Study of Modulation Instability and Solitary Waves in Nonlinear Optical Systems

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Supervisor

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Certificate

It is certified that the work contained in the thesis entitled "***Study of Modultion Instability and Solitary Waves in Nonlinear Optical Systems***" by Miss Manirupa Saha, a Ph.D student of the Department of Physics, Indian Institute of Technology Guwahati has been carried out under my supervision and this work has not been submitted elsewhere for award of any degree.

Dr. Amarendra Kumar Sarma

This thesis is dedicated to my parents

Mr. Sunil Kumar Saha (Bapi)

&

Mrs. Menaka Saha (Maa)

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Abstract

Modulation instability (MI) is a well known and most ubiquitous and widespread type of instability that appears in most nonlinear systems. In Physics, it appears in many branches of physics such as hydrodynamics, plasma physics, and electrodynamics, low temperature physics, and quite obviously in nonlinear optics. One of the most fascinating and important manifestations of the phenomenon of modulation instability is the solitary waves, commonly called solitons. Solitons are robust, localized travelling waves of permanent form. They are recognized as the modes of a nonlinear system. This thesis is primarily devoted towards study of moduational instability and solitary waves in the following nonlinear optical systems: nonlinear negative index metamaterials, non-Kerr media exhibiting power law nonlinearity and very briefly, silicon waveguides. The recent emergence of the so-called negative refractive index metamaterials (NIM) or simply known as metamaterials (MM) have made nonlinear optics research a very useful and exciting activity. This thesis proposes a new generalized coupled nonlinear field equations for pulse propagation in MM embedded into a Kerr medium. The model successfully recovers previously proposed models by other authors. Moreover, it contains some additional terms like magnetic self-steepening effect connecting both the electric and the magnetic field envelopes. MI analysis is carried out using this new model and is extended later to the case of an MM embedded into a medium with cubic-quintic nonlinearity. The generalized nonlinear Schrodinger equation, appropriate to model nonlinear pulse propagation in non-Kerr cubic quintic media is integrated to find the exact solitary wave solutions. The parameter domain restrictions have also been identified in the process of obtaining these solutions. The MI analysis is also carried out in this media and the effect of higher order dispersion and higher order nonlinearity on MI gain spectrum is discussed. The exact dark soliton solutions to the generalized nonlinear Schrödinger equation with coefficients dependent on the evolution parameters are obtained. Finally, a numerical investigation of solitary wave propagation in a silicon waveguide is reported.

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Chapter 1

Introduction

1.1 Background

Modulation instability (MI) is a well known and most ubiquitous and widespread type of instability that appears in most nonlinear systems [1-2]. In fact, a search on the Internet with the term 'Modulation instability' returns millions of hits! In Physics, it appears in many branches of physics such as hydrodynamics, plasma physics, and electrodynamics, low temperature physics, and quite obviously in nonlinear optics [2-15]. It is very difficult to say who started MI research first. However, Benjamin and Feir was the first to observe the MI process on deep water waves in 1967 [6-7]. Since then MI is also sometimes known as Benjamin-Feir instability. MI has been experimentally observed for electromagnetic waves in radio-wave signal by Zagryadskaya and Ostrovsky in 1969 [8]. Later, numerous studies on MI has been carried out in the field of nonlinear optics [2,14-22] In optics, the interest in MI stems from its possible applications and relevance in ultrafast pulse generation with high repetition rate, ultra-broadband super continuum generation etc.[15-19]. MI is useful to generate ultrashort pulses whose repetition rate could be externally controlled. Nowadays MI is used as a technique to generate ultra-short pulses with repetition rate higher than those attainable from mode locked-laser [2]. Also, MI is now widely recognized as the precursor of soliton formation. In the context of nonlinear fiber optics, the possibility of MI was predicted theoretically in optical fiber by Hasegawa in 1984 [20] and have been experimentally verified by Tai et al [21-22]. In general, MI occurs in the anomalous group velocity dispersion (GVD) regime for a focusing nonlinearity [2]. MI is possible in the normal GVD regime also but it is limited to some special cases only [23-27]. It is interesting to know that, recently, the study of MI process has even been extended to oceanography, to understand the phenomena behind the generation of the so-called rogue waves [28-31].

The essential physics behind MI could be understood as follows [32]. Let us consider a periodic wave train propagating, say in a cubic nonlinear media, along the z-direction represented by: $A = a_0\cos(k_0 z - \omega_0 t)$ where a_0 is the amplitude, k_0 and ω_0 are the fundamental wave number and frequency respectively. Now, a weak disturbance is introduced on it which consists of two waves, lower and upper sidebands, represented by $a_1\cos(k_1 z - \omega_1 t)$ and $a_2\cos(k_2 z - \omega_2 t)$ respectively with $a_1, a_2 << a_0$. The lower side band has wave number $k_1 = (1-\eta)k_0$ and frequency $\omega_1 = (1-\delta)\omega_0$ and an upper sideband has wave number $k_2 = (1+\eta)k_0$ and frequency $\omega_2 = (1+\delta)\omega_0$. η and δ are very small parameters, such that $\eta << 1$, $\delta << 1$. In a linear situation, i.e. when a_0 is small, the modes within the wave envelope would only spread out owing to dispersion. But when the amplitude is sufficiently large the modes undergo nonlinear interaction due to cubic nonlinearity. This would result in the generation of harmonics of the fundamental wave with phase $2(k_0 z - \omega_0 t), 3(k_0 z - \omega_0 t)$,........ etc. Now mixing of the harmonics with one another may take place. For example, a second harmonic wave, say $a_0^2 \cos 2(k_0 z - \omega_0 t)$, may interact with the lower sideband or upper sideband to give $a_0^2 a_1 \cos(k_2 z - \omega_2 t)$ or $a_0^2 a_2 \cos(k_1 z - \omega_1 t)$. Clearly, a mutual reinforcement of each sideband modes and second harmonic wave takes place triggering a kind of chain reaction! The resonant forcing effect felt by one side band (say, lower) increases the amplitude of the other side band (say, upper) and in this way, the two amplitudes of the weak perturbations grows exponentially due to nonlinear interaction. As time goes on, the fundamental wave train becomes unstable and which is known as MI. It should be noted that the dispersion relation for the resonant mechanism to occur is $k_1 + k_2 = 2k_0$ and $\omega_1 + \omega_2 = 2\omega_0$. This process is similar to the so-called four wave mixing (FWM) process in nonlinear optics [33]. Fig.1.1 explains this close analogy between FWM and MI process.

The fundamental wave with frequency ω_0 interact with the probe wave at frequency $\omega_0 + \delta$ and owing to nonlinear wave mixing process it generates new idler frequency $\omega_0 - \delta$ as shown in Fig.1.1 . Physically, two quanta at ω_0 interacts with another quanta at $\omega_0 + \delta$ to generate a quanta at $\omega_0 - \delta$ due to FWM process. Again, the generated quanta at $\omega_0 - \delta$ may interact through nonlinear process to create a quanta at $\omega_0 + \delta$. Depending upon the type of perturbation i.e. whether the perturbation is induced from outside or from inside the medium, say due to noise, MI is categorized as induced or spontaneous MI respectively.

