**INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI**
**SHORT ABSTRACT OF THESIS**

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| Name of the Student | : | DEVANG SINHA |
| Roll Number | : | 196123105 |
| Programme of Study | : | Ph.D. |

Thesis Title: Monte Carlo and Multilevel Monte Carlo Methods with Applications in Financial Engineering

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| Name of Thesis Supervisor(s) | : | Prof. Siddhartha Pratim Chakrabarty |
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**SHORT ABSTRACT**


We undertake an in-depth investigation of the Monte Carlo simulation approach and its various extensions to tackle computationally demanding problems that frequently arise in the field of Quantitative Finance. Specifically, we concentrate on three major domains: derivative pricing, risk management, and portfolio optimization, each of which involves significant computational complexity. By leveraging advanced Monte Carlo techniques, we aim to improve numerical efficiency, accuracy, and convergence rates in these financial applications.

In the context of derivative pricing, we explore an innovative hybrid algorithm that combines Multilevel Richardson-Romberg extrapolation (ML2R) with adaptive importance sampling to enhance the overall computational efficiency of Monte Carlo estimators. The ML2R technique provides a systematic means of reducing bias while improving the accuracy of numerical approximations, whereas adaptive importance sampling seeks to minimize estimator variance through an optimally chosen change of measure. We establish theoretical guarantees for the convergence of this hybrid methodology, ensuring that it remains robust even when applied to the pricing of financial derivatives. To validate our approach, we conduct numerical experiments within the quantitative finance framework, demonstrating the superior performance of our hybrid method in comparison to the standard ML2R method.

Regarding risk management, we delve into the role of stochastic optimization within a biased sampling framework, with a particular emphasis on the Sample Average Approximation (SAA) method. The SAA framework is widely used in stochastic programming to approximate the optimal value of a decision problem by replacing the expected value with a sample-based empirical mean. We investigate the uniform convergence properties of SAA and analyze the computational cost associated with achieving an accurate estimation of the optimal value. Additionally, we incorporate the Multilevel Monte Carlo (MLMC) method within the SAA framework to improve computational efficiency in solving stochastic optimization problems. We demonstrate that by leveraging MLMC one can reduce

computational costs to achieve desired accuracy. As part of our analysis, we conduct a root-mean-squared error (RMSE) study, assessing the trade-off between computational effort and estimation accuracy. To substantiate our theoretical insights, we perform numerical simulations in which we estimate Conditional Value-at-Risk (CVaR)—a widely used risk measure—in the context of a Geometric Brownian Motion (GBM) model and a nested expectation setting. Our empirical results illustrate the benefits of integrating MLMC with SAA, particularly in terms of reducing variance and improving the precision of CVaR estimation under stochastic dynamics.

Finally, in the domain of portfolio optimization, we focus on the efficient computation of the minimum-CVaR portfolio, an essential problem in financial risk management that involves constructing a portfolio that minimizes the risk measure CVaR. To this end, we study a variance-reduced variant of Stochastic Gradient Langevin Dynamics (SGLD) to solve the minimum-CVaR portfolio optimization problem efficiently. The SGLD algorithm is particularly well-suited for high-dimensional optimization problems with noisy gradient information, as it incorporates stochastic noise to improve convergence properties. By introducing a variance-reduction technique, we aim to enhance the stability and accuracy of the SGLD algorithm while ensuring faster convergence to the optimal portfolio allocation. We provide rigorous non-asymptotic error bounds for the Expected Excess Risk, quantifying the precision of our variance reduced SGLD method in solving the optimization problem. Furthermore, we conduct extensive numerical experiments to evaluate the practical effectiveness of our proposed methodology, demonstrating its ability to achieve improved portfolio allocations with lower computational costs. Our results underscore the potential of variance-reduced SGLD as a powerful tool for risk-averse portfolio optimization in high-dimensional settings.