INVESTIGATION ON SOME APPROACHES FOR IMPROVING CHANNEL EQUALIZATION IN WIRELESS ENVIRONMENT

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INVESTIGATION ON SOME APPROACHES FOR IMPROVING CHANNEL EQUALIZATION IN WIRELESS ENVIRONMENT

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Certificate

This is to certify that the thesis entitled "**INVESTIGATION ON SOME APPROACHES FOR IMPROVING CHANNEL EQUALIZATION IN WIRELESS ENVIRONMENT**," submitted by **Ramesh Chandra Mishra** (09610221), a research scholar in the *Department of Electronics and Electrical Engineering, Indian Institute of Technology Guwahati*, for the award of the degree of **Doctor of Philosophy**, is a record of an original research work carried out by him. Mr. Mishra started his research works under the supervision of Dr. Abhijit Mitra. After Dr. Mitra left the Institute, since December 2011, he has been working under my supervision and guidance. The thesis has fulfilled all requirements as per the regulations of the Institute and in my opinion, has reached the standard needed for submission. The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

Dated:

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Dedicated to

My dear parents

Late Shyam Sundar Mishra and Smt. Basantilata Mishra,

wife **Madhusmita** and son **Ayush**

for their love and support

&

My guide

Prof. Ratnajit Bhattacharjee

for his guidance and inspiration.

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Abstract

This thesis deals with channel equalization techniques for mitigating the effect of inter-symbol interference owing to the multipath propagation, and rapid channel variations due to the large Doppler spread in a band-limited wireless channel. In this work, we investigate the channel equalization performance using some algorithms, which to our best of knowledge have not been adequately explored in the context of realization of the decision feedback equalizer. We use data-reusing 'Affine Projection' (AP) and 'Binormalized least mean square' (BNLMS), and data-selective 'Set-membership Normalized least mean square' (SM-NLMS), 'Set-membership Affine Projection' (SM-AP) and 'Set-membership Binormalized least mean square' (SM-BNLMS) as adaptive algorithms to track the time-varying characteristics of the mobile channel. The aim of this work is to reduce the complexity of time domain equalizers, at the same time maintain their performance adequately in a fading environment.

In recent times, multiple-input multiple-output (MIMO) communication has come up as one of the most promising technologies due to its advantage in terms of high data rate and diversity gain without any additional bandwidth and power. MIMO figures prominently in the rapidly developing modern wireless systems such as WLAN, WiMAX, and Long-term evolution (LTE). Design of an equalizer for a MIMO receiver maintaining a balance between complexity and performance is quite challenging. This thesis also investigates the problem of channel equalization in MIMO dispersive fading environments. Performance evaluation of the decision feedback equalizer (DFE) for a MIMO system using data-reusing and data-selective adaptive algorithm has been carried out in this thesis, and some aspects of implementation of such equalizers have also been discussed.

The contribution of this thesis work includes performance investigation and feasibility assessment of data reusing and data selective algorithms for the realization of adaptive DFEs on a practical frequency selective single-input single-output (SISO) and MIMO channels.

The performance analysis includes evaluation of convergence speed, computation requirement, steady-state error and BER, when such algorithms are used in DFE for ITU-R wireless channel models recommended by the WiMAX forum.

Keywords: Affine projection; Binormalized least mean square; Data-reusing algorithm; Data-selective algorithm; Decision feedback equalizer; Multiple-input multiple-output systems; Normalized least mean square; Set-membership filtering.

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List of Acronyms

2G	Second Generation
3G	Third Generation
3GPP	The Third Generation Partnership Project
4G	Fourth Generation
ADFE	Adaptive Decision Feedback Equalizer
AP	Affine Projection
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BNLMS	Bi-normalized Least Mean Squares
BPSK	Binary Phase Shift Keying
CCI	Co-Channel Interference
CIR	Channel Impulse Response
dB	Decibel
DCD	Dichotomous Coordinate Descents
DD	Decision Directed
DFE	Decision Feedback Equalizer
FB	Feed Back
FBF	Feed Back Filter
FDE	Frequency Domain Equalizer
FF	Feed Forward
FFF	Feed Forward Filter
ICI	Inter-carrier Interference
i.i.d.	Independent and identically distributed
ISI	Inter Symbol Interference

ITU-R	International Telecommunication Union for Radiocommunication Sector
LMS	Least Mean Squares
LTE	Long-term Evolution
MIMO	Multiple-Input Multiple-Output
MLSE	Maximum-Likelihood Sequence Estimation
MMSE	Minimum Mean Square Error
MSE	Mean Square Error
NLMS	Normalized Least Mean Squares
OFDM	Orthogonal Frequency Division Multiplexing
Ped-A	Outdoor to Indoor and Pedestrian Test Environment with Low Delay Spread
RLS	Recursive Least Square
RMS	Root Mean Squared
Rx	Receiver
SC	Single Carrier
SISO	Single-Input Single-Output
SM	Set Membership
SMF	Set Membership Filtering
SNR	Signal to Noise Ratio
SS	Steady State
TDE	Time Domain Equalizer
Tx	Transmitter
V-BLAST	Vertical Bell Labs Layered Space Time
Veh-B	Vehicular Test Environment with Medium Delay Spread
WiMAX	Worldwide Interoperability for Microwave Access
WLAN	Wireless Local Area Network
ZF	Zero Forcing

List of Symbols

$\otimes$	Convolution operation
μ	Convergence factor or step size
Θ	Feasibility or solution set
$\Re^N$	Real vector space of dimension N
$\psi(k)$	Intersection of the constraint sets
γ	Upper bound on the magnitude of output estimation error
α	Regularization parameter, a small constant
$d_{0,k}$	Reference data signal at time instant k
$d_{i,k}$	Interfering signals at time instant k
e_k	Error at time instant k
$h_{0,k}$	Channel impulse response for the reference data signal at time instant k
$h_{i,k}$	Channel impulse response of interfering co-channels at time instant k
H_k	Constraint or observation set
L_{FF}	Length of feed-forward filter
L_{FB}	Length of feedback filter
$\mathcal{O}\left(P^3\right)$	Order of P^3
$\hat{x}_{k-1}$	Output of decision device at time instant $k-1$
y_k	Received signal at time instant k
$\boldsymbol{w_{FF}}$	Weight vector of feed-forward filter
$\boldsymbol{w_{FB}}$	Weight vector of feedback filter

1

Introduction

Contents

1.1 Introduction

Multipath propagation, a time-varying environment and the limited frequency spectrum are some of the features of land mobile radio channels. In such environment, the system performance gets affected due to inter-symbol interference (ISI) caused by the time-dispersive multipath channels, co-channel interference (CCI) resulting because of a reduction in cell size, rapid channel variations as a result of large Doppler spread and fading due to time-varying received signal strength. The channel equalization is necessary to mitigate the effects of ISI and track the channel variation in band-limited communication channels. The time-domain equalizers (TDE) are conventionally used for time-dispersive channels, but suffer from high complexity, convergence and tracking problems for channels with a large delay spread as the required number of taps increase significantly [1]. A single carrier frequency-domain equalizer (SC-FDE) and orthogonal frequency division multiplexing (OFDM) perform better in channels with a high delay spread [2]. However, the channel impulse response (CIR) during one OFDM symbol period may not remain constant in a channel having high Doppler spread, and it leads to inter-carrier interference (ICI) due to a loss of orthogonality between sub-carriers, and deteriorates in the BER performance [3]. Hence, based on channel conditions, the choice of equalization method which trades off the overall performance and complexity, is very important. Moreover, wireless communication in small indoor environments has been an active area of research due to large-scale deployment of micro and pico base stations within areas with high users density. In this work, we make an endeavor to reduce the complexity of TDE, maintaining their performance in the multipath fading environment having moderate delay spread.

In a mobile fading environment, adaptive equalizers are generally used to track the time-varying characteristics of the mobile channel [4]. In a wireless environment, where the channel spectrum exhibits nulls, the decision feedback equalizers (DFE) are often preferable compared to linear and maximum likelihood sequence estimation (MLSE) based equalizers on performance and complexity, respectively [5, 6]. In a rapidly varying mobile environment, the filter weights of a decision feedback equalizer are updated adaptively using some fast converging algorithms. The least mean square (LMS) and recursive least square (RLS) algorithms, or their variants are typically applied for realizing adaptive equalizers.

In recent times, multiple-input multiple-output (MIMO) system, which employs multiple antennas at both receiver and transmitter, have been introduced to achieve high spectral efficiencies. Such system exploits space-time diversity. The use of multiple antennas combined with signal processing techniques at the receiver increases the data rate and utilizes the spectrum very efficiently.

Channel equalization using LMS and RLS based algorithms on DFE for both single-input single-output (SISO) and MIMO systems in a wireless environment, is thoroughly investigated in literature [7–11]. DFE based on LMS algorithm is simple and easy to implement on the hardware, but the convergence speed of such an equalizer is slow and depends on the eigenvalue spread of the input correlation matrix. RLS algorithm based DFE offers fast convergence, and is independent of the eigenvalue spread of the input correlation matrix; however, while implementing on fixed-point hardware, it suffers from stability issues because of the fact that recursive calculations of the inverse of the input correlation matrix may diverge [12]. Kalman filter based DFE exhibits fast convergence, independent of the eigenvalue spread of the correlation matrix and assures stability when implementing on hardware at the expense of computational complexity [13].

Therefore, the motivation for this thesis is to explore efficient equalization techniques in terms of both complexity and convergence, for time dispersive SISO and MIMO wireless systems.

1.2 Literature Survey

Equalization in a frequency selective fading channel is necessary to facilitate high-speed data transmission. The adaptive equalizers [4, 14] can track the time-varying characteristics of the mobile channel. As already mentioned, adaptive DFE is often preferable over linear and MLSE equalizers in channels with a large delay spread [6]. Generally, the MLSE performs better, but in the fast fading environment, its performance is similar to a DFE [15]. To operate in a fast fading environment, the adaptive algorithm must have a fast convergence property. The LMS and RLS algorithms, or their variants are typically used in implementing DFEs [16, 17]. The LMS algorithm [18], has been widely used for adaptive filters due to its simplicity, but its convergence speed deteriorates significantly for colored input signals [1, 19–21]. The RLS algorithm originated from the use of the Kalman filtering algorithm for the estimation of equalizer coefficients [22], and Falconer et.al. [23] later recognized the Kalman filter to be a form of the RLS algorithm. It possesses fast convergence, but it has higher computational complexity compared to the LMS based equalizer [24, 25]. In literature, the LMS algorithm based DFE [7–9] and RLS algorithm based DFE [10, 11] have been well studied for the mobile fading channel.

Presently, MIMO has emerged as one of the most promising areas in wireless communication research due to its performance advantage over SISO systems [26,27]. For high data-rate transmission in the mobile fading environment between transmit and receive antennas of the MIMO system, MIMO-DFE can be used. In MIMO-DFE, multi-dimensional feed-forward filter (FFF) and multi-dimensional feedback filter (FBF) are used for performing multi-dimensional channel equalization. The performance of MMSE-DFE for the MIMO system has been studied in [28–31]. The adaptive MIMO channel equalization using RLS algorithm has been presented in [32], whose performance can be improved further by minimizing the interference from non-diagonal channel elements. By introducing a Kalman filter for tracking the channel, and an optimized MMSE-DFE to equalize the channel, inter-user interference can be suppressed as well as improved performance and good tracking can be achieved at the cost of increased complexity [33]. The use of a successive interference cancelation in RLS-DFE for vertical Bell Labs layered space time (V-BLAST) further improves the performance [34–36]. In [37], an adaptive MIMO DFE model has been reported using computationally-efficient RLS algorithm based on dichotomous coordinate descents (DCD) iterations. A low complexity soft-input soft-output DFE for the MIMO system has been reported in [38].

The data-reusing affine projection (AP) algorithm was proposed by Ozeki [39], where the recycling of old data signal is done for highly correlated input to improve the speed of convergence with less complexity in comparison to RLS based algorithms. In [40, 41], the performance of data-reusing binormalized LMS (BNLMS) algorithm, which utilizes the present as well as the immediate past input signal vector to achieve fast convergence, has been studied. In some literature, AP algorithm based channel equalization has also been discussed [42–44].

The set-membership (SM) filtering in an adaptive filter puts an upper bound on the magnitude of the estimation error while updating the filter coefficient. It reduces computational complexity due to data selective updates and exhibits better convergence and tracking properties due to the optimization of the weighting sequence [45]. The normalized least mean square (NLMS) algorithm with SM filtering [46] offers low computational complexity, but its convergence performance deteriorates for colored input signals. The SM-NLMS algorithm based DFE for the SISO system has been presented in [9]. The recursive least square (RLS) algorithm with SM filtering performs well, but at the cost of increased computational complexity [37,47,48]. The data-reusing affine projection (AP) algorithm with a SM filtering offers faster convergence with marginally higher computational complexity than the SM-NLMS based algorithms, but suffers from a trade-off between the convergence speed and the computational requirements in the information matrix inversion for highly correlated input [49, 50]. The set-membership binormalized data-reusing least mean square (SM-BNLMS) algorithm, which uses only two constraint sets for each update has been reported to have good convergence while keeping the lower implementation complexity [51]. A channel equalization technique using BNLMS algorithm with SM filtering has been discussed in [52].

1.3 Motivation

As discussed earlier, fast converging equalizers are desirable when they are to be operated in rapidly varying wireless channels. For such channels, although RLS based equalizers are often used [32]; due to its high computational complexity, a large amount of computational power is consumed in the receiver.

In the literature, data-reusing algorithms such as the affine projection algorithm [39] and BNLMS algorithm have been reported, which offer faster convergence in comparison to LMS algorithm and complexity much less than RLS algorithms for a highly correlated input. Channel equalization schemes using AP algorithm are available in literature [43,44,53]; however, the same have not been well explored for implementation in a frequency selective wireless channel.

Data-selective algorithms like SM-AP [49] and SM-BNLMS [51] exhibit faster convergence due to reusing of old data, and reduced complexity for data selective updates. Performance evaluation of DFEs employing such algorithms in both SISO and MIMO channels have not been well studied in literature. The motivation behind this thesis is to investigate approaches for improving channel equalization in the wireless environment with reduced computational complexity.

1.4 Problem Formulation

From the open literature, we note that sufficient studies are not reported for channel equalization using data-reusing algorithms. Moreover, channel equalization using data-selective algorithms for SISO and MIMO systems are not adequately investigated in the literature.

Hence, the following problems have been taken up for further investigation:

- Implementation and performance analysis of efficient equalizers using data-reusing and data-selective algorithms in time-varying SISO and MIMO wireless channels.

On the basis of the discussion presented earlier, equalizer configuration that will be considered in this thesis is an adaptive DFE, whose weights are to be updated using the data-reusing and data-selective algorithms. Performance analysis includes evaluation of convergence speed, computation requirement, steady-state error and bit error rate (BER), when such algorithms are used in DFE for ITU-R wireless channel models [54] recommended by the WiMAX forum, one for outdoor to indoor and pedestrian test environment with a low delay spread (Ped-A) and another for the vehicular test environment with a medium delay spread (Veh-B).

1.5 Thesis Contributions

In this thesis work, we have investigated the channel equalization performance using some algorithms which, to the best of our knowledge, have not been applied extensively for realizing decision feedback equalizers. In particular, the algorithms like AP, BNLMS, SM-AP and SM-BNLMS have been implemented for realizing DFE. Analytical formulation presented in this thesis are based on these algorithms which are already reported in literature. However, some modifications have been suggested for making them suitable for implementation of DFE. The performances of the proposed schemes are evaluated in both time-varying SISO and MIMO channels. It may also be noted that while implementing the concepts presented in these algorithms, appropriate representation of different signals has been used. As mentioned earlier, performance analysis includes evaluation of convergence speed, computation requirement, steady-state error and bit error rate (BER), when such algorithms are used in DFE for ITU-R recommended wireless channels. The important contributions of the thesis are stated below.

- Investigated channel equalization performances using the adaptive decision feedback equalizer in a time-varying wireless channel, where data-reusing 'Affine Projection Algorithm' and 'Binormalized LMS Algorithm' are used for updating filter weights.

- Proposed efficient channel equalization techniques using data-selective "Set-membership Affine Projection Algorithm" and "Set-membership Binormalized LMS Algorithm" for adaptive weight updating of DFE. We have modified the SM-AP and SM-BNLMS algorithms by introducing a convergence factor μ to achieve a trade-off between final misadjustment and convergence speed. An analytical derivation is carried out for the range of convergence factor μ for the SM-BNLMS algorithm.

- Evaluated performance of MIMO-DFE employing data-reusing 'Affine Projection' and 'Binormalized LMS' algorithm for MIMO wireless channel.

- Reduced complexity MIMO channel equalization technique, suitable for hand-held devices in an indoor and pedestrian environment based on AP and BNLMS algorithm with SM filtering is proposed.

1.6 Thesis Organization

The thesis comprises of seven chapters including the present one. The content of each chapter is summarized as follows:

Chapter-2:

Chapter 2 gives an overview of impairments observed in time-varying wireless communication channels and also illustrates how adaptive equalizers mitigate the effect of inter-symbol interference and track the channel variation. It also describes the basic theory and capacity of MIMO channels.

This chapter also provides the details about two WiMAX recommended frequency selective ITU-R wireless channel models, one for outdoor to indoor and pedestrian test environment for low delay spread (Ped-A) and other for vehicular test environment for medium delay spread (Veh-B), which is considered in the thesis.

Chapter-3:

This chapter investigates the performance of adaptive DFE using data-reusing algorithms in time dispersive wireless channels. Here, we also discuss the adaptive DFE system model adopted in this work. In recent years, the data-reusing algorithms have gained popularity because of their faster convergence speed than the stochastic gradient algorithms and lower computational requirement than the RLS based algorithms, in situations where the input signal is highly correlated. The data-reusing affine projection (AP) algorithm proposed by Ozeki and Umeda [39], utilizes present input signal vector along with past input signal vectors in the coefficient update conducive to achieve faster convergence. The Binormalized least mean square (BNLMS) algorithm introduced in [55], employs consecutive input signal vectors for updating coefficients. In this work, we consider AP and BNLMS algorithms for adaptively updating the filter weights of DFE in a mobile fading environment. Simulation results are presented to confirm the effectiveness of the introduced channel equalization methods.

Chapter-4:

This chapter proposes data-selective algorithms based channel equalization schemes in time-varying wireless communication channels. Set-membership (SM) filtering in an adaptive filter puts an upper bound on the magnitude of the estimation error while updating filter coefficients. It reduces computational complexity due to data-selective updates and exhibits better convergence and tracking properties due to the optimization of the weighting sequence [45]. In this chapter, we present channel equalization schemes for time-varying wireless channels, where filter weights of DFEs are updated using SM-AP and SM-BNLMS based algorithms. Here, we appropriately modify these algorithms and also introduce a convergence factor μ, which has been found to improve the channel equalization performance. The performance of the proposed schemes has been examined with different values of μ, upper bound γ on the estimation error and also projection order P in case of AP algorithm. Simulation results confirm that the proposed technique offers advantages of having fewer numbers of weight updating operations at moderate SNR with almost similar convergence as AP and BNLMS algorithms based channel equalizer. It is also found that the proposed equalizer converges faster than the SM-NLMS algorithm based equalizer. The choice of projection order and upper bound γ, which reflects a trade-off between the convergence speed, number of updates and the computational complexity, are very important. It is shown that the proposed equalizer works well even in frequency-selective channels.

Chapter-5:

The channel equalization scheme with an adaptive DFE for SISO system with data-reusing algorithms is presented in Chapter 3. The scheme performed reasonably well with low computational complexity. This chapter presents an extension of adaptive SISO DFE for frequency selective MIMO channels. Here, we discuss an adaptive MIMO-DFE system model and study the performance with data-reusing AP and BNLMS as adaption algorithms in time dispersive wireless channels.

Chapter-6:

In this chapter, we investigate the performance of an adaptive DFE based on SM-AP and SM-BNLMS algorithm for frequency selective MIMO wireless channels. The performance of the equalizer is investigated for a MIMO receiver in a multi-path fading environment as experienced in the indoor and pedestrian environment. The equalizer performance is also studied for channels having higher delay and Doppler spread. The convergence issues, BER performance and tracking capabilities are examined through computer simulations. Moreover, the computational complexity issue for this MIMO equalizer is compared with other existing data-selective algorithm based techniques.

Chapter-7:

This chapter presents the conclusion of the thesis with a brief summary of the works done in previous chapters. Besides, it introduces some future directions for extending this research work.

2

Equalization in Wireless Channels - An Overview

Contents

Objective

This chapter presents a brief introduction to the context of the thesis. Here, impairments observed in time-varying wireless channels are explained, and different channel equalization schemes are also presented. A MIMO system model is described. Channel models considered in this work are also discussed.

2.1 Introduction

In the last two decades, the rapid progress in wireless communications has triggered a communication revolution, making it possible for people to communicate reliably every-where. Nowadays, high data rate wireless communication system constitute a major research area both in academia and industry. The growth is driving technology to develop communication techniques, which can offer high quality of service at high data rates, with large number of users, under difficult propagation conditions. Wireless communication channels feature multi-path propagation, a time-varying environment and restricted frequency spectrum. Therefore, the major challenge in wireless communication research is to develop an efficient technique to deal with multi-path propagation in a limited frequency spectrum.

Multiple-input multiple-output (MIMO) communication technique uses multipath propagation to increase the data rate and improve the quality of link utilizing diversity without any additional bandwidth and power requirement. One of the major challenging aspects of MIMO receiver in a mobile fading environment is the channel equalization, due to the presence of inter-symbol interference (ISI) and co-channel interference (CCI) in addition to the noise, which deteriorates the BER performance significantly. This thesis deals with the reduced complexity equalization method for time dispersive wireless channels in both single-input single-output (SISO) and MIMO systems.

The rest of this chapter is organized as follows. Section 2.2 discusses the major impairments observed in wireless channels, Section 2.3 gives a brief introduction on channel equalization schemes and also discusses some primitive adaption algorithms. A generic MIMO system model and detail about its channel capacity is described in Section 2.4. The WiMAX forum recommended channel models considered in this work are presented in Section 2.5. Section 2.6 concludes the chapter.

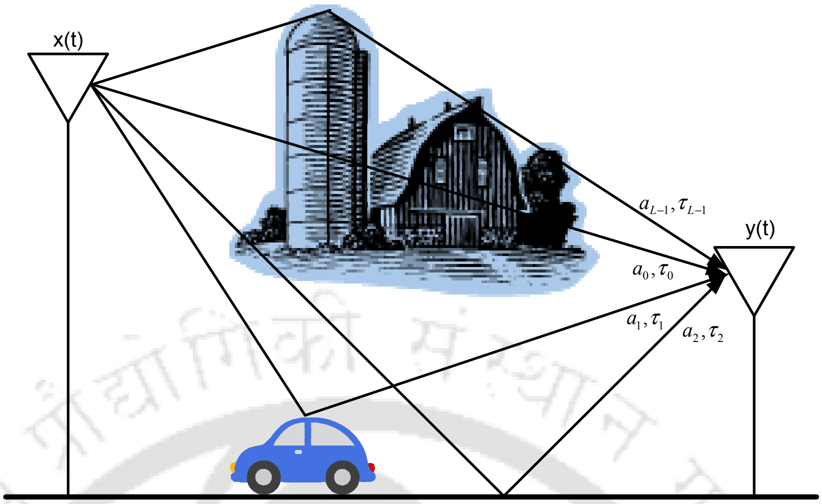


Figure 2.1: Multipath Propagation.

2.2 Major Impairments in Wireless Channels

Multipath propagation and Doppler spread are major sources of impairment in wireless channels, which can degrade the system performance.

The multipath propagation is a phenomenon that causes multiple versions of the transmitted signal to arrive at the receiver with different time delays. Reflecting objects and scatterers in the transmission environment generate multiple copies of the transmitted signal as shown in Fig. 2.1. Each of the paths will have different characteristics, such as amplitude, phase, arrival time, and angle of arrival. The multiple signals may constructively or destructively add up at the receiver, thus creating the rapid fluctuations in the received signal envelope. When the signals add up constructively it will increase the signal power at the receiver, but destructive summation will cause fading in the received signal, which corresponds to the sudden drops in received power. Multipaths not only cause fluctuations in the received power, but it also affects the shape of the pulse as it is transmitted through the channel. The arrival of the multiple versions will broaden the transmitted signal. The signals arriving on receiver at different times will overlap with each other and lead to broadening of the envelope of the received pulse. Hence, one characteristic of a multipath propagation is the delay spread, introduced in the signal that is transmitted through the channel. Delay spread causes interference between adjacent symbols, known as ISI.

We know that, mobile channels are time-varying in nature, as a result multipath varies with time. Each multipath is subjected to a frequency shift due to the relative motion between mobile station and base station. This frequency shift is called Doppler shift, which is proportional to the speed of the mobile unit. The Doppler shift in a multipath propagation environment spreads the bandwidth of the multipath waves, which results from frequency dispersion of the channel.

Moreover, system performance is also affected due to CCI resulting due to a reduction in cell size and additive noise, which is always present in any kind of transmission.

2.3 Adaptive Channel Equalization

ISI has been recognized as the major obstacle to high-speed data transmission over mobile radio channels. The mobile fading channels are also random and time varying. The purpose of communication channel equalization is to mitigate the effects of ISI and track the channel variation. Combining adaptive filters with channel equalizers increases the robustness of wireless communication systems.

The decision feedback equalizer (DFE) is preferable to the linear and maximum likelihood sequence estimation (MLSE) equalizers based on performance and complexity, respectively [56]. Moreover, in a fast fading environment, DFE performs almost same as the MLSE equalizer but with less complexity [15]. In addition, even though linear equalizers are alternative solutions to the ISI problem, they have the disadvantage of noise enhancement.

2.3.1 Decision Feedback Equalizer

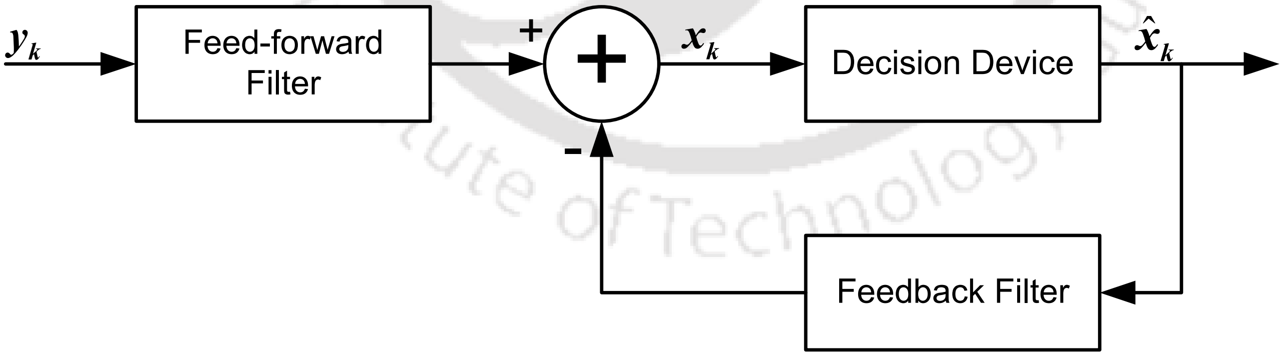


Figure 2.2: Block Diagram of a Generic DFE

A DFE is a nonlinear equalizer that employs previous decisions to eliminate the ISI. A simple block diagram of a generic DFE is shown in Fig. 2.2. It consists of a feed-forward (FF) and a feedback (FB) filter, as well as a decision device. Here, $\mathbf{y}_k$ and $\hat{\mathbf{x}}_{k-1}$ are the input signal vectors to FF and FB filters respectively. So, the input to the decision device at time instant k is:

$$x_k = \mathbf{y}_k^T \mathbf{w}_{FF} - \hat{\mathbf{x}}_{k-1}^T \mathbf{w}_{FB} \tag{2.1}$$

where $\mathbf{w}_{FF}$ and $\mathbf{w}_{FB}$ are tap weight vectors for the FF and FB filters, respectively [57]. The details are explained in chapter 3. The minimum mean squared error (MMSE) criterion is usually applied for minimization. The weights in the FF and FB filters are adjusted simultaneously to satisfy this criterion. The FF filter is designed to suppress the precursor ISI, and the FB filter suppresses the post-cursor ISI. DFE avoids amplifying noise, where channel has spectrum nulls, because of the use of combination of FF and FB filters. Although DFE outperforms a linear equalizer, it is not the optimum equalizer. The optimum detector in a digital communication system in the presence of ISI is a maximum likelihood symbol sequence detector. However, with appropriate design, a DFE can provide reasonably good performance and can satisfy the requirements of practical systems. Moreover, combining an adaptive technique with DFE improves the performance by updating the filter weights regularly according to the channel condition. Basically, an adaptive equalizer changes its filter weights based on signal inputs that have passed through an unknown channel by using specific algorithm. The basis of adaptive equalization is minimization of error, which is the difference between the desired and actual signal measured at each symbol. The error is used to determine the direction in which the tap-weights of the filter should be changed so as to approach an optimum set of values.

2.3.2 Adaptive Algorithms

In wireless communication, adaptive techniques in equalization helps to compensate for the unknown channel. It requires a specific algorithm to update the equalizer coefficients and track the channel variations. The performance of an algorithm is described in terms of various factors such as convergence rate, misadjustment factor, computational complexity and stability. The LMS and RLS algorithms, or their variants are mainly used in the field of adaptive equalization.

To explain the working of LMS and RLS algorithms, consider an adaptive FIR filter with N coefficients. The output of the adaptive filter at time index k can be expressed as:

$$\hat{v}_k = \mathbf{u}_k^T \mathbf{w}_k \tag{2.2}$$

where $\mathbf{w}_k$ and $\mathbf{u}_k$ are vectors of size $N \times 1$, containing the filter coefficients and input samples, respectively. So:

$$\mathbf{u}_k = \begin{bmatrix} u_{k,0} \\ u_{k,1} \\ \vdots \\ u_{k,N-1} \end{bmatrix} \qquad \mathbf{w}_k = \begin{bmatrix} w_{k,0} \\ w_{k,1} \\ \vdots \\ w_{k,N-1} \end{bmatrix} \tag{2.3}$$

2.3.2.1 LMS Algorithm

The LMS algorithm is a linear adaptive filtering algorithm that consists of two basic processes, i.e. filtering process and adaptive process. Filtering process involves the computation of the output of the transversal filter and the generation of the error by comparing the instantaneous values of desired and estimated output. In adaptive process, automatic adjustment of the tap weights are made according to the error produced. The prediction error is the difference between desired signal and estimated signal. So, the error at time instant k is:

$$e_k = d_k - \hat{v}_k = d_k - \mathbf{u}_k^T \mathbf{w}_k \tag{2.4}$$

Thus, the equalizer weights are updated as:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu e_k \mathbf{u}_k \tag{2.5}$$

LMS algorithm is convergent if the step size μ satisfies the condition

$$0 < \mu < \frac{2}{Trace\left(\boldsymbol{R_{uu}}\right)} \tag{2.6}$$

where $\boldsymbol{R_{uu}}$ is the autocorrelation matrix of input signal vector.

This is simple to implement, but it suffers from slow convergence when the eigenvalue spread of the autocorrelation matrix is large, which limits the real time application. As the filter coefficients are directly proportional to instantaneous value of the input, any fluctuation in the input results in divergence of the algorithm. Moreover, when the input signal is non-stationary, the eigenvalues also change with time and selection of step size becomes more difficult.

The normalized least mean square (NLMS) is an LMS based algorithm which eliminates gradient noise amplification problem by taking into account the variation of signal level at input and selecting the normalized step size parameter $\mu(k)$ as

$$\mu(k) = \frac{\beta}{\varepsilon + \|\mathbf{u}_k\|^2} \tag{2.7}$$

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu(k)\, e_k \mathbf{u}_k \tag{2.8}$$

where the β is chosen in the range $0 < \beta < 2$. Here, ε is a small number included in order to avoid large step sizes when $\mathbf{u}_k^T \mathbf{u}_k$ becomes small.

2.3.2.2 RLS Algorithm

The RLS algorithm is a special case of the Kalman filter, utilizing the least squares approach to optimize the equalizer coefficients [22]. The algorithm deals directly with the input data sequences and obtains estimates of correlations from the data. The RLS algorithm exploits the method of least squares and a relation in matrix algebra known as the matrix inversion lemma. An important feature of the RLS algorithm is that it makes use of an exponentially weighted sum of squared errors extending back to the instant of time when the algorithm is initiated.

The RLS algorithm calculates the tap weights such that $\mathbf{u}_k^T \mathbf{w}_k$ is an estimate of d_k. Here, the cost function J_k is exponentially weighted sum of squared errors. The λ is the forgetting factor, typically close to but less than 1. $\mathbf{P}_k$ is the inverse correlation matrix. The algorithm is initialized by setting $\mathbf{P}_0 = \delta^{-1} I$ where δ is a small positive constant, and $\mathbf{w}_0 = \mathbf{d}_0 = \mathbf{0}$. The gain vector is calculated as

$$\mathbf{g}_k = \frac{\mathbf{P}_{k-1}\mathbf{u}_k}{\lambda + \mathbf{u}_k^T \mathbf{P}_{k-1}\mathbf{u}_k}. \tag{2.9}$$

The filter parameters are updated as

$$\mathbf{w}_k = \mathbf{w}_{k-1} + \mathbf{g}_k e_k \tag{2.10}$$

where $e_k = d_k - \mathbf{u}_k^T \mathbf{w}_{k-1}$ and inverse correlation matrix is updated as:

$$\mathbf{P}_k = \frac{1}{\lambda}\left[\mathbf{P}_{k-1} - \mathbf{g}_k \mathbf{u}_k^T \mathbf{P}_{k-1}\right]. \tag{2.11}$$

RLS algorithm converges fast and which is independent of eigenvalue spread of the input correlation matrix with expense of high computational complexity. Various versions of fast RLS algorithm have been proposed [23, 58–60], but they may suffer from stability problem when implemented on real-time systems.

2.4 MIMO System

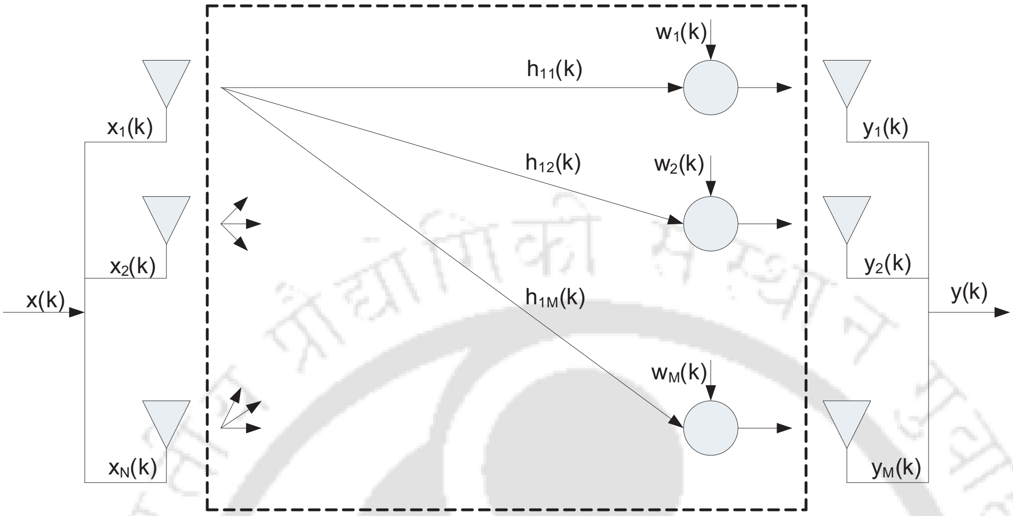


Figure 2.3: A Generic MIMO System Model.

A MIMO system can be defined as a wireless communication system, for which the transmitting end as well as the receiving end is equipped with multiple antenna elements. The idea behind MIMO is that the signals on the transmit antennas at one end and the receive antennas at the other end are combined in such a way that the quality (bit-error rate) or the data rate (bit/s) of the communication for each MIMO user will be improved [61–64]. The main advantages of MIMO system can be categorized as antenna array gain, spatial diversity gain (improves in link reliability obtained by transmitting the same data on independently fading branches) and spatial multiplexing gain (capacity gain at no additional power or bandwidth consumption obtained through the use of multiple antennas at both sides).

We have considered a system with N transmit and M receive antennas as shown in Fig. 2.3. Each element of the resulting MIMO channel is considered to be frequency selective and the noise is assumed to be AWGN. The discrete output of the j_{th} receiving antenna is:

$$y_j(k) = \sum_{i=1}^{N} h_{ji}(k)\, x_i(k) + n_j(k) \tag{2.12}$$

where $x_i(k)$ is the discrete input to the i_{th} transmitting antenna, $h_{ji}(k)$ is the channel response between the i^{th} transmitting antenna and j^{th} receiving antenna at time instant k and $n_j(k)$ is the AWGN at the output of j_{th} receiving antenna at time k.

In a vector form, the received signal can be expressed as

$$\boldsymbol{y}(k) = \boldsymbol{H}(k)\,\boldsymbol{x}(k) + \boldsymbol{n_e}(k)\,. \tag{2.13}$$

where

$$\boldsymbol{x}(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_N(k) \end{bmatrix}, \quad \boldsymbol{y}(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_M(k) \end{bmatrix}, \quad \boldsymbol{n_e}(k) = \begin{bmatrix} n_1(k) \\ n_2(k) \\ \vdots \\ n_M(k) \end{bmatrix} \tag{2.14}$$

and the complex channel matrix is:

$$\boldsymbol{H}(k) = \begin{pmatrix} h_{11}(k) & h_{12}(k)\cdots & h_{1N}(k) \\ h_{21}(k) & h_{22}(k)\cdots & h_{2N}(k) \\ \vdots & \cdots & \vdots \\ h_{M1}(k) & h_{M2}(k)\cdots & h_{MN}(k) \end{pmatrix}. \tag{2.15}$$

2.4.1 Channel Capacity

The channel capacity is a measure of how much information can be transmitted and received with a negligible probability of error. The multi-path characteristic of radio transmissions can be used to multiplicatively increase the capacity of a radio system, when MIMO is used [26,27]. For SISO system the mean channel capacity is given by

$$C = E\left[\log_2\left\{1 + \rho\,|h_{11}|^2\right\}\right]. \tag{2.16}$$

The mean capacity for MIMO system is given by

$$C = E\left[\log_2\left\{\det\left(I_M + \frac{\sigma_x^2}{\sigma_w^2} H H^H\right)\right\}\right] \tag{2.17}$$

where M elements of channel noise vector are independent and identically distributed (i.i.d.) complex Gaussian random variable with zero mean and common variance σ_w^2, N symbols constituting transmitted signal vector are i.i.d. complex Gaussian random variable with zero mean and common variance σ_x^2, and ρ is the average SNR at each receiver branch. MIMO outperforms SISO in terms of capacity. Increasing ρ increases C logarithmically for both SISO and MIMO.

2.5 Channel Models

Wireless environment between the transmitter antenna and the receiver antenna are too complex to model very accurately. Various empirical models have been developed considering measurements taken in several real environments. The International Telecommunication Union for Radiocommunication Sector (ITU-R) recommended set of path loss models in different test environments [54]. In this thesis, we consider WiMAX forum recommended two empirical ITU-R wireless channel models, one for pedestrian test environment with a low delay spread (Ped-A) and other for vehicular test environment with a medium delay spread (Veh-B). The multipath tap delay profile is shown in Table 2.1.

Table 2.1: ITU-R Channel Models for Ped-A and Veh-B

	Pedestrian Test Environment-A		Vehicular Test Environment-B	
Tap	Relative delay (ns)	Average power (dB)	Relative delay (ns)	Average power (dB)
1	0	0	0	-2.5
2	110	-9.7	300	0
3	190	-19.2	8900	-12.8
4	410	-22.8	12900	-10.0
5	-	-	17100	-25.2
6	-	-	20000	-16.0
	RMS Delay Spread = 45 ns		RMS Delay Spread = 4000 ns	

2.6 Summary

This chapter briefly discusses some major impairments which are often met in wireless channels due to the multipath propagation and the randomly changing medium characteristics. We have also discussed the channel equalization method, which includes the adaptive filters, equalizer structure and adaption algorithms, to remove the effect of ISI and track the channel variations effectively. Further, we have given a brief introduction on MIMO system and capacity calculation of MIMO channels. We have also discussed the WiMAX forum recommended wireless channel models for small as well as medium delay spreads.

3

Performance Investigation of Data-reusing Algorithms based Decision Feedback Equalizer

Contents

Objective

In this chapter, we investigate the performance of an adaptive decision feedback equalizer using data-reusing algorithms over time dispersive wireless channels. The performances are analyzed through computer simulations considering International Telecommunication Union for Radiocommunication Sector (ITU-R) wireless channel models, one for outdoor to indoor and pedestrian test environment for the low delay spread (Ped-A) and other for the vehicular test environment for medium delay spread (Veh-B).

