vertical velocities to synthesize various handwriting shape. The zero-crossing points of the vertical velocities correspond to the top and bottom corners of the handwriting trace.

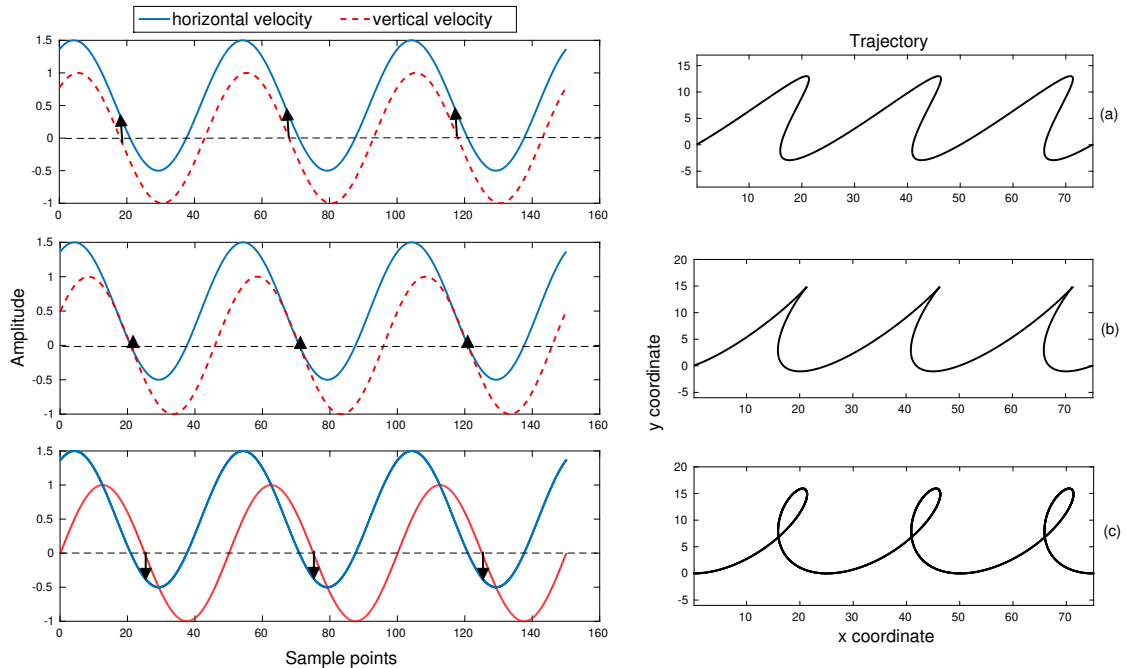
The two velocities along the horizontal and vertical directions  $v_x$  and  $v_y$  are defined as:

$$v_x = a \sin(\omega_x(t - t_0) + \phi_x) + c \quad (1.2.1)$$

$$v_y = b \sin(\omega_y(t - t_0) + \phi_y) \quad (1.2.2)$$

where  $a$  and  $b$  are the amplitudes,  $\omega_x$  and  $\omega_y$  are the frequencies,  $\phi_x$  and  $\phi_y$  are the phases of horizontal and vertical velocity, respectively. The initial time is represented by  $t_0$ , and  $c$  is the magnitude of the horizontal sweep, which controls the horizontal distance between the successive characters in the generated handwriting. The parameters of these oscillations are modulated after a short interval of time to produce the required character shape.

Figure 1.6 illustrates the variation in character due to the change in the horizontal velocity. As shown in subplot (a), a positive horizontal velocity at zero-crossing of vertical velocity forms the rounded corners. On the other hand, a zero and a negative horizontal velocity at zero-crossing of

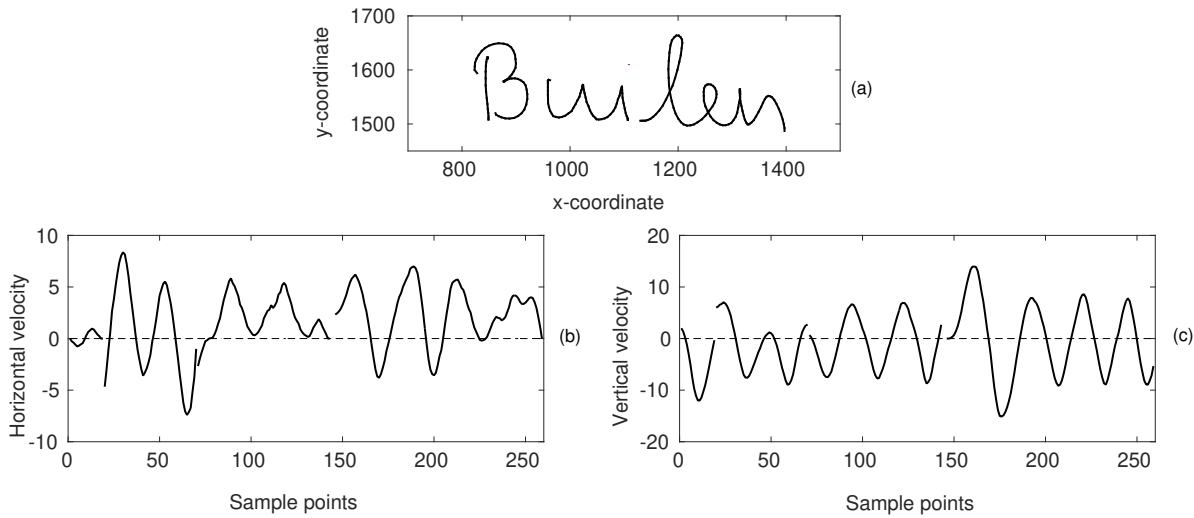


**Figure 1.6:** Illustration of different corner shapes (top corners) produced by different relation between horizontal and vertical velocities. The left panel of subplot (a) shows the velocity profiles with a positive horizontal velocity at vertical velocity zero-crossings. The right panel of subplot (a) shows the corresponding trajectory. Similarly, (b) and (c) shows zero and negative horizontal velocity at vertical velocity zero-crossing, resulting in a sharp edge and loop, respectively at the top corners.

vertical velocity result in sharp edge corners and loops, respectively, as depicted in subplot (b) and (c). This shows that the relation between the two velocities controls the trajectory shape. Therefore, it can be said that the sinusoidal oscillations that model these velocities can provide useful information about the handwriting, which can be used in different handwriting research areas.

### 1.3 Motivation of the thesis

The analysis and characterization of any physical phenomenon will be more effective if its underlying generation process is well understood. This understanding of the handwriting generation process can be achieved using handmovement models, which describe the handwriting generation process using mathematical representation. Therefore, the parameters of these models can define effective movement-related features of the handwriting. The analysis of the underlying movement-based information can provide valuable insights about the identity of the writer or into the written message for different handwriting applications. These models also provide the flexibility to transform the writing process for modifying handwriting or synthesize a new one. Although the handwriting analysis from



**Figure 1.7:** The subplot (a) shows an online handwritten word ‘Builen’ from UNIPEN-ICROW-03 word database. The velocities in horizontal and vertical direction are shown in (b) and (c), respectively.

the generation perspective has the capability to enhance the performance of handwriting systems significantly, only a few studies have made use of these models in handwriting system development.

The handwriting generation is a complex process that has several degrees of freedom. However, according to oscillatory theory by Hollerbach [47], the handwriting is predominantly controlled by the relation between the velocities corresponding to the wrist and finger movements. It considers that handwriting is produced by a continuous process comprising of two orthogonal oscillators whose parameters are modulated after every small interval. These two oscillators correspond to the two velocity profiles that produce the handwriting- one along the horizontal axis ( $x$ -velocity) and the other along the vertical axis ( $y$ -velocity). Figure 1.7 depicts the two velocity profiles for a handwritten word. It can be observed that the two velocity profiles are low-frequency oscillatory signals with invariant bell-shaped nature. The amplitude and time of the velocity profiles vary with trajectory shape between every successive zero-crossing points. Therefore, the velocity profiles can be modeled by sinusoidal signals of different amplitude, frequency, and phase.

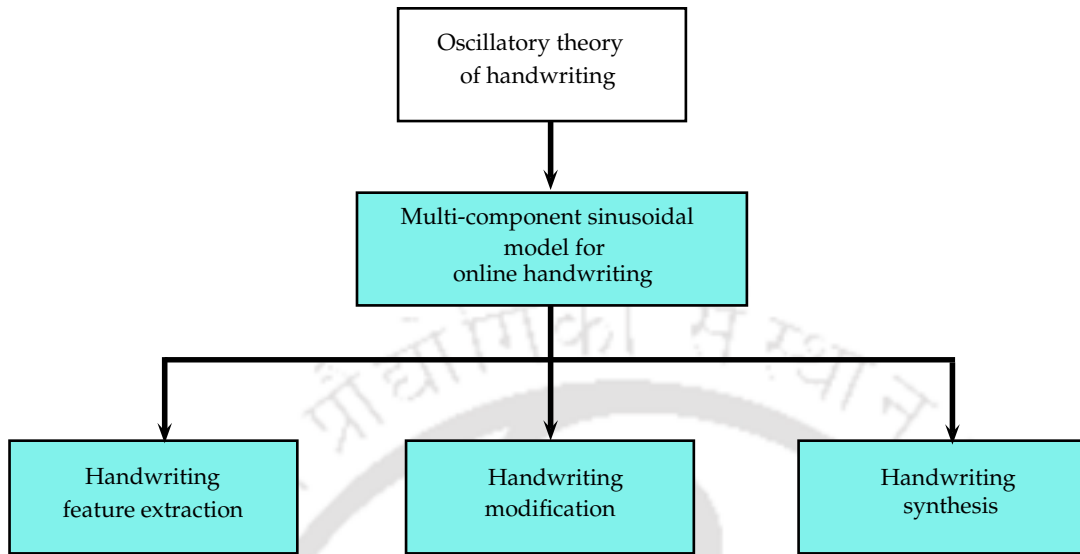
The sinusoidal modeling of online handwriting in this work is motivated by the parsimonious oscillatory model for handwriting (POMH) [51]. The POMH model is an improved version of the Hollerbach model. In this model, a half-wave of a sine function is used to model the velocity profiles between successive zero-crossing points. This allows efficient extraction of the parameters for any written trace and reproduces the dynamics of the handwritten traces. One of the advantages of the

sinusoidal model for online handwriting is that it is a movement based model whose parameters can help to extract handwriting's hidden movement-based information. Most of the handwriting systems utilize the conventional point-based features that capture the geometric shape of the handwriting. In contrast, the handmovement theory based model tries to capture movement-based features of the handwriting trajectory. Moreover, the flexibility to change model parameters and control handwriting trajectory shape can transform the handwriting and generate new samples. Therefore, it can be employed to create a synthetic database and add natural-like variations in the training data, which in turn increases the robustness of the systems. This model also provides a compact representation of handwriting data as the handwriting can be reconstructed using a small set of parameters. Accordingly, first, a detailed study of the existing sinusoidal model is presented, and then a new multi-component sinusoidal model is proposed for online handwriting. As a major work of this thesis, we explore multiple oscillations for improved modeling of online handwriting. The proposed sinusoidal model is explored in three different directions: handwriting representation for recognition, handwriting modification for data-augmentation, and personalized handwriting synthesis.

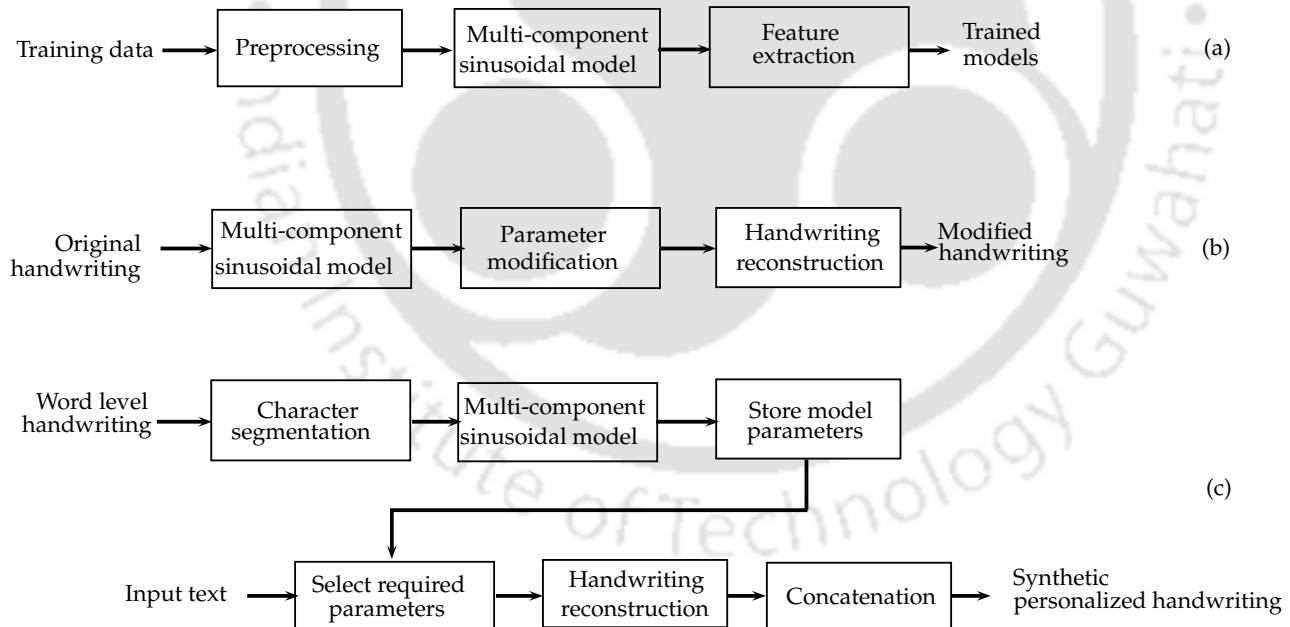
A similar concept has been widely used for analysis and modeling of speech and audio signals using the sinusoidal model [52]. Speech representation using the sinusoidal model has been applied to different areas of speech and audio, such as enhancement, transformation, and classification [53, 54]. The model parameters, frequency, amplitude, and phase have been analyzed and used as input features for speech classification systems such as emotion detection [55]. It is worth mentioning that the modeling approach and parameter extraction methods for handwriting and speech signals are distinct due to differences in the signal characteristics.

We summarize the contributions of the thesis in Figure 1.8, where the main contributions are depicted in the colored blocks. The first contribution of this thesis is a modeling approach for online handwriting using a multi-component sinusoidal model. The proposed model is explored in three different directions: handwriting feature extraction, handwriting modification, and handwriting synthesis. In the first direction, the multi-component sinusoidal model parameters are utilized to obtain sinusoidal model-based features from the online handwriting. The proposed feature set is employed to develop an online handwriting recognition system.

The block diagram of the same is shown in Figure 1.9 (a). In our next contribution, the effect of sinusoidal parameters on the handwriting shape is analyzed, and a method is proposed for handwriting



**Figure 1.8:** Overview of the contributions of this thesis. The contributions (shown in coloured blocks) are proposal of a sinusoidal model for online handwriting and its application in the three directions: feature extraction, modification, and synthesis.



**Figure 1.9:** The subplot (a) shows the block diagram for feature extraction using proposed multi-component sinusoidal model in recognition system. Similarly (b) and (c) show the block diagram for online handwriting modification and synthesis system, respectively.

modification. The original handwriting samples are first represented using the multi-component sinusoidal model to extract the control parameters. These parameters are then varied to reconstruct the modified handwriting. The block diagram of the modification technique is depicted in Figure 1.9 (b).

The third direction is personalized handwriting synthesis. The writer's handwriting is first segmented into character level, and the sinusoidal parameters are extracted and stored for each character sample. During synthesis, for a given text, the required sinusoidal parameters are selected, and the individual handwritten characters are reconstructed for the specific writer. The adjacent characters are then concatenated to obtain the personalized cursive handwriting for the given text. Figure 1.9 (c) illustrates the block diagram of the proposed personalized handwriting synthesis system. The details of each of these contributions are discussed in the following.

## 1.4 Contribution of the thesis

The following are the major contributions established in this thesis.

### 1.4.1 Multi-component sinusoidal model for online handwriting

In this work, a multi-component sinusoidal model is proposed for online handwriting. The method extracts sinusoidal parameters from handwriting by modeling its horizontal and vertical velocities between each successive pair of zero-crossing points with a half period of the sine function. Thus, each velocity profile is represented by the sinusoidal oscillations whose parameters are modulated at the zero-crossing points. The use of multiple oscillations to model the velocities results in a better representation of the complex trajectories. The parameters of the model are computed iteratively from its residual signals. We hypothesize that the analysis of the sinusoidal components and their parameters may provide added dynamic information about the handwriting. The efficacy of the proposal is evaluated by the reconstruction of online handwritten characters.

### 1.4.2 Sinusoidal model-based features

The representation of online handwriting is an important aspect of handwriting applications, which involves the extraction of various spatial and temporal attributes for analysis and individualization of handwritten patterns. In this work, we focus on improving online handwriting representation using the multi-component sinusoidal model. We propose a set of features by sinusoidal modeling of handwriting velocities for the online handwriting recognition task. Although the predominant information of handwriting is modeled in the velocity profiles, the acceleration and the  $(x, y)$  coordinates are also important in characterizing the handwriting. Accordingly, these signals are also modeled by sinusoidal oscillations, and the parameters are utilized as features to develop the handwriting recognition sys-

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tem. As these parameters are extracted from the model based on handmovement theory, it may also contain additional information describing the generation of the pattern along with its spatial shape information. Further, we also combined the proposed set of features with the existing popular set of features and compared it with other reported works in the literature to demonstrate the usefulness of sinusoidal parameters. The efficacy of the features is shown for character and word recognition tasks.

### 1.4.3 Modification of online handwriting

Handwriting modification has potential applications in areas such as handwriting recognition, synthesis, and signature verification. In this work, we propose and implement algorithms for handwriting modification based on the sinusoidal model. One of the advantages of the proposed method is its flexibility to vary the model parameters to introduce variability that may be found naturally in real handwriting. The extracted parameters from real data are modified accordingly to generate variations of the real sample. As it is based on the handmovement theory, we hypothesize that the transformed handwriting within certain limits does not lose the naturalness of handwriting. The proposed modification technique is also used for data augmentation to training a handwriting recognition system.

### 1.4.4 Personalized handwriting synthesis

Handwriting synthesis is the automatic generation of natural handwriting for a given text. It has different applications, such as the creation of personalized handwritten fonts, synthetic signatures, and keyword spotting. Handwriting synthesis can be writer specific or script specific. In this work, a writer specific approach is followed. We introduce a new method for personalized cursive handwriting synthesis using the sinusoidal model parameters. We hypothesize that it can synthesize natural looking handwriting and also respects the dynamic properties of the handwriting. Moreover, the model allows a compact representation of the character shapes using only the model parameters. An efficient method to synthesize the joining ligatures is also presented depending on the end and start position of the two neighboring letters.

## 1.5 Thesis organization

This thesis is organized into seven chapters, which are summarized as follows:

- In Chapter 2, we review the several existing methods for modeling handwriting using movement theories and its applications. Based on the literature survey, the scope of the thesis work is

formulated.

- In Chapter 3, modeling of online handwriting using a multi-component sinusoidal model is presented. The model is evaluated using the reconstruction quality of handwritten characters and words.
- In Chapter 4, a feature set is proposed for online handwriting recognition using sinusoidal parameters. The efficacy of the feature set is shown for character and word recognition tasks.
- In Chapter 5, a handwriting modification method is proposed using the multi-component sinusoidal representation. The modified handwriting is used for data-augmentation to improve the performance of the recognition system.
- In Chapter 6, a personalized cursive handwriting synthesis system is proposed using the sinusoidal model-based representation. The synthetic data is evaluated by subjective methods.
- Chapter 7 summarizes and concludes the work presented in this thesis, highlights the main contributions of the work and gives the future direction of research.

## 1.6 Summary

In this chapter, we presented the different challenges and research explorations in various online handwriting based systems. In this context, we also discussed the significance of handmovement based models in handwriting research and developments. Thereafter, we presented the main goal of this thesis with an elaboration of each contributing chapter. The main focus of the thesis, as discussed, is to propose a sinusoidal model for online handwriting based on the oscillatory theory and explore it for feature extraction, modification, and synthesis of online handwriting.

In the next chapter, we provide a detailed review of the various handmovement based models in the literature and their applications in handwriting representation and synthesis.



# 2

## Handmovement theory based models and its applications- A review

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### Overview

The analysis of online handwriting based on handmovement theory has gained significant attention in the past few years. These theories study the production process of handwriting to provide a mathematical representation of the underlying movement and generate handwriting trajectories. The models based on handmovement theory may help to analyze the handwriting from the perspective of its control mechanism. In this context, one of the successful handmovement theory is the oscillatory theory of handwriting. As outlined in Chapter 1, the motivation of this thesis is to analyze online handwriting using the sinusoidal model based on the oscillatory theory. We demonstrate its effectiveness for handwriting representation and explore it in three directions: handwriting feature extraction, handwriting modification, and personalized handwriting synthesis. In this chapter, we review the different handwriting models based on handmovement theories and their applications. The review is followed by a summary and a brief discussion about the scope of the present work.

### 2.1 Introduction

Research in online handwriting has been carried out for many decades to develop various handwriting systems. Due to a large amount of handwriting variability present both within and across different writers, handwriting analysis still remains a long-standing problem. A brief review of the challenges in handwriting related tasks and different ways to overcome these problems have been presented in Chapter 1. It is observed that most of these approaches study the geometric/spatial attributes of the handwriting or explore advanced modeling techniques. Alternatively, in the past few years, handwriting analysis from the perspective of generation using handmovement models has become popular in different areas [51, 56, 57].

The handmovement models have been intensively used and studied in many fields of handwriting research, such as online handwriting recognition, signature verification [15, 58], diagnosis of neurodegenerative diseases [18], handwriting synthesis [3], and analysis of handwriting variability [56]. We can broadly categorize the application of handmovement models into two directions. The first category includes handwriting analysis and representation using the handmovement models. The models are used to extract movement based characteristics from the handwriting, which are used for classification and verification purposes. In the second category, the handmovement models are utilized for the handwriting synthesis task. This motivated us to project the handmovement models as a prospective

approach for online handwriting representation and synthesis. In this chapter, we review the various handmovement models and their applications for handwriting representation and synthesis tasks in detail.

The remaining part of this chapter is organized as follows. In Section 2.2, we discuss the various handmovement models for handwriting. The works related to handwriting representation and synthesis are reviewed in Section 2.3 and 2.4, respectively. In Section 2.5, the summary and the possible advantages of the explored directions are discussed in comparison with the existing approaches for handwriting representation and synthesis tasks.

## **2.2 Movement based modeling approaches for online handwriting**

Handwriting is a symbolic representation of a message using a 2D trajectory that forms the required letters of the script. Several studies on hand movement have emerged over the years to understand the generation process of handwriting. These approaches model the handwriting primarily from the motor control point of view and can be broadly categorized into two families – discrete and continuous models. According to the discrete model, handwriting is generated by the superposition of several basic movements between target points. It considers that any handwritten shape can be produced by the combination of a fixed set of basic movements. The models such as lognormal models [42,59], the Edelman and Flash model [41], the VITEWRITE model [60], and the beta-function based models [46] are typical examples that follow discrete modeling paradigm. On the other hand, in a continuous model, the handwriting is considered to be produced from uninterrupted movement generators, which are typically oscillatory in nature [47,51]. The parameters of the generators are modulated after every small interval of time to produce the required handwriting trajectory. An example of this paradigm is Hollerbach’s oscillatory model for handwriting [47]. These models provide a different parametric representation of handwriting to give an insight into the relationship between control variables and the output handwriting. However, the fundamental studies of these models are motivated by the same invariant bell-shaped nature of the handwriting velocities [61]. A detailed description of the most explored handmovement models is given in the following.

### **2.2.1 Lognormal models**

The lognormal models for handwriting are based on the kinematic theory of rapid human movements. The kinematic theory is one of the well-followed approaches to study and analyze any rapid

## 2. Handmovement theory based models and its applications- A review

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human movements, including the handmovement for writing [42]. According to this theory, the handmovements are produced by two parallel neuromuscular systems- agonist and antagonist. In the agonist system, the muscles act in favor of the intended motion. Whereas in the antagonist system, the muscles work opposite to the desired motion. The kinematic theory considers that these individual systems consist of a large number of sub-systems, and following the central limit theorem, the impulse response of these systems converges to lognormal curves. Therefore, the velocity of any rapid human movement produced by the complete neuromuscular system is considered as the difference between the two lognormal curves corresponding to the impulse response of the two systems. This is referred to as the delta lognormal theory of rapid human movements. Based on this theory, there are different handwriting models such as vectorial delta lognormal model [48], sigma-lognormal model [56], and omega lognormal model [62].

The vectorial delta lognormal model considers handwriting as a sequence of rapid human movements to form segments joining a set of target points in the 2D space [48,63]. The resulting curvilinear velocity of the handwriting is a vectorial summation of the velocity of the individual segments. The curvilinear velocities of these segments are modeled by the delta lognormal theory. The lognormal segments (or the target points) are localized by inspecting the local maximum, minimum, and inflection points of the curvilinear velocity profile [64]. Similar to the delta lognormal model, the sigma lognormal also considers the lognormal segments as the primitive of handwriting. However, in the sigma lognormal model, the lognormal segments have only a single lognormal profile [56]. It does not assume that the two neuromuscular systems are acting exactly in opposite directions. This model is found to be more useful for complex trajectories, like in handwriting or signatures [27, 59, 65, 66]. The omega-lognormal model describes the oscillatory movement in one dimension by a sequence of alternating lognormal curve for analysis using the kinematic theory [62, 67].

### 2.2.2 Beta function based models

The modeling of velocities using the properties of Beta function [68] has been one of the popular handmovement theory based method for practical handwriting applications [37, 46, 69]. Similar to the lognormal models, the Beta function-based models also follows the kinematic theory of handmovement. It is considered that handwriting is produced by a sequence of rapid handmovements that are produced by two agonist and antagonist neuromuscular systems. As each of these systems comprises a large number of sub-systems, the global impulse response of the complete system is converged to a Beta curve

using the central limit theorem. A combination of the Beta function with geometric models, such as circular or elliptic models [68,69], are also explored to improve the modeling quality of the handwriting. The parameters of the Beta function can explain the different kinematic phenomena observed in handwriting production. On the other hand, the parameters of the geometric models characterize the handwriting in the space domain to capture the geometric properties of the muscles and joints involved in the handmovement. Some Beta function-based models are Beta velocity model [70], Beta-circular approach model [68], Beta-elliptic model [69], and neuro Beta elliptic model [71].

The Beta velocity model comprises of a velocity function, and a pen-lifting function [70]. The velocity function is a Beta curve that models the curvilinear velocity of handwriting segments. The pen-lifting function learns how the pen-lifting factor depends on the speed of movement. This model is useful for handwriting with many strokes, such as the Korean script, where the pen-lift information plays an important role in the handwriting analysis. In the Beta-circular modeling of handwriting, [68], cursive handwriting is described as the superimposition of basic segments with a circular form. Each segment is described by a set of parameters that characterizes the movement both in the kinematics and the static (2D shape) domains. The handwriting is first divided into elementary segments at local minimums of the curvilinear velocity. The velocity of each segment is then modeled by the Beta curve, and its trajectory shape is characterized by parameters of a circle, the initial angle, the circular radius, and the center of the circle. Similarly, in the Beta-elliptic models [46], the handwriting segments are considered to be of elliptic form. The curvilinear velocity of the segment is modeled by the Beta function parameters, and the trajectory is modeled by the parameters of the ellipse: major axis and minor axis, the center of the ellipse, and its angle. The Beta-elliptic model is also realized using neural networks and is known as the neuro Beta-elliptic model [72]. The model parameters are fed to the input layer of the model, and the output layer gives the sequence of  $x$  and  $y$  coordinates.

### 2.2.3 Oscillatory theory based models

Handwriting production includes mainly two degrees of freedom: the finger movement acting in the direction of letter height and the wrist movement acting along the writing direction (for a right-handed person <sup>1</sup>). The oscillatory theory, pioneered by Hollerbach, considers that this process is of continuous form [47]. According to this theory, the vertical and horizontal velocities associated with fingers and wrist movements, respectively, are produced by two orthogonal oscillations that are superimposed on

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<sup>1</sup>The role of the finger and wrist joints reverse for a left-handed person.

## 2. Handmovement theory based models and its applications- A review

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a constant magnitude horizontal sweep velocity. The relation between the two orthogonal oscillations is responsible for producing various trajectory shapes, and the horizontal sweep separates the adjacent letters in the writing direction. Therefore, the letter shapes are produced by parameter modulation of two oscillations at specific points depending on the required shape. The work carried out in this thesis is inspired by the oscillatory theory, and the details have already been discussed in Chapter 1. In the following, we discuss the various models based on the oscillatory theory of handwriting.

The oscillatory theory is utilized for representing online handwriting using a dynamic encoding model [73]. In this model, the horizontal and vertical velocities are modeled using two sinusoidal oscillations. The amplitude and phase lags between the two oscillations are modulated to encode the handwriting velocity profiles. Handwriting is then represented by the values of the model parameters quantized to a fixed set of levels. A neural network realization of Hollerbach's oscillatory model is presented in [74]. In this model, each node in the input layer represents a handwriting segment. The hidden layer is an oscillator layer, and the output layer generates horizontal and vertical velocities. Recently, a parsimonious oscillatory model of handwriting (POMH) is proposed based on the oscillatory theory of handwriting [51]. The POMH model is symmetric in the  $x$  and  $y$  coordinates and allows efficient extraction of the parameters for any written trace. Therefore, the dynamics of the handwritten traces can be reproduced using the model parameters.

The summary of the review carried out in this section is that a number of handwriting models are explored in the literature, which aims to describe the underlying movement of the handwriting. These models are developed for different interest and analyze the handwriting velocities differently using various mathematical representations. The mathematical description helps to understand the handwriting control factors, which can benefit many practical handwriting systems. Currently, pattern recognition and machine learning methods have become prevalent in several areas. The parameters associated with these models can be useful for different pattern recognition applications. However, a limited number of studies have developed a handmovement based model for handwriting from the perspective of pattern recognition systems. Therefore, one prospective direction of work in this thesis is the development of a handwriting model based on handmovement theory for pattern recognition systems. The other directions of this work dealing with the application of these models are discussed in the following sections.

## 2.3 Handwriting analysis and representation using handmovement models

Handwriting representation is an important part of any pattern recognition system. Various information extracted from the patterns or their representation is analyzed to describe and understand the handwriting. Some of the important criteria for a good representation are efficient retrieval accuracy, compactness, robustness to noise and distortion, and less computational complexity. One popular approach for online handwriting representation is the  $(x, y)$  coordinate sequence of the trajectory. Different features are extracted at the coordinate points (known as point-based features) to capture the shape information of the handwriting trajectory such as displacement, curvature, writing direction, writing speed, slope, linearity, and curliness at each sample point, i.e.,  $(x, y)$  coordinate location [23]. Besides these, additional features such as global features (geometric characteristics obtained by considering the entire pattern) [1], probabilistic features [7], and offline features (features extracted from the image form of the handwriting) [1, 25] are also investigated in various handwriting systems. Depending on the task, different features are chosen to analyze and represent the handwriting. For example, in the forensic systems, the writing speed provides important aspects of the writer [75]. Similarly, the probabilistic features from statistical models, such as Gaussian mixture model (GMM) posterior features, have proven to be a useful representation in classification systems [7]. Many works have also studied the combination of features with complementary information to extract maximum possible attributes of the writer or the message that is written [1, 76, 77]. Table 2.1 summarizes the features used for different handwriting tasks.

Coming to the handmovement based models, several works have considered this paradigm for handwriting representation. It has led to an improved characterization of the movement-based information [36–38]. The authors in [36] employed the Beta-elliptic model to represent the handwriting for a digit recognition system. The model parameters are used as features and employed to train multi-layer perceptron (MLP) for the recognition task. The Beta-elliptic features are also utilized for training a hybrid MLP-hidden Markov model (HMM) based Arabic handwriting recognition system [88]. It allows the extraction of combined features from the dynamics and geometry of the trajectories. The authors in [89] combined the features from the Beta-elliptic model with offline features from the convolutional deep belief network (CDBN) classifier for online Arabic handwriting recognition. In [90], the authors utilized the beta-elliptic approach for a multi-language recognition system dealing with Latin,

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**Table 2.1:** Summary of the various features extracted from online handwriting.

Study	Features
[Venugopal et. al. 2019] [78]	speed, writing direction, curvature, vicinity aspect, vicinity curliness and GMM based feature representation.
[Nguyen et. al. 2018], [79]	$x$ and $y$ difference of two adjacent coordinates, pen-up/pen-down, vicinity curvature, aspect, curliness, slope, linearity, offline context map, and online shape context.
[Keysers et. al., 2017], [1]	$(x, y)$ and its derivatives, curvature, curliness, context map, bounding box, mean, variance, stroke crossing, dots, the histogram of writing direction, water reservoir.
[Mukherjee et. al., 2017], [80]	polar coordinates, writing direction, curvature, aspect, velocity, $y$ -coordinates, average square distance, and Fourier coefficients.
[Du et. al., 2017], [25]	CNN features learned from offline image.
[Bhattacharya et. al., 2016], [81]	difference of initial and final respective $x$ and $y$ coordinates, basic unit aspect ratio, trajectory length, direction feature, number of points in the various zone, the second order derivatives.
[Abdelaziz et. al., 2016], [82]	chain code, curliness, aspect ratio, writing direction, curvature, baseline, zones, loop, hat, and few derived features.
[Yang et. al., 2015], [83]	path signature and directional features.
[Samanta et. al., 2014], [84]	$(x, y)$ coordinates, writing direction, angular displacement, vicinity aspect ratio, curliness, linearity and circular features.
[Du et. al., 2014], [85]	bottleneck features.
[Chowdhury et. al., 2013], [86]	directional and positional information based string, horizontal and vertical distance features.
[Bharath et. al., 2012], [87]	$y$ -coordinates, writing direction, curvature, aspect, curliness, linearity and slope.

Arabic, and digit scripts. Beta-elliptic features are used to train a Time Delay Neural Network. The Beta-elliptic model is also used for feature extraction in a text-independent online writer identification system [37]. The authors in [38] represented the signatures as a sequence of lognormal segments in the form of six sigma lognormal parameters and then matched using string edit distance. O'Reilly et al. presented a sigma-lognormal model to represent an online signature in [49]. This model was later employed in a signature verification system for improved characterization of the signee's hidden motor control information [91].

The works discussed in this section highlight the importance of handmovement based handwriting models for handwriting analysis and representation. These advantages project the use of handmovement based models for handwriting as an insightful direction to improve a handwriting system.

## 2.4 Handwriting synthesis using handmovement based models

Handwriting synthesis refers to the artificial generation of data that resembles natural handwriting. It has become an active area for research due to the increasing need for creating synthetic database for recognition system [26, 92–95], personalized handwritten fonts [12, 96, 97], and synthetic CAPTCHAs [13, 98]. One of the pioneering work in handwriting synthesis is generation of realistic cursive handwriting using long short-term memory recurrent neural network [99]. Now, it is worth mentioning that a synthesis system for creating data to improve a recognition system should generate handwriting with all possible writer-independent variations [93]. On the other hand, a personalized handwriting synthesis system should be able to produce any handwriting with the writing style of the user [12, 97]. Thus, the requirement of a synthesis system varies depending on its interest or goal. Various techniques have been explored for synthesizing handwriting data and discussed in the following.

Many researchers have proposed methods to model the handwriting shape, i.e., the 2D  $(x, y)$  trajectory. These methods rely entirely on the outcome of the writing process and are called shape simulation methods [93]. One approach for shape simulation is the perturbation method, which applies transformations on real samples to generate synthetic data. Various deformation methods have been proposed to bring variations in handwriting shape, such as scaling, rotation, and size variation. Another approach for shape simulation is to train statistical models, such as the Bayesian model and HMM, which are capable of producing synthetic samples [100]. These models are trained using a large number of samples to learn the natural variations in the handwriting shape. The advantage of using a statistical model-based method is that it captures the natural variations from the data set and can generate handwriting from the learned parameters. However, it requires a large number of samples to train the model. On the other hand, although the perturbation based methods are simple and require low data, it may result in unnatural handwriting due to the random variations.

On the other hand, handmovement based models have more prominent theories for handwriting production and can produce natural handwriting with fewer samples and have been utilized in many works [15, 26–29]. This technique is called the movement simulation, which models the writing process instead of the shape. These models have well-established theories for handwriting production that can produce natural handwriting that follows the dynamic characteristics of handwriting. Therefore, these methods can facilitate the synthesis of online handwriting with both natural-like shapes and veloci-

## 2. Handmovement theory based models and its applications- A review

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ties. In this section, we review the different works related to handwriting synthesis using movement simulation techniques.

A synthesis system may output handwriting at different levels, such as stroke, character, glyph, word, or sentence. The goal may be to generate character handwriting for given ASCII code. Moreover, it may aim to synthesize handwritten words by concatenation of characters or characters by concatenation of strokes. Based on the input and output granularity level, explorations in synthesis systems can be broadly divided into two categories.

- The first category of works comprises of synthesis system where the input and output unit has the same level of granularity [97], which are called generation techniques. One approach for these synthesis systems is to generate new samples of handwriting by changing the geometric or movement properties of the original sample. Another approach is to create a new sample from a model, which is capable of producing handwriting. These systems are used mostly for synthesizing handwriting databases [94]
- The second category of work deal with concatenation techniques of the input handwriting where the output unit has a higher level of granularity than the input. The concatenation may be of original handwriting collected from the users or handwriting synthesized using the generation techniques. These methods are mostly used for personalized handwriting synthesis, where the system produces handwriting for any text from a small set of handwriting collected from the user [101].

Handmovement based models have been utilized for both generation and concatenation of handwriting and are discussed in the following. In [27], the authors presented a system for synthesizing an extensive database of handwriting from a few specimens using the sigma lognormal model. The system could be used to generate writing samples for a single writer, as well as in a multi-writer setup. The authors showed that variability observed in handwriting data can be regenerated by varying the sigma-lognormal parameters around their mean values within limits fixed by their standard deviations. The authors in [28] presented an automated handwritten CAPTCHA generation system by adding distortions to the sigma lognormal representation of a handwritten word sample. Distorted samples are generated by modifying the parameters of the sigma lognormal model. The authors in [29] proposed a method to generate skilled synthetic forgeries from a single online genuine specimen using the sigma lognormal model. The genuine signature trajectory is modified by distorting the lognormal

parameters to synthesize forgeries that are much harder to detect by a signature verification system. In [15], the authors describe a methodology to generate a large database of synthetic samples from a small set of original online handwriting specimens using the sigma lognormal model.

In the concatenation based synthesis, the strokes, letters, glyphs may be simply aligned without connection, direct connection, or using concatenation models [96, 102, 103]. Although no connection and direct connection are simple methods, they result in unnatural handwriting. Therefore, different concatenation models have been used in recent works [12, 102, 104, 105] for generating ligatures (segment joining neighboring letters). Some of the works that have used handmovement based models modify the ending and beginning regions between two adjacent letters for smooth joining [102, 104]. The authors in [26] synthesized text lines from letter templates using delta lognormal velocities. A prototype of each letter is formed by a series of Bezier arcs. The letters are then synthesized from the prototypes using delta lognormal velocities of strokes. The synthetic data is used to train an HMM-based recognition system.

Thus, as discussed in this section, various attempts have been made to synthesize natural handwriting using handmovement models. These models can provide the flexibility to modify the control parameters of the handwriting and produce different variations of the original handwriting. Moreover, the appropriate selection of the model parameters can synthesize a new handwriting trajectory. Therefore, handmovement generation models can also synthesize joining ligatures in the concatenation based synthesis system. The efficacy of the handmovement model can be explored in these two potential directions, i.e., handwriting modification and synthesis.

## 2.5 Summary and discussion

The review in this chapter focused on the study of handmovement based models and their applications in the development of handwriting systems. The literature review shows that the movement-based information obtained from these models are utilized in different areas to improve handwriting representation and synthesis. The model parameters mainly describe the controlling factors of the handwriting process. Therefore, these parameters can extract movement-related features and modify the writing process to transform the original trace. Moreover, being generative models, it can also synthesize personalized handwriting using appropriate parameter selection.

In the literature, various handmovement models have been proposed, which are broadly categorized

## 2. Handmovement theory based models and its applications- A review

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into discrete and continuous types. We reviewed both discrete and continuous handmovement based models in this chapter. In the discrete models, such as lognormal and beta functions models, the handwriting process is considered as the concatenation of small handwriting segments both in space and time. However, there is an uncertainty in the discrete model about the definition of these small segments, which changes depending on the task – a stroke, letter, grapheme, or even a word. On the other hand, continuous models which are based on oscillatory theory consider handwriting as an outcome of a nonstop generative process. From these models, one can obtain various kinds of graphic shapes (such as letters, words, or sentences) efficiently by intermittently updating a limited set of parameters. This ability of oscillatory models can be beneficial for various handwriting related applications. Recently, the authors in [51] proposed the POMH model where a linear and symmetrical model with a simple parameter extraction method is presented for online handwriting. It decomposes handwriting into horizontal and vertical velocities and represents them with sinusoidal oscillations. The parameters amplitude, frequency, and phase are modulated after every zero-crossings. The authors demonstrated that handwriting is reconstructed efficiently by using these parameters.

It is to be noted that most of the reported works on the sinusoidal oscillations are mainly developed with an interest in understanding motor control. Moreover, there are not enough studies showing its usability in practical handwriting systems. Therefore, there is a definite need to explore the sinusoidal modeling for handwriting from a practical handwriting system development point of view. With this motivation, we investigate three possible directions of work for exploring this sinusoidal model framework in an online handwriting system: handwriting representation, modification, and synthesis.

The most popular approach for handwriting representation is using the  $(x, y)$  coordinates of the trajectory and various shape attributes extracted at each point (point-based features) [1,7,23]. These features provide the geometric characteristics of the trajectory through various measures such as curvature, displacement, writing direction, and slope. In some recent works, an alternative approach for feature extraction is observed using handmovement models [58,68]. These features provide movement-related information of the handwriting in contrast to the point-based features. Moreover, we hypothesize that the basic hand movements for generating a pattern remain more stable compared to the variability present in its final outcome, i.e., the 2D trajectory. Therefore, movement-based features extracted from the sinusoidal model may well represent the intra-class variability and can be a suitable representation scheme for handwriting related tasks.