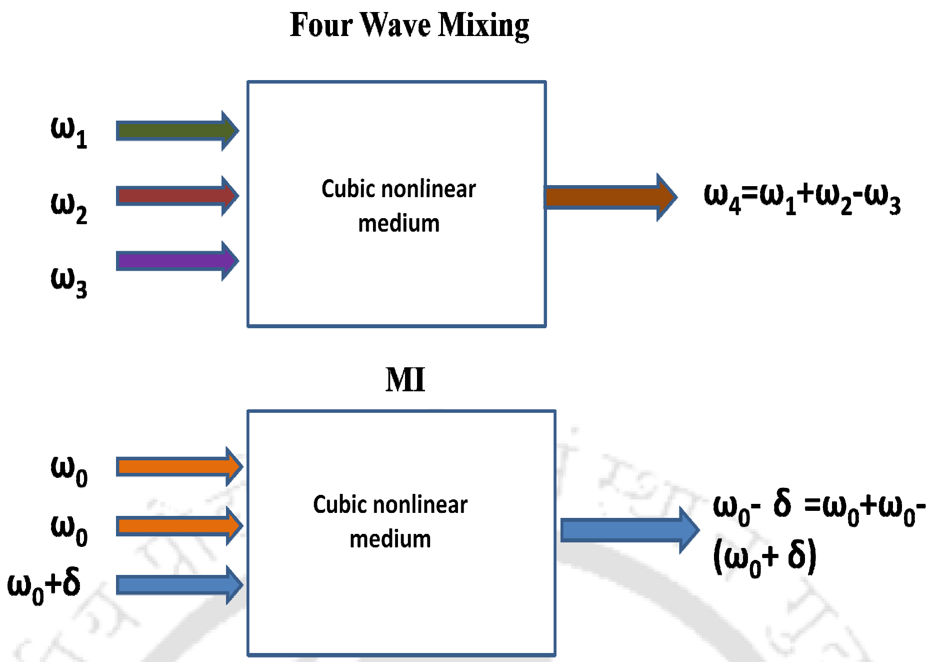

Fig1.1: Schematic diagram of Four wave mixing (FWM) and MI process.

One of the fascinating manifestations of the phenomenon of modulation instability is the solitary waves, commonly called solitons [34]. Solitons are universal in nature in the sense that, they exist in the sky as density waves in spiral galaxies; they exist in the ocean as waves bombarding oil wells; they exist in laser pulses propagating in solids, magnetic systems, liquid crystals, polymers, as well as elementary particles etc. [35-41]. Solitons are recognized as the modes of a nonlinear system. Hence, solitons appear in any branch of science and mathematics which discuss nonlinearity [42-43]. Solitons are robust, localized travelling waves of permanent form. Mathematically, they are the solutions of some nonlinear partial differential equations with the following properties: solitons are spatially localized; a single soliton is a travelling wave; they are stable; when a single soliton collides with another one, both of them retain identities after collision [34-35]. The last property is related to the integrability of the system and appears very rarely in real physical systems. Mathematicians insist that all the four properties should be there for a solution to be called soliton. However, a solitary wave may be defined more generally than a soliton. Any solution of a nonlinear system which represents a hump-shaped wave of permanent form, whether it is a soliton or not can be termed as a solitary wave. In this sense, in physics it is preferable to use the word 'solitary wave', rather than 'soliton'. However, nowadays it is common, especially in physics to use the word 'soliton'. And we are also no exception to this! In this thesis we would discuss solitons primarily in the context of nonlinear optics [45-47]; however, some of the results may be quite general. The term 'soliton' was introduced in 1965 [44], but the scientific research of solitons had started way back in 1834 when John Scott-Russell observed a large solitary wave propagating at a constant speed without changing its shape over a long distance in a canal near Edinburgh [48]. In the days of Scott Russell, there was much debate from the leading scientific scholars of the day regarding the very existence of this kind of solitary waves. In 1845, Airy’s nonlinear shallow-water wave theory predicted that a wave with elevation of finite amplitude cannot propagate without change of its form i.e. solitary waves could not exist [49]. Later on in 1849, Stokes showed that it is possible to have solitary waves with finite amplitude and permanent shape in deep water and they are periodic wave trains [50]. Then it was followed by water tank experiments by H. Brazin [51] and the theoretical developments by Joseph Boussinesq in 1871 and Lord Rayleigh in 1876 independently[52,53]. In 1895, Korteweg and de Vries derived a mathematical model equation, popularly known as the KdV equation, which describes the unidirectional propagation of shallow water waves [54]. The KdV equation successfully explained the 1834 observations of J. S. Russel. Though the existence of solitary waves were proved beyond doubt, not much progress have been made until the 1960s when N. Zabusky and M. Kruskal numerically re-investigated the KdV equation and discovered the elastic collision between KdV solitary waves [44]. It is worth noting that Zabusky and Kruskal were motivated to study the KdV equation numerically by the so-called Fermi, Pasta and Ulam (FPU) problem [55]. In 1967, Gardner et al obtained the analytical solution of the KdV equation for the localized solitary waves by using the idea of Inverse Scattering method and their results agreed well with the experimental results obtained by John Scott Russell [56]. This pioneering work triggered unprecedented burst of research activities on nonlinear waves and which is continuing till today. One year later, Lax generalized their results [57] and in 1972, Zakharov and Shabat exactly solved the nonlinear evolution equation known as nonlinear Schrodinger equation (NLS) for weakly nonlinear deep water waves by the same method and they termed the solutions as envelope soliton [58]. In the context of nonlinear optics, Hasegawa and Tappert in 1973 showed theoretically the existence of optical solitons inside an optical fiber [59] where the propagation of light is modeled by the NLS equation and it was experimentally observed by Mollenauer et al in 1980[60]. Since then optical solitons have found practical applications in long distance communicational systems, producing short pulse lasers, pulse compression technique, electronic devices like optical switching and optical logic gates etc [61-66].

Optical solitons are classified as temporal or spatial depending on the confinement of light occurs in time or space during wave propagation. Optical solitons originate from the delicate balance between the dispersion effect and the nonlinear effect, manifesting through the so-called optical Kerr effect [2, 41]. The Kerr effect leads to the intensity dependent refractive index, which results in self-phase modulation (SPM) of an optical pulse. As a result of the time varying nonlinear phase shift, the instantaneous frequency becomes time dependent as $\omega_{ins} = \omega_0 + (d\phi_{NL}/dt)$ with $\phi_{NL} = n_2 I z$. ω_0 is the carrier frequency, I is the intensity of the pulse, z is the distance of propagation and n_2 is the nonlinear-index coefficient of the medium [2]. For a medium with $n_2 > 0$, due to SPM the leading edge of the pulse get red shifted while the trailing edge gets blue shifted. This chirping effect could be exactly cancelled if the medium exhibits anomalous group velocity dispersion (GVD) and an appropriately shaped optical pulse is used. In the presence of anomalous dispersion, higher frequency components of a pulse travels faster and lower ones travel slower, resulting in blue-shift at the leading edge and red-shift in the trailing edge. Thus the effects of both anomalous group velocity dispersion and nonlinearity could be neutralized to establish undistorted propagation of optical pulse with special pulse shape. Such special optical pulses are termed as bright optical solitons [59]. It is interesting to note that in some systems, showing normal GVD, an absence of energy could behave like a soliton, called dark solitons. These darks solitons appears as a region of reduced amplitude set against a continuous background. Dark solitons have better stability in presence of perturbation like noise, mutual interaction between two pulses etc. The dark soliton has been observed experimentally first in optical fiber in 1988[67]. Very recently, for the first time dark soliton has been experimentally observed on the surface of water also [68]. The shape and width of these water-wave dark solitons depend on the water depth, carrier frequency, and the amplitude of the background wave. This recent experimental result excellently matches the theoretical results as predicted by the nonlinear Schrodinger (NLS) equation. As discussed above, temporal soliton is formed when SPM balances the temporal broadening of the pulse. On the other hand, spatial soliton represents self-guided beam that remain confined in the transverse direction orthogonal to the direction of wave propagation. A spatial soliton is formed when the natural diffraction induced spreading balances the self-focusing, arising due to nonlinearity, of an optical beam.