3.1 Introduction

In recent years, the data-reusing algorithms have gained popularity because of their faster convergence speed than the stochastic gradient algorithms and lower computational requirement than the RLS based algorithms, in situations where the input signal is highly correlated. The data-reusing affine projection (AP) algorithm proposed by Ozeki and Umeda [39], utilizes present input signal vector along with past input signal vectors in the coefficient update to achieve faster convergence. The data-reusing binormalized LMS (BNLMS) algorithm introduced in [55] employs present as well as immediate past input signal vectors for updating coefficients. Its performance have been studied in [41, 65].

Many literature reported equalization schemes for time-varying wireless channels using the adaptive DFE [8, 9]. Although some basic studies on equalizers employing data-reusing algorithms have been presented in [42, 44, 66], there is scope for further investigation on performance of such algorithms in equalizing signals when standard wireless channel models are considered. This work presents channel equalization techniques which uses data-reusing algorithms for adaptively updating weights of decision feedback equalizers (DFE) operating in wireless channels modeled according to ITU-R recommendations.

The rest of the chapter is organized as follows: Section 3.2 discusses about data-reusing algorithms. Section 3.3 describes the adaptive DFE model adopted in this work. Section 3.4 presents the affine projection (AP) algorithm based DFE. Simulation results are provided to confirm the effectiveness of the introduced channel equalization methods. The performance of the BNLMS algorithm based DFE is investigated in Section 3.5. Finally, the summary of work presented in this chapter is provided in Section 3.6.

3.2 Data-reusing Algorithms

Data-reusing algorithms use the past input signals in addition to the present input signal for improving the speed of convergence where the input signal is correlated [39, 55, 67, 68]. The data-reusing may increase the misadjustment; introducing a convergence factor accomplishes a trade-off between final misadjustment and convergence speed. Here, we explain the data-reusing AP and BNLMS algorithms. For this purpose, we consider an adaptive FIR filter with N coefficients.

3.2.1 The Affine Projection Algorithm

Here, we present an overview of the AP algorithm. The AP algorithm is a data-reusing algorithm, where it utilizes past data along with present data on the coefficient update, leading to fast convergence for highly correlated input [39, 69, 70]. It combines data-reusing, orthogonal projections in $P-1$ consecutive gradient directions, and normalization in order to achieve faster convergence. The AP algorithm updates its coefficient vector such that the new solution belongs to the intersection of P hyper-planes defined by the present and the $P-1$ previous data pairs [39,69,70]. The hyperplane $\mathbf{Q}_k$ is defined as:

$$\mathbf{Q}_k = \{\mathbf{w}_{k+1} \in \Re^N : d_k - \mathbf{u}_k^T \mathbf{w}_{k+1} = 0\} \tag{3.1}$$

where $\mathbf{u}_k$ is the input signal vector, $\mathbf{w}_k$ is the weight vector and d_k is the desired signal. At time instant $(k+1)$, the affine projection algorithm results weight vector $\mathbf{w}_{k+1}$ which is at the intersection of hyperplanes $\mathbf{Q}_k$, $\mathbf{Q}_{k-1}$,...., $\mathbf{Q}_{k-P+1}$ and is as close as possible to $\mathbf{w}_k$. So, the objective function used for the derivation of the AP algorithm requires to maintain the next coefficient vector as close as possible to the current one, while forcing the a-posteriori error to be zero [71]. Mathematically, it can be written as:

$$\begin{aligned} &\min \|\mathbf{w}_{k+1} - \mathbf{w}_k\|^2 \\ &\text{subject to:} \quad \mathbf{d}_k - \mathbf{U}_k^T \mathbf{w}_{k+1} = 0 \end{aligned} \tag{3.2}$$

where the desired signal vector is:

$$\mathbf{d}_k = \begin{bmatrix} d_k \\ d_{k-1} \\ \cdots \\ d_{k-P+1} \end{bmatrix} \tag{3.3}$$

and $\mathbf{U}_k$ is a matrix of size $N \times P$ containing present input signal vector along with $P-1$ past input signal vectors.

$$\mathbf{U}_k = \begin{bmatrix} \mathbf{u}_k & \mathbf{u}_{k-1} & \cdots & \mathbf{u}_{k-P+1} \end{bmatrix}. \tag{3.4}$$

This optimization problem can be solved by using the method of Lagrange multipliers. The derivation of the weight update equation is shown in Section 3.4.

3.2.2 The Binormalized Least Mean Square Algorithm

The binormalized data-reusing LMS algorithm was introduced in [55], where present input signal vector as well as immediate past input signal vector are utilized for updating the filter coefficients. It is similar to AP algorithm of projection order 2, but with reduced complexity due to absence of matrix inversion term. In each update, normalization is employed on two orthogonal directions from consecutive data pairs for achieving faster convergence [41, 65].

At time instant $(k+1)$, the weight vector $\mathbf{w}_{k+1}$ is updated in such a way that, it lies at the intersection of hyperplanes $\mathbf{Q}_k$ and $\mathbf{Q}_{k-1}$, as well as at a minimum distance from $\mathbf{w}_k$. So, the objective function of this algorithm can be written as:

$$\begin{aligned} & \min \|\mathbf{w}_{k+1} - \mathbf{w}_k\|^2 \\ \text{subject to:} \quad & d_k - \mathbf{u}_k^T \mathbf{w}_{k+1} = 0 \\ \text{and} \quad & d_{k-1} - \mathbf{u}_{k-1}^T \mathbf{w}_{k+1} = 0 \end{aligned} \tag{3.5}$$

The weight updating equation can be derived by solving this constraint minimization problem. Alternatively, it can also be derived using geometrical interpretations as shown in Fig. 3.1, where weight updating is done by employing two orthogonal projections [71]. In this figure, input signal vectors $\mathbf{u}_k$ and $\mathbf{u}_{k-1}$ are orthogonal to hyperplanes $\mathbf{Q}_k$ and $\mathbf{Q}_{k-1}$, respectively. The derivation of the weight updating equation using geometrical interpretations is explained below:

Step-1: Perform orthogonal projection of $\mathbf{w}_k$ onto the closest boundary of $\mathbf{Q}_k$, that is from position a to b, to reach a value $\hat{\mathbf{w}}_{k+1}$ (say).

$$\hat{\mathbf{w}}_{k+1} = \mathbf{w}_k + \frac{e_k}{\|\mathbf{u}_k\|^2}\mathbf{u}_k \tag{3.6}$$

where

$$e_k = d_k - \mathbf{u}_k^T \mathbf{w}_k \tag{3.7}$$

Step-2: Perform orthogonal projection with respect to first step, that is from position b to c, so that the solution $\mathbf{w}_{k+1}$ lies at the intersection of hyperplanes $\mathbf{Q}_k$ and $\mathbf{Q}_{k-1}$, that is $\mathbf{w}_{k+1} \in \mathbf{Q}_k \cap \mathbf{Q}_{k-1}$.

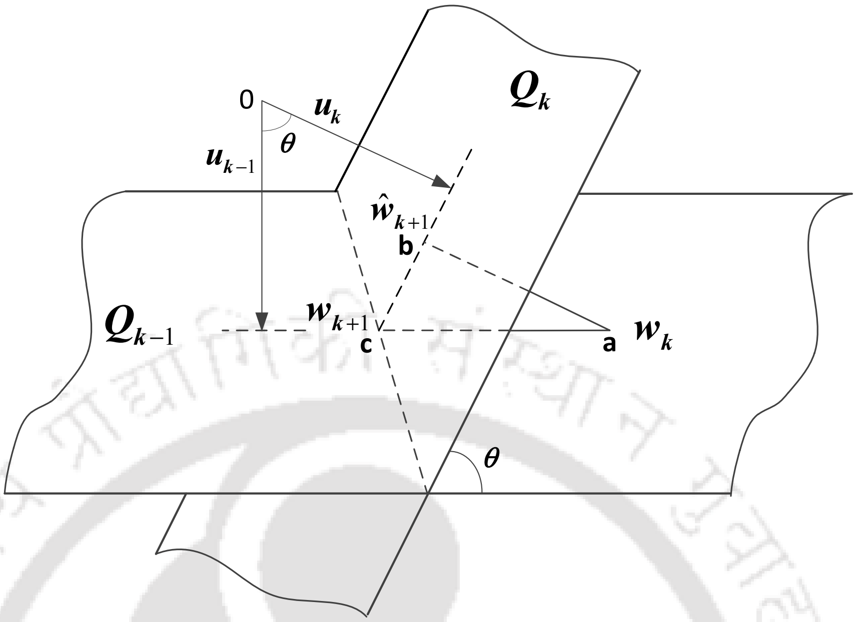


Figure 3.1: Geometrical Interpretation of Weight Updating using BNLMS Algorithm.

So, this is realized by commencing from $\hat{\mathbf{w}}_{k+1}$ in the direction of $\hat{\mathbf{u}}_k$, which is the projection of $\mathbf{u}_{k-1}$ onto $\mathbf{Q}_k$.

$$\mathbf{w}_{k+1} = \hat{\mathbf{w}}_{k+1} + \frac{\varepsilon_{k-1}}{\|\hat{\mathbf{u}}_k\|^2}\hat{\mathbf{u}}_k \tag{3.8}$$

where

$$\hat{\mathbf{u}}_k = \mathbf{u}_{k-1}\left[I - \frac{\mathbf{u}_k^T\mathbf{u}_k}{\|\mathbf{u}_k\|^2}\right] \tag{3.9}$$

and

$$\varepsilon_{k-1} = d_{k-1} - \mathbf{u}_{k-1}^T\hat{\mathbf{w}}_k \tag{3.10}$$

The derivation of the weight updating equation using BNLMS algorithm is elaborated in Section 3.5.

3.3 Adaptive DFE Model

A discrete time model of data transmission system in the wireless channel is considered at time instant k with a reference data signal $d_{0,k}$ and the interfering signals as $d_{i,k}$, where $i = 1, 2, ..., N_i$ and N_i is the number of interfering signals. So the received signal at time instant k is:

$$y_k = d_{0,k} \otimes h_{0,k} + \sum_{i=1}^{N_i} d_{i,k} \otimes h_{i,k} + n_k \tag{3.11}$$

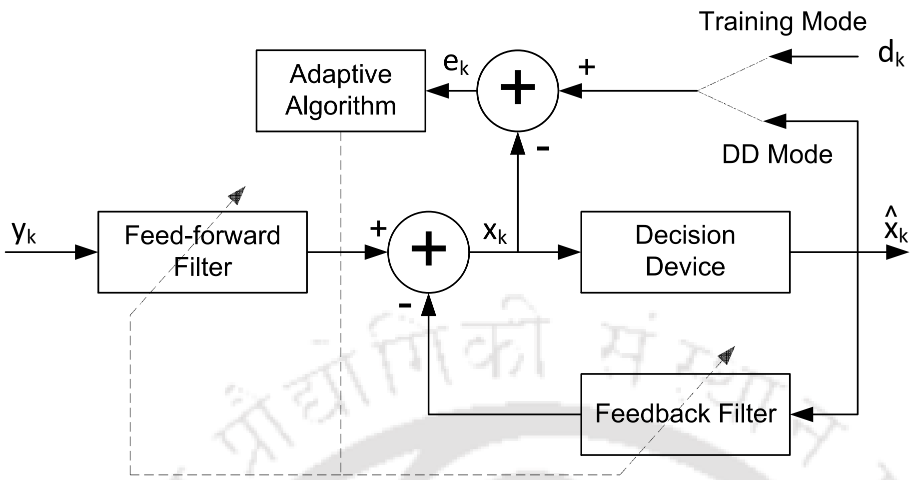


Figure 3.2: Block diagram of a generic adaptive DFE model.

where $h_{0,k}$ is the channel impulse response for the reference data signal, $h_{i,k}$ is the channel impulse response of interfering co-channels, $\otimes$ represents the convolution operation and n_k is additive white Gaussian noise (AWGN). Here, $d_{0,k} \otimes h_{0,k}$ is the desired channel output and $\sum_{i=1}^{N_i} d_{i,k} \otimes h_{i,k}$ describes interference resulted by N_i co-channels.

The block diagram of a generic adaptive DFE consisting of a linear feed-forward filter (FFF), a linear feedback filter (FBF) and a nonlinear decision device, is shown in Fig. 3.2. The input signal vector to FFF ($\mathbf{y}_k$) and input signal vector to FBF ($\hat{\mathbf{x}}_{k-1}$) at time instant k are given as:

$$\mathbf{y}_k = \begin{bmatrix} y_k \\ y_{k-1} \\ \cdots \\ y_{k-L_{FF}+1} \end{bmatrix} \quad and \quad \hat{\mathbf{x}}_{k-1} = \begin{bmatrix} \hat{x}_{k-1} \\ \hat{x}_{k-2} \\ \cdots \\ \hat{x}_{k-L_{FB}} \end{bmatrix} \tag{3.12}$$

where $\hat{x}_{k-1}$ is the output of decision device at time instant $k-1$, L_{FF} and L_{FB} are length of FFF and FBF respectively. The weight vector $\mathbf{w}_{FF}$ and $\mathbf{w}_{FB}$ for FFF and FBF respectively, are given as:

$$\mathbf{w}_{FF} = \begin{bmatrix} w_{FF,0} \\ w_{FF,1} \\ \cdots \\ w_{FF,L_{FF}-1} \end{bmatrix} \quad and \quad \mathbf{w}_{FB} = \begin{bmatrix} w_{FB,1} \\ w_{FB,2} \\ \cdots \\ w_{FB,L_{FB}} \end{bmatrix}. \tag{3.13}$$

The signal applied to the decision device at time instant k is:

$$x_k = \sum_{m=0}^{L_{FF}-1} w_{FF,m} y_{k-m} - \sum_{n=1}^{L_{FB}} w_{FB,n} \hat{x}_{k-n}. \tag{3.14}$$

It can also be expressed as a product of two concatenated vectors. The input signal vectors $\mathbf{y}_k$ and $\hat{\mathbf{x}}_{k-1}$ are concatenated to produce $\mathbf{z}_k$, and the weight vectors $\mathbf{w}_{FF}$ and $\mathbf{w}_{FB}$ are concatenated to produce $\mathbf{w}_k$. So,

$$x_k = \mathbf{w}_k^T \mathbf{z}_k = \begin{bmatrix} \mathbf{w}_{FF} \\ \mathbf{w}_{FB} \end{bmatrix}^T \begin{bmatrix} \mathbf{y}_k \\ -\hat{\mathbf{x}}_{k-1} \end{bmatrix}. \tag{3.15}$$

The adaptive equalization is a minimization problem, which is carried out by adaptive algorithms in an iterative manner. In an adaptive equalization process, a training sequence is used. In practical systems only fixed number of training symbols can be used as the reference and these are known at the receiver during a training phase. After the training period, the output from the decision device is used as the reference signal for estimating error. This mode is called the decision directed (DD) mode. The equalizer periodically switches between training and DD mode, and the error at time instant k can be written as:

$$e_k = \begin{cases} d_k - x_k & : \text{Training Mode} \\ \hat{x}_k - x_k & : \text{DD Mode} \end{cases} \tag{3.16}$$

3.4 The Affine Projection Algorithm based DFE

The data-reusing affine projection algorithm is discussed in Section 3.2. Here, we illustrate how AP algorithm can be used for adaptively updating the weights of a DFE.

For implementing AP algorithm in an adaptive SISO-DFE system, we have modified it appropriately as detailed below. The objective function of this algorithm at time instant k can be expressed as:

$$\begin{gathered} \min \|\mathbf{w}_{k+1} - \mathbf{w}_k\|^2 \\ \text{subject to:} \quad \mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_{k+1} = 0 \end{gathered} \tag{3.17}$$

where $\mathbf{d}_k$ is a vector of size $P \times 1$ containing desired signals and $\mathbf{w}_k$ is weight vector of size $(L_{FF} + L_{FB}) \times 1$. A matrix $\mathbf{Z}_k$ of size $(L_{FF} + L_{FB}) \times P$ is considered, which contains present and previous $P-1$ input signal vectors:

$$\mathbf{Z}_k = \begin{bmatrix} \mathbf{z}_k & \mathbf{z}_{k-1} & \cdots & \mathbf{z}_{k-P+1} \end{bmatrix} = \begin{bmatrix} y_k & y_{k-1} & \cdots & y_{k-P+1} \\ y_{k-1} & y_{k-2} & \cdots & y_{k-P} \\ \vdots & \vdots & \ddots & \vdots \\ y_{k-L_{FF}+1} & y_{k-L_{FF}} & \cdots & y_{k-P-L_{FF}+2} \\ -\hat{x}_{k-1} & -\hat{x}_{k-2} & \cdots & -\hat{x}_{k-P} \\ -\hat{x}_{k-2} & -\hat{x}_{k-3} & \cdots & -\hat{x}_{k-P-1} \\ \vdots & \vdots & \ddots & \vdots \\ -\hat{x}_{k-L_{FB}} & -\hat{x}_{k-L_{FB}-1} & \cdots & -\hat{x}_{k-P-L_{FB}+1} \end{bmatrix}. \tag{3.18}$$

where concatenated input vector $\mathbf{z}_k$ is,

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{y}_k \\ -\hat{\mathbf{x}}_{k-1} \end{bmatrix}. \tag{3.19}$$

Further, $\mathbf{d}_k$ is expressed as,

$$\mathbf{d}_k = \begin{bmatrix} d_k \\ d_{k-1} \\ \vdots \\ d_{k-P+1} \end{bmatrix}. \tag{3.20}$$

The constrained minimization problem of equation 3.17 is solved by applying Lagrange multiplier method. The unconstrained function to be minimized is:

$$f\left[\mathbf{w}_{k+1}\right] = \|\mathbf{w}_{k+1} - \mathbf{w}_k\|^2 + \boldsymbol{\lambda}_k^T \left[\mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_{k+1}\right] \tag{3.21}$$

where Lagrange multipliers $\boldsymbol{\lambda}_k$ is

$$\boldsymbol{\lambda}_k = \begin{bmatrix} \lambda_{k1} \\ \lambda_{k2} \\ \cdots \\ \lambda_{kP} \end{bmatrix}. \tag{3.22}$$

Setting the gradient of the function $f\left[\mathbf{w}_{k+1}\right]$ with respect to $\mathbf{w}_{k+1}$ equal to zero, we obtain:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mathbf{Z}_k \frac{\boldsymbol{\lambda}_k}{2}. \tag{3.23}$$

The $\boldsymbol{\lambda}_k$ is derived from constraint condition in equation 3.17 and the weight updating expression in equation 3.23, as given below.

$$\begin{aligned} &\mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_{k+1} = 0 \\ \Rightarrow \quad &\mathbf{d}_k - \mathbf{Z}_k^T \left[\mathbf{w}_k + \mathbf{Z}_k \frac{\boldsymbol{\lambda}_k}{2} \right] = 0 \\ \Rightarrow \quad &\mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_k = \mathbf{Z}_k^T \mathbf{Z}_k \left[\frac{\boldsymbol{\lambda}_k}{2} \right]. \end{aligned} \tag{3.24}$$

So, from equation 3.24, $\boldsymbol{\lambda}_k$ can be expressed as:

$$\boldsymbol{\lambda}_{\boldsymbol{k}} = 2 \left[\mathbf{Z}_k^T \mathbf{Z}_k + \alpha \boldsymbol{I} \right]^{-1} \mathbf{e}_k \tag{3.25}$$

where the error vector is

$$\mathbf{e}_k = \mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_k \tag{3.26}$$

and the matrix $\alpha \boldsymbol{I}$ is added to avoid the problem in inversion of $\mathbf{Z}_k^T \mathbf{Z}_k$ in a noisy environment.

In decision directed mode, the error vector can be written as:

$$\mathbf{e}_k = \hat{\mathbf{x}}_k - \mathbf{Z}_k^T \mathbf{w}_k = \begin{bmatrix} e_k \\ \varepsilon_{k-1} \\ \vdots \\ \varepsilon_{k-P+1} \end{bmatrix}. \tag{3.27}$$

where $\hat{\mathbf{x}}_k$ is a vector of size $P \times 1$ consisting of present and previous $P-1$ decisions. The e_k is the a-priori error at iteration k and ε_{k-j} is the a-posteriori error at iteration $k-j$ (for $j = 1, 2, \cdots, P-1$), given below for reference.

$$e_k = \hat{x}_k - \mathbf{z}_k^T \mathbf{w}_k \tag{3.28}$$

$$\varepsilon_{k-j} = \hat{x}_{k-j} - \mathbf{z}_{k-j}^T \mathbf{w}_k. \tag{3.29}$$

Finally, the weight updating equation is expressed as:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mathbf{Z}_k \left[\mathbf{Z}_k^T \mathbf{Z}_k + \alpha \boldsymbol{I} \right]^{-1} \mathbf{e}_k. \tag{3.30}$$

The algorithm is modified by introducing a convergence factor (step size) μ to achieve a trade-off between final misadjustment and convergence speed. The μ should be chosen in the range $0 < \mu < 2$, so that the a-priori error remain bounded [71]. So, the modified weight updating equation is:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu \mathbf{Z}_k \left[\mathbf{Z}_k^T \mathbf{Z}_k + \alpha \boldsymbol{I} \right]^{-1} \mathbf{e}_k. \tag{3.31}$$

The summary of the AP algorithm based channel equalization scheme on SISO systems, is shown in Table 3.1.

Table 3.1: Summary of the Affine Projection Algorithm based DFE

Step-1: Initialization
Weight vector $\mathbf{w}_0$ of size $(L_{FF} + L_{FB}) \times 1$ is initialized as zero vector.
Input signal vector $\mathbf{z}_k$ of size $(L_{FF} + L_{FB}) \times 1$ is initialized as zero vector.
$\boldsymbol{I}$ is an identity matrix of size $P \times P$, where P is the projection order.
α is a small constant.
Step size be in the range $0 < \mu < 2$.
Step-2: Processing for $k > 0$
(a) Computation of Filter Output:
$x_k = \mathbf{w}_k^T \mathbf{z}_k$
(b) Computation of Error Vector:
In training mode: $\mathbf{e}_k = \mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_k$
In DD mode: $\mathbf{e}_k = \hat{\mathbf{x}}_k - \mathbf{Z}_k^T \mathbf{w}_k$.
(c) Filter Weight Updating:
$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu \mathbf{Z}_k \left[\mathbf{Z}_k^T \mathbf{Z}_k + \alpha \boldsymbol{I}\right]^{-1} \mathbf{e}_k$
(d) Increment k **and go back to 'for' loop.**

3.4.1 Computational Complexity

Table 3.2 compares the computational complexity per update for implementing the AP algorithm based channel equalization schemes with the existing schemes such as NLMS and RLS algorithm based equalization techniques.

Table 3.2: Computational Complexity Comparison of DFE using NLMS, RLS and AP Algorithm.

Algorithm	Multiplication	Addition	Division
NLMS	$3L + 1$	$3L$	1
RLS	$L^3 + 4L^2 + 4L + 2$	$L^3 + 2L^2 + L$	2
AP	$P^2\left(\frac{L}{2} + 1\right) + \frac{5}{2}LP + L + \mathcal{O}\left(P^3\right)$	$\frac{P^2}{2}(L + 1) + \frac{5}{2}LP - \frac{P}{2} + \mathcal{O}\left(P^3\right)$	-

The computational complexity is expressed in terms of the number of complex multiplication, complex addition and division per update. Here, we take $L = L_{FF} + L_{FB}$. It may be seen that computation involved per update in this AP algorithm based DFE is higher than NLMS algorithm based scheme, but much less compared to DFE based on RLS algorithms. The main complexity of this scheme is in computing the matrix inversion, which is of the order $\mathcal{O}\left(P^3\right)$, where P is the projection order. Usually P value is kept small compared to $L_{FF} + L_{FB}$. So, small value of P reduces the computational requirement of the proposed scheme.

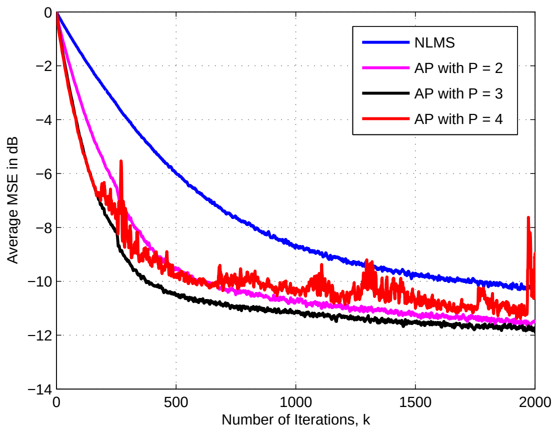
3.4.2 Simulation Results and Discussion

In the earlier section, we discussed how an adaptive DFE can be implemented using an AP algorithm. In this subsection we present the details of simulation carried out using a DFE which employs AP for weight update and operates in a wireless channel. We study the performance of this DFE in terms of convergence speed, computation requirement, stability, tracking capability and bit error rate (BER). It may be noted that the performance of the AP-DFE depends on the parameters, such as step size μ, filter lengths (L_{FF}, L_{FB}) and projection order P. The choice of these parameters are crucial to obtain a balance between complexity and performance. In our simulation study, the transmitted symbols are randomly generated, then modulated by the binary phase shift keying (BPSK) modulation scheme, and the noise is considered to be additive white Gaussian. We consider each packet consisting of 2048 bits, which includes 256 bits training sequence, and 3000 such packets are transmitted to evaluate the performance of DFE.

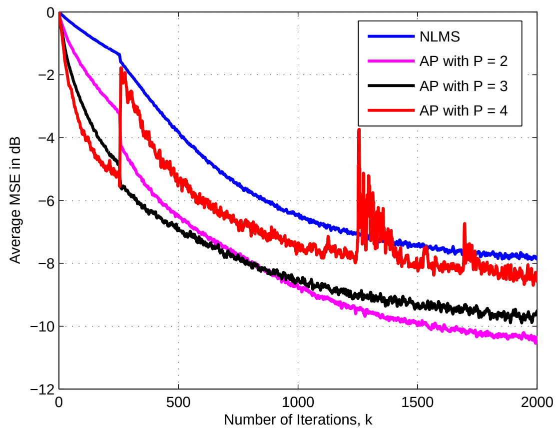
In this work, we have considered WiMAX forum recommended ITU-R wireless channel models [54], one for outdoor to indoor and pedestrian test environment for low delay spread (Ped-A) and other for the vehicular test environment with medium delay spread (Veh-B). Taking into consideration the delay spread of the two channel models, the simulations are done with sample time of the signal t_s as .01 μsec for Ped-A and .3 μsec for Veh-B so that the channel will be frequency selective. The carrier frequency is considered as 2.5 GHz and the maximum doppler frequency shift f_d is considered as 5 Hz for Ped-A and 70 Hz for Veh-B. The MSE and BER curves are plotted taking an average of the results obtained from 3000 independent runs. The MSE curves are plotted with SNR of 20 dB..

3.4.2.1 Performance of DFE with AP and NLMS as Adaption Algorithms

The convergence and BER performance of adaptive DFEs using NLMS and AP algorithm of projection order 2, 3 and 4 are discussed here. The study is performed with varying projection order P keeping filter lengths and step size fixed for carrying out simulation. The step size μ has been taken as 0.01 and filter lengths of DFE are taken as $L_{FF} = 3, L_{FB} = 1$, in both Ped-A and Veh-B environments.



(a) Pedestrian Test Environment-A



(b) Vehicular Test Environment-B

Figure 3.3: Convergence Performances of DFE using AP and NLMS as Adaption Algorithms.

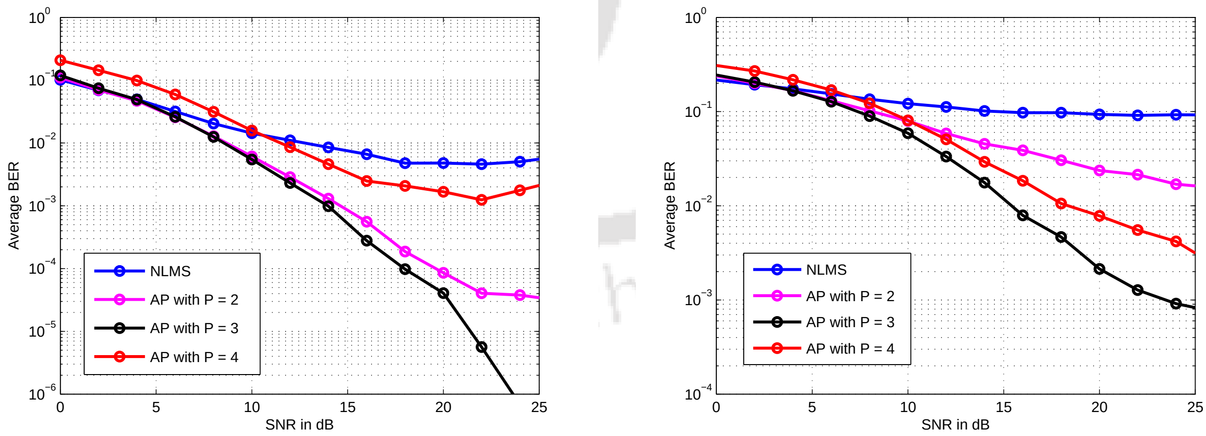
The MSE performance of DFEs using NLMS algorithm and AP algorithm of projection order 2, 3 and 4 are presented in Fig. 3.3. It is observed from MSE curves that, AP based DFE of projection order 2 offers the better rate of convergence and lower steady-state MSE (SS-MSE) than NLMS algorithm based scheme for both Ped-A and Veh-B environments. It is also seen in Ped-A environment that, by increasing projection order from 2 to 3 improves the convergence performance as well as SS-MSE value, but abrupt changes in MSE are observed for AP based DFE with projection order 4. In Veh-B environment, convergence performance of the proposed scheme improves for projection order 3 with a decrease in SS-MSE value, and increasing projection order from 3 to 4 results in instantaneous error terms. As already discussed, a wireless fading channel results a higher misadjustment, which increases with the eigenvalue spread. Further, in data-reusing algorithms, misadjustment also increases with the number of data reuses. Though increase in projection order improves the performance, but for higher projection order performance deteriorates substantially for higher misadjustment.

Table 3.3 shows SS-MSE of DFEs using NLMS algorithm and AP algorithm for the given simulation

Table 3.3: Steady-state MSE Performance of DFE using AP and NLMS as Adaption Algorithms.

Adaptive Algorithm	Pedestrian Test Environment-A	Vehicular Test Environment-B
	Steady-state MSE	Steady-state MSE
NLMS	-10.2 dB	-7.8 dB
AP with P=2	-11.5 dB	-10.3 dB
AP with P=3	-11.7 dB	-9.6 dB

parameters. It is observed that, SS-MSE of the AP based DFE with projection order 2 is approximately 1.3 dB less than NLMS algorithm based scheme in Ped-A and 2.5 dB less in Veh-B environments, which reflects better tracking. Furthermore, changing projection order from 2 to 3, improves SS-MSE by 0.2 dB in Ped-A environment, whereas deteriorates by 0.7 dB in Veh-B environment. For projection order 4, the MSE performance shows fluctuations and its SS-MSE is worst in comparison to the performance obtained with projection order 2 and 3. As the channel is frequency selective as well as time-varying, AP algorithm with higher projection order deteriorates the performance due to undesired interference of previous input signal vectors. For this reason, with projection order 4, average MSE curve contains spikes in both Ped-A as well as Veh-B environments.



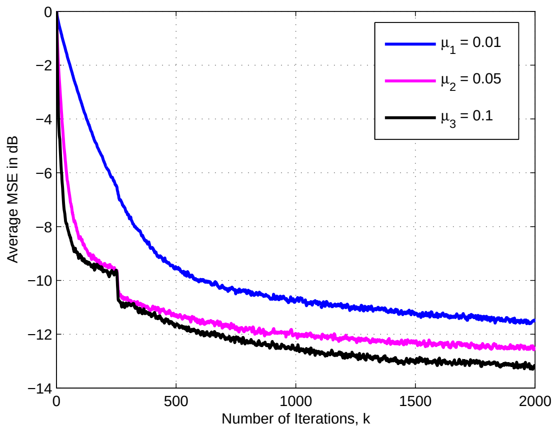
(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 3.4: BER Performance of DFE using AP and NLMS as Adaption Algorithms.

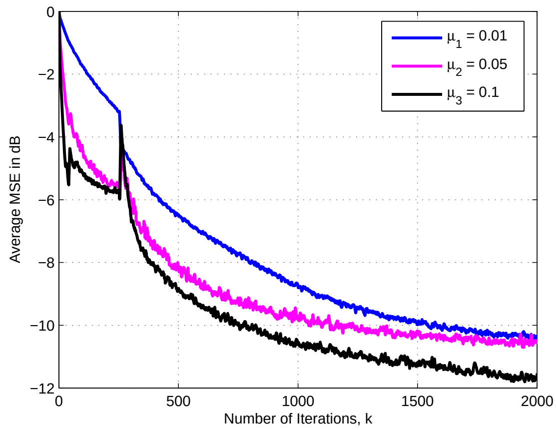
The BER performance comparison of NLMS algorithm based DFEs with AP algorithm based DFE of projection order 2, 3 and 4 is shown in Fig. 3.4. Simulation results show improved performance of the AP algorithm based DFEs over NLMS algorithm based scheme. It is also observed that, increasing projection order in AP algorithm from 2 to 3 improves the BER performance, whereas increasing further from 3 to 4 deteriorates the performance due to higher misadjustment.

3.4.2.2 Performance of AP Algorithm based DFE with Different Values of Step Size

The convergence and BER performance of AP algorithm based DFE is investigated for different values of step size.



(a) Pedestrian Test Environment-A

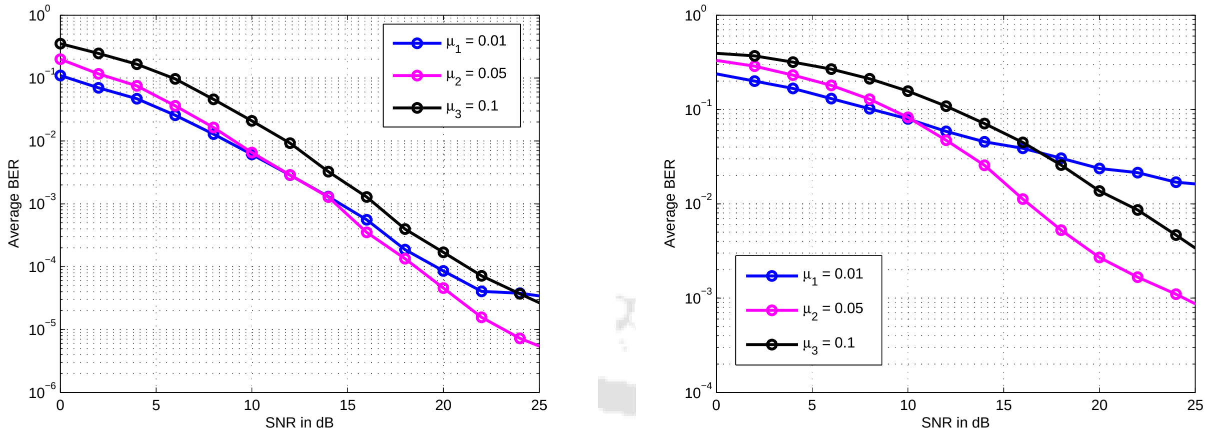


(b) Vehicular Test Environment-B

Figure 3.5: MSE Performance of AP Algorithm based DFE with Different Values of Step Size.

Fig. 3.5 shows the MSE performance of the AP algorithm based DFE as a function of step size μ. In this simulation, we consider projection order as 2, L_{FF} and L_{FB} as 3 and 1, respectively and SNR is set to 20 dB. The simulation results are obtained for step size values $\mu_1 = 0.01$, $\mu_2 = 0.05$ and $\mu_3 = 0.1$. It is observed that, increasing step size improves the convergence performance as well as reduces the SS-MSE value. However, for μ value 0.1, MSE curve contains more fluctuations in steady-state.

The BER performance of the proposed channel equalization scheme for different values of μ is shown in Fig. 3.6. In this simulation, projection order is taken as 2, and L_{FF} and L_{FB} of DFE are taken as 3 and 1 respectively. It may be seen that, BER performance improves with increasing step size from 0.01 to 0.05. Further, it is observed that performance is degraded for μ value as 0.1. Higher value of step size increases the fluctuations in the instantaneous error terms as observed in Fig. 3.5, and it may be attributed to the poor BER performance. We know, in a mobile fading environment small step size is needed to estimate the filter coefficient more precisely according to the time-varying



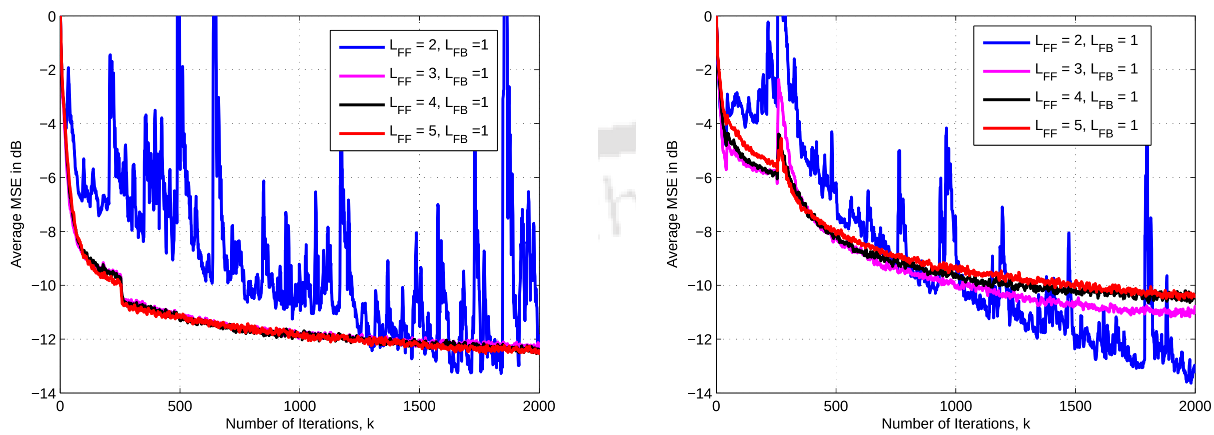
(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 3.6: BER Performance of AP Algorithm based DFE with Different Values of Step Size.

channel condition. In both Ped-A and Veh-B environments, the step size 0.05 is found to have good overall performance in terms of convergence, low residual error, better tracking capability and BER.

3.4.2.3 Performance of AP Algorithm based DFE with Different Filter Lengths

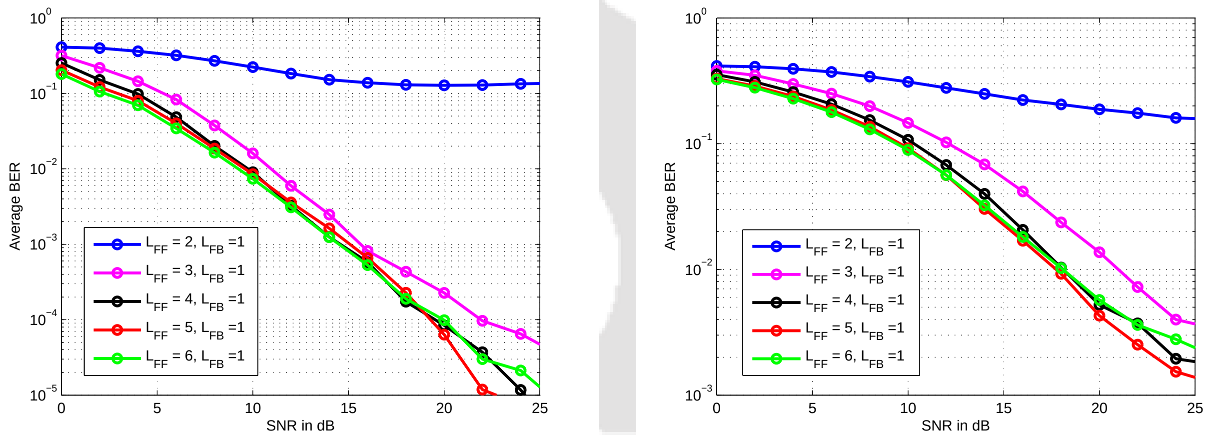
The MSE and BER performance of AP algorithm based adaptive DFE is studied for different set of filter lengths for FFF and FBF.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 3.7: MSE Performance of Affine Projection Algorithm based DFE with Different Filter Lengths.

Fig. 3.7 demonstrates the convergence performance of the proposed affine projection algorithm based DFE with change in filter length. MSE curves are drawn in both Ped-A and Veh-B environments with different sets of FFF length L_{FF} and FBF length L_{FB} i.e. (2,1), (3,1), (4,1),and (5,1). The simulations are done with projection order 3, step size μ as 0.05, and received SNR of 20 dB. In Ped-A environment, it is observed that the convergence performance is almost same for L_{FF} and L_{FB} as (3,1), (4,1),and (5,1), and SS-MSE is -12.2 dB; whereas for L_{FF}, 2 and L_{FB}, 1, performance is deteriorated and MSE curve shows instantaneous fluctuations as time dispersive channels with higher delay spread requires more filter taps for channel equalization. In Veh-B environment, with L_{FF} and L_{FB} of DFE 2 and 1, the average MSE performance contains spikes as observed in Ped-A environment. MSE performance of DFE with L_{FF} and L_{FB}, (3,1), (4,1),and (5,1) are more or less same in terms of convergence as well as SS-MSE, and their SS-MSE values are -11 dB, -10.5 dB and -10.4 dB respectively.