Apart from handwriting representation, the application of handmovement based models is also demonstrated for handwriting synthesis. As reviewed in this chapter, one popular approach for creating modified handwriting is the perturbation method, which applies different transformations on 2-D trace of real samples. These methods are simple and require low data. However, it may result in unnatural handwriting due to the random variations of the  $(x, y)$  coordinates. In some works, statistical models, such as the Bayesian model, HMM are trained using a large number of samples to learn the natural variations in the handwriting shape. The advantage of using a statistical model-based method is that it captures the natural variations from the data set and can generate handwriting from the learned parameters. However, it requires a large number of samples to train the model. On the other hand, handmovement based models provide flexibility to transform the handwriting process. It can help understand the handwriting variability and synthesize handwriting samples by modifying the original handwriting trace [56]. It is worth mentioning that training a handwriting system with a database consisting of more variations can increase the system's robustness. Therefore, we can improve system performance by training data augmentation with the synthetically generated samples [15]. The augmentation of the training dataset also helps modern data-driven classifiers such as HMM and deep neural network (DNN), which require a large training dataset for proper training. Therefore, another important direction for the application of the sinusoidal model is the modification of handwriting for data augmentation and can be explored.

The sinusoidal model can also be utilized for personalized handwriting synthesis. An efficient system for personalized handwriting synthesis should generate natural-looking handwriting similar to the user's handwriting for a given text. In this context, the handmovement models can provide a compact representation for storing a person's handwriting using the model parameters. Moreover, as it is a movement based model, it can generate natural-looking handwriting. One important issue in personalized handwriting synthesis is the formation of ligatures joining between two neighboring characters. An improper ligature can result in unnatural synthesized handwriting. The use of a handmovement based model can produce natural-looking ligature by considering the handwriting dynamics in the preceding and succeeding characters. Therefore, the potential of the sinusoidal model can also be explored for personalized handwriting synthesis.

In the next chapter, we present the details of our first contribution in this thesis – the multi-component sinusoidal model for online handwriting.



# 3

## Multi-component sinusoidal model for online handwriting

### Publications

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1. H. Choudhury, and S.R.M. Prasanna, "Representation of online handwriting using multi-component sinusoidal model," *Pattern Recognition*, vol. 91, pp. 200-215, 2019
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## Overview

In this chapter, we first study the existing works related to the sinusoidal modeling of handwriting, where the horizontal and vertical velocities are represented using two orthogonal sinusoidal oscillations. It is observed that modeling the individual velocities using a single sinusoidal oscillation is inadequate, especially for complex handwriting trajectories. Therefore, to overcome this problem, we propose a multi-component sinusoidal model that represents each velocity profile with multiple sinusoidal oscillations. The model parameters—amplitude, frequency, and phase are extracted by fitting a half period of the sine function between each successive pair of zero-crossing points of the velocity profiles. The parameters of the multiple oscillations are computed iteratively from the residual signals. The proposed model is evaluated using two methods. In the first method, we measure reconstructed handwriting quality using signal to noise ratio (SNR) obtained from the difference between the original and reconstructed velocities. In the second method, we compare the recognition performance of a hidden Markov model (HMM) based character recognizer on the reconstructed handwriting using a different number of sinusoidal components. A better recognition performance indicates better reconstruction quality. The experiments are performed on English and Assamese handwriting.

### 3.1 Introduction

One crucial factor for the success of any handwriting system or handwriting analysis is its representation scheme. Conventionally, online handwriting is represented in the form of  $(x, y)$  coordinates and various shape attributes derived from the 2D trace. An alternative representation can be based on its generation mechanism, which may provide additional movement-based information. This generation based representation can be obtained using handmovement models. The handmovement models represent the handwriting in the form of mathematical equations that describe its production process. The movement information obtained from these models can be beneficial for both the synthesis and analysis of handwriting. In Chapter 1, we discussed the potential uses of handmovement based models for developing practical handwriting systems. Further, we reviewed some of the important handmovement based models and their applications in Chapter 2. The movement information obtained from these models and its parameters have been utilized to develop various handwriting systems such as signature verification, handwriting recognition, and writer identification [36–38].

The work in this thesis is also in this direction, motivated by the oscillatory theory of handwriting [47]. According to the oscillatory theory, the generation of handwriting is associated with mainly two degrees of freedom, one towards the letter height in the vertical direction and the other along the writing line in the horizontal direction. A planned coupling of the orthogonal velocities in the vertical and horizontal direction associated with fingers and wrist generates different letter shapes. The two velocities and their relation vary across writers as well as with various letter shapes. Therefore, an analysis of the two orthogonal velocities of online handwriting may provide added information of the writing and can be used to enhance the handwriting representation. The two velocity profiles are low-frequency oscillatory signals with an invariant bell-shaped nature. At the same time, the amplitude and time between two successive zero crossings of the velocities vary with trajectory shape. Therefore, owing to the invariant bell-shaped and oscillatory nature, we explore online handwriting representation by modeling the velocity profiles using sinusoidal oscillations.

In the literature, a limited number of works have explored the oscillatory theory to represent online handwriting [51, 73, 106]. The pioneering work in this direction is the Hollerbach model [47]. Simard et al. reformulated the Hollerbach model to extract the sinusoidal parameters from handwriting [106]. The authors also demonstrated the importance of phase difference between horizontal and vertical velocity profiles for the handwriting shapes. Singer et al. presented a discrete motor control representation for encoding the online handwriting using amplitudes, and phase parameters of the oscillations [73]. However, the methodologies employed to extract sinusoidal parameters from velocity profiles were abortive until the recent parsimonious oscillatory model for handwriting (POMH model) [51]. A half-wave of a sine function is used to model the velocity profiles between successive zero-crossing points. The POMH model allows efficient extraction of the parameters for any written trace between zero-crossing points and reproduces its dynamics accurately. The image reconstruction using zero-crossing points have also been carried out in [107]. In this work, we propose an improved sinusoidal model using multiple oscillations from the point of view of practical applications.

We explore the existing sinusoidal representation of the POMH model for online handwriting. It is observed that the reconstructed velocities are unable to capture the wide variations in the trajectory. Thus, it may result in an insufficient representation of the handwriting from a practical application point of view. Therefore, as a contribution, we propose a multi-component sinusoidal model to enhance online handwriting representation. Accordingly, an iterative algorithm is presented for extracting the

### 3. Multi-component sinusoidal model for online handwriting

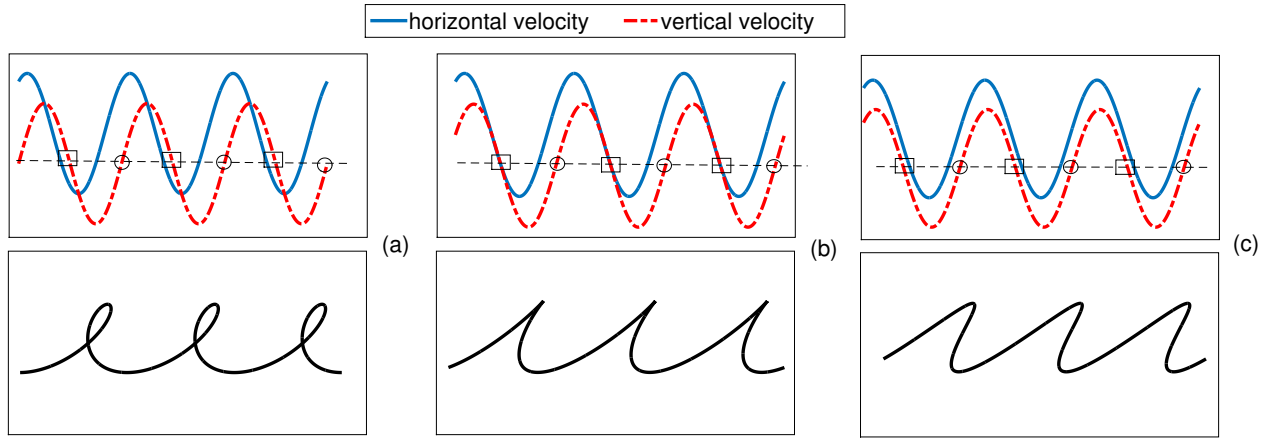
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multi-component sinusoidal parameters from the handwritten data. It is to be noted that unlike the works in [47, 51], we do not model the hand movement; we directly use this model to represent handwriting for practical system development. Finally, the efficacy of the proposed model is demonstrated by reconstructing online handwriting characters and words. We evaluate the reconstructed handwriting using two methods: using reconstruction SNR and using recognition accuracy. In the first method, we measure the quality of reconstructed handwriting using SNR obtained by computing the difference between the original and reconstructed velocities. A higher SNR value indicates better reconstruction quality. In the second method, we compute the recognition performance of an HMM based online handwriting character recognizer for the reconstructed samples of the test set. A better recognition performance will be obtained if the reconstructed handwriting is similar to the natural handwriting. The experiments are performed on English and Assamese handwriting.

This chapter is organized in the following order. Firstly, in Section 3.2, we investigate the existing Hollerbach model and the POMH model for handwriting. Section 3.3 presents the details of the proposed model and the parameter extraction method for handwriting. The evaluation methods are demonstrated in Section 3.4. Section 3.5 elucidates the online handwritten character/word recognition system. The results obtained from the various experiments are discussed in Section 3.7. Finally, Section 3.8 summarizes the proposals made in this chapter.

## 3.2 Oscillatory theory of handwriting

According to oscillatory theory, the vertical and horizontal velocities associated with fingers and wrist movements are produced by two orthogonal oscillations superimposed on a constant magnitude horizontal sweep velocity. The oscillatory theory considers that handwriting is a continuous process where the relationship between these oscillations controls the handwriting shape. Thus, the parameters of the oscillations are modulated after every small interval of time as the shape of the trajectory changes. In the following, we discuss the oscillatory theory and POMH model in detail.



**Figure 3.1:** Three types of handwriting trajectories– loop, sharp edge and rounded corners generated using  $\phi = 90^\circ$ ,  $\phi = 60^\circ$ ,  $\phi = 0^\circ$  are shown in the subplot (a), (b), and (c), respectively. Here,  $\omega_x = \omega_y = 10\pi$ ,  $a = b = 4c = 1$ . The corresponding horizontal velocity  $v_x$  (solid line) and vertical velocity  $v_y$  (dotted line) are shown above the pattern where *square* and *circle* denotes vertical velocity zero crossing point corresponding to top and bottom corner, respectively.

### 3.2.1 Hollerbach model

The oscillatory theory was proposed by Hollerbach in [47]. Based on this theory Hollerbach modeled the horizontal and vertical velocities of handwriting using sinusoidal oscillations as follows :

$$v_x = a \sin(\omega_x(t - t_0) + \phi_x) + c \quad (3.2.1)$$

$$v_y = b \sin(\omega_y(t - t_0) + \phi_y) \quad (3.2.2)$$

where  $a$  and  $b$  are the amplitudes,  $\omega_x$  and  $\omega_y$  are the frequencies,  $\phi_x$  and  $\phi_y$  are the phases of horizontal and vertical velocity, respectively.  $t$  represents the time with reference to initial time  $t_0$ , and  $c$  is the magnitude of horizontal sweep which controls the horizontal distance between the successive letters in the generated handwriting. Temporal modulation of these parameters at particular time instants generates different horizontal and vertical velocity profiles to form the required handwriting. According to Hollerbach, these modulation instants are the zero-crossing instants of the vertical velocity, which corresponds to the corner points of the handwriting. The corner shapes at these points are controlled by the horizontal velocity at that particular instant.

In Figure 3.1, we illustrate the different handwriting shapes obtained using the Hollerbach model. Three patterns and their corresponding velocity profiles produced using (3.2.1) and (3.2.2) by changing the phase difference  $\phi = \phi_x - \phi_y$  are shown in the Figure3.1(a), (b) and (c). The frequencies  $\omega_x$  and  $\omega_y$

### 3. Multi-component sinusoidal model for online handwriting

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are assumed to be equal. In the figure, the vertical velocity corresponding to top corners and bottom corners are marked by square and round dots, respectively, in the top panels. It can be observed that the horizontal velocity value changes for different corner shapes, which are negative, zero, and positive for the loop, the sharp edge, and rounded corner, as shown in Figure 3.1(a), (b) and (c), respectively.

At the vertical velocity zero crossing points for the top and bottom corners we have  $\omega(t-t_0) + \phi_y = (2n+1)\pi$  and  $\omega(t-t_0) + \phi_y = 2n\pi$ , respectively (marked by squares and round dots, respectively in the vertical velocity profiles of Figure 3.1).  $n$  represents any integer value. At these points, the corresponding horizontal velocity from (3.2.1) is given by:

$$k_1 = c - a \sin \phi, \quad \text{top corner} \quad (3.2.3)$$

$$k_2 = c + a \sin \phi, \quad \text{bottom corner} \quad (3.2.4)$$

Therefore, the main controlling parameters for the corner shapes are the amplitude of horizontal oscillation and phase difference  $\phi$  between horizontal and vertical oscillations. Hollerbach demonstrated that the writing slant is effected by amplitudes of both vertical and horizontal oscillations and their phase difference mathematically represented as follows [47]:

$$k_3 = \tan \beta = \frac{b}{a \cos \phi} \quad (3.2.5)$$

where  $\beta$  is the slant of the handwriting.

Thus, handwriting can be generated from the sinusoidal model by modulation of its parameters such that it satisfies the horizontal velocity constraint ( $k_1$  or  $k_2$  depending on corner location) and slant constraint ( $k_3$ ).

#### 3.2.1.1 Handwriting generation using Hollerbach model

We demonstrate the generation of different letter shapes using the Hollerbach model by parameter modulation of the two orthogonal oscillations. The required sinusoidal parameters are computed by solving equations (3.2.3)(or (3.2.4)) and (3.2.5) where the horizontal velocity constraint  $k_1$  (or  $k_2$ ) and slant constraint  $k_3$  are chosen according to the required shape and slant, respectively as follows:

- In the first step,  $\phi_y$  is considered depending on the starting point of the letter (top or bottom). It controls the initial direction of trajectory, i.e., upward or downward.
- Next, the vertical velocity amplitude  $b$  is fixed depending on letter height. The height of a letter increases as the value of  $b$  increases and vice versa.

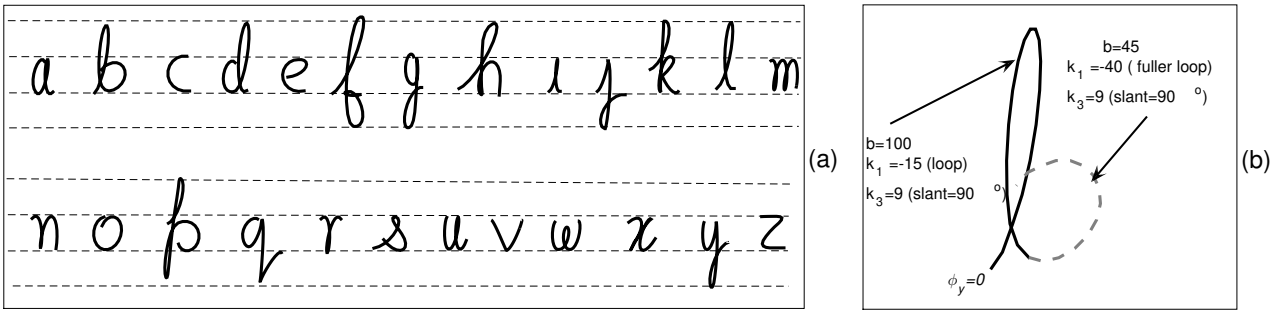
**Table 3.1:** Sinusoidal parameters for generating different English letter shape.

Letter/ initial phase of vertical velocity ( $\phi_y$ ) in radian	time in seconds	a	b	$\phi$	Letter/ initial phase of vertical velocity in radian	time in seconds	a	b	$\phi$
a / 2.51	0.0	30	45	1.05	o/3.14	0.0	33	45	1.43
	0.3	11	45	1.14		0.0	30	70	1.06
b / 0.0	0.0	23	100	1.06		0.30	30	150	6.24
	0.6	45	45	1.5	p/0.0	0.45	17	150	5.93
c / 2.8	0.0	45	45	1.5		0.60	23	90	5.22
d / 2.51	0.0	36	45	1.32		0.85	40	45	4.83
	0.36	23	45	1.06		0.0	38	45	1.16
e / 0.0	0.0	40	25	1.2	q/2.51	0.30	17	45	0.85
	0.3	45	45	1.5		0.66	12	100	5.86
	0.0	40	100	0.64		1.0	27	50	5.90
f / 0.0	0.30	30	140	1.01	r/2.51	0.0	11	45	5.17
	0.60	30	45	2.11		0.0	36	45	0.92
	0.90	15	5	1.50	s/0.0	0.35	50	45	2.72
	0.0	20	45	1.32		0.4	40	45	4.14
g / 2.20	0.42	9.15	40	2.07		0.0	13	100	5.78
	0.72	13.6	110	5.82	t/3.14	0.30	18	45	2.92
	0.9	27	60	5.15		0.60	35	0.0	4.71
	0.0	54	90	0.6	u/0.0	0.0	11	45	1.01
h / 0.0	0.35	10	90	0.50		0.0	16	45	3.27
	0.60	20	45	4.95	v/3.14	0.3	16	45	0.50
	1.1	7	45	5.5		0.0	16	45	1.25
i / 0.0	0.0	10	45	4.01	w/3.14	0.3	16	35	1.31
	0.0	12	45	0.66		0.9	16	45	1.25
j / 0.0	0.3	15	100	5.55		0.0	28	40	5.50
	0.6	40	50	5.15	x/2.20	0.3	25	45	6.08
	0.0	35	70	1.04		0.65	23	45	0.65
	0.33	11	100	0.09		0.0	11	45	1.10
k / 0.0	0.63	16	50	5.06	y/3.14	0.63	15	100	5.55
	0.96	55	45	5.81		0.91	30	70	5.32
	1.06	21	45	2.35		0.0	45	1	1.54
l / 0.0	0.0	22	100	1.06		0.19	50	45	0.4
m / 2.51	0.0	11	45	5.15	z/1.57	0.42	40	1	1.54
n / 2.51	0.0	11	45	5.15					

- Depending on the required top (bottom) corner shape value of  $k_1$  ( $k_2$ ) is chosen. A more positive value results in a more rounded shape, while a more negative value results in a fuller loop.
- The value of  $k_3$  is fixed according to the required slant.
- Finally, the horizontal velocity amplitude  $a$  and phase difference  $\phi$  are computed by solving the (3.2.3) (or (3.2.4)) for top (or bottom) corner shape constraint and (3.2.5) for slant constraint.

Table 3.1 presents the model parameters computed to generate the English lowercase letters fol-

### 3. Multi-component sinusoidal model for online handwriting



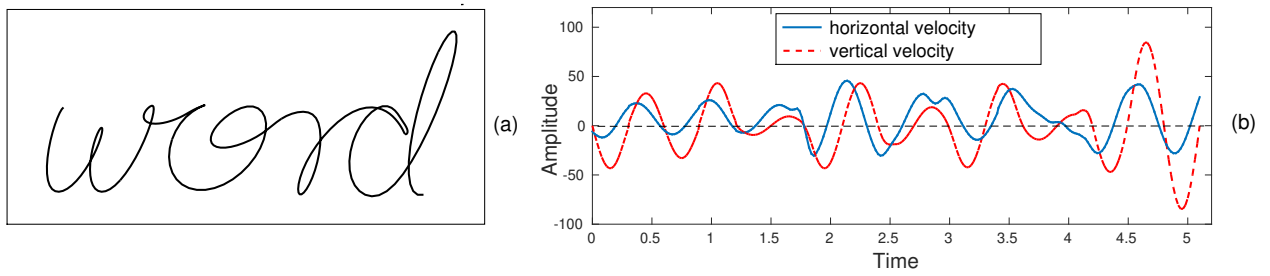
**Figure 3.2:** The subplot (a) shows the English lowercase letters ( $a - z$ ) generated from sinusoidal model. The subplot (b) shows the generation of letter ‘ $b$ ’ using two sets of parameters forming two loops. The handwriting trajectories formed using the two different sets of parameters are shown using solid and dotted lines.

lowing the above steps.

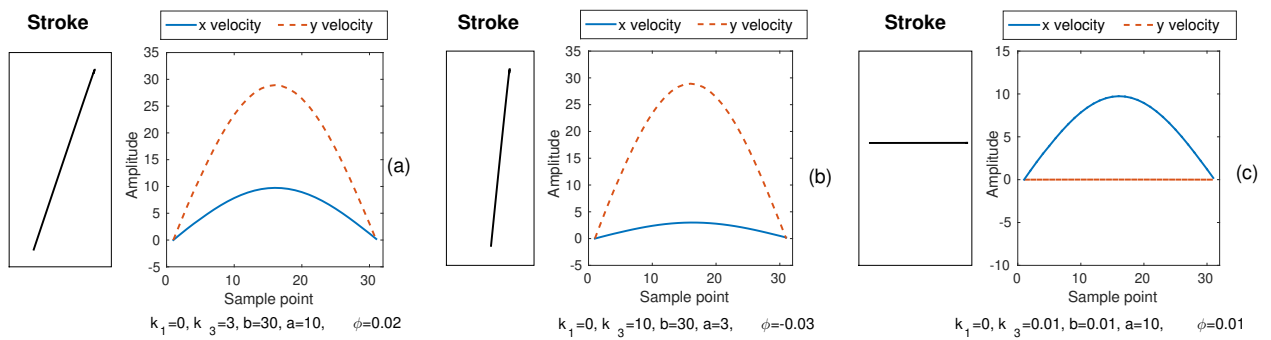
Figure 3.2(a) shows the letter shapes generated from (3.2.1) and (3.2.2) using the parameters given in Table 3.1. We consider the frequency  $\omega_x = \omega_y = \omega = 3.33\pi$  Hz. For an illustration of the parameter selection method, we considered an English lowercase letter ‘ $b$ ’, which consists of two corner loops, as shown in Figure 3.2(b). The generation of ‘ $b$ ’ requires two modulations, including the initial parameters. The first corner from the start point is at greater height as compared to the second corner. Therefore the value of parameter  $b$  is 100 and 40 for the first and second corner, respectively. The slant factor  $k_3 = 9$  for both the corners for a slant equals to  $90^\circ$ . On the other hand, the value of  $k_1$  is  $-15$  and  $-40$  for the first and second corners, respectively, as the latter is more fuller loop.

It is observed that the letters with a simple shape such as ‘ $o$ ’, ‘ $e$ ’, and ‘ $n$ ’ are generated using only one set of initial parameters. On the other hand, a higher number of modulations are required to create complex shape letters like ‘ $f$ ’, ‘ $k$ ’, and ‘ $p$ ’. Moreover, in addition to the vertical velocity zero-crossings, parameter modulations are required at other points also to produce complex shapes. A word generated using the Hollerbach model is shown in Figure 3.3 along with its velocity profiles. The letter parameters are chosen similar to the values shown in Table 3.1 with a different slant condition. Additional parameter modulations are used to form the joining ligatures between letters using the method described in the previous text.

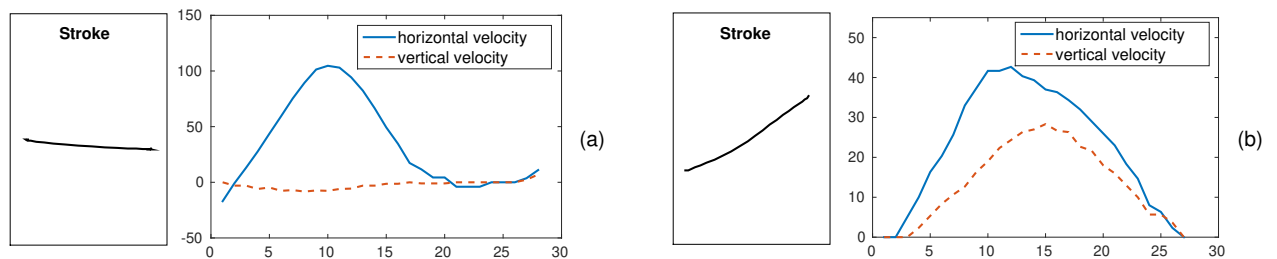
It is to be noted that, although the sinusoidal model has been used to generate the cursive writing, the same method may also be employed to generate straight-line strokes that are frequently used in the Chinese and Indic scripts. The ends of the straight-line stroke can be considered as sharp edges, and hence the value of  $k_1 = 0$ . The value of  $k_3$  is chosen from the required slant. Figure 3.4 shows



**Figure 3.3:** (a) Illustration of a word ‘word’ generated from the sinusoidal model. The generated horizontal and vertical velocities are also shown in the subplot (b).



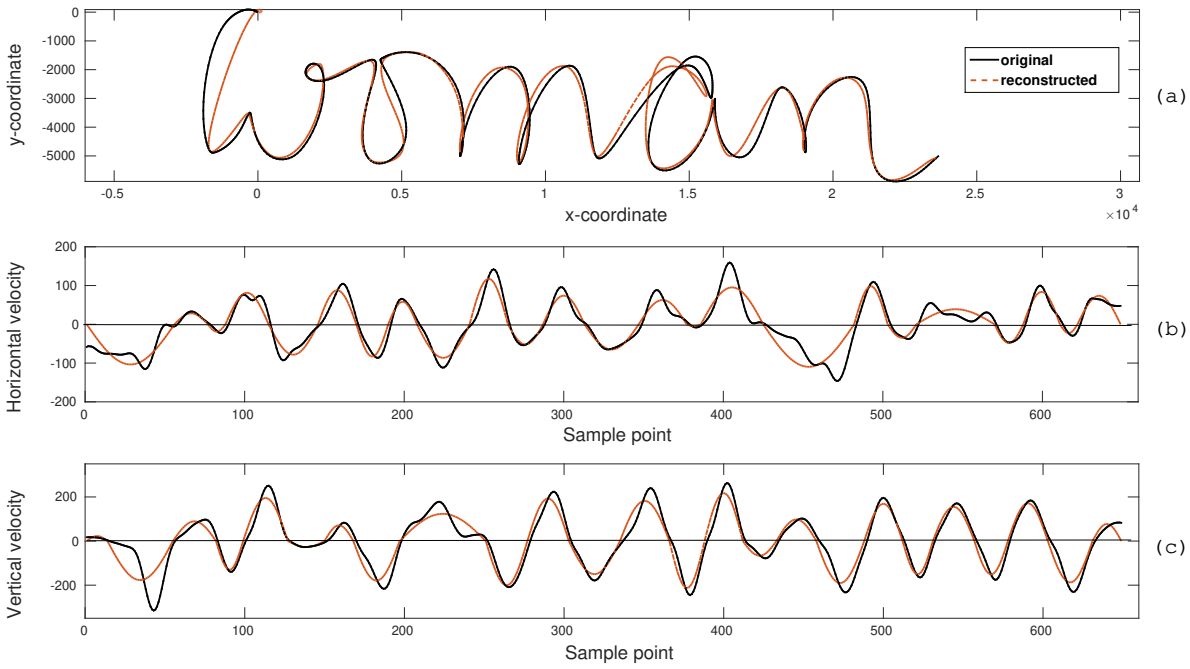
**Figure 3.4:** (a)–(c) show three synthetic straight strokes in the left panel and the velocity profiles generated using sinusoidal model in the right panel.



**Figure 3.5:** The subplots (a) and (b) show two real online handwritten strokes (a horizontal line and a slanted straight stroke) along with the velocity profiles.

three generated straight line strokes along with its velocity profiles and the sinusoidal parameters using the sinusoidal model. It can be observed that the phase difference between the two velocity profiles is almost zero, while the slant varies depending on the ratio of their amplitudes. We plot two real handwritten strokes and the velocity profiles from the Assamese database in Figure 3.5. It can be visualized that similar to the synthetic data, the phase difference between two velocities of real

### 3. Multi-component sinusoidal model for online handwriting



**Figure 3.6:** The subplot depict (a) depicts an original (solid line) and reconstructed (dash line) handwriting using sinusoidal representation of word ‘woman’ using POMH model. The subplots (b) and (c) represent original (solid line) and reconstructed (dash line) horizontal and vertical velocity, respectively using POMH model.

handwriting is close to zero, and the writing slant varies with the ratio of the amplitude. Therefore, this model can produce handwriting trajectory for various letter shapes if appropriate parameters and modulation instants are chosen.

#### 3.2.2 POMH model

The most recent work on the sinusoidal representation is the parsimonious oscillatory model for handwriting (POMH) [51], which is essentially an improved version of the Hollerbach model. The horizontal and vertical velocities of the handwriting are modeled by fitting half a period of sine function between two successive zero-crossing points. The authors omitted the horizontal drift velocity parameter  $c$  in the Hollerbach model to obtain symmetric equations, making the parameter extraction method simple. The velocities  $v_x$  and  $v_y$  along the horizontal and vertical directions, respectively are defined as follows:

$$v_x(t) = a_{v_x}(t) \sin(\omega_{v_x}(t).t + \phi_{v_x}(t)) \quad (3.2.6)$$

$$v_y(t) = a_{v_y}(t) \sin(\omega_{v_y}(t).t + \phi_{v_y}(t)) \quad (3.2.7)$$

where  $a_{v_x}$ ,  $\omega_{v_x}$  and  $\phi_{v_x}$  are the sinusoidal oscillation parameters for horizontal oscillation and similarly  $a_{v_y}$ ,  $\omega_{v_y}$  and  $\phi_{v_y}$  are parameters for the vertical oscillation. Figure 3.6.(a) depicts an original handwritten pattern and its reconstructed pattern using POMH model. The respective original and reconstructed velocity profiles  $v_x$  and  $v_y$  are given in Figure 3.6.(b)-(c).

In the following, we describe the parameter extraction method of the POMH model. Suppose  $S$  denotes an oscillatory signal with the number of points  $T$  and  $N_S$  zero-crossings at time instants  $z_{S,1}$ ,  $z_{S,2}, \dots, z_{S,N_S}$ . The sinusoidal parameters i.e. the amplitude, frequency and phase denoted by  $a_{S,i}$ ,  $\omega_{S,i}$  and  $\phi_{S,i}$  between two successive zero crossing points  $z_{S,i}$  and  $z_{S,i+1}$  are computed by the following equations [51]:

$$a_{S,i} = \frac{\pi}{2} \text{mean} (S|_{z_{S,i}^{z_{S,i+1}}}), \quad \omega_{S,i} = \frac{\pi}{z_{S,i+1} - z_{S,i}}, \quad \phi_{S,i} = -\frac{\pi z_{S,i}}{z_{S,i+1} - z_{S,i}},$$

Accordingly, the sinusoidal parameters for the entire signal is given by:

$$a_S(t) = a_{S,i}, \quad z_{S,i} < t \leq z_{S,i+1} \tag{3.2.8}$$

$$\omega_S(t) = \omega_{S,i}, \quad z_{S,i} < t \leq z_{S,i+1} \tag{3.2.9}$$

$$\phi_S(t) = \phi_{S,i}, \quad z_{S,i} < t \leq z_{S,i+1} \tag{3.2.10}$$

where,  $1 \leq i \leq (N - 1)$ ,  $z_{S,1} = 1$ ,  $z_{S,N_S} = T$  and  $1 \leq t \leq T$ . Now, the sinusoidal representation of velocity profiles (denoted by  $S = v_x$  and  $S = v_y$ ) of an online handwritten data using the parameters given in (3.2.8) - (3.2.10) are as follows:

$$v_x(t) = a_{v_x}(t) \sin(\omega_{v_x}(t).t + \phi_{v_x}(t)) \tag{3.2.11}$$

$$v_y(t) = a_{v_y}(t) \sin(\omega_{v_y}(t).t + \phi_{v_y}(t)) \tag{3.2.12}$$

### 3.3 Proposed multi-component sinusoidal model

As discussed in the previous section, the POMH model is a linear and symmetrical model for online handwriting with a simple parameter extraction method. It decomposes handwriting into horizontal and vertical velocities and represents them with sinusoidal oscillations. However, it is observed that the complete handwriting characteristics are not captured by this representation and hence may be inadequate for practical applications such as handwriting recognition or writer identification, where

### 3. Multi-component sinusoidal model for online handwriting

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the handwriting characteristics play an important role. For example, an insufficient representation of the handwriting may increase the confusion between similar-looking characters. It may result in the loss of important writer-specific information for forensics systems.

To illustrate the handwriting representation using the POMH model, we refer to Figure 3.6(a). It shows the original and reconstructed handwriting for a word using the POMH model. It can be observed from the reconstructed handwriting that although the POMH model can capture the overall letter shapes, it is not sufficient for representing the full characteristics of the handwriting. Hence, it may be inadequate for practical applications such as handwriting recognition or writer identification, where the handwriting characteristics play an important role. For illustration, the error between the original and reconstructed horizontal and vertical velocities of Figure 3.6(b) and 3.6(c) are shown in Figure 3.7(a) and 3.7(b), respectively. The reconstruction error indicates that further improvement is required to obtain a robust representation of handwriting. Therefore, the error signals can be considered as additional sinusoidal components which are not accommodated in the first oscillation. In this direction, an improved handwriting representation is presented in this work by using multi-component sinusoidal oscillations for modeling the velocity profiles. Thus, instead of only a single oscillation, multiple oscillations are modulated at different time instants for  $v_x$  and  $v_y$  as follows:

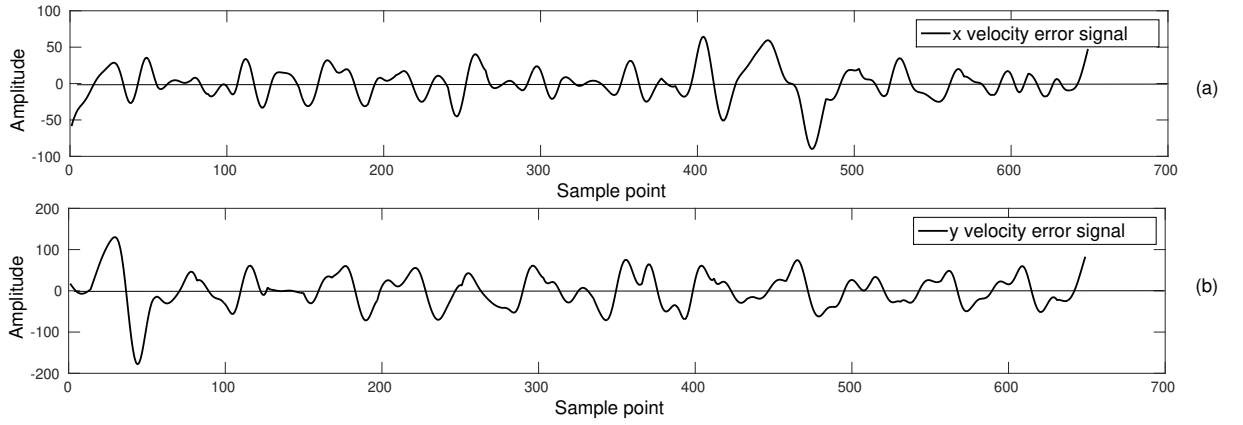
$$v_x(t) = \sum_{m=1}^K a_{v_x}^m(t) \sin(\omega_{v_x}^m(t).t + \phi_{v_x}^m(t)) \quad (3.3.1)$$

$$v_y(t) = \sum_{m=1}^K a_{v_y}^m(t) \sin(\omega_{v_y}^m(t).t + \phi_{v_y}^m(t)) \quad (3.3.2)$$

where  $a_{v_x}^m$ ,  $\omega_{v_x}^m$  and  $\phi_{v_x}^m$  are the sinusoidal parameters amplitude, frequency and phase of the  $m^{th}$  component from horizontal velocity. Similarly,  $a_{v_y}^m$ ,  $\omega_{v_y}^m$  and  $\phi_{v_y}^m$  are the sinusoidal parameters of the  $m^{th}$  component from vertical velocity.

#### 3.3.1 Extraction of model parameters

In the following, we describe the parameter extraction method used for the proposed handwriting representation. The overall framework of the method is shown in Figure 3.8. First, the online handwriting is decomposed into its horizontal velocity  $v_x$  and vertical velocity  $v_y$ . In the first iteration, each of the velocities  $v_x$  and  $v_y$  are modeled by sinusoidal parameters and reconstructed to obtain the horizontal and vertical residuals signals. These residual signals are fed as input to the next iteration



**Figure 3.7:** (a) Represents the reconstruction error signal of horizontal velocity for the word shown in Figure 3.6 and (b) represents the same as in (a) for vertical velocity.

to model it by another sinusoidal component. Thus, the parameters of the sinusoidal components are estimated iteratively. Details of the steps involved in this method are given in the following.

- **Addition of imaginary strokes**

A handwriting data may contain multiple strokes. Therefore, adjacent strokes of handwriting samples are joined using imaginary strokes to form a single stroke, which is generated using interpolation. Continuous velocity profiles are obtained like the handwriting written in a single stroke, as shown in Figure 3.9 for a handwritten word.

- **Horizontal and vertical velocities**

Let  $P = \{(x_1, y_1), (x_2, y_2), \dots, (x_t, y_t), \dots, (x_T, y_T)\}$  denote a coordinate sequence of online handwriting where  $(x_t, y_t)$  denotes the  $t^{th}$  coordinate point. The velocity profiles are computed by finding the difference between its consecutive coordinate values as follows.

$$\text{Horizontal velocity, } v_x(t) = x_{t+1} - x_t \quad (3.3.3)$$

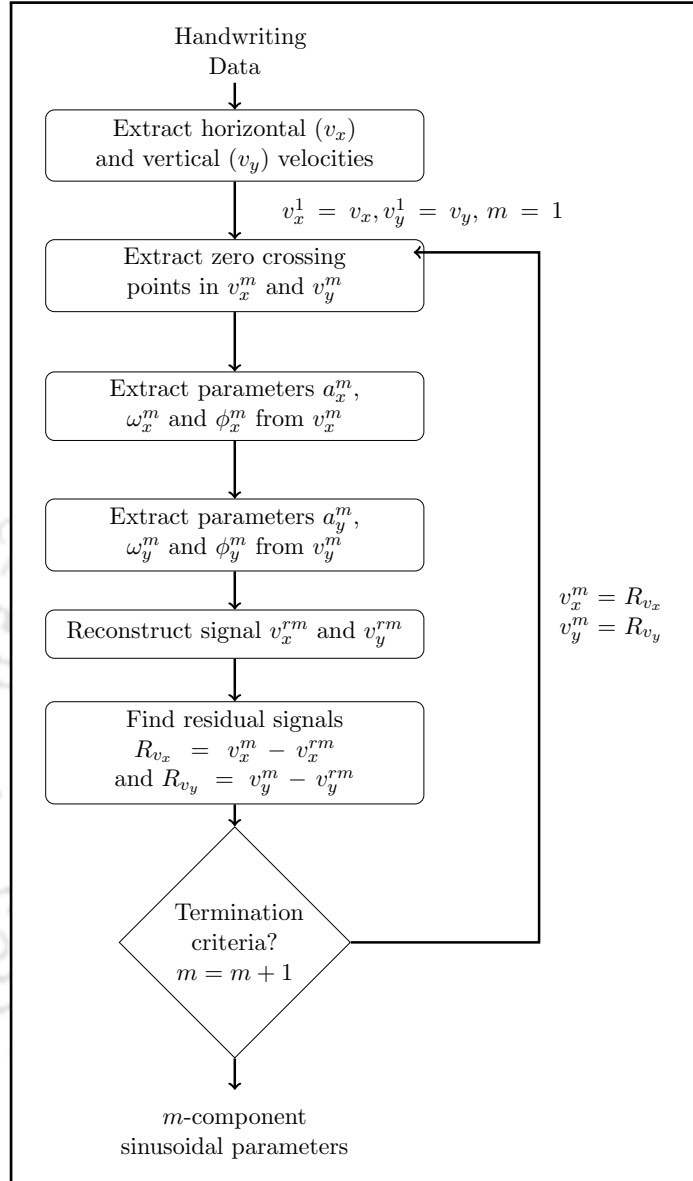
$$\text{Vertical velocity, } v_y(t) = y_{t+1} - y_t \quad (3.3.4)$$

We consider  $v_x(T) = v_x(T - 1)$  and  $v_y(T) = v_y(T - 1)$ .

- **Zero crossing points**

At each iteration, zero crossing points are extracted from the input signals. Let  $v_x^m$  and  $v_y^m$  be the input signals at  $m^{th}$  iteration in horizontal and vertical direction, respectively. Now, in the first iteration ( $m = 1$ ), the input signals  $v_x^1$  and  $v_y^1$  are equal to the  $v_x$  and  $v_y$ , whereas in all other

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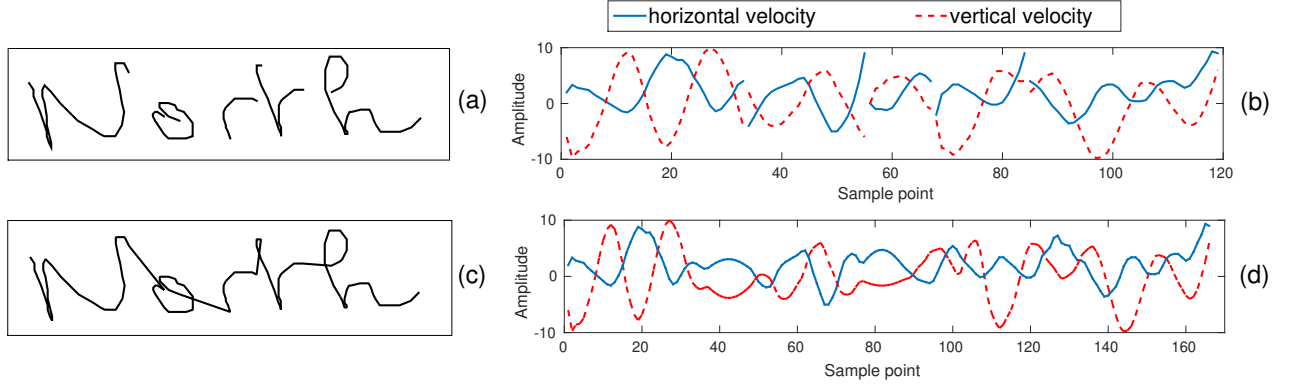


**Figure 3.8:** Illustration of the steps involved in the parameter extraction of the proposed multi-component sinusoidal model.

iterations ( $m > 1$ ) the input signals  $v_x^m$  and  $v_y^m$  are equal to the residual signal at the previous iteration. We denote the zero crossing instants of  $v_x^m$  and  $v_y^m$  by  $\{z_{v_x,1}^m, z_{v_x,2}^m, \dots, z_{v_x,N_x^m}^m\}$  and  $\{z_{v_y,1}^m, z_{v_y,2}^m, \dots, z_{v_y,N_y^m}^m\}$ , respectively.  $N_x^m$  and  $N_y^m$  are the number of zero crossings in  $m^{th}$  component of horizontal and vertical velocities, respectively.