The nonlinear Schrödinger equation (NLSE) is used to describe the pulse propagation in an optical medium and is written in the following form:

$$i\frac{\partial A}{\partial z}\mp\frac{\beta_2}{2}\frac{\partial^2 A}{\partial T^2}+\gamma|A|^2 A=0 \tag{1.1}$$

Here A is the pulse envelope, γ is the nonlinear parameter arising due to the intensity dependent refractive index, z is the propagation distance and T is the evolution time. This equation explains the propagation of temporal solitons; while the same equation, with an interchange of the z and T variables, could also be used to describe spatial solitons. Bright solitons are the solution of the NLS equation in the anomalous GVD regime ($\beta_2 < 0$). In the normal dispersion regime with $\beta_2 > 0$, the corresponding solution of NLSE, supports the so-called dark solitons.

It is worthwhile to mention that the NLSE described Eq. (1.1) cannot be used to model propagation of optical pulse with pulse width $T_0 < 1ps$ [2]. For such short optical pulses, the nonlinear effects like self-steepening, intrapulse Raman scattering etc. become extremely relevant [69-72]. Also one must have to consider higher order dispersion effects. Hence, a more generalized evolution equation for pulse propagation, known as the generalized nonlinear Schrodinger equation (GNLSE) is used. In normalized units, it is given by:

$$i\,q_Z + a\,q_{tt} + b|q|^2 q = i\,\alpha q - i\,\delta\,q_{ttt} + i\,\lambda(|q|^2 q)_t + i\nu\left(|q|^2\right)_t q \tag{1.2}$$

Here $q(z,t)$ is the pulse envelope, z is the normalized distance along the fiber and t is the normalized time with the frame of reference moving along the fiber at the group velocity. The co-efficient $a, b, \alpha, \lambda, \nu$ and δ represent GVD, SPM, self-steepening , self-frequency shift due to intrapulse Raman scattering and third order dispersion (TOD) respectively in the normalized units. A brief outline of some of these various effects may be useful.

The self-steepening effect arises owing to the intensity dependence of the group velocity. Physically it represents for $n_2 > 0$, the peak of the pulse is slowed down more than the edges of the pulse and this leads to the progressive steepening of the trailing edge of the pulse. It introduces asymmetry in the SPM broadened pulse spectrum. The self-steepening effect would continue until the pulse develops a region where the temporal intensity becomes discontinuous and form an optical shock front. A schematic is shown in Fig. 1.2 to illustrate the effect of SS on the pulse envelope.

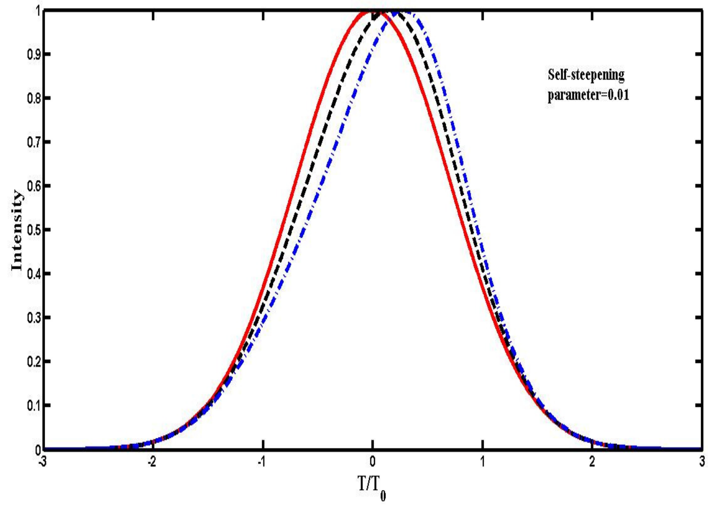

Fig 1.2: Self-steepening of a Gaussian pulse without dispersion effect.

The last term on the RHS of Eq.(1.2) incorporates the so-called intra-pulse Raman scattering. For pulses with a wide spectrum and $T_0 < 1fs$, the Raman gain can amplify the low-frequency components of a pulse by transferring energy from the high-frequency components of the same pulse. As a result of it, the pulse spectrum shifts toward the low-frequency (redshifted) side as the pulse propagates inside a medium; this phenomenon is referred to as the self-frequency shift also [73-74]. The physical origin of this effect is related to the delayed nature of the Raman (vibrational) response. On the other hand, the TOD results owing to the frequency dependence of the GVD parameter. TOD generally distorts the pulse such that it become asymmetric and oscillations occur near the trailing edge or leading edge depending on the sign of the TOD parameter. When the pulse width $T_0 < 0.01ps$, the fourth order dispersion (FOD) term also become relevant. The exact solitary wave solutions of the generalized NLSE are not easy to find because of the non integrable nature of the equation. But, in recent years a lot of research effort is made for various reasons to get the exact solitary wave solutions of Eq. (1.2) [75-79].

The Kerr effect is not the only nonlinear effect that produces the optical solitons. Solitons have been reported in media lacking inversion symmetry, such as lithium niobate or $\beta-$barium-borate, also [80]. Even in systems where the source of nonlinearity is the so called photon-phonon interactions, solitons have been observed [81]. In this thesis, we would be interested in the so called power-law nonlinearity and cubic-quintic nonlinearities [41, 82]. Power-law nonlinearity is exhibited in various materials including semiconductors. This law also occurs in a media for which higher order photon processes dominate at different intensities. Moreover in nonlinear plasmas, the power law solves the problem of small-K condensation in weak turbulence theory [82-83]. For power law the refractive index is given by $n = n_0 + n_2 |E|^{2p}$, n_0 is the linear refractive index and n_2 is the nonlinear refractive index arising due to the third order polarization. On the other hand, for cubic-quintic nonlinearity the refractive index is given by: $n = n_0 + n_2 |E|^2 + n_4 |E|^4$ with $n_2 = 3\chi^{(3)}/8n_0$ and $n_4 = 3\chi^{(5)}/16n_0$. Here n_2 and n_4 represents the nonlinear refractive indexes arising due to the third and fifth order nonlinear susceptibilities respectively. Recently, it was experimentally observed that CdS_xSe_{1-x} doped glasses shows a significant amount of fifth order nonlinearity. It was also reported that a considerable amount of fifth order nonlinearity exists in a transparent glass in intense femto second pulses at 620 nm. The organic nonlinear material polydiacetylene para-toluene sulfonate (PTS) is another material which exhibits cubic-quintic nonlinearity [84-86].