(a) Pedestrian Test Environment-A

(b) Vehicular Test Environment-B

Figure 3.8: BER Performance of Affine Projection Algorithm based DFE with Different Filter Lengths.

The BER performance of the proposed channel equalization schemes as a function of filter length is shown in Fig. 3.8. Here we had taken five different sets of filter length, i.e. L_{FF},L_{FB} as (2,1), (3,1), (4,1), (5,1),and (6,1). The step size and projection order are chosen as 0.05 and 3 respectively. It is noticed that, when the set of filter length is 2 and 1, the BER performance is very poor, that is also observed in MSE performance shown in Fig. 3.7. It is further observed that BER performance improves a lot with filter lengths as (3,1), (4,1), (5,1),and (6,1). The BER performance with filter length (4,1) is comparatively better than (3,1) but quite close to the performance achieved with (5,1),and (6,1).

Higher values of L_{FF} and L_{FB} can provide lower BER at the cost of higher complexity. In design of

DFE, it is important to have a balance between convergence speed, performance and complexity. From the various case studies presented above, one can fix that a DFE with $\mu = 0.05$, $L_{FF} = 3, L_{FB} = 1$ and $P = 3$ gives a balanced performance in terms of MSE, BER and complexity.

3.5 The Binormalized LMS Algorithm based DFE

The performance of adaptive DFE using an AP algorithm in a time-varying wireless channel is discussed in last section. It provides advantage of good convergence as well as better BER performance but with more complexity compared to NLMS algorithm based scheme. The major complexity lies with computing the matrix inversion. In this section, we propose a channel equalization scheme for time-varying wireless channels, which does not require any inversion of matrix for its operations. It uses a data-reusing binormalized LMS (BNLMS) algorithm for updating the filter weights of the DFE.

A basic introduction of BNLMS algorithm is given in Section 3.2. It utilizes the present input signal vector as well as immediate past input signal vector for updating the filter coefficient. Its characteristics are similar to AP algorithm of projection order 2. In each update, normalization is employed on two orthogonal directions from consecutive data pairs for achieving faster convergence [55, 65].

Here, we discuss how BNLMS algorithm can be modified and used for adaptive weight update of a SISO-DFE configuration as shown in Fig. 3.2. The objective of this algorithm is to maintain the next coefficient vector as close as possible to the current one, to maintain a-posteriori errors to zero. So, at time instant k in DD mode, the objective function can be expressed as:

$$\begin{aligned} &\min \left\|\mathbf{w}_{k+1} - \mathbf{w}_k\right\|^2 \\ &\text{subject to:} \quad \hat{x}_k - \mathbf{z}_k^T \mathbf{w}_{k+1} = 0 \\ &\text{and} \quad \hat{x}_{k-1} - \mathbf{z}_{k-1}^T \mathbf{w}_{k+1} = 0 \end{aligned} \tag{3.32}$$

where $\hat{x}_k$ is the output of the decision device and $\mathbf{z}_k$ is the present input signal vector of dimension $(L_{FF} + L_{FB}) \times 1$. The concatenated input vector $\mathbf{z}_k$ contains $\mathbf{y}_k$ and $-\hat{\mathbf{x}}_{k-1}$, where $\mathbf{y}_k$ represents the input vector to FFF and $\hat{\mathbf{x}}_{k-1}$ the input vector to FBF as given in equation 3.19.

The constrained minimization problem of equation 3.32 is solved by applying Lagrange multiplier method. The unconstrained function to be minimized is:

$$f\left(\mathbf{w}_{k+1}\right) = \left\|\mathbf{w}_{k+1} - \mathbf{w}_k\right\|^2 + \lambda_1(k)\left[\hat{x}_k - \mathbf{z}_k^T \mathbf{w}_{k+1}\right] + \lambda_2(k)\left[\hat{x}_{k-1} - \mathbf{z}_{k-1}^T \mathbf{w}_{k+1}\right] \tag{3.33}$$

where $\lambda_1(k)$ and $\lambda_2(k)$ are the Lagrange multipliers at time instant k. Setting the gradient of the function $f\left[\mathbf{w}_{k+1}\right]$ with respect to $\mathbf{w}_{k+1}$ equal to zero, it is obtained as:

$$2\left(\mathbf{w}_{k+1}-\mathbf{w}_{k}\right)-\lambda_{1}(k)\,\mathbf{z}_{k}-\lambda_{2}(k)\,\mathbf{z}_{k-1}=0 \tag{3.34}$$

So,

$$\mathbf{w}_{k+1}=\mathbf{w}_{k}+\frac{\lambda_{1}(k)}{2}\mathbf{z}_{k}+\frac{\lambda_{2}(k)}{2}\mathbf{z}_{k-1} \tag{3.35}$$

To find the value of $\lambda_1(k)$ and $\lambda_2(k)$, we substitute the value obtained for $\mathbf{w}_{k+1}$ in equation 3.35, in the constraint conditions of equation 3.32, as given below.

$$\begin{aligned} &\hat{x}_{k}-\mathbf{z}_{k}^{T}\mathbf{w}_{k+1}=0\\ \text{so,}\quad &\hat{x}_{k}-\mathbf{z}_{k}^{T}\left[\mathbf{w}_{k}+\frac{\lambda_{1}(k)}{2}\mathbf{z}_{k}+\frac{\lambda_{2}(k)}{2}\mathbf{z}_{k-1}\right]=0 \end{aligned} \tag{3.36}$$

and

$$\begin{aligned} &\hat{x}_{k-1}-\mathbf{z}_{k-1}^{T}\mathbf{w}_{k+1}=0\\ \text{so,}\quad &\hat{x}_{k-1}-\mathbf{z}_{k-1}^{T}\left[\mathbf{w}_{k}+\frac{\lambda_{1}(k)}{2}\mathbf{z}_{k}+\frac{\lambda_{2}(k)}{2}\mathbf{z}_{k-1}\right]=0 \end{aligned} \tag{3.37}$$

Equations 3.36 and 3.37 can be rewritten as below.

$$\frac{\lambda_{1}(k)}{2}\left\|\mathbf{z}_{k}\right\|^{2}+\frac{\lambda_{2}(k)}{2}\mathbf{z}_{k}^{T}\mathbf{z}_{k-1}=\hat{x}_{k}-\mathbf{z}_{k}^{T}\mathbf{w}_{k} \tag{3.38}$$

and

$$\frac{\lambda_{1}(k)}{2}\mathbf{z}_{k-1}^{T}\mathbf{z}_{k}+\frac{\lambda_{2}(k)}{2}\left\|\mathbf{z}_{k-1}\right\|^{2}=\hat{x}_{k-1}-\mathbf{z}_{k-1}^{T}\mathbf{w}_{k} \tag{3.39}$$

Solving these equations 3.38 and 3.39, the value of $\lambda_1(k)$ and $\lambda_2(k)$ are obtained.

$$\frac{\lambda_{1}(k)}{2}=\frac{\left[\hat{x}_{k}-\mathbf{z}_{k}^{T}\mathbf{w}_{k}\right]\left\|\mathbf{z}_{k-1}\right\|^{2}-\left[\hat{x}_{k-1}-\mathbf{z}_{k-1}^{T}\mathbf{w}_{k}\right]\mathbf{z}_{k-1}^{T}\mathbf{z}_{k}}{\left\|\mathbf{z}_{k}\right\|^{2}\left\|\mathbf{z}_{k-1}\right\|^{2}-\left[\mathbf{z}_{k}^{T}\mathbf{z}_{k-1}\right]^{2}} \tag{3.40}$$

$$\frac{\lambda_{2}(k)}{2}=\frac{\left[\hat{x}_{k-1}-\mathbf{z}_{k-1}^{T}\mathbf{w}_{k}\right]\left\|\mathbf{z}_{k}\right\|^{2}-\left[\hat{x}_{k}-\mathbf{z}_{k}^{T}\mathbf{w}_{k}\right]\mathbf{z}_{k-1}^{T}\mathbf{z}_{k}}{\left\|\mathbf{z}_{k}\right\|^{2}\left\|\mathbf{z}_{k-1}\right\|^{2}-\left[\mathbf{z}_{k}^{T}\mathbf{z}_{k-1}\right]^{2}} \tag{3.41}$$

If a successive optimized steps are taken for updating the weight vector $\mathbf{w}_k$ for all k, then the a-posteriori error ε_{k-1} at time instant $(k-1)$ can be assumed zero.

$$\varepsilon_{k-1}=\hat{x}_{k-1}-\mathbf{z}_{k-1}^{T}\mathbf{w}_{k}=0 \tag{3.42}$$

So, Lagrange multipliers obtained in equations 3.40 and 3.41 can be simplified further and can be written as:

$$\frac{\lambda_{1}(k)}{2}=\frac{\left[\hat{x}_{k}-\mathbf{z}_{k}^{T}\mathbf{w}_{k}\right]\left\|\mathbf{z}_{k-1}\right\|^{2}}{\left\|\mathbf{z}_{k}\right\|^{2}\left\|\mathbf{z}_{k-1}\right\|^{2}-\left[\mathbf{z}_{k}^{T}\mathbf{z}_{k-1}\right]^{2}} \tag{3.43}$$

and

$$\frac{\lambda_2(k)}{2} = \frac{-\left[\hat{x}_k - \mathbf{z}_k^T \mathbf{w}_k\right] \mathbf{z}_{k-1}^T \mathbf{z}_k}{\|\mathbf{z}_k\|^2 \|\mathbf{z}_{k-1}\|^2 - \left[\mathbf{z}_k^T \mathbf{z}_{k-1}\right]^2} \tag{3.44}$$

A convergence factor μ is incorporated in 3.35 to control the excess MSE in BNLMS algorithm [41]. Though rate of convergence is maximum with $\mu = 1$, but the smaller value of μ is required where measurement error is high. Further, the μ value is kept within the range $0 < \mu < 2$ to achieve convergence in BNLMS algorithm.

So, the weight updating equation can be written as:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu \left[\frac{\lambda_1(k)}{2}\mathbf{z}_k + \frac{\lambda_2(k)}{2}\mathbf{z}_{k-1}\right] \tag{3.45}$$

The step by step implementation of BNLMS algorithm based channel equalization scheme on SISO systems, is shown in Table 3.4.

Table 3.4: Summary of the Binormalized LMS Algorithm based DFE

Step-1: Initialization
Weight vector $\mathbf{w}_0$ of size $(L_{FF} + L_{FB}) \times 1$ is initialized as zero vectors.
Input signal vector $\mathbf{z}_0$ of size $(L_{FF} + L_{FB}) \times 1$ is initialized as zero vector.
Convergence factor be in the range $0 < \mu < 2$.
Step-2: Processing for $k > 0$
(a) Filter weight updating
$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu \left[\frac{\lambda_1(k)}{2}\mathbf{z}_k + \frac{\lambda_2(k)}{2}\mathbf{z}_{k-1}\right]$.
where $\frac{\lambda_1(k)}{2} = \frac{\left[\hat{x}_k - z_k^T w_k\right] \|z_{k-1}\|^2}{\|z_k\|^2 \|z_{k-1}\|^2 - \left[z_k^T z_{k-1}\right]^2}$, $\frac{\lambda_2(k)}{2} = \frac{-\left[\hat{x}_k - \mathbf{z}_k^T \mathbf{w}_k\right] \mathbf{z}_{k-1}^T \mathbf{z}_k}{\|\mathbf{z}_k\|^2 \|\mathbf{z}_{k-1}\|^2 - \left[\mathbf{z}_k^T \mathbf{z}_{k-1}\right]^2}$.
(b) Increment k **and go back to 'for' loop.**

3.5.1 Computational Complexity Issues

This subsection compares the computational complexity for implementing the proposed BNLMS algorithm based channel equalization schemes with the existing schemes such as NLMS and AP algorithm based equalization techniques.

Table 3.5 shows the comparison of computational complexity in terms of the number of complex multiplication, complex addition and division per update. It is observed that computation involved per update in BNLMS algorithm based DFE is slightly higher than NLMS based DFE, but very less compared to DFE using an AP algorithm.

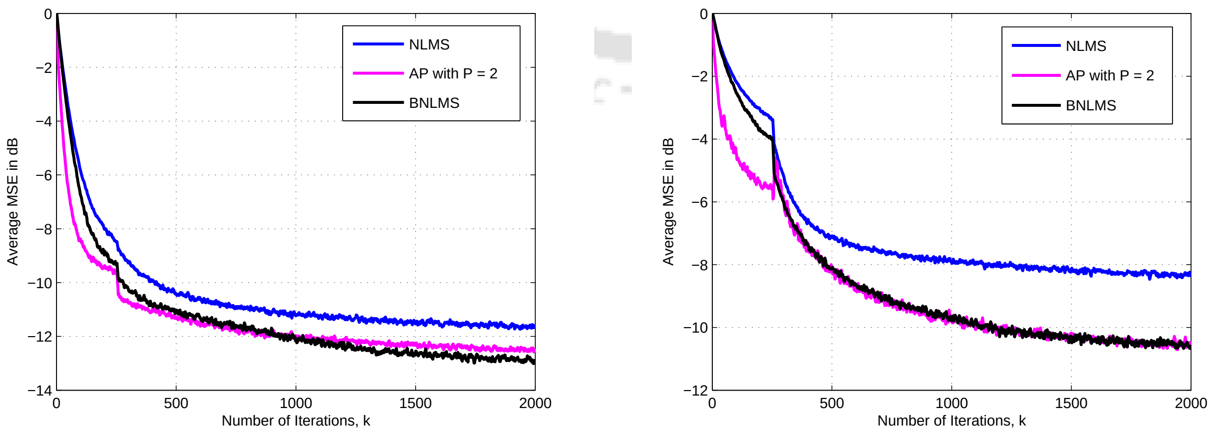
Table 3.5: Computational Complexity Comparison of DFE using NLMS, BNLMS and AP Algorithm.

Algorithm	Addition	Multiplication	Division
NLMS	$3L + 1$	$3L$	1
BNLMS	$5L + 1$	$5L + 7$	1
AP	$P^2\left(\frac{L}{2}+1\right) + \frac{5}{2}LP + L + \mathcal{O}\left(P^3\right)$	$\frac{P^2}{2}(L+1) + \frac{5}{2}LP - \frac{P}{2} + \mathcal{O}\left(P^3\right)$	-

3.5.2 Simulation Results and Discussion

In this subsection, the performance of a DFE which employs data-reusing BNLMS algorithm is investigated through simulations in Ped-A and Veh-B environments. For the simulation work, we have considered the same values of different parameters as taken for AP algorithm based DFE in Section 3.4. Moreover, the filter lengths of DFE are chosen as $L_{FF} = 3$ and $L_{FB} = 1$ for both Ped-A and Veh-B environments for evaluating the MSE and BER performance of the equalizer.

3.5.2.1 Performance of DFE using BNLMS, NLMS and AP ($P = 2$) Algorithm.



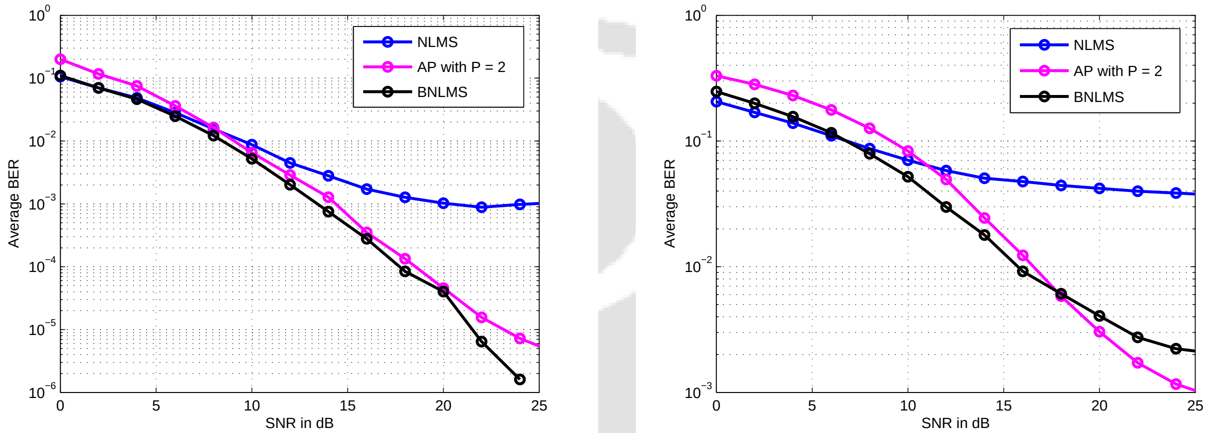
(a) Pedestrian Test Environment-A

(b) Vehicular Test Environment-B

Figure 3.9: Convergence Performances of DFE using BNLMS, NLMS and AP ($P = 2$) as Adaption Algorithms.

The convergence issues as well as BER performance of the proposed BNLMS-DFE is compared with DFE using NLMS and AP ($P = 2$) as adaption algorithm for a convergence factor $\mu = 0.05$.

It is observed in Fig. 3.9 that, the convergence performance of the proposed scheme is almost similar to DFE using an AP algorithm of projection order 2, and much better than NLMS-DFE. We know the complexity involved with BNLMS algorithm based channel equalization is substantially less compared to AP algorithm based scheme.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

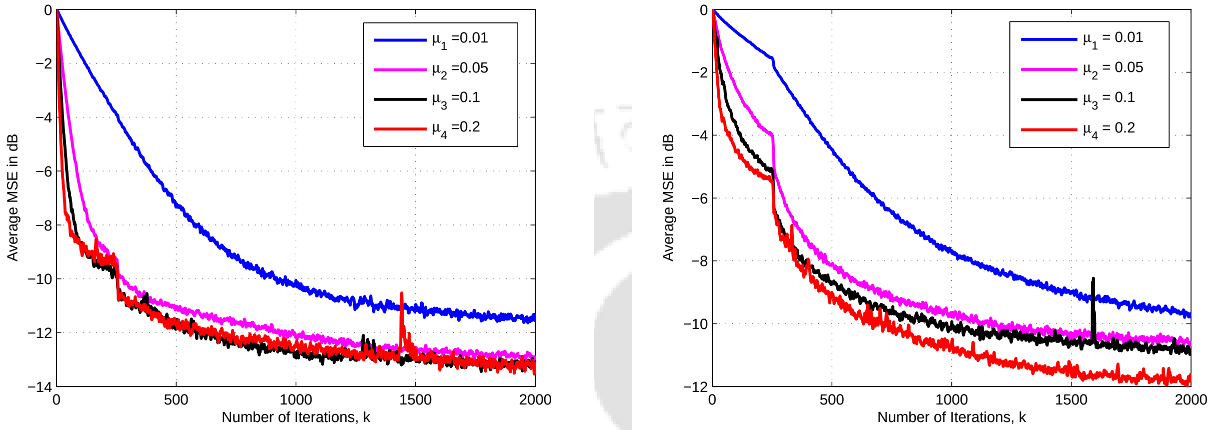
Figure 3.10: BER Performance Comparison of DFE using BNLMS, NLMS and AP ($P = 2$) as Adaption Algorithms.

The BER performance comparison of BNLMS, NLMS and AP ($P = 2$) algorithm based DFE is shown in Fig. 3.10. We can see that, BER performance of BNLMS-DFE is almost similar as NLMS-DFE for SNR value less than 8 dB, but performance improves remarkably in the high SNR region for both Ped-A and Veh-B environments. Moreover, the BER performance of this channel equalizer is quite similar as that of DFE using an AP algorithm with projection order 2 for Ped-A environment; but its performance is comparatively better in Veh-B environment for SNR $\leq$ 18 dB.

3.5.2.2 Performance of BNLMS Algorithm based DFE with Different Values of Step Size.

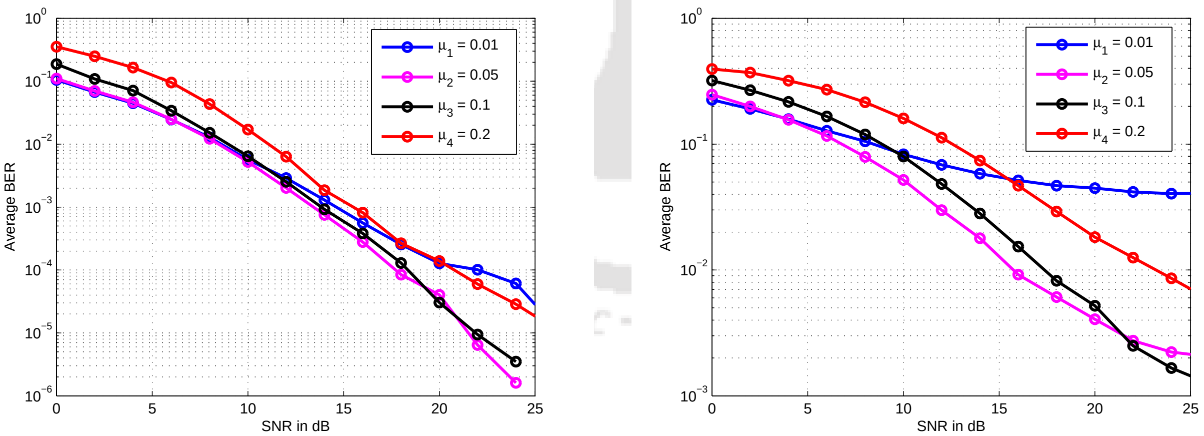
The MSE and BER performance of the proposed DFE using BNLMS algorithm are evaluated for different values of step size. Here, the simulation results are obtained for step sizes $\mu_1 = 0.01$, $\mu_2 = 0.05$, $\mu_3 = 0.1$ and $\mu_4 = 0.2$.

The MSE curves are shown in Fig. 3.11. It is observed that increase in step size improves the convergence performance. It is also observed that MSE curves for step size 0.2 contain fluctuations. As discussed earlier, small step size in a mobile fading environment performs better for tracking the channel variation effectively, hence step size is to be chosen judiciously.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 3.11: MSE Performance of Binormalized LMS Algorithm based DFE with Different Values of Step Size.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 3.12: BER Performance of Binormalized LMS Algorithm based DFE with Different Values of Step Size.

The BER performance of the proposed channel equalization scheme for different values of μ is shown in Fig. 3.12. It may be observed that variation in step size parameter effects the BER performance. For both Ped-A and Veh-B environments, the better BER performance is obtained for μ values 0.05

and 0.1. Further, it is observed that higher μ value results higher misadjustment, deteriorates the performance.

3.6 Summary

In this chapter, performance analysis of data-reusing AP algorithm based DFE and also BNLMS algorithm based DFE in a low delay spread Ped-A environment as well as medium delay spread Veh-B environment are presented. The MSE and BER performances as a function of projection order, step size and filter length were shown in simulation results. Simulation have also been performed for such adaptive DFE system using NLMS algorithm.

It is observed that AP algorithm is suitable for adaptive DFE systems in time-varying channels, as it offers significant improvement in convergence characteristics over NLMS based algorithm with marginal increase in computational complexity, also its complexity is very less over RLS algorithm. It is further observed that in both Ped-A and Veh-B environments, by increasing the projection order from 2 to 3, a significant improvement in convergence is achieved whereas increasing P further from 3 to 4 results in instantaneous error terms which deteriorates the performance as well as increases computational complexity. In this scheme, the main complexity is in computing the matrix inversion, which is of the order $\mathcal{O}\left(P^3\right)$. So, complexity increases with projection order; however, keeping P value small compared to $L_{FF} + L_{FB}$ can reduce the computational burden. Hence, choice of projection order which reflects a trade-off between the convergence speed, and the computational complexity, is very important. The selection of step size μ is very important with respect to convergence speed and stability. The choice of filter length is also very important, which decides the complexity and performance of the system.

The performance of a DFE with data-reusing BNLMS as adaption algorithm is found to be having advantages of (i) almost similar performance with reduced computational burden compared to DFE using an AP algorithm of projection order 2, due to absence of matrix inverse operation, (ii) convergence performance is much better than NLMS algorithm based DFE, and (iii) BER performance is similar as NLMS-DFE in the low SNR region, but performance improves remarkably for SNR value more than 8 dB, in both Ped-A and Veh-B environments. Further, it is observed that higher values of step size increases fluctuations in the instantaneous error terms, and it may be attributed to the poor BER performance. Hence, step size μ value is kept within the range of $0.05 < \mu \leqslant 0.1$ to achieve better performance in terms of convergence and BER.

Though both the equalizers discussed in this chapter are found to perform well in low delay spread Ped-A environment, their performance degraded in Veh-B environment.

4

Reduced Complexity Channel Equalization using Data-selective Algorithm

Contents

Objective

This chapter presents equalization techniques for time dispersive wireless channels, where different data-selective algorithms are used to adaptively update the coefficients of a decision feedback equalizer. The performance of the proposed approach is found to be quite similar to the corresponding schemes without data selective updating, but with almost 40 % less computational burden due to decrease in the number of updates.

4.1 Introduction

In wireless communication scenario, adaptive filtering algorithms are used to update the equalizer coefficients and track the channel variations. The efficacy of such algorithms are judged in terms of various factors such as convergence rate, misadjustment factor, computational complexity etc. Further, with an assumption that the noise added in wireless communication channel is bounded, and the bound is either known or can be estimated [72]; an upper bound on the estimation error can be introduced so that the filter coefficients are updated only when the output estimated error is higher than the pre-determined upper bound. This feature is called as the set-membership (SM) filtering. Introducing set-membership (SM) filtering in an adaptive algorithm reduces the computational complexity with an overall improvement in the system performance due to data selective updates [45]. The SM filtering technique has been discussed exhaustively in the literature [46,49,51,73]. As a standard procedure, the filter coefficients are updated such that the output estimation error is upper bounded by a prescribed threshold. The SM in adaptive filter employs a deterministic objective function related to a bounded error constraint on the filter output such that the updates belong to a set of feasible solutions. The SM adaptive filtering algorithms are inherently data selective, whereby a given rule decides if updates are required or not.

Although the data-reusing algorithms discussed in Chapter-3 found to be quite appealing in channel equalization, but it requires a trade-off between convergence speed and computational complexity. Moreover, these algorithms may have high misadjustment. Therefore, their combination with deterministic objective functions, leading to data selective updating, results in computationally efficient algorithms with low misadjustment and high convergence speed. These characteristics of SM-AP algorithm [49] and SM-BNLMS algorithm [51] motivates us to study these algorithms in detail and through simulation evaluate the performance a DFE employing them as the adaption algorithm when the equalizer operates in a frequency selective wireless environment.

The rest of the chapter is organized as follows; Section 4.2 discusses about data-selective algorithms. Section 4.3 investigates the performance of DFE using SM-AP algorithm through simulation. Section 4.4 presents the channel equalization performance of SM-BNLMS algorithm based DFE. Finally, the summary of work presented in this chapter is provided in Section 4.5.

4.2 Data-selective Algorithms

In this section, we discuss briefly about different data-selective algorithms such as 'Set-membership Normalized least mean square' (SM-NLMS), 'Set-membership Affine Projection' (SM-AP) and 'Set-membership Binormalized least mean square' (SM-BNLMS) algorithms.

4.2.1 Set-Membership Filtering

The use of SM filtering in an adaptive algorithm reduces the computational complexity with an overall improvement in the system performance due to data selective updates [45]. To explain this, we can take an adaptive FIR filter with N coefficient as considered in Section 2.3. The output of the filter at time instant k given in equation 2.2, is mentioned here for clarity.

$$\hat{v}_k = \mathbf{u}_k^T \mathbf{w}_k \tag{4.1}$$

where $\mathbf{u}_k$ is the input signal vector and $\mathbf{w}_k$ is the filter coefficient vector. The estimation error at time instant k is calculated as:

$$e_k = d_k - \hat{v}_k = d_k - \mathbf{u}_k^T \mathbf{w}_k \tag{4.2}$$

The vectors $\mathbf{u}_k$ and $\mathbf{w}_k$ are of size $N \times 1$. In SM filtering, the weight vector $\mathbf{w}_k$ is updated such that the magnitude of estimation error $|e_k|$ is less than or equal to a threshold value γ.

The data space, denoted as S is defined as the set of all possible input-desired signal pairs $(\mathbf{u}, d)$ of interest. The set of all possible vectors $\mathbf{w}$ that satisfy the above equation is referred as feasibility set Θ. It can be expressed as:

$$\Theta = \bigcap_{(\mathbf{u},d)\in S} \{\mathbf{w} \in \Re^N : |e_k| \le \gamma\} \tag{4.3}$$

If the adaptive filter is trained with k input-desired data pairs $\{u_i, d_i\}_{i=1}^k$, then the set containing all vectors $\mathbf{w}$ for which estimation error at iteration k is upper bounded in magnitude γ which is the region enclosed by a set of hyperplanes upper bounded in γ, is called constraint set H_k, where

$$H_k = \left\{ \mathbf{w} \in \Re^N : |e_k| \leq \gamma \right\}. \tag{4.4}$$

We know each constraint set is associated with a data-pair. The exact membership set ψ_k is the intersection of the constraint sets over all the available time instants $i = 0, 1, ..., k$. So, Ω_k results a polygon in the parameter space.

$$\psi_k = \bigcap_{i=0}^{k} H_i \tag{4.5}$$

4.2.2 Set-membership Normalized LMS Algorithm

The normalized LMS (NLMS) is an algorithm based on LMS, which utilizes a variable convergence factor for reducing the instantaneous output error. It converges faster than the LMS algorithm. Employing SM filtering in NLMS algorithm reduces the computational complexity due to data selective updates [46]. The SM-NLMS algorithm performs coefficient update if $\mathbf{w}_k$ lies outside the constraint set H_k, i.e., $\left|d_k - \mathbf{u}_k^T \mathbf{w}_k\right| > \gamma$, so that the new estimate $\mathbf{w}_{k+1}$ will lie on the closest boundary of H_k. It is performed by an orthogonal projection of $\mathbf{w}_k$ onto the closest boundary of H_k as shown in Fig. 4.1.

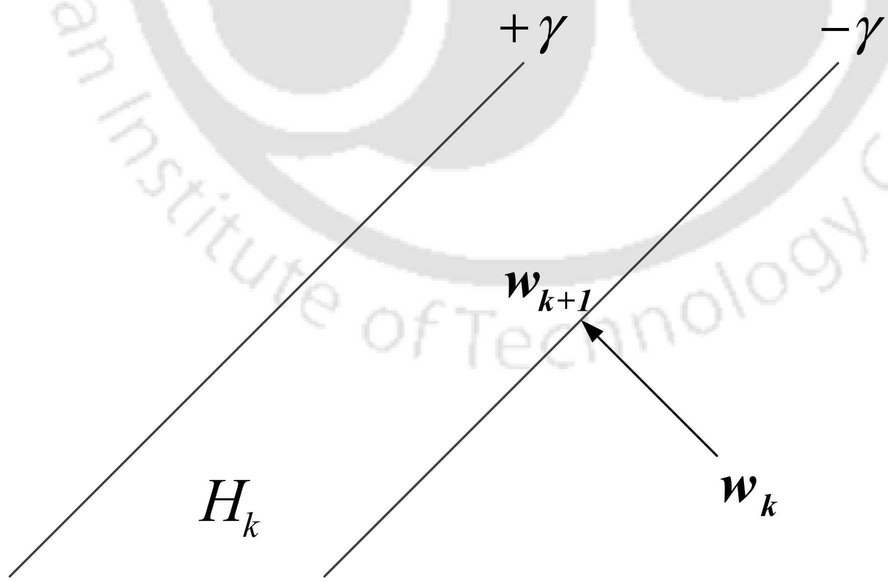


Figure 4.1: Graphical Visualization of the Updating Procedure of SM-NLMS Algorithm.

The filter coefficient can be updated if $|e_k| > \gamma$ by using this equation:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mu \frac{e_k - \gamma \text{ sign}[e_k]}{\|\mathbf{u}_k\|^2 + \varepsilon} \mathbf{u}_k \tag{4.6}$$

where μ is the step size.

4.2.3 Set-membership Affine Projection Algorithm

The data-reusing affine projection algorithm reuses the past data along with the present data for updating the filter coefficients which yields fast convergence even for highly correlated input [39]. The AP algorithm with SM filtering reduces the computational burden compared to AP algorithm due to data-selective updates [49, 50]. The SM-AP algorithm utilizes the present input signal vector as well as $P-1$ past input signal vectors on the coefficient update. The coefficient update is done when $\mathbf{w}_k \notin \psi_k^P$, where ψ_k^P is the intersection of last P constraint sets. The membership set ψ_k is expressed as:

$$\psi_k = \left(\bigcap_{i=0}^{k-P} H_i\right) \bigcap \left(\bigcap_{j=k-P+1}^{k} H_j\right) = \psi_k^{k-P} \bigcap \psi_k^P \tag{4.7}$$

where ψ_k^{k-P} is the intersection of the first $k-P+1$ constraint sets. Each constraint set specifies a region between two parallel hyperplanes where the estimation error is upper bounded in magnitude by a prescribed threshold. So, mathematically the objective function for SM-AP algorithm can be evaluated as described below.

Perform a coefficient update whenever $\mathbf{w}_k \notin \psi_k^P$ such that,

$$\begin{aligned} &\min \|\mathbf{w}_{k+1} - \mathbf{w}_k\|^2 \\ &\text{subject to:} \quad \mathbf{d}_k - \mathbf{U}_k^T \mathbf{w}_{k+1} = \boldsymbol{\gamma}_{\boldsymbol{k}} \end{aligned} \tag{4.8}$$

where $\mathbf{U}_k$ is the matrix of size $(N \times P)$ containing input signal vectors, $\mathbf{w}_k$ is coefficient vector of size $(N \times 1)$ and $\mathbf{d}_k$ is a vector of size $(P \times 1)$ containing desired signals, as discussed in the affine projection algorithm of section 3.2. Moreover, $\boldsymbol{\gamma}_k$ is a vector of size $(P \times 1)$ containing the threshold values on estimation error. The equation for coefficient updating using SM-AP algorithm can be derived by solving the minimization problem for the given constraints using Lagrange multiplier method as elaborated in Section 4.3.

4.2.4 Set-membership Binormalized LMS Algorithm

The SM-NLMS algorithm has low complexity per update, but convergence speed depends on the eigenvalue spread of the input signal correlation matrix, whereas SM-AP algorithm offers better convergence but the computational complexity is directly related to the number of data reuses. The

SM-BNLMS algorithm which can be seen as the extensions of the SM-NLMS algorithm, which mitigate these drawbacks. It uses two constraint sets for each update. In this case, the coefficient update is done whenever $\mathbf{w}_k \notin H_k \cap H_{k-1}$ so that the new update $\mathbf{w}_{k+1} \in H_k \cap H_{k-1}$, where H_k and H_{k-1} are constraint sets at iteration k and $k-1$, respectively. So, the objective function for coefficient updating at $(k+1)^{th}$ instant can be written as:

$$\begin{aligned} &\min \|\mathbf{w}_{k+1} - \mathbf{w}_k\|^2 \\ &\text{subject to:} \quad d_k - \mathbf{u}_k^T \mathbf{w}_{k+1} = \gamma_{k1} \\ &\text{and} \quad d_{k-1} - \mathbf{u}_{k-1}^T \mathbf{w}_{k+1} = \gamma_{k2} \end{aligned} \tag{4.9}$$

where γ_{k1} and γ_{k2} are threshold value on estimation error at instants k and $k-1$, respectively. Let these threshold values are upper bounded in magnitude to a value γ. The co-efficient update using SM-BNLMS algorithm can be done in two ways.

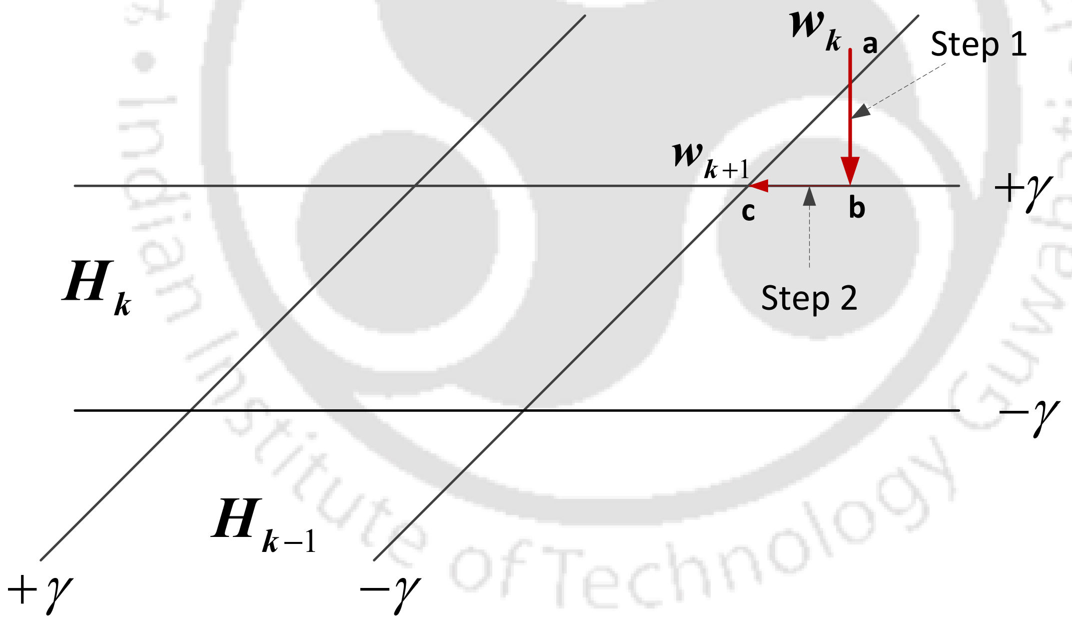


Figure 4.2: Graphical Visualization of the Updating Procedure of SM-BNLMS Algorithm (Method-1).

Method I: In this case, coefficient updating is done in two steps as shown in Fig. 4.2.

(i)Perform orthogonal projection of $\mathbf{w}_k$ onto the closest boundary of H_k, that is from position 'a' to 'b'.

(ii) Check whether the new estimate lies inside the constraint set H_{k-1} i.e. the region between two parallel hyperplanes where $\left|d_{k-1} - \mathbf{u}_{k-1}^T \mathbf{w}_k\right| \leq \gamma$. If the new estimate lies inside the constraint set, no further update is needed, else the second step is performed in the orthogonal direction with respect to the first step, that is from position 'b' to 'c' as shown in Fig. 4.2. So, the solution lies at the intersection of two sets of two parallel hyperplanes, that is $\mathbf{w}_{k+1} \in H_k \cap H_{k-1}$.

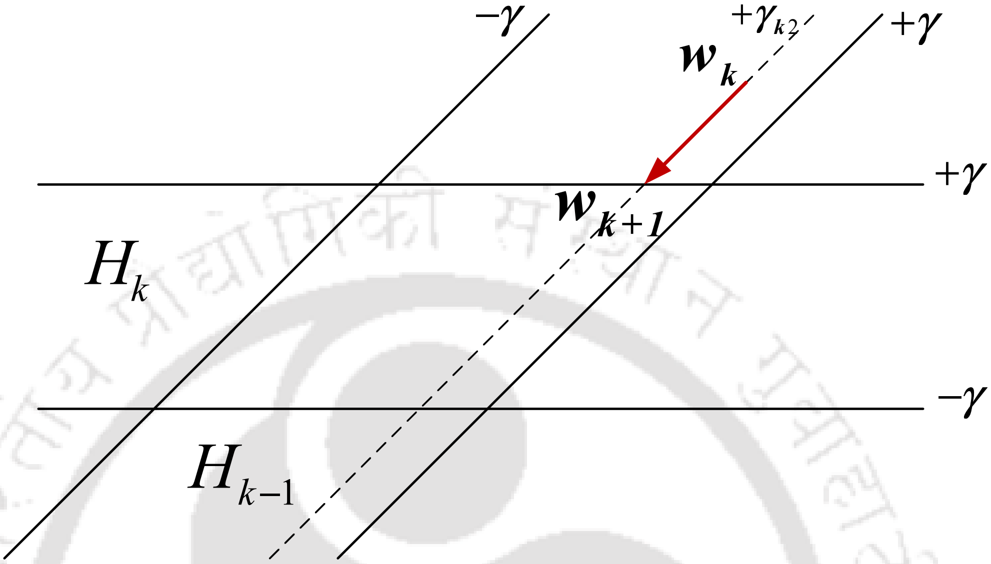


Figure 4.3: Graphical Visualization of the Updating Procedure of SM-BNLMS Algorithm (Method-2).

Method II: In this case, coefficient update is done in a single step by avoiding the intermediate constraint check, hence reduces the computational complexity per update. Fig. 4.3 shows the graphical representation of this type of updating approach of SM-BNLMS algorithm. This method is explained below.