- **Parameter estimation**

The sinusoidal parameters for the  $m^{th}$  component are extracted by fitting a half period of sine



**Figure 3.9:** The subplots (a) and (b) show an English word written using multiple pen-ups and its velocities, respectively. The subplots (c) and (d) illustrates the same handwriting and velocities, respectively after addition of imaginary strokes.

function between each successive zero-crossing points of the velocity profiles. Thus, velocities between two zero-crossing points are represented by the amplitude, phase, and frequency of the sine function. The number of parameters set representing a velocity profile is one less than the number of its zero-crossing points.

The parameters for the input signals  $v_x^m$ , between zero crossing points  $z_{v_x,i}^m$  and  $z_{v_x,i+1}^m$  are computed by using the equations given below [51]:

$$\omega_{v_x,i}^m = \frac{\pi}{z_{v_x,i+1}^m - z_{v_x,i}^m} \quad (3.3.5)$$

$$\phi_{v_x,i}^m = -\frac{\pi t_{x,i}^m}{z_{v_x,i+1}^m - z_{v_x,i}^m} \quad (3.3.6)$$

$$a_{v_x,i}^m = \frac{\pi}{2} \text{mean}(S_x^m) \Big|_{z_{v_x,i}^m}^{z_{v_x,i+1}^m} \quad (3.3.7)$$

where  $1 \leq i \leq N_x^m - 1$

Similarly the sinusoidal parameters for  $v_y^m$  between  $z_{v_y,i}^m$  and  $z_{v_y,i+1}^m$  are computed as,

$$\omega_{v_y,i}^m = \frac{\pi}{z_{v_y,i+1}^m - z_{v_y,i}^m} \quad (3.3.8)$$

$$\phi_{v_y,i}^m = -\frac{\pi t_{y,i}^m}{z_{v_y,i+1}^m - z_{v_y,i}^m} \quad (3.3.9)$$

$$a_{v_y,i}^m = \frac{\pi}{2} \text{mean}(S_y^m) \Big|_{z_{v_y,i}^m}^{z_{v_y,i+1}^m} \quad (3.3.10)$$

where  $1 \leq i \leq N_y^m - 1$

### 3. Multi-component sinusoidal model for online handwriting

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- **Reconstruction and residual signal**

The input signals  $v_x^m$  and  $v_y^m$  are reconstructed using the extracted parameters to obtain  $v_x^{rm}$  and  $v_y^{rm}$ , respectively as follow:

$$v_x^{rm}(t) = a_{v_x}^m(t) \sin(\omega_{v_x}^m(t).t + \phi_{v_x}^m(t)), \quad 1 \leq t \leq T \quad (3.3.11)$$

where

$$\begin{aligned} a_{v_x}^m(t) &= a_{v_x,i}^m, \quad t = [z_{v_x,i}^m, z_{v_x,i+1}^m] \\ \omega_{v_x}^m(t) &= \omega_{v_x,i}^m, \quad t = [z_{v_x,i}^m, z_{v_x,i+1}^m] \\ \phi_{v_x}^m(t) &= \phi_{v_x,i}^m, \quad t = [z_{v_x,i}^m, z_{v_x,i+1}^m], \quad 1 \leq i \leq N_x^m - 1 \\ v_y^{rm}(t) &= a_{v_y}^m(t) \sin(\omega_{v_y}^m(t).t + \phi_{v_y}^m(t)), \quad 1 \leq t \leq T \end{aligned} \quad (3.3.12)$$

where

$$\begin{aligned} a_{v_y}^m(t) &= a_{v_y,i}^m, \quad t = [z_{v_y,i}^m, z_{v_y,i+1}^m] \\ \omega_{v_y}^m(t) &= \omega_{v_y,i}^m, \quad t = [z_{v_y,i}^m, z_{v_y,i+1}^m] \\ \phi_{v_y}^m(t) &= \phi_{v_y,i}^m, \quad t = [z_{v_y,i}^m, z_{v_y,i+1}^m], \quad 1 \leq i \leq N_y^m - 1 \end{aligned}$$

The residual signals at  $m^{th}$  iteration are computed as:

$$R_{v_x}^m = v_x^m - v_x^{rm} \quad \text{and} \quad R_{v_y}^m = v_y^m - v_y^{rm} \quad (3.3.13)$$

These residual signals are fed to the next iteration to find parameters of the  $(m+1)^{th}$  component.

Therefore,

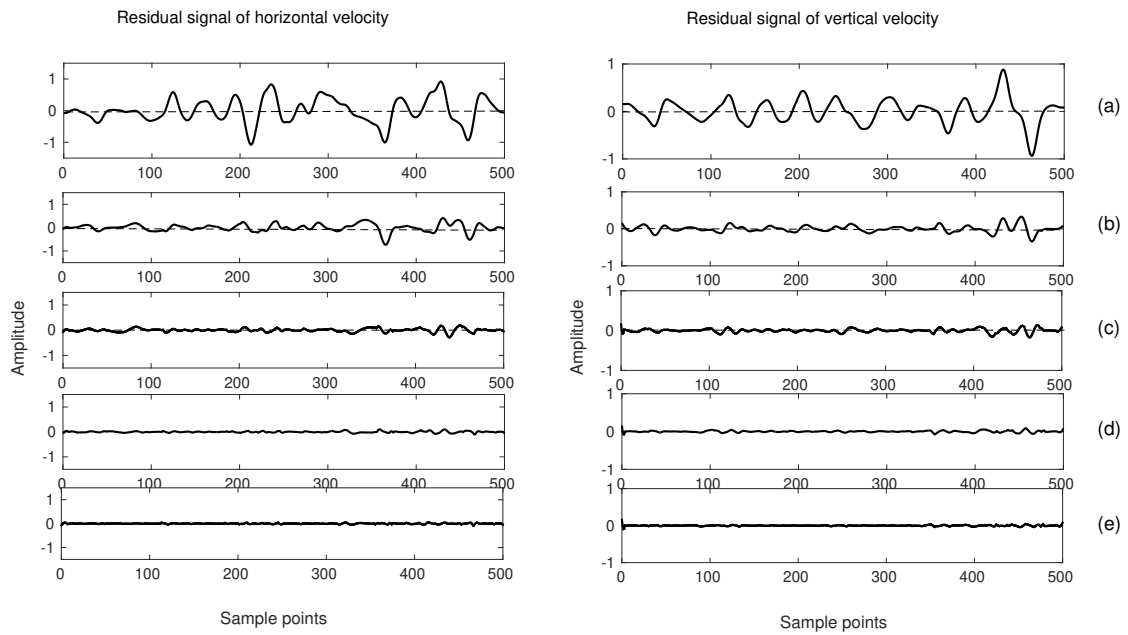
$$v_x^{m+1} = R_{v_x}^m \quad (3.3.14)$$

$$v_y^{m+1} = R_{v_y}^m \quad (3.3.15)$$

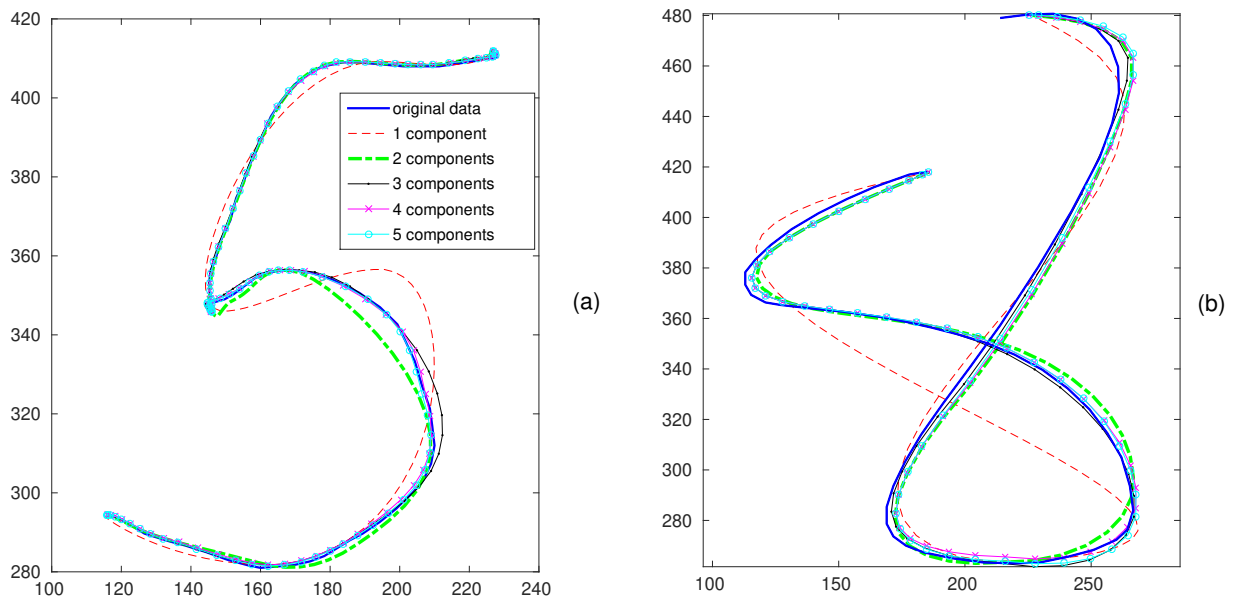
Finally after  $K$  iterations,

$$v_x^r(t) = \sum_{m=1}^K a_{v_x}^m(t) \sin(\omega_{v_x}^m(t).t + \phi_{v_x}^m(t)) \quad (3.3.16)$$

$$v_y^r(t) = \sum_{m=1}^K a_{v_y}^m(t) \sin(\omega_{v_y}^m(t).t + \phi_{v_y}^m(t)) \quad (3.3.17)$$



**Figure 3.10:** The left column and right column show the horizontal and vertical velocity components of a handwritten word. The subplot (a) represents first iteration input i.e.,  $v_x$  and  $v_y$  and subplot (b)-(e) are residual signals after each iterations. The  $x$ -axis represents time in seconds and  $y$ -axis represents amplitude.



**Figure 3.11:** The subplots (a) and (b) represent the original and reconstructed handwriting samples using different sinusoidal components for English digits ‘five’ and ‘eight’, respectively.

#### 3.3.2 Illustration

Figure 3.10 shows the input signals at each iteration with an increasing number of components. It can be observed that the amplitude of the residual signal decreases as the number of iterations increases and may be considered as noise signals after two or three iterations depending on the complexity of the handwriting. For illustration, in Figure 3.11, we depict two online handwriting samples of English digits ‘five’ and ‘eight’ along with their reconstructed handwriting by varying the number of components in the sinusoidal representation. It can be observed that the first component mainly captures the overall structure of the character, and with the increase in the number of components, the model captures precise details of the character shape. Thus, the use of multi-component may enhance the representation beyond a single component.

### 3.4 Evaluation methods

The proposed model is evaluated using two methods. The first method is the computation of SNR between original and reconstructed velocities. The second method is using an online handwriting recognition system. The experiments are performed using English and Assamese databases. The details of the databases are given in Section 3.6.

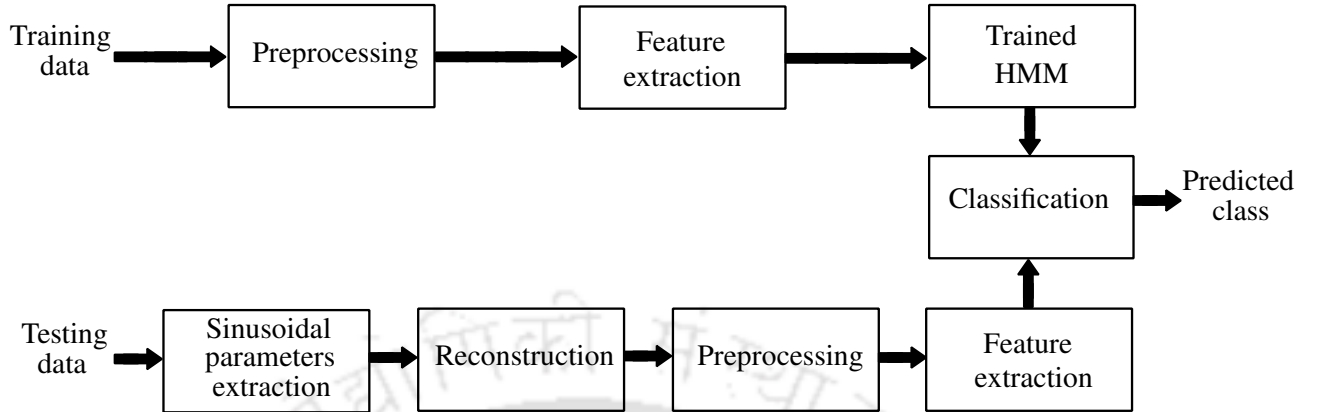
#### 3.4.1 Evaluation using reconstruction SNR

An efficient model-based representation is expected to have a small reconstruction error. Therefore, the performance of the proposed model is evaluated by measuring the reconstruction quality. In this experiment, the reconstruction quality is measured in terms of the SNR value computed between the reconstructed handwriting and the original sample. The SNR value is computed for the velocity signals using different number of sinusoidal components (from  $K = 1$  to 5). The model parameters are extracted from the word samples using the method described in Section 3.3.1. The SNR of the horizontal and vertical velocities reconstruction of the English and Assamese handwriting are computed using the definition [49]:

$$\text{SNR}_{v_x, \text{dB}} = 20 \log \left( \frac{\int v_x^2(t) dt}{\int (v_x(t) - v_x^r(t))^2 dt} \right) \quad (3.4.1)$$

$$\text{SNR}_{v_y, \text{dB}} = 20 \log \left( \frac{\int v_y^2(t) dt}{\int (v_y(t) - v_y^r(t))^2 dt} \right) \quad (3.4.2)$$

where  $v_x$  and  $v_y$  are the original velocities and  $v_x^r$  and  $v_y^r$  are the corresponding reconstructed signals.



**Figure 3.12:** Illustration of recognition experiment for evaluation of the multi-component sinusoidal model. The training samples are the original data from the database and the testing samples are the reconstructed data. The experiment uses point-based features to train HMM for the character recognition task.

### 3.4.2 Evaluation using online handwriting recognition system

In the second evaluation method, the efficacy of the proposed representation is demonstrated using the recognition performance of HMM based online handwriting recognition system. The recognition system is trained using original handwriting, i.e., the training set of the databases. On the other hand, the test data are reconstructed handwriting using the multi-component sinusoidal model. We compare the recognition performance on the reconstructed data with different number of sinusoidal components ( $K$ ). The poor reconstruction will result in dissimilarity between the training and testing data. Moreover, it may also increase confusion between similar-looking characters if the handwriting is not reconstructed efficiently. Therefore, a better recognition performance indicates better reconstruction quality. Figure 3.12 illustrates the recognition experiment carried out for the evaluation of the multi-component sinusoidal model. The recognition system comprises of the following stages: preprocessing, feature extraction, and modeling. In the following, we describe the recognition system in detail.

## 3.5 HMM based online handwriting recognition system

In this section, we describe the details of preprocessing, feature set, and modeling approach for the HMM character and word recognition systems used in this thesis.

### 3.5.1 Preprocessing

The following preprocessing steps are performed before extracting the handwriting attributes: smoothing, size normalization, and resampling, as described in [23]. Smoothing reduces the amount

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of high-frequency noise in the input data resulting from either the capturing device or due to jitters in writing. We apply a moving average filter with a window size of three points for smoothing the data. The size normalization steps are different for the character and word recognition systems. The size variation among the character samples is removed by normalizing the  $y$  coordinates of the pattern to  $[0, 1]$  range, maintaining the aspect ratio. To normalize the size of a word sample, first, the baseline and corpus line for the word are determined [108]. The region between these two lines is normalized to the  $[0, 1]$  range. The remaining portion is normalized proportionally by preserving the aspect ratio. Thereafter, we resample the character and word data to obtain a constant spacing between two consecutive points [108].

#### 3.5.2 Feature extraction

The recognition system is trained using the most popular point-based features for online handwriting. These features are obtained locally at each point  $(x_t, y_t)$  of the preprocessed handwriting within a small vicinity between  $(x_{t-r}, y_{t-r})$  and  $(x_{t+r}, y_{t+r})$  as described in the following. The value of  $r$  in this work is considered as two.

- the preprocessed coordinates  $(x, y)$ ,
- the first and second derivative of  $x$  and  $y$  coordinates for measuring the slope and change of slope at each point in horizontal and vertical direction, respectively,
- aspect ratio at each point  $(x_t, y_t)$  is measured by the ratio of height and width of the bounding box  $B$ , formed by containing the points between  $(x_{t-r}, y_{t-r})$  and  $(x_{t+r}, y_{t+r})$ ,
- curliness at point  $(x_t, y_t)$  is the ratio between length of the trajectory and maximum height or width of the bounding box  $B$ ,
- the writing direction is described by cosine and sine of the angle  $\alpha(t)$  made by the line joining  $(x_{t-r}, y_{t-r})$  and  $(x_{t+r}, y_{t+r})$ , and the horizontal axis,
- the linearity which is average of squared distance of points between  $(x_{t-r}, y_{t-r})$  and  $(x_{t+r}, y_{t+r})$ , to the straight line joining them,
- the curvature at each point of the handwritten pattern is described by the cosine and sine of the angle made by the sequence of points  $(x_{t-r}, y_{t-r})$ ,  $(x_t, y_t)$  and  $(x_{t+r}, y_{t+r})$ , at  $(x_t, y_t)$ ,
- trajectory slope at point  $(x_t, y_t)$  is given by cosine of the angle made by straight line joining  $(x_{t-r}, y_{t-r})$  and  $(x_{t+r}, y_{t+r})$ , with the horizontal axis.

### 3.5.3 Classifier

In this work, we have employed the HMM framework to build the online handwriting character and word recognition systems. The number of states in each HMM model is denoted  $N_{states}$  which are trained using a linear left to the right topology. The initial probabilities for each state are  $\pi = \pi_i$  and the transition matrix  $A = a_{ij}$  where  $1 \leq i \leq N_{state}$ ,  $1 \leq j \leq N_{state}$ . The observation probabilities  $B = b_i(o_t)$  for a sample with  $T$  observations  $\mathbf{o}_1, \mathbf{o}_2, \dots, \mathbf{o}_T$  are modeled using a Gaussian mixture model (GMM).

For character recognition system an HMM is trained for each class  $C$  denoted by  $\lambda_j$ ,  $j = [1, 2, \dots, C]$  using a left to right topology. The HMM parameters, i.e., the number of states and mixture components in each state, are optimized during training. For recognition, first, the log-likelihood score of each model  $\lambda_j$  is computed for a given test sample  $\mathbf{O}$ . The class label with the highest log-likelihood score is considered as a recognized class and is given by

$$\hat{c} = \arg \max_{1 \leq j \leq C} P(\mathbf{O}|\lambda_j) \quad (3.5.1)$$

For word recognition, the segmentation free HMM-based framework is considered in this work. In this framework, the character HMMs are trained using word databases that comprise of isolated word samples. These databases consist of handwriting transcription, but it does not contain the exact character boundaries. Therefore, the initial character models are trained using uniform character boundaries known as flat-start initialization. In this initialization method, each character in the transcription of a word sample is assigned an equal number of points. Then, over iteration, the segmentation boundaries are reestimated, and the models are retrained to obtain refined character boundaries. The iteration terminates when a convergence criterion is met, and the final character HMMs for word recognition are achieved.

During testing, given the sequence of feature vectors of a test word, the Viterbi algorithm is used to find the log-likelihood of each lexicon word. The word HMMs for each lexicon word are constructed by concatenating character HMMs corresponding to the letters present in the transcription of the word. Accordingly, the lexicon word with the highest log-likelihood is declared as the recognized word.

$$\hat{w} = \arg \max_{1 \leq j \leq W} P(\mathbf{O}|\hat{\lambda}_j) \quad (3.5.2)$$

where,  $P(\mathbf{O}|\hat{\lambda}_j)$  is computed for each word HMM  $\hat{\lambda}_j$  and  $\hat{w}$  is the recognized word,  $W$  is the total

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number of words in the dictionary. For evaluating the performance of the proposed feature set and its robustness towards the size of the lexicon, we conducted the experiments by varying the lexicon size. The words in the lexicon for Assamese word database and UNIPEN-ICROW-03 word database are selected from the Assamese OCR data and “google-book-common-word” list (Google, 2012) [109], respectively. The IRONOFF word database consists of both English and French words. Therefore, for the experiments on IRONOFF word database, the lexicon uses words from the French dictionary [110]. The most frequent words are chosen to form the required lexicon size.

## 3.6 Databases for present study

This section describes the various databases that are used for the experiments in this work. It consists of three databases: the UNIPEN database, the IRONOFF database, and the Assamese database. The details are given in the following.

### 3.6.1 English database

**UNIPEN database** This is a publicly available database that consists of both isolated character and word databases: UNIPEN character database and UNIPEN-ICROW-03 word database.

- **Character database:** The UNIPEN character database contains English digit, uppercase, and lowercase letters [111]. The database is cleaned by applying an aspect-ratio based cleaning method [112]. Each category of data is further split into disjoint training, validation, and testing sets. The total number of samples present in each category is given in Table 3.2.
- **Word database:** The UNIPEN-ICROW-03 database is an English word database which contains freestyles writing (hand-print, cursive and mixed styles) of 72 writers [113]. This database is divided into a disjoint training set (7689 words from 41 writers), a validation set (1334 words from 8 writers), and a testing set (3943 words from 23 writers). The transcription of the database contains a total of 44 characters, including all lowercase letters, some uppercase letters, and one special symbol.

**IRESTE On/Off (IRONOFF) database** The IRESTE On/Off (IRONOFF) is a dual handwriting database that contains both online and offline handwriting for the same writing samples [114]. It consists of both character and word level handwriting.

**Table 3.2:** Description of the classes and number of samples in the UNIPEN character and UNIPEN-ICROW-03 word databases.

UNIPEN character database					UNIPEN-ICROW-03 word database				
Category	Class	Train	Validation	Test	Category	Class	Train	Validation	Test
Digit	10	8061	1992	4887	Word	884	7689	1334	3943
Uppercase	26	13334	3330	8168					
Lowercase	26	24929	6122	15352					

**Table 3.3:** Description of the classes and number of samples in the IRONOFF database.

IRONOFF character database					IRONOFF word database				
Category	Class	Train	Validation	Test	Category	Class	Train	Validation	Test
Digit	10	2860	406	820	Cheque	30	8390	1200	1402
Lowercase	26	1070	2140	7475	English	26	1880	267	540
Uppercase	26	7479	1065	2135	French	171	20050	2874	5731
Euro sign	1	290	40	80					

- **Character database** The character database contains isolated digits, uppercase, lowercase, and the Euro sign with 4,086, 10,679, 10,685, and 410 samples. The samples are collected from more than 700 writers of French nationality. Each category is divided into training, testing, and validation sets as given in Table 3.3
- **Word database** The word database consists of 10,992 cheque words, 28,657 isolated French words, and 2689 isolated English words. The lexicon size for the cheque words, French words, and English words are 30, 197, and 26, respectively. The details are given in Table 3.3

### 3.6.2 Assamese database

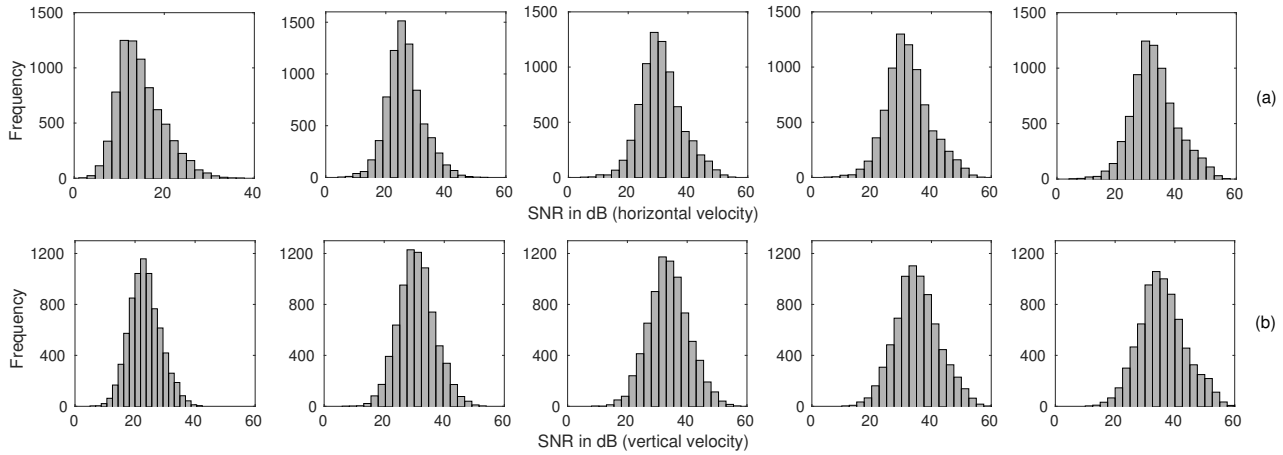
Assamese is the script of Assam, a northeastern state in India. It is written from left to right and consists of 11 vowels, 41 consonants, 10 digits, 10 vowel modifiers, and 2 consonant modifiers. Almost all consonants can combine with each other to form compound characters or conjuncts. An Assamese character may be written using a single stroke or multiple numbers of strokes. The database

**Table 3.4:** Description of the classes and number of samples in the Assamese character and word databases.

Assamese character database					Assamese word database				
Category	Class	Train	Validation	Test	Category	Class	Train	Validation	Test
Digit	10	1630	404	1066	Word	182	16208	2941	8414
Basic character	52	7238	1753	4366					
Conjuncts	104	16271	3803	9817					
Modifiers	18	2814	735	1811					

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**Figure 3.13:** The subplot (a) and (b) show the histogram of reconstruction SNR (in dB) of horizontal and vertical velocities, respectively for UNIPEN word database. The number of components is increased from  $K = 1$  to 5 (left to right)

is collected using a Lenovo Tablet PC- X230 and an open-source tool. The Tablet-PC captures the  $(x, y)$  coordinate of pen movement, pen-down/pen-up status, and the writer information.

- **Character database:** The character database consists of 10 digits, 52 basic characters comprising of vowels and consonants, 104 conjuncts, and 18 modifiers. The handwriting is collected from a total of 200 writers. Each writer was asked to write each character in separate boxes displayed on the tablet PC screen using the stylus. Two samples were collected from each writer in two different sessions. It is further divided into three disjoint sets, namely, training, testing, and validation sets. The number of samples present in each of these sets is given in Table 3.4.
- **Word database:** The word database contains samples for 182 unique words, collected from 163 writers. Two samples are collected for each word in two different sessions. Each participant is asked to write the words in the respective box displayed on the screen of the tablet PC. Similar to the character database, the samples are divided into disjoint training, validation, and testing sets in a writer independent way. The statistics of each of these sets is shown in Table 3.4.

## 3.7 Results and discussion

In the following, we present the experimental results of the two evaluation methods discussed in Section 3.4: reconstruction SNR and recognition performance on reconstructed data.

**Table 3.5:** Mean and standard deviation of  $\text{SNR}_{\text{dB}}$  for horizontal ( $\text{SNR}_{v_x, \text{dB}}$ ) and vertical velocity ( $\text{SNR}_{v_y, \text{dB}}$ ) for UNIPEN word database.

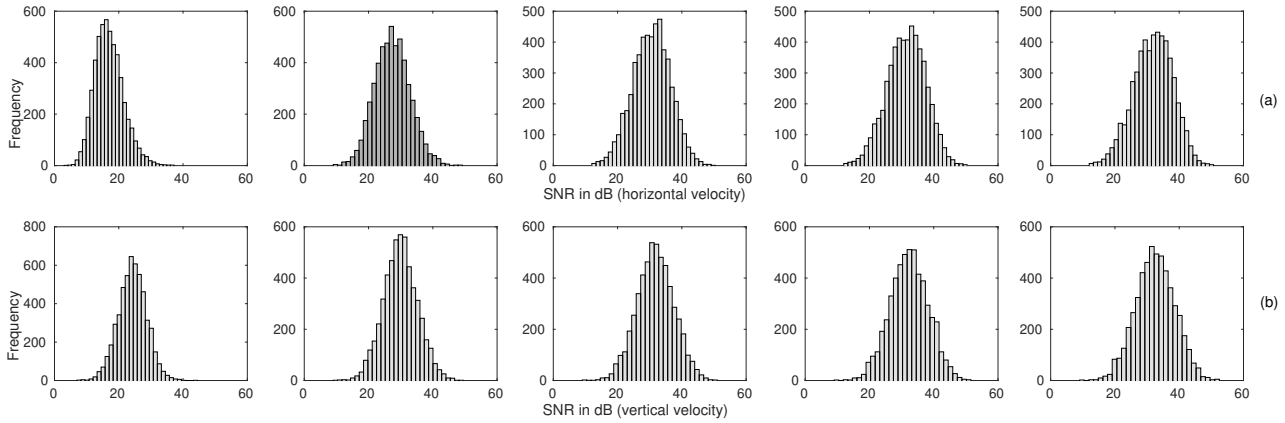
Sinusoidal Components	Mean $\text{SNR}_{v_x, \text{dB}}$	Mean $\text{SNR}_{v_y, \text{dB}}$	Standard Deviation, $\text{SNR}_{v_x, \text{dB}}$	Standard Deviation, $\text{SNR}_{v_y, \text{dB}}$
1	14.81	20.34	5.00	5.21
2	26.42	30.84	5.93	6.06
3	30.99	33.86	6.91	6.70
4	32.26	35.23	7.29	7.28
5	32.66	35.78	7.45	7.63

### 3.7.1 Reconstruction SNR results

Figure 3.13 shows the histogram of the reconstruction SNR for online handwriting samples of the UNIPEN word database with a varying number of components from one to five (left to right). The first and second row (a) and (b) in the figure corresponds to  $v_x$  and  $v_y$ , respectively. Additionally, the statistics of SNR of reconstruction is also described in Table 3.5. As expected, with the increase in the number of sinusoidal components, the  $\text{SNR}_{\text{dB}}$  of reconstruction increases. It can be observed that the improvement in the  $\text{SNR}_{\text{dB}}$  is not much significant above three components. Moreover, the greatest improvement is obtained when the number of components is increased from one to two in the representation. Therefore, we hypothesize that two to three sinusoidal components may be sufficient for obtaining an acceptable reconstruction quality using the proposed representation. It is also observed that the SNR for vertical velocity reconstruction is less as compared to horizontal velocity for all the cases. This shows that vertical velocity fits the sinusoidal nature better than the horizontal velocity. The reconstruction SNR for IRONOFF and Assamese word databases are shown in Figure 3.14 and Figure 3.15, respectively. The figure demonstrates the histograms of the reconstruction SNR. The statistics of SNR of reconstruction for the IRONOFF and Assamese word databases are shown in Table 3.6 and Table 3.7, respectively.

It is to be noted that the reconstruction quality of the velocities depends on the complexity of the written trajectory. For illustration, we show the original and reconstructed handwriting of a word in Figure 3.16 with low reconstruction SNR using a single sinusoidal component. It can be observed that the complex trajectory word in Figure 3.16(a) have a very low  $\text{SNR}_{\text{dB}}$  using only a single component. Here, the addition of one more component results in a better reconstruction, as shown in Figure 3.16(d). Thus, the multi-component sinusoidal model improves the handwriting

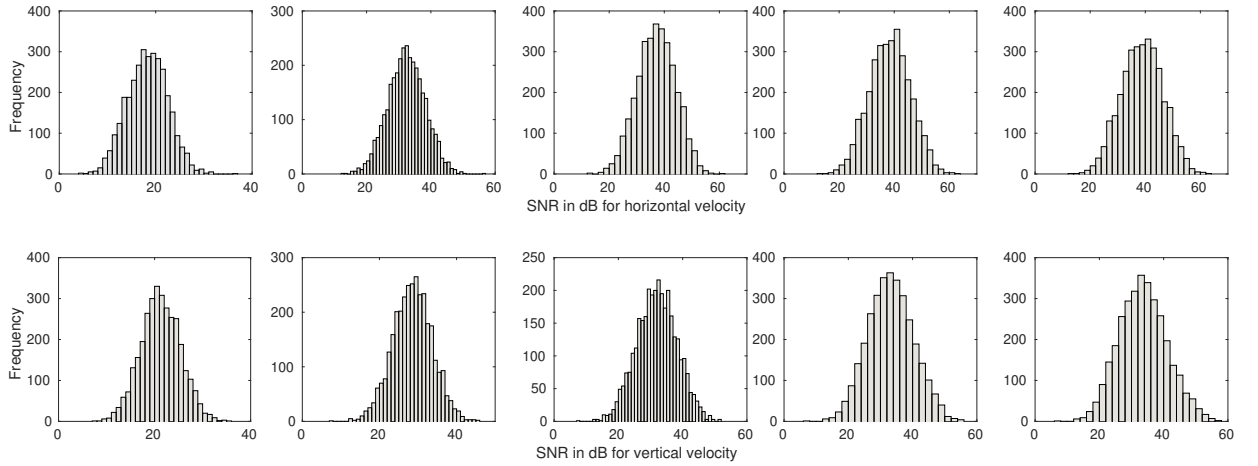
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**Figure 3.14:** The subplot (a) and (b) show the histogram plot of reconstruction SNR (in dB) with increasing number of components from  $K=1$  to 5 (left to right) for horizontal and vertical velocity, respectively of IRONOFF word database.

**Table 3.6:** Mean and standard deviation of  $\text{SNR}_{\text{dB}}$  of reconstruction for horizontal velocity ( $\text{SNR}_{v_x, \text{dB}}$ ) and vertical velocity ( $\text{SNR}_{v_y, \text{dB}}$ ) for IRONOFF word database.

Sinusoidal Components	Mean $\text{SNR}_{v_x, \text{dB}}$	Mean $\text{SNR}_{v_y, \text{dB}}$	Standard Deviation, $\text{SNR}_{v_x, \text{dB}}$	Standard Deviation, $\text{SNR}_{v_y, \text{dB}}$
1	17.00	24.23	4.45	4.39
2	27.40	29.93	5.48	5.28
3	30.55	31.77	5.54	5.69
4	31.46	32.34	5.98	5.02
5	31.55	32.85	6.07	6.03



**Figure 3.15:** The subplot (a) and (b) show the histogram of reconstruction SNR (in dB) with increasing number of components from  $K = 1$  to 5 (left to right) for horizontal and vertical velocities, respectively for Assamese word database.

**Table 3.7:** Mean and standard deviation of  $\text{SNR}_{\text{dB}}$  for reconstruction of horizontal velocity ( $\text{SNR}_{v_x, \text{dB}}$ ) and vertical velocity ( $\text{SNR}_{v_y, \text{dB}}$ ) for Assamese word database.

Sinusoidal Components	Mean $\text{SNR}_{v_x, \text{dB}}$	Mean $\text{SNR}_{v_y, \text{dB}}$	Standard Deviation, $\text{SNR}_{v_x, \text{dB}}$	Standard Deviation, $\text{SNR}_{v_y, \text{dB}}$
1	18.65	20.62	4.58	4.07
2	32.75	28.36	6.22	5.06
3	37.72	32.32	7.26	6.24
4	38.07	33.20	7.63	7.11
5	38.55	33.59	7.75	7.47

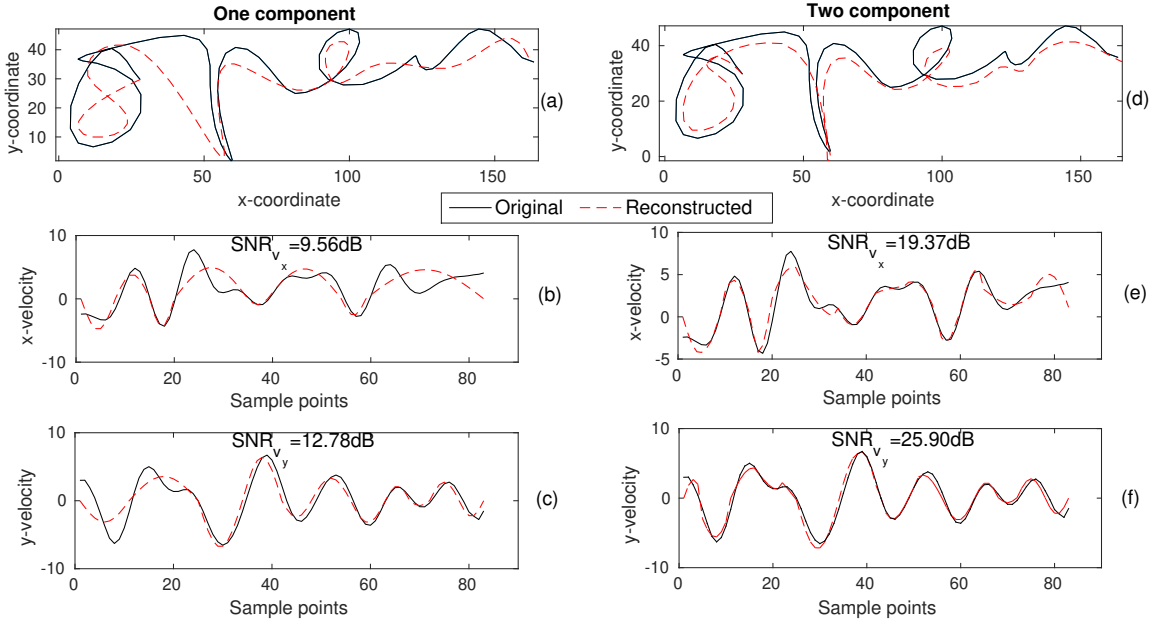
representation for complex handwriting trajectory.

We also demonstrate the reconstruction SNR for UNIPEN and Assamese character databases in Figure 3.17. Similar to words, it can be observed from the increase in SNR values that multiple oscillations can better model the character velocities as compared to one component.

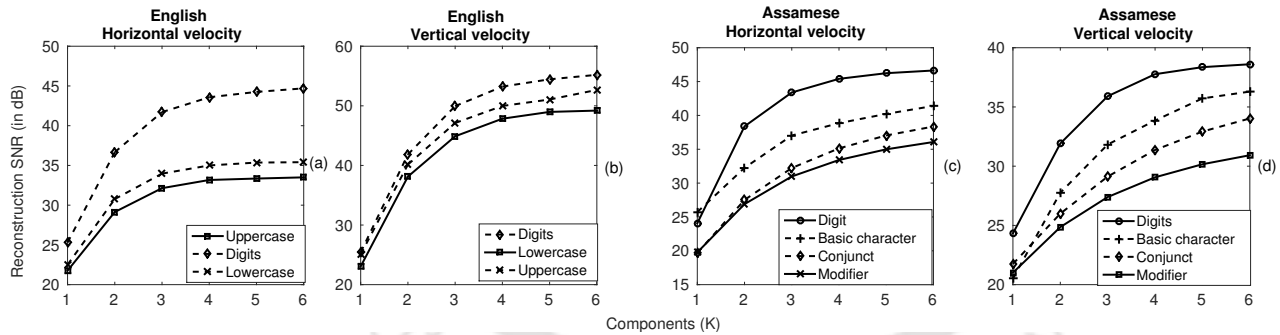
### 3.7.2 Recognition performance on reconstructed samples

Table 3.8 depicts the recognition performance for English character databases using HMM. The number of sinusoidal components ( $K$ ) is varied from 1 to 4 for the reconstruction of the test data. The number of states and mixtures for the HMM system are optimized to 11 and 20, respectively. It is observed that the recognition performance is less when the number of components  $K = 1$ , which corroborates with our hypothesis that more number of components are required for representing handwriting data. It is observed that the recognition performance improves significantly when two

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**Figure 3.16:** The subplots (a)–(c) show the word sample from the UNIPEN-ICROW-03 word database and its velocities with low reconstruction SNR value using the single component. The subplot (d)–(f) show the same in (a)–(c) using two components.



**Figure 3.17:** Plot of mean reconstruction SNR (in dB) with increasing number of components from  $K = 1$  to 6 for various character databases. (a) and (b) illustrates the SNR for horizontal and vertical velocities, respectively for UNIPEN character databases. Similarly, (c) and (d) show the SNR for horizontal and vertical velocities, respectively for Assamese character database.

components are used to model the handwriting and remains almost the same with the addition of higher components. Similar results are obtained for Assamese databases, as shown in Table 3.9. The optimum number of states for the Assamese databases is 15, and the number of mixtures is 20.

To find the statistical significance between the recognition error rate obtained with various  $K$ , we compute the 'two-sample paired t-test' as shown in Table 3.10. The null hypothesis of the t-test considers that the two groups of results (e.g., 1-component vs. 2-components) are equal. The statistical significance level is indicated by the ' $p$ ' value associated with the t-test where a larger

**Table 3.8:** Error rate (%) of character recognition system with different number of components ( $K$ ) for UNIPEN and IRONOFF character database (test set).

Number of components ( $K$ )	UNIPEN database			IRONOFF database		
	Digit	Uppercase	Lowercase	Digit	Uppercase	Lowercase
1	1.90	4.70	9.16	3.29	8.00	12.62
2	1.43	3.80	6.96	1.95	6.13	8.68
3	1.42	3.75	6.74	2.07	6.18	8.45
4	1.42	3.75	6.71	2.07	6.22	8.45

**Table 3.9:** Error rate (%) of character recognition system with different number of components ( $K$ ) for Assamese database (test set).

Number of components ( $K$ )	Assamese character database			
	Digit	Basic character	Conjuncts	Modifiers
1	2.63	6.21	6.42	5.24
2	1.31	4.16	4.29	3.09
3	1.21	4.08	4.22	2.98
4	1.12	3.94	4.13	2.87

**Table 3.10:** Statistical significance test (two-sample paired t-test) for recognition performance obtained by reconstruction with various number of components ( $K$ ).

	UNIPEN			IRONOFF			Assamese			
	Digit	Upper case	Lower case	Digit	Upper case	Lower case	Digit	Basic Char	Conjunct	Modifier
$K = 1$ vs $K = 2$	0.049	0.029	0.017	0.039	0.017	0.008	0.006	0.038	0.041	0.010
$K = 1$ vs $K = 3$	0.057	0.028	0.009	0.040	0.018	0.007	0.004	0.033	0.034	0.008
$K = 1$ vs $K = 4$	0.057	0.028	0.008	0.040	0.030	0.007	0.003	0.022	0.027	0.004

‘ $p$ ’ value indicates the two groups are not statistically significant and vice-versa. When the value of  $p < 0.1$ , it tells that the t-test rejects the null hypothesis and suggests that the two groups of results are different and statistically significant with a 10% significance level. Following the same, we tabulated the  $p$ -value of the  $t$ -test carried out between the recognition error rates with various components  $K$ . It can be observed that the results obtained with  $K = 2$  is statistically significant over  $K = 1$ .