1.2 The relevance and the aim of the topic of research

In the given thesis we focus mainly on the following nonlinear optical systems: nonlinear negative index metamaterials, non-Kerr media exhibiting power law nonlinearity and very briefly, silicon waveguides.

Recently, the emergence of a new type of novel artificially structured materials called negative refractive index metamaterials (NIM) or simply known as metamaterials (MM) have made nonlinear optics research a very useful and exciting activity [87]. Study of solitary waves in such novel nonlinear optical systems is extremely challenging and fruitful. *Considering the significance and importance of this newly structured material and its relevance to the present thesis we discuss negative index metamaterials, in some details, in the next section*. Because of the recent experimental realization of MM at infrared and optical frequencies, MM research is now getting a new dimension [88-89]. The possibility of nonlinear MMs is demonstrated by embedding split-ring resonators (SRR) in nonlinear Kerr type dielectrics [90]. In this context, study of linear and nonlinear pulse propagation in MMs is of paramount importance owing to their numerous possible practical applications. Recently, many authors have investigated and proposed various nonlinear pulse propagation models [91-92]. One of our research goals is to study the pulse propagation models more thoroughly. We would derive an appropriate pulse propagation equation, starting with the so called Maxwell's equations, in order to model pulse propagation in a MM embedded into a Kerr and as well as a non-Kerr medium. Also we intend to carry out MI analysis and look for solitary wave solutions. Linear stability analysis technique has been used to carry out the modulation instability analysis [2].

The existence of solitons in Kerr-like media, e.g. optical fiber, is now a well established fact both from theoretical and experimental view point. The NLS equation is routinely used to describe the pulse propagation dynamics in such media. However, it turns out that in many practical situations, due to the non-saturable nature of the Kerr-nonlinearity, it is inadequate to describe the soliton dynamics in non-Kerr media such as silicate and chalcogenide based fibers [82]. Also, non-Kerr nonlinearity becomes more important when higher order nonlinearities and higher order dispersions needs to be considered for femtosecond pulse propagation through optical media [71,93]. In order to realize practical low power optical devices, whose operating principles are based on nonlinear phenomena, materials with higher nonlinearities must be used. The conventional optical fiber (silica) has very low Kerr-nonlinearity. Various dopants are added to fibers to greatly increase their nonlinearities. Recently, it has been experimentally observed that semiconductor doped glasses and chalcogenide fibers exhibit higher order nonlinearity at very low peak power level [94-95]. Motivated by these developments, in this thesis, we have carried out an analysis to study solitary wave solutions in a non-Kerr medium. We would like to emphasize that one of the very important aspects of soliton research is the issue of integrability in the presence of perturbation terms, specifically for non-Kerr law nonlinearity [82]. Generally, these perturbative effects are studied using numerical simulation [96-97]. But exact soliton solutions always offer many useful pieces of information, thereby supplementing the numerics. This is why one of the primary theme in this thesis is to obtain exact solitary wave solutions to the nonlinear evolution equations appropriate to a particular nonlinear optical system. However, there are situations where these methods could not be applied and one needs to rely on numerical methods. One such example would be found in the context of silicon photonics. Silica based fibers are never suitable for applications related to optical devices based on nonlinear phenomena. This is due to the extremely low nonlinear parameter exhibited by silica at low optical power [2]. However due to significant technological advancements in the fabrication of silicon-on-insulator (SOI) waveguides, they are getting considerable attention from the researchers [98-102]. It is to be noted that because of the large values of the Kerr parameter and the tight mode confinement of the optical mode, nonlinear effects are enhanced considerably in SOI waveguides. The formation of optical soliton in an SOI waveguide is reported [103] which may lead to new applications of SOI waveguides related to high-speed optical switching and optical interconnects [104-105].Unlike silica, in silicon based waveguides, the so-called two photon absorption (TPA) would not be neglected [106-107]. In fact the presence of TPA parameter in NLSE makes the equation non-integrable and we need to use numerical methods to investigate solitons in a silicon-based waveguide. In the final chapter of the thesis, we would primarily focus on studying the effect of TPA on soliton propagation in Si waveguide. In order to study solitary waves, we have adopted solitary wave ansatz method [108]. However, analytical methods could not be applied to study the soliton propagation in the context of silicon photonics and we have to rely on the numerical techniques. The well known split-step Fourier method is been used to study the pulse propagation numerically in silicon waveguide [2].

1.3 Negative refractive index metamaterials

Novel electromagnetic properties could be realized by creating an artificially structured composite consisting of well-arranged functional inclusions of subwavelength dimensions. Generally, the unit cell size of such artificial structures are much smaller than the wavelength of interest and the electromagnetic response of such structures are expressed in terms of homogenized, effective material parameters [87]. These artificial, man-made, structures are termed as metamaterials. Although there is no universal definition of the term 'metamaterial', we prefer the following definition put forward by W. Cai and V. Shalaev [109]: "A metamaterial is an artificially structured material which attains its properties from the unit structure rather than the constituent materials. A metamaterial has an inhomogeneity scale that is much smaller than the wavelength of interest, and its electromagnetic response is expressed in terms of homogenized material parameters". One of the most fascinating example of such structures is the so called negative-index metamaterials (NIM). In this thesis, by metamaterials (MM) we would mean negative refractive index metamaterials.

In 1904, H. Lamb for the first time mentioned the possibility of the existence of backward propagating waves [110]. This work remained unnoticed for nearly forty years until the famous Mandelshtam's work [111] was published, in which he predicted and gave a detailed explanation of a surprising occurrence, i.e., the negative refraction of rays. The next considerable step was made by Sivuhin [112]; in this work it was first shown that the group and phase velocities are directed opposite to each other in a medium with simultaneously negative values of permittivity ε and permeability μ.The concept of negative refractive index was studied very systematically by the Russian physicist Victor G. Veselago in 1968 [113]. He started by asking a question on the ambiguity in choosing the sign of refractive index 'n', defined by $n = \pm\sqrt{\varepsilon\mu}$, where ε and μ are permittivity and permeability respectively. For all naturally occurring transparent materials both ε and μ are positive and thus we need to choose the positive sign. Veselago pointed out that if both ε and μ are negative, one must choose the negative sign and write $n = -\sqrt{\varepsilon\mu}$. As at that time and even afterwards, no naturally occurring materials were discovered, this study of Veselago lied as a mere theoretical exercise for a long time. In 2000, a group at University of California led by D. R. Smith demonstrated a composite medium, based on a periodic array of interspaced conducting nonmagnetic split ring resonators and continuous wires, that exhibits a frequency region in the microwave regime with simultaneously negative values of effective permittivity $\varepsilon(\omega)$ and permeability $\mu(\omega)$[114]. This demonstration revived Veselago's idea and gave rise to explosive research activities in the area of negative refractive index metamaterials. Specifically, J. B. Pendry's work on the possibility of realizing a perfect lens [115] added another dimension to the area. Exhibition of various exotic and unusual properties, yet to be found in naturally occurring materials, by MMs at microwave frequencies, inspired the researchers to extend MMs to the terahertz, infrared and visible bands of frequencies [88-89, 116]. Today, the artificial structures exhibiting electromagnetic responses at light frequencies, known as optical metamaterials, are considered to be one of the most fascinating and fruitful area of MMs research.