(a) As the previous estimate $\mathbf{w}_k \in H_{k-1}$, the a-posteriori error $\varepsilon_{k-1} = d_{k-1} - \mathbf{u}_{k-1}^T \mathbf{w}_k$ will remain upper bounded by γ. Hence, $\gamma_{k2} = \varepsilon_{k-1}$ and $\gamma_{k2} \leq \gamma$ can be chosen.

(b) Moreover, if we consider $\gamma_{k1} = \gamma \operatorname{sign}[e_k]$, then the new update $\mathbf{w}_{k+1}$ lies on the nearest boundary of H_k.

The implementation of SM-BNLMS algorithm is further elaborated in Section 4.4.

4.3 Set-membership Affine Projection Algorithm based DFE

In this subsection, we consider a DFE where the coefficients of both the feed-forward filter (FFF) and the feedback filter (FBF) are adaptively updated using set-membership affine projection (SM-AP) algorithm. The system model used in this work and the equations describing the model are similar to the adaptive DFE model adopted in Chapter 3.

The SM-AP algorithm is already discussed in Section 4.2. This algorithm utilizes the present input signal vector as well as $P-1$ past input signal vectors for coefficient updating and it is done when

$\boldsymbol{w_k} \notin \psi_k^P$, where ψ_k^P is the intersection of last P constraint sets. For implementing this algorithm in an adaptive SISO DFE system, we have modified it appropriately as detailed below. The objective function of SM-AP algorithm at time instant k can be expressed as:

$$\begin{aligned} &\min \|\mathbf{w}_{k+1} - \mathbf{w}_k\|^2 \\ &\text{subject to:} \quad \mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_{k+1} = \boldsymbol{\gamma_k} \end{aligned} \tag{4.10}$$

where $\boldsymbol{d_k}$ is a vector containing desired signals of size $P \times 1$ and $\mathbf{w}_k$ is the weight vector of size $(L_{FF} + L_{FB}) \times 1$. The $\boldsymbol{\gamma}_k$ is the vector containing bound constraints. So,

$$\boldsymbol{\gamma}_k = \begin{bmatrix} \gamma_{k,1} \\ \gamma_{k,2} \\ \cdots \\ \gamma_{k,P} \end{bmatrix}. \tag{4.11}$$

All $\gamma_{k,i}$ are chosen such that $|\gamma_{k,i}| \leqslant \gamma$, where γ is the upper bound on the magnitude of output estimation error and $i = 1, 2, \cdots, P$. A matrix $\mathbf{Z}_k$ of size $(L_{FF} + L_{FB}) \times P$ is considered, which contains present and previous $P-1$ input signal vectors as expressed in equation 3.18 for AP algorithm based DFE.

$$\mathbf{Z}_k = \begin{bmatrix} \mathbf{z}_k & \mathbf{z}_{k-1} & \cdots & \mathbf{z}_{k-P+1} \end{bmatrix} \tag{4.12}$$

where concatenated input vector $\mathbf{z}_k$ is:

$$\mathbf{z}_k = \begin{bmatrix} \mathbf{y}_k \\ -\hat{\mathbf{x}}_{k-1} \end{bmatrix}. \tag{4.13}$$

The constrained minimization problem of equation 4.10 is solved by applying Lagrange multiplier method. The unconstrained function to be minimized is:

$$f\left[\mathbf{w}_{k+1}\right] = \|\mathbf{w}_{k+1} - \mathbf{w}_k\|^2 + \boldsymbol{\lambda}_k^T \left[\mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_{k+1} - \boldsymbol{\gamma}_k\right] \tag{4.14}$$

where Lagrange multipliers $\boldsymbol{\lambda}_k$ is:

$$\boldsymbol{\lambda}_k = \begin{bmatrix} \lambda_{k1} \\ \lambda_{k2} \\ \cdots \\ \lambda_{kP} \end{bmatrix}. \tag{4.15}$$

Setting the gradient of the function $f[\mathbf{w}_{k+1}]$ with respect to $\mathbf{w}_{k+1}$ equal to zero, we obtain:

$$\mathbf{w}_{k+1} = \mathbf{w}_k + \mathbf{Z}_k \frac{\boldsymbol{\lambda}_k}{2}. \tag{4.16}$$

The $\boldsymbol{\lambda}_k$ is derived from equations 4.10 and 4.16, as given below.

$$\begin{aligned} & \mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_{k+1} = \gamma_k \\ \Rightarrow \quad & \mathbf{d}_k - \mathbf{Z}_k^T \left[\mathbf{w}_k + \mathbf{Z}_k \frac{\boldsymbol{\lambda}_k}{2} \right] = \gamma_k \\ \Rightarrow \quad & \mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_k - \gamma_k = \mathbf{Z}_k^T \mathbf{Z}_k \left[\frac{\boldsymbol{\lambda}_k}{2} \right]. \end{aligned} \tag{4.17}$$

Solving equation 4.17, the $\boldsymbol{\lambda}_k$ is derived.

$$\boldsymbol{\lambda}_k = 2 \left[\mathbf{Z}_k^T \mathbf{Z}_k \right]^{-1} [\mathbf{e}_k - \gamma_k] \tag{4.18}$$

where the error vector is:

$$\mathbf{e}_k = \mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_k = \begin{bmatrix} e_k \\ \varepsilon_{k-1} \\ \cdots \\ \varepsilon_{k-P+1} \end{bmatrix} \tag{4.19}$$

and a-posteriori error is calculated as:

$$\varepsilon_{k-j} = d_{k-j} - \mathbf{z}_{k-j}^T \mathbf{w}_k \tag{4.20}$$

where $j = 1, 2, \cdots, P-1$.

In decision directed mode, the output of the decision device is used as the reference signal for estimating error. So, the error vector in DD mode can be expressed as:

$$\mathbf{e}_k = \hat{\mathbf{x}}_k - \mathbf{Z}_k^T \mathbf{w}_k. \tag{4.21}$$

where $\hat{\mathbf{x}}_k$ is a vector of size $P \times 1$, consists of present and previous $P-1$ decisions. Hence, the weight updating equation can be expressed from equations 4.16 and 4.18 as:

$$\mathbf{w}_{k+1} = \begin{cases} \mathbf{w}_k + \mathbf{Z}_k \left[\mathbf{Z}_k^T \mathbf{Z}_k + \alpha \mathbf{I} \right]^{-1} (\mathbf{e}_k - \gamma_k) & , \text{ if } \quad |e_k| > \gamma \\ \mathbf{w}_k & , \text{ otherwise} \end{cases} \tag{4.22}$$

where the matrix $\alpha\mathbf{I}$ is added to avoid the problem in inversion of $\mathbf{Z}_k^T \mathbf{Z}_k$ in a noisy environment.

The weight updating expression derived in equation 4.22, can be simplified further considering following steps.

(a) The weight update is done considering previous data pairs. So, $\mathbf{w}_k \in H_{k-j}$, where $j = 1, 2, \cdots, P-1$. Hence, the a-posteriori error ε_{k-j} will be equal to respective bound constraints $\gamma_{k,(j+1)}$, i.e.

$$\varepsilon_{k-j} = d_{k-j} - \mathbf{z}_{k-j}^T \mathbf{w}_k = \gamma_{k,(j+1)} \tag{4.23}$$

(b) We know $|\gamma_{k,1}| \leqslant \gamma$, where γ is the upper bound on the magnitude of output estimation error. Hence, the updated $\mathbf{w}_{k+1}$ lies on the nearest boundary of H_k if:

$$\gamma_{k,1} = \gamma \frac{e_k}{|e_k|} = \gamma \text{ sign} [e_k] \tag{4.24}$$

Thus, the simplified weight updating equation can be expressed as:

$$\mathbf{w}_{k+1} = \begin{cases} \mathbf{w}_k + \mathbf{Z}_k \left[\mathbf{Z}_k^T \mathbf{Z}_k + \alpha \mathbf{I}\right]^{-1} (e_k - \gamma \text{ sign} [e_k]) \mathbf{v} & , \text{ if } \quad |e_k| > \gamma \\ \mathbf{w}_k & , \text{ otherwise} \end{cases} \tag{4.25}$$

where $\mathbf{v} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T$ of size $P \times 1$.

Through simulation, it is observed that by introducing a convergence factor μ in equation 4.25 improves the convergence as well as BER performance of a DFE when operated in a standard wireless channel modeled according to [54]. The μ should be chosen in the range $0 < \mu < 2$, so that the a-priori error remain bounded, as in affine projection algorithm [71]. However μ value is kept small to achieve better performance. So, the modified weight updating equation is:

$$\mathbf{w}_{k+1} = \begin{cases} \mathbf{w}_k + \mu \mathbf{Z}_k \left[\mathbf{Z}_k^T \mathbf{Z}_k + \alpha \mathbf{I}\right]^{-1} (e_k - \gamma \text{ sign} [e_k]) \mathbf{v} & , \text{ if } \quad |e_k| > \gamma \\ \mathbf{w}_k & , \text{ otherwise} \end{cases} \tag{4.26}$$

The summary of the adaptive DFE using SM-AP algorithm is shown in Table 4.1.

4.3.1 Simulation Results and Discussion

In this subsection, we provide simulation results and analyze the same to apprise the performance of a DFE which operates in a wireless channel and employs SM-AP based algorithm for update of filter coefficients. In performing simulation, we have considered the same set of parameters as taken in Chapter 3 for data-reusing algorithm based DFE; however, the same is included here for the sake of continuity. In the simulations, WiMAX forum recommended ITU-R wireless channel models (Ped-A and Veh-B environments) are considered. The transmitted signals are binary phase shift keying

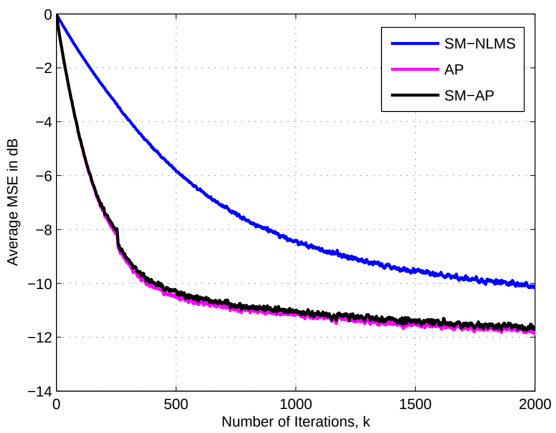
Table 4.1: Summary of the Adaptive DFE Implementation using SM-AP Algorithm

Step-1: Initialization
Weight vector $\mathbf{w}_0$ is initialized as zero vector.
Input signal vector $\mathbf{z}_0$ is initialized as zero vector.
The regularization parameter α is a small constant.
Convergence factor be in the range $0 < \mu < 2$.
Step-2: Processing for $k > 0$
(a) Computation of Error Vector:
$\mathbf{e}_k = \mathbf{d}_k - \mathbf{Z}_k^T \mathbf{w}_k$.
A-posteriori errors, $\varepsilon_{k-j} = d_{k-j} - \boldsymbol{z}_{k-j}^T \boldsymbol{w}_k$,
where $j = 1, 2, \cdots, P-1$.
In decision directed mode $\mathbf{d}_k$ is replaced by $\hat{\mathbf{x}}_k$.
(b) Setting an Upper Bound on Estimation Error:
The $\boldsymbol{\gamma}_k$ is the vector containing bound constraints, where
$\gamma_{k,1} = \gamma \text{ sign}[e_k]$ and $\gamma_{k,(j+1)} = \varepsilon_{k-j}$.
(c) Filter Weight Updating:
$\mathbf{w}_{k+1} = \begin{cases} \mathbf{w}_k + \mu \mathbf{Z}_k \left[\mathbf{Z}_k^T \mathbf{Z}_k + \alpha \mathbf{I}\right]^{-1} (e_k - \gamma \text{ sign}[e_k]) \mathbf{v} & , \text{ if } \lvert e_k \rvert > \gamma \\ \mathbf{w}_k & , \text{ otherwise} \end{cases}$
where $\mathbf{v} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T$ of size $P \times 1$.
(d) Increment k and go back to 'for' loop.

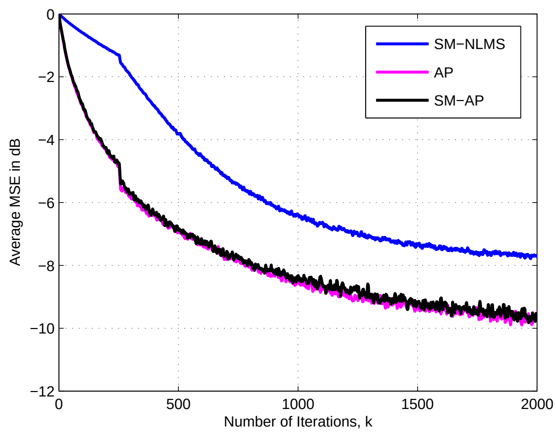
(BPSK) symbols, which are generated randomly and noise is additive white Gaussian. Here, each transmit data packet is composed of 2048 bits including 256 training bits, and 3000 such packets are transmitted. The sample time t_s of the signal is considered as .01 μsec for Ped-A and .3 μsec for Veh-B, and the carrier frequency f_c is taken as 2.5 GHz. Further, the maximum doppler frequency shift f_{dm} is considered as 5 Hz for Ped-A and 70 Hz for Veh-B. The MSE and BER curves are plotted taking an average of the results obtained from 3000 independent runs. The SNR value is taken as 20 dB to plot MSE curves.

4.3.1.1 Performance of DFE using SM-NLMS, AP and SM-AP Algorithms

The convergence and BER performance of adaptive DFEs using SM-NLMS, AP and SM-AP algorithms are compared here. For this study, the filter lengths $L_{FF} = 3, L_{FB} = 1$ and the projection order $P = 3$ are employed. Further, the upper bound γ on the estimation error is set to $\sqrt{1.5}\sigma_n$, where σ_n^2 is the variance of additive white Gaussian noise (AWGN). The σ_n^2 is calculated from SNR value, e.g. for 20 dB SNR, the value of σ_n^2 is 0.01, considering normalized signal power.



(a) Pedestrian Test Environment-A



(b) Vehicular Test Environment-B

Figure 4.4: MSE Performance Comparison of DFE with SM-NLMS, AP and SM-AP as Adaption Algorithm.

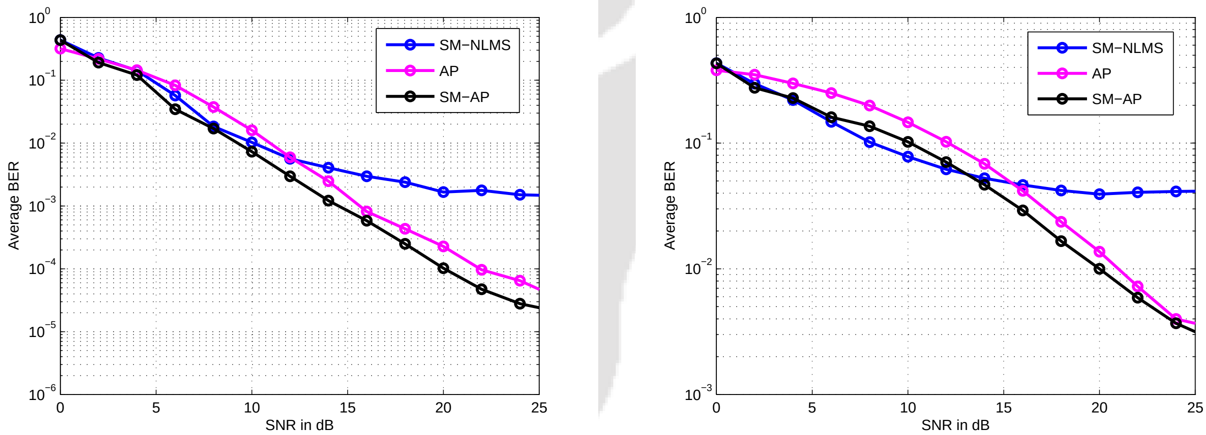
Fig. 4.4 shows the MSE performance curves for DFE using SM-NLMS, AP and SM-AP algorithm. The curves are plotted for a convergence factor value 0.01. It is observed that, the convergence performance of the proposed SM-AP algorithm based DFE is almost similar to AP algorithm based scheme. Further, it is noticed that the proposed technique offers a better rate of convergence than SM-NLMS algorithm based scheme.

The steady-state MSE (SS-MSE) values and number of updates associated with DFE using different adaptive algorithms are presented in Table 4.2. It is observed that, this scheme shows similar convergence with almost same SS-MSE compared to AP algorithm based DFE. It requires 1288 and 1462 number of updates in a packet of 2048 symbols, respectively in Ped-A and Veh-B environments, which leads to 37 % (in Ped-A) and 29 % (in Veh-B) reduction in computational complexity. It is noticed that, SS-MSE value of the DFE using SM-AP algorithm is 1.6 dB and 1.9 dB less than with SM-NLMS algorithm in Ped-A and Veh-B environments, respectively, which reflects better tracking. Moreover, this technique requires 279 (in Ped-A) and 300 (in Veh-B) fewer numbers of updates

Table 4.2: SS-MSE and Number of Updates Associated with DFE using Different Adaptive Algorithms.

Adaptive Algorithm	Ped-A		Veh-B	
	SS-MSE	Number of Updates	SS-MSE	Number of Updates
SM-NLMS	-10 dB	1567	-7.6 dB	1762
AP (P=3)	-11.7 dB	2048	-9.6 dB	2048
SM-AP (P=3)	-11.6 dB	1288	-9.5 dB	1462

compared to SM-NLMS algorithm based DFE.



(a) Pedestrian Test Environment-A

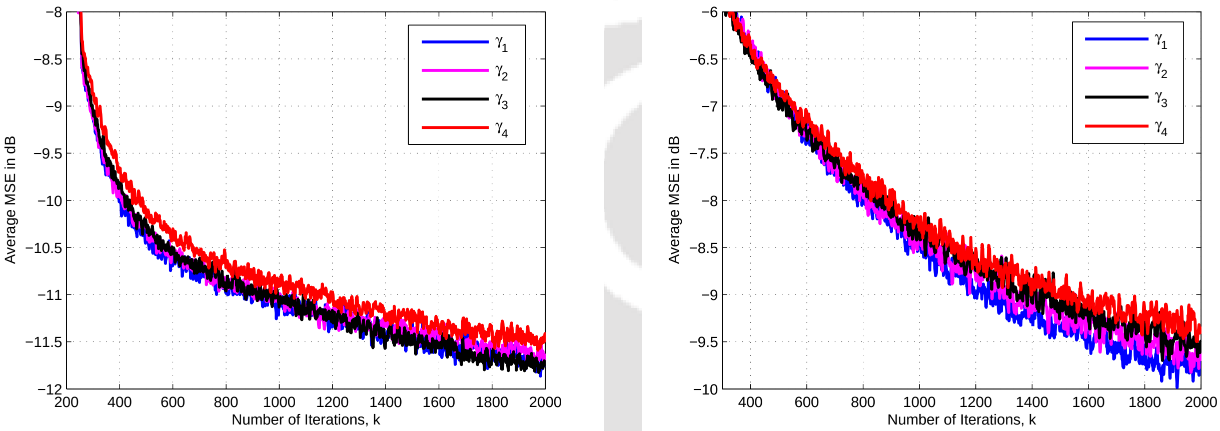
(b) Vehicular Test Environment-B

Figure 4.5: BER Performance Comparison of DFE with SM-NLMS, AP and SM-AP as Adaption Algorithm.

Fig. 4.5 displays the BER performance comparison of DFE with SM-NLMS, AP and SM-AP as the adaption algorithm in a time-varying wireless channel. Here, the convergence factor μ value is taken as 0.05. It is observed that, BER performance of SM-AP algorithm based DFE is similar, even slightly better than the DFE using AP algorithm in both Ped-A and Veh-B environments. The improvement in performance is obtained for low misadjustment due to the data selective updating. Further, it is observed that the BER performance of the proposed channel equalization is much better than SM-NLMS based DFE for SNR value more than 8 dB and 12 dB respectively, in Ped-A and Veh-B environments, but similar in low SNR region.

4.3.1.2 Performance of SM-AP Algorithm based DFE for Different Values of Upper Bounds on Estimation Error

The performance of the proposed equalizer is tested for different values of upper bounds on estimation error. In this test, we consider $\mu = 0.01$, $P = 3$, and filter lengths as $L_{FF} = 3, L_{FB} = 1$. The simulations are conducted for upper bound γ as $\gamma_1 = \sqrt{0.7}\sigma_n$, $\gamma_2 = \sqrt{1.5}\sigma_n$, $\gamma_3 = \sqrt{3}\sigma_n$ and $\gamma_4 = \sqrt{5}\sigma_n$.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

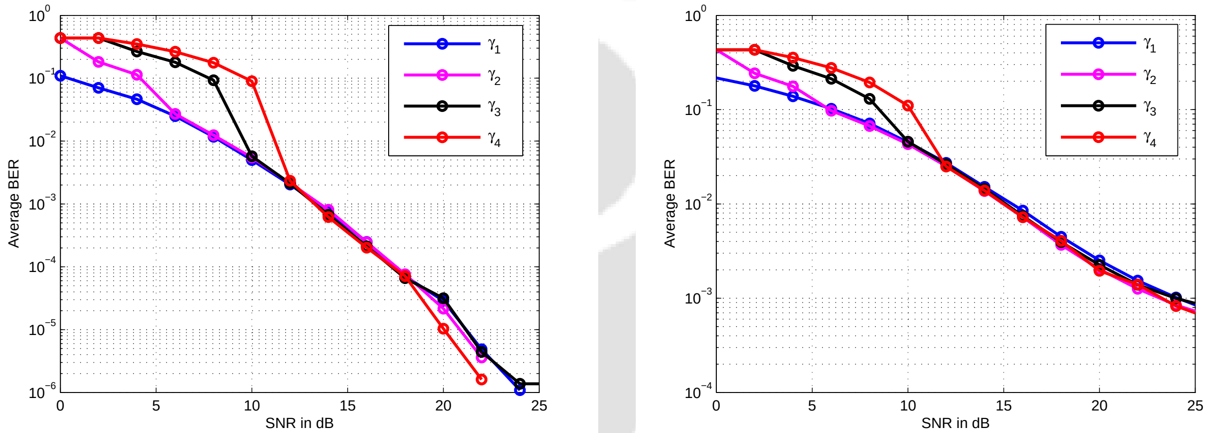
Figure 4.6: MSE Performance of SM-AP Algorithm based DFE with Different γ Values.

The MSE curves for SM-AP Algorithm based adaptive DFE for different values of upper bounds are displayed in Fig. 4.6. It can be observed that increasing the value of upper bound deteriorates the convergence performance.

Table 4.3: SS-MSE and Number of Updates of SM-AP algorithm based DFE for different γ values.

Upper Bound γ	Ped-A		Veh-B	
	SS-MSE	Number of Updates	SS-MSE	Number of Updates
$\gamma_1 = \sqrt{0.7}\sigma_n$	-11.6 dB	1496	-9.6 dB	1622
$\gamma_2 = \sqrt{1.5}\sigma_n$	-11.6 dB	1288	-9.5 dB	1462
$\gamma_3 = \sqrt{3}\sigma_n$	-11.6 dB	1064	-9.3 dB	1295
$\gamma_4 = \sqrt{5}\sigma_n$	-11.4 dB	900	-9.2 dB	1152

The SS-MSE values with corresponding number of updates for this channel equalization scheme for different γ values are given in Table 4.3. It can be noticed that increase in upper bound results a decrease in the number of updates, for example changing γ from $\sqrt{0.7}\sigma_n$ to $\sqrt{1.5}\sigma_n$ results almost 10 % and 8 % reduction in number of updates, respectively, in Ped-A and Veh-B environments. Furthermore, higher value of γ deteriorates the convergence performance as well SS-MSE value. It can be noticed that, employing SM filtering with γ as $\sqrt{1.5}\sigma_n$ requires 760 (Ped-A) and 586 (Veh-B) less number of updates in a packet of 2048 symbols compared to AP algorithm based DFE.



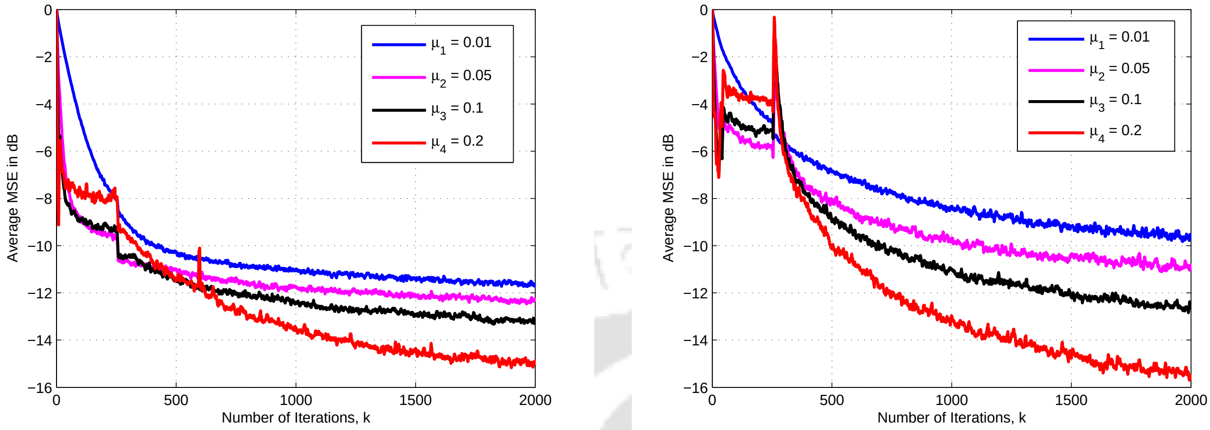
(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 4.7: BER Performance of SM-AP Algorithm based DFE with Different γ Values.

Fig. 4.7 shows the BER performance of SM-AP Algorithm based DFEs for different values of γ. The figure reveals that, in low SNR region increasing upper bound degrades BER performance, whereas for SNR above 12 dB, performance is almost identical. Therefore the choice of γ, which reflects as a trade-off between computational requirement linked to number of updates, and MSE as well as BER performance, is of great significance. The upper bound γ in the range of $\sqrt{1.5}\sigma_n \leqslant \gamma \leqslant \sqrt{3}\sigma_n$ is found to be suitable for achieving the same.

4.3.1.3 Performance of SM-AP Algorithm based DFE with Different Values of Convergence Factor

In this study, the performance of SM-AP algorithm based DFE for different values of convergence factor μ are compared. The performance includes convergence and BER as well as steady-state MSE value and requirement of number of updates. The simulations are done for convergence factor values $\mu_1 = 0.01$, $\mu_2 = 0.05$, $\mu_3 = 0.1$ and $\mu_4 = 0.2$. Moreover, we consider $P = 3$, $\gamma = \sqrt{1.5}\sigma_n$ and filter lengths as $L_{FF} = 3, L_{FB} = 1$ in this simulation.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

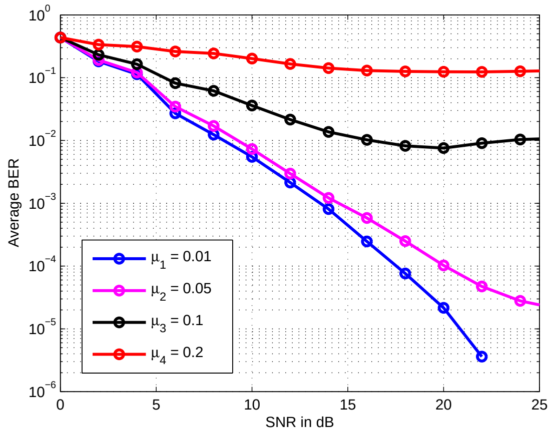
Figure 4.8: MSE Performance Comparison of SM-AP Algorithm based DFE for Different Values of Convergence Factor.

Fig. 4.8 displays the MSE learning curves of the proposed channel equalization scheme for different values of convergence factor μ. It can be noticed that, increasing the μ value from 0.01 to 0.05 and also from 0.05 to 0.1 improves the rate of convergence, whereas μ value 0.2, increases the fluctuations in the error curve as well as takes more time to reach steady-state value in both Ped-A and Veh-B environment.

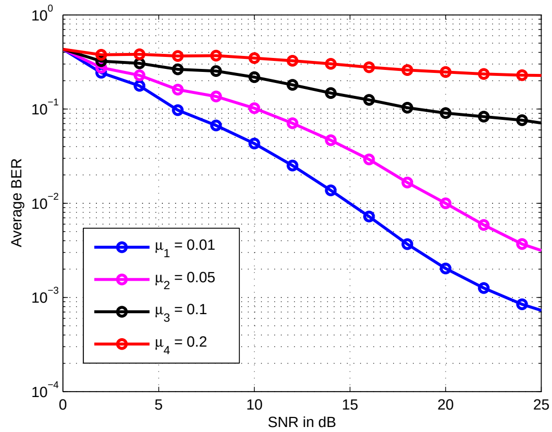
Table 4.4: 'SS-MSE' and 'Number of Updates' of SM-AP Algorithm Based DFE for Different Values of Convergence Factor.

Convergence Factor	Ped-A		Veh-B	
	SS-MSE	Number of Updates	SS-MSE	Number of Updates
0.01	-11.6 dB	1288	-9.5 dB	1474
0.05	-12.3 dB	1129	-10.6 dB	1249
0.1	-13 dB	1060	-12.5 dB	1097
0.2	-15 dB	890	-15 dB	848

The SS-MSE and number of updates associated with SM-AP algorithm based DFE for different values of convergence factor are given in Table 4.4. It is observed that, increase in μ results reduction in SS-MSE and number of updating operations, for example increasing μ from 0.01 to 0.05 results almost 8 % and 11 % reduction in number of updates, respectively, in Ped-A and Veh-B environments. Moreover, through simulations it is found that if μ value is made greater than 0.1, the performance degrades due to instantaneous error terms.



(a) Pedestrian Test Environment-A



(b) Vehicular Test Environment-B

Figure 4.9: Comparison of BER Performance for SM-AP Algorithm based DFE for Different Values of Convergence Factor.

The comparison of BER performance for SM-AP algorithm based DFE with different values of convergence factor μ are shown in Fig. 4.9. It is observed that, BER performance degrades with increase in values of μ. We know, in a time dispersive channel a small step size is needed to estimate the filter co-efficient more precisely according to the time-varying channel condition. Hence, the higher values of μ increases the fluctuations in the instantaneous error terms, and it may be attributed to the poor BER performance. Moreover, convergence factor $\mu = 0.05$ is found to be suitable both in terms of BER performance and requirement of number of updates in both Ped-A and Veh-B environment.

4.4 Set-Membership Binormalized LMS Algorithm based DFE

In this section, an adaptive DFE using set-membership binormalized least mean square (SM-BNLMS) algorithm is considered. We adopt the adaptive DFE system model described in Chapter 3 for implementing this algorithm.

The data selective SM-BNLMS algorithm and a graphical visualization of its updating procedure is already given in Section 4.2. As explained earlier, this algorithm performs a coefficient update whenever the weight vector $\mathbf{w}_k \notin H_k \cap H_{k-1}$, where H_k and H_{k-1} are constraint sets at iteration k and $k-1$, respectively. As defined earlier, H_k is the set containing all vectors $\mathbf{w}$ for which estimation error at iteration k is upper bounded in magnitude by a prescribed quantity γ, that is the region enclosed by a set of hyperplanes upper bounded by γ.

For implementing this algorithm in an adaptive SISO DFE system, we have modified it appropriately as detailed below. The objective of this algorithm is to maintain the next coefficient vector as close as possible to the current one, while maintaining a-posteriori errors prescribed by thresholds $(\gamma_{k1}, \gamma_{k2})$. So, the objective function at time instant k in DD mode can be expressed as:

$$\begin{aligned} &\min \left\|\mathbf{w}_{k+1} - \mathbf{w}_k\right\|^2 \\ \text{subject to:} \quad &\hat{x}_k - \mathbf{z}_k^T \mathbf{w}_{k+1} = \gamma_{k1} \\ \text{and} \quad &\hat{x}_{k-1} - \mathbf{z}_{k-1}^T \mathbf{w}_{k+1} = \gamma_{k2} \end{aligned} \tag{4.27}$$

where $\hat{x}_k$ is the output of the decision device and the input signal vectors $\mathbf{y}_k$ and $\hat{\mathbf{x}}_{k-1}$ are concatenated to produce $\mathbf{z}_k$ of dimension $(L_{FF} + L_{FB}) \times 1$. The thresholds $(\gamma_{k1}, \gamma_{k2})$ are chosen in such a way that it is upper bounded in magnitude γ. The constrained minimization problem of equation 4.27 is solved by applying Lagrange multiplier method. The unconstrained function to be minimized is:

$$\begin{aligned} f\left(\mathbf{w}_{k+1}\right) = \left\|\mathbf{w}_{k+1} - \mathbf{w}_k\right\|^2 + \lambda_1(k)\left[\hat{x}_k - \mathbf{z}_k^T \mathbf{w}_{k+1} - \gamma_{k1}\right] \\ + \lambda_2(k)\left[\hat{x}_{k-1} - \mathbf{z}_{k-1}^T \mathbf{w}_{k+1} - \gamma_{k1}\right] \end{aligned} \tag{4.28}$$

where $\lambda_1(k)$ and $\lambda_2(k)$ are the Lagrange multipliers for the two consecutive constraints at time instant k. Setting the gradient of the unconstrained function in equation 4.28 with respect to $\mathbf{w}_{k+1}$ equal to zero, we obtain the weight update equation as:

$$\mathbf{w}_{k+1} = \begin{cases} \mathbf{w}_k + \frac{\lambda_1(k)}{2}\mathbf{z}_k + \frac{\lambda_2(k)}{2}\mathbf{z}_{k-1} & \text{, if} \quad |e_k| > \gamma \\ \mathbf{w}_k & \text{, otherwise} \end{cases} \tag{4.29}$$

where $\lambda_1(k)$ and $\lambda_2(k)$ are expressed as:

$$\begin{aligned} \frac{\lambda_1(k)}{2} &= \frac{[e_k - \gamma_{k1}]\,\|\mathbf{z}_{k-1}\|^2 - [\varepsilon_{k-1} - \gamma_{k2}]\,\mathbf{z}_k^T\mathbf{z}_{k-1}}{\|\mathbf{z}_k\|^2\,\|\mathbf{z}_{k-1}\|^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2} \\ \frac{\lambda_2(k)}{2} &= \frac{[\varepsilon_{k-1} - \gamma_{k2}]\,\|\mathbf{z}_k\|^2 - [e_k - \gamma_{k1}]\,\mathbf{z}_{k-1}^T\mathbf{z}_k}{\|\mathbf{z}_k\|^2\,\|z_{k-1}\|^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2} \end{aligned} \tag{4.30}$$

with a-priori error at iteration k as $e_k = \hat{x}_k - \mathbf{z}_k^T\mathbf{w}_k$ and a-posteriori error at iteration $k-1$ as $\varepsilon_{k-1} = \hat{x}_{k-1} - \mathbf{z}_{k-1}^T\mathbf{w}_k$.

The weight updating equation for the SM-BNLMS algorithm based DFE is simplified further by considering the following assumptions [51];

(i) As the previous estimate $\mathbf{w}_k \in H_{k-1}$, so the a-posteriori error ε_{k-1} will remain upper bounded by γ. Hence, $\gamma_{k2} = \varepsilon_{k-1}$ and $\gamma_{k2} \leq \gamma$ can be chosen.

(ii) Moreover, if we consider $\gamma_{k1} = \gamma \operatorname{sign}[e_k]$, then the new update $\mathbf{w}_{k+1}$ lies on the nearest boundary of H_k.

So, the Lagrange multipliers given in equation 4.30, are modified as:

$$\begin{aligned} \frac{\lambda_1(k)}{2} &= \frac{[e_k - \gamma_{k1}]\,\|\mathbf{z}_{k-1}\|^2}{\alpha + \|\mathbf{z}_k\|^2\,\|\mathbf{z}_{k-1}\|^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2} \\ \frac{\lambda_2(k)}{2} &= \frac{-[e_k - \gamma_{k1}]\,\mathbf{z}_{k-1}^T\mathbf{z}_k}{\alpha + \|\mathbf{z}_k\|^2\,\|\mathbf{z}_{k-1}\|^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2} \end{aligned}. \tag{4.31}$$

where regularization parameter α is a small constant.

It is found through simulation that the introducing a convergence factor μ in the weight update equation 4.29, improves the overall performance. So, the modified weight updating equation can be expressed as:

$$\mathbf{w}_{k+1} = \begin{cases} \mathbf{w}_k + \mu\left(\frac{\lambda_1(k)}{2}\mathbf{z}_k + \frac{\lambda_2(k)}{2}\mathbf{z}_{k-1}\right) & \text{, if} \quad |e_k| > \gamma \\ \mathbf{w}_k & \text{, otherwise} \end{cases} \tag{4.32}$$

The summary of the proposed SM-BNLMS algorithm based channel equalization scheme is shown in Table 4.5.

Table 4.5: Summary of the SM-BNLMS Algorithm based DFE Implementation Scheme.

Step-1:Initialization
Weight vector $\mathbf{w}_0$ is initialized as zero vector.
Input signal vector $\mathbf{z}_0$ is initialized as zero vector.
The regularization parameter α is a small constant.
Convergence factor be in the range $0 < \mu < 2$.
Step-2: Processing for $k > 0$
(a) Computation of Error in DD Mode
A-priori Error: $e_k = \hat{x}_k - \mathbf{z}_k^T \mathbf{w}_k$.
A-posteriori Error: $\varepsilon_{k-1} = \hat{x}_{k-1} - \mathbf{z}_{k-1}^T \mathbf{w}_k$.
(b) Setting an Upper Bound
$\gamma_{k1} = \gamma \text{ sign}\left[e_k\right]$.
$\gamma_{k2} = \varepsilon_{k-1} \leqslant \gamma$.
(c) Filter Weight Updating
$\mathbf{w}_{k+1} = \begin{cases} \mathbf{w}_k + \mu\left(\frac{\lambda_1(k)}{2}\mathbf{z}_k + \frac{\lambda_2(k)}{2}\mathbf{z}_{k-1}\right) & \text{, if } \quad \lvert e_k \rvert > \gamma \\ \mathbf{w}_k & \text{, otherwise} \end{cases}$
where $\begin{aligned} \frac{\lambda_1(k)}{2} &= \frac{\left[e_k - \gamma_{k1}\right]\lVert\mathbf{z}_{k-1}\rVert^2}{\alpha + \lVert\mathbf{z}_k\rVert^2\lVert\mathbf{z}_{k-1}\rVert^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2} \\ \frac{\lambda_2(k)}{2} &= \frac{-\left[e_k - \gamma_{k1}\right]\mathbf{z}_{k-1}^T\mathbf{z}_k}{\alpha + \lVert\mathbf{z}_k\rVert^2\lVert\mathbf{z}_{k-1}\rVert^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2} \end{aligned}$.
(d) Increment k **and go back to 'for' loop.**

4.4.1 Convergence Analysis

In this work, we employ a SM-BNLMS algorithm [51] based adaptive channel equalization technique to estimate the coefficient vector $\boldsymbol{w_0}$ of an unknown FIR filter. Let the desired signal is,

$$d_k = \mathbf{z}_k^T \mathbf{w}_0 + n_k \tag{4.33}$$

where $\mathbf{z}_k$ is concatenated input signal vector and n_k is measurement noise. Here, we assume that: (a) measurement noise is Gaussian with zero mean and variance σ_n^2, (b) input signal vector and measurement noise are independent, and (c) variance of input signal vector is σ_z^2. Our purpose is to analyze the convergence behavior of the coefficient vector $\mathbf{w}_k$ as a function of convergence factor μ.