We also perform the recognition experiments on reconstructed word handwriting from English and Assamese databases. The number of sinusoidal components is varied from  $K = 1$  to  $K = 4$ . We conducted the experiments by varying the lexicon size 5000, 10000, and 20000. The words in the lexicon for Assamese and English word databases are selected from the Assamese OCR data and “google-book-common-word” list (Google, 2012) [109], respectively. Like character recognition, the number of mixtures for both English and Assamese handwriting is optimized to 20. On the other hand, the

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**Table 3.11:** Error rate (%) of the HMM based word recognition system evaluated on the reconstructed data of different databases (test sets).

Database	Lexicon	$K = 1$	$K = 2$	$K = 3$	$K = 4$
UNIPEN	5000	30.20	27.46	26.93	26.70
	10000	34.23	30.94	30.28	29.98
	20000	38.04	34.13	33.35	33.04
IRONOFF	5000	26.1	23.72	23.42	23.35
	10000	30.04	29.45	29.05	28.82
	20000	39.88	35.91	35.46	35.27
Assamese	5000	26.80	24.08	23.52	23.40
	10000	30.74	27.75	26.96	26.80
	20000	33.16	30.03	29.19	28.94

number of states is optimized to 11 and 15 for the English and Assamese word recognition, respectively. Similar results are obtained for word-level handwriting. Table 3.11 depicts the recognition performance of an HMM system for the reconstructed handwriting of UNIPEN and Assamese word databases. The recognition error obtained for the UNIPEN word database on handwriting reconstructed using one component is 30.20%, 34.23%, and 38.04% for 5000, 10000, and 20000 lexicon size, respectively. This improved to 27.46%, 30.94%, and 34.13% for reconstructed handwriting using two components. The recognition performance does not change much between the reconstructed handwriting with three and four components. Similar results are also obtained for the IRONOFF and Assamese word databases. The recognition results show that important information of the handwriting is lost when only one sinusoidal component is used to model the handwriting (POMH model).

### 3.8 Summary

We proposed a multi-component sinusoidal model for online handwriting based on the oscillatory theory of handwriting in this work. As demonstrated in this theory, it is observed that the handwriting velocities in the horizontal and vertical directions are oscillatory in nature with an invariant bell-shape. Therefore, we modeled the handwriting velocities using sinusoidal oscillations, whose parameters are modulated after every short interval of time. The sinusoidal parameters are computed at every zero-crossing points of the velocity profiles. One important observation is that modeling each velocity signal using a single sinusoidal oscillation (POMH model) is not sufficient to represent complex handwriting trajectories in pattern recognition systems. Therefore, an improved sinusoidal model using multiple oscillations is proposed for online handwriting. As this model uses multiple sinusoidal oscillations

to model each velocity profile, it is termed as the multi-component sinusoidal model. A parameter extraction method is proposed to extract the model parameters iteratively using the residual signals. We evaluate the proposed model using the reconstruction SNR and recognition system on both English and Assamese databases. The experimental results showed that adequate reconstruction quality is obtained using two to three sinusoidal components.

As the proposed multi-component model can describe online handwriting efficiently, its parameters can also be used for handwriting representation in a pattern recognition system. Therefore, in the following chapter, we explore the proposed model for feature extraction in an online handwriting recognition system.



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# 4

## Sinusoidal model based features

### Publications

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1. H. Choudhury, and S.R.M. Prasanna. "Handwriting recognition using sinusoidal model parameters," *Pattern Recognition Letter*, vol. 121, pp. 87-96, 2019
- 

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### Overview

In the previous chapter, we demonstrated that the multi-component sinusoidal model could represent online handwriting efficiently. The model parameters can reconstruct the handwriting with complex trajectories, as shown for English and Assamese scripts. These parameters can be considered as the control variable of the handwriting trajectory shape and explored as features for online handwriting tasks. In this chapter, we propose a feature representation using the multi-component sinusoidal model for online handwriting recognition. The predominant information about handwriting is modeled in the velocity profiles itself, which can completely reconstruct the handwriting trajectory. However, the acceleration and the  $(x, y)$  coordinates also contain important information that can improve the discriminability between different handwriting shapes. Therefore, the acceleration and normalized  $(x, y)$  position are also modeled by the proposed method to improve the recognition performance. The efficacy of the proposed features is shown for online handwritten character and word recognition tasks. Moreover, the proposed feature representation is combined with the existing point-based features to develop the handwriting recognition system. The experiments are conducted on English and Assamese databases, and promising results are obtained.

### 4.1 Introduction

Online handwriting recognition is one of the most active areas of research in document analysis. As handwriting is a natural and convenient mode of communication, therefore efforts have been made by different researchers to replace the keyboard interface with natural handwriting. This flexibility to input data using handwriting is provided by an online handwriting recognition system. Handwriting comprises various trajectory shapes forming the letters depending on their lexicon and writing script. A recognition system learns these shapes based on different features extracted from the handwriting, which discriminates the letter classes. Now, although the letter shapes are fixed for a particular script, in the process of learning, different writers develop a different writing style, and thus high variability of shape, slant, and speed can be observed in the handwriting data. Therefore, it has been a longstanding challenge in the development of a handwriting recognition system to extract adequate features that can maximize the discriminability between various handwriting shapes.

As discussed in Section 2.3 of Chapter 2, various features have been extracted from the online handwriting 2D trace for describing the different letter shapes. Some features are extracted at each

coordinate point of the trajectory to capture the local variations of the shape, which includes vertical position, writing direction, curvature, pen-up/pen-down, aspect ratio, curliness, linearity, slope, and ascenders/descenders [23]. On the other hand, some features are computed considering from the complete pattern to capture global handwriting attributes [1]. Combination of different features with complementary information extracted from alternative perspectives)are also used to represent the writer's handwriting or the message that is written [1, 76]. Among the different features, the most popular feature representation for online handwriting is the point-based features computed locally within a small vicinity of each coordinate point. In contrast, the handmovement theory based model tries to capture movement-based information of the handwriting trajectory. Moreover, as the basic set of hand movements for generating a handwritten pattern mostly remain identical, compared to the natural variabilities in the handwritten 2D trace, therefore, features extracted from a generation model may well represent the intra-class variability.

In this work, we propose a set of features derived from the multi-component sinusoidal model. The sinusoidal parameters obtained by modeling the handwriting velocities help to extract attributes from the generation perspective, unlike the conventional shape-based features computed at each point of the 2D trace. This information may provide interesting aspects of the handwriting for the field of pattern recognition. Therefore, we explore the proposed sinusoidal parameters based features for online handwriting recognition. Further, we also model the acceleration and  $(x, y)$  coordinates of handwriting, along with velocity profiles. This helps to combine both dynamic and shape characteristics of the handwriting in the proposed feature set.

Various experiments are performed to find the optimal set of parameters and the number of components for the feature representation in the recognition system. We demonstrate the effect of different parameters and components on the recognition performance. The sinusoidal parameter based feature set is used for hidden Markov model (HMM) and support vector machine (SVM) based character recognition. We develop a large vocabulary HMM-based word recognition system using the proposed feature set. Moreover, the proposed sinusoidal features and the shape-based features are combined as both are extracted from two different representation schemes. The results are shown for both Assamese and English scripts.

In the perspective of the aforementioned discussion, the contributions of this study include:

- Proposal of a sinusoidal model-based feature set for online handwriting recognition.

## 4. Sinusoidal model based features

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- Extraction of the sinusoidal parameter from acceleration and normalized  $(x, y)$  coordinates of handwriting, along with the conventional horizontal and vertical velocities.
- An extensive validation is performed for character and word recognition tasks to demonstrate the usefulness of the proposed features.

The chapter is organized in the following order. Section 4.2 details the sinusoidal features for handwriting recognition extracted from velocity, acceleration, and position information of handwriting. In Section 4.3, we discuss various experiments performed for finding the optimal sinusoidal model-based feature set. We also present the results obtained for the character and word recognition tasks using the proposed feature. Finally, the concluding remarks are drawn in Section 4.4.

### 4.2 Proposed sinusoidal model based features

The multi-component sinusoidal model describes the horizontal and vertical velocities  $v_x$  and  $v_y$ , respectively using sinusoidal oscillations as given below:

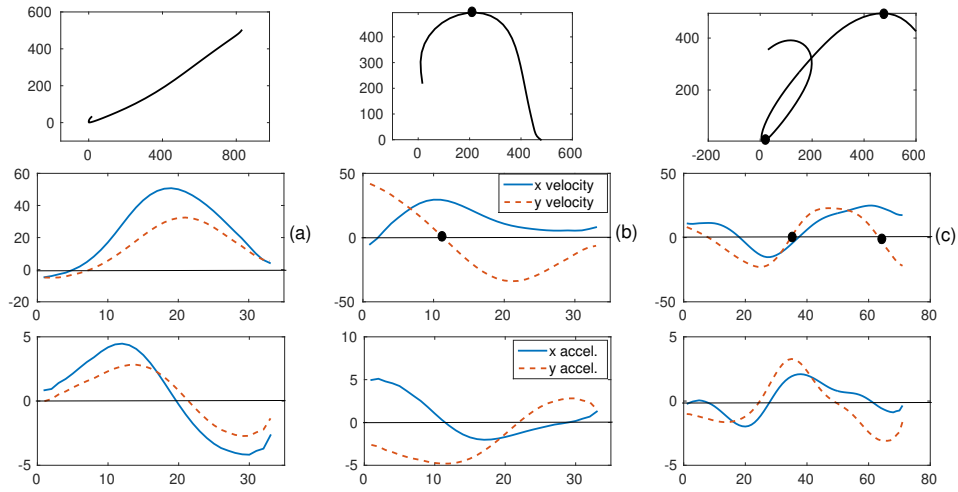
$$v_x(t) = \sum_{m=1}^K a_{v_x}^m(t) \sin(\omega_{v_x}^m(t).t + \phi_{v_x}^m(t)) \quad (4.2.1)$$

$$v_y(t) = \sum_{m=1}^K a_{v_y}^m(t) \sin(\omega_{v_y}^m(t).t + \phi_{v_y}^m(t)) \quad (4.2.2)$$

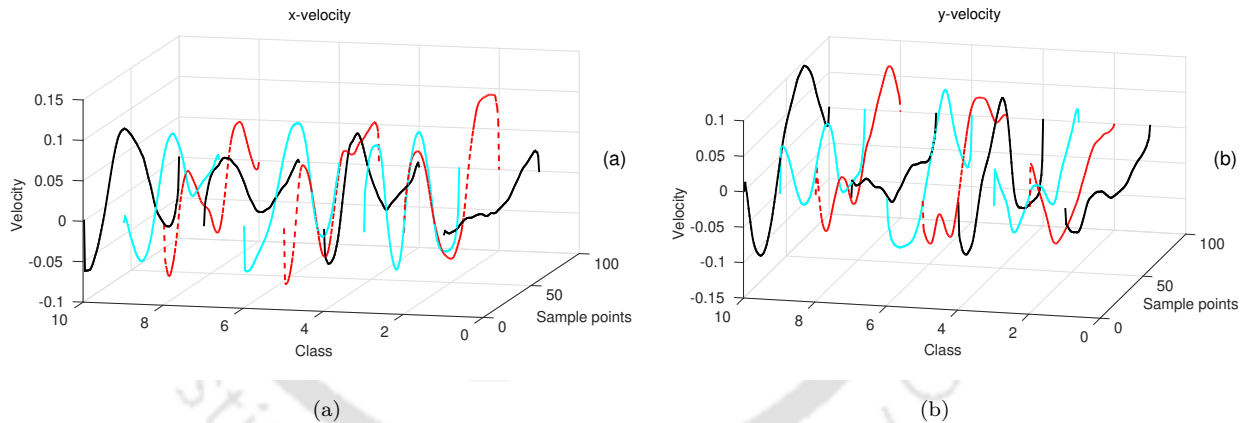
where  $a^m(t)$ ,  $\omega^m(t)$  and  $\phi^m(t)$  are the amplitude, frequency and phase of the  $m^{th}$  component sinusoid. Although these equations are used to model the velocity signals, it can also be used to model any oscillatory signal, such as the acceleration and position signals. In the following, we discuss in detail the sinusoidal parameters obtained from the different signals that are used for the proposed feature representation.

#### 4.2.1 Sinusoidal parameters extracted from velocity profiles

It can be recalled from Section 3.2 of the previous chapter that the relation between the horizontal and vertical velocity profiles characterize the trajectory shape of the handwritten pattern. For illustration, we plot three handwritten strokes along with the velocities and acceleration signals in Figure 4.1. A linear trajectory is generated if the two velocity profiles change proportionally, as shown in Figure 4.1(a). In a curved trajectory (where the curve is appearing at top or bottom), the  $x$ -velocity amplitude at the  $y$ -velocity zero-crossing point determines the curvature shape. If the  $x$ -velocity is positive, the trajectory has a rounded shape, and when it decreases to a negative value,



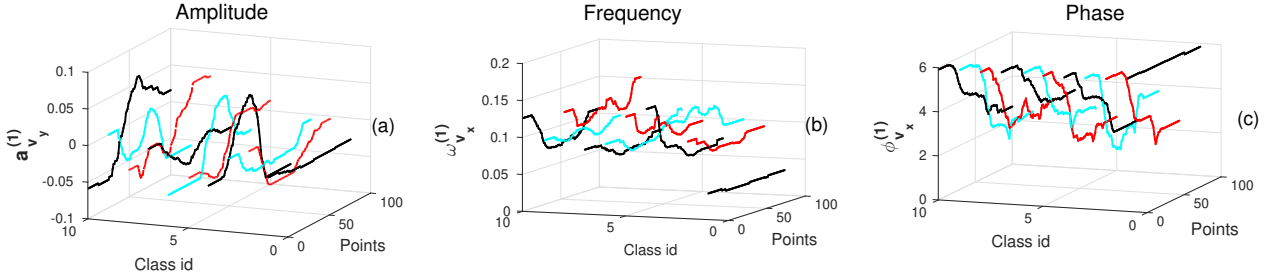
**Figure 4.1:** Three trajectories with different corner shapes are shown in the first row of subplots (a)–(c). The corresponding velocities and accelerations are shown in second and third row, respectively. The vertical velocity zero crossing are shown using the solid dots.



**Figure 4.2:** The subplots (a) and (b) show the  $x$ - and  $y$ -velocity profiles for the UNIPEN digit database averaged across the samples of individual classes.

the trajectory shape changes from a rounded shape to a loop. Figure 4.1(b) and (c) show trajectories with different corner shapes along with their velocity and acceleration profiles. To visualize the discriminating ability of velocity profiles for classifying different classes, we plot the  $x$ - and  $y$ -velocity profiles of English digits in Figure 4.2 by averaging across the training samples. It can be observed that the velocity profiles of each class are significantly different, having a sinusoidal nature of various amplitudes and frequencies. Now, given a handwritten sample with coordinates sequence

#### 4. Sinusoidal model based features



**Figure 4.3:** The subplot (a), (b) and (c) depict the amplitude, frequency and phase parameters of first component ( $a_{v_y}^1$ ,  $\omega_{v_y}^1$  and  $\phi_{v_y}^1$ ) of  $y$ -velocity profiles averaged across training samples of the ten digit classes. Colors are used for better clarity of the plots.

$P = \{(x(1), y(1)), (x(2), y(2)), \dots, (x(T), y(T))\}$ , the  $x$ - and  $y$ -velocity profiles are computed as:

$$v_x(t) = \frac{x(t+1) - x(t-1)}{2}, \quad v_y(t) = \frac{y(t+1) - y(t-1)}{2},$$

where  $2 < t < T - 1$  and  $T$  is the total points in the trajectory.

The sinusoidal parameters, i.e., amplitude, frequency and phase (denoted by  $a_{v_x}$ ,  $a_{v_y}$ ,  $\omega_{v_x}$ ,  $\omega_{v_y}$ ,  $\phi_{v_x}$ , and  $\phi_{v_y}$ ) of these velocity profiles are computed, using the method described in Section 3.3.1 of Chapter 3. As the velocity profiles are modeled using of  $K$  components, therefore,  $K$  sets of sinusoidal parameters are obtained at each point. A character sample having  $T$ -points is represented by the sinusoidal feature vector (obtained from the velocity profiles) at  $t^{th}$  point as:

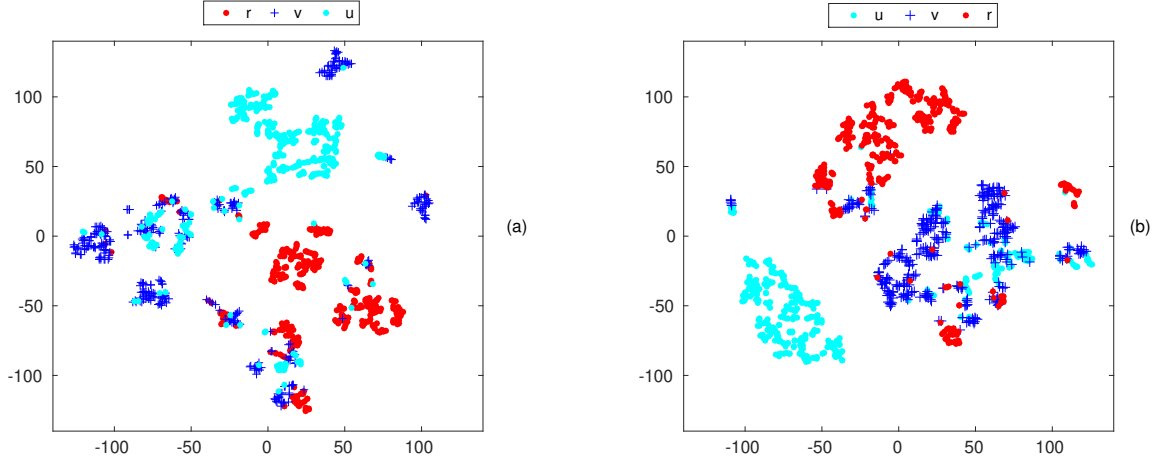
$$\begin{aligned} \hat{v}_x(t) &= [a_{v_x}^1(t) \ \omega_{v_x}^1(t) \ \phi_{v_x}^1(t) \ \dots \ a_{v_x}^K(t) \ \omega_{v_x}^K(t) \ \phi_{v_x}^K(t)] \\ &= [a_{v_x}^{(1-K)}(t), \ \omega_{v_x}^{(1-K)}(t), \ \phi_{v_x}^{(1-K)}(t)] \end{aligned} \quad (4.2.3)$$

where  $1 \leq t \leq T$  and  $a_{v_x}^{(1-K)}(t)$ ,  $\omega_{v_x}^{(1-K)}(t)$  and  $\phi_{v_x}^{(1-K)}(t)$  are the amplitudes, frequencies and phases of  $K$  sinusoidal components of  $x$  velocity ( $v_x$ ).

$$\begin{aligned} \hat{v}_y(t) &= [a_{v_y}^1(t) \ \omega_{v_y}^1(t) \ \phi_{v_y}^1(t) \ \dots \ a_{v_y}^K(t) \ \omega_{v_y}^K(t) \ \phi_{v_y}^K(t)] \\ &= [a_{v_y}^{(1-K)}(t), \ \omega_{v_y}^{(1-K)}(t), \ \phi_{v_y}^{(1-K)}(t)] \end{aligned} \quad (4.2.4)$$

where  $1 \leq t \leq T$  and  $a_{v_y}^{(1-K)}(t)$ ,  $\omega_{v_y}^{(1-K)}(t)$  and  $\phi_{v_y}^{(1-K)}(t)$  are the amplitudes, frequencies and phases of  $K$  sinusoidal components of  $y$  velocity ( $v_y$ ).

It is to be noted that the shape of the handwritten pattern depends on each of these parameters. For instance, the amplitude of  $y$ -velocity  $a_{v_y}$  rises as the height of the trajectory increases



**Figure 4.4:** The visualization of the feature distribution of three confusing letters ( $r$ ,  $u$ ,  $v$ ) using t-SNE algorithm. The features of the three classes are marked with red, green, and blue, respectively. The subplot (a) depicts the distribution for the four dimensional feature vector  $(a_{v_x}^{(1)}, \omega_{v_x}^{(1)}, a_{v_y}^{(1)}, \omega_{v_y}^{(1)})$  using only one component and (b) depicts the distribution for the eight dimensional feature vector  $(a_{v_x}^{(1-2)}, \omega_{v_x}^{(1-2)}, a_{v_y}^{(1-2)}, \omega_{v_y}^{(1-2)})$  using two components.

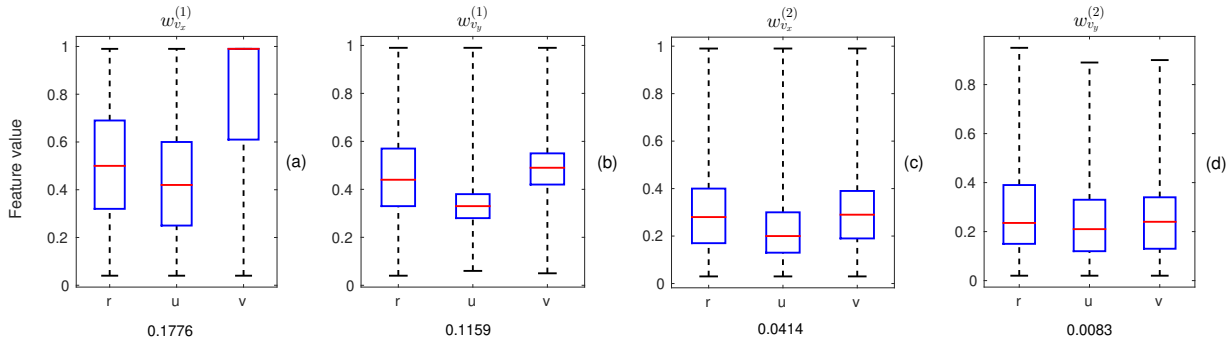
and vice versa. Similarly,  $a_{v_x}$  also increases proportionally with letter width. Now, as proposed in the literature [47], the trajectory distance traversed is a combined effect of both amplitude and frequency parameters of the velocities. Therefore, for recognition, these parameters may contain the most significant information about the trajectory shape.

Figure 4.3 depicts the averaged  $a_{v_y}^1$ ,  $\phi_{v_y}^1$  and  $\omega_{v_y}^1$  obtained from UNIPEN English digit classes. The parameters are averaged across all the training samples. It can be observed from the figure that the amplitudes and frequencies may contain the most discriminating information in classifying the different digit classes as compared to the phase parameter. This is further corroborated by the experimental results of Section 4.3 of this chapter.

#### 4.2.2 Visualization of intra-class and inter-class variability of sinusoidal parameters

The usefulness of the sinusoidal parameters extracted from the velocity profiles for handwriting recognition is demonstrated by visualization of intra-class and inter-class variability induced by the parameters. Three confusing lowercase English letters are chosen ( $r, u, v$ ) for this visualization. Figure 4.4 shows the feature distribution of the three classes using the t-SNE algorithm [115] where the feature space is mapped to a two-dimensional plane. The features of the three classes  $r$ ,  $u$ , and  $v$  are

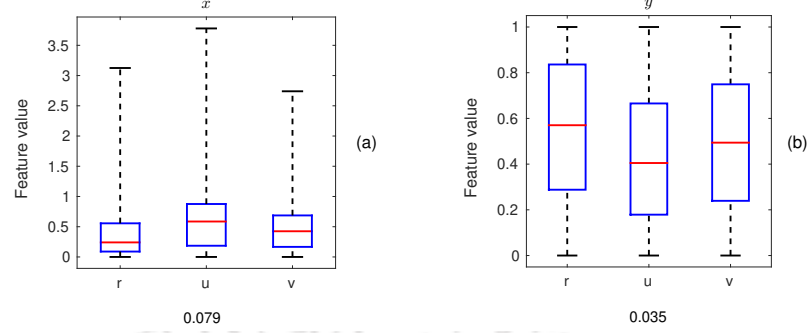
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**Figure 4.5:** The feature distribution for the three confusing letters ( $r$ ,  $u$ ,  $v$ ) employing the proposed sinusoidal model. The subplots (a)-(d) depict distribution for the frequency parameter  $\omega_{v_x}^{(1)}$ ,  $\omega_{v_y}^{(1)}$ ,  $\omega_{v_x}^{(2)}$ , and  $\omega_{v_y}^{(2)}$ , respectively. The divergence value, which measures the discriminating ability of the feature is provided below every panel.

marked with red, green, and blue, respectively. We visualize the feature distribution using amplitude and frequency parameters of (a) one component and (b) two components. It can be observed that the parameters can separate the three classes in the feature space. Moreover, the separation between the three classes is higher for the feature space using two components as compared to one component. Therefore, this shows that the proposed features can be useful for discriminating the classes in the recognition system.

We also visualize the distribution of each feature individually extracted from the first and second components. The sub-plots (a)-(d) in Figure 4.5 depict the distribution for  $\omega_{v_x}^{(1)}$ ,  $\omega_{v_y}^{(1)}$ ,  $\omega_{v_x}^{(2)}$ , and  $\omega_{v_y}^{(2)}$ , respectively. The frequency parameter is chosen here as it is found to be the most discriminating feature for the three classes. The divergence value, which measures the discriminating ability of the feature, is also given below each plot [7]. It can be observed that the divergence score of the first-component frequency parameter  $\omega_{v_x}^{(1)}$  is highest for the three confusing letters. It is to be noted that the divergence values of the second component parameters are less as compared to the first component. Therefore, we compare the divergence of these parameters with two point-based features. The two point-based features that are chosen for the illustration are the first derivative of  $x$ - and  $y$ - coordinates of the online handwriting. The subplots (a) and (b) in Figure 4.6 depict the box-plots for the first derivative of  $x$ - coordinate and  $y$ - coordinate, respectively. It can be observed that  $\omega_{v_x}^{(2)}$  and  $\omega_{v_y}^{(2)}$  have comparable divergence score with the two point-based features.



**Figure 4.6:** The feature distribution for the three confusing letters ( $r$ ,  $u$ ,  $v$ ) employing first derivative of  $x$ - and  $y$ - coordinate are shown in (a) and (b), respectively along with divergence value at the below each panel.

### 4.2.3 Sinusoidal parameters extracted from acceleration and position

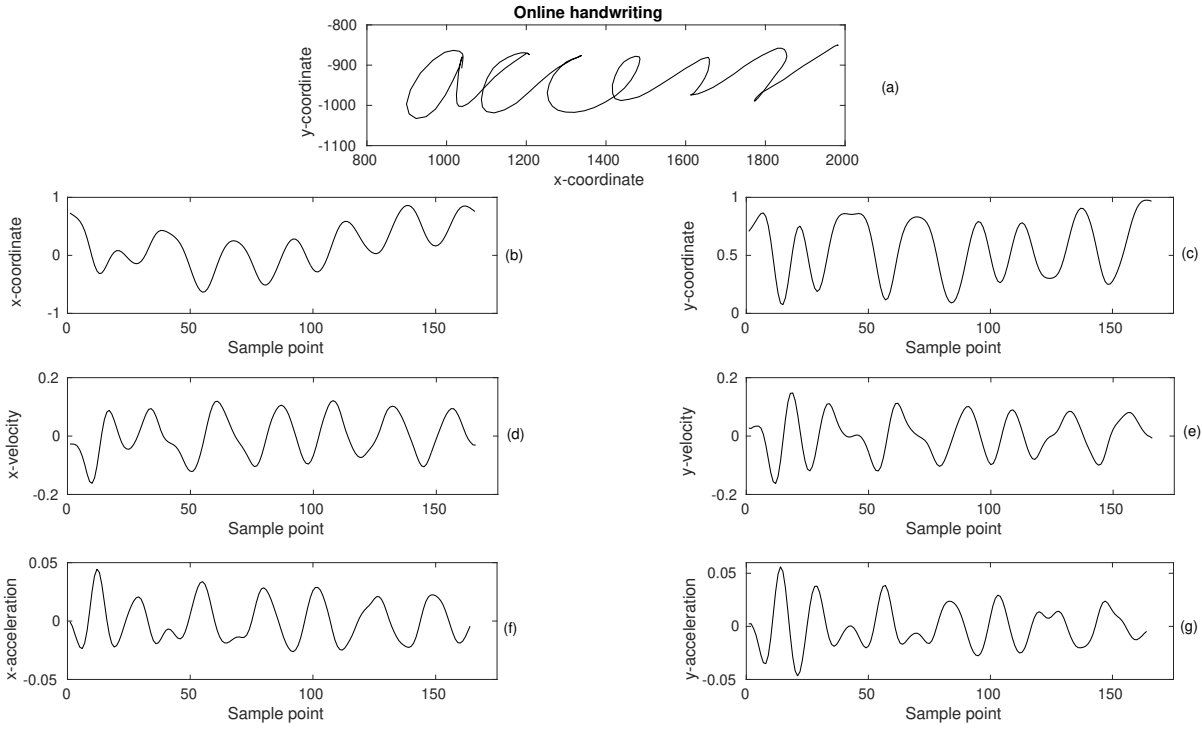
The parameters extracted from the velocity are sufficient for reconstructing the handwriting trajectory. However, we also model the acceleration of handwriting using the multi-component sinusoidal model to extract additional attributes. These attributes, in terms of the model parameters, may help to discriminate the letter shapes. The spatial position of a pattern contains the output trajectory shape information. The sinusoidal parameters are also extracted from horizontal and vertical positions (i.e.,  $x$ -coordinates and  $y$ -coordinates) of the handwriting. Finally, a set of parameters obtained from the velocity, acceleration, and position signals are combined and utilized as features for the handwriting recognition task.

**Acceleration based sinusoidal features** The acceleration describes the change in the handwriting velocities with time, which plays an important role in the writing shape. Similar to velocity profiles, it is also oscillatory, therefore, we can extract alternative information by modeling the acceleration signals. The horizontal and vertical acceleration signals (denoted by  $r_x$  and  $r_y$ ) are obtained as follows:

$$r_x(t) = \frac{v_x(t+1) - v_x(t-1)}{2}, \quad r_y(t) = \frac{v_y(t+1) - v_y(t-1)}{2},$$

where,  $2 < i < T - 1$ ,  $v_x$  and  $v_y$  are respective velocity profiles and  $T$  is total points. The acceleration signals are also modeled using the sinusoidal oscillations with  $K$  components. Now, similar to (4.2.3)–

#### 4. Sinusoidal model based features



**Figure 4.7:** The subplot (a) represents  $(x, y)$  coordinate of a word sample from UNIPEN-ICROW-03 database. The subplots (b) and (c) represent the normalized horizontal and vertical position ( $x$ - and  $y$ -coordinates) of the word. The subplots (d) and (e) represent the horizontal and vertical velocities, (f) and (g) represent the the horizontal and vertical acceleration signals.

(4.2.4), the sinusoidal feature vector for acceleration signals at  $t^{th}$  point can be written as:

$$\hat{r}_x(t) = [a_{r_x}^{(1-K)}(t), \omega_{r_x}^{(1-K)}(t), \phi_{r_x}^{(1-K)}(t)] \quad (4.2.5)$$

$$\hat{r}_y(t) = [a_{r_y}^{(1-K)}(t), \omega_{r_y}^{(1-K)}(t), \phi_{r_y}^{(1-K)}(t)] \quad (4.2.6)$$

**Position based sinusoidal features** The velocity and acceleration are dynamic information related to the hand movement process for generating handwriting. The output of the hand movements is the handwriting coordinate positions. The vertical position ( $y$ -coordinates) of the trajectory oscillates along the letter heights, while the horizontal position ( $x$ -coordinates) oscillates along the letter width with a constant drift as letters are spaced with a horizontal distance between them. Therefore, unlike the  $y$ -coordinates, the  $x$ -coordinates do not have oscillatory nature and hence difficult to model the  $x$ -coordinate using sinusoids. Therefore, a normalized  $x$ -coordinate is derived by computing a zero mean velocity signal. This results in an oscillatory  $x$ -coordinates signal. Figure 4.7 shows the  $x$ -coordinate,  $y$ -coordinate, along with the velocity and acceleration signals for an online handwritten word. Next,

the  $y$ -coordinates are normalized between -1 to 1. Both the derived  $x$ -coordinate and  $y$ -coordinate signals are modeled by sinusoidal model using (4.2.1) and (4.2.2). Now, similar to (4.2.3)–(4.2.4), we derive the sinusoidal feature vector for position signal at  $t^{th}$  point as:

$$\hat{s}_x(t) = [a_{s_x}^{(1-K)}(t), \omega_{s_x}^{(1-K)}(t), \phi_{s_x}^{(1-K)}(t)] \quad (4.2.7)$$

$$\hat{s}_y(t) = [a_{s_y}^{(1-K)}(t), \omega_{s_y}^{(1-K)}(t), \phi_{s_y}^{(1-K)}(t)] \quad (4.2.8)$$

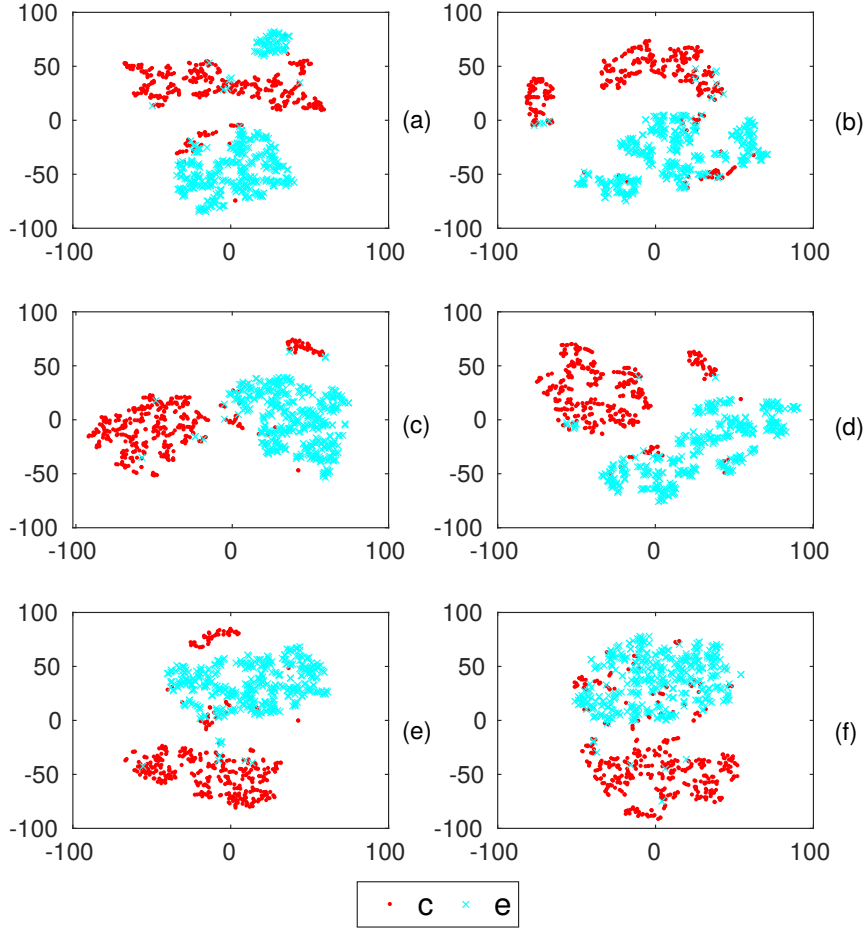
Finally, the parameters obtained from velocity, acceleration, and position are combined as

$$F(t) = [\hat{v}_x(t) \ \hat{v}_y(t) \ \hat{r}_x(t) \ \hat{r}_y(t) \ \hat{s}_x(t) \ \hat{s}_y(t)]^T \quad (4.2.9)$$

where  $1 \leq t \leq T$  and dimension of  $F(t) = 6 \times 3 \times K$ . The value six corresponds to the two velocities, the two accelerations, and the two position signals in  $x$  and  $y$  directions; the value three represents amplitude, frequency, and phase, and lastly, the term  $K$  indicates the number of sinusoidal components used for representation. The feature vector given in (4.2.9) is the proposed sinusoidal feature vector and employed in this work to develop an online handwriting recognition system. As the sinusoidal components  $K$  increases, the parameters obtained from higher components capture small variations in data, which may be considered as noise and thus can increase intra-class variability. Therefore, only the first few components contain the most important characteristic of the pattern.

Next, we present the feature distribution of normalized  $(x, y)$  coordinates, velocity, and acceleration signals for classifying a confusing English character pair ‘ $c$ ’-‘ $e$ ’ in Figure 4.8 employing t-SNE algorithm [115]. The feature distribution of the sinusoidal feature set obtained by modeling the signals are also given. It is to be noted that, for recognition, ideal features should have small intra-class variability and high inter-class variability. Now, one can observe in Figure 4.8 that the sinusoidal features lead to higher inter-class distance. This shows the discriminating ability of the sinusoidal features in classification.

The sinusoidal feature set, unlike the point-based features, is based on the generation theory of handwriting. The main motivation for the proposal of the sinusoidal model-based feature set is to use the generation based information for boosting handwriting recognition performance. While the existing point-based feature set captures the geometric characteristics of a handwriting trajectory locally at each point, the proposed features are computed for each stable movement instead of each

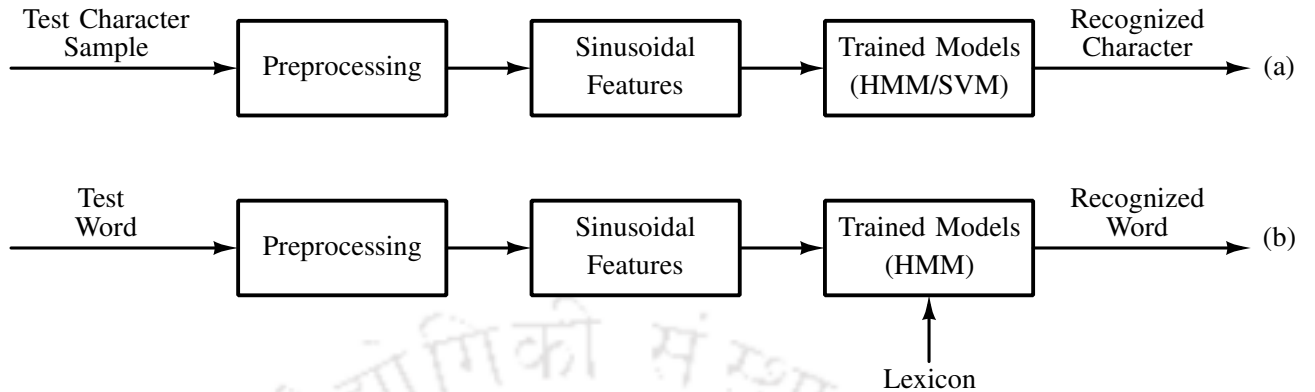


**Figure 4.8:** Represent t-SNE feature visualization for two confusing lowercase letters ‘c’ and ‘e’. The subplots (a), (c), and (e) depict the feature distribution of normalized  $(x, y)$  coordinates, first-derivative  $(x', y')$ , and second derivative  $(x'', y'')$ , respectively. The subplots (b), (d), and (f) depict the feature distribution of sinusoidal parameters extracted from normalized  $(x, y)$  coordinates, velocity, and acceleration signals, respectively using two components.

point. As an added advantage, this also helps the representation of complex trajectories using only a few sets of parameters.

### 4.3 Results and discussion

The efficacy of the proposed sinusoidal features is evaluated for both character and word recognition tasks. The block schematic of the online handwritten character and word recognition systems employing proposed features is given in Figure 4.9. The character recognition system consists of three blocks: preprocessing, extraction of sinusoidal features, and modeling. The details of the HMM-based character recognition system are discussed in Section 3.5 of Chapter 3. In this chapter, we also consid-



**Figure 4.9:** Block diagram of an online handwritten (a) character recognition and (b) word recognition systems using proposed sinusoidal features.

ered the SVM classifier for the character recognition system. The SVM is one of the supervised and discriminative machine learning technique which is well suited for character recognition. There are two most popular strategies aiming to extend the use of SVM for multi-class classification problems, namely one-vs-all and one-vs-one strategies. It is shown in [116] that the one-vs-one strategy is more suitable for the development of a practical system. Therefore, this work uses the one-vs-one strategy for training the SVM-based system. The radial basis function (RBF) kernel is used to train the SVM, and its parameters are optimized through a grid search approach.

In the word recognition system, the sinusoidal features are extracted from an input word sample after preprocessing, and thereafter, the same word recognition system described in Section 3.5 of Chapter 3 is used to find the recognized output. In addition to character recognition blocks, it also consists of the lexicon module, which matches the test sample with different words present in the lexicon.

