



**INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI**  
**SHORT ABSTRACT OF THESIS**

Name of the Student : Jibrail Ali  
Roll Number : 146123009  
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Name of Thesis Supervisor(s) : Rafikul Alam  
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The thesis addresses three broad problems in the area of theory and numerics of nonlinear eigenvalue problems. The first problem concerns with spectral analysis of holomorphic and meromorphic eigenvalue problems. The second problem concerns with perturbation theory of holomorphic eigenvalue problems as well as realization and linearization of meromorphic matrices. The third problem concerns with numerical solution of holomorphic and meromorphic eigenvalue problems.

Canonical forms such as Smith forms of matrix polynomials and Smith-McMillan forms of rational matrices are important tools for spectral analysis of matrix polynomials and rational matrices. We present, among other things, Smith forms for holomorphic matrices and Smith-McMillan forms for meromorphic matrices, which are akin Smith forms and Smith-McMillan forms of matrix polynomials and rational matrices. Also, we present a detailed system theoretic analysis (e.g. matrix fraction descriptions (MFDs), realizations, and analytic system matrices) of meromorphic matrices and establish relationships between canonical forms of meromorphic matrices, their MFDs and analytic system matrices.

Linearization is an important method for solving and analyzing polynomial and rational eigenvalue problems. We utilize linearization of holomorphic operator-valued functions and develop perturbation theory for holomorphic eigenvalue problems. We derive perturbation bounds for (discrete) eigenvalues and associated eigenvectors of holomorphic operator-valued functions. We analyze local minimal realizations of meromorphic matrices and compute such realizations from Markov parameters. We utilize local minimal realization of a meromorphic matrix to define and construct a linearization.

Finally, we show that numerical methods for solving holomorphic eigenvalue problems can be developed from pole finding methods for meromorphic matrices. Utilizing this point of view, we provide a unified framework for contour integration based methods for solving nonlinear eigenvalue problems. We propose and solve spectral recovery problems, which are actually moment problems of certain kinds, for linear and nonlinear eigenvalue problems.