We can write the weight updating equation mentioned in equation 4.32 as:

$$\mathbf{w}_{k+1} = \begin{cases} \mathbf{w}_k + \mu\left(\frac{\lambda_1(k)}{2}\mathbf{z}_k + \frac{\lambda_2(k)}{2}\mathbf{z}_{k-1}\right) & , \text{ if } \quad |e_k| > \gamma \\ \mathbf{w}_k & , \text{ otherwise} \end{cases} \tag{4.34}$$

where

$$\begin{aligned} \frac{\lambda_1(k)}{2} &= \frac{[e_k - \gamma_{k1}] \|\mathbf{z}_{k-1}\|^2}{\alpha + \|\mathbf{z}_k\|^2 \|\mathbf{z}_{k-1}\|^2 - \left[\mathbf{z}_{k-1}^T \mathbf{z}_k\right]^2} \\ \frac{\lambda_2(k)}{2} &= \frac{-[e_k - \gamma_{k1}] \mathbf{z}_{k-1}^T \mathbf{z}_k}{\alpha + \|\mathbf{z}_k\|^2 \|\mathbf{z}_{k-1}\|^2 - \left[\mathbf{z}_{k-1}^T \mathbf{z}_k\right]^2} \end{aligned}. \tag{4.35}$$

where $\gamma_{k1} = \gamma$ sign $[e_k]$ is the threshold value on estimation error which is a function of σ_n^2. We can also write the weight updating equation as:

$$\mathbf{w}_{k+1} = \begin{cases} \mathbf{w}_k + \mu(e_k - \gamma)\boldsymbol{D} & , \text{ if } \quad e_k > \gamma \\ \mathbf{w}_k + \mu(e_k + \gamma)\boldsymbol{D} & , \text{ if } \quad e_k > -\gamma \\ \mathbf{w}_k & , \text{ otherwise} \end{cases} \tag{4.36}$$

where

$$\boldsymbol{D} = \frac{\|\mathbf{z}_{k-1}\|^2 \mathbf{z}_k - \mathbf{z}_{k-1}^T \mathbf{z}_k \mathbf{z}_{k-1}}{\alpha + \|\mathbf{z}_k\|^2 \|\mathbf{z}_{k-1}\|^2 - \left[\mathbf{z}_{k-1}^T \mathbf{z}_k\right]^2} \tag{4.37}$$

We can write the weight error vector at time instant $k+1$ as:

$$\tilde{\mathbf{w}}_{k+1} = \begin{cases} (\boldsymbol{I} + \mu\boldsymbol{C})\tilde{\mathbf{w}}_k + \mu(n_k - \gamma)\boldsymbol{D} & , \text{ if } \quad e_k > \gamma \\ (\boldsymbol{I} + \mu\boldsymbol{C})\tilde{\mathbf{w}}_k + \mu(n_k + \gamma)\boldsymbol{D} & , \text{ if } \quad e_k > -\gamma \\ \tilde{\mathbf{w}}_k & , \text{ otherwise} \end{cases} \tag{4.38}$$

where $e_k = d_k - \mathbf{z}_k^T\mathbf{w}_k = \mathbf{z}_k^T\mathbf{w}_0 + n_k - \mathbf{z}_k^T\mathbf{w}_k = n_k - \mathbf{z}_k^T\tilde{\mathbf{w}}_k$, weight error vector at time instant k is $\tilde{\mathbf{w}}_k = \mathbf{w}_k - \mathbf{w}_0$ and

$$\boldsymbol{C} = \frac{\mathbf{z}_{k-1}^T \mathbf{z}_k \mathbf{z}_{k-1} \mathbf{z}_k^T - \|\mathbf{z}_{k-1}\|^2 \mathbf{z}_k \mathbf{z}_k^T}{\alpha + \|\mathbf{z}_k\|^2 \|\mathbf{z}_{k-1}\|^2 - \left[\mathbf{z}_{k-1}^T \mathbf{z}_k\right]^2}. \tag{4.39}$$

Our purpose is to find the range of convergence factor for which:

$$E[\tilde{\mathbf{w}}_{k+1}] = 0 \qquad \text{as} \quad k \to \infty \tag{4.40}$$

Taking expected value on both sides of equation 4.38, we will get:

$$E[\tilde{\mathbf{w}}_{k+1}] = \begin{cases} E[(\boldsymbol{I} + \mu\boldsymbol{C})\tilde{\mathbf{w}}_k] + \mu E[n_k\boldsymbol{D}] - \mu\gamma E[\boldsymbol{D}] & , \text{ if } \quad e_k > \gamma \\ E[(\boldsymbol{I} + \mu\boldsymbol{C})\tilde{\mathbf{w}}_k] + \mu E[n_k\boldsymbol{D}] + \mu\gamma E[\boldsymbol{D}] & , \text{ if } \quad e_k > -\gamma \\ E[\tilde{\mathbf{w}}_k] & , \text{ otherwise} \end{cases} \tag{4.41}$$

For simplifying the evaluation of the expected value mentioned above, we assume:

- Elements of $\mathbf{z}_k$ are samples of a white Gaussian process.
- $\boldsymbol{w_k}$ is statistically independent of $\mathbf{z}_k\mathbf{z}_k^T$ [74].
- $E\left[\frac{P}{Q}\right] \approx \frac{E[P]}{E[Q]}$ where P and Q are terms in the numerator and denominator respectively, which shows independence of P and Q, as well as a 1st order approximation of $E\left[\frac{1}{Q}\right]$ [41].

The individual term of equation 4.41 is evaluated as follows.

(i) The second term $\mu E\left[n_k\boldsymbol{D}\right]$:

$$\begin{aligned}\mu E\left[n_k\boldsymbol{D}\right] &= \mu E\left[\frac{n_k\left\|\mathbf{z}_{k-1}\right\|^2\mathbf{z}_k - n_k\mathbf{z}_{k-1}^T\mathbf{z}_k\mathbf{z}_{k-1}}{\left\|\mathbf{z}_k\right\|^2\left\|\mathbf{z}_{k-1}\right\|^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2}\right] \\ &\approx \mu\frac{E\left[n_k\left\|\mathbf{z}_{k-1}\right\|^2\mathbf{z}_k\right] - E\left[n_k\mathbf{z}_{k-1}^T\mathbf{z}_k\mathbf{z}_{k-1}\right]}{E\left[\left\|\mathbf{z}_k\right\|^2\left\|\mathbf{z}_{k-1}\right\|^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2\right]} = 0\end{aligned} \tag{4.42}$$

as $\mathbf{z}_k$ and n_k are independent, zero mean random processes. So the numerator term of equation 4.42 will be zero.

(ii) The third term $\mu\gamma E\left[\boldsymbol{D}\right]$:

$$\begin{aligned}\mu\gamma E\left[\boldsymbol{D}\right] &= \mu\gamma E\left[\frac{\left\|\mathbf{z}_{k-1}\right\|^2\mathbf{z}_k - \mathbf{z}_{k-1}^T\mathbf{z}_k\mathbf{z}_{k-1}}{\left\|\mathbf{z}_k\right\|^2\left\|\mathbf{z}_{k-1}\right\|^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2}\right] \\ &\approx \mu\gamma\frac{E\left[\left\|\mathbf{z}_{k-1}\right\|^2\mathbf{z}_k\right] - E\left[\mathbf{z}_{k-1}^T\mathbf{z}_k\mathbf{z}_{k-1}\right]}{E\left[\left\|\mathbf{z}_k\right\|^2\left\|\mathbf{z}_{k-1}\right\|^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2\right]} = 0\end{aligned} \tag{4.43}$$

As we know from Isserlis theorem [75] that for stationary Gaussian distributed signal all odd moments are zero and even moments is $(t-1)!!\sigma^t$ where $(t-1)!!$ denotes the product of all odd integers up to and including $(t-1)$. So, the two numerator terms in equation 4.43 which contains odd moments, will be zero.

Thus, for $|e_k| > \gamma$, the equation 4.41 can be written as:

$$\begin{aligned}E\left[\tilde{\mathbf{w}}_{k+1}\right] &= E\left[\left(\boldsymbol{I} + \mu\boldsymbol{C}\right)\tilde{\mathbf{w}}_k\right] \\ &= E\left[\left(I + \mu\frac{\mathbf{z}_{k-1}^T\mathbf{z}_k\mathbf{z}_{k-1}\mathbf{z}_k^T - \left\|\mathbf{z}_{k-1}\right\|^2\mathbf{z}_k\mathbf{z}_k^T}{\left\|\mathbf{z}_k\right\|^2\left\|\mathbf{z}_{k-1}\right\|^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2}\right)\tilde{\mathbf{w}}_k\right] \\ &= E\left[\tilde{\mathbf{w}}_k\right] + \mu\left[\frac{E\left[\mathbf{z}_{k-1}^T\mathbf{z}_k\mathbf{z}_{k-1}\mathbf{z}_k^T\right] - E\left[\left\|\mathbf{z}_{k-1}\right\|^2\mathbf{z}_k\mathbf{z}_k^T\right]}{E\left[\left\|\mathbf{z}_k\right\|^2\left\|\mathbf{z}_{k-1}\right\|^2 - \left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2\right]}\right]E\left[\tilde{\mathbf{w}}_k\right]\end{aligned} \tag{4.44}$$

Expected values mentioned in equation 4.44, are evaluated as follows.

(a)

$$\begin{aligned}
E\left[\|\mathbf{z}_k\|^2\|\mathbf{z}_{k-1}\|^2\right] &= E\left[\sum_{i=0}^{L}\sum_{j=0,j\neq i-1}^{L} z_{k-i}^2 z_{k-1-j}^2+\sum_{i=1}^{L} z_{k-i}^4\right] \\
&= \sum_{i=0}^{L} E\left[z_{k-i}^2\right]\sum_{j=0,j\neq i-1}^{L} E\left[z_{k-1-j}^2\right]+E\left[\sum_{i=1}^{L} z_{k-i}^4\right] \\
&= \sum_{i=1}^{L}\sigma_z^2\sum_{j=0,j\neq i-1}^{L}\sigma_z^2+\sigma_z^2\sum_{j=0}^{L}\sigma_z^2+LE\left[z_k^4\right] \\
&== L^2\left(\sigma_z^2\right)^2+(L+1)\left(\sigma_z^2\right)^2+3L\left(\sigma_z^2\right)^2=\left(L^2+4L+1\right)\left(\sigma_z^2\right)^2
\end{aligned} \tag{4.45}$$

Here, using the fourth moment factoring theorem for stationary Gaussian distributed signal, we have $E\left[z_k^4\right]=3\left(\sigma_z^2\right)^2$.

(b)

$$\begin{aligned}
&E\left[\left[\mathbf{z}_{k-1}^T\mathbf{z}_k\right]^2\right] \\
&= E\left[\sum_{i=0}^{L} z_{k-i}^2 z_{k-i-1}^2+2\sum_{i=0}^{L}\sum_{j=0,i\neq j}^{L} z_{k-i}z_{k-i-1}z_{k-j}z_{k-j-1}\right] \\
&= \sum_{i=0}^{L} E\left[z_{k-i}^2\right]E\left[z_{k-i-1}^2\right]+0=(L+1)\left(\sigma_z^2\right)^2
\end{aligned} \tag{4.46}$$

(c)

$$E\left[\mathbf{z}_{k-1}^T\mathbf{z}_k\mathbf{z}_{k-1}\mathbf{z}_k^T\right]=\begin{cases}\left(\sigma_z^2\right)^2 & \text{, if} \quad i=j \quad \text{or} \quad i=j-2 \\ 0, \text{ otherwise} & \end{cases} \tag{4.47}$$

Here, (i,j) are representing i^{th} row and j^{th} column of the matrix.

(d)

$$E\left[\|\mathbf{z}_{k-1}\|^2\mathbf{z}_k\mathbf{z}_k^T\right]=\begin{cases}(L+1)\left(\sigma_z^2\right)^2 & \text{, if} \quad i=j=1 \\ (L+3)\left(\sigma_z^2\right)^2 & \text{, if} \quad i=j\neq 1 \\ 0 & \text{, if} \quad i\neq j\end{cases} \tag{4.48}$$

So, putting the above values, $E\left[\tilde{\mathbf{w}}_{k+1}\right]$ in equation 4.44 can be rewritten as:

$$\begin{aligned}
E\left[\tilde{\mathbf{w}}_{k+1}\right] &= E\left[\tilde{\mathbf{w}}_k\right]+\mu\begin{bmatrix}-\frac{1}{L+3} & 0 & \frac{1}{L(L+3)} & \cdots & 0 \\ 0 & -\frac{L+2}{L(L+3)} & 0 & \cdots & 0 \\ 0 & 0 & -\frac{L+2}{L(L+3)} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -\frac{L+2}{L(L+3)}\end{bmatrix}E\left[\tilde{\mathbf{w}}_k\right] \\
&\approx \left[1-\frac{\mu}{L}\boldsymbol{I}\right]E\left[\tilde{\mathbf{w}}_k\right] \quad \text{, as} \quad L\gg 1
\end{aligned} \tag{4.49}$$

Our requirement is to choose the value of convergence factor μ such that:

$E\left[\tilde{\mathbf{w}}_{k+1}\right]=0 \quad$ as $\quad k \rightarrow \infty$. It can be achieved if the pole is inside unit circle. So,

$$\left|1-\frac{\mu}{L}\mathbf{I}\right|<1 \tag{4.50}$$

That is possible for $L \geq 1$ and for μ values in the range:

$$|1-\mu|<1 \quad \Rightarrow \quad 0<\mu<2 \tag{4.51}$$

4.4.2 Computational Complexity

In this subsection, we compare the computational complexity for implementing the proposed SM-BNLMS algorithm based channel equalization schemes with the existing schemes such as SM-NLMS and SM-AP algorithm based equalization techniques. Here, the computational complexity is expressed in terms of the number of complex multiplication, complex addition and division per update. We consider two cases of weight update; in one, new updating takes place without knowing the preceding values, and second when updating occurs at successive time instants with known values computed at earlier time instants.

Table 4.6: Comparison of Computational Complexity per Update in DFE using Data-selective Algorithms.

SM-NLMS Algorithm			
Updation	Addition	Multiplication	Div
New	$3L$	$3L+1$	1
Succesive	$2L+2$	$2L+6$	1
SM-AP Algorithm			
Updation	Addition	Multiplication	Div
New	$P^2\left(\frac{L}{2}+1\right)+L+\frac{5}{2}LP+\mathcal{O}\left(P^3\right)$	$\frac{P^2}{2}\left(L+1\right)-\frac{P}{2}+\frac{5}{2}LP+\mathcal{O}\left(P^3\right)$	
Proposed SM-BNLMS Algorithm			
New	$5L+1$	$5L+7$	1
Succesive	$3L+10$	$3L+15$	1

Table 4.6 compares the complexity per update of the proposed SM-BNLMS algorithm based DFE, with the corresponding SM-AP algorithm as well as SM-NLMS algorithm based DFE. Here, we take $L=L_{FF}+L_{FB}$. It may be seen that computation involved per update in the proposed scheme is higher than SM-NLMS algorithm based scheme. However, equalization with SM-BNLMS algorithm

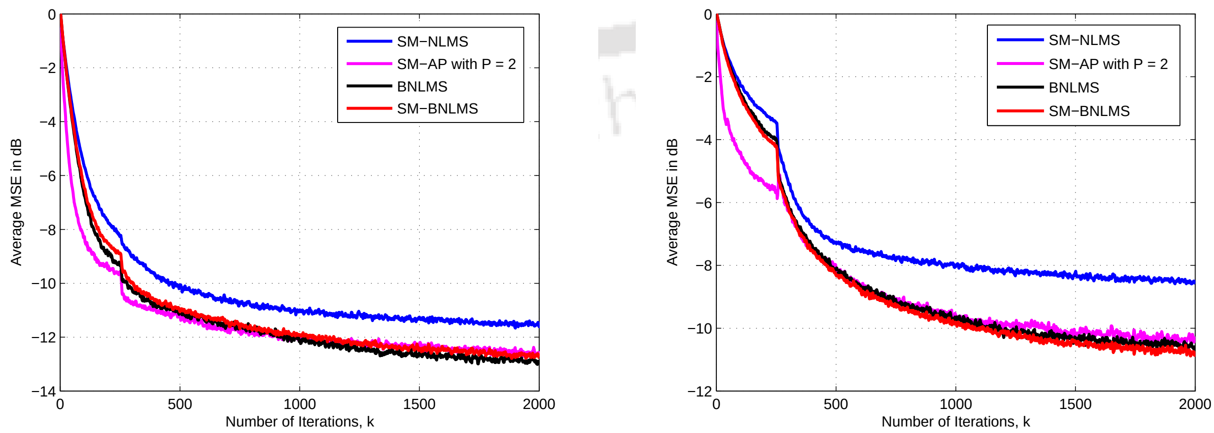
converges considerably faster than the scheme with SM-NLMS algorithm and number of updates required is comparatively less in case of SM-BNLMS, which makes overall computational load slightly higher to SM-NLMS while better convergence is obtained in case of SM-BNLMS. This issue is further elaborated in the next subsection. It is observed that, the main complexity terms of SM-AP algorithm based DFE is in computing the matrix inversion, which is of the order $\mathcal{O}(P^3)$, where P is the projection order. For highly correlated input, due to increase of P, complexity becomes high. The proposed technique offers reduced computation in comparison to SM-AP algorithm based equalization schemes.

4.4.3 Simulation Results and Discussion

The computer simulations with analysis of results are presented in this subsection to illustrate the effectiveness of the proposed SM-BNLMS algorithm based channel equalization technique. The simulation studies are performed with same parameter values as considered in Section 4.3 for implementation of SM-AP algorithm based DFE.

4.4.3.1 Performance Comparison with other Data-selective Algorithms based DFEs

The performance of SM-BNLMS algorithm based DFE is compared with DFE using BNLMS algorithm as well as with other data-selective algorithms such as SM-NLMS and SM-AP with projection order 2. In this simulation study, we consider L_{FF} and L_{FB} of DFE as 3 and 1, respectively for both Ped-A and Veh-B environments, and convergence factor μ as 0.05. Here, the upper bound γ on estimation error is set to $\sqrt{1.5}\sigma_n$, where σ_n^2 is the variance of additive white Gaussian noise.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 4.10: MSE Performance of DFEs using SM-NLMS, SM-AP ($P = 2$), BNLMS and SM-BNLMS as Adaption Algorithm.

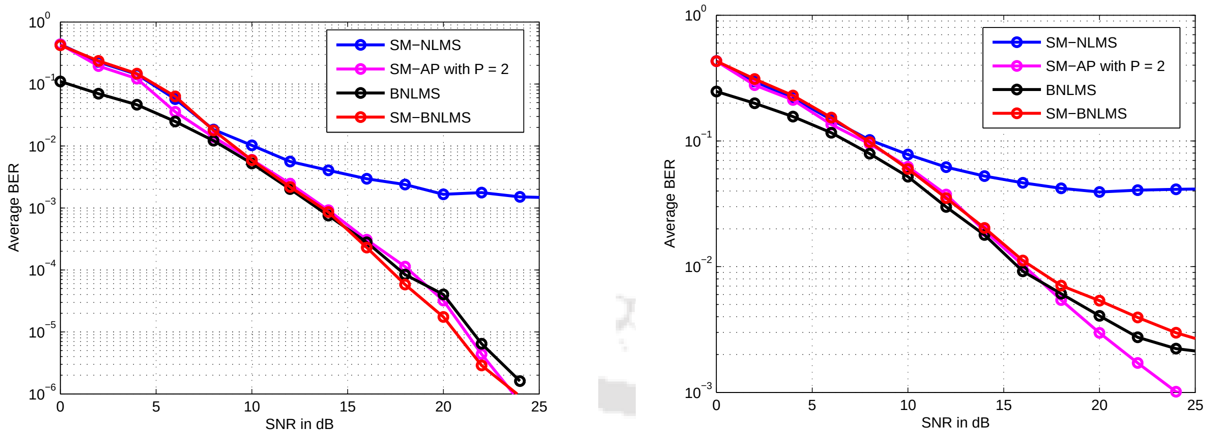
Fig. 4.10 compares the MSE performance of DFE using SM-BNLMS, BNLMS, SM-NLMS and SM-AP (projection order 2) algorithms. It is observed that the proposed technique offers better rate of convergence than SM-NLMS algorithm based scheme. Further, its convergence performance is almost similar to techniques with BNLMS algorithm and also SM-AP algorithm of projection order 2.

Table 4.7: Steady-state MSE and Number of Updates for Different Data-selective Algorithms based DFEs.

Algorithm	Ped-A		Veh-B	
	SS-MSE	Number of Updates	SS-MSE	Number of Updates
SM-NLMS	-11.5 dB	1254	-8.5 dB	1576
SM-AP (P=2)	-12.6 dB	1116	-10.3 dB	1333
SM-BNLMS	-12.6 dB	1129	-10.7 dB	1355
BNLMS	-12.7dB	2048	-10.5 dB	2048

Table 4.7 shows steady-state (SS) average MSE value and number of updating operations required in a packet of 2048 symbols for different data-selective algorithms based DFEs. It is observed that, SS-MSE of the proposed scheme is 1.1 dB and 2.2 dB less than SM-NLMS algorithm based scheme in Ped-A and Veh-B environments respectively, which reflects better tracking. It is also seen that, the proposed technique requires only 10 % and 15 % fewer numbers of updating operations in Ped-A and Veh-B environments respectively than SM-NLMS algorithm based DFE; that results overall computation to remain small, although marginally higher than SM-NLMS algorithm based scheme. Further, it is observed that SS-MSE and number of updates associated with this is almost similar to channel equalization with SM-AP algorithm of projection order 2. Its performance is quite similar to DFEs using BNLMS algorithm but the amount of computation required is reduced by 40 % due to lesser number of updates. It may however be noted that reduction in computation that can be achieved depends on the severity of the channel as well as lengths of the filters used.

Fig. 4.11 shows a comparison in BER performance of DFEs with different adaptive algorithms. It is observed that, all data-selective algorithms have a similar BER performance, when SNR is less than 8 dB, but in the high SNR region, performance of DFE with SM-BNLMS is better than that with SM-NLMS. However, BER performance of DFE employing SM-BNLMS algorithm is close to the one with SM-AP algorithm with projection order 2. It is further noticed that, BNLMS algorithm based DFE provides slightly better performance for SNR less than 8 dB, but for higher SNR, the



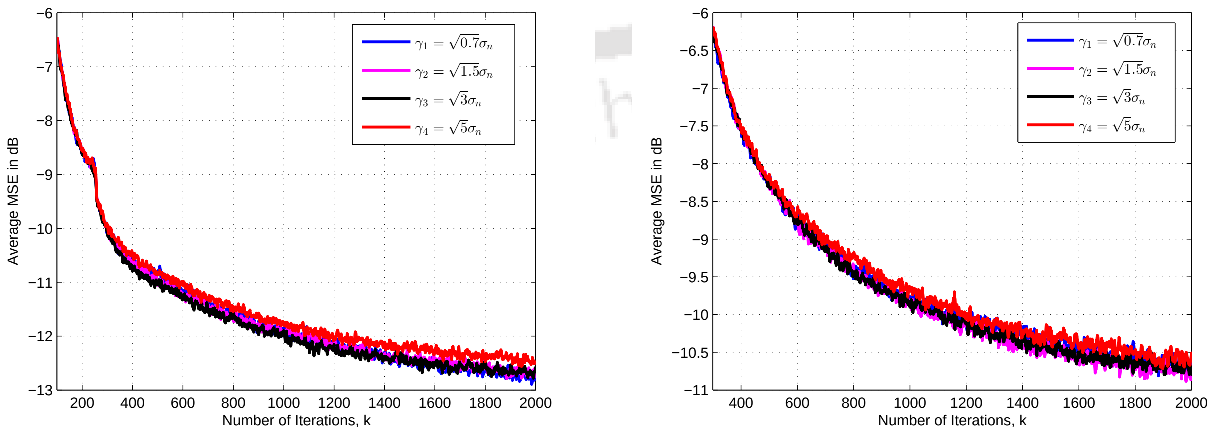
(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 4.11: BER Performance Comparison of SM-NLMS, SM-AP ($P = 2$), BNLMS and SM-BNLMS Algorithm based DFEs.

performance of the two schemes are almost similar.

4.4.3.2 Performance of SM-BNLMS Algorithm based DFE with Different Values of Upper Bounds on Estimation Error

The performance of the SM-BNLMS Algorithm based DFE is studied for different values of upper bounds on estimation error. Here, we consider convergence factor $\mu = 0.05$ and filter lengths as $L_{FF} = 3, L_{FB} = 1$. The simulations are performed for upper bound γ as $\gamma_1 = \sqrt{0.7}\sigma_n$, $\gamma_2 = \sqrt{1.5}\sigma_n$, $\gamma_3 = \sqrt{3}\sigma_n$ and $\gamma_4 = \sqrt{5}\sigma_n$.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

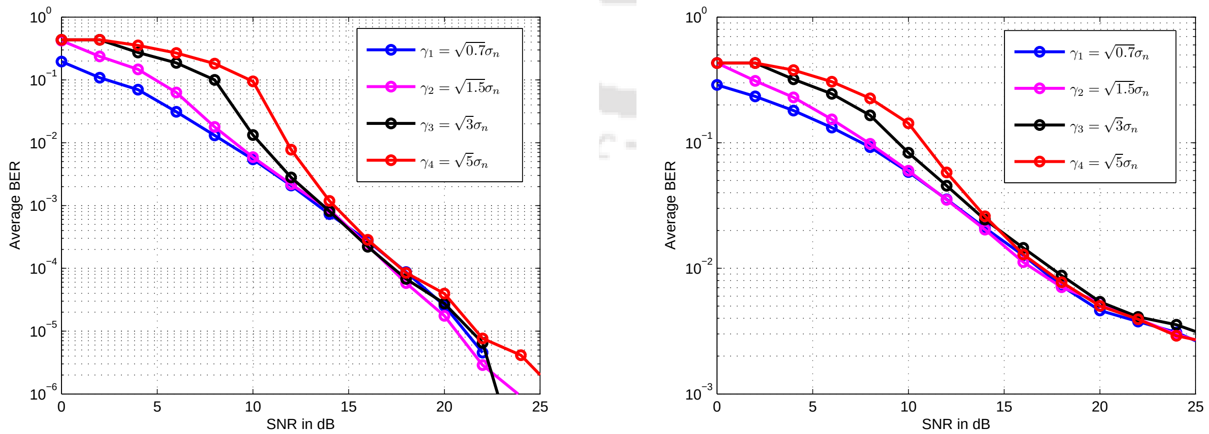
Figure 4.12: MSE Performance of SM-BNLMS Algorithm based DFE with Different γ Values.

Fig. 4.12 shows the convergence performance comparison of the proposed scheme with upper bound γ. It can be observed that, an increase in upper bound value on estimation error degrades the convergence performance. Further, the convergence performance with γ values $\sqrt{0.7}\sigma_n$, $\sqrt{1.5}\sigma_n$ and $\sqrt{3}\sigma_n$ are almost similar.

Table 4.8: SS-MSE and Number of Updates of SM-BNLMS Algorithm Based DFE with Different γ Values.

Upper bound γ	Ped-A		Veh-B	
	SS-MSE	Number of Updates	SS-MSE	Number of Updates
$\gamma_1 = \sqrt{0.7}\sigma_n$	-12.6 dB	1431	-10.7 dB	1527
$\gamma_2 = \sqrt{1.5}\sigma_n$	-12.6 dB	1129	-10.7 dB	1355
$\gamma_3 = \sqrt{3}\sigma_n$	-12.6 dB	949	-10.7 dB	1172
$\gamma_4 = \sqrt{5}\sigma_n$	-12.4 dB	767	-10.6 dB	1022

Table 4.8 shows the SS-MSE values and number of updates associated with SM-BNLMS algorithm based DFE for different γ values. It is noticed that, increase in γ value results a decrease in the number of updates. Further, it is found that, SS-MSE values for γ values $\sqrt{0.7}\sigma_n$, $\sqrt{1.5}\sigma_n$ and $\sqrt{3}\sigma_n$ are same, i.e. -12.6 dB and -10.7 dB in Ped-A and Veh-B environments, respectively. However, performance degrades for γ value of $\sqrt{5}\sigma_n$.



(a) Pedestrian Test Environment-A

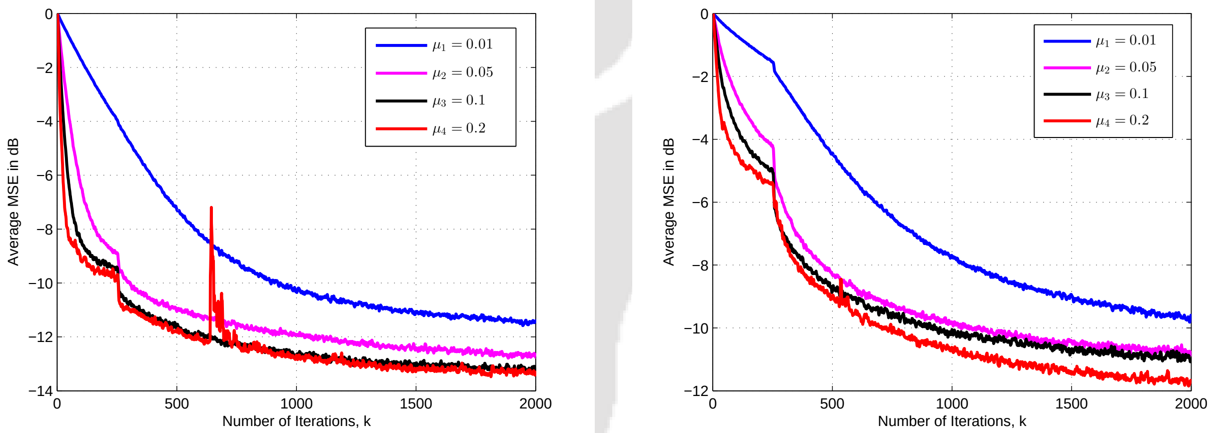
(b) Vehicular Test Environment-B

Figure 4.13: BER Performance of SM-BNLMS Algorithm based DFE with Different γ Values.

The BER performance of SM-BNLMS algorithm based DFE for different upper bound, γ is shown in Fig. 4.13. It is observed that, increasing upper bound degrades BER performance for SNR values below 12 dB, but performance is similar in high SNR region. Table 4.8 shows that an increase in upper bound reduces the number of updates. So, the choice of upper bound on estimation error as $\sqrt{1.5}\sigma_n$, is found to be suitable in terms of complexity and BER performance.

4.4.3.3 Performance of SM-BNLMS Algorithm based DFE with Different Values of Convergence Factor

In this study, the performance of SM-BNLMS Algorithm based DFE for different convergence factor is compared. The simulations are done for convergence factor μ values as $\mu_1 = 0.01$, $\mu_2 = 0.05$, $\mu_3 = 0.1$ and $\mu_4 = 0.2$. In this simulation, we consider $\gamma = \sqrt{1.5}\sigma_n$ and $L_{ff} = 3, L_{fb} = 1$.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 4.14: Convergence Performance of SM-BNLMS Algorithm based DFE with Different μ Values.

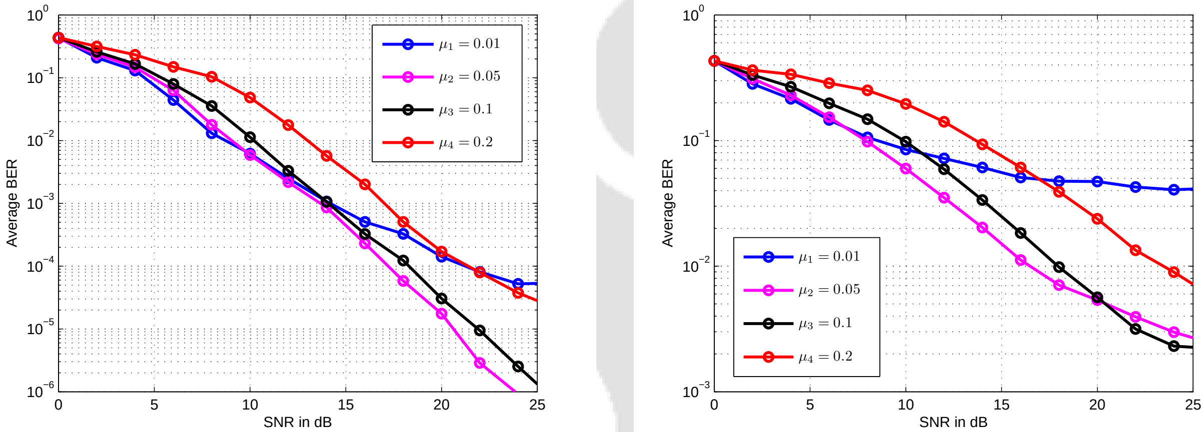
The convergence performance of the proposed SM-BNLMS algorithm based DFE as a function of convergence factor is shown in Fig. 4.14. It is observed that, increase in μ value improves the convergence performance, but for $\mu = 0.2$, the MSE curve contains fluctuations in the error terms.

Table 4.9 compares 'steady-state MSE' and 'number of updates' of SM-AP algorithm based DFE for different values of convergence factor. It is observed that, increase in μ result reduction in SS-MSE and number of updating operations, for example increasing μ from 0.01 to 0.05 reduces 359 and 281 number of updates in Ped-A and Veh-B environments, respectively; which reflects substantial reduction in complexity.

BER performance of the proposed channel equalization scheme for different values of convergence factor μ is shown in Fig. 4.15. It may be seen that, BER performance improves with convergence factor

Table 4.9: 'SS-MSE' and 'Number of Updates' of SM-BNLMS Algorithm based DFE with Different μ Values.

Step Size μ	Ped-A		Veh-B	
	SS-MSE	Number of Updates	SS-MSE	Number of Updates
0.01	-11.5 dB	1488	-9.5 dB	1636
0.05	-12.6 dB	1129	-10.7 dB	1355
0.1	-13.2 dB	1098	-10.9 dB	1284
0.2	-13.3 dB	1053	-11.6 dB	1188



(a) Pedestrian Test Environment-A

(b) Vehicular Test Environment-B

Figure 4.15: BER Performance of SM-BNLMS Algorithm based DFE with Different μ Values.

for μ value less than or equal to 0.1, but degrades for higher values of μ. In a mobile fading environment, keeping μ value small results better estimation of the filter coefficient according to the time-varying channel condition, and hence higher values of μ increases the fluctuations in the instantaneous error terms, and it may be attributed to the poor BER performance. Moreover, convergence factor $\mu = 0.05$ is found to be suitable both in terms of BER performance and number of updates.

4.5 Summary

In this chapter, we have investigated the MSE and BER performance of adaptive DFE with data selective SM-AP as well as with SM-BNLMS as adaption algorithms in a practical time dispersive multi-path channels.

The SM-AP algorithm based DFE is found to have advantages of having (i) almost similar convergence speed and BER performance as AP based DFE but with almost 30 % less computational requirements, (ii) performs better than SM-NLMS based DFE with comparatively less updates though with slight increase in complexity, and (iii) of good tracking ability. We also studied this channel equalization scheme with various factors like upper bound on estimation error γ and convergence factor μ. Through simulations, we have established that, the convergence factor $\mu = 0.05$ and the upper bound $\gamma = \sqrt{1.5}\sigma_n$ are found good enough in terms of convergence, computational complexity and BER performance of this method in both Ped-A and Veh-B environments.

Furthermore, SM-BNLMS algorithm based DFE presented in this chapter converges faster, provides much better BER performance and requires almost 10 % fewer number of updates than the SM-NLMS algorithm based methods. Its performance is quite similar to DFEs using BNLMS algorithm but with 40 % less computations due to reduced number of updates. Moreover, its overall performance is more or less similar to SM-AP algorithm (with projection order 2) based technique, but with reduced complexity due to non-existence of matrix inversion component. However, higher projection orders SM-AP algorithm based DFE performs better, but with more complexity. The selection of convergence factor and upper bound on estimation error, are very important with respect to convergence speed, number of updates and instability.

5

Data-reusing Algorithm based MIMO-DFE

Contents

Objective

Channel equalization scheme with adaptive decision feedback equalizer (DFE) for single input single output (SISO) system is presented in chapter 3 and 4. This chapter is an extension of adaptive SISO-DFE for multiple-input multiple-output (MIMO) wireless channels. Here, we discuss an adaptive MIMO-DFE system model and studied the channel equalization performance with the data-reusing 'Affine projection' (AP) and 'Binormalized LMS' (BNLMS) based adaption algorithms in time-dispersive MIMO wireless channel.

5.1 Introduction

Multiple-input multiple-output (MIMO) is one of the most promising areas in cellular communication research due to the huge requirement for better quality of service and throughput in fourth generation (4G) and fifth generation (5G) of mobile telecommunications technology. MIMO system in a wireless communication system is defined as a radio link with multiple antennas on the transmitter and receiver [61–64]. The major advantages of MIMO system can be categorized as antenna array gain, spatial diversity gain and spatial multiplexing gain.

The performance of MIMO system is degraded, mainly by two types of interference; one is inter-symbol interference (ISI) due to channel multi-path dispersion, and the other is cross-talk or co-channel interference due to the simultaneous transmissions from the multiple transmit antennas. Channel equalization in a time-varying MIMO environment becomes a difficult task in the presence of co-channel interference (CCI) and rapid changes in the channel. The adaptive DFE performs better in canceling the effect of lSI and CCI for MIMO systems operating in frequency selective channels [6, 14, 32, 34]. From the literature [76, 77], it is observed that LMS algorithm based MIMO-DFE is simpler, but it has the convergence problem in colored environment, whereas MIMO-DFE with RLS based adaptive algorithms, converges fast but computational complexity can be significant in applications where the order of the adaptive filter is high. The data-reusing algorithms based adaptive SISO DFE in a practical time dispersive multi-path channel is discussed in Chapter 3. It is found to be attractive in terms of overall performance as well as complexity. This chapter presents the extension of DFE using data-reusing AP and BNLMS algorithms for time dispersive MIMO channels. In literature [43], a solution of adaptive equalization for MIMO channel using AP algorithms is available; however, data-reusing algorithm based MIMO channel equalization is not well investigated in a standard time-varying wireless environment. Here, we use a time domain adaptive MIMO-DFE for equalization of time-varying wireless channels, where data-reusing AP and BNLMS algorithms are used for updating coefficients.

The rest of this chapter is organized as follows; A generic MIMO system is described in Section 5.2, and Section 5.3 discusses the adopted adaptive MIMO-DFE model for this work. The performance of the MIMO-DFE using the AP algorithm is investigated in Section 5.4 and the performance analysis of BNLMS algorithm based MIMO-DFE is presented in Section 5.5. Finally, Section 5.6 provides the summary of work presented in this chapter.

5.2 Multiple-input Multiple-output System

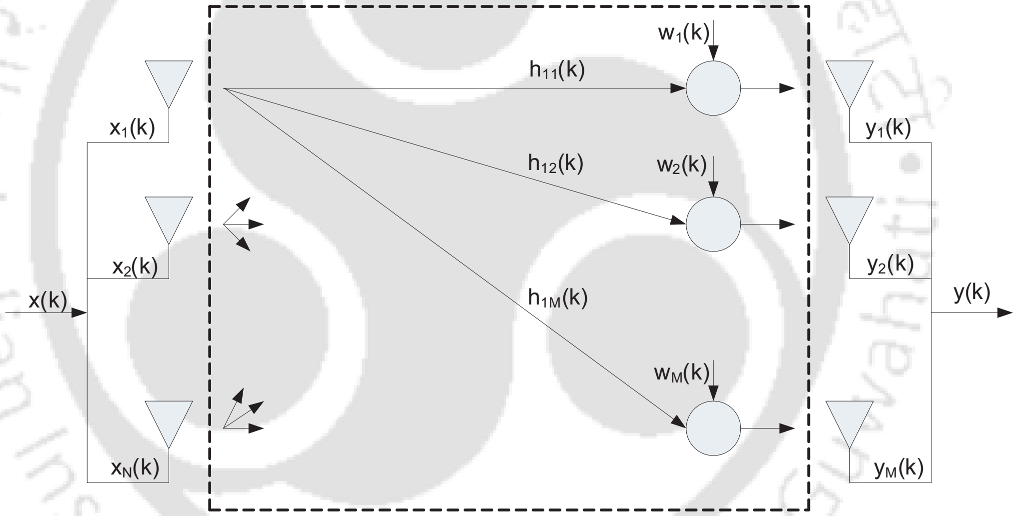


Figure 5.1: A Generic $N \times M$ MIMO System Model.

A generic MIMO system model is discussed in details in Chapter 2. A brief overview is presented here for the purpose of continuity. We have considered a system with N transmit and M receive antennas as shown in Fig. 5.1. The discrete output of the j_{th} receiving antenna is:

$$y_j(k) = \sum_{i=1}^{N} h_{ji}(k)\, x_i(k) + n_j(k) \tag{5.1}$$

where $x_i(k)$ is the discrete input to the i_{th} transmitting antenna, $h_{ji}(k)$ is the channel response between the i^{th} transmitting antenna and j^{th} receiving antenna at time instant k and $n_j(k)$ is the AWGN at the output of j_{th} receiving antenna at time k.

5.3 Adaptive MIMO DFE Model

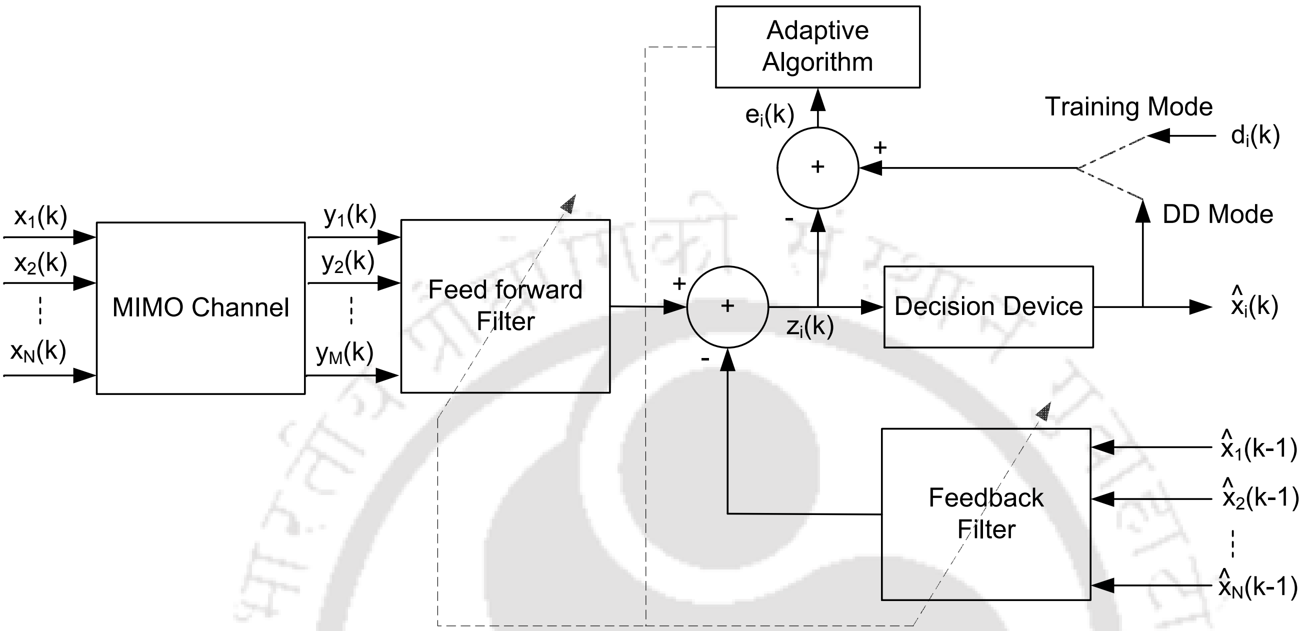


Figure 5.2: Block Diagram of Adaptive DFE for $M \times N$ MIMO Channel.