#### 4.3.1 Extraction of sinusoidal features

In the feature extraction stage, first, the velocity, acceleration, and position of the handwriting are computed in both horizontal and vertical directions for the handwriting trajectory. The sinusoidal feature set is obtained by representing each of these signals using the multi-component sinusoidal model. A signal is considered to be formed of  $K$  sinusoidal oscillations where the parameters are extracted iteratively from the residue signal obtained from the previous iteration. Three parameters, i.e., amplitude, frequency, and phase, are extracted between every successive zero-crossing point by

#### 4. Sinusoidal model based features

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fitting half cycle of a sine wave. Finally, we obtain the sinusoidal parameter set from (4.2.9) as follows:

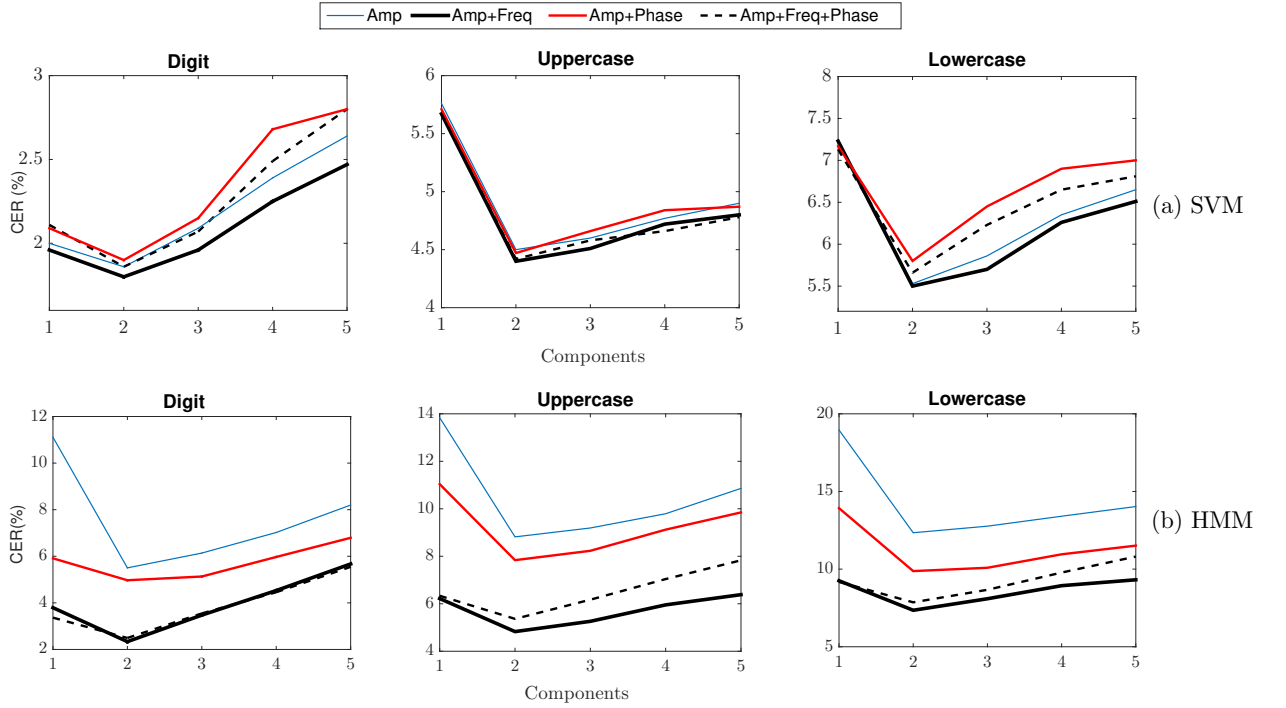
$$F(t) = [\hat{v}_x(t) \ \hat{v}_y(t) \ \hat{r}_x(t) \ \hat{r}_y(t) \ \hat{s}_x(t) \ \hat{s}_y(t)]^T$$

The obtained features  $F$  are then optimized with respect to the number of components ( $K$ ) and parameters. Although all the parameters, i.e., amplitude, frequency, and phase, are important for the reconstruction of the signals, however, there may be redundant information in the parameters, which may not help in improving the recognition performance. Moreover, the higher  $K$  may represent the noise in the data; therefore, the value of  $K$  also needs to be optimized. The optimum value of  $K$  for velocity, acceleration, and position are found experimentally and are discussed in Section 4.3. Further, as there are repeated values of the parameters between zero crossing instants in  $F$ , instead of considering complete sequence, i.e.,  $T$  points, we can use sample few points between zero-crossings to train the models. Accordingly, we tune the different parameters, components  $K$ , and the number of sampled points,  $Q$ , for optimal recognition performance.

In the following, we discuss the recognition results obtained for both validation and test sets of different databases. We optimize the number of sinusoidal components  $K$ , its parameters, and the sampled points  $Q$  for achieving the best performance using the validation sets. On the other hand, the final results are shown on the test sets for different databases.

##### 4.3.2 Sinusoidal parameters for character recognition

Figure 4.10 shows the error rate using different parameter combinations and sinusoidal components extracted from  $v_x, v_y$  for both HMM and SVM classifiers. The experimental results are shown for the UNIPEN character databases. To optimize the number of components, different combinations of its parameters, and the number of components  $K$  varying from 1 to 5 are employed as features for training the character models. It can be observed that the error rate reduces significantly on increasing from one to two components, i.e.,  $K = 1$  to  $K = 2$ , and then starts increasing with the addition of higher component parameters after  $K = 2$ . This is because, with an increase of  $K$ , fine details of the data are captured, which does not contain significant discriminatory information. Moreover, it may also increase the intra-class variabilities, which reduces the recognition performance. We also demonstrate the performance using different parameters in Figure 4.10. It is found that among various parameters, the combination of amplitude and frequency gives the best result. The same trend is observed for both HMM and SVM frameworks. The obtained results corroborate with the



**Figure 4.10:** Tuning of sinusoidal components and its parameters to obtain optimal feature set for the online handwriting recognition task. The subplot (a) represents the error rate (%) with different sinusoidal components for different tasks with SVM classifier. The subplot (b) represents the same as in (a) with HMM. For this experiment, 20 points feature vector ( $Q = 20$ ) is considered.

discussion in Section 4.2. Therefore, only amplitude and frequency parameters of the velocity signals for  $K = 2$  (i.e.  $a_{v_x}^{(1-2)}, \omega_{v_x}^{(1-2)}, a_{v_y}^{(1-2)}, \omega_{v_y}^{(1-2)}$ ) are employed in feature vector to train the handwriting recognition system. All the subsequent experiments are conducted considering only the amplitudes and frequencies and  $K \leq 2$ .

In Table 4.1, we compare the error rate (%) obtained using extracted sinusoidal parameters with that obtained by using the original signals, i.e., position, velocity, and acceleration directly, for the validation set of UNIPEN character databases using HMM and SVM. It can be observed that for the velocity and acceleration signals, the recognition performance obtained using sinusoidal parameters have improved. However, in the case of the position signal, the recognition performance using a single component ( $K = 1$ ) reduces significantly and improves when two components ( $K = 2$ ) are used for the feature representation. It may be because, unlike the velocity and acceleration signals, the nature of the  $x$  and  $y$  position signals  $s_x$  and  $s_y$  are not entirely sinusoidal and hence may not be characterized by the parameter set adequately. The improvement in the case of the HMM-based system is much better compared to the SVM-based system as the latter maps the feature vector to a higher dimension

#### 4. Sinusoidal model based features

**Table 4.1:** Error rate (%) obtained using the velocity, acceleration, and position signals directly as features and that obtained using the sinusoidal parameters extracted from the corresponding signals as features for UNIPEN character database (validation sets) in HMM and SVM character recognition framework.

	Feature Set	Dim	HMM			SVM		
			Digit	Upper case	Lower case	Digit	Upper case	Lower case
Position	$s_x, s_y$	2	10.49	15.43	17.10	2.86	5.91	7.48
	$a_{s_x}^{(1)}, \omega_{s_x}^{(1)}, a_{s_y}^{(1)}, \omega_{s_y}^{(1)}$	4	8.88	10.66	12.51	8.98	9.12	10.14
	$a_{s_x}^{(1-2)}, \omega_{s_x}^{(1-2)}, a_{s_y}^{(1-2)}, \omega_{s_y}^{(1-2)}$	8	2.81	6.09	9.11	2.15	4.41	5.81
Velocity	$v_x, v_y$	2	8.03	13.45	15.81	2.20	5.10	8.21
	$a_{v_x}^{(1)}, \omega_{v_x}^{(1)}, a_{v_y}^{(1)}, \omega_{v_y}^{(1)}$	4	2.96	6.12	9.50	1.80	4.53	6.86
	$a_{v_x}^{(1-2)}, \omega_{v_x}^{(1-2)}, a_{v_y}^{(1-2)}, \omega_{v_y}^{(1-2)}$	8	2.15	5.17	7.28	1.76	4.11	5.91
Accel.	$r_x, r_y$	2	9.13	15.22	16.79	3.11	9.81	8.72
	$a_{r_x}^{(1)}, \omega_{r_x}^{(1)}, a_{r_y}^{(1)}, \omega_{r_y}^{(1)}$	4	4.16	13.72	14.52	2.96	9.12	8.10
	$a_{r_x}^{(1-2)}, \omega_{r_x}^{(1-2)}, a_{r_y}^{(1-2)}, \omega_{r_y}^{(1-2)}$	8	2.61	12.82	13.41	2.51	9.81	7.85

**Table 4.2:** Selection of optimum sinusoidal feature set and the corresponding error rate (%) for UNIPEN character database (validation set) employing HMM and SVM.

Notation	Feature Set	Dim	HMM			SVM		
			Digit	Upper	Lower	Digit	Upper	Lower
F1	$a_{v_x}^{(1-2)}, \omega_{v_x}^{(1-2)}, a_{v_y}^{(1-2)}, \omega_{v_y}^{(1-2)}$	8	2.16	5.17	7.29	1.76	4.11	5.91
F2	$F1 + a_{r_x}^{(1)}, \omega_{r_x}^{(1)}, a_{r_y}^{(1)}, \omega_{r_y}^{(1)}$	12	1.61	4.52	6.94	1.56	3.81	5.41
F3	$F1 + a_{r_x}^{(1-2)}, \omega_{r_x}^{(1-2)}, a_{r_y}^{(1-2)}, \omega_{r_y}^{(1-2)}$	16	1.61	4.58	7.01	1.61	3.99	5.47
F4	$F2 + a_x^{(1)}, \omega_x^{(1)}, a_y^{(1)}, \omega_y^{(1)}$	16	1.46	4.23	6.70	1.36	3.45	4.92
F5	$F2 + a_x^{(1-2)}, \omega_x^{(1-2)}, a_y^{(1-2)}, \omega_y^{(1-2)}$	20	1.31	3.93	6.53	1.26	3.15	4.56

and hence is not much affected by the input feature dimension. In comparison, the HMM system performs poorly on less feature dimension, such as when the signals are used directly, which has two dimensions.

##### 4.3.2.1 Optimum feature set

To find the complete feature set, we combine the parameters extracted from the velocity profiles  $(v_x, v_y)$ , acceleration  $(r_x, r_y)$  and normalized position  $(s_x, s_y)$  signals. The first two columns of Table 4.2 denotes the notation and its constitute feature set that are evaluated in this study.

- $F1$  represents the features that are generated by combining amplitude and frequency of  $K = 2$  components of  $v_x$  and  $v_y$ .
- $F2$  is generated by combining  $F1$  with the amplitude and frequency parameters of acceleration

signals  $(r_x, r_y)$  with  $K = 1$ .

- $F3$  is generated by combining  $F1$  with amplitude and frequency parameters of  $K = 2$  components of the acceleration signal. The respective error rate for each of these feature set as well as for different databases employing both SVM and HMM are given in Table 4.2.
- It is found that the obtained error rate using  $F2$  is better than  $F3$  for all the cases, which shows that the second component of the acceleration signal does not contain much discriminating information for handwriting recognition task when combined with the velocity parameters.
- We next add the feature set  $F2$  comprising of velocity and acceleration parameters with one and two components of position signals denoted by  $F4$  and  $F5$ , respectively.
- The feature set  $F5$  with dimension  $d = 20$  performs best for all the tasks.

Thus, the final feature set comprises of the amplitude and frequency parameters of two sinusoidal components of velocity  $(v_x, v_y)$  and position  $(s_x, s_y)$ , and one component of acceleration  $(r_x, r_y)$ . Thus, the total dimension of proposed feature is 20.

#### 4.3.2.2 Number of sample point

In this experiment, we observe the change in error rate with the number of points  $Q$  in the feature vector (section 4.3.1) used to train the models. Ideally, the number of  $Q$  should be equal to the number of movements. However, as the letter shape varies, we need to find an optimum number of sampled points to achieve the best performance. The number of sampled points  $Q$  is varied from 5 to 100, and the respective error rates employing both SVM and HMM are given in Table 4.3. It is observed that with the increase in points ( $Q$ ), the error rate reduces significantly up to  $Q = 25$ , then the improvement in error rate is very negligible. On the other hand, as the feature vector length increases, the processing time also increases. Figure 4.11 shows the error rate and processing time comparison for the different number of points. Therefore,  $Q = 25$  is considered the optimal choice for the proposed features.

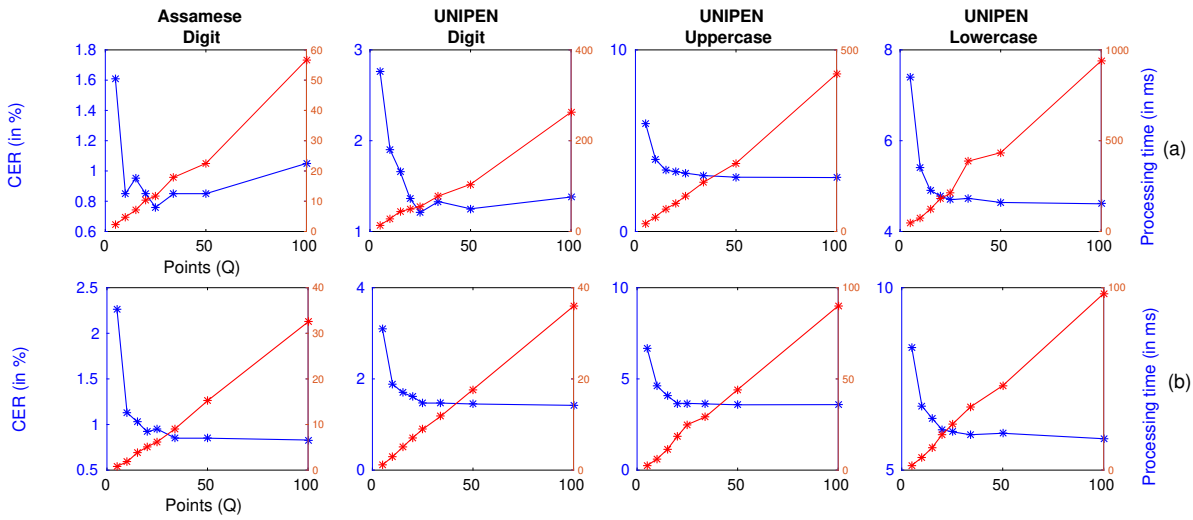
#### 4.3.3 Comparison with existing point-based features

We compare the proposed sinusoidal features with the existing point-based features that are mostly used for online handwriting recognition. Some of the most popular point-based features are normalized  $(x, y)$  coordinates and their first and second derivative, linearity, writing direction, curvature, aspect ratio, and slope. We train five systems using combinations of existing features, proposed features, and

#### 4. Sinusoidal model based features

**Table 4.3:** The variation in error rate (%) with number of points ( $Q$ ) for UNIPEN character database using SVM and HMM based system.

(Points $Q$ )	HMM			SVM		
	Digit	Uppercase	Lowercase	Digit	Uppercase	Lowercase
5	7.08	8.26	10.54	2.26	6.25	6.80
10	2.41	5.59	7.25	1.81	3.51	4.80
15	1.46	4.23	7.11	1.31	3.24	4.66
20	1.31	3.93	6.53	1.26	3.15	4.56
25	1.31	3.81	6.32	1.10	3.03	4.44
35	1.20	3.72	6.31	1.10	3.00	4.41
50	1.20	3.72	6.27	1.10	3.06	4.43
100	1.15	3.69	6.09	1.05	3.06	4.39



**Figure 4.11:** Plot depicting relation between number of feature points ( $Q$ ) and processing time.

a combination of proposed and existing features, as shown in Table 4.4.

- System 1 is trained using the position, i.e., normalized  $(x, y)$  coordinates, velocities, and acceleration signal from which the sinusoidal parameters are extracted.
- System 2 is trained using the point-based features which capture the geometric characteristics like curvature and linearity.
- The combination of the two feature sets used in System 1 and System 2 is used to train System 3.
- System 4 is trained using the proposed sinusoidal parameters based feature set.
- The proposed feature set is combined with the point-based features to train System 5.

The recognition performances obtained for the different systems for both validation and test sets of UNIPEN character database are given in the Table 4.5 employing SVM and HMM based system.

**Table 4.4:** Description of different systems trained using various existing feature sets, proposed features, and combined features along with the feature dimensions.

Sys. Notation	Feature Set	Feature Dim.
System 1	$s_x, s_y, v_x, v_y, r_x, r_y$	6
System 2	$LN, AR, \cos\alpha, \sin\alpha, \cos\beta, \sin\beta, \cos\gamma$	7
System 3	$s_x, s_y, v_x, v_y, r_x, r_y, LN, AR, \cos\alpha, \sin\alpha, \cos\beta, \sin\beta, \cos\gamma$	13
System 4	$F5, Q=25$ ( <b>Proposed</b> )	20
System 5	$(F5, Q = 25) + LN, AR, \cos\alpha, \sin\alpha, \cos\beta, \sin\beta, \cos\gamma$ ( <b>Combined</b> )	27

**Table 4.5:** The error rate (%) of different systems defined in Table 4.4 for UNIPEN character database using HMM and SVM.

	HMM						SVM					
	Digit		Uppercase		Lowercase		Digit		Uppercase		Lowercase	
	Val	Test	Val	Test	Val	Test	Val	Test	Val	Test	Val	Test
System 1	1.95	1.80	4.89	4.52	7.04	7.50	1.45	1.31	3.93	3.65	5.54	5.66
System 2	3.66	3.01	6.92	6.59	9.78	10.52	2.25	1.94	4.86	4.58	6.96	7.30
System 3	1.36	1.25	3.84	3.61	6.44	6.75	1.20	1.21	3.18	3.10	4.61	4.89
System 4	1.31	1.21	3.69	3.50	6.26	6.54	1.10	1.15	3.03	2.90	4.44	4.50
System 5	0.95	0.88	3.10	2.85	5.45	5.86	0.85	0.76	2.10	2.20	3.72	3.84

**Table 4.6:** The error rate (%) of different systems defined in Table 4.4 for IRONOFF character database (test set) using HMM and SVM.

	HMM			SVM		
	Digit	Uppercase	Lowercase	Digit	Uppercase	Lowercase
System 1	2.93	7.12	8.48	2.07	5.98	7.63
System 2	5.00	9.59	11.52	3.05	7.58	10.30
System 3	2.07	6.12	8.24	1.46	5.14	6.89
System 4	1.95	5.86	7.92	1.34	4.96	6.65
System 5	1.83	5.25	6.86	1.22	4.02	5.34

**Table 4.7:** The error rate (%) of different systems defined in Table 4.4 for Assamese character database (test set) using HMM and SVM.

	HMM				SVM			
	Digit	Basic char.	Conjunct	Modifier	Digit	Basic char.	Conjunct	Modifier
System 1	1.59	4.90	5.20	3.64	1.13	5.25	5.92	2.98
System 2	3.10	7.64	5.78	4.03	2.72	7.95	8.48	4.97
System 3	1.21	3.96	4.16	2.81	0.94	4.60	4.82	2.32
System 4	1.13	3.85	4.00	2.72	0.84	4.44	4.65	2.26
System 5	0.94	2.84	3.10	2.15	0.75	3.44	3.50	1.71

#### 4. Sinusoidal model based features

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**Table 4.8:** Performance comparison (% error rate) with other works reported on UNIPEN character database.

Method	Digit	Uppercase	Lowercase
DTW [117]	2.90	7.20	9.30
OnSNT [112]	1.10	4.30	7.90
ANN [1]	0.80	3.10	5.10
GP-SVM [7]	0.73	2.33	3.95
CNN-features [118]	0.63	2.16	3.45
Two-stage system [119]	0.86	3.58	4.07
this work (System 4-SVM)	1.15	2.90	4.50
this work (System 5-SVM)	0.76	2.20	3.84

**Table 4.9:** Performance comparison (% error rate) with other works reported on IRONOFF character database.

Method	Digit	Uppercase	Lowercase
TDNN and SDNN [120]	1.60	5.80	9.30
SVM [121]	1.67	4.54	7.53
k-NN [122]	6.50	11.40	15.10
this work (System 4-SVM)	1.34	4.96	6.65
this work (System 5-SVM)	1.22	4.02	5.34

It can be seen that improved performance is obtained using the proposed features. Further, to show the advantage of the additional information present in the proposed feature set, we have combined the sinusoidal features with the existing point-based features. The results show that the performance of combined features reduces the error rate significantly than individual feature representation. Similarly, Table 4.6 and Table 4.7 depict the results obtained for IRONOFF and Assamese database, respectively. The results show that the proposed feature set gives better recognition performance for all the databases.

Finally, we compare the performance of the SVM-based handwriting recognition system employing proposed features and combined features with the works reported on UNIPEN and IRONOFF character databases in Table 4.8 and Table 4.9, respectively. It can be seen that improved character recognition performance is achieved with the proposed sinusoidal features to the existing works.

#### 4.3.4 Results of the word recognition system

The proposed feature set is also evaluated for online handwriting word recognition. The experiments are performed using the Assamese word database, UNIPEN word database, and IRONOFF word database. For evaluating the performance of the proposed feature set and its robustness towards the size of the lexicon, we conducted the experiments by varying the lexicon size as 5000, 10000, and

**Table 4.10:** Error rate (%) of the HMM-based word recognition system evaluated on the test sets of different databases.

Database	Lexicon	System-1	System-2	System-3	System-4	System-5
UNIPEN-ICROW-03	5000	28.20	34.11	26.43	24.70	20.04
	10000	31.55	35.86	29.80	28.18	23.23
	20000	34.59	38.12	32.92	31.04	25.36
IRONOFF	5000	25.80	30.23	23.14	21.05	18.97
	10000	29.90	33.45	28.34	26.50	22.44
	20000	35.45	39.56	34.98	31.98	27.46
Assamese	5000	24.80	39.24	23.02	21.00	18.21
	10000	27.91	32.45	26.57	25.03	20.77
	20000	30.45	35.56	28.96	26.18	22.55

**Table 4.11:** Comparison of error rate (%) with performance of reported system on the UNIPEN-ICROW-03 word database.

Lexicon Size	System-4	System-5	[123]	[118]	[7]
5000	24.70	20.04	29.16	14.35	25.21
10000	28.18	23.23	32.98	16.84	29.22
20000	31.04	25.36	37.41	19.32	33.27

20000. The most frequent words are chosen to form the required lexicon size. The system is trained using the words corresponding to the training set, whereas the validation set is used to tune parameter HMM parameters, and the test set is used for final performance evaluation.

The number of states and number of mixtures for Assamese word recognition are optimized to 15 and 20, respectively, using the validation set. For UNIPEN and IRONOFF word recognition, the number of states and mixtures are optimized to 10 and 20, respectively. For word recognition also the same feature sets given in Table 4.4 are extracted from the word data. System 1–3 are trained using existing features except for the  $x$  coordinate signal. On the other hand, System 4 is trained using the proposed feature set, and System-5 utilizes the combination of proposed and existing point-based features used in System-2. In Table 7.3, we present the error rate (%) of the word recognition systems on validation sets of the different databases. It can be observed that the word recognition performance obtained using the proposed sinusoidal features in System-4 is better compared to the existing features for different lexicon size. Further improvement is obtained by combining the sinusoidal and existing features, i.e., System 5.

Table 4.11 compares the error rate of the proposed features on the UNIPEN-ICROW-03 database with the performance obtained in the work [123]. The results are given for different lexicon sizes. In

#### 4. Sinusoidal model based features

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**Table 4.12:** Comparison of error rate (%) with performance of reported system on the IRONOFF word database.

Lexicon Size	System-4	System-5	[124]
1000	21.00	18.21	24.60
2000	25.03	20.77	30.00
10000	26.18	22.55	36.20

**Table 4.13:** Error rate (%) for IRONOFF word database using lexicon with only the words present in the database.

Database	Lexicon	System-4	System-5	[125]
Cheque	30	3.56	2.85	3.70
English	26	2.96	2.59	3.33
French	171	11.20	10.46	12.19

Table 4.12, the efficacy obtained using our proposed feature is compared with the performance obtained in other reported work on IRONOFF word database [124]. For comparison with the reported result, we consider the lexicon size in this experiment as 1000, 2000, and 10000. Another work in the IRONOFF word database uses a fixed lexicon size, which consists of the words present in the database [125]. We compare the results obtained using our proposed feature with [125] in Table 4.13. It can be observed that better recognition performance is obtained using the proposed feature for all the databases.

#### 4.4 Summary

This work presents a new approach for feature extraction using the multi-component sinusoidal model for the online handwriting recognition task. Handwriting can be reconstructed efficiently from the parameters of the multi-component sinusoidal model extracted from the horizontal and vertical velocities. Therefore, these parameters can also be utilized for representing the handwriting in a recognition system. It is observed that the parameters reduce the intra-class variability compared to using the handwriting velocity directly and hence improves the performance of a recognition system. Similar to the horizontal and vertical velocities, the position and acceleration signals are also oscillatory in nature. Therefore, these signals can also be modeled using the multi-component sinusoidal model. Although the handwriting can be reconstructed completely using the velocity profiles, the  $(x, y)$  coordinate and acceleration signals can also provide useful information for recognition. Therefore, the model parameters extracted from the position and acceleration signals are also utilized as features

for online handwriting recognition. The parameters (i.e., amplitude, phase, and frequency) for each of these signals are extracted by fitting half cycles of sine wave between its successive zero crossings points.

It has been found that representing the handwriting using model parameters instead of these signals give better recognition performance. Extensive experiments have been carried out to find the optimal set of parameters as features for the online handwriting recognition task. The results show that the amplitude parameter contains the most discriminating information about the characters, while the phase contains the least discriminatory information. The sinusoidal model-based features are combined with point-based features to obtain an improved performance for both character and word recognition system. This shows that the proposed feature set extracts complementary information from the handwriting.

In the following chapter, we present another application of the multi-component sinusoidal model for handwriting modification and generating synthetic samples.



# 5

## Modification of online handwriting

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### Overview

Handwriting modification refers to the change in various shape characteristics of the handwriting trajectory. The handwriting shape is primarily controlled by the relation between its horizontal and vertical velocities. The multi-component sinusoidal model represents these velocities using sinusoidal oscillations by modulating its parameters— amplitude, frequency, and phase at different time instants. As it is a mathematical representation, it provides flexibility to vary the model parameters and introduce variability found naturally in the real handwriting by altering the parameters extracted from the original handwriting. Based on this motivation, we propose a method for handwriting modification using the multi-component sinusoidal model. We hypothesize that the modified handwriting obtained by parameter variation within a limit does not lose the structural and dynamic characteristics of natural handwriting. Therefore, we first demonstrate the effect of varying the sinusoidal model parameters on the handwriting shape and then present a model-based approach for handwriting modification. The proposed modification technique is used for creating synthetic variabilities of handwriting. The efficacy of the proposed method is shown by using the modified handwriting for data-augmentation in a recognition system. It is worth mentioning that advanced classifiers, mostly based on deep neural networks, need a considerable amount of data samples for proper training. However, it requires extensive time and effort to collect and annotate data for all possible writing styles from different writers. Therefore, our proposed modification technique for data augmentation can be beneficial to train such systems. Accordingly, we train a Gaussian mixture model (GMM)-hidden Markov model (HMM) and a deep neural network (DNN)-HMM based recognition system using both original handwriting and its synthetic variations. The addition of the modified handwriting improves the performance of both the recognition systems.

### 5.1 Introduction

In the present time, when technology is advancing at such a rapid pace, researchers have come up with many modern classifiers and modeling techniques to develop different handwriting systems. The importance of feature representation for the efficient performance of the handwriting systems has already been discussed in the previous chapter. In addition to the feature representation, the performance of these modern modeling techniques also depends on the size of the training set. Most of these techniques are data-driven and require a large amount of data to work efficiently, such as the

models based on neural networks. A good database should consist of all the variabilities that may occur naturally in handwriting. However, the collection of a large database with all possible variabilities requires extensive time and effort. Moreover, it is not always easy to find writers, especially for low resource language and scripts. Therefore, the availability of a large labeled database is not always possible, which results in the classifier's sub-optimal training.

One approach to solving this data shortage problem is to augment the data-set with synthetic handwriting. It can help increase the amount of training and testing data-set size without collecting data from users and labeling them. This makes handwriting modification an important research area to create synthetic data by changing the properties of the original handwriting [15,29]. The handwriting can be modified using various transformation directly on 2D trace such as rotation, scaling, shearing, noise addition to the  $(x, y)$  sequence [100]. However, these geometric transformations or perturbation of the  $(x, y)$  coordinates do not describe the control process of handwriting. Another approach for modifying the handwriting as discussed in Section 2.4 of Chapter 2 is based on the handmovement theory. In this approach, the modified sample can be generated by changing the underlying movement that produces the handwriting 2D trace. The work presented in this chapter is also along the same line and explores the multi-component sinusoidal model for changing the movement information to modify the online handwriting.

The handwriting trajectory is controlled by the hand movement in the horizontal and vertical directions. The change in the relation between the velocities in these two directions changes the handwriting output in a 2D trace. The multi-component sinusoidal model represents these velocities using a set of parameters, i.e., amplitude, frequency, and phase. Therefore, it allows us to change the velocities by varying the parameters extracted from the original handwriting. The new handwriting samples generated from the modified parameters contains the natural handwriting dynamic characteristics, which is not ensured by geometric transformations and perturbation on the 2D space. Similar approach have also been used for generation of new motion sequences in [126]. A statistical model of human motion is learned such that it identifies common choreographic elements across sequences. This model can generate new graceful dance by adjusting a small number of parameters.

In this work, first, we analyze the effect of the sinusoidal parameters on the handwriting shape. The handwriting undergoes different variability by the change of different parameters in the model. These effects are investigated by varying the parameters in a small amount and visualizing the change

## 5. Modification of online handwriting

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in final handwriting outcome. Accordingly, a method is proposed to change the model parameters extracted from the handwriting to generate different variations of handwriting. Further, we also determine a permissible range for varying the different parameters by comparing the synthetic variation with natural variations of handwriting using the dynamic time warping (DTW) distance. In the experimental analysis, we demonstrate the handwriting modification result for English and Assamese handwriting. The effectiveness of the modified handwriting is demonstrated by using the synthetic data for training the handwriting recognition system in addition to the original data. The recognition system's performance has been found to be improved and shown for different amounts of original and synthetic data obtained from the proposed method.

To summarize, the main contributions of this work are as follows.

- Analysis of effect of sinusoidal parameters on handwriting shape
- Proposal of a handwriting modification method using multi-component sinusoidal representation
- Addition of synthesized handwriting to the training set of GMM-HMM and DNN-HMM based online recognition system improve its accuracy

The rest of this chapter is organized as follows. In Section 5.2, we analyze and demonstrate the effect of various sinusoidal parameters on handwriting shape. Based on this analysis, we propose a handwriting modification method, which is described in Section 5.3. The results obtained for the various experiments performed in this work are presented in Section 5.4. Finally, the chapter is concluded with a summary in Section 5.5.

### 5.2 Effect of sinusoidal model parameters

The multi-component sinusoidal model proposed in Chapter 3 describes the horizontal and vertical velocities of handwriting using a set of sinusoidal parameters as described in the following.

$$v_x(t) = \sum_{m=1}^K a_{v_x}^m(t) \sin(\omega_{v_x}^m(t).t + \phi_{v_x}^m(t)) \quad (5.2.1)$$

$$v_y(t) = \sum_{m=1}^K a_{v_y}^m(t) \sin(\omega_{v_y}^m(t).t + \phi_{v_y}^m(t)) \quad (5.2.2)$$

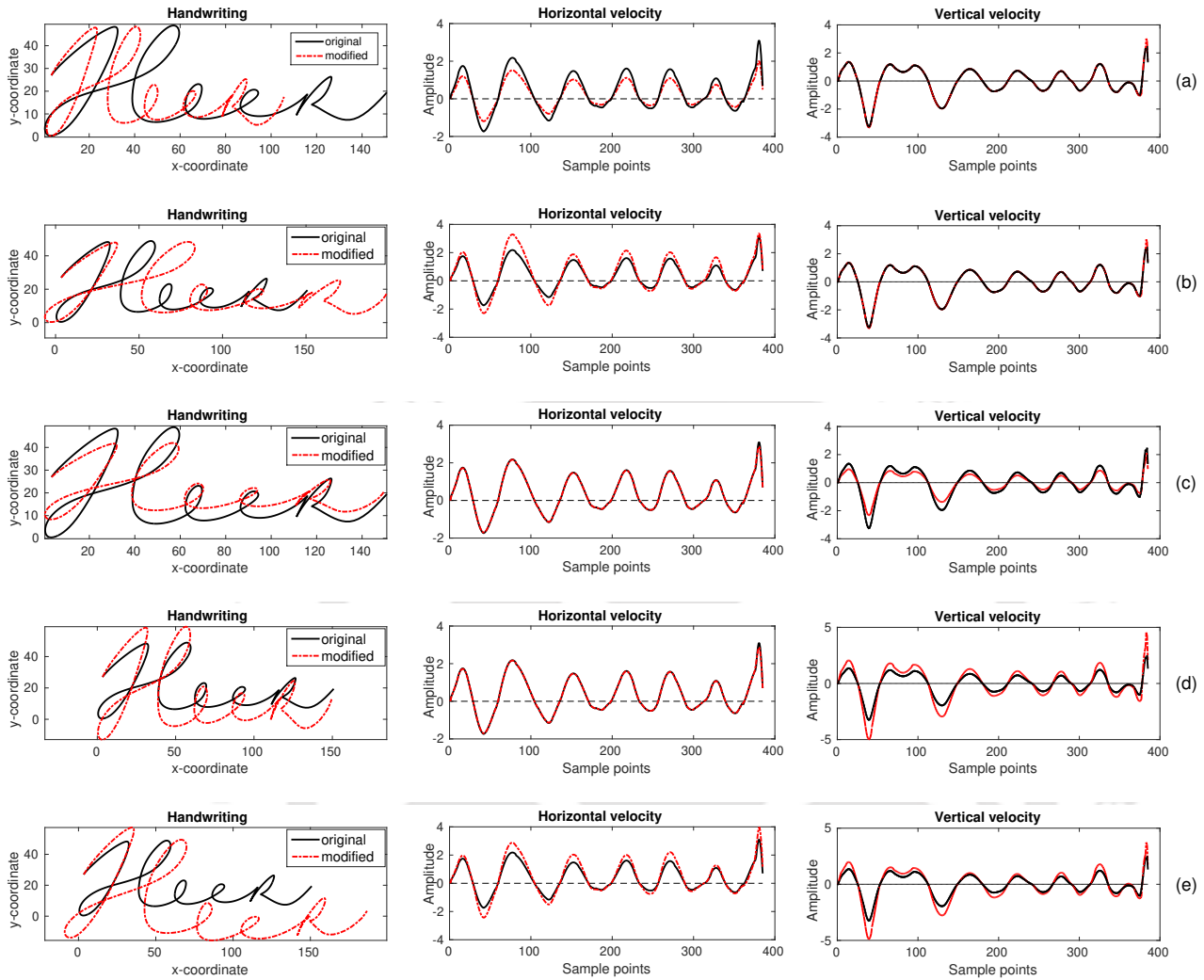
where  $K$  is the number of components and  $a_{v_x}^m$ ,  $\omega_{v_x}^m$ ,  $\phi_{v_x}^m$ ,  $a_{v_y}^m$ ,  $\omega_{v_y}^m$  and  $\phi_{v_y}^m$  are the sinusoidal parameters of the  $m^{th}$  component extracted from the velocities. The shape of a handwriting trajectory depends on the handwriting velocities. Therefore, this mathematical representation of the velocities using

the multi-component sinusoidal model gives us the flexibility to modify the handwriting dynamics by acting on its parameters. In the following, we discuss the effect of various sinusoidal parameters on handwriting shape. We demonstrate the modification of a real handwritten word by varying the multi-component sinusoidal model parameters given in (5.2.1) and (5.2.2). The model parameters are extracted from the real handwriting using the parameter extraction method described in Section 3.3.1 of Chapter 3. We illustrate the effect of each parameter— amplitude, phase, and frequency separately. The parameters are varied both globally and locally, where the global variation modifies the complete handwriting in a uniform manner with an equal amount of parameter variation at each point. Whereas by the local variation, the parameters at different points are varied independently.

**Amplitude variation** The amplitude of the horizontal and vertical velocities control the trajectory length in the corresponding direction. An increase in the amplitude of horizontal velocity increases the trajectory width and vice-versa. Similarly, an increase in vertical velocity amplitude increases the trajectory height and vice-versa.

Figure 5.1 illustrates the handwriting modification achieved by changing the amplitude parameters extracted from the original velocities. The left panel of each subplot shows the original and the reconstructed handwriting obtained after amplitude modification. The middle and last panels show the reconstructed horizontal and vertical velocities, respectively, along with the corresponding original velocities. To begin with, we first vary the amplitude of the horizontal velocity and reconstruct the handwriting with the modified velocities. Subplot (a) shows the handwriting reconstructed after increasing the amplitude of the horizontal velocity components by 30%, whereas subplot (b) shows the same after decreasing it by 30%. It can be observed that only the width of the word varies, and the height does not change as the vertical velocity remains the same. Next, we illustrate the effect of variation in amplitude of the vertical velocity component in subplot (c), where the handwriting is reconstructed after increasing the amplitude of vertical velocity components by 30%. Similarly, the subplot (d) shows the same after decreasing it by 30%. The change in vertical amplitude modifies the trajectory length between top and bottom corner points, while the word's width remains the same. Thus, the global variation modifies the width and height of the word equally at every point. Finally, we vary the amplitudes of both the horizontal and vertical velocity components with local variation. The subplot (e) shows the handwriting reconstructed after increasing the amplitude of the horizontal and vertical velocities randomly between 10 to 30%. It can be observed that individual letters are

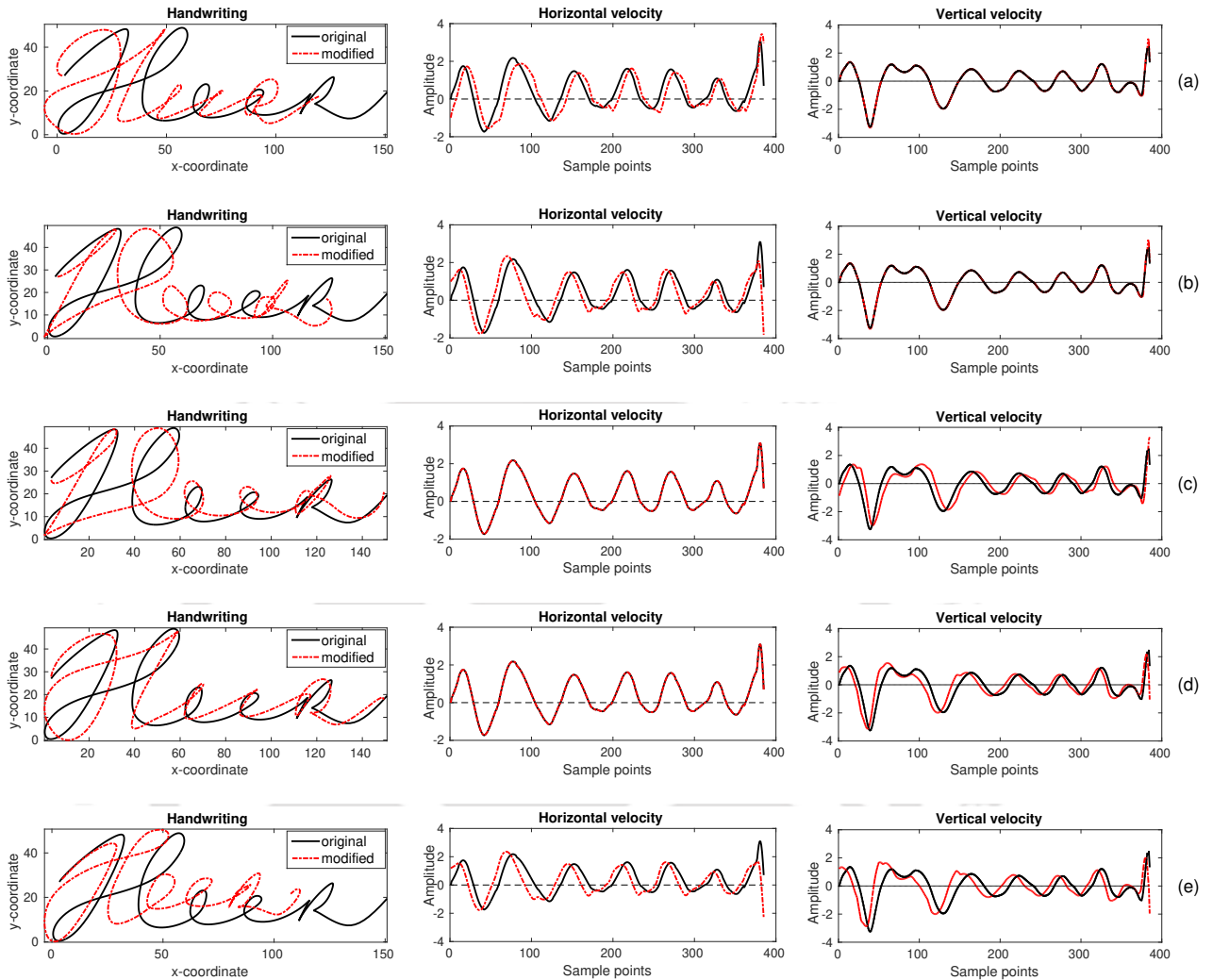
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**Figure 5.1:** Illustration of variation of amplitude parameter. The panels from left to right show the online handwriting, horizontal velocity, and vertical velocity, respectively. The original and reconstructed handwriting are represented using solid and dotted lines. The subplot (a) shows the effect of increasing horizontal velocity amplitude by 30%. The subplot (b) shows the effect of decreasing the horizontal velocity amplitude by 30%. The subplot (c) shows the effect of increasing the vertical velocity amplitude by 30%. The subplot (d) shows the effect of decreasing vertical velocity amplitude by 30%. The subplot (e) shows the effect when the amplitudes of both horizontal and vertical velocities increased locally with random values between 10% to 30%.

modified differently by local distortions. Therefore, both global and local variations of the amplitude parameter can be utilized for handwriting modification.

**Phase variation** The phase modification changes the relation between the two velocities resulting in variation of the handwriting corners. A change in the phase difference between the velocities can transform a rounded corner into a loop and a loop to a sharp edge. In Figure 5.2, the phase parameter of the horizontal and vertical velocities are varied both locally and globally to analyze the variations



**Figure 5.2:** Illustration of the effect of phase parameter on handwriting shape. The panel from left to right shows the online handwriting, horizontal velocity, and vertical velocity, respectively. The original and reconstructed handwriting are represented using solid and dotted lines. The subplot (a) shows the effect of decreasing horizontal velocity phase by  $30^\circ$ , (b) shows the effect of increasing horizontal velocity phase by  $30^\circ$ , (c) and (d) show the effect when vertical velocity phase is decreased by  $30^\circ$  and increased by  $30^\circ$ , respectively. In subplot (e), the phase of both horizontal and vertical velocities increased locally with random values between  $10^\circ$  to  $30^\circ$ .

obtain in the handwriting shape. The change in phase between the two velocities can change the hand movement from clockwise to anti-clockwise and vice-versa. Therefore, we vary the phase parameter by a small amount such that the modified shape of the handwriting remains meaningful. The subplot (a) and (b) show the reconstructed handwriting reconstructed after increasing and decreasing the phase of horizontal velocity components by  $30^\circ$ . The subplot (c) and (d) show the handwriting reconstructed after increasing and decreasing the phase of vertical velocity components by  $30^\circ$ . The variation in

the phase parameter changes the horizontal and vertical velocity relation at the zero-crossings. Thus, altering the corner shapes in the handwriting, such as a loop, may change into a sharp edge, as observed from the illustration. In the subplot (e), we illustrate the effect of varying the phase parameter of both vertical and horizontal velocity together. The phase of the horizontal and vertical velocities are varied randomly between  $10^\circ$  to  $30^\circ$ . Thus, the phase modification of the multi-component sinusoidal model within a small range can transform the corner shapes of the handwriting.

