INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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Programme of Study	:	Ph.D.
Thesis Title:		A Study of Class Number of Real Quadratic and Cubic Fields
Name of Thesis Supervisor(s)	:	Prof. Anupam Saikia
Thesis Submitted to the Department/ Center	:	Department of Mathematics
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SHORT ABSTRACT

The primary goal of the thesis is to study class number of the ring of integers of a number field and related arithmetical properties.

The first part of the thesis provides a solution to a classical problem posed by Dirichlet. Dirichlet asked whether exist infinitely many real quadratic fields with 1 as relative class number. It has been shown in the thesis that a real quadratic field will always have 1 as relative class number for the conductor 3, depending om certain conditions on the field . The thesis also provides a necessary and sufficient condition for a real quadratic field to have relative class number 1. Moreover, the thesis contains significant generalization of the continued fraction approach of A. Furness and E. A. Parker towards relative class number. Using this approach, the thesis also shows the existence of another infinite family of real quadratic fields with relative class number 1 with an odd prime dividing the discriminant as conductor.

The thesis also shows a simple relation between the fundamental unit of a real cubic field and its class number. It shows that the fundamental unit of a real cubic field must satisfy certain congruences if the class number is not divisible by 3. As a consequence of the methods employed, one can obtain classes of real cubic fields of certain forms which have class number not divisible by 3. The latter is in agreement with a classical result of F. Gerth stated in terms of the discriminant of the cubic fields. The approach also yields an elementary proof for certain congruence properties satisfied by the fundamental unit of a real quadratic field of odd class number which were recently proved by Z. Zhe and Q. Yue.

The final part of the thesis gives a construction for an unramified quadratic extension of a biquadratic field from a non-torsion point on a suitable elliptic curve. Using the duplication formula, this result also extends the construction for an infinite family of biquadratic fields starting from a non-torsion point on the curve. In particular, every field of this biquadratic family has even class number.