INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI

SHORT ABSTRACT OF THESIS

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Programme of Study	: Ph.D.
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The main objective of this thesis is to examine purely critical and supercritical exponent problems involving nonlocal operator in the symmetric domain. The nonlocal superlinear semipositone problem is also investigated.

In the first chapter, we talk about the rationale behind writing the thesis as well as the primary goals that it aims to achieve. After that, we provide a precise summary of the most important aspects of our primary difficulties as well as the significance of our works.

The preliminaries are presented in the second chapter. The fractional Sobolev spaces and embedding results are discussed. We look at how different solutions concepts, such as viscosity and weak solutions, are connected. We also examine topological techniques like genus theory used in later chapters.

In the third chapter, we establish Struwe's type global compactness result for the following critical exponent problem involving fractional p-Laplace operator in a bounded domain $\Omega \subset \mathbb{R}^N$ which is invariant under the action of a group G of orthogonal transformations

$$(P^s_{p,Q,\Omega}) \begin{cases} (-\Delta)^s_p u &= Q(x)|u|^{p^*_s-2}\,u \quad \text{in } \Omega, \\ u &= 0 \qquad\qquad\qquad\quad \text{on } \Omega^c, \end{cases}$$

where $0 < s < 1$, $1 < p < \infty$ such that $sp < N$, $p_s^* := \frac{Np}{N-ps}$ is fractional critical Sobolev exponent, Q is continuous and strictly positive in $\overline{\Omega}$. We provide a detailed description of all G-invariant Palais Smale sequences for the energy functional associated with the problem $(P^s_{p,Q,\Omega})$.

In the fourth chapter of the thesis, we study Coron's type problems involving nonlocal operators with critical nonlinearities.
In the first part of this chapter, we consider the following problem

$$(P^s_\Omega)\begin{cases} (-\Delta)^s u &= |u|^{2_s^*-2}\, u \quad \text{in } \Omega, \\ u &= 0 \qquad\quad \text{on } \Omega^c, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded annular domain which is invariant under a group G of orthogonal transformations of $\mathbb{R}^N$. We establish the existence of a positive and multiple sign-changing solutions to the problem (P^s_Ω).
In the second part of the fourth chapter, we show the existence of a positive and multiple sign-changing solutions to problem $(P^s_{p,\Omega})$

$$(P^s_{p,\Omega})\begin{cases} (-\Delta)^s_p u &= |u|^{p_s^*-2}\, u \quad \text{in } \Omega, \\ u &= 0 \qquad\quad \text{on } \Omega^c, \end{cases}$$

in some bounded domain $\Omega \subset \mathbb{R}^N$ with nontrivial topology under some symmetry assumptions.

The fifth chapter of the thesis is dedicated to the study of supercritical exponent problem involving fractional Laplace operator. We consider the following nonlocal problem

$$(P^s_{b,\Omega})\begin{cases} (-\Delta)^s u &= b(x)|u|^{q-2}\, u \quad \text{in } \Omega, \\ u &= 0 \qquad\quad\ \ \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with some symmetry assumptions, $N \geq 2s$, $s \in (0,1)$, $b \in C^{0,\alpha}(\overline{\Omega})$ and positive, $q > 2_s^*$. Here, we show the existence of a positive and multiple sign-changing solutions to problem $(P^s_{b,\Omega})$.

The sixth chapter deals with study of nonlocal superlinear subcritical semipositone problem. We prove the existence of a positive solution to the following Dirichlet boundary value problem

$$\begin{cases} (-\Delta)^s_p u &= \mu(u^r - 1) \text{ in } \Omega, \\ u &> 0 \qquad\quad \text{in } \Omega, \\ u &= 0 \qquad\quad \text{on } \Omega^c, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with C^2 boundary, $p - 1 < r < p_s^* - 1$, $\mu > 0$ is a parameter.

Finally, the last chapter of the thesis deals with study of nonlocal superlinear critical semipositone problem. We show the existence of a positive solution to the following Dirichlet boundary value problem

$$\begin{cases} (-\Delta)^s_p u &= u^{p_s^*-1} - \mu \text{ in } \Omega, \\ u &> 0 \qquad\quad \text{in } \Omega, \\ u &= 0 \qquad\quad \text{on } \Omega^c, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with C^2 boundary and $\mu > 0$ is a parameter.