In wireless communication, use of adaptive technique in equalization helps to compensate for the affect of unknown channel. Channel variation is tracked by adaptively updating the filter coefficients. The block diagram of adaptive DFE for $M \times N$ MIMO channel is shown in Fig. 5.2. In general, the DFE is a nonlinear equalizer, consists of a decision device, a feed-forward filter and a feedback filter to which the input is previously detected symbols. The basis of adaptive equalization is the implementation of any adaptive algorithm on the DFE to solve the minimization problem. The concept of an adaptive DFE discussed in Chapter 3 can be extended to the multivariate DFE by following the same approach for the SISO-DFE using normal equations. Such equations are generated by concatenating the input vectors and weight vectors. The normal equations allow the implementation of any adaptive algorithm on the DFE to solve the minimization problem [78].

For the i_{th} DFE, we define $z_i(k)$ and $\hat{x}_i(k)$ as the signals before and after the decision device respectively. The $z_i(k)$ is expressed as:

$$z_i(k) = \sum_{u=1}^{M} \mathbf{y}_u(k)\mathbf{w}_{u,i}^{FF}(k) - \sum_{v=1}^{N} \hat{\mathbf{x}}_v(k-1)\mathbf{w}_{v,i}^{FB}(k) \tag{5.2}$$

where, the input vector to feed-forward filter (FFF) is $\mathbf{y}_u(k)$ and input vector to feedback filter (FBF) is $\hat{\mathbf{x}}_v(k-1)$ as given below.

$$\begin{aligned} \mathbf{y}_u(k) &= \begin{bmatrix} y_u(k) & y_u(k-1) & \cdots & y_u(k-L_{FF}+1) \end{bmatrix} \\ \hat{\mathbf{x}}_v(k-1) &= \begin{bmatrix} \hat{x}_v(k-1) & \hat{x}_v(k-2) & \cdots & \hat{x}_v(k-L_{FB}) \end{bmatrix} \end{aligned} \tag{5.3}$$

Further, the weight vector for the FFF, $\mathbf{w}_{u,i}^{FF}(k)$ is of size $L_{FF}\times 1$ and the weight vector for the FBF $\mathbf{w}_{v,i}^{FB}(k)$, is of size $L_{FB}\times 1$, which can be expressed as:

$$\mathbf{w}_{u,i}^{FF}(k) = \begin{bmatrix} w_{u,i}^{FF}(k) \\ w_{u,i}^{FF}(k-1) \\ \vdots \\ w_{u,i}^{FF}(k-L_{FF}+1) \end{bmatrix} \text{ and } \mathbf{w}_{v,i}^{FB}(k) = \begin{bmatrix} w_{v,i}^{FB}(k) \\ w_{v,i}^{FB}(k-1) \\ \vdots \\ w_{v,i}^{FB}(k-L_{FB}+1) \end{bmatrix} \tag{5.4}$$

The input vectors to FFF and FBF are concatenated to obtain input signal vector $\mathbf{s}(k)$, of size $1\times(ML_{FF}+NL_{FB})$, given by:

$$\mathbf{s}(k) = \begin{bmatrix} \mathbf{y}(k) & -\hat{\mathbf{x}}(k-1) \end{bmatrix} \tag{5.5}$$

where

$$\begin{aligned} \mathbf{y}(k) &= \begin{bmatrix} \mathbf{y}_1(k) & \mathbf{y}_2(k) & \cdots & \mathbf{y}_M(k) \end{bmatrix} \\ &\text{and} \\ \hat{\mathbf{x}}(k-1) &= \begin{bmatrix} \hat{\mathbf{x}}_1(k-1) & \hat{\mathbf{x}}_2(k-1) & \cdots & \hat{\mathbf{x}}_N(k-1) \end{bmatrix}. \end{aligned} \tag{5.6}$$

Further, concatenated filter weight vectors $\mathbf{w}_i(k)$, which is of size $(ML_{FF}+NL_{FB})\times 1$, can be expressed as:

$$\mathbf{w}_i(k) = \begin{bmatrix} \mathbf{w}_i^{FF}(k) \\ \mathbf{w}_i^{FB}(k) \end{bmatrix} \tag{5.7}$$

where

$$\mathbf{w}_i^{FF}(k) = \begin{bmatrix} \mathbf{w}_{1,i}^{FF}(k) \\ \mathbf{w}_{2,i}^{FF}(k) \\ \vdots \\ \mathbf{w}_{M,i}^{FF}(k) \end{bmatrix} \quad and \quad \mathbf{w}_i^{FB}(k) = \begin{bmatrix} \mathbf{w}_{1,i}^{FB}(k) \\ \mathbf{w}_{2,i}^{FB}(k) \\ \vdots \\ \mathbf{w}_{N,i}^{FB}(k) \end{bmatrix} \tag{5.8}$$

Using the above concatenated vectors, we can rewrite the equation 5.2 as:

$$z_i(k) = \begin{bmatrix} \mathbf{y}(k) & -\hat{\mathbf{x}}(k-1) \end{bmatrix} \begin{bmatrix} \mathbf{w}_i^{FF}(k) \\ \mathbf{w}_i^{FB}(k) \end{bmatrix} = \mathbf{s}(k)\,\mathbf{w}_i(k) \tag{5.9}$$

The adaptive algorithm can minimize the error between desired signal and filter output in a mean square sense by optimizing the filter weights iteratively. So, the error signal obtained at time instant k for i_{th} DFE is:

$$e_i(k) = \begin{cases} d_i(k) - z_i(k) & : \text{ Training Mode} \\ \hat{x}_i(k) - z_i(k) & : \text{ DD Mode} \end{cases} \tag{5.10}$$

where $d_i(k)$ is the desired signal at time instant k and $\hat{x}_i(k)$ is the output of the i_{th} decision device at time instant k.

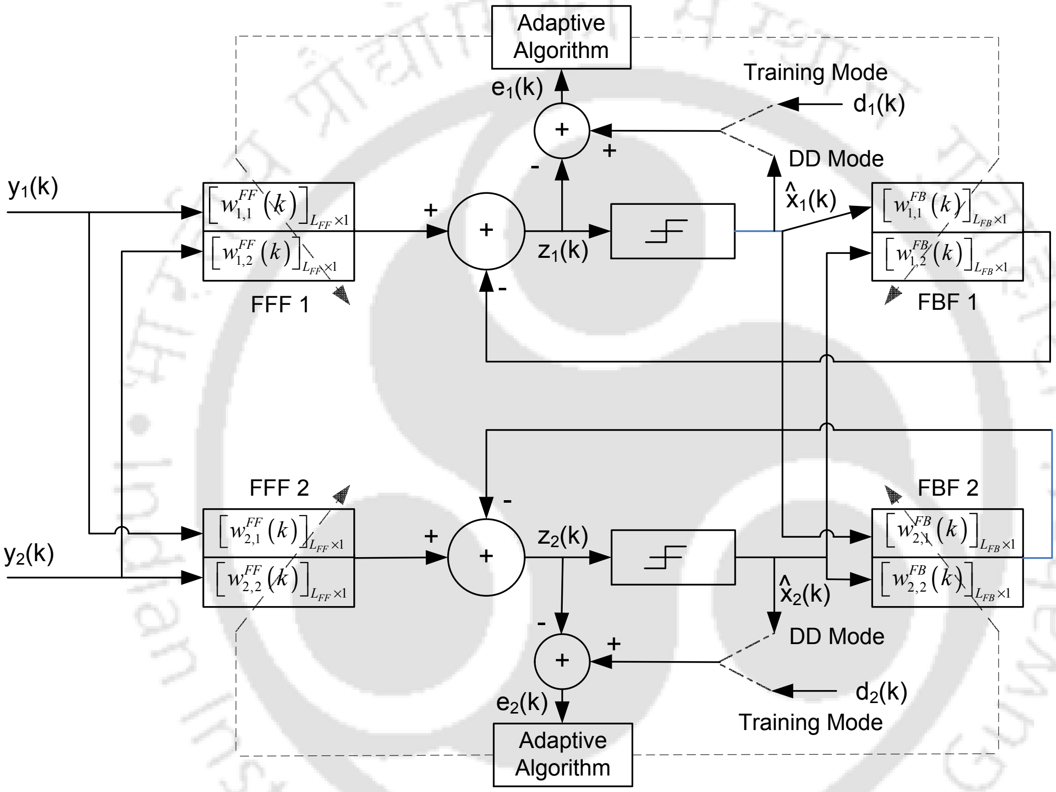


Figure 5.3: A Generic 2×2 Adaptive MIMO-DFE Architecture.

Here, we consider the adaptive DFE scheme for 2×2 MIMO channel, as shown in Fig.5.3. So, the MIMO system is having $N = 2$ transmit and $M = 2$ receive antennas. Each link between transmit and receive antenna is an independent Rayleigh fading channel. For this 2×2 MIMO-DFE scheme, we can rewrite the equation 5.2 for $z_i(k)$ as:

$$z_i(k) = \sum_{u=1}^{M} \mathbf{y}_u(k)\mathbf{w}_{u,i}^{FF}(k) - \sum_{v=1}^{N} \hat{\mathbf{x}}_v(k-1)\mathbf{w}_{v,i}^{FB}(k) \tag{5.11}$$

We can express $z_i(k)$ using concatenated input signal vector and weight vector as:

$$z_i(k) = \begin{bmatrix} \mathbf{y}(k) & -\hat{\mathbf{x}}(k-1) \end{bmatrix} \begin{bmatrix} \mathbf{w}_i^{FF}(k) \\ \mathbf{w}_i^{FB}(k) \end{bmatrix}$$

$$= \begin{bmatrix} y_1(k) & y_1(k-1) & \cdots & y_1(k-L_{FF}+1) & y_2(k) & y_2(k-1) & \cdots & y_2(k-L_{FF}+1) \end{bmatrix} \begin{bmatrix} w_{1,i}^{FF}(k) \\ w_{1,i}^{FF}(k-1) \\ \vdots \\ w_{1,i}^{FF}(k-L_{FF}+1) \\ w_{2,i}^{FF}(k) \\ w_{2,i}^{FF}(k) \\ \vdots \\ w_{2,i}^{FF}(k-L_{FF}+1) \end{bmatrix}$$

$$- \begin{bmatrix} \hat{x}_1(k-1) & \hat{x}_1(k-2) & \cdots & \hat{x}_1(k-L_{FB}) & \hat{x}_2(k-1) & \hat{x}_2(k-2) & \cdots & \hat{x}_2(k-L_{FB}) \end{bmatrix} \begin{bmatrix} w_{1,i}^{FB}(k) \\ w_{1,i}^{FB}(k-1) \\ \vdots \\ w_{1,i}^{FB}(k-L_{FB}+1) \\ w_{2,i}^{FB}(k) \\ w_{2,i}^{FB}(k) \\ \vdots \\ w_{2,i}^{FB}(k-L_{FB}+1) \end{bmatrix} \tag{5.12}$$

where

$$\mathbf{y}(k) = \begin{bmatrix} y_1(k) & y_1(k-1) & \cdots & y_1(k-L_{FF}+1) & y_2(k) & y_2(k-1) & \cdots & y_2(k-L_{FF}+1) \end{bmatrix} \tag{5.13}$$

and

$$\hat{\mathbf{x}}(k-1) = \begin{bmatrix} \hat{x}_1(k-1) & \hat{x}_1(k-2) & \cdots & \hat{x}_1(k-L_{FB}) & \hat{x}_2(k-1) & \hat{x}_2(k-2) & \cdots & \hat{x}_2(k-L_{FB}) \end{bmatrix} \tag{5.14}$$

This 2×2 adaptive MIMO-DFE system model is considered in this chapter to analyze the performance of data-reusing AP and BNLMS algorithm based MIMO channel equalizer.

5.4 Affine Projection Algorithm based Adaptive MIMO-DFE

In this section, we discuss an equalization scheme using DFE with data-reusing affine projection algorithm for MIMO channel. As already mentioned, the AP algorithm utilizes past data along with the present data for coefficient update, leading to fast convergence for highly correlated input. It updates its coefficient vector such that the new solution belongs to the intersection of P hyperplanes defined by the present and the $P-1$ previous data pairs [71].

For implementing this algorithm in an adaptive MIMO DFE, we have modified it appropriately as detailed below. The objective function for i^{th} DFE at time instant k for this AP algorithm based MIMO-DFE is:

$$\begin{aligned} &\min \left\|\mathbf{w}_i\left(k+1\right)-\mathbf{w}_i\left(k\right)\right\|^2 \\ &\text{subject to:} \quad \mathbf{d}_i\left(k\right)-\mathbf{S}\left(k\right)\mathbf{w}_i\left(k+1\right)=0 \end{aligned} \tag{5.15}$$

where $\mathbf{S}\left(k\right)$ is a matrix of size $P \times \left(ML_{FF}+NL_{FB}\right)$, which contains present input signal vector defined in equation 5.5 and $P-1$ previous input signal vectors as follows.

$$\mathbf{S}\left(k\right)=\begin{bmatrix} \mathbf{s}\left(k\right) \\ \mathbf{s}\left(k-1\right) \\ \vdots \\ \mathbf{s}\left(k-P+1\right) \end{bmatrix} \tag{5.16}$$

The $\boldsymbol{d_i}\left(\boldsymbol{k}\right)$ is the desired signal vector of size $P \times 1$, expressed as:

$$\mathbf{d}_i\left(k\right)=\begin{bmatrix} d_i\left(k\right) \\ d_i\left(k-1\right) \\ \vdots \\ d_i\left(k-P+1\right) \end{bmatrix} \tag{5.17}$$

The constrained minimization problem given in equation 5.15, is solved by the Lagrange multiplier method, as done in Section 3.4 for implementing affine projection algorithm based DFE. So, we can write the unconstrained function as:

$$f\left[\mathbf{w}_i\left(k+1\right)\right]=\left\|\mathbf{w}_i\left(k+1\right)-\mathbf{w}_i\left(k\right)\right\|^2+\boldsymbol{\lambda}_i^T\left(k\right)\left[\mathbf{d}_i\left(k\right)-\mathbf{S}\left(k\right)\mathbf{w}_i\left(k+1\right)\right] \tag{5.18}$$

where Lagrange multipliers $\boldsymbol{\lambda}_i\left(k\right)$ is

$$\boldsymbol{\lambda}_i\left(k\right)=\begin{bmatrix} \lambda_i\left(k,1\right) \\ \lambda_i\left(k,2\right) \\ \cdots \\ \lambda_i\left(k,P\right) \end{bmatrix}. \tag{5.19}$$

Setting the gradient of the function $f\left[\mathbf{w}_i\left(k+1\right)\right]$ with respect to $\mathbf{w}_i\left(k+1\right)$ cited in equation 5.18, equal to zero, we obtain:

$$\mathbf{w}_i\left(k+1\right)=\mathbf{w}_i\left(k\right)+\mathbf{S}^T\left(k\right)\frac{\boldsymbol{\lambda}_i\left(k\right)}{2}. \tag{5.20}$$

The value of $\mathbf{w}_i(k+1)$ obtained in equation 5.20 is put in the constraint condition in equation 5.15.

$$\begin{aligned} & \mathbf{d}_i(k) - \mathbf{S}(k)\,\mathbf{w}_i(k+1) = 0 \\ \Rightarrow \quad & \mathbf{d}_i(k) - \mathbf{S}(k)\left[\mathbf{w}_i(k) + \mathbf{S}^T(k)\,\frac{\boldsymbol{\lambda}_i(k)}{2}\right] = 0 \end{aligned} \tag{5.21}$$

So, from equations 5.21, $\frac{\boldsymbol{\lambda}_i(k)}{2}$ can be obtained.

$$\frac{\boldsymbol{\lambda}_i(k)}{2} = \left(\mathbf{S}(k)\,\mathbf{S}^T(k)\right)^{-1}\mathbf{e}_i(k) \tag{5.22}$$

The error vector $\mathbf{e}_i(k)$ is given as:

$$\mathbf{e}_i(k) = \mathbf{d}_i(k) - \mathbf{S}(k)\,\mathbf{w}_i(k) = \begin{bmatrix} e_i(k) \\ \varepsilon_i(k-1) \\ \vdots \\ \varepsilon_i(k-P+1) \end{bmatrix} \tag{5.23}$$

where $e_i(k)$ is the a-priori error at iteration k and $\varepsilon_i(k-j)$ is the a-posteriori error at iteration $k-j$ (for $j = 1, 2, \cdots, P-1$) which are expressed as:

$$e_i(k) = d_i(k) - \mathbf{s}(k)\,\mathbf{w}_i(k) \tag{5.24}$$

$$\varepsilon_i(k-j) = d_i(k-j) - \mathbf{s}(k-j)\,\mathbf{w}_i(k)\,. \tag{5.25}$$

From equation 5.20 and 5.22, the filter weight updating expression can be expressed as:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mathbf{S}^T(k)\left(\mathbf{S}(k)\,\mathbf{S}^T(k) + \alpha\boldsymbol{I}\right)^{-1}\mathbf{e}_i(k) \tag{5.26}$$

where α is a small constant. The matrix $\alpha\mathbf{I}$ is added to avoid the problem in inversion of $\mathbf{S}(k)\,\mathbf{S}(k)^T$ in a noisy environment.

As discussed in AP algorithm based DFE implementation scheme in Section 3.4, a step size μ is introduced in equation 5.26 to achieve a trade-off between final misadjustment and convergence speed [71]. So, the modified weight updating equation is:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu\mathbf{S}^T(k)\left(\mathbf{S}(k)\,\mathbf{S}^T(k) + \alpha\boldsymbol{I}\right)^{-1}\mathbf{e}_i(k) \tag{5.27}$$

The summary of step by step implementation of affine projection algorithm based MIMO channel equalization scheme for i^{th} DFE is given in Table 5.1.

Table 5.1: Summary of the Affine Projection Algorithm based MIMO Channel Equalization for i^{th} DFE.

Step-1: Initialization
Weight vector $\mathbf{w}_i(k)$ of size $(ML_{FF} + NL_{FB}) \times 1$, is initialized as zero vector.
Input signal vector $\mathbf{s}(k)$ of size $1 \times (ML_{FF} + NL_{FB})$, is initialized as zero vector .
Regularization parameter α is a small constant, and $\boldsymbol{I}$ is an unit matrix of size $P \times P$.
Convergence factor be in the range $0 < \mu < 2$.
Step-2: Processing for $k > 0$
(a) Computation of Error:
A-priori error $e_i(k) = d_i(k) - \mathbf{s}(k)\mathbf{w}_i(k)$
A-posteriori error $\varepsilon_i(k-j) = d_i(k-j) - \mathbf{s}(k-j)\mathbf{w}_i(k)$ for $j = 1, 2, \cdots, P-1$.
(b) Filter Weight Updating:
$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu \mathbf{S}^T(k)\left(\mathbf{S}(k)\mathbf{S}^T(k) + \alpha \boldsymbol{I}\right)^{-1} \mathbf{e}_i(k)$
(c) Increment k **and go back to 'for' loop.**

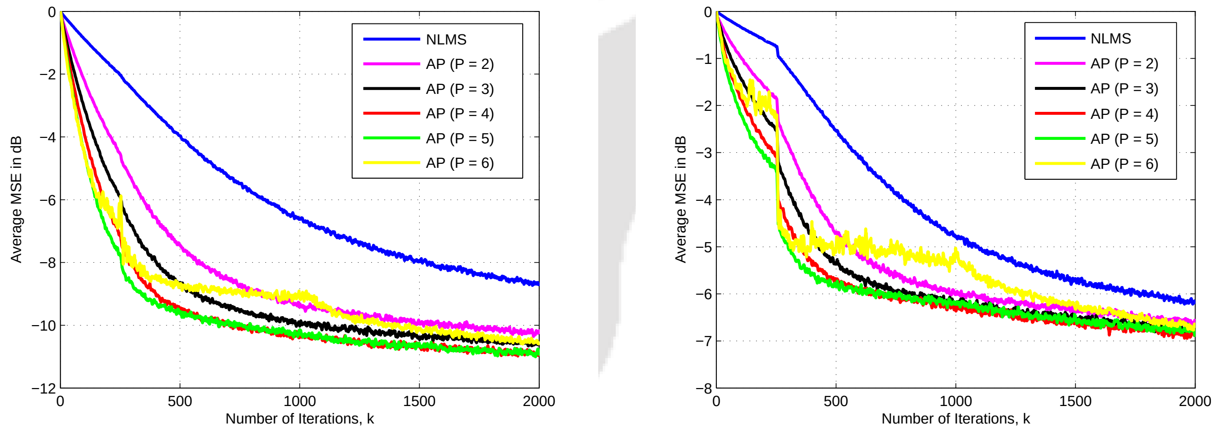
It may be mentioned that an AP algorithm based MIMO-DFE is reported in [43]. In the following subsection we carry out the simulation studies on the performance of an AP based MIMO-DFE employing a practical wireless channel model as recommended in [54].

5.4.1 Simulation Results and Discussion

In this subsection, we present simulation results illustrating the effectiveness of the AP algorithm based MIMO-DFE in a frequency selective MIMO wireless channel. We consider the adaptive DFE scheme for 2×2 MIMO channel as shown in Fig. 5.3. The transmitted symbols are binary phase shift keying (BPSK) signals, and the noise is assumed to be additive white Gaussian. In our work, we consider WiMAX recommended ITU-R wireless channel models; one for outdoor to indoor and pedestrian test environment with a low delay spread (Ped-A) and another for the vehicular test environment with a medium delay spread (Veh-B) as shown in Table 2.1. Here, the carrier frequency is considered as 2.5 GHz. The rms delay spread for Ped-A and Veh-B environments are 45 nanosec and 4 μsec, respectively. Considering the delay spread of channel models, the simulations are done with sample time t_s of the signal as .01 μsec for Ped-A and .3 μsec for Veh-B so that the channel will be frequency selective. Further, taking into account the test environment as well as the delay spread, we consider the maximum doppler frequency shift f_d as 5 Hz for Ped-A and 70 Hz for Veh-B. We also consider each packet consists of 2048 bits, which includes 256 bits as training sequence, and 3000 such packets are transmitted to evaluate the performance of MIMO-DFE. In the simulation, SNR is maintained at 20 dB, and the MSE curves are plotted on the average obtained from 3000 runs. For channel equalization, we consider DFEs with a feed-forward filter of length L_{FF} as 3 and a feedback filter of length L_{FB} as 1.

5.4.1.1 Performance of MIMO-DFE using AP and NLMS as Adaption Algorithm

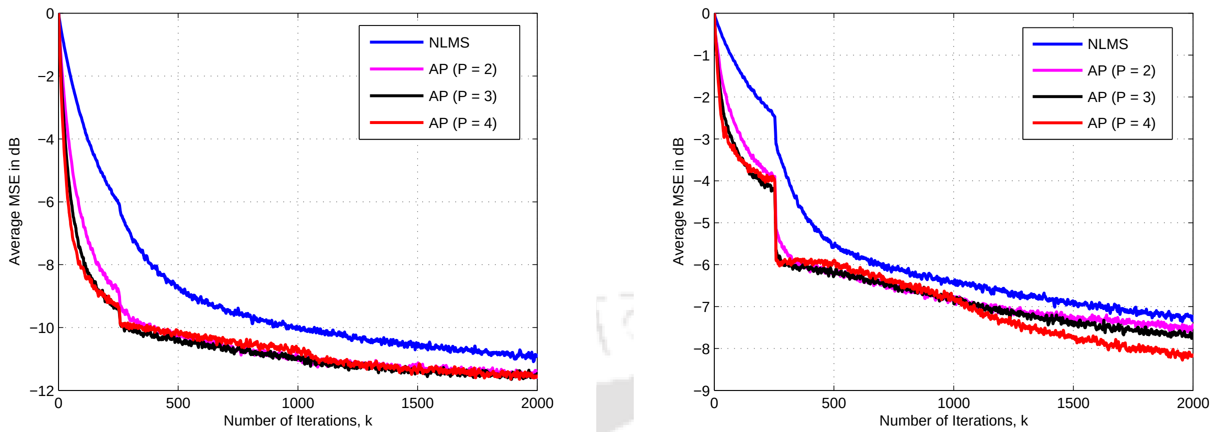
The convergence characteristics along with BER performance of the adaptive MIMO-DFE using AP algorithm of different projection order are compared with NLMS algorithm based MIMO channel equalizer.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 5.4: MSE Performance Comparison of MIMO-DFE using AP and NLMS as Adaption Algorithms with $\mu = 0.01$.

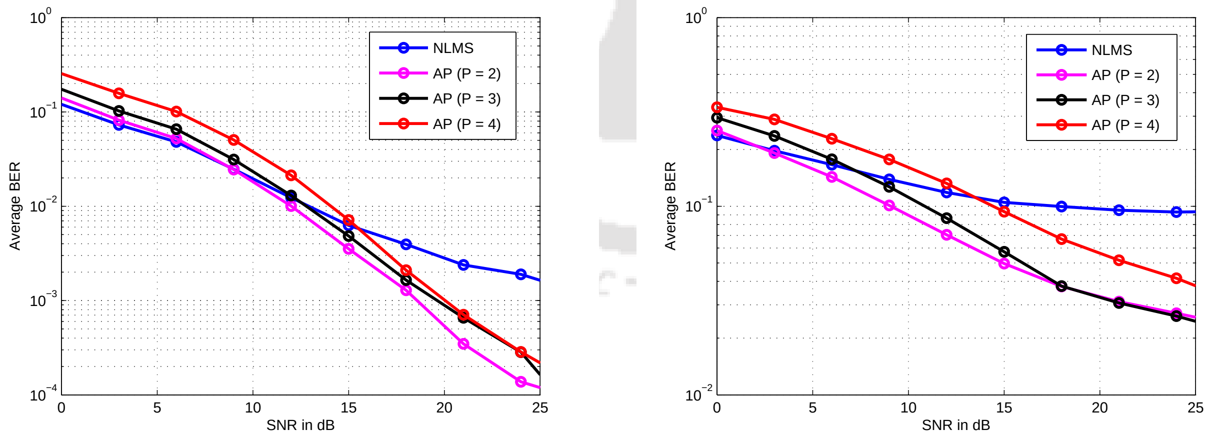
The convergence performance comparison with step size μ as 0.01 and 0.05 are shown in Fig. 5.4 and Fig. 5.5 respectively. The MSE curves for AP algorithm based equalizer is shown with projection order P as 2, 3, 4, 5 and 6. The simulation result shows that MIMO channel equalization scheme with AP algorithm converges faster in comparison to NLMS algorithm based techniques. It is observed in Fig. 5.4 for $\mu = 0.01$ that, convergence performance improves with increase in projection order from 2 to 3 as well as 3 to 4, but deteriorates for P value more than 5; whereas as seen in Fig. 5.5 for $\mu = 0.05$, minor improvement in the convergence is achieved when P increases from 2 to 3, but further increase of projection order does not yield much improvement due to increase in misadjustment. We know the



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 5.5: MSE Performance Comparison of MIMO-DFE using AP and NLMS as Adaption Algorithms with $\mu = 0.05$.

main complexity terms of AP algorithm is in computing the inverse of $(\boldsymbol{S}(\boldsymbol{k})\,\boldsymbol{S}(\boldsymbol{k})^{\boldsymbol{T}} + \alpha \boldsymbol{I})$, which is of the order $\mathcal{O}(P^3)$. Keeping P small compared to $(ML_{FF} + NL_{FB})$, the computational complexity can be reduced. So, the projection order is to be chosen properly considering its convergence and computational complexity.



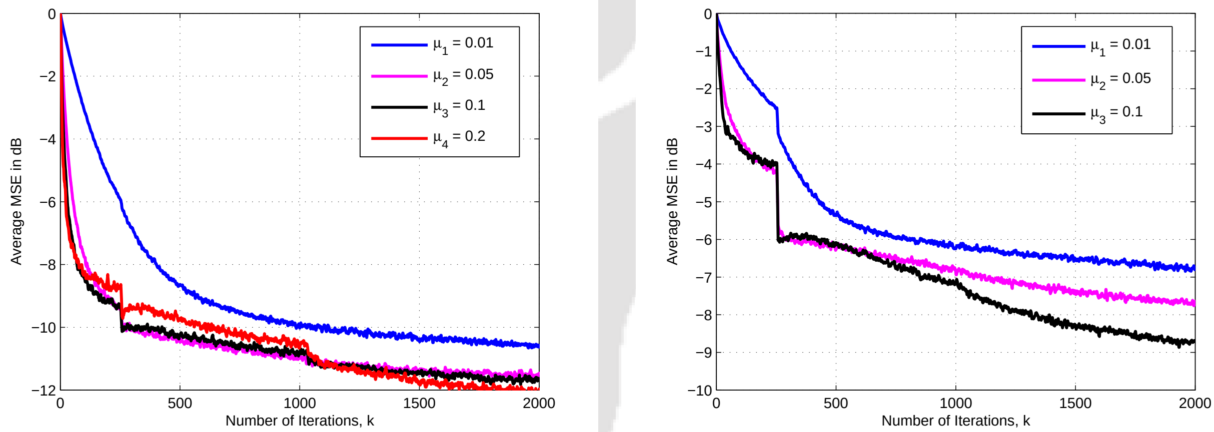
(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 5.6: BER Performance Comparison of MIMO-DFE using AP and NLMS as Adaption Algorithms with $\mu = 0.05$.

Fig. 5.6 compares the BER performance of this MIMO channel equalizer with NLMS algorithm based scheme for μ value 0.05. It is observed that, the BER performance of this equalization scheme with projection order 2 and 3 has identical performance as NLMS algorithm based MIMO-DFE in low SNR region, but improves significantly for SNR more than 8 dB in both Ped-A and Veh-B environments. Moreover, it is observed that the MIMO-DFE with AP algorithm of projection order 2 and 3 shows similar BER performance. It is also seen that, the BER performance deteriorates for P value 4.

5.4.1.2 Performance Study of AP Algorithm based MIMO-DFE for Different Values of Step Size

The MSE and BER performance of this MIMO channel equalizer using AP algorithm is studied for different values of step size μ. The simulations are conducted for μ values as $\mu_1 = 0.01$, $\mu_2 = 0.05$, $\mu_3 = 0.1$ and $\mu_4 = 0.2$ taking projection order $P = 3$.

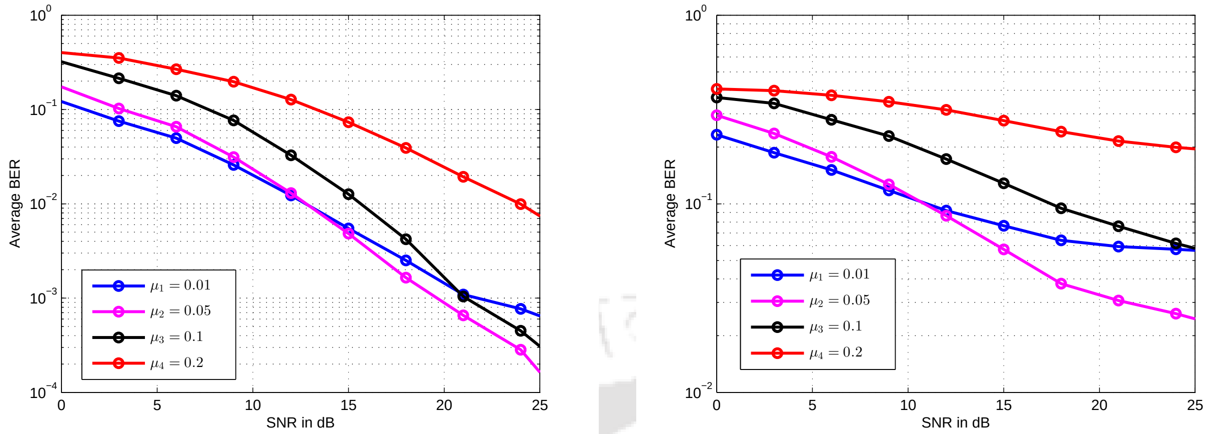


(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 5.7: MSE Performance of MIMO-DFE with AP Algorithms with Different Values of Step Size.

Fig. 5.7 shows the MSE curves for different μ values. It is observed that, increasing μ from 0.01 to 0.05 significantly improves the convergence speed as well as the MSE performance. Further, it is seen that with μ value 0.2 in Ped-A environment and 0.1 in Veh-B environment, the MSE curve contains more fluctuations.

Fig. 5.8 compares the BER performance of MIMO-DFE with AP Algorithm for different step sizes. It is seen that, increasing μ from 0.01 to 0.05 shows similar performance in low SNR region, whereas performance improves for SNR values more than 14 dB. Further, it is observed that BER performance of this MIMO-DFE degrades significantly for step size more than 0.1. Hence, the step size is kept small in a mobile fading environment to track the channel variation more accurately.



(a) Pedestrian Test Environment-A

(b) Vehicular Test Environment-B

Figure 5.8: BER Performance of MIMO-DFE with AP Algorithms with Different Values of Step Size.

5.5 Binormalized LMS Algorithm based Adaptive MIMO-DFE

The AP algorithm based MIMO channel equalization presented in Section 5.4 offers significant improvement in the convergence as well as BER performance over NLMS algorithm based equalizer with a marginal increase of computational load. As already mentioned for the SISO case, the computational complexity of AP is $\mathcal{O}(P^3)$. The computational complexity can be reduced if data-reusing BNLMS algorithm is used. Therefore, we also investigate BNLMS algorithm in a MIMO framework to determine its performance.

As already discussed, the data-reusing BNLMS algorithm utilizes the present input signal vector as well as the immediate past input signal vector for updating the filter coefficient. In each update, normalization is employed in two orthogonal directions from consecutive data pairs to achieve faster convergence [55], [41]. Further, its performance is almost similar or even slightly better than data-reusing AP algorithms for the case of two data pairs. This algorithm is implemented in an adaptive MIMO-DFE system, where its objective is to keep the next coefficient vector as close as possible to the current one, to maintain a-posteriori errors to be zero. So, we can express the objective function for i^{th} DFE at time instant k in DD mode as:

$$\begin{aligned} &\min \left\|\mathbf{w}_i\left(k+1\right)-\mathbf{w}_i\left(k\right)\right\|^2 \\ \text{subject to:}\quad &\hat{x}_i\left(k\right)-\mathbf{s}\left(k\right)\mathbf{w}_i\left(k+1\right)=0 \\ \text{and}\quad &\hat{x}_i\left(k-1\right)-\mathbf{s}\left(k-1\right)\mathbf{w}_i\left(k+1\right)=0 \end{aligned} \tag{5.28}$$

The Lagrange multiplier method is used to solve the constrained minimization problem cited in equation 5.28. So, the unconstrained function can be expressed as:

$$\begin{aligned} f\left[\mathbf{w}_i\left(k+1\right)\right] = \left\|\mathbf{w}_i\left(k+1\right)-\mathbf{w}_i\left(k\right)\right\|^2 + \lambda_{1,i}\left(k\right)\left[\hat{x}_i\left(k\right)-\mathbf{s}\left(k\right)\mathbf{w}_i\left(k+1\right)\right] \\ +\lambda_{2,i}\left(k\right)\left[\hat{x}_i\left(k-1\right)-\mathbf{s}\left(k-1\right)\mathbf{w}_i\left(k+1\right)\right] \end{aligned} \tag{5.29}$$

where $\lambda_{1,i}\left(k\right)$ and $\lambda_{2,i}\left(k\right)$ are Lagrange multipliers. Then the weight update equation can be derived by setting the gradient of the unconstrained function $f\left[\mathbf{w}_i\left(k+1\right)\right]$ with respect to $\mathbf{w}_i\left(k+1\right)$ cited in equation 5.29, equal to zero. So:

$$\begin{aligned} &\tfrac{\partial\left(f\left[\mathbf{w}_i\left(k+1\right)\right]\right)}{\partial\left(\mathbf{w}_i\left(k+1\right)\right)} = 0 \\ &\Rightarrow 2\left(\mathbf{w}_i\left(k+1\right)-\mathbf{w}_i\left(k\right)\right)-\mathbf{s}^T\left(k\right)\lambda_{1,i}\left(k\right)-\mathbf{s}^T\left(k-1\right)\lambda_{2,i}\left(k\right)=0 \end{aligned} \tag{5.30}$$

From the above equation, we can write the expression for filter weight update as:

$$\mathbf{w}_i\left(k+1\right) = \mathbf{w}_i\left(k\right) + \frac{\lambda_{1,i}\left(k\right)}{2}\mathbf{s}^T\left(k\right) + \frac{\lambda_{1,i}\left(k\right)}{2}\mathbf{s}^T\left(k-1\right) \tag{5.31}$$

The value of Lagrange multipliers are calculated from equations 5.28 and 5.31. So:

$$\begin{aligned} \frac{\lambda_{1,i}\left(k\right)}{2} &= \frac{e_i\left(k\right)\left\|\mathbf{s}\left(k-1\right)\right\|^2-\varepsilon_i\left(k-1\right)\mathbf{s}\left(k\right)\mathbf{s}^T\left(k-1\right)}{\alpha+\left\|\mathbf{s}\left(k\right)\right\|^2\left\|\mathbf{s}\left(k-1\right)\right\|^2-\left[\mathbf{s}\left(k\right)\mathbf{s}^T\left(k-1\right)\right]^2} \\ \frac{\lambda_{2,i}\left(k\right)}{2} &= \frac{\varepsilon_i\left(k-1\right)\left\|\mathbf{s}\left(k\right)\right\|^2-e_i\left(k\right)\mathbf{s}\left(k\right)\mathbf{s}^T\left(k-1\right)}{\alpha+\left\|\mathbf{s}\left(k\right)\right\|^2\left\|\mathbf{s}\left(k-1\right)\right\|^2-\left[\mathbf{s}\left(k\right)\mathbf{s}^T\left(k-1\right)\right]^2} \end{aligned} \tag{5.32}$$

where regularization parameter α is a small constant. The $e_i\left(k\right)$, the a-priori error at iteration k and $\varepsilon_i\left(k-1\right)$, a-posteriori error at iteration $k-1$, are written in DD mode as:

$$\begin{aligned} e_i\left(k\right) &= \hat{x}_i\left(k\right)-\mathbf{s}\left(k\right)\mathbf{w}_i\left(k\right) \\ \varepsilon_i\left(k-1\right) &= \hat{x}_i\left(k-1\right)-\mathbf{s}\left(k-1\right)\mathbf{w}_i\left(k\right) \end{aligned} \tag{5.33}$$

If optimization is achieved, then a-posteriori error $\varepsilon_i\left(k-1\right)=0$. It will further simplify the value of Lagrange multipliers as:

$$\begin{aligned} \frac{\lambda_{1,i}\left(k\right)}{2} &= \frac{e_i\left(k\right)\left\|\mathbf{s}\left(k-1\right)\right\|^2}{\alpha+\left\|\mathbf{s}\left(k\right)\right\|^2\left\|\mathbf{s}\left(k-1\right)\right\|^2-\left[\mathbf{s}\left(k\right)\mathbf{s}^T\left(k-1\right)\right]^2} \\ \frac{\lambda_{2,i}\left(k\right)}{2} &= \frac{-e_i\left(k\right)\mathbf{s}\left(k\right)\mathbf{s}^T\left(k-1\right)}{\alpha+\left\|\mathbf{s}\left(k\right)\right\|^2\left\|\mathbf{s}\left(k-1\right)\right\|^2-\left[\mathbf{s}\left(k\right)\mathbf{s}^T\left(k-1\right)\right]^2} \end{aligned} \tag{5.34}$$

A step size μ is introduced in equation 5.31, as discussed earlier in the implementation of BNLMS algorithm in Section 3.5.

So, the modified weight updating equation is:

$$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu\left[\frac{\lambda_{1,i}(k)}{2}\mathbf{s}^T(k) + \frac{\lambda_{1,i}(k)}{2}\mathbf{s}^T(k-1)\right] \tag{5.35}$$

The summary of the proposed MIMO channel equalization with BNLMS Algorithm for i^{th} DFE is given in Table 5.2.