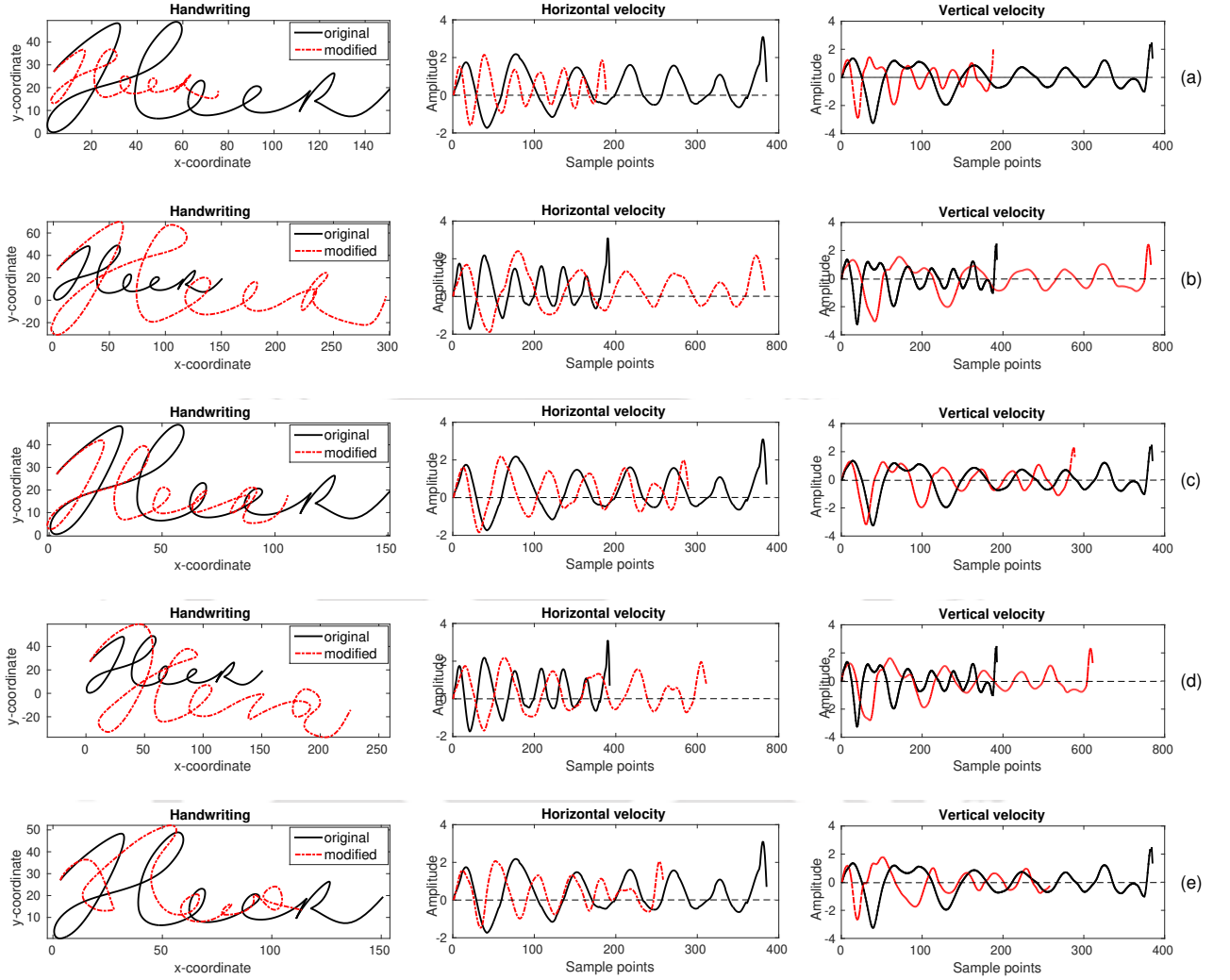
**Frequency variation** The frequency parameter is inversely proportional to the time taken to produce the handwriting. Thus, with a constant amplitude, an increase in the frequency parameter decreases the handwriting size and vice-versa. We show the effect of frequency modification in reconstructed handwriting in Figure 5.3. The subplot (a) and (b) show the handwriting reconstructed after increasing and decreasing the frequency parameter by 100% and 50%, respectively. Here the frequency parameter of both the horizontal and vertical velocities is varied with an equal amount such that the number of sample points in both the signals remains the same. It is observed that the global increase of the frequency parameter results in an overall decrease in the handwriting size and vice-versa. The shape of the handwriting remaining identical. However, the trajectory shape changes on a random local variation of the frequency parameter. The subplots (c)–(e) show the reconstructed handwriting after changing the frequency parameter locally with different variation range 1 – 5%, 5 – 10%, and 10 – 30%, respectively. Here the two modified velocities are sampled to an equal number of points before reconstructing the handwriting. It is observed that local variation above 10% results in highly distorted handwriting. Therefore, in this work, we do not consider the change in the frequency parameter for the handwriting modification.

### 5.3 Proposed framework for handwriting modification

The proposed method generates synthesized variations for online handwriting data using the multi-component sinusoidal model. The steps involved in the proposed method are given in the following:

- The first step is the addition of an imaginary pen-up stroke to form continuous handwriting.
- The second step is the computation of the horizontal and vertical velocities from the coordinate sequence. Suppose,  $P = \{(x_1, y_1), (x_2, y_2), \dots, (x_T, y_T)\}$  is a coordinate sequence of an online

### 5.3 Proposed framework for handwriting modification



**Figure 5.3:** Illustration of the effect of frequency parameter on handwriting shape. The panels from left to right show the online handwriting, horizontal velocity, and vertical velocity, respectively. The subplot (a) shows the effect of increasing the frequency of horizontal and vertical velocities by 100%. The subplot (b) shows the effect of decreasing frequency of horizontal and vertical velocities by 50%. The subplots (c)–(d) show the effect of increasing the frequency of horizontal and vertical velocities randomly between 1 to 10%, 10 to 20%, and 10% to 30%, respectively.

handwriting data. The horizontal and vertical velocity  $v_x$  and  $v_y$ , respectively, are given by

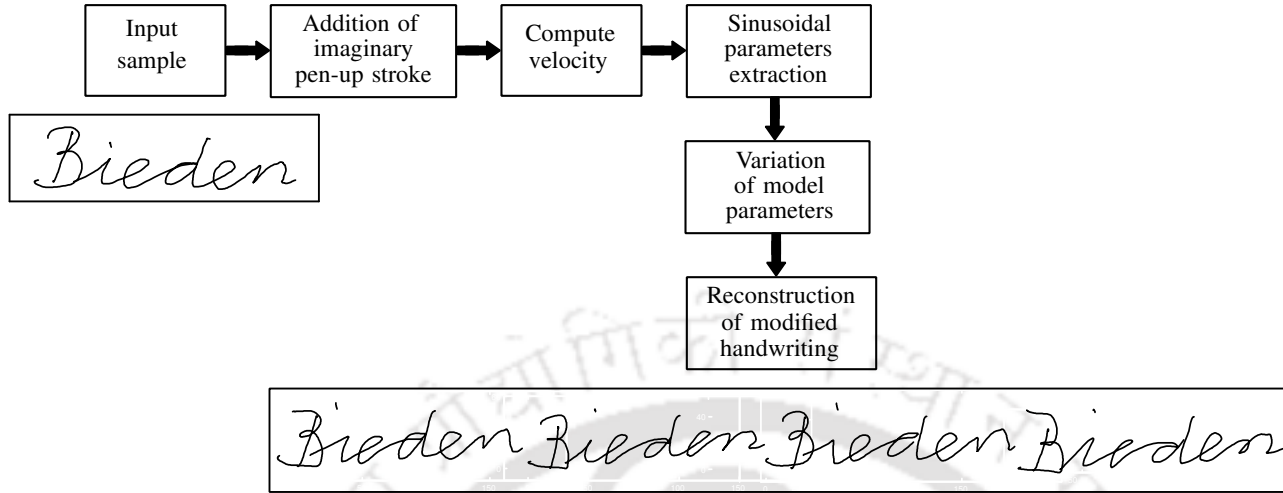
$$v_x(t) = \frac{x_{t+1} - x_{t-1}}{2} \quad (5.3.1)$$

$$v_y(t) = \frac{y_{t+1} - y_{t-1}}{2} \quad (5.3.2)$$

where  $1 < t < T$  and  $v_x(1) = v_x(2)$ ,  $v_y(1) = v_y(2)$

- In the third step, we represent the horizontal and vertical velocities  $v_x$  and  $v_y$  using the multi-component sinusoidal model with  $K = 2$  in 5.2.1 and 5.2.2 as follows:

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**Figure 5.4:** Illustration of the proposed handwriting modification framework.

$$v_x(t) = \sum_{m=1}^2 a_{v_x}^m(t) \sin(\omega_{v_x}^m(t).t + \phi_{v_x}^m(t)) \quad (5.3.3)$$

$$v_y(t) = \sum_{m=1}^2 a_{v_y}^m(t) \sin(\omega_{v_y}^m(t).t + \phi_{v_y}^m(t)) \quad (5.3.4)$$

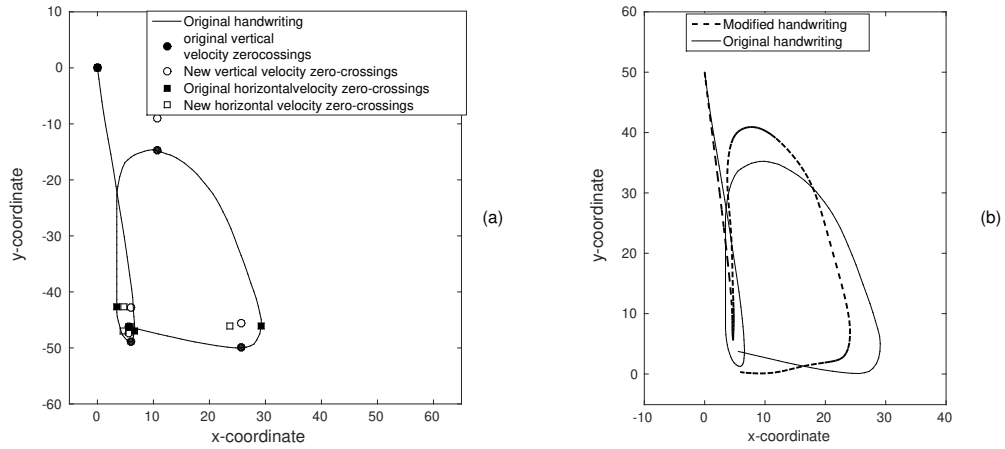
The parameters  $a_{v_x}^m$ ,  $\omega_{v_x}^m$ ,  $\phi_{v_x}^m$ ,  $a_{v_y}^m$ ,  $\omega_{v_y}^m$  and  $\phi_{v_y}^m$  are the sinusoidal parameters of the  $m^{th}$  component extracted from both the horizontal and vertical velocities by the method described in Section 3.3.1 of Chapter 3.

- The fourth step is the variation of the extracted sinusoidal parameters which control the handwriting shape.
- Finally, the modified handwriting velocities are reconstructed using the varied model parameters to obtain the modified handwriting.

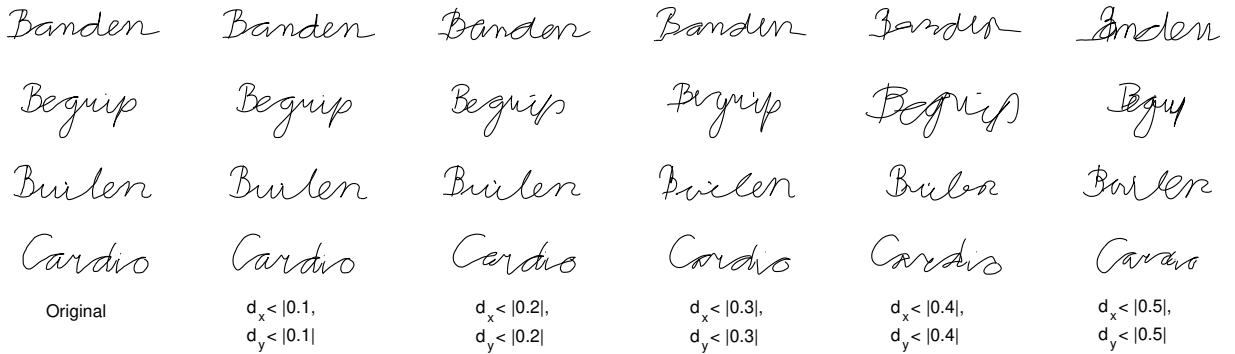
Figure 5.4 shows the block diagram for the proposed online handwriting modification framework where input is the original word, and the output is five modified handwriting. In the following, we describe the variation of different parameters in detail.

### 5.3.1 Amplitude modification

The modification of amplitudes of the sinusoidal oscillations in (5.3.3) and (5.3.4) changes the trajectory distance traversed in horizontal and vertical directions between the corners. The left and right corners of handwriting are the zero-crossing points of horizontal velocity, while the top and



**Figure 5.5:** Illustration of the amplitude variation using handwritten character ‘b’. The subplot (a) shows the original and the modified corner locations indicated by different markers. The target corners corresponding to the horizontal and vertical velocity zero-crossings are represented by squares and red circles, respectively. The subplot (b) shows the original and modified handwriting after amplitude variation.



**Figure 5.6:** Illustration of the variations in original handwriting with increasing range of  $d_x$  and  $d_y$ .

bottom corners correspond to the vertical velocity zero-crossings. A random change in the amplitude parameter may result in an unnatural and deformed handwriting shape. Therefore, first, the target corner locations are varied within a permissible range for the handwriting modification. These points are changed by altering the  $x$  coordinate of the horizontal velocity zero-crossings and  $y$  coordinate of vertical velocity zero-crossings. The distances between the modified target corners are then used to compute the corresponding amplitude parameters.

Suppose,  $z_{v_x,1}, z_{v_x,2}, \dots, z_{v_x,N_x}$  are horizontal velocity zero-crossings corresponding to left or right corners and  $z_{v_y,1}, z_{v_y,2}, \dots, z_{v_y,N_y}$  are vertical velocity zero-crossings corresponding to top or bottom corners. Here the number of horizontal and vertical velocity zero-crossings are denoted by  $N_x$  and  $N_y$ ,

## 5. Modification of online handwriting

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respectively. The position of these points are modified as follows:

$$x_{z_{v_x},i}^{new} = x_{z_{v_x},i} + \min(\tau_x^{p,i}, \tau_x^{n,i}) \times d_{x,i} \quad (5.3.5)$$

$$y_{z_{v_y},i}^{new} = y_{z_{v_y},i} + \min(\tau_y^{p,i}, \tau_y^{n,i}) \times d_{y,i} \quad (5.3.6)$$

$$\text{where } \tau_x^{p,i} = \text{abs}(x_{z_{v_x},i} - x_{z_{v_x},i-1}) \text{ and } \tau_x^{n,i} = \text{abs}(x_{z_{v_x},i} - x_{z_{v_x},i+1})$$

$$\text{and } \tau_y^{p,i} = \text{abs}(y_{z_{v_y},i} - y_{z_{v_y},i-1}) \text{ and } \tau_y^{n,i} = \text{abs}(y_{z_{v_y},i} - y_{z_{v_y},i+1})$$

The variation of the reconstructed handwriting using amplitude modification depends on  $d_{x,i}$  and  $d_{y,i}$ . The values of  $d_x = \{d_{x,i}\}$ ,  $d_y = \{d_{y,i}\}$  are chosen randomly within a given range of upper and lower bounds which are determined experimentally. Now, depending on the change in a corner location, the amplitudes of the first sinusoidal component are computed as follows.

$$a_{v_x,i}^{m,new} = \frac{x_{z_{v_x},i+1}^{new} - x_{z_{v_x},i}^{new}}{x_{z_{v_x},i+1} - x_{z_{v_x},i}} \times a_{v_x,i}^m \quad (5.3.7)$$

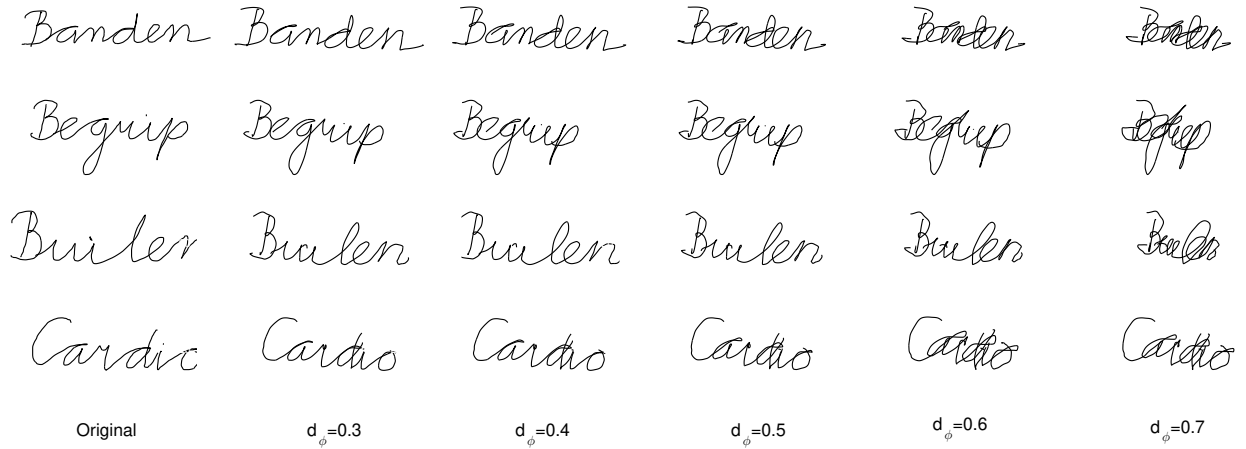
$$a_{v_y,i}^{m,new} = \frac{y_{z_{v_y},i+1}^{new} - y_{z_{v_y},i}^{new}}{y_{z_{v_y},i+1} - y_{z_{v_y},i}} \times a_{v_y,i}^m \quad (5.3.8)$$

We reconstruct the modified handwriting using the new set of amplitude parameters. Figure 5.5 demonstrates the variation in corner positions for a character 'b' and the final synthesized handwriting. It shows the original and modified corner points in the subplot (a). The modified handwriting is shown in subplot (b) along with the original handwriting.

We visualize the handwriting variations obtained with a different range of  $d_x$  and  $d_y$  in Figure 5.6 for four different English words from the UNIPEN word database. The first panel of each row in the figure shows the original handwriting, and the other panels show the synthesized handwriting using the modified parameters. The ranges of  $d_x$  and  $d_y$  are given at the bottom. It can be observed that difference between the original and the modified handwriting increases as the value of  $d_x$  and  $d_y$  rises. Therefore, a permissible range for variation of these parameters is needed to be computed.

### 5.3.2 Phase modification

The handwriting shape is controlled by the phase relation between the two velocities. Phase modification of the sinusoidal oscillations in (5.3.3) and (5.3.4) modify the corner shapes of the handwriting by changing the trajectory direction. In this work, we modify the phase parameter of the horizontal velocity to change its value at the vertical velocity zero-crossings. This results in the variation of the



**Figure 5.7:** Illustration of the variations in original handwriting with increasing of  $d_\phi$ .



**Figure 5.8:** Illustration of the variations in original handwriting with decreasing  $d_\phi$ .

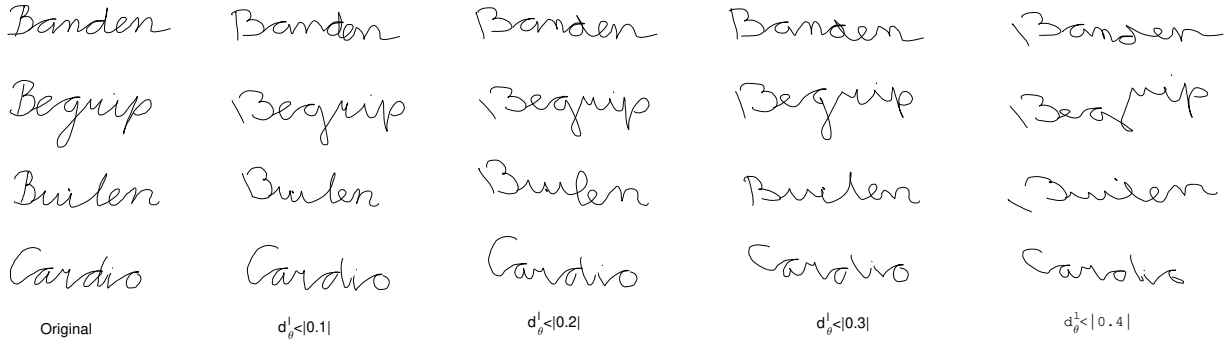
shape of the top and bottom corners of the handwriting. The phase parameter is varied equally across the handwriting as follows:

$$\phi_{v_x,i}^{m,new} = \phi_{v_x,i}^m + \frac{\pi}{2} \times d_\phi \quad (5.3.9)$$

$$\phi_{v_y,i}^{m,new} = \phi_{v_y,i}^m \quad (5.3.10)$$

Figure 5.7 and Figure 5.8 illustrates the handwriting modification obtained by increasing and decreasing  $d_\phi$ , respectively. The horizontal velocity is varied with different amounts, and it can be observed that different handwriting variations are obtained by phase modification. The change in corner shapes of the handwriting increases with an increase in the value of  $d_\phi$ . It can be observed that the handwriting is highly distorted with  $|d_\phi|$  above 0.5.

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**Figure 5.9:** Illustration of the variations in original handwriting with increasing range of  $d_\theta^l$ .

### 5.3.3 Slant variation

The writing slant is effected by amplitudes of both vertical and horizontal oscillations and their phase difference, mathematically represented as follows [47]:

$$\tan \beta = \frac{a_{v_y}}{a_{v_x} \cos \phi} \quad (5.3.11)$$

Other explanations for writing slant that has been given in [47] is non-orthogonal axes disposition. The slant of the handwriting follows the direction of the slanted vertical axis. If  $\theta$  is the angle the vertical axis makes with the horizontal axis, (5.3.3) and (5.3.4) becomes

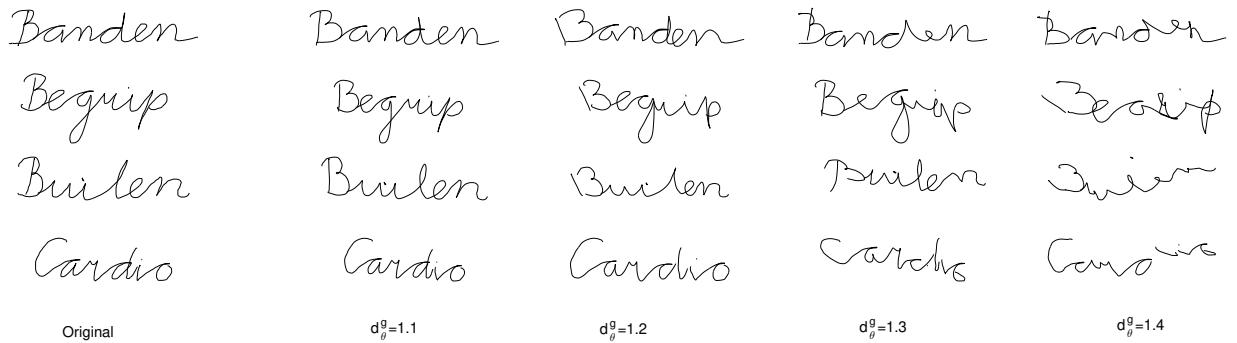
$$\begin{aligned} v_x &= \sum_{m=1}^2 \left[ a_{v_x}^m(t) \sin(\omega_{v_x}^m(t) \cdot t + \phi_{v_x}^m(t)) + a_{v_y}^m(t) \cos \theta(t) \sin(\omega_{v_y}^m(t) + \phi_{v_y}^m(t)) \right] \\ v_y &= \sum_{m=1}^2 a_{v_y}^m(t) \sin \theta(t) \sin(\omega_{v_y}^m(t) + \phi_{v_y}^m(t)) \end{aligned} \quad (5.3.12)$$

One advantage of using this method is different segments between corner points can be changed with different angles as follows.

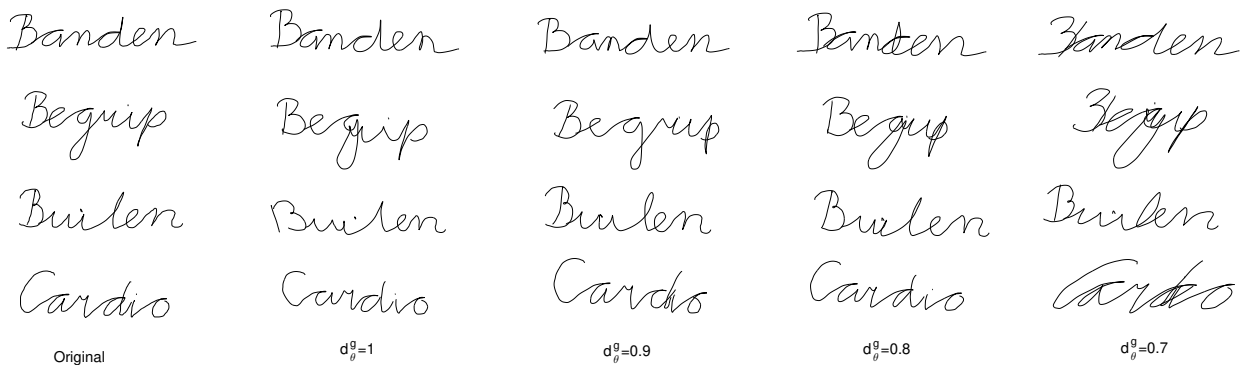
$$\theta(t) = \theta_i \text{ for } t = [z_{v_y,i} z_{v_y,i+1}], \quad 1 < i < N_y \quad (5.3.13)$$

$$\theta_i = \frac{\pi}{2} \times (d_\theta^g + d_{\theta,i}^l) \quad (5.3.14)$$

The value of  $d_\theta^g$  and  $d_\theta^l = \{d_{\theta,i}^l\}$  are the global and local changes in slant angle, i.e., the value of  $d_\theta^g$  remains same for complete handwriting while  $d_\theta^l$  is varies across the signal. Figure 5.9 shows the modified handwriting with different range of  $d_\theta^l$  varying from  $|d_\theta^l| < 0.1$  to  $|d_\theta^l| < 0.4$  Figure 5.10 and Figure 5.11 shows the handwriting variations obtained for different values of  $d_\theta^g$  between [1.1 1.4] and



**Figure 5.10:** Illustration of the variations in original handwriting with increasing of  $d_{\theta}^g$ .



**Figure 5.11:** Illustration of the variations in original handwriting with decreasing of  $d_{\theta}^g$ .

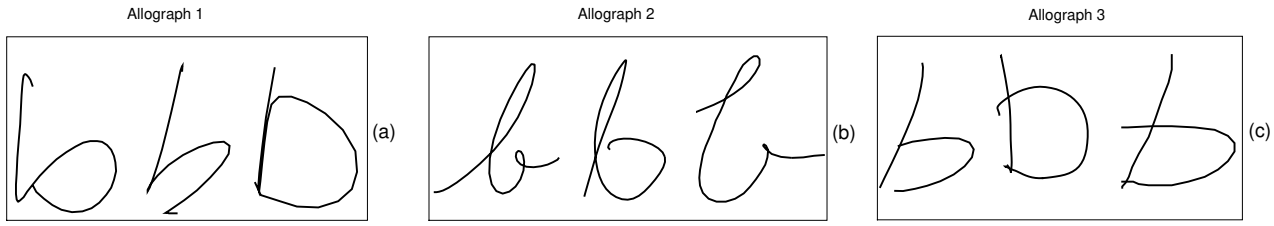
[0.7 1], respectively. In the following, we determine the various permissible ranges for the different parameter variation.

### 5.3.4 Estimation of variation range for amplitude, phase, and slant modification

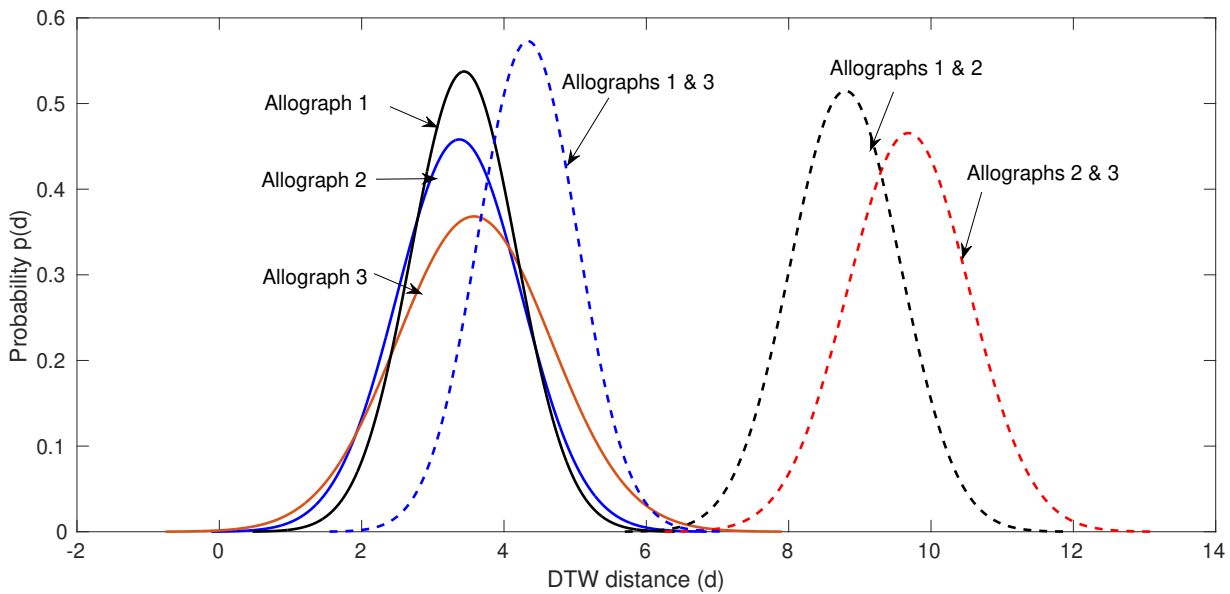
As discussed earlier, the modified handwriting should not lose any meaningful characteristics of the original handwriting. Therefore, a permissible range for varying the different model parameters needs to be estimated such that the deviation of original handwriting is within the natural variation. These variations present in natural handwriting is analyzed by computing the dynamic time warping (DTW) distance between the real handwriting samples. We consider that the modification can be carried out such that the DTW distance between the original and modified handwriting is within the limit found in natural handwriting. The DTW distance is measured in the feature space between the size normalized samples using the point-based features described in Chapter 3.

We consider the UNIPEN lowercase character database for the first experiment. It contains a large

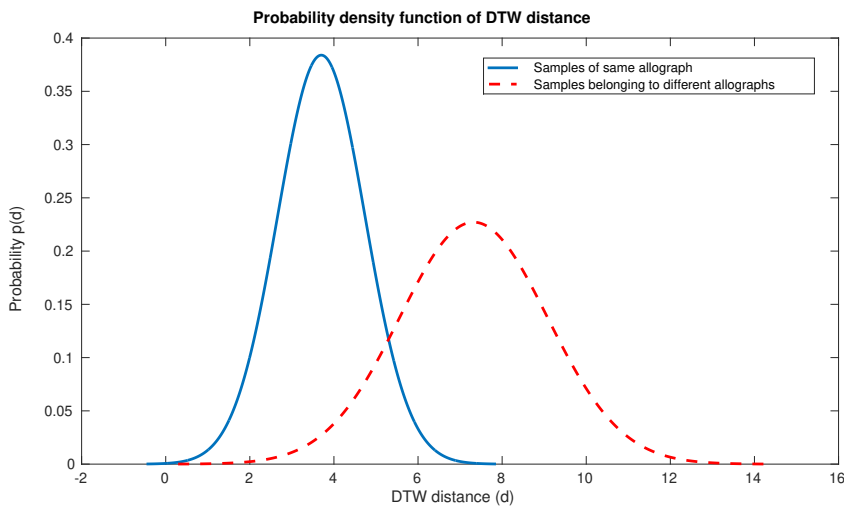
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**Figure 5.12:** The subplots (a)–(c) show three allographs of letter *b* from the UNIPEN database. For each allograph three samples are depicted. The allograph in (a) and (b) have only one pen-up and allograph in (c) has two pen-ups.



**Figure 5.13:** Probability density functions of the DTW distance between samples of same allograph and different allograph of letter ‘*b*’ of Figure 5.12.



**Figure 5.14:** Probability density functions of the DTW distance between samples of same allograph (solid line) and different allographs (dotted) averaged across the samples in UNIPEN character database.

### 5.3 Proposed framework for handwriting modification

**Table 5.1:** The mean and standard deviation of DTW distance between real samples of different databases. The DTW distance is normalized by dividing with number of points in the handwriting trajectory.

	UNIPEN				Assamese			
	Digits	Uppercase	Lowercase	Words	Digits	Vowels	Consonants	Words
Mean	3.88	5.65	5.42	5.78	5.14	5.94	5.92	5.80
Std. deviation	1.34	1.76	1.02	1.96	1.34	1.60	1.35	2.50

**Table 5.2:** The mean and standard deviation of DTW distance between original and modified handwriting for words in UNIPEN-ICROW-03 and Assamese databases with different range of  $d_x$  and  $d_y$ .

		$d_x <  0.1 $	$d_x <  0.2 $	$d_x <  0.3 $	$d_x <  0.4 $	$d_x <  0.5 $
		$d_y <  0.1 $	$d_y <  0.2 $	$d_y <  0.3 $	$d_y <  0.4 $	$d_y <  0.5 $
UNIPEN-ICROW-03	Mean	3.50	<b>5.12</b>	6.05	8.80	10.60
	Std. deviation	2.30	2.66	3.05	3.45	4.30
Assamese word	Mean	3.98	<b>5.48</b>	7.35	9.16	11.26
	Std. deviation	2.80	1.86	1.48	2.05	3.08

number of samples with a high amount of intra-class variability and allographs. As the structural characteristics change for different allographs, the samples of each letter are first clustered using the agglomerative hierarchical cluster tree [35] to obtain the different allographs. For illustration, the allographs obtained for the letter ‘b’ are shown in Figure 5.12. The probability density functions of the DTW distance between samples belonging to the same allograph of letter ‘b’ are shown in Figure 5.13 denoted by the solid lines. It can be observed that the DTW distance for samples of allograph-1, 2, and 3 overlaps. We also compute the variations between samples belonging to different allographs. The probability density function of the DTW distance for different allographs is shown using a dotted line in Figure 5.13. A much higher DTW distance is obtained between samples of the different allographs. The same is also analyzed for the other lowercase letters. Figure 5.14 shows the DTW probability density functions of samples within the same allograph (solid line), and different allographs (dotted) averaged across different characters in the UNIPEN database. A similar experiment is performed for UNIPEN and Assamese word databases. We measure the DTW distance between samples written by different writers for each word in the databases.

This DTW distance shows the natural variation present in the handwritten characters and words due to different writing styles. Table 5.1 shows the mean and standard deviation of the DTW distance between real samples belonging to the same character allograph and words in different databases. We can observe that similar values are obtained for different databases. It is to be noted that the DTW distance is normalized by dividing it by the number of points in the handwriting. Therefore,

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**Table 5.3:** The mean and standard deviation of DTW distance between original and modified handwriting with different amount of phase modification  $d_\phi$ .

$d_\phi$		-0.1	-0.2	-0.3	-0.4	-0.5	0.1	0.2	0.3	0.4	0.5
UNIPEN-	Mean	2.61	3.26	<b>4.92</b>	7.78	12.40	2.12	2.61	4.16	<b>5.65</b>	9.19
ICROW-03	Std. deviation	1.83	2.24	2.96	3.75	5.22	1.55	1.72	2.44	3.36	4.29
Assamese	Mean	2.40	2.79	<b>4.08</b>	6.21	11.84	1.90	2.40	3.80	<b>5.12</b>	9.10
word	Std. deviation	1.06	2.68	3.17	3.52	4.62	1.06	1.88	2.94	4.06	4.75

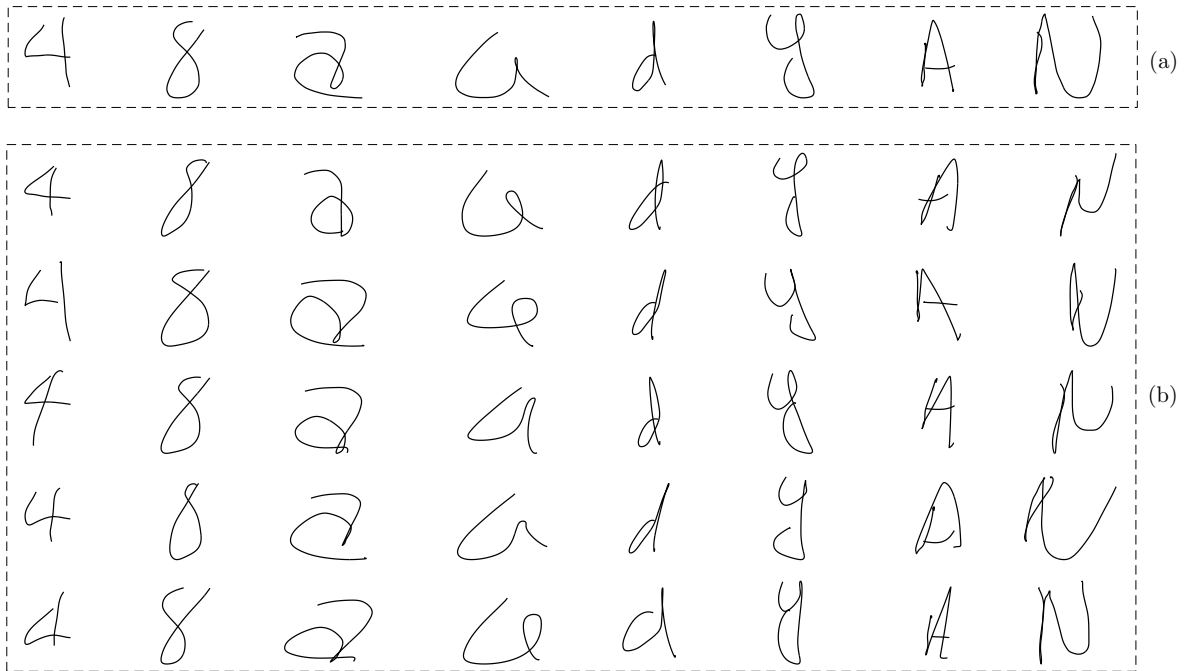
**Table 5.4:** The mean and standard deviation of DTW distance between original and modified handwriting with different range of  $d_\theta^l$ .

		$d_\theta^l <  0.1 $	$d_\theta^l <  0.2 $	$d_\theta^l <  0.3 $	$d_\theta^l <  0.4 $	$d_\theta^l <  0.5 $
UNIPEN-ICROW-03	Mean	4.64	<b>5.14</b>	8.36	10.10	12.00
	Std. deviation	2.62	2.33	2.51	2.62	3.14
Assamese word	Mean	5.12	<b>5.78</b>	8.66	10.84	12.52
	Std. deviation	2.06	2.29	2.76	3.56	3.80

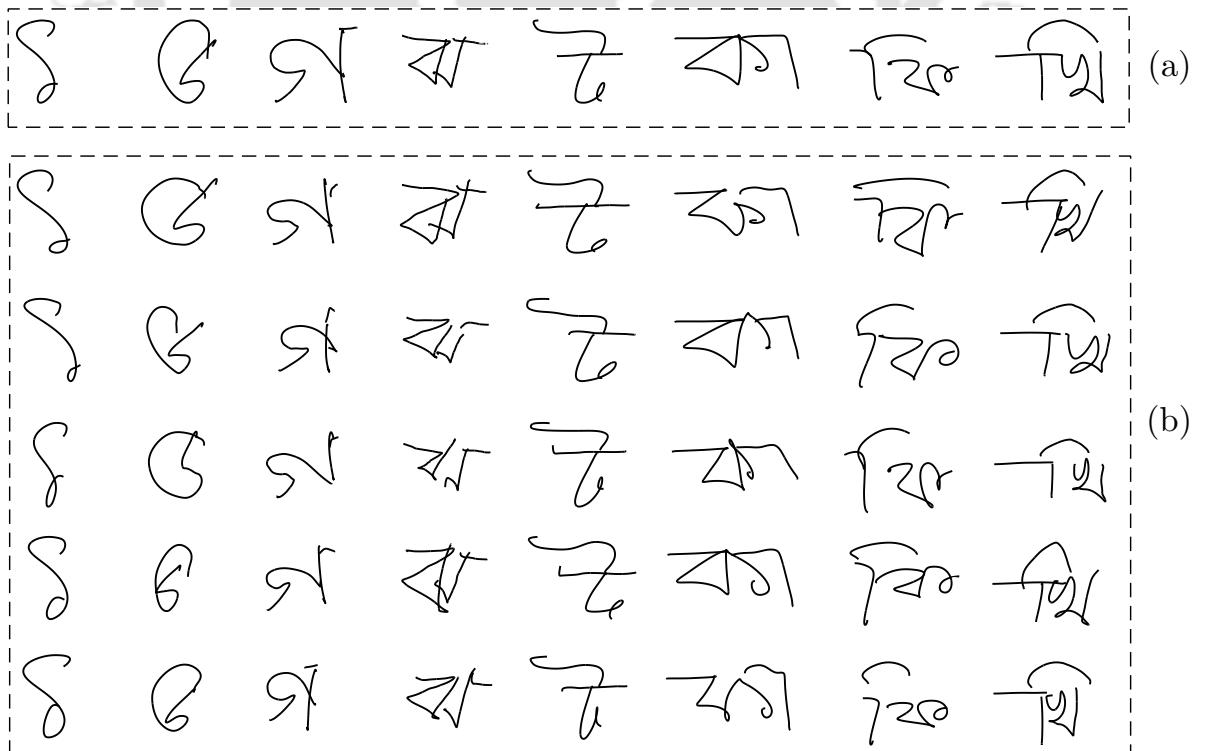
based on this analysis, a permissible range for parameter modification can be estimated such that the DTW distance original and modified samples should not be above a certain limit. In the following, we present the variation range for the different parameters.