It is well understood in elementary physics that if either the permittivity ε or the permeability μ is negative, while the other one is positive, then the refractive index is purely imaginary resulting in no propagating waves. If we assume that materials with both $\varepsilon < 0$ and $\mu < 0$ exists, then one can show that propagating waves are possible and one must write $n = -\sqrt{\varepsilon\mu}$ due to causality condition. It may be explained as follows. Let us consider a passive medium with $\varepsilon = -\varepsilon_1 + i\varepsilon_2$ and $\mu = -\mu_1 + i\mu_2$, where $\varepsilon_2 \ll \varepsilon_1$ and $\mu_2 \ll \mu_1$. Then we have

$$n = \pm\sqrt{\varepsilon\mu} = \pm\sqrt{(-\varepsilon_1 + i\varepsilon_2)(-\mu_1 + i\mu_2)}$$
$$\approx \pm\sqrt{\varepsilon_1\mu_1}\left[1 - i\frac{\varepsilon_1\mu_2 + \varepsilon_2\mu_1}{2\varepsilon_1\mu_1}\right] \tag{1.3}$$

Causality requires that the imaginary part of the refractive index must be positive for any passive medium and hence we must choose the negative sign, when the real parts of both ε and μ are negative. One can immediately see some of the consequences of negative refractive index and of the fact that $\varepsilon, \mu < 0$.

Left-handedness:

Let us consider a plane electromagnetic wave propagating through a medium, where the electric field is written as: $\vec{E} = \vec{E}_0 e^{i(\vec{k}.\vec{r}-\omega t)}$ where $\vec{E}_0, \vec{k}$ and ω are electric field amplitude, wave vector and angular frequency of the wave. Maxwell's curl equations $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \vec{\nabla}\times\vec{H} = \frac{\partial\vec{D}}{\partial t}$ take the following form in an isotropic medium: $\vec{k}\times\vec{E} = \omega\mu(\omega)\vec{H}, \vec{k}\times\vec{H} = -\omega\varepsilon(\omega)\vec{E}$. For a common material, $\varepsilon(\omega), \mu(\omega) > 0$ and so the vectors $\vec{E}, \vec{H}$ and $\vec{k}$ form a right-handed coordinate system. Also the wave vector $\vec{k}$ is parallel to the Poynting vector $\vec{S} = \vec{E}\times\vec{H}$. On the other hand, for a negative index material, for which $\varepsilon(\omega), \mu(\omega) < 0$, the vectors $\vec{E}, \vec{H}$ and $\vec{k}$ form a left-handed coordinate system. This is reason NIMs are known as left-handed materials also. It is interesting to note that in

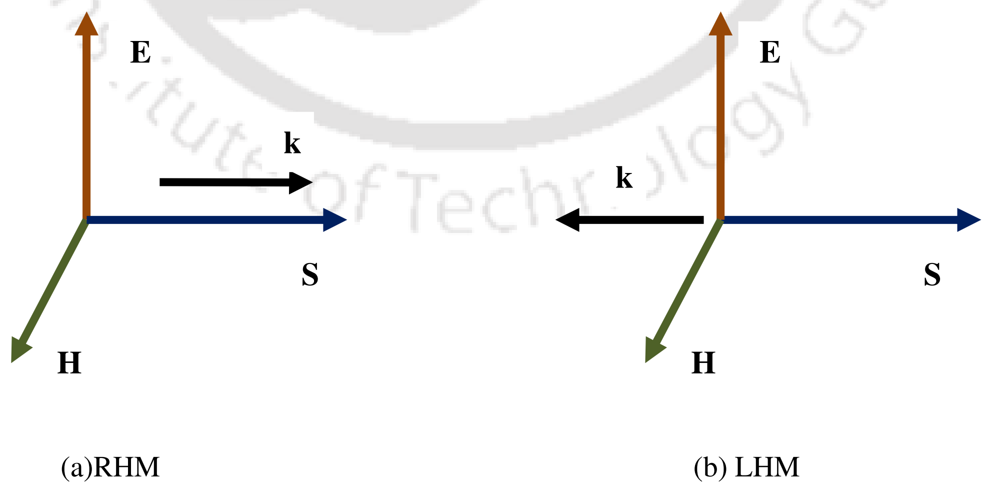

Fig.1.3: Orientation of **E**, **H**, **k** and **S** in (a) Right handed medium (RHM) (b) Left handed medium (LHM).

NIM the wave vector is directed opposite the Poynting vector. This physically means that in MM, the energy propagates opposite to the wave vector direction and the phase is advanced in the propagation direction.

Reversal of Snell's law:

Let us consider a plane wave incident from a positive index medium (i.e. $n > 0$), say medium 1, to a medium with negative index (i.e. $n < 0$), say medium 2. If θ_1 and θ_2 are the angle of incidence and angle refraction respectively, then according to Snell's law we have:

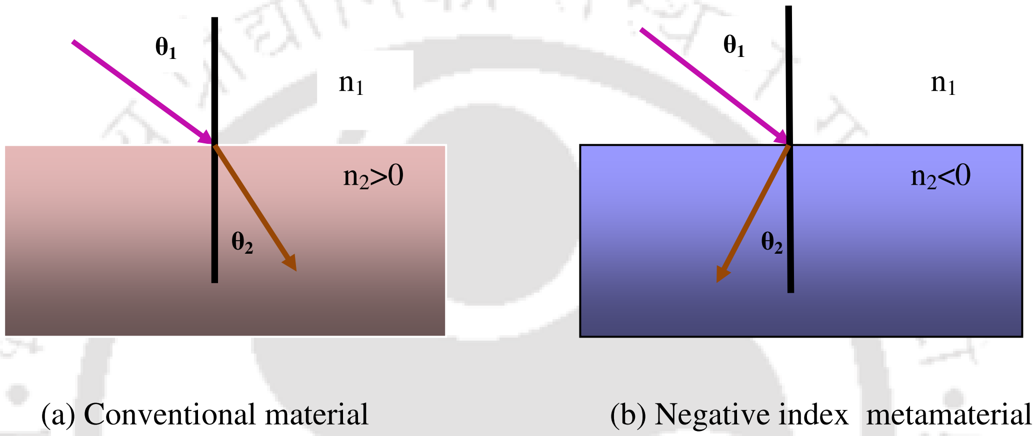

Fig.1.4: Ray diagram of Snell’s law in (a) conventional material (b) negative index metamaterial

$n_1 \sin\theta_1 = n_2 \sin\theta_2$. Now $n_2 < 0$ implies that we must have $\sin\theta_2 < 0$, which in turn implies that the refracted ray makes a negative angle with respect to the normal to the interface. Physically speaking, the refracted beam is bent at the same side of the normal to the interface. Reversal of Snell's law in MM is already demonstrated experimentally in the microwave frequency domain [117].