Table 5.2: Summary of the Proposed MIMO Channel Equalization with BNLMS Algorithm for i^{th} DFE.

Step-1: Initialization
Weight vector is initialized as zero vector $\mathbf{w}_i(0)$.
Input signal vector is initialized as zero vector $\mathbf{s}(0)$.
The α a small constant, is regularization parameter .
Convergence factor be in the range $0 < \mu < 2$.
Step-2: Processing for $k > 0$
(a) Computation of Error Vector
A-priori error $e_i(k) = \hat{x}_i(k) - \mathbf{s}(k)\mathbf{w}_i(k)$
A-posteriori error $\varepsilon_i(k-1) = \hat{x}_i(k-1) - \mathbf{s}(k-1)\mathbf{w}_i(k)$.
(b) Filter Weight Updating
$\mathbf{w}_i(k+1) = \mathbf{w}_i(k) + \mu\left[\frac{\lambda_{1,i}(k)}{2}\mathbf{s}^T(k) + \frac{\lambda_{1,i}(k)}{2}\mathbf{s}^T(k-1)\right]$
$\frac{\lambda_{1,i}(k)}{2} = \frac{e_i(k)\left\Vert\mathbf{s}(k-1)\right\Vert^2}{\alpha + \left\Vert\mathbf{s}(k)\right\Vert^2\left\Vert\mathbf{s}(k-1)\right\Vert^2 - \left[\mathbf{s}(k)\mathbf{s}^T(k-1)\right]^2}$
$\frac{\lambda_{2,i}(k)}{2} = \frac{-e_i(k)\mathbf{s}(k)\mathbf{s}^T(k-1)}{\alpha + \left\Vert\mathbf{s}(k)\right\Vert^2\left\Vert\mathbf{s}(k-1)\right\Vert^2 - \left[\mathbf{s}(k)\mathbf{s}^T(k-1)\right]^2}$
(c) Increment k and go back to 'for' loop.

5.5.1 Computational Complexity

Table 5.3 shows the computational complexity in terms of the number of complex multiplication, complex addition and division per update involved with the MIMO equalizer with different adaptive algorithms. Here, we take $B = ML_{FF} + NL_{FB}$. It may be seen that computation involved per update in the proposed scheme is higher than NLMS algorithm based scheme. However, equalization with BNLMS algorithm converges faster than the scheme with NLMS algorithm. Moreover, it is observed that, the main complexity terms of AP algorithm based DFE is in computing the matrix inversion,

Table 5.3: Comparison of Computational Complexity per Update in Implementing i^{th} MIMO-DFE using BNLMS, NLMS and AP Algorithms.

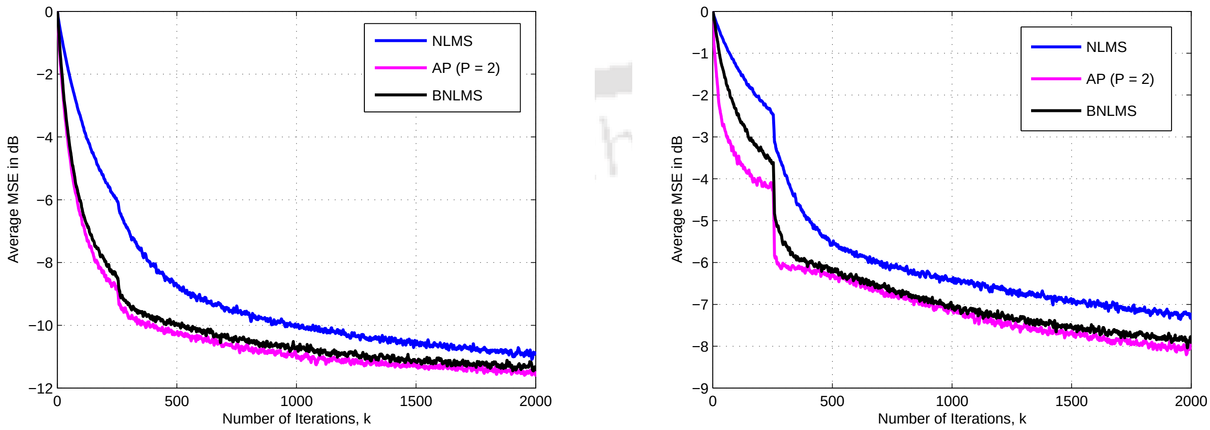
Adaptive Algorithms	Addition	Multiplication	Division
NLMS	$3BN$	$3BN + N$	N
AP	$N(2BP-1) + BP^2 + \mathcal{O}(P^3)$	$NP(2B+1) + 2BP^2 + \mathcal{O}(P^3)$	
BNLMS	$6BN$	$6BN - N^2$	N

which is of the order $\mathcal{O}(P^3)$, where P is the projection order. For colored input, due to increase of P, complexity becomes high. The proposed technique offers reduced computation in comparison to AP algorithm based equalization schemes.

5.5.2 Simulation Results and Discussion

The performance of a MIMO-DFE based on BNLMS algorithm is evaluated through simulation using MATLAB R2010a. In this work, we consider the same set of parameters as taken for studying the performance of AP algorithm based MIMO-DFE in Section 5.4. We plot MSE curves to demonstrate the convergence performance and BER curves to show the throughput of the proposed equalizer.

5.5.2.1 Performance Comparison of MIMO-DFE with BNLMS, NLMS and AP ($P = 2$) as Adaption Algorithm

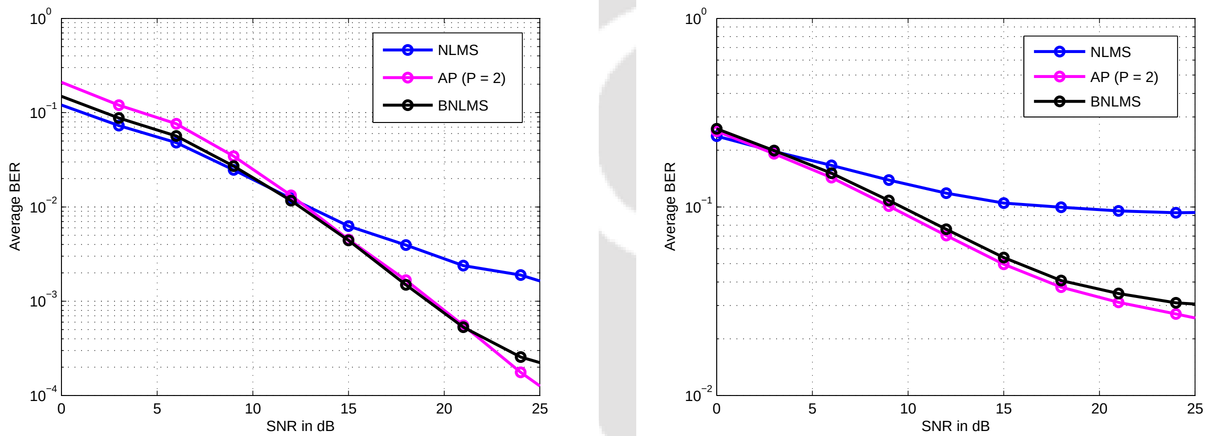


(a) Pedestrian Test Environment-A   (b) Vehicular Test Environment-B

Figure 5.9: MSE Performance Comparison of BNLMS, NLMS and AP (P = 2) Algorithm based MIMO-DFE.

The convergence characteristics and BER performance of the proposed MIMO-DFE using BNLMS algorithm is compared with other adaptive algorithms such as NLMS, and AP with projection order 2.

Fig. 5.9 shows the MSE performance comparison of BNLMS, NLMS and AP algorithm based MIMO channel equalizer with step size μ as 0.1 in both Ped-A and Veh-B environments. It is observed that for the specified test environment, MIMO channel equalizer employing BNLMS algorithm converges faster than NLMS algorithm based equalizer, and have convergence rate almost similar to equalizer employing SM-AP algorithm of projection order 2.



(a) Pedestrian Test Environment-A

(b) Vehicular Test Environment-B

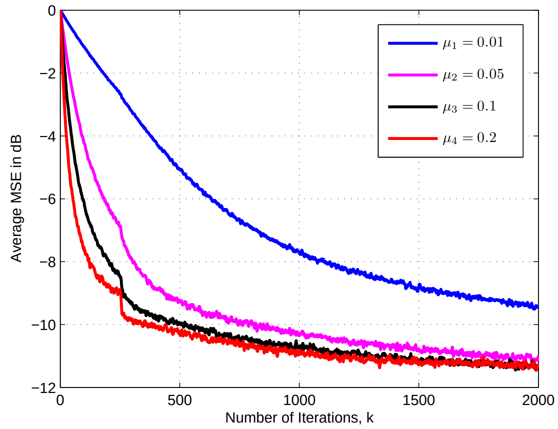
Figure 5.10: BER Performance Comparison of BNLMS, NLMS and AP (P = 2) Algorithm based MIMO-DFE.

The BER performance comparison of BNLMS, NLMS and AP algorithm based MIMO-DFE with step size μ as 0.1 is shown in Fig. 5.10. It is observed that the BER performance of the proposed equalizer is similar to the one with AP algorithm having projection order 2 in both the test environments. Further, it is found that, though the BER performance of BNLMS algorithm based MIMO-DFE is similar to NLMS algorithm based equalizer for SNR values below 12 dB in Ped-A and 4 dB in Veh-B; it performs much better in the high SNR region.

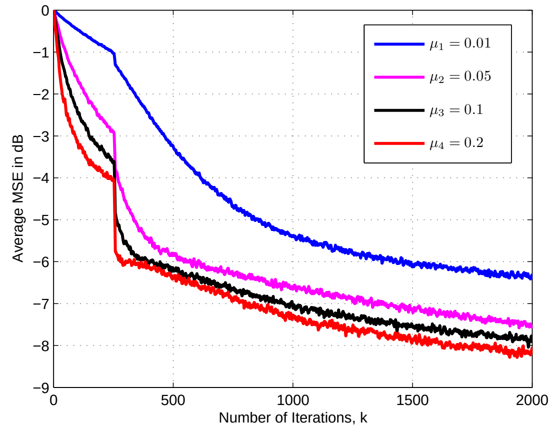
5.5.2.2 Performance Study of BNLMS Algorithm based MIMO-DFE with Different Values of Step Size

The performance of the proposed MIMO-DFE employing BNLMS algorithm are studied for different values of step size μ. The simulations are conducted for step size μ as $\mu_1 = 0.01$, $\mu_2 = 0.05$, $\mu_3 = 0.1$ and $\mu_4 = 0.2$.

The convergence performance of BNLMS algorithm based MIMO-DFE with different step size



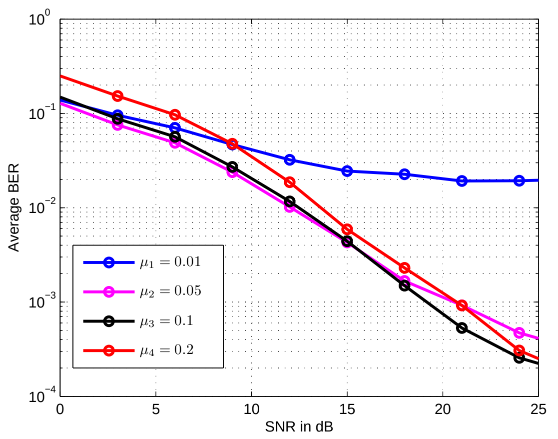
(a) Pedestrian Test Environment-A



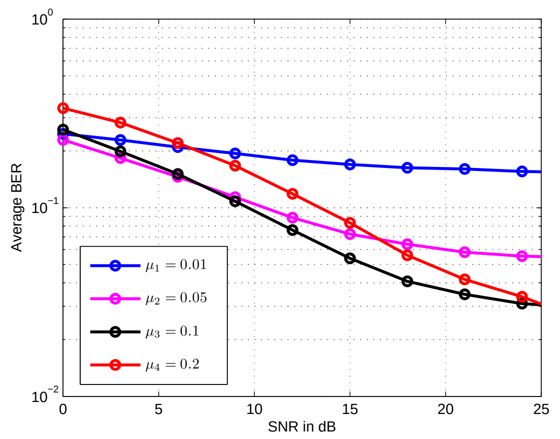
(b) Vehicular Test Environment-B

Figure 5.11: MSE Performance of BNLMS Algorithm based MIMO-DFE with Different Values of Step Size.

values are shown in Fig. 5.11. It is observed that, convergence performance improves with increase in step size. Moreover, it is seen that, in MSE curves for step size 0.2 contains comparatively more fluctuations in the error terms.



(a) Pedestrian Test Environment-A



(b) Vehicular Test Environment-B

Figure 5.12: BER Performance of BNLMS Algorithm based MIMO-DFE with Different Values of Step Size.

Fig. 5.12 shows the BER performance of the proposed equalization scheme for different μ values. As seen from figure, BER performance improves with increase in μ value till 0.1, but starts deteriorating for higher μ value. Higher values of μ increases the fluctuations in error terms, and it may be attributed to the poor BER performance.

5.6 Summary

In this chapter, we have investigated the performance of adaptive MIMO-DFE with data reusing AP and BNLMS as adaption algorithms in a practical frequency selective MIMO channel. The convergence as well as BER performances as a function of projection order (in case of AP algorithm) and step size were shown in simulation results. Simulation have also been performed for such adaptive MIMO-DFE system using NLMS algorithm.

It is observed that AP algorithm is suitable for MIMO-DFE systems in time-varying channels, as it offers significant improvement in the convergence as well as BER performance over NLMS based algorithm with marginal increase in computational complexity. It is observed that, choice of the projection order P is important, which reflects the performance as well as complexity. Moreover, step size $\mu = 0.05$ is found to be suitable both in terms of convergence as well as BER performance.

Further, the performance of MIMO-DFE with data-reusing BNLMS algorithm is also investigated. The performance of the proposed scheme is studied in low as well as medium delay spread channel. It is observed that, MIMO-DFE employing BNLMS algorithm has similar performance with reduced computational requirement in comparison with an equalizer using AP algorithm with projection order 2. In a mobile fading environment small step size is needed to estimate the filter coefficient more precisely according to the time-varying channel condition. Further, we find step size $\mu = 0.05$ is suitable to achieve better performance in terms of convergence as well as BER performance.

6

Data-selective Algorithm based MIMO-DFE

Contents

Objective

This chapter investigates channel equalization scheme using adaptive MIMO-DFE based on SM-AP algorithm as well as SM-BNLMS algorithm for frequency selective MIMO channels. It is found that, SM-AP and SM-BNLMS algorithm based proposed MIMO channel equalization scheme exhibits almost similar performance respectively, as data-reusing AP and BNLMS algorithm based adaptive MIMO-DFE but with a significant reduction in computational load due to the data selective updating.

6.1 Introduction

The data-reusing algorithm based adaptive MIMO-DFE schemes investigated in the previous chapter are found to provide better convergence and BER performance than NLMS based equalizer with a marginal increase in computational complexity. Since equalization is a receiver end technique, reducing computation load is very useful particularly for hand-held devices. Recently, attention has been given to the schemes using the set-membership (SM) adaptive filtering algorithm due to reduced computational requirement with improvement in overall performance.

In this chapter, we present MIMO channel equalization schemes with SM-AP and SM-BNLMS as the adaption algorithm. The system model used in this work and the equations describing the model are similar to the adaptive MIMO-DFE model described in Chapter 5. The performances of these equalizers are investigated through simulation in a multi-path fading environment as experienced in the indoor and pedestrian environment. The equalizer performances are also studied for channels having higher delay and Doppler spread. The convergence issues, BER performance and tracking capabilities are examined through computer simulations. Moreover, the computational complexity issue for such MIMO equalizer is compared with MIMO equalizer employing AP, BNLMS and SM-NLMS algorithm.

The rest of the chapter is organized as follows: Section 6.2 presents the adaptive MIMO-DFE using SM-AP algorithm; numerical and simulation results are given to confirm the effectiveness of the introduced channel equalization methods. The performance of the SM-BNLMS algorithm based MIMO-DFE is investigated in Section 6.3. Finally, the summary of this chapter is mentioned in Section 6.4.

6.2 Set-Membership Affine Projection Algorithm based MIMO-DFE.

In this section, we discuss the use of SM-AP algorithm for adaptively updating coefficients of a DFE. Here, we first present an overview of SM-AP algorithm and highlight the modifications needed to use such algorithm in realizing a MIMO-DFE equalizer. The SM-AP algorithm utilizes present input signal vector as well as the $P-1$ past input signal vectors on the coefficient update, leading to fast convergence for highly correlated input signals [49].

We consider a generic adaptive MIMO-DFE system model as discussed in Section 5.3 for implementing SM-AP algorithm. We have modified the basic SM-AP algorithm appropriately for applying in MIMO-DFE, as detailed below. The objective function for i^{th} MIMO-DFE at time instant k is:

$$\begin{aligned} &\min \left\|\mathbf{w}_i(k+1) - \mathbf{w}_i(k)\right\|^2 \\ &\text{subject to:} \quad \mathbf{d}_i(k) - \mathbf{S}(k)\,\mathbf{w}_i(k+1) = \boldsymbol{\gamma}_i(k) \end{aligned} \tag{6.1}$$

where $\mathbf{S}(k)$ is a matrix of size $P \times (ML_{FF} + NL_{FB})$, which contains present input signal vector and $P-1$ previous input signal vectors expressed as:

$$\mathbf{S}(k) = \begin{bmatrix} \mathbf{s}(k) \\ \mathbf{s}(k-1) \\ \vdots \\ \mathbf{s}(k-P+1) \end{bmatrix} \tag{6.2}$$

The $\mathbf{d}_i(k)$ is the desired signal vector of size $P \times 1$ and $\boldsymbol{\gamma}_i(k)$ is a column vector of size $P \times 1$ containing $\gamma_{il}(k)$ for $l = 1, 2, \cdots, P$ which represents the upper bound constraint to be met by the output error magnitudes after weight updating of i_{th} DFE.

By using the Lagrange multiplier method, the constrained minimization problem of equation 6.1 is solved. Then the weight update equation can be derived by setting the gradient of the unconstrained function with respect to $\mathbf{w}_i(k+1)$ equal to zero. So, expression for the filter weight update can be expressed as:

$$\mathbf{w}_i(k+1) = \begin{cases} \mathbf{w}_i(k) + \mathbf{S}^T(k)\left(\mathbf{S}(k)\,\mathbf{S}^T(k) + \alpha\boldsymbol{I}\right)^{-1}\!(\mathbf{e}_i(k) - \boldsymbol{\gamma}_i(k)), & \text{if } |e_i(k)| > \gamma \\ \mathbf{w}_i(k) & , \text{ otherwise} \end{cases} \tag{6.3}$$

where γ value is chosen slightly higher than the bound constraints $\gamma_{il}(k)$. The $\mathbf{e}_i(k)$ is the error vector of size $P \times 1$, which contains a-priori error $e_i(k)$ and $(P-1)$ a-posteriori errors $\varepsilon_i(k-j)$ (for $j = 1, 2, \cdots, P-1$), where:

$$e_i(k) = d_i(k) - \mathbf{s}(k)\,\mathbf{w}_i(k) \tag{6.4}$$

$$\varepsilon_i(k-j) = d_i(k-j) - \mathbf{s}(k-j)\,\mathbf{w}_i(k) \tag{6.5}$$

As we know the weights are updated considering previous data pairs, hence the a-posteriori errors value will be nearly equal to the respective bound constraints. So, we can write as:

$$\mathbf{e}_i(k) - \boldsymbol{\gamma}_i(k) = \begin{bmatrix} e_i(k) - \gamma_{i1}(k) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{6.6}$$

As $\gamma_{i1}(k)$ value is slightly less than γ, so, we can consider:

$$\gamma_{i1}(k) = \gamma \text{ sign}\left[e_i(k)\right]. \tag{6.7}$$

Hence, the weight update equation can be simplified as:

$$\mathbf{w}_i(k+1) = \begin{cases} \mathbf{w}_i(k) + \mathbf{S}^T(k)\left(\mathbf{S}(k)\,\mathbf{S}^T(k) + \alpha \boldsymbol{I}\right)^{-1}\left(e_i(k) - \gamma \text{ sign}\left[e_i(k)\right]\right)\mathbf{v}_i, & \text{if } |e_i(k)| > \gamma \\ \mathbf{w}_i(k) & , \text{ otherwise} \end{cases} \tag{6.8}$$

where $\mathbf{v}_i = \begin{bmatrix} 1 & 0 & \ldots & 0 \end{bmatrix}^T$ of size $P \times 1$.

Through simulation, it has been observed that in SM-AP algorithm based MIMO-DFE implementation, introducing a convergence factor μ in equation 6.8 results in better performance. So, the weight update equation can be rewritten as:

$$\mathbf{w}_i(k+1) = \begin{cases} \mathbf{w}_i(k) + \mu\mathbf{S}^T(k)\left(\mathbf{S}(k)\,\mathbf{S}^T(k) + \alpha \boldsymbol{I}\right)^{-1}\left(e_i(k) - \gamma \text{ sign}\left[e_i(k)\right]\right)\mathbf{v}_i, & \text{if } |e_i(k)| > \gamma \\ \mathbf{w}_i(k) & , \text{ otherwise} \end{cases} \tag{6.9}$$

The step by step implementation of SM-AP algorithm based MIMO-DFE is given in Table 6.1.

Table 6.1: Summary of SM-AP algorithm based MIMO-DFE Implementation Scheme for i^{th} DFE.

1. Initialization

Weight vector $\mathbf{w}_i(k)$ is initialized as a zero vector.

Input signal vector $\mathbf{s}(k)$ is initialized as a zero vector.

Regularization parameter α is a small constant.

$\boldsymbol{I}$ is an unit matrix of size $P \times P$.

Convergence factor be in the range $0 < \mu < 2$.

2. Processing for the time instant $k > 0$

(a) Error Computation and Upper bound:

A-priori Error: $e_i(k) = d_i(k) - \mathbf{s}(k)\mathbf{w}_i(k)$

A-posteriori Error: $\varepsilon_i(k-j) = d_i(k-j) - \mathbf{s}(k-j)\mathbf{w}_i(k)$ for $j = 1, 2, \cdots, P-1$.

Upper Bound on Estimation Error: $\gamma_{i1}(k) = \gamma \text{ sign}[e_i(k)]$

(b) Weight updating:

$$\mathbf{w}_i(k+1) = \begin{cases} \mathbf{w}_i(k) + \mu\mathbf{S}^T(k)\left(\mathbf{S}(k)\mathbf{S}^T(k) + \alpha\boldsymbol{I}\right)^{-1}(e_i(k) - \gamma \text{ sign}[e_i(k)])\mathbf{v}_i, & \text{if } |e_i(k)| > \gamma \\ \mathbf{w}_i(k), & \text{otherwise} \end{cases}$$

(c) Increment k and go back to 'for' loop.

6.2.1 Comparison of Computational Complexity

Table 6.2 compares the computational complexity of MIMO-DFE using SM-AP algorithm with SM-NLMS algorithm based scheme. Here, we take $B = ML_{FF} + NL_{FB}$.

Table 6.2: Comparison of Computational Complexity of SM-NLMS and SM-AP Algorithm based MIMO-DFE.

Algorithm	Addition	Multiplication	Division
SM-NLMS	$3B$	$3B+1$	1
SM-AP	$P^2\left(\frac{B}{2}+1\right) + B + \frac{5}{2}BP + \mathcal{O}\left(P^3\right)$	$\frac{P^2}{2}(B+1) - \frac{P}{2} + \frac{5}{2}BP + \mathcal{O}\left(P^3\right)$	

It may be seen that computation involved per update in the proposed scheme is higher than SM-NLMS algorithm based scheme. However, SM-AP algorithm based MIMO-DFE requires less number of updates than the scheme with SM-NLMS algorithm, which makes the overall computational load slightly higher to SM-NLMS, while better convergence is obtained in case of SM-AP. This issue is further elaborated in the result section. Moreover, it is observed in the implementation of SM-AP algorithm based MIMO-DFE that, the major computational complexity terms are in computing the matrix inversion, which is of the order $\mathcal{O}(P^3)$, where P is the projection order. Hence, P is to be chosen carefully considering the performance as well as the complexity involved.

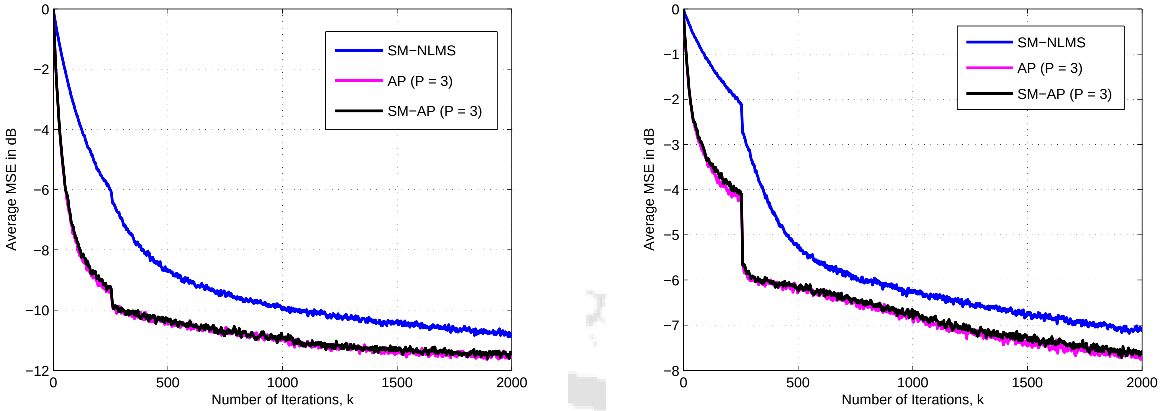
6.2.2 Simulation Results and Discussion

Here, we present the simulation results and discuss the performance of the SM-AP algorithm based MIMO-DFE. We consider the adaptive MIMO-DFE scheme for a 2×2 MIMO channel as shown in Fig. 5.3. Simulations are performed with the same set of parameters considered for AP algorithm based MIMO-DFE in Chapter 5; however the same is included here for continuity. A BPSK modulation scheme has been used and the noise has been considered to be additive white Gaussian. We consider each data packet consisting of 2048 bits, which includes 256 bits training sequence, and the performances were evaluated by taking average of 3000 such independent runs. We consider WiMAX forum recommended ITU-R wireless channel models; one for outdoor to indoor and pedestrian test environment with a low delay spread (Ped-A), and another for the vehicular test environment with a medium delay spread (Veh-B) [54]. Taking into consideration the delay spread of the above two channel models, the simulations are carried out with sample time t_s of the signal as .01 μsec for Ped-A and .3 μsec for Veh-B so that the channel behaves as frequency selective. Further, simulations are performed assuming the operation in 2.5 GHz band and the maximum doppler frequency shift f_d as 5 Hz for Ped-A and 70 Hz for Veh-B. Moreover, the filter lengths of DFE are chosen as $L_{FF} = 3$ and $L_{FB} = 1$, and MSE curves are plotted with an SNR of 20 dB.

6.2.2.1 Performance Comparison of MIMO-DFE using SM-NLMS, AP and SM-AP Algorithm

The MSE and BER performance of MIMO-DFE using SM-NLMS, AP and SM-AP algorithms are compared here. In the study, projection order $P = 3$ and step size $\mu = 0.05$ are employed. Further, the upper bound γ on the estimation error is set to $\sqrt{1.5}\sigma_n$, where σ_n^2 is the variance of additive white Gaussian noise (AWGN).

Fig. 6.1 compares the MSE performance of MIMO-DFE using SM-AP algorithm with AP algorithm as well as SM-NLMS algorithm. It is observed that, this scheme offers better convergence compared to MIMO-DFE using SM-NLMS algorithm, but similar convergence as AP algorithm based scheme.



(a) Pedestrian Test Environment-A

(b) Vehicular Test Environment-B

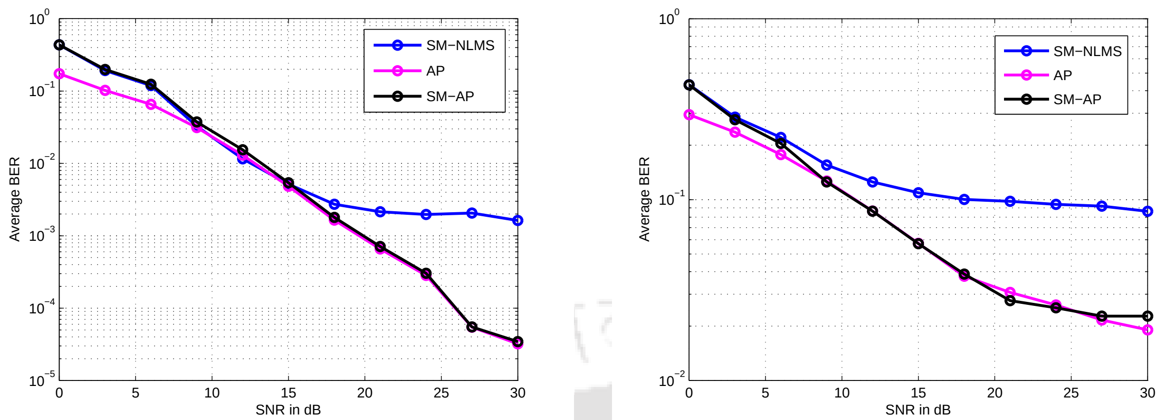
Figure 6.1: MSE Performance Comparison of MIMO-DFE for SM-NLMS, AP and SM-AP Algorithms.

Table 6.3: Comparison of Number of Updates in SM-NLMS, SM-AP and AP Algorithm based MIMO-DFE.

Adaptive Algorithm	Pedestrian-A	Vehicular-B
SM-NLMS	1424	1708
SM-AP (P=3)	1250	1624
AP	2048	2048

Table 6.3 compares the number of updates required in MIMO-DFE with SM-NLMS, AP and SM-AP as adaption algorithm. It is observed that, the number of updates associated in a packet of 2048 symbols are 1250 and 1624 respectively, in Ped-A and Veh-B environments due to data selective updating, which reduces the computational load compared to AP algorithm based technique. Further, SM-AP algorithm based MIMO-DFE requires less number of updates than with SM-NLMS algorithm, which reduces the difference of computational requirement between them.

Fig. 6.2 shows the BER performance comparison of MIMO-DFE using SM-NLMS, AP and SM-AP algorithm with $\mu = 0.05$ and γ as $\sqrt{1.5}\sigma_n$. It is observed that both SM-NLMS and SM-AP algorithms are having similar BER performance in the low SNR region, but SM-AP performs well for higher SNR values. Moreover, BER performance of AP and SM-AP algorithm are quite close for SNR values above 8 dB, however AP algorithm performs better for low SNR values.



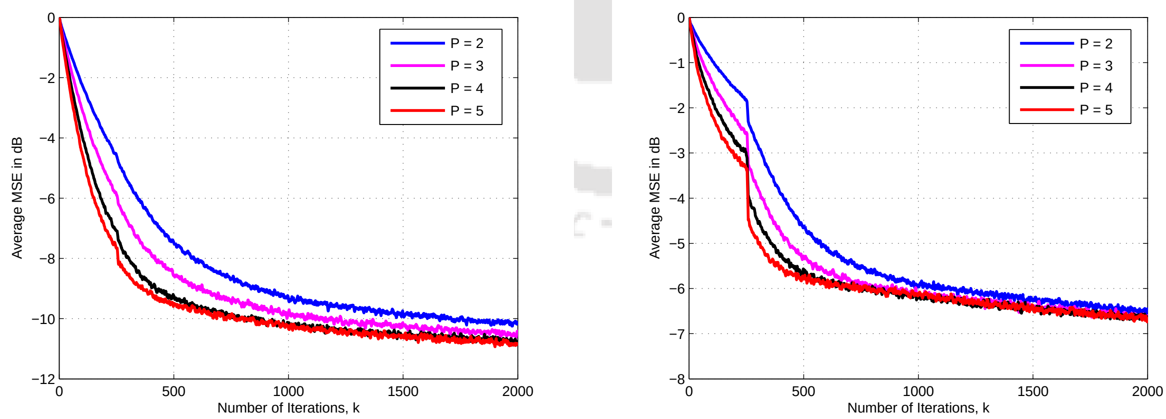
(a) Pedestrian Test Environment-A

(b) Vehicular Test Environment-B

Figure 6.2: BER Performance Comparison of MIMO-DFE using SM-NLMS, AP and SM-AP Algorithms.

6.2.2.2 Performance of SM-AP Algorithm based MIMO-DFE for Different Projection Order Values

The convergence as well as BER performance of SM-AP algorithm based MIMO-DFE is studied for different projection order (P) values in both Ped-A and Veh-B environments. Simulations are done for P values 2, 3, 4 and 5. For this study, the upper bound on the estimation error $\gamma = \sqrt{1.5}\sigma_n$ and step size $\mu = 0.01$ are employed.



(a) Pedestrian Test Environment-A

(b) Vehicular Test Environment-B

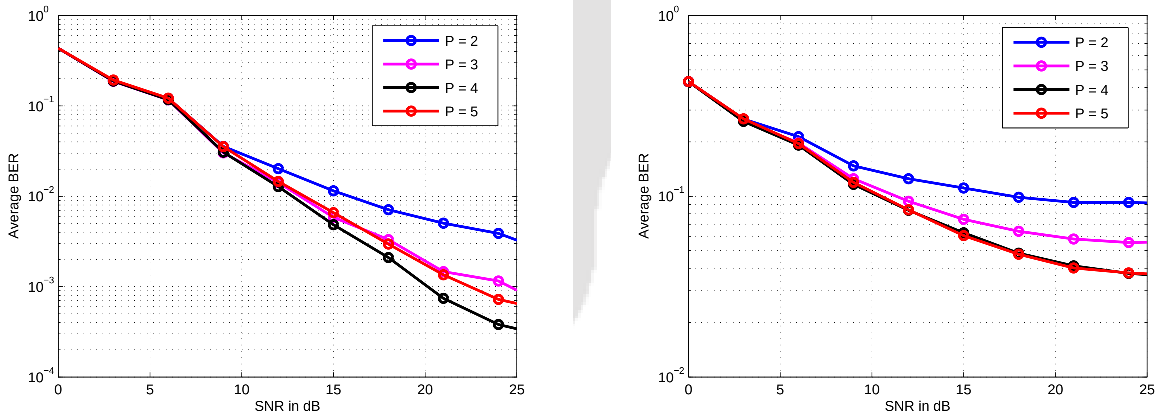
Figure 6.3: MSE Performance of SM-AP Algorithm based MIMO-DFE for Different Projection Order.

Fig. 6.3 shows the convergence performance comparison of MIMO-DFE with SM-AP Algorithms for different P values. It is observed in Fig. 6.3 that, the convergence performance improves with increase in projection order from 2 to 3 as well as from 3 to 4. Further, it is observed that the convergence performance of the proposed MIMO equalizer with P values 4 and 5 are quite close.

Table 6.4: Comparison of Number of Updates in SM-AP Algorithm based MIMO-DFE with Projection Order.

Projection Order	Pedestrian-A	Vehicular-B
P = 2	1524	1770
P = 3	1447	1741
P = 4	1397	1714
P = 5	1370	1701

Table 6.4 compares the number of updates required in SM-AP algorithm based MIMO-DFE for different P values. Though increase in the projection order is found to have slightly less number of updates, it substantially increases the complexity.



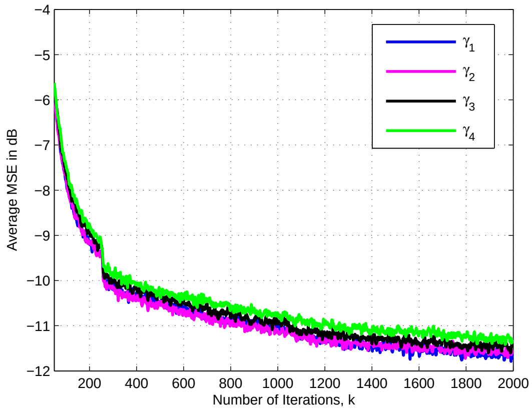
(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 6.4: BER Performance of SM-AP Algorithm based MIMO-DFE for Different Projection Order.

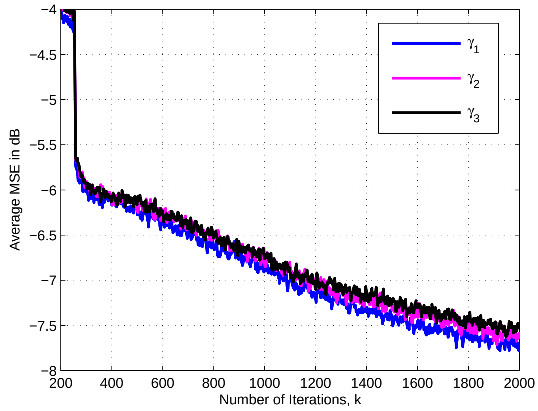
Fig. 6.4 shows the comparison of BER performance of SM-AP algorithm based MIMO-DFE for different values of projection order. It is noticed that, increasing projection order results no change in BER performance for low SNR value, but improves for SNR values more than 8 dB and 3 dB respectively, in Ped-A and Veh-B environments. Further, it is observed that increasing P value from 4 to 5 slightly degrades the BER performance in Ped-A environment.

6.2.2.3 Performance of SM-AP Algorithm Based MIMO-DFE for Different Values of Upper Bounds on Estimation Error

The performance of the proposed equalizer is investigated for different values of upper bound on estimation error γ. We consider γ values as, $\gamma_1 = \sqrt{0.7}\sigma_n$, $\gamma_2 = \sqrt{1.5}\sigma_n$, $\gamma_3 = \sqrt{3}\sigma_n$ and $\gamma_4 = \sqrt{5}\sigma_n$. The simulations are carried out with $P = 3$ and $\mu = 0.05$.



(a) Pedestrian Test Environment-A



(b) Vehicular Test Environment-B

Figure 6.5: MSE Performance of SM-AP Algorithm based MIMO-DFE with upper bound γ.

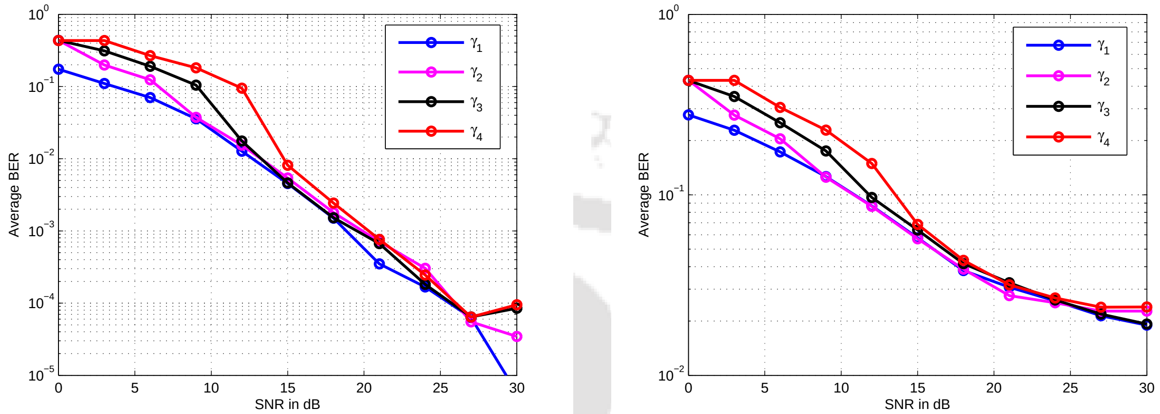
Fig. 6.5 shows the convergence performance of SM-AP algorithm based MIMO-DFE with different values of γ, the upper bound. It is observed that increase in γ deteriorates the convergence performance.

Table 6.5: Number of Updates Required in SM-AP algorithm based MIMO-DFE for Different γ Values.

Threshold Value on Estimation Error (γ)	Number of Updates	
	Pedestrian-A	Vehicular-B
$\sqrt{0.7}\sigma_n$	1470	1753
$\sqrt{1.5}\sigma_n$	1250	1624
$\sqrt{2.3}\sigma_n$	1103	1525
$\sqrt{3}\sigma_n$	991	1456
$\sqrt{5}\sigma_n$	814	1310

Table 6.5 shows the number of updates required in this MIMO equalizer with different γ values. It is found that, increasing γ value results in decrease in number of updates, for example, increasing γ

value from $\sqrt{0.7}\sigma_n$ to $\sqrt{1.5}\sigma_n$ results a decrease in the number of updates from 1470 to 1250 and 1753 to 1624 respectively, in Ped-A and Veh-B environments; but convergence performance deteriorates with increase in γ.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

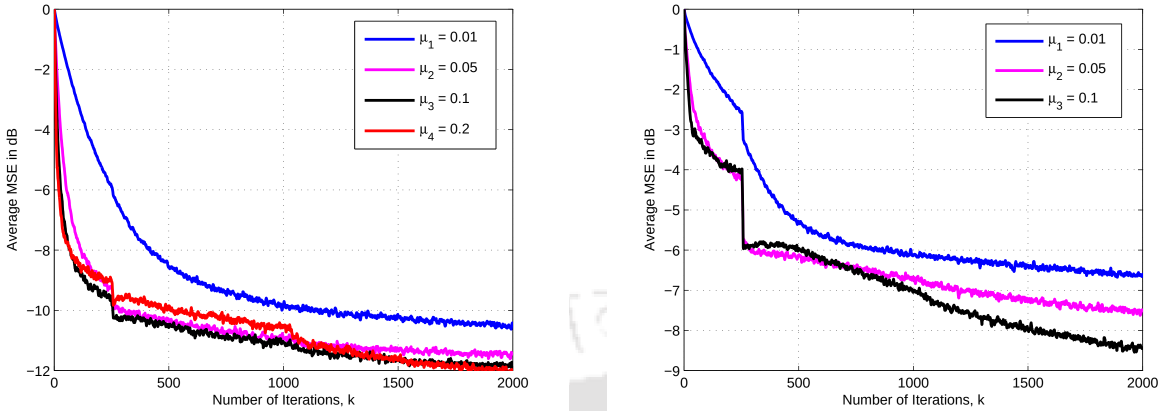
Figure 6.6: BER Performance of SM-AP Algorithm based MIMO-DFE with Upper Bound γ.