**Range for amplitude, phase, and slant variation** Experiments are carried on the UNIPEN English and Assamese word databases. Twenty-five different words are selected from each of the databases, and first, the amplitude is varied using different values of  $d_x$ ,  $d_y$ . The mean and standard deviation of the DTW distance between the original and modified handwriting for the English and Assamese word databases are given in Table 5.2. Based on the analysis carried out on real handwriting, it can be concluded that a range of  $[-0.3 \ 0.3]$  is suitable for  $d_x$ ,  $d_y$ . Above this range, the synthesized handwriting is distorted as it can also be observed from Figure 5.6.

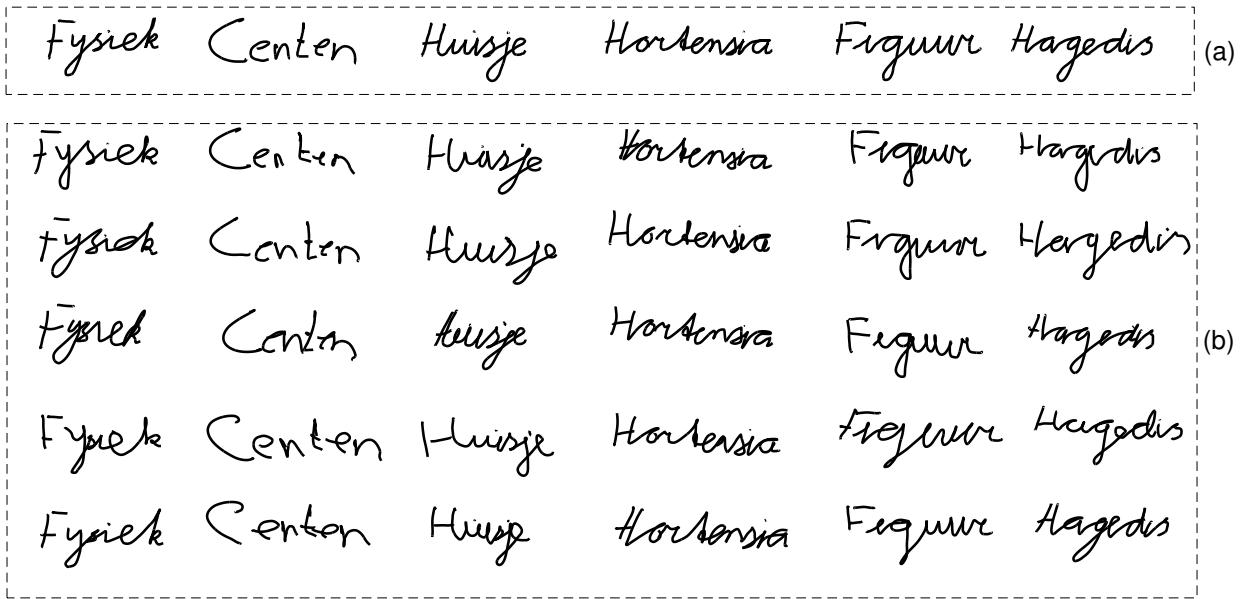
Similar experiments are carried out for phase and slant variations. The mean and standard deviation of the DTW distance between the original and modified handwriting obtained by phase variation is given in Table 5.3. The permissible range for  $d_\phi$  is found to be  $[-0.3 \ 0.4]$  based on the DTW distance. Table 5.4 presents the statistics of DTW distance obtained by local variation of the slant using  $d_\theta^l$ . The permissible range for variation of  $d_\theta^l$  is found to be  $[-0.2 \ 0.2]$ . We vary the global variation parameter between  $[0.8 \ 1.1]$  to get a overall slant variation between  $[80^\circ \ 110^\circ]$ .



**Figure 5.15:** The subplot (a) and (b) show the original and synthesized variations of English handwritten characters, respectively.



**Figure 5.16:** The subplot (a) and (b) show the original and synthesized variations, respectively of Assamese digits, consonants, and consonant with modifiers.



**Figure 5.17:** The subplot (a) shows the original handwriting and (b) shows the synthesized variations of English words from the UNIPEN-ICROW-03 database.

## 5.4 Results and discussion

We illustrate the modified handwriting obtained using the proposed method and present the recognition performance obtained by data-augmentation for English and Assamese databases.

Firstly, we demonstrate the modifications obtained using our proposed technique for character handwriting. A few samples are chosen from the English and Assamese character databases for the illustration. The original handwriting is first represented using the multi-component sinusoidal model, and the three types of modifications, i.e., amplitude, phase, and slant, are carried out to modify the original samples. Each parameter is varied according to the proposed method within the permitted range, as discussed in the previous section. Figure 5.15 and 5.16 show the modified handwritten characters obtained for English and Assamese script, respectively. The subplot (a) in each figure shows the original handwritten characters from which we extract the sinusoidal parameters. The subplot (b) shows the different reconstructed characters obtained using the proposed modification framework.

Similarly, we also demonstrate the handwriting variation obtained for English and Assamese handwritten words in Figure 5.17 and 5.18, respectively. The original samples are shown in the subplot (a) of each figure. Five modified samples for each word are illustrated in the subplot (b). It can be



**Figure 5.18:** The subplot (a) shows the original of Assamese handwritten words. The subplot (b) shows its synthesized variations obtained using the proposed method.

observed different variations are obtained for both Assamese and English handwriting, which are similar to the natural handwriting. We evaluate the naturalness of the proposed modified handwriting by using it for data-augmentation to train a recognition system. In the following, we describe the various recognition experiments performed in this work and the obtained results.

#### 5.4.1 Data-augmentation in online handwriting recognition system

We develop two online handwriting recognition systems for the data-augmentation experiments. The first is GMM-HMM based online handwriting recognition system, which is described in Section 3.5 of Chapter 3. The second system is the DNN-HMM based online handwriting recognition system, which is a neural network-based system. In this system, instead of GMM, a DNN is used to output the observation probabilities [127]. The number of input units in the neural network is three times the feature dimension (one central feature vector and a context of one vector at each side) and two hidden layers.

The recognition experiments are performed using two different sets of features. First is point-based features, including the preprocessed coordinates  $(x, y)$ , its first and second derivatives, aspect ratio, curliness, the writing direction, linearity, curvature, trajectory slope. The details of the point-based features are given in Section 3.5.2 of Chapter 3. The second feature set is a combination of point-based

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**Table 5.5:** Error rate (%) of the GMM-HMM based character recognition system where R denotes using only real data and R+S denotes using both real and synthetic data. The system is trained using different amount of data by considering 30%, 50%, and 100% of the real training set. The details of the point-based and combined feature sets are given in Section 3.5.2 and Section 4.3.3, respectively.

Database	Feature	Percentage of real training set, $p_{real}$					
		30%		50%		100%	
		R	R+S	R	R+S	R	R+S
UNIPEN	Digit	6.81	4.73	3.78	2.98	1.25	1.14
	Uppercase	13.56	9.44	6.75	5.37	3.61	2.97
	Lowercase	17.10	11.87	10.80	9.07	6.75	5.23
	Digit	4.40	3.10	2.93	2.40	0.88	0.73
	Uppercase	9.50	6.67	4.71	3.74	2.85	2.34
	Lowercase	15.42	10.07	8.60	6.63	5.86	4.77
IRONOFF	Digit	10.35	7.15	2.82	2.49	2.07	1.55
	Uppercase	12.24	8.57	8.73	6.97	6.12	5.04
	Lowercase	16.48	10.95	11.74	9.08	8.24	6.76
	Digit	7.32	5.10	2.54	2.10	1.83	1.38
	Uppercase	10.50	7.32	7.50	6.23	5.25	4.34
	Lowercase	13.72	9.06	9.35	7.29	6.86	5.48
Assamese	Digit	5.76	4.05	1.75	1.49	1.21	0.92
	Basic character	13.20	9.20	7.17	5.56	3.96	3.04
	Conjunct	13.87	9.54	6.49	5.40	4.16	3.42
	Modifier	8.26	5.73	4.54	3.64	2.81	2.21
	Digit	4.48	3.19	1.48	1.20	0.94	0.76
	Basic character	10.22	7.16	5.52	4.33	2.84	2.25
	Conjunct	11.79	7.97	5.74	4.68	3.10	2.50
	Modifier	7.17	5.10	3.64	3.10	2.15	1.69

and sinusoidal features comprising the amplitude and frequency parameters of the multi-component sinusoidal model. The combination of point-based and sinusoidal features is described in Section 4.3.3 of Chapter 4. We train the systems using different amounts of real training data to observe the effect of the data-augmentation. Suppose the total number of samples in the original training set of a database is  $N_{samples}$ . The number of samples used to train the system  $N_{train}$  is given by

$$N_{train} = p_{real}\% \text{ of } N_{samples}$$

The different amount of training samples are chosen by varying  $p_{real}$  as 30%, 50%, and 100%. The real samples are then augmented with the ten synthetically modified samples obtained from each of its real samples.

#### 5.4.1.1 Recognition performance of GMM-HMM based system

In the following, we describe the recognition performance of the GMM-HMM based character and word recognition systems using the modified samples for data-augmentation. The number of states and mixtures for Assamese character and word recognition are optimized to 15 and 20, respectively, using the validation set. On the other hand, for UNIPEN and IRONOFF character and word recognition, the number of states and mixtures are optimized to 11 and 20, respectively. As described earlier, two different feature sets are used in this experiment: point-based and combined sinusoidal features.

The results obtained from the character recognition experiment are described in Table 5.5. First, we describe the recognition performance improvement in the system trained using 30% of the real training data for the UNIPEN database trained using the point-based features <sup>1</sup>. It can be observed that the recognition performance improved by 2.08%, 4.12%, and 5.23% for the digit, uppercase, and lowercase recognition system. Similarly, for the system trained using combined features <sup>2</sup>, the improvement in performance on the digit, uppercase, and lowercase recognition system are 1.30%, 2.83%, and 5.35%, respectively. Next, data-augmentation is carried out for system trained using a higher number of real samples, i.e., 50% of the real training set. The recognition improvement obtained for the digit, uppercase, and lowercase recognition system using point-based and combined features are 0.80%, 1.38%, 1.73%, and 0.53%, 0.97%, 1.97%, respectively. Similarly, for the system trained using 100% of the real training set and point-based features, the recognition of digit, uppercase, and lowercase handwriting improves by 0.11%, 0.64%, and 1.52%. In the case of the combined feature set, the recognition performance improves by 0.15%, 0.51%, and 1.09%. This result shows that the proposed modification method can produce natural variations for the English characters and hence improves the recognition performance when used for training the system. We also perform the data-augmentation experiment for the IRONOFF and Assamese character databases. The results obtained are shown in Table 5.5.

Table 5.6 shows the recognition results obtained for the English word database UNIPEN-ICROW-03 using the GMM-HMM based system. First, we describe the results obtained for the system trained using point-based features. It can be observed that for the system trained using a small amount of real data, i.e., 30% of the complete real training set, the recognition performance improves by 4.82%,

<sup>1</sup>The details of the point-based features are given in Section 3.5.2 of Chapter 3

<sup>2</sup>The details of the point-based features are given in Section 4.3.3 of Chapter 4

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**Table 5.6:** Error rate (%) of the GMM-HMM based word recognition system evaluated on English UNIPEN-ICROW-03 database. R denotes trained using only real data and R+S denotes trained using both real and synthetic data.

Feature set	Lexicon	Percentage of real training set ( $p_{real}$ )					
		30%		50%		100%	
		R	R+S	R	R+S	R	R+S
Point-based	5000	41.44	36.62	32.03	28.63	26.42	24.73
	10000	45.09	39.53	37.56	33.32	29.79	26.51
	20000	49.88	42.81	41.77	36.72	32.91	30.41
Combined	5000	37.20	33.17	27.94	25.20	20.03	18.72
	10000	40.78	35.98	33.32	29.79	23.24	21.10
	20000	43.31	37.73	35.40	31.60	25.36	23.13

**Table 5.7:** Error rate (%) of the GMM-HMM based word recognition system evaluated on IRONOFF word database where R denotes only real data and R+S denotes real and synthetic data.

Feature set	Lexicon	Percentage of real training set ( $p_{real}$ )					
		30%		50%		100%	
		R	R+S	R	R+S	R	R+S
Point-based	5000	37.57	33.81	26.93	24.44	23.14	21.57
	10000	44.18	38.65	32.78	29.45	28.34	26.44
	20000	50.87	42.45	37.57	32.15	34.28	33.09
Combined	5000	33.59	30.89	23.14	21.89	18.97	17.43
	10000	39.84	35.43	28.19	26.04	22.44	21.89
	20000	44.18	37.98	32.01	27.93	27.46	25.90

5.56%, and 7.07%, for the lexicon sizes 5000, 10000, and 20000, respectively. Similarly, for the system trained using 50% and 100% of the real training set, modified synthetic data improves the performance by 3.4%, 2.38%, and 2.50%, and 1.69%, 2.38%, and 2.50%, respectively. Similarly the improvement obtained for the proposed combined features for 30%, 50%, and 100% are 3.57%, 4.27%, and 5.02%, 2.1%, 2.85%, and 3.16%, and 0.26%, 1.05%, and 1.07%. The improvement in recognition results for the IRONOFF and Assamese databases are shown in Table 5.7 and Table 5.8, respectively.

This result shows that the proposed method can produce natural variations in the generated handwriting, which can improve recognition performance when used for training the system. The recognition results obtained for the system trained with less real data (30% of the real training set) show the greatest improvement after the addition of synthetic handwriting. This is because a small amount of training samples contains a very less amount of natural variabilities resulting in a weak or inefficient model. Therefore, the modified handwriting added to this training set increases the robustness of the system and the recognition performance.

**Table 5.8:** Error rate (%) of the GMM-HMM based word recognition system evaluated on Assamese word database where R denotes only real data and R+S denotes real and synthetic data.

Feature set	Lexicon	Percentage of real training set ( $p_{real}$ )					
		30%		50%		100%	
		R	R+S	R	R+S	R	R+S
Point-based	5000	39.12	34.41	28.32	25.05	23.02	20.93
	10000	42.31	36.64	32.51	28.57	26.58	23.86
	20000	45.34	38.92	35.99	31.32	28.96	26.71
Combined	5000	36.21	32.74	22.05	19.95	18.21	17.95
	10000	39.13	34.56	25.77	22.92	20.78	19.73
	20000	41.19	36.17	28.61	25.45	22.55	21.48

**Table 5.9:** Error rate (%) of the DNN-HMM based character recognition system evaluated on English databases where R denotes trained using only real data and R+S denotes trained using both real and synthetic data.

Database	Feature	Percentage of real training set						
		30%		50%		100%		
		R	R+S	R	R+S	R	R+S	
UNIPEN	Digit	3.75	2.44	1.67	1.17	0.75	0.68	
	Uppercase	Point-based	8.71	5.82	5.09	3.62	2.70	2.22
	Lowercase		9.95	6.53	7.37	5.50	3.98	3.08
	Digit	Combined	2.86	1.88	1.09	0.80	0.60	0.51
	Uppercase		7.50	4.92	4.25	3.01	2.25	1.85
	Lowercase		9.90	5.52	6.43	4.44	3.60	2.93
IRONOFF	Digit	3.83	2.48	2.02	1.57	1.15	0.86	
	Uppercase	Point-based	13.33	8.72	7.69	5.49	4.00	3.29
	Lowercase		17.66	11.00	11.88	8.24	6.18	5.07
	Digit	Combined	3.40	2.22	1.82	1.34	1.02	0.77
	Uppercase		11.19	7.29	6.51	4.81	3.58	2.96
	Lowercase		16.94	10.49	10.67	7.44	5.76	4.60
Assamese	Digit	3.24	2.13	1.58	1.20	0.68	0.52	
	Basic character	Point-based	9.35	6.09	3.91	2.89	2.15	1.65
	Conjunct		10.39	6.69	4.60	3.40	2.39	1.96
	Modifier		4.92	3.19	2.28	1.61	1.23	0.97
	Digit	Combined	2.90	1.93	1.32	0.95	0.58	0.47
	Basic character		6.96	4.56	3.35	2.35	1.74	1.38
	Conjunct		8.16	5.17	4.00	2.90	2.04	1.65
	Modifier		4.48	2.98	1.93	1.46	1.12	0.88

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**Table 5.10:** Error rate (%) of the DNN-HMM based word recognition system evaluated on UNIPEN-ICROW-03 database where R denotes only real data and R+S denotes real and synthetic data.

Feature set	Lexicon	Percentage of real training set ( $p_{real}$ )					
		30%		50%		100%	
		R	R+S	R	R+S	R	R+S
Point-based	5000	28.15	22.03	18.36	15.13	13.18	11.86
	10000	31.60	24.04	24.39	19.80	15.49	13.72
	20000	34.89	25.56	26.93	21.50	17.72	14.81
Combined	5000	25.05	20.39	15.49	13.72	10.44	9.57
	10000	27.31	21.65	19.95	17.32	11.51	10.42
	20000	29.67	23.00	22.03	18.36	12.55	11.31

**Table 5.11:** Error rate (%) of the DNN-HMM based word recognition system evaluated on IRONOFF word database where R denotes only real data and R+S denotes real and synthetic data.

Feature set	Lexicon	Percentage of real training set ( $p_{real}$ )					
		30%		50%		100%	
		R	R+S	R	R+S	R	R+S
Point-based	5000	23.43	18.43	16.58	13.76	12.01	10.53
	10000	27.14	20.53	20.02	16.17	14.29	12.75
	20000	32.89	23.43	25.12	19.43	16.58	13.78
Combined	5000	21.24	17.64	14.94	12.75	10.01	9.34
	10000	24.07	18.97	17.29	14.45	11.81	10.81
	20000	29.46	21.61	21.57	17.64	13.05	11.04

### 5.4.1.2 Recognition performance of DNN-HMM based system

In the following, we describe the recognition performance of the DNN-HMM based character and word recognition systems using the modified samples for data-augmentation. The number of states for the DNN-HMM based systems is the same as the GMM-HMM system, which is 11 and 15 for English and Assamese, respectively. Each hidden layer of the DNN has 300 neurons [127].

Similar to GMM-HMM, the data-augmentation is performed using 30%, 50%, and 100% of samples in the real training set of the corresponding database. The character recognition results are depicted in Table 5.9 for the character databases UNIPEN, IRONOFF, and Assamese. The improvement in the character recognition performance trained using 30% of the samples present in the real training set are 1.31%, 2.89%, and 3.42% for UNIPEN digit, uppercase, and lowercase databases, respectively. Similarly, for systems trained using 50% and 100% of the real training set are 0.49%, 1.47%, and 1.87% and 0.08%, 0.48%, and 0.90%, respectively.

Table 5.10 shows the recognition results on English word databases UNIPEN-ICROW-03. The lex-

**Table 5.12:** Error rate (%) of the DNN-HMM based word recognition system evaluated on Assamese word database where R denotes only real data and R+S denotes real and synthetic data.

Feature set	Lexicon	Percentage of real training set ( $p_{real}$ )					
		30%		50%		100%	
		R	R+S	R	R+S	R	R+S
Point-based	5000	31.17	22.03	20.58	17.01	11.22	10.82
	10000	31.61	24.04	22.15	17.89	11.95	10.42
	20000	34.89	25.56	24.55	19.53	12.72	11.01
Combined	5000	28.77	20.09	18.39	16.01	9.76	9.09
	10000	30.82	22.27	19.84	16.84	10.12	9.24
	20000	31.98	23.41	21.81	18.29	10.79	9.71

icon size for the word recognition is varied with 5000, 10000, and 20000 words. As shown in Table 5.10, the word recognition performance of the system trained using 30% of real training data improved by 6.12%, 7.56%, 9.33%, for 5000, 10000, and 20000 lexicon size, respectively. The performance of the system trained using combined features also improved by 4.67%, 5.66%, 6.67%. Similarly, the performance of system trained using 100% of the real training data improved by 1.32%, 1.77%, and 2.91% using the point-based features and 0.87%, 1.09%, and 1.24% using combined features. Similar results are obtained for the IRONOFF and Assamese word databases. The results of the IRONOFF and Assamese word databases are shown in Table 5.11 and Table 5.12, respectively.

**Table 5.13:** Performance comparison (error rate %) with other works reported on UNIPEN character database. R+S denotes using both real and synthetic data.

Method	Digit	Uppercase	Lowercase
DTW [117]	2.90	7.20	9.30
OnSNT [112]	1.10	4.30	7.90
ANN [1]	0.80	3.10	5.10
GP-SVM [7]	0.73	2.33	3.95
CNN-features [118]	0.63	2.16	3.45
Two-stage system [119]	0.86	3.58	4.07
this work (DNN-HMM 100% R+S)	0.51	1.85	2.93

**Table 5.14:** Comparison of error rate (%) with performance of reported system on the UNIPEN-ICROW-03 word database. R+S denotes using both real and synthetic data.

Lexicon Size	GMM-HMM (R+S)	DNN-HMM (R+S)	[123]	[118]	[7]
5000	18.72	9.57	29.16	14.35	25.21
10000	21.10	10.42	32.98	16.84	29.22
20000	23.13	11.31	37.41	19.32	33.27

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Therefore, the proposed modification method can be utilized for improving the system performance of both English and Assamese scripts. The recognition results show that the data-augmentation can improve the recognition performance significantly in case of a low amount of training data. We compare the character recognition performance on the UNIPEN character database obtained in this work with the reported results in Table 5.13. Finally, a comparison of the word recognition results obtained in this work with reported results in the literature is made in Table 5.14 and Table 5.15 for the UNIPEN-ICROW-03 and IRONOFF word databases, respectively. It can be observed that the best recognition performance is obtained for the DNN-HMM based system with data augmentation for both character and word databases.

**Table 5.15:** Comparison of error rate (%) with performance of reported system on the IRONOFF word database. R+S denotes using both real and synthetic data.

Lexicon Size	GMM-HMM (R+S)	DNN-HMM (R+S)	[124]
1000	17.43	9.34	24.60
2000	21.89	10.81	30.00
10000	25.90	11.04	36.20

## 5.5 Summary

This work presents a handwriting modification method using the multi-component sinusoidal model. We analyze the effect of variation of different model parameters on the handwriting shape. It is observed that the amplitude of horizontal and vertical velocity affects the width and height of the trajectory, respectively. Both local and global variations of amplitude within a permissible range result in a change of the handwriting shape that could be attributed to the natural handwriting variations. Similarly, the phase parameter controls the relationship between the horizontal and vertical velocity. In the proposed method, the phase parameter of the horizontal velocity is modified to obtain different variations of the original handwriting. On the other hand, it is observed that the global variation of frequency results in an increase or decrease of the handwriting size, while the shape remains the same. In contrast, a local variation of frequency within a small amount results in highly distorted handwriting. Based on this analysis, a handwriting modification method is proposed using the sinusoidal model by varying amplitude and phase. Moreover, we also vary the handwriting slant, where the parameters are varied both globally and locally to generate different variability.

The range of allowed variation for each parameter is analyzed using DTW distance. A permissible range is obtained for each parameter modification by comparing the deviation between the original and modified handwriting with natural variation. In this work, the DTW distance is used to measure the handwriting deviation between two samples. Finally, the synthesized variations of handwriting are used for data-augmentation in GMM-HMM and DNN-HMM based word recognition systems. An improvement in recognition performance is obtained for both the systems on the addition of the synthetically modified handwriting to the training data. Thus, the proposed method can successfully generate natural variations of the handwriting by modification of the different sinusoidal parameters.

As shown in the previous chapters, the multi-component sinusoidal model can represent complex handwriting trajectories adequately. It can extract efficient features from online handwriting and also modify the writing's control process. Another advantage of the model is that its velocities follow the natural handwriting characteristics. Therefore, the sinusoidal model can store a person's handwriting style and synthesize handwriting efficiently. In the next chapter, we explore the multi-component sinusoidal model for personalized handwriting synthesis using a concatenation based approach.



# 6

## Personalized handwriting synthesis

### Publications

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1. H. Choudhury, and S.R.M. Prasanna, “Synthesis of handwriting dynamics using the sinusoidal model,” in *Proc. of International Conference on Document Analysis and Recognition (ICDAR)*, pp. 870-875, 2019
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### Overview

In the previous chapters, we demonstrated that the multi-component sinusoidal model could efficiently reconstruct the handwriting (2D trace) and its velocities. The model also proved to be useful for synthesizing modified handwriting samples by variation of its parameters. Thus, these parameters can also be utilized to store the writer-specific handwriting styles efficiently in a compact manner. This motivates us to propose a personalized handwriting synthesis system using the multi-component sinusoidal model. In the first step, the writer's handwriting is segmented into characters using a hidden Markov model (HMM), which are then represented using the multi-component sinusoidal model and stored in the system. The first y-coordinate of each segmented sample is also stored in the system as it is required for vertical alignment of the characters during synthesis. The proposed method follows the concatenation-based approach. In concatenation-based synthesis, a generation model produces handwriting at smaller units that are concatenated to synthesize handwriting at a higher-level. The generation model in this work is at the character level stored in form the sinusoidal parameters. The output of the system is produced at word-level formed by the concatenation of the generated characters. Given an input text in the form of ASCII code, the isolated handwriting for the characters is first generated using the model parameters. These characters are then joined using a sinusoidal model based method for joining characters. This method estimates the model parameters such that it generates smooth dynamics between the ending and beginning regions of the two characters. The experiments are performed for English handwriting using the UNIPEN-ICROW-03 word database. The personalization and the naturalness of the synthesized handwriting are evaluated using subjective evaluation methods.

### 6.1 Introduction

In the present computer age, almost all text information is transmitted virtually in the digital form using the typeface letter fonts. These fonts lack the uniqueness and personal touch present in the handwriting of a person. A personalized handwriting synthesis system can incorporate the user's writing style into the digital information. It can add a personal touch to the emails or messages by enabling a person to send information in handwritten fonts. Moreover, it can also facilitate the font personalization in devices using handwriting, similar to the personalized wallpaper and other settings. Thus, personalized handwriting synthesis has many applications and is an important research area.

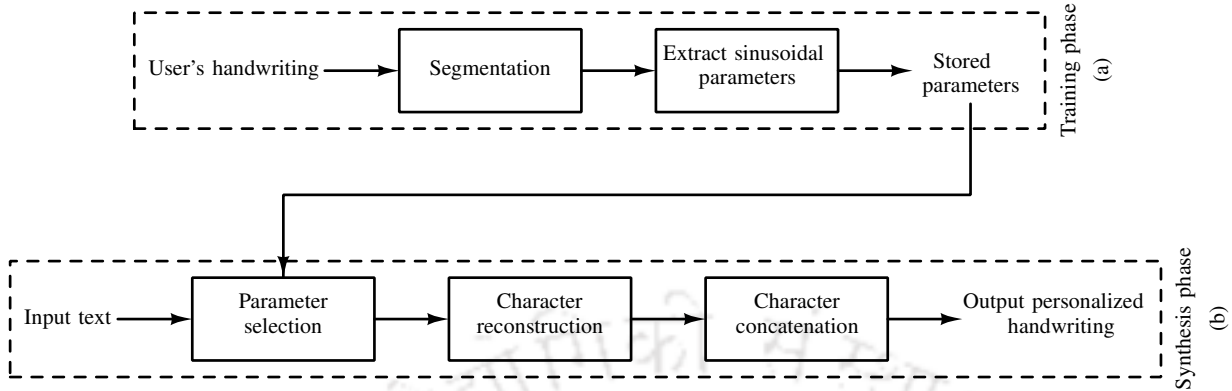
The personalized handwriting synthesis system usually follows a concatenation based technique where smaller units (stroke, characters, or sequence of characters) are generated using various methods and then joined to form the handwritten words or sentences [12, 96, 97, 101, 102, 104, 128]. We reviewed this in detail in Section 2.4 of Chapter 2. The major challenge of personalized handwriting synthesis is to learn the writing style of the targeted writer using a less number of samples. Broadly, two approaches are used for capturing the shape of these smaller handwriting units of the target writer. The first approach is to store the handwriting samples of the targeted writer in the form of its  $(x, y)$  coordinates [12, 96, 128]. These  $(x, y)$  coordinates are used directly for synthesizing the personalized handwriting. As a result, the synthesized handwriting lacks natural variations present in the real handwriting [96]. Some works proposed perturbation models or geometric deformation models on the  $(x, y)$  sequence to generate the new samples [12, 93, 94]. The second approach is based on statistical models [97, 102, 129] to capture the natural variability and shape of the handwriting. These models capture the statistics on the displacements of the sample points from a template representation. New sample points are then drawn from the statistical model to generate the handwriting shapes. However, a major limitation for this approach is the requirement of a large amount of training data [97, 102]. Moreover, none of these approaches consider the dynamic properties of the handwriting.

In this chapter, we explore the multi-component sinusoidal model to develop a personalized handwriting synthesis system. The proposed system follows the concatenation approach for handwriting synthesis. As shown in Chapter 3, this model can compactly represent the handwriting using the model parameters. The 2D trace of the handwriting and its horizontal and vertical velocities can be reconstructed accurately using the model parameters. Therefore, it can also be used to store the handwriting samples of the target writer in a compact format.

In the proposed system, the target writer's word-level handwriting is used to train the system. The handwritten words are segmented into characters by using a Gaussian mixture model (GMM)-HMM based forced alignment method. The GMM-HMM models used for segmenting the writer's handwriting are recognition models built using the UNIPEN-ICROW-03 word database. The segmented character data are modeled using the multi-component sinusoidal model to store the parameters. One advantage of using word-level data for training the system is that unconstrained handwriting contains more information about the writer's handwriting, such as the effect of the neighboring characters.

Now, the naturalness of the synthesized handwriting also depends on the connection between

## 6. Personalized handwriting synthesis



**Figure 6.1:** Block diagram of the personalized handwriting synthesis. The subplot (a) shows the training phase and (b) shows the synthesis phase.

the generated samples. Therefore, we also present a concatenation technique using the sinusoidal model. The proposed method estimates the model parameters between the end region of the previous character and the beginning region of the next character to obtain smooth joining ligatures between the adjacent characters. We hypothesize that as the sinusoidal model is based on movement theory, it will produce natural-looking ligatures. Another advantage of using the proposed method is that the synthesized handwriting follows the dynamic characteristics of natural handwriting. Therefore, these methods can facilitate the synthesis of online handwriting with both natural-like shapes and velocities. The synthesized handwriting looks natural and is similar to the user's original handwriting, as shown by the experimental results. We evaluate the synthesized handwriting using two subjective evaluation methods. Human subjects are asked to (i) identify the synthesized handwriting from a collection of real and synthesized handwriting and (ii) identify the writer of the synthesized handwriting.

The rest of the chapter is organized as follows. The proposed personalized handwriting synthesis system is described in Section 6.2. In Section 6.3, we present the proposed concatenation method for generating the joining ligatures between neighboring characters. The experimental set-up and the results obtained in this chapter are described in Section 6.4. Finally, the work in this chapter is concluded by a summary in Section 6.5.

## 6.2 Proposed personalized handwriting synthesis system

The proposed personalized handwriting synthesis system consists of the training and synthesis phases, as depicted in Figure 6.1. The input given to the training phase is the user's word-level handwriting. The word handwriting is first segmented into character level, followed by the extraction

of sinusoidal parameters stored in the system. In the synthesis phase, the text for which the user's handwriting needs to be generated is fed to the system. Depending on the letters present in the input text, sinusoidal parameters are chosen from the system to reconstruct the character's handwriting in an isolated form. These isolated characters are then joined using a sinusoidal model based concatenation method to synthesize cursive personalized handwriting. In the following, we give a detailed description of each step of the system.

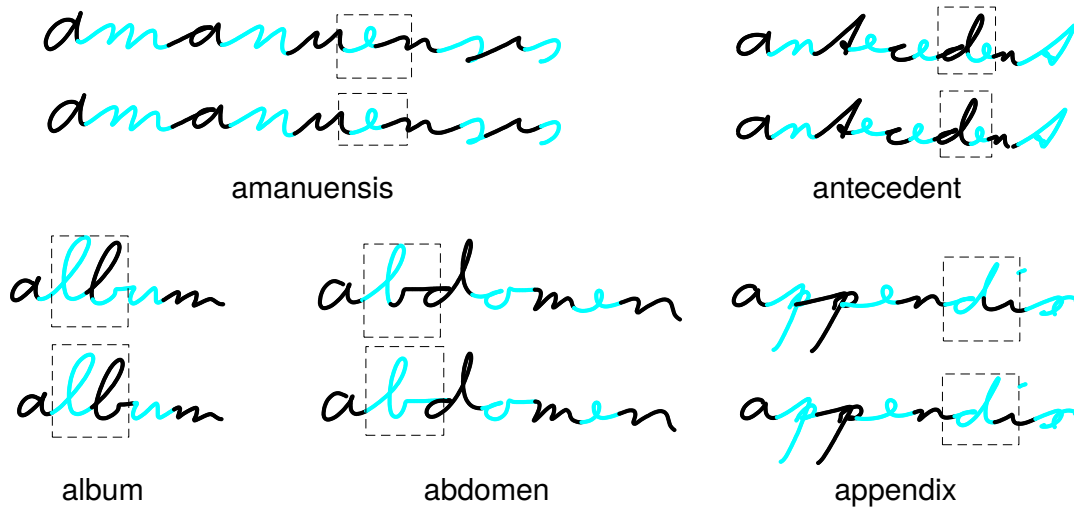
### **6.2.1 Training phase**

The training phase of the proposed system consists of the segmentation of the handwritten words followed by parameter extraction, as described in the following.

#### **6.2.1.1 Segmentation using GMM-HMM forced alignment**

The quality of synthesized handwriting depends on the segmented characters obtained from the training data. One of the popular methods used for automatic segmentation of handwriting given the correct transcription is the GMM-HMM framework. In this method, the feature vector corresponding to handwriting is forced aligned with the states of the GMM-HMM word model. The word models are formed by the concatenation of character models corresponding to the transcription letters. In this method, the initial models are trained using either a flat start or bootstrap initialization. In flat start initialization, the HMMs are trained with uniform segmentation of the data, i.e., each character in transcription is assigned an equal number of feature vectors. On the other hand, in bootstrap initialization, a small amount of data with ground truth boundaries are employed to train the initial character models. An iterative process of training and boundary re-estimation using Baum-Welch re-estimation method [130] is carried out such that the HMMs are well trained. The iteration terminates when a convergence criterion, such as a maximum number of iteration, is fulfilled.

Conventionally, the forced alignment in the standard system is carried out in the forward direction. However, this method often yields inaccurate boundaries, where the segmented characters do not contain the complete pattern or contain some portion of its neighboring character. In other words, the segmentation boundaries are improper, which may not be admissible as the ground truth, or may increase the manual effort in case of semi-automatic segmentation scenario. The segmentation errors obtained from this standard GMM-HMM based forced alignment are examined, and an early-boundary problem is observed. Due to this early-boundary problem, the character segmentation boundaries



**Figure 6.2:** Character boundaries obtained by the proposed forward-reverse alignment technique. The top figure shows the result obtained from the conventional forward method, and the bottom figure shows the result from the proposed method. The rectangles mark the correctly segmented boundaries by the proposed method.

appear before their actual boundaries. To improve the character segmentation boundaries, we use an automatic boundary correction technique in the HMM framework that aligns the handwritten data from both forward and backward directions [131]. The reverse alignment results in a late segmentation boundary, i.e., the segmentation point is detected after the actual boundary point. Finally, boundaries obtained from both forward and reverse alignments are averaged for better localization of character boundaries. The improved character boundaries obtained using this approach are shown in Figure 6.2. It illustrates the character boundaries obtained from the HMM-based segmentation method after ten iterations with and without boundary correction. The regions enclosed by the rectangles at the top show the boundary misalignment due to the early-boundary problem, and the corrected boundary is shown at the bottom.

### 6.2.1.2 Sinusoidal parameter extraction

Suppose a segmented handwriting sample has coordinate sequence  $P = \{(x_1, y_1), (x_2, y_2), \dots, (x_T, y_T)\}$ , the horizontal and vertical velocity profiles are computed as:

$$v_x(t) = \frac{x_{t+1} - x_{t-1}}{2}, \quad v_y(t) = \frac{y_{t+1} - y_{t-1}}{2},$$

where  $2 < t < T - 1$  and  $T$  is the total points in the trajectory.

The horizontal and vertical velocities  $v_x$  and  $v_y$  are represented using  $K = 2$  sinusoidal components

as follows:

$$v_x(t) = \sum_{m=1}^K a_{v_x}^m(t) \sin(\omega_{v_x}^m(t).t + \phi_{v_x}^m(t)) \quad (6.2.1)$$

$$v_y(t) = \sum_{m=1}^K a_{v_y}^m(t) \sin(\omega_{v_y}^m(t).t + \phi_{v_y}^m(t)) \quad (6.2.2)$$

where  $a_{v_x}^m$ ,  $\omega_{v_x}^m$  and  $\phi_{v_x}^m$  are the amplitude, frequency, and phase of the  $m^{th}$  component from horizontal velocity. Similarly,  $a_{v_y}^m$ ,  $\omega_{v_y}^m$  and  $\phi_{v_y}^m$  are the sinusoidal parameters of the  $m^{th}$  component from vertical velocity.

The parameters of the multi-component sinusoidal model are extracted iteratively by fitting half a period of sine wave between every successive zero-crossing points. The detail of the parameter extraction method described in Section 3.3 of Chapter 3. The sinusoidal parameters– amplitude, frequency, and phase of the two components are stored for the segmented character samples. We also store the y-coordinate ( $y_1$ ) of the character’s first point to store the vertical or baseline-alignment information. The number of samples for each character depends on the training data collected from the writer. Therefore, for each segmented characters we store  $a_{v_x}^{(1)}$ ,  $a_{v_x}^{(2)}$ ,  $\omega_{v_x}^{(1)}$ ,  $\omega_{v_x}^{(2)}$ ,  $\phi_{v_x}^{(1)}$ ,  $\phi_{v_x}^{(2)}$ ,  $a_{v_y}^{(1)}$ ,  $a_{v_y}^{(2)}$ ,  $\omega_{v_y}^{(1)}$ ,  $\omega_{v_y}^{(2)}$ ,  $\phi_{v_y}^{(1)}$ ,  $\phi_{v_y}^{(2)}$ , and ( $y_1$ ).

### 6.2.2 Synthesis phase

The synthesis phase comprises of generation and concatenation of the character samples using the sinusoidal representation.

**Parameter selection** As discussed earlier, the sinusoidal parameters of each segmented character from the user’s handwriting are stored during the training, which are used to reconstruct the handwriting during synthesis. Therefore, the first step to synthesize personalized handwriting is the selection of sinusoidal parameters for each character present in the text that is to be synthesized. Now, for each letter, there can be multiple segmented characters for which sinusoidal parameters are stored in the system. In this work, the parameters for the required letters are selected randomly from the stored multiple samples. These parameters are then fed to the reconstruction module to generate the required handwriting in an isolated manner.

**Reconstruction using sinusoidal parameters** The handwritten character in the user’s writing style is synthesized using the reconstructed velocity profiles  $v_x^r$  and  $v_y^r$  obtained from the sinusoidal

parameters. The reconstructed velocity profiles are given by:

$$v_x^r(t) = \sum_{m=1}^2 a_{v_x}^m(t) \sin(\omega_{v_x}^m(t).t + \phi_{v_x}^m(t)), \quad 1 \leq t \leq T \quad (6.2.3)$$

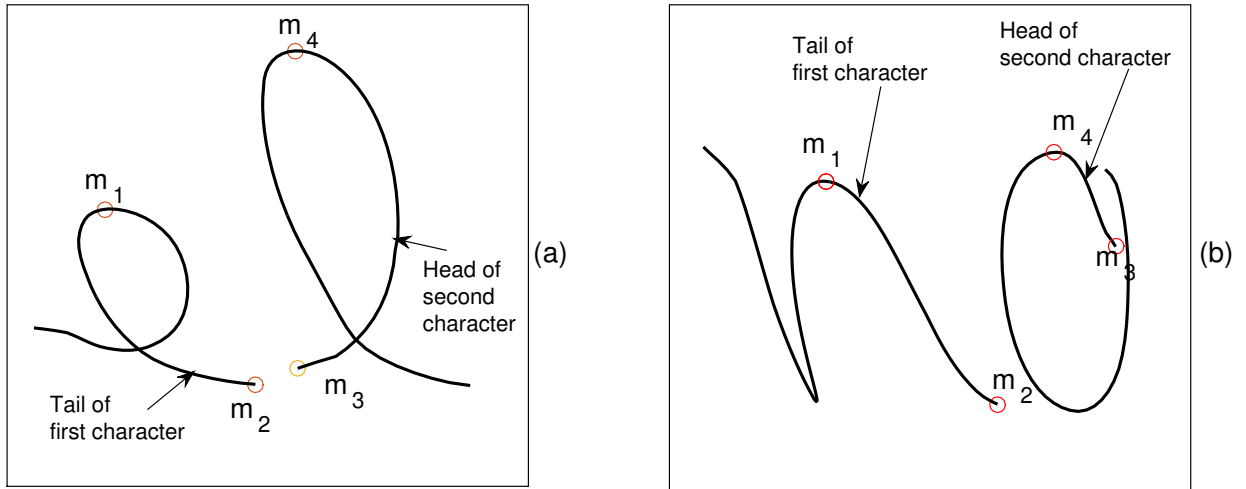
$$v_y^r(t) = \sum_{m=1}^2 a_{v_y}^m(t) \sin(\omega_{v_y}^m(t).t + \phi_{v_y}^m(t)), \quad 1 \leq t \leq T \quad (6.2.4)$$

where  $K$  is the number of components and the parameters  $a_{v_x}^m, a_{v_y}^m, \omega_{v_x}^m, \omega_{v_y}^m, \phi_{v_x}^m, \phi_{v_y}^m$  are the sinusoidal parameters stored for the  $m^{\text{th}}$  component selected from the stored parameters. For vertical alignment,  $y_1$  stored for the corresponding character is added to the  $y$ -coordinate of the synthesized character. The horizontal alignment of the synthesized characters is carried out using the method followed in [102].

