INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

The central theme of this thesis is to study the moments of certain exponential sums, which we primarily capture by some asymptotic formulas. Our results are conceptual advancements to previously known methods and results on Kloosterman sums, generalized Gauss sums, and double exponential sums. Firstly, we introduce some new techniques to find the number of solutions of certain congruence equations modulo a prime p, which eventually allow us to obtain a sharper bound compared to previously known bounds for the fourth power mean of the 3-dimensional Kloosterman sum. We further employ our techniques to find an asymptotic formula for the fourth power mean of the 4-dimensional Kloosterman sum. Secondly, we study a conjecture of Wenpeng Zhang on higher order moments of the generalized quadratic Gauss sums weighted by L-functions. Previously known works on this conjecture used Weil's bound on curves, which seems to be insufficient to make further progress in proving the conjecture. We estimate some complicated character sums involving quadratic characters and find asymptotic formulas for such sums by relating these sums to traces of cohomology sheaves. With the help of the new estimates, we establish Zhang's conjecture upto weight 4. Thirdly, we study a problem on double exponential sums. The problem of estimating double exponential sums in prime fields is of a great interest in analytic number theory. We consider a double exponential sum where the range of the summation depends on some subsets X and Y of finite fields. As usually happens in this kind of sums, the problem is relatively easy when one or both subsets are initial intervals of the form {1, 2,…,N}; for some integer N; as one can control the size of product of elements in the range. In our work we consider intervals of arbitrary position and propose a new approach which leads to a new estimate of the given sum.