Apart from the above two phenomena, many interesting phenomena such as reversal of the Doppler effect [118-119], reversal of Cerenkov radiation [120], reversal of Goos-Hanchen shift [121] etc. have been realized in negative index metamaterials.

Dispersion and loss in MM:

The energy density of an electromagnetic wave propagating through a non-dispersive and no-loss media is given by

$$u = \frac{1}{4}\left[\varepsilon\left|\vec{E}\right|^2 + \mu\left|\vec{H}\right|^2\right] \tag{1.4}$$

Clearly, because $\varepsilon,\ \mu < 0$, for a NIM, we get a non-physical result of negative energy density if this expression is used. On the other hand, the correct expression for the energy density, away from any absorption resonances, for a quasi-monochromatic wave-packet propagating through a dispersive media is:

$$u = \frac{1}{4}\left[\frac{\partial}{\partial\omega}\left(\omega\,\varepsilon(\omega)\right)\left|\vec{E}\right|^2 + \frac{\partial}{\partial\omega}\left(\omega\,\mu(\omega)\right)\left|\vec{H}\right|^2\right] \tag{1.5}$$

where the derivatives are evaluated at the central frequency of the wavepacket. The physical requirement of $u > 0$ implies that

$$\frac{\partial}{\partial\omega}\left(\omega\,\varepsilon(\omega)\right) > 0 \text{ and } \frac{\partial}{\partial\omega}\left(\omega\,\mu(\omega)\right) > 0 \tag{1.6}$$

Above conditions are compatible with $\varepsilon, \mu < 0$ if $\partial\varepsilon/\partial\omega > |\varepsilon|/\omega$ and $\partial\mu/\partial\omega > |\mu|/\omega$. This means that NIMs must be highly dispersive. It should also be noted that due to causality, generally well explained through the so called Kramers-Kronig relations, the negative NIMs must be lossy.

In order to achieve negative refractive index, the material must possesses simultaneously negative permittivity and negative permeability at the same frequency. There are natural materials like gold, silver and other novel metals which possess ε<0 and few antiferromagnets and insulating ferromagnets that show μ<0. But, the resonance frequencies for ε<0 occur mostly at few THz while the magnetic resonances limit to relatively very low frequencies for μ<0. One can overcome this problem using artificially engineered metamaterials where 'meta-atoms' are designed to exhibit both electric and magnetic responses in the same frequency range. John Pendry proposed a structure of very thin conducting wires arranged in a periodic lattice to achieve ε<0 at microwave frequencies [122]. The plasma frequency of this type of thin wires of radius ‘r’ and lattice spacing ‘a’ is reduced significantly due to the spatial confinement of the electrons to the thin wires, the effective electron density, N_{eff}, is decreased and moreover the self-inductance of the wires greatly enhances the effective mass m_{eff}. It could be understood from the expression of plasma frequency given by: $\omega_{pe}^2 = N_{eff}e^2 / m_{eff}\varepsilon_0 = 2\pi c^2 / a^2 \ln(a/r)$, where ε_0 is the permittivity of free space. Clearly, ω_{pe} depends only on the wire radii and their spacing and with judicious engineering frequency in the Giga-Hertz range could be obtained. But, the most critical issue is to achieve the negative magnetic permeability at microwave frequency.

The first attempt to solve this problem was made by Pendry who suggested use of split ring resonator (SRR) structures as meta atoms to have negative permeability [123]. Following this idea, Smith and his coworkers combined the two structures, SRRs and thin wires, and experimentally demonstrated the realization of simultaneous negative ε and μ over a frequency band [114, 117]. The first experimental realization of negative refractive index was demonstrated by Shelby et al at microwave frequencies [124]. In recent years, various 2D and 3D composite structures have been proposed as well as experimentally realized by various research groups to achieve negative refractive index at microwave frequency [87].

For practical applications it is very important to achieve negative refractive index in optical domain. Also, it is a very challenging affair to extend negative refractive index to optical frequency as it requires scaling down of the artificial composite structure from millimeter size to nanometer size. Owing to the rapidly developing nanotechnology and subwavelegth imaging techniques this goal is already achieved in some laboratories, though commercialization would take a while. Researchers have fabricated MM at infrared frequency by using double SRRs structure [88-89]. But at optical frequency, the electromagnetic response is totally different as the metal required for fabrication deviates from an ideal conductor behavior. This distinction prohibits scaling down the same structure used in microwave frequency. The first experimental realization of optical negative index material was demonstrated by Shalaev et al in 2005 designing a structure of double periodic array of pairs of parallel gold nano rods[89]. However, in the same year a new design called double fishnet structure, which contains a pair of metal fishnet, was used to achieve negative refractive index [125]. After that using these structures several research group reported NIM in optical frequency [126-130]. One of the most critical issues of optical negative index metamaterial is the significant loss in optical frequencies. This is a major hindrance in using optical metamaterials for the purpose of practical applications. At optical wavelength MM suffers from very high dissipative losses due to the metallic nature of their constituents metamolecules. But recently in 2010, Shalaev and his co-workers experimentally demonstrated that by inserting gain materials in the high-local-field areas of fishnet structure it is possible to fabricate an extremely low loss and active optical negative index MM [130].

A great deal of research has been exclusively focused on the linear MMs both theoretically and experimentally [131-135]. In the linear region, it has been assumed that the electric permittivity ε and magnetic permeability μ does not depend on the intensity of incident radiation. Recently, the nonlinear effects in MMs become interesting field of research particularly among the theoretical physicists. As the linear responses of MMs show various unusual properties which are not found in naturally occurring materials, in the same way the study of nonlinear MMs may have a path breaking impact in context of nonlinear optics. Kivshar and his colleagues were the first to explore theoretically the nonlinear properties of metamaterial [90]. The nonlinear MMs are created by the combination of arrays of wires and SRRs embedded in a nonlinear Kerr dielectric medium or by inserting certain nonlinear elements in the SRR's path [90,136]. The nonlinear response of this structure arises from two different contributions. Firstly, the nonlinear electric response of the electric permittivity is intensity dependent Kerr type nonlinearity which arises due to embedding the metamaterial in an optical Kerr medium. The effective dielectric permittivity (ε_{eff}) of the nonlinear composite material is given by:

$$\varepsilon_{eff}(\omega) = \varepsilon_0 \left(\varepsilon_{L0} + \alpha |E|^2\right) \tag{1.7}$$

where $\alpha = \pm 1/E_c^2$ represents focusing and defocusing nonlinearity, E_c is the characteristic electric field, ε_{L0} is linear dielectric permittivity. Secondly, the nonlinear response of the magnetic permeability shows very complicated behavior. The response comes from the resonance behavior of effective permeability (μ_{eff}) as SRR capacitance or SRR eigen frequency depends on the strength of the magnitude of electric field ($\mathbf{E_g}$), localized within the SRR gap. The intensity of the electric field in turn depends on the electromotive force inside the resonator loop, which is generated by the external magnetic field (**H**). The effective nonlinear permeability is given by:

$$\mu_{eff}(\omega) = \mu_0 \left(1 - \frac{F\omega^2}{\omega^2 - \omega_{0NL}^2(|H|^2)}\right) \tag{1.8}$$

where $\omega_{0NL} = \omega_{0NL}(|H|^2)$ is nonlinear resonant frequency which depends on $|H|^2$ as [132]:

$$\alpha\Omega^2 X^6 |H|^2 = A^2 E_c^2 (1 - X^2)(X^2 - \Omega^2)^2 \tag{1.9}$$

where $X = \omega_{0NL}/\omega_0$, $\Omega = \omega/\omega_0$ and A is a function of the physical and the geometrical parameters of the composite structure. For relatively small fields, when μ_{eff} is truly field dependent, one may consider the nonlinear behavior of μ_{eff} as : $\mu_{eff} = \mu + \mu_{NL}(|H|^2)$ i.e. Kerr type of magnetic nonlinearity. Hence, one of the most of striking feature of nonlinear MM is that the magnitude of magnetic field plays a significant role on the nonlinear behavior of magnetic permeability and the magnetic nonlinearity has been found to be much stronger than the nonlinearity in the dielectric response owing to the enhancement of field in the gaps of SRRs. The intensity dependence of magnetic permeability can be allowed to switch from positive to negative values which in turn can tune the material properties from positive index materials to negative index materials. These may have useful application in nonlinear optical devices like optical memory and optical diodes. Cloaking devices are another peculiar example of the application of negative index materials that has been first demonstrated in 2006 [137] i.e. MM could be used to hide objects. Recently, MM cloak is experimentally designed for microwaves with the help of transformation optics formulation [138].

1.4 Thesis Overview

The contents of chapters in the remaining part of the present thesis are described very briefly below:

Chapter 2: This chapter begins with the literature review of various nonlinear pulse propagation models for Metamaterials. Then starting with the Maxwell's equation and considering the lossy Drude model to describe the permittivity and permeability in MM, we derive a generalized coupled nonlinear field equations for pulse propagation in a MM embedded into a Kerr medium. Our model successfully recovers previously proposed models to describe pulse propagation in MMs exhibiting Kerr nonlinearity. Moreover, it contains a few additional terms like magnetic self-steepening effect connecting both the electric and magnetic field envelopes in an MM. We have studied MI using these coupled equations and it is found that one could control the MI gain in an MM by tuning the initial electric or magnetic field amplitudes. We have also reported solitary wave solutions to the propose model.

Chapter 3: In this chapter, we have focused on the study of exact solitary wave solutions and MI of the generalized nonlinear Schrödinger equation in the context of a non-Kerr medium. The nonlinear Schrodinger equation, appropriate to model nonlinear pulse propagation in a cubic-uintic non-kerr media, is studied to obtain the exact bright and dark solitary wave solutions. We have also carried out the MI analysis of the model equation and discuss the effect of higher order dispersion and nonlinear effect on MI both in the anomalous and the normal dispersion regime.

Chapter 4: This chapter is aimed at studying the MI analysis in an MM embedded in a cubic-quintic non-Kerr medium. We have derived a theoretical model appropriate for pulse propagation in this nonlinear media with higher order dispersion effect and cubic quintic nonlinear effect. We have discussed the issue of loss in MMs in the context of MI. The effect of higher order dispersion effects and the cubic-quintic nonlinear effect has also been discussed.

Chapter 5: The generalized NLSE with variable coefficients is studied in this chapter. The coefficients corresponding to the group velocity dispersion, the self-phase modulation, the self-steepening etc. are considered to be dependent on the evolution parameter. The dynamics of dark optical solitons with power-law nonlinearity in the presence of linear attenuation, TOD and self-steepening term, all with evolution parameter dependent coefficient is studied. We have used solitary wave ansatz method to carry out the integration and obtain the exact solitary wave solutions. As a special case, we have also reported the exact bright and dark solitary wave solutions of the generalized NLSE with constant co-efficient of the above mention higher order dispersion and nonlinear effects.

Chapter 6: This chapter begins with an overview of silicon waveguide. Silicon photonics has received considerable attention in recent times due to its capability for IC integration via SOI technology. In silicon waveguide, nonlinear absorption via the so called TPA is an extremely important effect that limits the extent of SPM. We demonstrate the formation of a fundamental soliton from a higher order soliton in silicon waveguide by exploiting the TPA effect. Soliton-soliton interaction is also discussed.

Chapter 7: We give a brief summary of the results and analysis of the problems considered in the thesis. Also, we give the future directions of research in this ever expanding area of research.

Chapter 2

Modulation Instability of coupled nonlinear field equations for pulse propagation in Metamaterial embedded into a Kerr medium*

2.1 Introduction

Recently, negative index metamaterials (NIMs) have been demonstrated experimentally at infrared (IR) and optical frequencies [88-89], thereby motivating the researchers to study pulse propagation in such media [123-130]. A great deal of research has been carried out on the propagation of electromagnetic waves in the linear regime, specially the basic dynamics of ultrashort pulses undergoing negative refractive index has been discussed [131-135]. Study of nonlinear pulse propagation in Metamaterials(MMs) have started after the demonstration of nonlinear MMs, created by arrays of wires and split-ring resonators embedded in a Kerr dielectric [90] or by inserting certain nonlinear elements like diodes in the split ring resonator's path [136]. Very recently, many authors have investigated and proposed nonlinear ultrashort pulse propagation models in NIMs in various contexts [91,139-147]. Scalora et al.[91] first derived a new generalized nonlinear Schrodinger (NLS) equation, to describe the propagation of ultrashort pulses in bulk negative index media exhibiting frequency dependent dielectric susceptibility and magnetic permeability. Going beyond the usual slowly varying envelope approximation (SVEA), they investigated the propagation of pulses for at least a few tens of optical cycles in MMs. They did not consider magnetic nonlinearity in their approach. Wen et al. have derived a (3+1) dimensional evolution equation for NIM with a Kerr nonlinear polarization by following the same procedure as applied to the case of ordinary materials [139]. Based on the propagation equation, they studied the MI analysis and summed up with the conclusion that MI in MMs may occur for all the combinations of dispe-

*Part of the results presented in this chapter have been published in the paper, A. K. Sarma and M. Saha, "Modulational instability of coupled nonlinear field equations for pulse propagation in a negative index material embedded into a Kerr medium" *Journal of Optical Society of America B* Vol. 28, p.944 (2011)

-rsion and nonlinearities, unlike ordinary materials for which MI can occur only for some specific combination of dispersion and nonlinearities. On the other hand, Lazarides and Tsironis[141] derived a system of coupled nonlinear Schrodinger equations (NLSE) for the envelopes of the propagating electric and magnetic fields in an isotropic and homogeneous nonlinear left-handed materials taking both nonlinear polarization and magnetization into account. However, they used perturbative approach and focused only on a completely integrable Manakov-type model. They also studied the bright and dark solitary wave solutions of their proposed model equations. Based on this model, Kourikas and Shukla investigated the stability of the nonlinear system [142]. Later, Wen et al. have derived a more generalized coupled NLSE suitable for few cycle pulse propagation in a NIM considering both nonlinear electric and magnetic polarization [143-147]. In this context optical soliton is a new and exciting field of research [148-149].