**Concatenation of characters** In the next step, the reconstructed handwritten characters are joined using synthesized ligatures. The joining of two characters in natural handwriting depends on the shape of adjacent letters. Therefore, we propose a method for estimating the parameters of the sinusoidal oscillations based on the corner shapes at joining regions of the two characters. The proposed method synthesizes joining ligatures with natural-looking shape and velocities. A detailed description is given in the next section.

### 6.3 Concatenation using sinusoidal model

In this work, we propose a method to synthesize natural-like joining ligatures between two characters by using the sinusoidal model. The proposed method modifies the tail and head regions of the adjacent characters to form a smooth joining trajectory. The head region lies between the first point of the character and its successive vertical velocity zero-crossing point. Similarly, the tail region lies between its last vertical velocity zero-crossing point and the endpoint of the character. Now, as discussed in Section 1.2.3 of Chapter 1, the oscillatory theory considers the handwriting as a continuous process where the parameters of the two oscillations are modulated at the zero-crossings of the vertical velocity. Therefore, in our proposed concatenation method, we consider a sequence of modulation points  $m_1, m_2, m_3$ , and  $m_4$  consisting of the start point and endpoint of the tail region of the first character followed by the start point and endpoint of the head region of the second character. The illustration of the head and tail regions of two character pairs are given in Figure 6.3 (a) and (b). We estimate the model parameters for horizontal and vertical velocities between each pair of modulation points  $m_i$  and  $m_{i+1}$  based on the corner shape at  $m_i$  and  $m_{i+1}$ .



**Figure 6.3:** The subplots (a) and (b) show the head and tail regions of two different adjacent character pairs along with the modulation points  $m_1$ ,  $m_2$ ,  $m_3$ , and  $m_4$ .

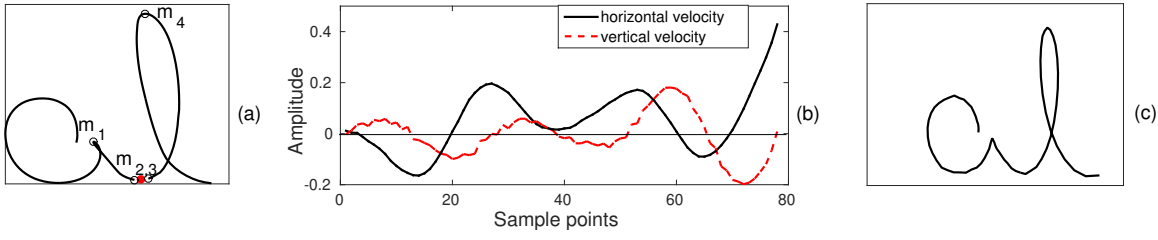
It is observed that to obtain a smooth transition between the characters, a deletion or insertion of a modulation point is required between the head and tail regions if their corresponding vertical velocities are of opposite phases or signs. This deletion or insertion is carried out to maintain the oscillatory nature in the velocity profiles as described in the following:

- **Deletion:** We remove the two modulation points consisting of the endpoint of the tail region and start point of the head region if the horizontal velocity is positive at both the joining corners. These points are replaced by a modulation point obtained by the average of the two points. In this case, the joining ligature comprises only the tail and head region of the adjacent characters. The points  $m_2$  and  $m_3$  are merged to obtain a modulation point at its average location such that coordinates of  $m_{2,3}$  is given by

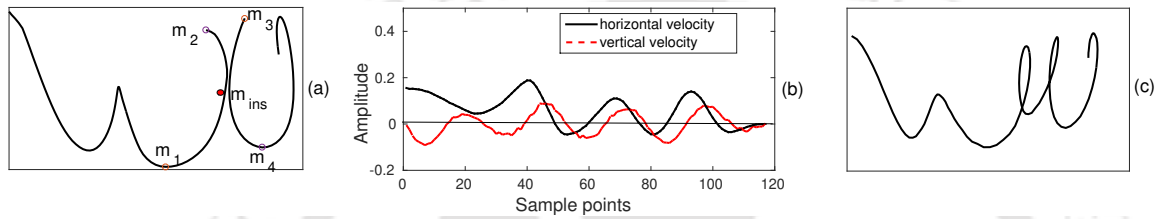
$$(x_{m_{2,3}}, y_{m_{2,3}}) = \left( \frac{x_{m_2} + x_{m_3}}{2}, \frac{y_{m_2} + y_{m_3}}{2} \right)$$

Thus, the ligature can be synthesized using three modulation points. The same is illustrated in Figure 6.4 for real handwriting samples where the horizontal velocity is positive at the joining corner denoted by  $m_{2,3}$ . The subplot (a) illustrates the two characters that are concatenated. The subplots (b) and (c) show concatenated velocities and handwriting obtained after insertion of the modulation point.

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**Figure 6.4:** Illustration of deletion of modulation point to synthesize joining ligature. The subplot (a) shows the three modulation points between ‘a’ and ‘l’ after deletion. The subplots (b) and (c) show the velocity profiles and final handwriting outcome of the proposed framework, respectively.



**Figure 6.5:** Illustration of insertion of modulation point  $m_{ins}$  to synthesize joining ligature. The subplot (a) shows the five modulation points between ‘w’ and ‘o’ after insertion. The subplots (b) and (c) show the velocity profiles and final handwriting outcome of the proposed framework, respectively.

- Insertion: A modulation point is inserted between the points  $m_2$  and  $m_3$  of tail and head regions, respectively if the horizontal velocity at any of these points is negative.

The inserted point  $m_{ins}$  is given by

$$x_{m_{ins}} = \frac{x_{m_2} + x_{m_3}}{2}$$

$$y_{m_{ins}} = y_{m_2} + (y_{m_1} - y_{m_2}) * \zeta$$

where  $\zeta \in [0.1 \ 0.3]$

Thus, in this case, the joining ligature comprises a vertical velocity full cycle in addition to the tail and head region of the adjacent characters. Therefore, the joining ligature is synthesized using five modulation points. The same is demonstrated in Figure 6.5 for a character pair ‘wo’.

Now, after insertion or deletion let  $m_1 = (x_{m_1}, y_{m_2}), \dots, m_N = (x_{m_N}, y_{m_N})$  be the sequence of modulation points for which the vertical and horizontal velocities need to be synthesized, where  $3 \leq N \leq 5$ . It is to be noted for synthesizing the joining ligature velocities we use a handwriting trajectory synthesized with only one sinusoidal component  $K = 1$  in this work.

### 6.3.1 Vertical velocity synthesis

The proposed method considers a vertical velocity zero-crossing at each of these modulation points and therefore synthesizes the vertical velocity by considering a sine half-wave between two successive modulation points. The amplitude of the half-wave is computed based on the height of the handwriting, and the time period is computed based on the length of the trajectory and the specific user's writing speed. In the following, we describe the steps performed to estimate the sinusoidal parameters between  $m_i$  and  $m_{i+1}$ .

- The first step is computation of frequency.

Let  $d_i$  be the approximate length of the trajectory between  $m_i$  and  $m_{i+1}$ . The time duration to write the trajectory  $t_{d_i}$  is computed as

$$t_{d_i} = d_i \times t_{writer} \quad (6.3.1)$$

where  $t_{writer}$  is the average time or sample points per unit trajectory length of the target writer. The time period of vertical oscillation between  $m_i$  and  $m_{i+1}$  is given by  $T_\omega = 2 \times t_{d_i}$ . Therefore the  $\omega_y$  is given by,

$$\omega_{v_y}^{(1)} = 2\pi \frac{1}{T_\omega} \quad (6.3.2)$$

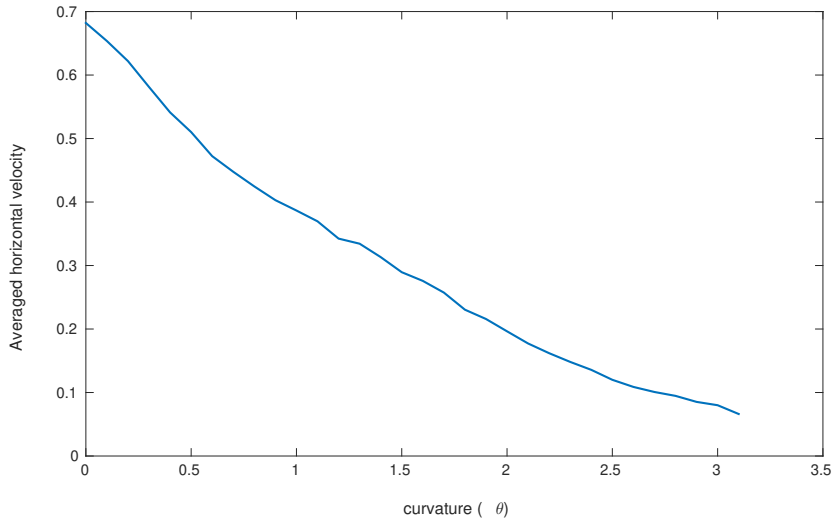
- The second step is computation of amplitude considering area under the half cycle of vertical velocity is equal to the vertical distance  $h$  between the two modulation points  $m_i$  and  $m_{i+1}$ .

$$a_{v_y}^{(1)} = \frac{2 \times h}{\omega_{v_y}^{(1)}}, \quad (6.3.3)$$

- Next, the phase  $\phi_{v_y}^{(1)}$  is chosen depending on the writing direction. If the trajectory is moving upward at the  $i^{th}$  modulation instant, then  $\phi_{v_y}^{(1)}$  is 0, and else  $\phi_{v_y}^{(1)}$  is  $\pi$ .

### 6.3.2 Horizontal velocity synthesis

The horizontal velocity controls the corner shapes at the modulation points. Therefore, the sinusoidal parameters for the synthesis of horizontal velocity need to be computed based on the required corner shapes at points  $m_i$  and  $m_{i+1}$ . The corner shape can be defined by the curvature of the trajectory. Therefore, we first analyze the relationship between the curvature and horizontal velocity in handwriting. Figure 6.6 shows the plot between the curvature angle and horizontal velocity at



**Figure 6.6:** The plot shows the relation between the corner shape and horizontal velocity at vertical velocity zero-crossings. The horizontal axis shows the curvature of the trajectory, and the vertical axis shows the corresponding horizontal velocity value averaged across all the samples in the UNIPEN database for lowercase letters.

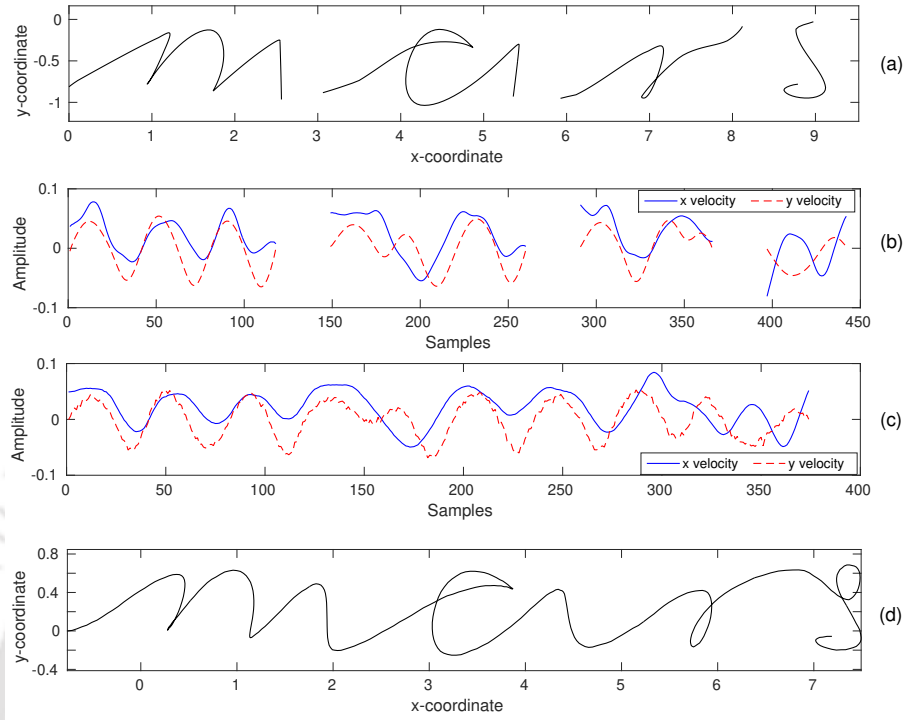
vertical velocity zero-crossing points averaged across all samples for the UNIPEN-ICROW-03 word database. It can be observed that the magnitude of horizontal velocity increases when the curvature angle decreases and vice-versa. The horizontal velocity decreases when the curvature angle is high, i.e., a sharp change in direction. Therefore, based on the required curvature at the  $m_i$  and  $m_{i+1}$ , the horizontal velocity can be computed. It is to be noted that the corner shapes or the horizontal velocity at  $m_1$  and  $m_N$  are considered from the reconstructed velocities of individual characters. On the other hand, we choose a low curvature value at the rest of the modulation points randomly between 0.3 to 0.8.

Let the values of horizontal velocity at the  $m_i$  and  $m_{i+1}$  be  $q_i$  and  $q_{i+1}$ , respectively and  $A$  is the horizontal distance between  $m_i$  and  $m_{i+1}$ . Therefore, the parameters for synthesis of horizontal velocity between  $m_i$  and  $m_{i+1}$  is computed as follows:

$$a_{v_x}^{(1)} \sin(\omega_{v_x}^{(1)} t_i + \phi_x) = q_i \quad (6.3.4)$$

$$a_{v_x}^{(1)} \sin(\omega_{v_x}^{(1)} t_{i+1} + \phi_x) = q_{i+1} \quad (6.3.5)$$

where  $t_i = 0$  and  $t_{i+1} = t_{d_i}$  (obtained from vertical velocity frequency computation (6.3.1)). Now, the



**Figure 6.7:** Illustration of proposed concatenation method for word ‘mars’. The subplots (a) and (b) show the characters and corresponding velocities reconstructed using the multi-component sinusoidal parameters. The subplot (c) shows the continuous velocities synthesized using the proposed method by concatenating velocities shown in (b). The final synthetic cursive handwritten word is shown in subplot (d).

total distance traversed by the trajectory in the horizontal direction between  $t_i$  and  $t_{i+1}$  is equal to the area under the curve given by

$$\int_{t_i=0}^{t_{i+1}=t_{d_i}} a_{v_x}^{(1)} \sin(\omega_{v_x}^{(1)} t + \phi_x) dt = A \quad (6.3.6)$$

where  $A$  is the total horizontal distance traversed between  $m_i$  and  $m_{i+1}$ , i.e.,  $A = x_{m_{i+1}} - x_{m_i}$ .

Suppose,

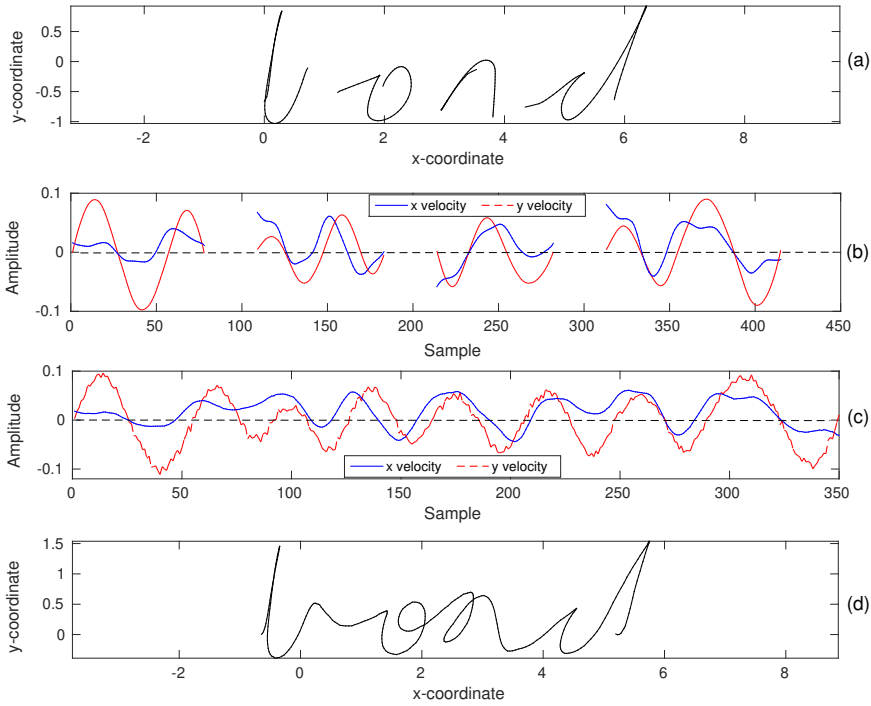
$$\begin{aligned} \theta_1 &= \omega_{v_x}^{(1)} t_i + \phi_x = \phi_x \\ \theta_2 &= \omega_{v_x}^{(1)} t_{i+1} + \phi_x = \omega_{v_x}^{(1)} t_{d_i} + \phi_x \\ \omega_{v_x}^{(1)} &= \frac{\theta_{i+1} - \theta_i}{t_{i+1} - t_i} = \frac{\theta_{i+1} - \theta_i}{t_{d_i}} \end{aligned}$$

Therefore, from (6.3.4) and (6.3.5)

$$a_{v_x}^{(1)} \sin(\theta_i) = q_i \quad (6.3.7)$$

$$a_{v_x}^{(1)} \sin(\theta_{i+1}) = q_{i+1} \quad (6.3.8)$$

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**Figure 6.8:** Illustration of proposed concatenation method for word 'bond'. The subplots (a) and (b) show the characters and corresponding velocities reconstructed using the multi-component sinusoidal parameters. The subplot (c) shows the concatenated continuous velocities synthesized using proposed method. The final synthetic cursive handwritten word is shown in subplot (d).

Equation (6.3.6) can be simplified as

$$\begin{aligned}
 \frac{a_{v_x}^{(1)}}{\omega_{v_x}^{(1)}} [\cos(\theta_i) - \cos(\theta_{i+1})] &= A \\
 \implies \frac{a_{v_x}^{(1)} \times (t_{d_i})}{\theta_{i+1} - \theta_i} [\cos(\theta_i) - \cos(\theta_{i+1})] &= A \\
 \implies a_{v_x}^{(1)} \cos(\theta_i) - a_{v_x}^{(1)} \cos(\theta_{i+1}) - \frac{A}{t_{d_i}} (\theta_{i+1} - \theta_i) &= 0 \quad (6.3.9)
 \end{aligned}$$

Now an optimum solution for (6.3.7)-(6.3.9) can be found to obtain the required horizontal velocity profile. The value of  $q_i$  and  $q_{i+1}$  decides the shape of the trajectory at  $t_i$  and  $t_{i+1}$ , respectively. As the handwriting is written from left to right in English, therefore, the total displacement by the ligature in the horizontal direction,  $A$  in (6.3.6) is always positive. Figure 6.7 and 6.8 describes the cursive handwriting synthesized using the proposed method. It shows isolated reconstructed handwriting and their corresponding velocities in the subplot (a) and (b), respectively. The concatenated continuous velocities and the cursive handwriting are shown in subplots (c) and (d), respectively.



**Figure 6.9: Writer-A** The subplot (a) shows the real handwriting, (b) shows the synthesized handwriting for words which are present in the training database, and (c) shows the synthesized handwriting for words which are not present in the training database.

## 6.4 Results and discussion

This section describes the experiments that are performed in this work and present the results obtained from the various experiments.

### 6.4.1 Database and experimental set-up

In this work, we use word data from the UNIPEN-ICROW-03 database [113]. As this work mainly focuses on cursive handwriting synthesis, the writers with only cursive writing style are chosen for the experiments. It is to be noted that the discrete style of writing is not considered in this work. For the GMM-HMM based segmentation, the handwriting between baseline to the headline is normalized from



**Figure 6.10: Writer-B** The subplot (a) shows the real handwriting, (b) depicts the synthesized handwriting for words which are present in the training database, and (c) depicts the synthesized handwriting for words which are not present in the training database.

0 to 1. A 14-dimensional geometric feature is extracted at each point of the normalized handwriting. The number of states and mixtures is optimized to 12 and 20, respectively. The details of the GMM-HMM model is given in Chapter 3.

### 6.4.2 Illustration of personalized handwriting synthesis

For an illustration of the synthetic personalized handwriting, the synthetic handwriting of three different writers A, B, and C, are shown in Figure 6.9, 6.10, and 6.11, respectively. In Figure 6.9, the subplot (a) shows the real handwriting of the writer-A for twelve different words. The subplot (b) demonstrates the synthesized handwriting of text for which the writer’s handwriting is present in the training data. On the other hand, the synthesized handwriting for the text (not present in the



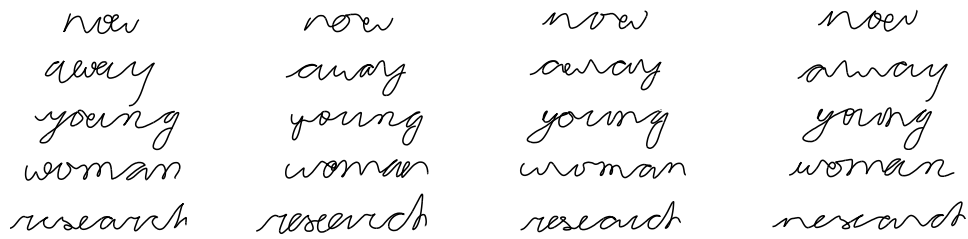
**Figure 6.11: Writer-C** The subplot (a) shows the real handwriting, (b) shows the synthesized handwriting for words that are present in the training database, and (c) shows the synthesized handwriting for words that are not present in the training database.

training data) is shown in subplot (c). It can be observed that the proposed method can synthesize handwriting for text, both present and absent in the training data. Similarly, the real and synthesized handwriting for writer-B and writer-C are demonstrated in Figure 6.10, and 6.11, respectively. It can be observed that the synthesized handwriting is similar to the person's real handwriting.

The cursive handwriting is synthesized using the sinusoidal parameters extracted from character level data segmented from the writer's real handwriting. As the reconstruction quality using the multi-component sinusoidal model is efficient, it is obvious that the writing style of the targeted person remains preserved in the synthesized handwriting. Therefore, the proposed method can produce readable handwriting in a particular person's writing style using only a few handwritten words of the

## 6. Personalized handwriting synthesis

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**Figure 6.12:** Each row shows four different synthesized samples for same word.

person. Depending on the selection of the particular template in the form of the stored parameters, different handwriting variations can be synthesized. Figure 6.12 shows different handwriting variations of the same word produced for the same writer. Finally, the synthetic handwriting generated using the proposed framework is evaluated using subjective methods, i.e., using human perception. In the following, we describe the subjective evaluation methods and their results in detail.

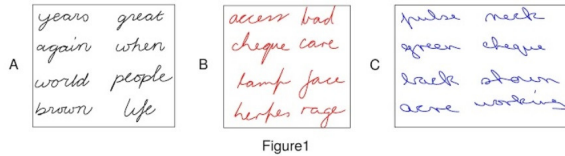
### 6.4.3 Subjective evaluation

In this work, two tests are performed to evaluate the proposed personalized handwriting system: (a) writer personalization test and (b) naturalness test of the synthetic handwriting. The details of these evaluation methods are given in the following.

**Writer personalization test** A subjective test is a traditional evaluation method for synthesis tasks [12, 104]. In this work, we ask 25 persons/subjects to evaluate our results. The handwriting of three writers A, B, and C, is shown to the subjects. The subjects are then asked to identify the writer of 50 words (out of which 20 are real and 30 are synthetic handwriting) by comparing with the writer's handwriting. Figure 6.14 (a) shows a questionnaire example for the personalization test. The subjects do not know if a word is real or synthetic handwriting. We compare the accuracy of writer identification obtained for the synthetic words and original words. A similar identification rate will indicate that the proposed method can successfully imitate the writer's handwriting.

**Naturalness test** In this test, we evaluate the naturalness of the synthesized handwriting. For this test, we prepared a list of 50 words comprising both real and synthetic handwriting produced by our proposed method. We asked 25 subjects to choose if they are real or synthetic (machine-generated). There are 20 real online handwriting samples and 30 synthetic samples in the evaluation questionnaire.

1. Select which writer the handwriting resembles by matching it with the handwriting given in Figure 1. \* 5 points



Select if you think the given handwritten words in Figure 1 are real or synthesized \* 10 points

- 1. warm 2. please 3. car 4. query 5. humble
- 6. name 7. inform 8. doll 9. herpes 10. camera

1. peak 2. had 3. please 4. press 5. fever

	Writer A	Writer B	Writer C
1. peak	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2. had	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3. please	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4. press	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5. fever	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

(a)

	Real	Synthesised
1. warm	<input type="radio"/>	<input type="radio"/>
2. please	<input type="radio"/>	<input type="radio"/>
3. car	<input type="radio"/>	<input type="radio"/>
4. query	<input type="radio"/>	<input type="radio"/>
5. humble	<input type="radio"/>	<input type="radio"/>

(b)

Figure 6.13: Example of subjective test questionnaire for (a) writer personalization test (b) naturalness test.

Table 6.1: Writer identification error (%) obtained from subjective personalization test shown separately for the real and synthetic samples.

	Writer Identification Error (%)			Overall Identification Error
	Writer A	Writer B	Writer C	
Real handwriting	24.08	40.28	19.45	27.93
Synthetic handwriting	38.89	31.79	11.50	27.40

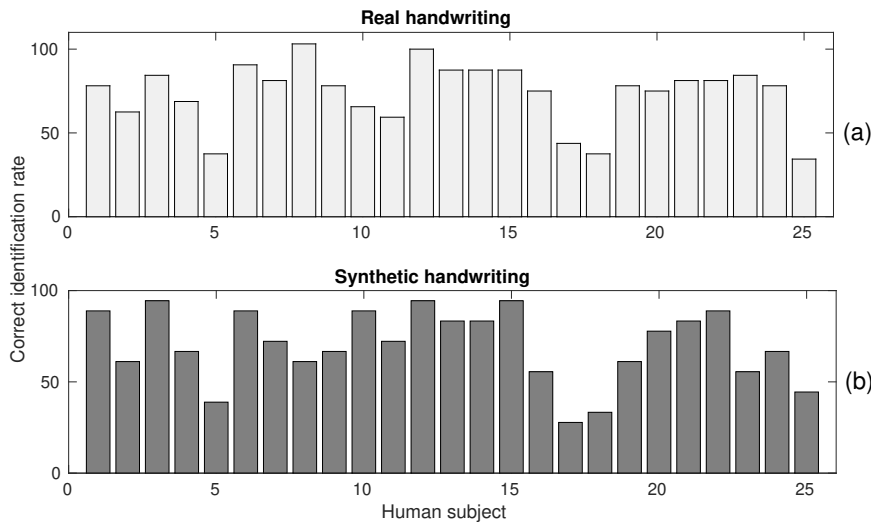
Figure 6.14 (b) shows a part of this questionnaire. The human subjects are unaware of the number of real handwriting given for the evaluation. Two types of error can be made here: (i) real handwriting identified as synthetic and (ii) synthetic handwriting identified as real data. The average of these two errors defines the overall classification error. A high classification error rate indicates the confusion between the real and synthetic handwriting, i.e., the synthetic handwriting is similar to the natural data. The results of the evaluation test are demonstrated in the following.

The detailed result of the personalization test is given in Table 6.1. The subjects were asked to identify the writer of the given samples in a questionnaire by comparing it with the writers' original handwriting. The questionnaire consisted of both original and real handwriting of the writers. The error rate for the individual writers is shown separately, for both real and synthetic data. A similar

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**Table 6.2:** Comparison of writer confusion (%) shown for both real and synthetic handwriting, obtained from the subjective personalization test.

Identification error	Real Handwriting	Synthetic Handwriting
Writer-A identified as Writer-B	20.37	33.33
Writer-A identified as Writer-C	3.70	5.55
Writer-B identified as Writer-A	34.25	19.44
Writer-B identified as Writer-C	6.01	12.34
Writer-C identified as Writer-A	7.40	3.33
Writer-C identified as Writer-B	12.03	8.14



**Figure 6.14:** Histogram showing the correct writer identification rate of the 25 human subjects on (a) real data (b) synthetic data.

overall error rate is obtained for both real and synthetic handwriting. The confusion rates for the different writers are presented in Table 6.2. It can be observed that the confusion between Writer A and Writer B is high for both real and synthetic handwriting. Figure 6.14 (a) and (b) show the correct writer identification rate of each subject individually for the real and synthetic samples, respectively. It shows that the identification rates of the subjects are almost similar for real and synthetic handwriting. Therefore, it can be said that the proposed personalized handwriting synthesis system can synthesize writer specific handwriting.

The results for the naturalness test are presented in Table 6.3. It can be observed that the false synthetic error rate and false real error rate are 32.60 % and 64.80 %, respectively. The high value

**Table 6.3:** Evaluation results of the naturalness test.

Average rate of real handwritten samples identified as real:	67.39 %
Average rate of synthetic handwritten samples identified as synthetic:	35.20 %
Average rate of real handwritten samples identified as synthetic : (False synthetic error rate)	32.60%
Average rate of synthetic handwritten samples identified as real : (False real error rate)	64.80 %
Overall error rate:	48.70 %

(47.39%) of average classification error shows that the subjects are highly confused in identifying the synthetic ones due to their naturalness when synthetic and original samples are mixed together. This empirically establishes that the synthetic samples look as natural as the original ones.

## 6.5 Summary

In this work, we introduced a personalized handwriting synthesis system using the multi-component sinusoidal model. The multi-component sinusoidal model provides a compact representation of the handwriting using the model parameters. As this model is based on the hand-movement theory and can control the handwriting trajectory shape, it has been utilized for synthesizing personalized handwriting. The input required to train the system is a set of words written by the target person consisting of at least one sample for each letter. The proposed method segments the target writer's handwriting into a character level using HMM-based forced alignment method. This is followed by the extraction of sinusoidal model parameters from the segmented characters. At the time of synthesis, the handwriting for any given text is synthesized for a particular person by using these stored sinusoidal parameters. A concatenation method is also proposed using a sinusoidal model. It synthesizes the joining ligature's velocity profile depending on the ending and starting region of adjacent letters. The results show that the proposed method can synthesize natural-looking personalized handwriting. We performed two subjective tests to evaluate the personalization and naturalness of the synthetic handwriting. All these experiments indicate that the generated samples are clear enough for humans to recognize and imitate the targeted person's writing style.



# 7

## Summary and conclusions

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### Overview

In this chapter, we summarize the work presented in this thesis towards developing a multi-component sinusoidal model for online handwriting. The proposed model has been explored in three different directions: feature representation for online handwriting recognition, handwriting modification for data augmentation, and personalized handwriting synthesis. In the last section of this chapter, we outline different future directions based on these contributions.

#### 7.1 Summary of contributions and results

In the following, we summarize the contributions and results of each working chapter of this thesis.

- In Chapter 3, we proposed a multi-component sinusoidal model for online handwriting based on the oscillatory theory. The model parameters are extracted from the horizontal and vertical velocities between successive zero-crossing points. An iterative method is presented to extract the different components from the residual signals. It is observed that the low-frequency components represent the basic shape and high-frequency components represent the finer details of the character shapes. Therefore, the use of multiple sinusoidal components results in a better representation of complex handwriting trajectories. The proposed model is evaluated in terms of the reconstruction quality using two approaches. First is the SNR of the reconstructed velocities, and the second is the performance of a recognition system trained on natural handwriting and tested on the reconstructed samples. It is observed that the SNR value increases significantly up to three sinusoidal components. Moreover, the HMM-based recognition system also showed better performance on the reconstructed handwriting obtained using the multi-component sinusoidal model as compared to only a single component. The experiments are carried out on both English and Assamese handwriting databases. Table 7.1 shows the SNR for the reconstruction of horizontal and vertical velocities using the different number of sinusoidal components for the UNIPEN English word database. The recognition performance on the reconstructed samples for the same database is also shown in Table 7.2. The number of states and mixtures for the GMM-HMM is optimized to 11 and 20, respectively, for English word recognition.
- In Chapter 4, we used the multi-component sinusoidal model for extracting sinusoidal model-based features from online handwriting. We illustrated that the multiple sinusoidal oscillations could better represent the handwriting for recognizing confusing handwritten character pairs.

**Table 7.1:** Mean and standard deviation of  $\text{SNR}_{\text{dB}}$  for reconstruction of horizontal ( $\text{SNR}_{v_x, \text{dB}}$ ) and vertical velocity ( $\text{SNR}_{v_y, \text{dB}}$ ) for UNIPEN-ICROW-03 word database (Chapter 3).

Sinusoidal Components	Mean $\text{SNR}_{v_x, \text{dB}}$	Mean $\text{SNR}_{v_y, \text{dB}}$	Standard Deviation, $\text{SNR}_{v_x, \text{dB}}$	Standard Deviation, $\text{SNR}_{v_y, \text{dB}}$
1	14.81	20.34	5.00	5.21
2	26.42	30.84	5.93	6.06
3	30.99	33.86	6.91	6.70
4	32.26	35.23	7.29	7.28
5	32.66	35.78	7.45	7.63

**Table 7.2:** Error rate (%) of the GMM-HMM based word recognition system evaluated on the reconstructed data of UNIPEN-ICROW-03 database (Chapter 3).

Lexicon	$K=1$	$K=2$	$K=3$	$K=4$
5000	30.20	27.46	26.93	26.70
10000	34.23	30.94	30.28	29.98
20000	38.04	34.13	33.35	33.04

Thus, improved recognition performance is obtained for both GMM-HMM and SVM-based recognition systems. Similar to the velocities, the oscillatory behavior is also observed in the acceleration and position signals. Therefore, to enhance the handwriting representation for the recognition system, we modeled the acceleration and normalized position signals. The final feature set comprises of parameters extracted from the velocities, accelerations, and position signals of the handwriting. The sinusoidal model-based features are also combined with point-based features to obtain the best recognition performance. Table ?? shows the recognition error rate of the GMM-HMM based system for the UNIPEN English word database with baseline point-based features, sinusoidal features, and combined features.

- In Chapter 5, we proposed a handwriting modification method using the multi-component sinusoidal model for data-augmentation. The multi-component sinusoidal model provides an understanding of the control variables of handwriting and the flexibility to change it. Therefore, we explored this advantage of the model for online handwriting modification. First, a permissible range is determined for variation of each sinusoidal parameter by analyzing the DTW distance between the original and the modified handwriting. The variation between the real and modified handwriting should be within the range of natural variation present in real handwriting. Therefore, the model parameters are varied within the permissible limits to synthesize modified handwriting samples. The modified handwriting is then used for data augmentation in GMM-

## 7. Summary and conclusions

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**Table 7.3:** Error rate (%) of the HMM-based word recognition system evaluated on the test sets of UNIPEN-ICROW-03 English word database.

	Lexicon	Features		
		Point-based	Sinusoidal	Combined
GMM-HMM (Chapter 4)	5000	26.43	24.70	20.04
	10000	29.80	28.18	23.23
	20000	32.92	31.04	25.36
GMM-HMM (Chapter 5)	5000	24.73	—	18.72
	10000	26.51	—	21.10
	20000	30.41	—	23.13
DNN-HMM (Chapter 5)	5000	11.86	—	9.57
	10000	13.57	—	10.42
	20000	14.81	—	11.31

HMM and DNN-HMM based word recognition systems. We obtained an improved recognition performance for both the systems on the addition of the modified handwriting to the training data. The experiments are performed for both point-based and combined (point-based and sinusoidal model-based ) features. Table ?? shows the recognition error rate for the UNIPEN word database. The number of states for the DNN-HMM system is 11, and each hidden layer has 300 neurons.

- In Chapter 6, we proposed a personalized handwriting synthesis system using the multi-component sinusoidal model parameters. First, the word-level handwriting from the target writer is segmented into characters using GMM-HMM based forced alignment method. The segmented characters are then stored in the form of parameters of multi-component sinusoidal model with two components. During synthesis, the required characters are reconstructed from these parameters and concatenated to form a cursive handwriting. A character concatenation method is also proposed using the sinusoidal model to synthesize natural-looking handwriting. The model parameters for the connecting ligature between characters are estimated based on the joining regions (the ending and beginning regions) of two adjacent characters. We demonstrated that as the sinusoidal model is based on generation theory, it can synthesize handwriting that follows the dynamic properties of the handwriting. Two subjective tests are performed to evaluate the naturalness and personalization of the synthesized handwriting. In the first evaluation test, 25 persons are asked to identify the real and machine generated handwriting from a list comprising of both real and synthetic handwriting. It is observed from the error rate that the synthetic

**Table 7.4:** Evaluation results of the naturalness test.

Average rate of real handwritten samples identified as real:	67.39 %
Average rate of synthetic handwritten samples identified as synthetic:	35.20 %
Average rate of real handwritten samples identified as synthetic : (False synthetic error rate)	32.60%
Average rate of synthetic handwritten samples identified as real : (False real error rate)	64.80 %
Overall error rate:	48.70 %

**Table 7.5:** Writer identification error (%) obtained from subjective personalization test shown separately for the real and synthetic samples.

	Writer Identification Error (%)			Overall Identification Error
	Writer A	Writer B	Writer C	
Real handwriting	24.08	40.28	19.45	27.93
Synthetic handwriting	38.89	31.79	11.50	27.40

samples look as natural as real handwriting. In the second evaluation method, the subjects were asked to identify the writer for a list of 50 words comprising both real and synthetic handwriting. The results shows that a similar identification rate is obtained for real and synthesized handwriting. Tables 7.4 and 7.5 shows the subjective evaluation results for the naturalness and personalization test of the synthesized handwriting

## 7.2 Conclusions

The main conclusions of this thesis are as follows:

- In the first work, we observed that modeling each velocity signal using a single sinusoidal oscillation (POMH model) is not sufficient for representing complex handwriting trajectories. It could not capture the detailed shape of the characters. This can result in confusion between different classes in pattern recognition systems. Therefore, an improved sinusoidal model using multiple oscillations is proposed for online handwriting..
- It is observed that the improvement in SNR of the velocity reconstruction is greatest when the number of components is increased from one to two.
- The sinusoidal parameters are explored as features for the recognition system, and it is observed that the amplitude and frequency parameters of the multi-component sinusoidal model contain

## 7. Summary and conclusions

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more important information for describing the handwriting trajectory.

- As the sinusoidal feature set is different from the conventional geometric features and extracts complementary information. Therefore, a combination of the two feature set results in improved recognition performance. The efficacy of the proposed features has been shown for character recognition and word recognition tasks.
- We analyzed the effect of varying the sinusoidal parameters on the handwriting trajectory shape. It is observed that the amplitude parameter of the sinusoidal model controls the amount of distance traversed by the trajectory between the zero-crossing locations. Similarly, the change in phase parameters of the sinusoidal oscillations changes the relation between velocities at the zero-crossing locations, which alter the corner shapes of the handwriting.
- As this model is based on the handmovement theory, the transformed handwriting with parameter variation within certain limits does not lose the naturalness of handwriting. The modified handwriting can improve the recognition performance of a system when added to the training set along with the original handwriting. This shows that natural handwriting variations can be obtained using the proposed method. However, not much improvement in recognition is obtained after data-augmentation when the original training set is large.
- The multi-component sinusoidal model is also explored to store the writer's handwriting in a personalized handwriting synthesis system. The character data can be stored in a compact form using the model parameters instead of the whole 2D trace. Moreover, an appropriate selection of model parameters can also synthesize the joining ligatures between characters to obtain cursive writing.

### 7.3 Directions for future work

Based on the work of this thesis work, several topics for future directions for presented below.

- In Chapter 3 of the thesis, we have shown that a single sinusoidal oscillation is not sufficient for modeling the velocity of complex handwriting. We proposed the multi-component sinusoidal model, where the parameters are extracted iteratively. Instead of using the iterative method, other approaches may be investigated similar to the works in speech processing [132].
- Further, the multi-component sinusoidal model is explored mainly for English and Assamese handwriting. A possible investigation would be to analyze the model for scripts with more

pen-lifts like Chinese and Korean scripts.

- In Chapter 4, we proposed representation for online handwriting recognition using the sinusoidal model parameters. We utilized these model parameters to understand the letter shape from the movement information. However, writer-specific information can also be captured from the parameters, which can be employed for signature verification and writer identification tasks. As the proposed features are based on parametric representation, the conventional dynamic time warping (DTW) approach for measuring distance may not be suitable in this case. Therefore, an effort can be made to explore a new framework for verification tasks to utilize the proposed model representation.
- In Chapter 5, we modified the handwriting to synthesize multiple samples from the original handwriting. The modified handwriting samples are utilized for data augmentation in the handwriting recognition system. Another application of the proposed modification method may be to synthesize signature databases. The writer-specific and writer-independent variability of the parameters can be analyzed as a future direction, which can then be helpful for synthesizing genuine and forgery signatures.
- In Chapter 6, we proposed a personalized handwriting synthesis method, where the sinusoidal model parameters are stored instead of the 2D trace of segmented letters from the user's writing. At the time of synthesis, the selection of letter samples (i.e., its parameters) is performed randomly. It will be interesting if some method is proposed for the selection of the letters based on the parameters of the adjacent letters so that the joining ligature is close to natural writing.
- The proposed sinusoidal model may also be explored for writer adaptation. Instead of storing the parameters for all the characters for the targeted writer, the parameters may be adapted from another writer. Thus, an efficient synthesis system for personalized handwriting can be developed using a small set of user's handwriting.
- The proposed sinusoidal representation may also be explored for the beautification of the online handwriting written on touch-screen devices by modification of the original parameters extracted from the handwriting.





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- H. Choudhury, and S.R.M. Prasanna, “Representation of online handwriting using multi-component sinusoidal model,” *Pattern Recognition*, vol. 91, pp. 200-215, 2019.
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- H. Choudhury, S. R. M. Prasanna, and S. Dandapat, “Modification of handwriting using sinusoidal representation and its application for training data augmentation in recognition system”. ( **To be submitted**)
- H. Choudhury, S. R. M. Prasanna, and S. Dandapat, “Personalized handwriting synthesis using sinusoidal model”. ( **To be submitted**)

### Not related to thesis

- H. Choudhury, S. Mandal, and S. R. M. Prasanna. “Exploiting forced alignment of time-reversed data for improving HMM-based handwriting segmentation,” *Expert Systems with Applications*, vol. 121, pp. 158-169, 2019.
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## List of Publications

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- A. Pal, B. K. Khonglah, S. Mandal, H. Choudhury, S. R. M. Prasanna, H. L. Rufiner and V. N Balasubramanian. “Online Bengali handwritten numerals recognition using Deep Autoencoders,” in *Proc. of National Conference on Communication (NCC)*, pp. 1-6, 2016.