Fig. 6.6 compares the BER performance of this MIMO-DFE for different γ values. It is observed that, increase of γ degrades the BER performance in low SNR region; however shows similar performance for SNR values more than 15 dB. It is found that; $\sqrt{1.5}\sigma_n \leqslant \gamma \leqslant \sqrt{3}\sigma_n$ is suitable in terms of performance as well as complexity.

6.2.2.4 Performance of SM-AP Algorithm based MIMO-DFE with Different Values of Convergence Factor.

Here, the performance of SM-AP Algorithm based MIMO-DFE for different values of convergence factor are investigated. The performance includes convergence, BER as well as the requirement of number of updates. The simulations are carried out for convergence factor μ values as $\mu_1 = 0.01$, $\mu_2 = 0.05$, $\mu_3 = 0.1$ and $\mu_4 = 0.2$ considering $P = 3$ and $\gamma = \sqrt{1.5}\sigma_n$.

Fig. 6.7 shows the MSE performance of this MIMO equalizer with different μ values. It is observed that, increase in convergence factor from 0.01 to 0.05, and also from 0.05 to 0.1 improves the convergence performance, but deteriorates for μ value 0.2. The number of updates in SM-AP algorithm based MIMO-DFE for different values of convergence factor are shown in Table 6.6. It is found that, increase in μ results in reduction of number of updating operations, and offers faster convergence; and if μ value is made greater than 0.1, the performance degrades due to instantaneous error terms.



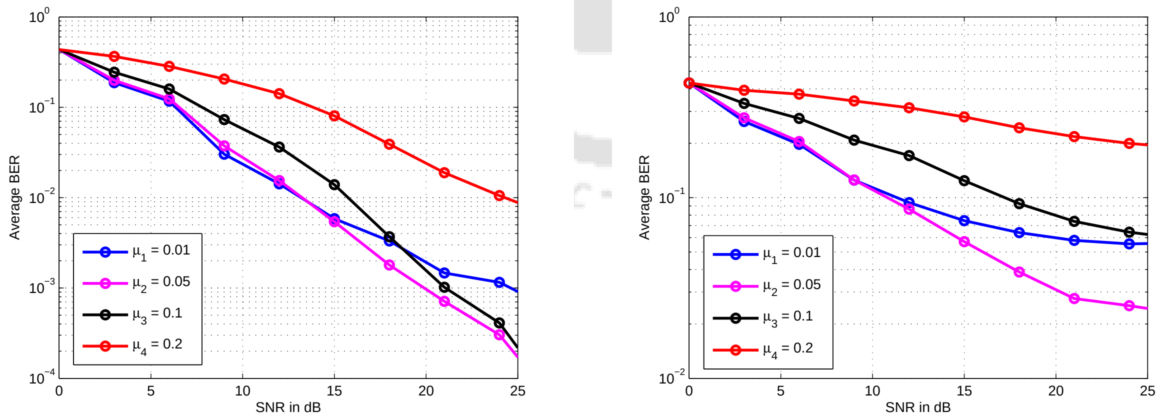
(a) Pedestrian Test Environment-A

(b) Vehicular Test Environment-B

Figure 6.7: MSE Performance of SM-AP Algorithm based MIMO-DFE with Different Values of Convergence Factor.

Table 6.6: Number of Updates in SM-AP Algorithm based MIMO-DFE with Different Values of Convergence Factor.

Convergence Factor (μ)	Pedestrian-A	Vehicular-B
0.01	1447	1741
0.05	1250	1624
0.1	1218	1579
0.2	1227	1591



(a) Pedestrian Test Environment-A

(b) Vehicular Test Environment-B

Figure 6.8: BER Performance SM-AP Algorithm based MIMO-DFE with Different Values of Convergence Factor.

The BER performance of this channel equalization scheme for different values of μ are shown in Fig. 6.8. It is observed that, increasing μ values from 0.01 to 0.05 improves the BER performance, but degrades for μ values 0.1 and 0.2. The higher values of μ increases the fluctuations in the instantaneous error terms, and it may be attributed to the poor BER performance. Moreover, step size $\mu = 0.05$ is found to be suitable both in terms of performance as well as requirement of number of updates.

6.3 SM-BNLMS Algorithm based Equalizer for MIMO Channel

In this Section, we propose a MIMO channel equalization scheme which uses a set-membership BNLMS algorithm for updating the filter weights of the DFEs. As discussed in Chapter 3, data-reusing BNLMS algorithm utilizes the present input signal vector as well as the immediate past input signal vector for updating the filter coefficient. In each update, normalization is employed in two orthogonal directions from consecutive data pairs to achieve faster convergence [41, 55]. The SM-BNLMS algorithm uses two consecutive constraint sets for allowing each data selective update [51]. It performs a coefficient updating whenever the weight vector $\mathbf{w}_i(k) \notin H_k \cap H_{k-1}$, where H_k and H_{k-1} are constraint sets at iteration k and $k-1$, respectively. H_k is the set containing all $\mathbf{w}_i(k)$ for which estimation error at iteration k is upper bounded in magnitude by a prescribed quantity γ.

For implementing this algorithm in an adaptive MIMO-DFE system model discussed in Section 5.3, we have modified it appropriately as detailed below. The objective of this algorithm is to maintain the next coefficient vector as close as possible to the current one, to maintain a-posteriori errors prescribed by the bound constraints $(\gamma_{1,i}(k), \gamma_{2,i}(k))$. So, the objective function for i^{th} MIMO-DFE is:

$$\begin{aligned} &\min \left\| \mathbf{w}_i(k+1) - \mathbf{w}_i(k) \right\|^2 \\ \text{subject to:}\quad &\hat{x}_i(k) - \mathbf{s}(k)\,\mathbf{w}_i(k+1) = \gamma_{1,i}(k) \\ &\hat{x}_i(k-1) - \mathbf{s}(k-1)\,\mathbf{w}_i(k+1) = \gamma_{2,i}(k) \end{aligned} \tag{6.10}$$

The Lagrange multiplier method is used to solve the constrained minimization problem given in equation 6.10. So, we can write the unconstrained function as:

$$\begin{aligned} f\left[\mathbf{w}_i(k+1)\right] = &\left\| \mathbf{w}_i(k+1) - \mathbf{w}_i(k) \right\|^2 + \lambda_{1,i}(k)\left[\hat{x}_i(k) - \mathbf{s}(k)\,\mathbf{w}_i(k+1) - \gamma_{1,i}(k)\right] \\ &+\lambda_{2,i}(k)\left[\hat{x}_i(k-1) - \mathbf{s}(k-1)\,\mathbf{w}_i(k+1) - \gamma_{2,i}(k)\right] \end{aligned} \tag{6.11}$$

Then the weight update equation is derived by setting the gradient of $f\left[\mathbf{w}_i(k+1)\right]$ with respect to

$\mathbf{w}_i(k+1)$ cited in equation 6.11, equal to zero. So, we get:

$$\mathbf{w}_i(k+1) = \begin{cases} \mathbf{w}_i(k) + \frac{\lambda_{1,i}(k)}{2}\mathbf{s}^T(k) + \frac{\lambda_{2,i}(k)}{2}\mathbf{s}^T(k-1) & , \text{ if } |e_i(k)| > \gamma \\ \mathbf{w}_i(k) & , \text{ otherwise} \end{cases} \tag{6.12}$$

where the γ value is slightly higher than the bound constraints $(\gamma_{1,i}(k), \gamma_{2,i}(k))$ and Lagrange multipliers are obtained as:

$$\begin{aligned} \frac{\lambda_{1,i}(k)}{2} &= \frac{[e_i(k) - \gamma_{1,i}(k)]\,\|\mathbf{s}(k-1)\|^2 - [\varepsilon_i(k-1) - \gamma_{2,i}(k)]\,\mathbf{s}(k)\,\mathbf{s}^T(k-1)}{\alpha + \|\mathbf{s}(k)\|^2\,\|\mathbf{s}(k-1)\|^2 - [\mathbf{s}(k)\,\mathbf{s}^T(k-1)]^2} \\ \frac{\lambda_{2,i}(k)}{2} &= \frac{[\varepsilon_i(k-1) - \gamma_{2,i}(k)]\,\|\mathbf{s}(k)\|^2 - [e_i(k) - \gamma_{1,i}(k)]\,\mathbf{s}(k)\,\mathbf{s}^T(k-1)}{\alpha + \|\mathbf{s}(k)\|^2\,\|\mathbf{s}(k-1)\|^2 - [\mathbf{s}(k)\,\mathbf{s}^T(k-1)]^2} \end{aligned} \tag{6.13}$$

where α is the regularization parameter, $e_i(k)$ is the a-priori error at iteration k and $\varepsilon_i(k-1)$ is the a-posteriori error at iteration $k-1$. So,

$$\begin{aligned} e_i(k) &= d_i(k) - \mathbf{s}(k)\,\mathbf{w}_i(k) \\ \varepsilon_i(k-1) &= d_i(k-1) - \mathbf{s}(k-1)\,\mathbf{w}_i(k) \end{aligned} \tag{6.14}$$

The above expressions for the filter weight update can be simplified further by avoiding the intermediate constraint check. We know that at time instant k, $\mathbf{w}_i(k) \in H_{k-1}$ which implies $\varepsilon_i(k-1) \leqslant \gamma$. Hence, the value of a-posteriori error $\varepsilon_i(k-1)$ is maintained as equal to the constraint threshold $\gamma_{2,i}(k)$. Moreover, if we consider $\gamma_{1,i}(k) = \gamma \text{ sign}[e_i(k)]$, then the new update lies on the closest boundary of H_k [71]. Thus, the value of $\lambda_{1,i}(k)$ and $\lambda_{2,i}(k)$ can be simplified as:

$$\begin{aligned} \frac{\lambda_{1,i}(k)}{2} &= \frac{[e_i(k) - \gamma_{1,i}(k)]\,\|\mathbf{s}(k-1)\|^2}{\alpha + \|\mathbf{s}(k)\|^2\,\|\mathbf{s}(k-1)\|^2 - [\mathbf{s}(k)\,\mathbf{s}^T(k-1)]^2} \\ \frac{\lambda_{2,i}(k)}{2} &= \frac{-[e_i(k) - \gamma_{1,i}(k)]\,\mathbf{s}(k)\,\mathbf{s}^T(k-1)}{\alpha + \|\mathbf{s}(k)\|^2\,\|\mathbf{s}(k-1)\|^2 - [\mathbf{s}(k)\,\mathbf{s}^T(k-1)]^2} \end{aligned} \tag{6.15}$$

As observed in Section 4.5, introducing a convergence factor μ in the weight update equation, improves the convergence as well as BER performance in a WiMAX forum recommended ITU-R wireless channel model. So, the equation 6.12 for weight updating of MIMO-DFEs using SM-BNLMS algorithm can be modified as:

$$\mathbf{w}_i(k+1) = \begin{cases} \mathbf{w}_i(k) + \mu\left[\frac{\lambda_{1,i}(k)}{2}\mathbf{s}^T(k) + \frac{\lambda_{2,i}(k)}{2}\mathbf{s}^T(k-1)\right] & , \text{ if } |e_i(k)| > \gamma \\ \mathbf{w}_i(k) & , \text{ otherwise} \end{cases} \tag{6.16}$$

where

$$\begin{aligned}\frac{\lambda_{1,i}(k)}{2} &= \frac{[e_i(k) - \gamma \text{ sign}[e_i(k)]] \|\mathbf{s}(k-1)\|^2}{\alpha + \|\mathbf{s}(k)\|^2 \|\mathbf{s}(k-1)\|^2 - [\mathbf{s}(k)\mathbf{s}^T(k-1)]^2} \\ \frac{\lambda_{2,i}(k)}{2} &= \frac{-[e_i(k) - \gamma \text{ sign}[e_i(k)]] \mathbf{s}(k)\mathbf{s}^T(k-1)}{\alpha + \|\mathbf{s}(k)\|^2 \|\mathbf{s}(k-1)\|^2 - [\mathbf{s}(k)\mathbf{s}^T(k-1)]^2}\end{aligned} \tag{6.17}$$

with $\gamma_{1,i}(k) = \gamma \text{ sign}[e_i(k)]$.

Table 6.7: Summary of the Proposed MIMO Channel Equalization with SM-BNLMS Algorithm for i^{th} DFE.

Step-I: Initialization

Weight vector is initialized as zero vector $\mathbf{w}_i(0)$.

Input signal vector is initialized as zero vector $\mathbf{s}(0)$.

The α a small constant, is regularization parameter .

Convergence factor be in the range $0 < \mu < 2$.

Step-II: Processing for $k > 0$

(a): Computation of Error Vector

A priori error $e_i(k) = d_i(k) - \mathbf{s}(k)\mathbf{w}_i(k)$.

A posteriori error $\varepsilon_i(k-1) = d_i(k-1) - \mathbf{s}(k-1)\mathbf{w}_i(k)$.

(b): Setting Upper Bound

$\gamma_{1,i}(k) = \gamma \text{ sign}[e_i(k)]$.

$\gamma_{2,i}(k) = \varepsilon_i(k-1) \leqslant \gamma$.

(c): Filter Weight Updating

$$\mathbf{w}_i(k+1) = \begin{cases} \mathbf{w}_i(k) + \mu\left[\frac{\lambda_{1,i}(k)}{2}\mathbf{s}^T(k) + \frac{\lambda_{2,i}(k)}{2}\mathbf{s}^T(k-1)\right] & \text{, if } |e_i(k)| > \gamma \\ \mathbf{w}_i(k) & \text{, otherwise} \end{cases}$$

$$\frac{\lambda_{1,i}(k)}{2} = \frac{[e_i(k) - \gamma \text{ sign}[e_i(k)]] \|\mathbf{s}(k-1)\|^2}{\alpha + \|\mathbf{s}(k)\|^2 \|\mathbf{s}(k-1)\|^2 - [\mathbf{s}(k)\mathbf{s}^T(k-1)]^2}$$

$$\frac{\lambda_{2,i}(k)}{2} = \frac{-[e_i(k) - \gamma \text{ sign}[e_i(k)]] \mathbf{s}(k)\mathbf{s}^T(k-1)}{\alpha + \|\mathbf{s}(k)\|^2 \|\mathbf{s}(k-1)\|^2 - [\mathbf{s}(k)\mathbf{s}^T(k-1)]^2}$$

(d): Increment k and go back to 'for' loop.

6.3.1 Computational Complexity Issues

Here, we compare the computation involved in implementing the proposed MIMO-DFE using SM-BNLMS algorithm with existing SM-NLMS and SM-AP algorithm based techniques. Table 6.8 shows the computational complexity in terms of the number of complex multiplication, complex addition

Table 6.8: Computational Complexity per Update for Implementing SM-NLMS, SM-AP and SM-BNLMS Algorithm Based MIMO-DFE.

Algorithm	Addition	Multiplication	Division
SM-NLMS	$3BN$	$3BN + N$	N
SM-AP	$N\,(2BP - 1) + BP^2 + \mathcal{O}\left(P^3\right)$	$NP\,(2B + 1) + 2BP^2 + \mathcal{O}\left(P^3\right)$	
SM-BNLMS	$6BN$	$6BN - N^2$	N

and division per update involved with the MIMO equalizer with different data-selective algorithms. It may be seen that computation involved per update in the proposed scheme is higher than SM-NLMS algorithm based scheme. However, equalization with SM-BNLMS algorithm converges considerably faster than the scheme with SM-NLMS algorithm and number of updates required is comparatively less in case of SM-BNLMS, which makes the overall computational load slightly higher to SM-NLMS, while better convergence is obtained in case of SM-BNLMS. This issue is further elaborated in the next subsection. Moreover, it is observed that, the complexity involved per update in the proposed MIMO channel equalizer is substantially less compared to SM-AP algorithm based MIMO-DFE.

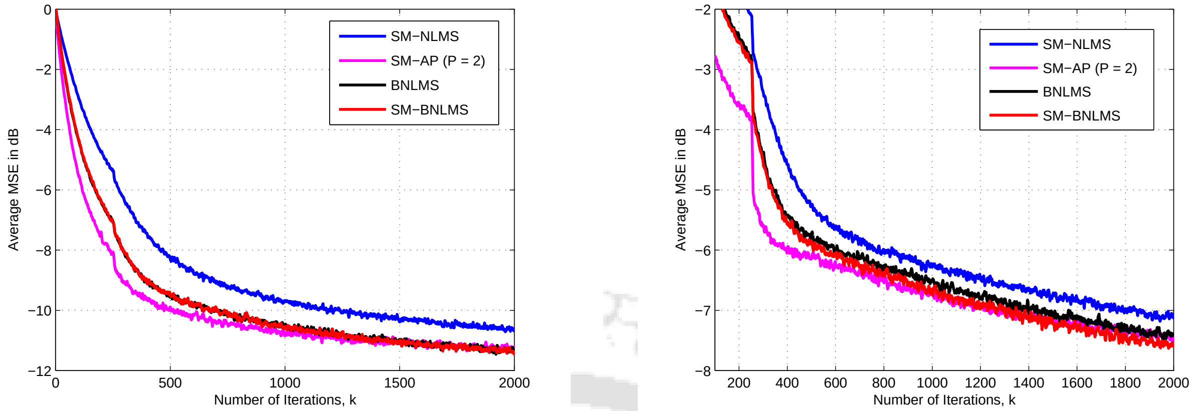
6.3.2 Results and Discussion

Here, we present the simulation results to apprise the performance of the proposed SM-BNLMS algorithm based MIMO-DFE. The simulation studies are performed with same parameter values as considered in Section 6.2 for implementation of SM-AP algorithm based MIMO-DFE.

6.3.2.1 Performance of Different Data-selective Algorithms based MIMO-DFE

The convergence and BER performance of the equalizer are tested for different adaptive algorithms: (i) SM-BNLMS, (ii) SM-NLMS, (iii) SM-AP with projection order 2 and (iv) BNLMS. The threshold value γ on the estimation error was set to $\sqrt{1.5}\sigma_n$ for SM filtering, where σ_n^2 is the variance of additive white Gaussian noise (AWGN). The convergence factor $\mu = 0.05$ is employed.

In Fig. 6.9, the MSE curves for MIMO-DFE using BNLMS, SM-BNLMS, SM-NLMS and SM-AP (with projection order 2) as adaption algorithm are plotted. It is observed that, the proposed equalizer offers a better rate of convergence than equalizer employing SM-NLMS algorithm. Further, its convergence is almost similar to MIMO-DFE using BNLMS and SM-AP algorithm of projection order 2.



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

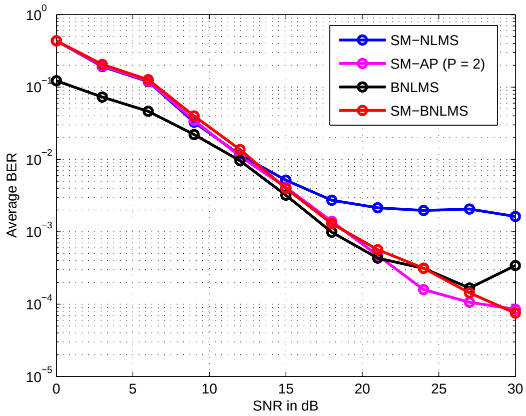
Figure 6.9: Convergence Performance of MIMO-DFE with BNLMS, SM-BNLMS, SM-NLMS and SM-AP (P = 2) Algorithm.

Table 6.9: No. of Updates in MIMO-DFE with BNLMS, SM-BNLMS, SM-NLMS and SM-AP(P = 2) Algorithm.

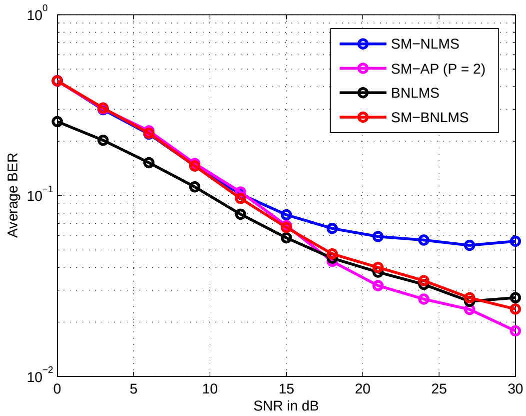
Adaptive Algorithm	Pedestrian-A	Vehicular-B
SM-NLMS	1424	1708
SM-AP (P=2)	1284	1646
SM-BNLMS	1321	1672
BNLMS	2048	2048

The number of updates required in a packet of 2048 symbols for different data-selective algorithms based DFE is shown in the Table 6.9. It is observed that, in comparison to MIMO-DFE using BNLMS algorithm, the proposed equalizer requires 35 % and 18 % fewer computations due to the reduced number of updates in Ped-A and Veh-B environments, respectively. Though this MIMO channel equalizer requires a slightly higher number of updates than MIMO-DFE with SM-AP algorithm of projection order 2, the computational requirement is quite less as discussed in the previous section. Furthermore, this equalizer offers better convergence with a nominal increase in computational complexity as compared to DFE using SM-NLMS algorithm, because of approximately 100 (Ped-A) and 50 (Veh-B) fewer numbers of weight updating operations.

BER performance of MIMO-DFE with different adaptive algorithms are shown in Fig. 6.10. It is observed that: (a) performance of the proposed equalizer is almost similar to one with SM-AP algorithm with projection order 2, and (b) although equalizer using SM-NLMS algorithm has identical



(a) Pedestrian Test Environment-A



(b) Vehicular Test Environment-B

Figure 6.10: BER Performance of MIMO-DFE with BNLMS, SM-BNLMS, SM-NLMS and SM-AP (P = 2) Algorithm.

performance in low SNR region, it deteriorates significantly for SNR values greater than 15 dB. It is further noticed that, BNLMS algorithm based MIMO-DFE provides slightly better performance for SNR less than 12 dB due to absence of SM filtering, but for higher SNR, the performance of the two schemes are almost similar.

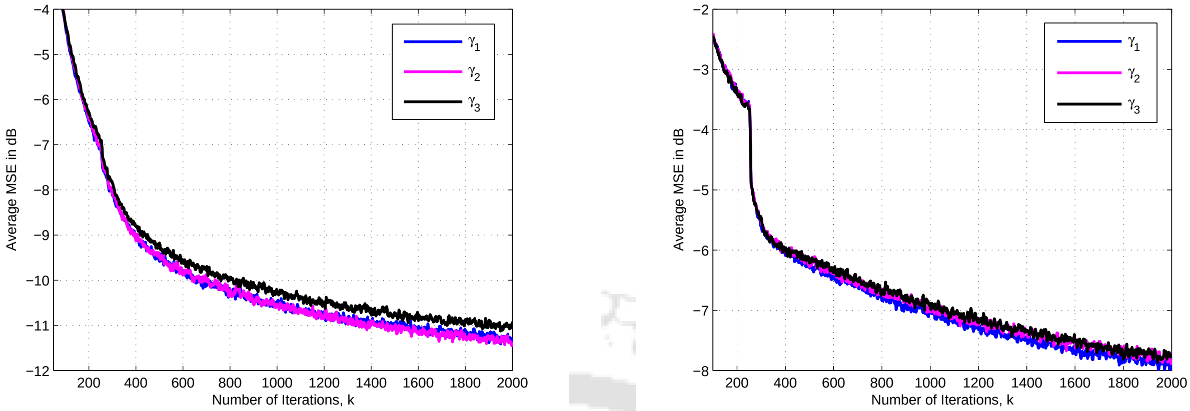
6.3.2.2 Performance of SM-BNLMS Algorithm based MIMO-DFE with Different Values of Upper Bounds on Estimation Error

In this simulation study, the performance of the proposed MIMO channel equalizer with the different threshold value γ on the estimation error is analyzed. Simulations are carried out for γ values as $\gamma_1 = \sqrt{0.7}\sigma_n$, $\gamma_2 = \sqrt{1.5}\sigma_n$ and $\gamma_3 = \sqrt{5}\sigma_n$, with convergence factor μ set to 0.05.

Fig. 6.11 shows the convergence performance of SM-BNLMS algorithm based MIMO-DFE for different γ values. It is observed that, the performance are almost similar for γ values $\sqrt{0.7}\sigma_n$ and $\sqrt{1.5}\sigma_n$ in both in Ped-A and Veh-B environments, but degraded for $\sqrt{5}\sigma_n$.

Table 6.10 displays the percentage reduction in the number of updates with different threshold values in estimation error, for this method. It is seen that, increase in γ value results in a decrease in the number of weight updating operations.

It is observed in Fig. 6.12 that, in the low SNR region, the BER performance deteriorates with an increase in γ; however, BER is almost same for SNR above 15 dB in both Ped-A and Veh-B environments. Hence, the γ value within the range $\sqrt{1.5}\sigma_n \leqslant \gamma \leqslant \sqrt{3}\sigma_n$, is found to be suitable considering the complexity as well as BER performance.

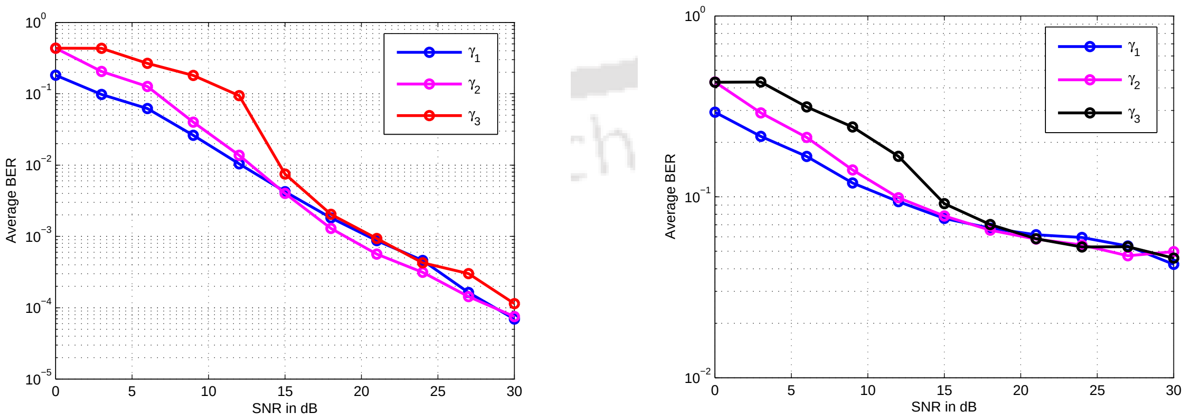


(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 6.11: MSE Performance of SM-BNLMS Algorithm based MIMO-DFE with Different γ values.

Table 6.10: Percentage Reduction in Number of Updates in SM-BNLMS Algorithm based MIMO-DFE with Different γ Values.

Threshold Value on Estimation Error	Percentage Reduction in Updates	
	Pedestrian-A	Vehicular-B
$\sqrt{0.7}\sigma_n$	23.7%	12.4%
$\sqrt{1.5}\sigma_n$	35%	18.4%
$\sqrt{2.3}\sigma_n$	40%	22.2%
$\sqrt{3}\sigma_n$	44.3%	25.3%
$\sqrt{5}\sigma_n$	53.3%	32%



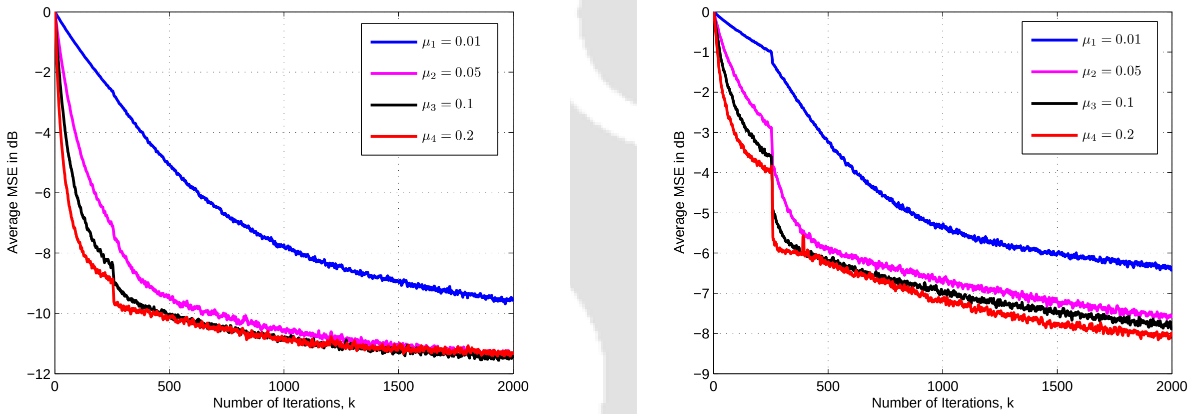
(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 6.12: BER Performance of the Proposed MIMO-DFE with Different γ Values.

6.3.2.3 Performance of SM-BNLMS Algorithm based MIMO-DFE with Different Values of Convergence Factor

In this study, the performance of the proposed equalizer is investigated for different values of convergence factor μ. The simulation are carried out for μ values as $\mu_1 = 0.01$, $\mu_2 = 0.05$, $\mu_3 = 0.1$ and $\mu_4 = 0.2$ with γ value as $\sqrt{1.5}\sigma_n$.

The MSE results are given in Fig. 6.13. As seen from the figure that, increase in μ improves the convergence performance, but for μ value greater than 0.1, the performance degrades due to instantaneous error terms. Table 6.11 shows that, increase in step size μ, results in reduction of number of updates.



(a) Pedestrian Test Environment-A

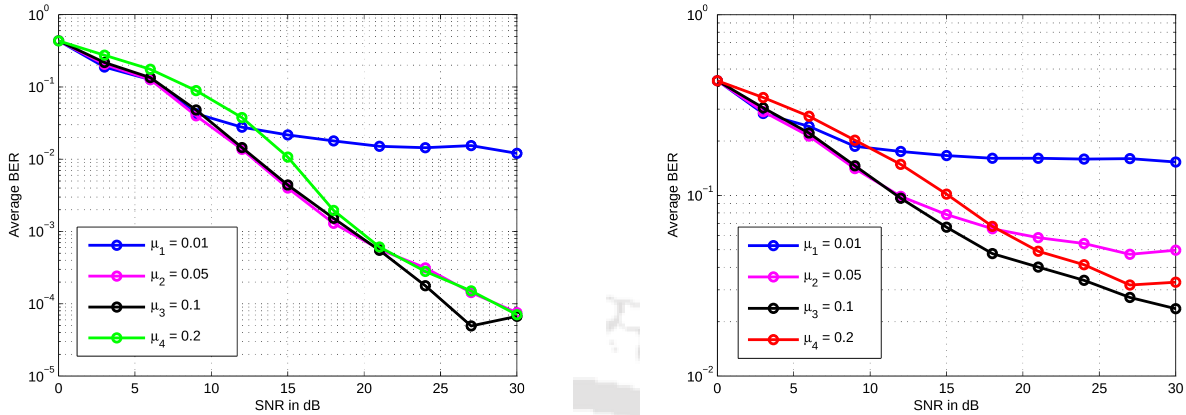
(b) Vehicular Test Environment-B

Figure 6.13: MSE Performance of SM-BNLMS Algorithm based MIMO-DFE with Different Values of Convergence Factor.

Table 6.11: Number of updates in SM-BNLMS Algorithm based MIMO-DFE with Different Values of Convergence Factor.

Convergence Factor μ	Pedestrian-A	Vehicular-B
0.01	1671	1828
0.05	1321	1672
0.1	1301	1628
0.2	1264	1592

The BER performance of SM-BNLMS algorithm based MIMO-DFE with different μ values are shown in Fig. 6.14. It may be seen that, BER performance improves for μ value less than or equal to 0.1, but degrades for μ value 0.2. Higher values of μ increases the fluctuations in instantaneous error



(a) Pedestrian Test Environment-A (b) Vehicular Test Environment-B

Figure 6.14: BER Performance of SM-BNLMS Algorithm based MIMO-DFE with Different Values of Convergence Factor.

terms, and it may be attributed to the poor BER performance. Hence, step size μ value is kept within the range of $0.05 \leqslant \mu \leqslant 0.1$ to achieve better performance in terms of BER and number of updates.

6.4 Summary

In this chapter, the performance of adaptive MIMO-DFE with data-selective SM-AP and SM-BNLMS as adaption algorithms in a low delay spread Ped-A environment as well as medium delay spread Veh-B environment are investigated. The convergence as well as BER performances as a function of upper bound on estimation error, convergence factor and projection order (in case of SM-AP algorithm) were shown in simulation results. Simulation have also been performed for SM-NLMS algorithm based MIMO-DFE system.

The performance of adaptive MIMO-DFE using SM-AP algorithm is found to have similar as that of an AP algorithm based MIMO equalizer but with substantial reduction in computational requirement. Further, its performance is better than SM-NLMS algorithm based MIMO-DFE. Moreover, it is shown that for a given filter length, the choice of projection order P, upper bound γ and convergence factor μ are important, which reflects the performance as well as complexity.

The SM-BNLMS algorithm based MIMO-DFE discussed in this chapter is found to, (i) perform well in low delay spread Ped-A environment, with performance degrading in Veh-B environment, (ii) provide better convergence, tracking capability and BER performance with marginally higher complexity than SM-NLMS algorithm based MIMO-DFE, (iii) have almost similar performance as that of an adaptive MIMO-DFE with SM-AP algorithm of projection order 2, but with much fewer computational requirement, (iv) provide more than 35% reduction in filter weight updating operations with almost same convergence speed as well as BER compared to data-reusing BNLMS algorithm based MIMO channel equalizer, and (v) have good overall performance in terms of convergence, low residual error, better tracking capability, BER and computational complexity for a γ value in the range of $\sqrt{1.5}\sigma_n \leqslant \gamma \leqslant \sqrt{3}\sigma_n$ and for a step size μ within the range of $0.05 \leqslant \mu \leqslant 0.1$. With good convergence rates and a low implementation complexity, the proposed equalizer is expected to be useful for hand-held devices of MIMO wireless system operating in an indoor or pedestrian environment experiencing time-varying and frequency selective fading.

7

Summary and Conclusions

Contents

Objective

In this chapter, we give a summary of the work presented in this thesis and provides the possible future directions for research based on the outcome of the thesis.

7.1 Summary

The work carried out in this thesis primarily focused on reduced complexity equalization for frequency selective channels. Adaptive equalization is a key component in wireless communication to mitigate the effect of ISI and track the channel variation, especially in a time-varying environment. It requires an adaptive filtering algorithm, which is to be chosen considering its convergence speed, computational complexity and misadjustment. For practical wireless receivers, training-data may be limited, hence the adaptive filtering algorithm having a fast convergence speed is necessary for performing the task of equalization. Further, we know that in wireless communication applications, especially for mobile hand-held devices; low computational complexity is desirable to reduce power consumption and enhance battery life.

This thesis work concentrated on decision feedback equalizers based on data-reusing and data-selective algorithms, to investigate their suitability in performing channel equalization with reduced computational complexity at the same time achieving reasonably fast convergence speed and maintaining adequate performance. In this thesis work equalizer performance have been evaluated for channels having low delay spread with low Doppler shift as well as channels having medium delay spread with large Doppler shift. ITU-R recommended channel model has been considered. Summary of the works reported in this thesis are presented in the remaining part of this section.

The channel equalization technique is introduced, the state-of the art literature in this area is discussed, and the problem has been formulated in Chapter 1. An overview of impairments in time-varying wireless communication channels, and adaptive equalizers to mitigate the effect of ISI and track the channel variation has been presented in Chapter 2. A generic MIMO system and channel model adopted in this work has also been discussed.

In Chapter 3, DFE with data-reusing AP and BNLMS algorithms for adaptively updating the filter weights for time dispersive channels have been presented. This equalizer has been found to be suitable in terms of performance and complexity, preferably in indoor and pedestrian scenarios. In Chapter 4, the data-selective SM-NLMS, SM-AP and SM-BNLMS algorithms have been discussed. The reduced complexity channel equalization schemes, with SM-AP and SM-BNLMS as adaption algorithms, have been proposed.

The time-domain adaptive MIMO-DFE based on AP and BNLMS algorithm have been presented in Chapter 5. The performance of the equalizer has been investigated throgh simulation for MIMO receiver in a multi-path fading environment as experienced in the indoor and pedestrian environment also for channels having higher delay and Doppler spread. The convergence issues, BER performance, computational complexity issue and tracking capabilities have been examined through computer simulations.

In Chapter 6, low complexity MIMO channel equalization technique, suitable for hand-held devices in an indoor and pedestrian environment, has been proposed. The equalizers are based on data selective AP and BNLMS algorithms. It is observed that, this channel equalization scheme has almost similar performance respectively, as data-reusing AP and BNLMS algorithm based adaptive MIMO-DFE but with a significant reduction in the computational load due to data-selective updates. Further, it has been found to offer better BER and convergence performance compared to SM-NLMS algorithm based equalizer with a nominal increase in the computational load.

7.2 Contributions

The major contributions of the research work reported in this thesis includes:

- Investigated channel equalization performances of data-reusing algorithm based adaptive decision feedback equalizer in a time-varying wireless channel.

- Proposed channel equalization techniques using data-selective 'Set-membership Affine Projection Algorithm' and 'Set-membership Binormalized LMS Algorithm' for weight updating of DFE. We have modified the SM-AP and SM-BNLMS algorithms by introducing a convergence factor μ to achieve a trade-off between final misadjustment and convergence speed. An analytical derivation is carried out for the range of convergence factor μ for the SM-BNLMS algorithm.

- Evaluated performance of MIMO-DFE employing data-reusing 'Affine Projection' and 'Binormalized LMS' algorithm for MIMO wireless channel.

- Reduced complexity MIMO channel equalization technique, suitable for hand-held devices in an indoor and pedestrian environment based on SM-AP and SM-BNLMS algorithm is proposed.

7.3 Directions for Future Work

In this section, we will provide some suggestions for further research.

(i) Steady-state MSE of an adaptive algorithm decides the tracking capability, which is important in a time-varying communication. Hence, a detailed steady-state MSE analysis for SM-AP and SM-BNLMS algorithms when implemented in a DFE, is proposed as a future extension of the studies reported in this thesis.

(ii) Indoor wireless communication, large scale deployment of micro and pico base stations and use of MIMO are some of the emerging trends. In such scenario, the smaller cell size and rapid frequency reuse gives rise to co-channel interference (CCI). The performance evaluation of the of equalizers discussed in this thesis in a CCI environment can be another logical extension of this work.

(iii) A linearly-constrained version of SM-AP algorithm would be of interest due to its convergence speed and low computational complexity. Exploring the channel equalization using such algorithm will be an interesting extension of our work.

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List of Publications

Journal Publication

1. Mishra R. C., Bhattacharjee R., "Performance Analysis of Adaptive DFE using Set-membership Binormalized Data-reusing LMS Algorithm for Frequency Selective MIMO Channels," AEU-International Journal of Electronics and Communications, Volume 77, Pages 91-99, 2017.

Conference Publications

1. Mishra R. C., Bhattacharjee R., "Decision Feedback Equalizer with Set-membership Affine Projection Algorithm for Frequency Selective MIMO Channels," Fourth International Conference on Signal Processing and Integrated Networks, Delhi, India, Feb. 2017.

2. Mishra R. C., Bhattacharjee R., "Adaptive Decision Feedback Equalizer with Set-membership Affine Projection Algorithm," The Eleventh International Conference on Industrial and Information Systems (ICIIS-2016), IIT Roorkee, India, December 2016.

3. Mishra R. C., Bhattacharjee R., Mitra A., "An affine projection algorithm based DFE with set-membership filtering," 5th IEEE Int. Conf. on Computers and Devices for Commun. (CODEC); pp. 1-4, Dec. 2012.

4. Mishra R. C., Bhattacharjee R., Mitra A., "MIMO channel equalization using set-membership affine projection algorithm," TENCON 2012 IEEE Region 10 Conference;1-5, 19-22 Nov. 2012.

5. Mishra R. C., Mitra A., "Affine projection algorithm based DFE for MIMO systems," NCSIP - 2012, Gandhigram, India, Feb. 2012.