In this chapter, we derive a mathematical model describing pulse propagation in an MM created from split-ring resonators and arrays of wires embedded in a Kerr medium, exhibiting both electric and magnetic nonlinearity of Kerr type. We assume that the unit cell size of the MM is considerably smaller than the operating wavelength and thereby take an effective medium approach [87]. Unlike other authors, we invoke the electric and magnetic Kerr effect quite early into the derivation. Previously proposed models, under appropriate approximations, could be successfully reproduced from our newly derived propagation equation. Our model predicts some new additional features as well. We carry out a brief MI analysis, considering its significance in relation to the existence of optical solitons in NIMs, using the newly derived model equations.We have also reported an exact solitary wave solutions to the proposed model in this chapter.

2.2 Generalized Theoretical Model

One of the most fundamental differences between an ordinary material and MM is that MM exhibits strong dispersive behaviors both in electric permittivity and in magnetic permeability while in ordinary materials only one of them is dominant at a time. We consider that the nonlinear MM is created by arrays of metallic wires and split ring resonators embedded in a nonlinear Kerr dielectric. This MM has Kerr type nonlinear polarization but the nonlinear magnetization shows a complicated behavior. However, for a relatively small magnetic field intensity **H**, as justified in Ref.[141], the nonlinear magnetization could also be taken as the

Kerr-type. Hence, we consider a nonlinear negative index material embedded in a Kerr medium characterized by the following forms of nonlinear electric polarization ($\mathbf{P}_{NL}$) and nonlinear magnetization ($\mathbf{M}_{NL}$): $\mathbf{P}_{NL} = \varepsilon_{NL}\mathbf{E} = \varepsilon_0 \chi_P^{(3)} |E|^2 \mathbf{E}, \mathbf{M}_{NL} = \mu_{NL}\mathbf{H} = \mu_0 \chi_M^{(3)} |H|^2 \mathbf{H}$; where ε_{NL} is nonlinear electric permittivity while $\chi_P^{(3)}$, $\chi_M^{(3)}$ are the respective third order electric and magnetic susceptibility. **E** and **H** are the electric and magnetic fields respectively. The nonlinear pulse propagation through MMs is characterized by electric flux density (**D**) and magnetic induction (**B**) which depends on electric (**E**) and magnetic (**H**) field intensities as $\mathbf{D} = \varepsilon * \mathbf{E} + \mathbf{P}_{NL}$ and $\mathbf{B} = \mu * \mathbf{H} + \mathbf{M}_{NL}$ [150-151]. Here, the asterisk refers to convolution. The dielectric permittivity (ε) and magnetic permeability (μ) are dispersive in NIM due to the reasons described in chapter 1. The frequency dispersion of ε and μ are given by the lossy Drude models as follows [133]:

$$\varepsilon(\tilde{\omega}) = \varepsilon_0 \left[1 - 1/\left(\tilde{\omega}(\tilde{\omega} + i\tilde{\gamma}_e)\right)\right]; \ \mu(\tilde{\omega}) = \mu_0 \left[1 - \tilde{\omega}_{pm}^2 / \left(\tilde{\omega}(\tilde{\omega} + i\tilde{\gamma}_m)\right)\right] \tag{2.1}$$

Here $\tilde{\omega} = \omega/\omega_{pe}, \tilde{\omega}_{pm} = \omega_{pm}/\omega_{pe}$ with ω_{pe} and ω_{pm} are the respective electric and magnetic plasma frequencies. $\tilde{\gamma}_e = \gamma_e/\omega_{pe}$ and $\tilde{\gamma}_m = \gamma_m/\omega_{pe}$ are the electric and magnetic loss respectively normalized with respect to the electric plasma frequency. ε_0 and μ_0 are respectively the free space electric permittivity and magnetic permeability. It should be noted that loss is an extremely important and relevant issue in MMs as shown by the recent work of Stockman [152]. In this work we are taking $\tilde{\gamma}_e \sim \tilde{\gamma}_m \sim 0.01$, a value two orders of magnitude greater than that of the one taken by D'Aguanno et al. [133]. In this way, we keep our model as close to reality as possible.

We assume that both the electric and magnetic field is propagating in a uniform, bulk NIM containing no free charge and no free current. It is straightforward to get the following nonlinear pulse propagation equations from the Maxwell equations:

$$\begin{aligned}
&\left(\frac{\partial^2}{\partial z^2} + \nabla_\perp^2\right)\mathbf{E}(r,t) - \nabla\left(\nabla.\mathbf{E}(r,t)\right) = \frac{\partial^2}{\partial t^2}[\mu * \varepsilon * \mathbf{E}(r,t)] + \frac{\partial^2}{\partial t^2}[\mu * \mathbf{P}_{NL}(r,t)] + \frac{\partial}{\partial t}\left(\nabla \times \mathbf{M}_{NL}(r,t)\right) \\
&\left(\frac{\partial^2}{\partial z^2} + \nabla_\perp^2\right)\mathbf{H}(r,t) - \nabla\left(\nabla.\mathbf{H}(r,t)\right) = \frac{\partial^2}{\partial t^2}[\mu * \varepsilon * \mathbf{H}(r,t)] + \frac{\partial^2}{\partial t^2}[\varepsilon * \mathbf{M}_{NL}(r,t)] - \frac{\partial}{\partial t}\left(\nabla \times \mathbf{P}_{NL}(r,t)\right)
\end{aligned} \tag{2.2}$$

In order to simplify the above complex equations we make a couple of approximations: Firstly we assume that both the electric and magnetic fields are propagating along the z direction and are linearly polarized. Secondly, there is negligible transverse inhomogeneities of the medium polarization and magnetization so that $\nabla(\nabla.\mathbf{E}(r,t))=0=\nabla(\nabla.\mathbf{H}(r,t))$. Moreover we treat ε_{NL} and μ_{NL} as constants so that the last two terms of the coupled equations could be put in the following form:

$$\frac{\partial}{\partial t}(\nabla\times\mathbf{M}_{NL}(r,t))\approx\mu_{NL}\frac{\partial}{\partial t}(\nabla\times\mathbf{H}) \quad \text{and} \quad \frac{\partial}{\partial t}(\nabla\times\mathbf{P}_{NL}(r,t))\approx\varepsilon_{NL}\frac{\partial}{\partial t}(\nabla\times\mathbf{E}) \tag{2.3}$$

Using the Fourier transformation of the fields as :

$\mathbf{E}(r,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{E}(r,\omega)e^{-i\omega t}d\omega$ and $\mathbf{H}(r,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{H}(r,\omega)e^{-i\omega t}d\omega$, where $\tilde{E}(r,\omega)$ and $\tilde{H}(r,\omega)$ are respectively the envelope amplitude of the electric and the magnetic fields in the frequency domain, the set of coupled equations for the electric and magnetic fields in frequency domain could be expressed as follows :

$$\begin{aligned}\left(\frac{\partial^2}{\partial z^2}+\nabla_\perp^2\right)\tilde{E}(r,\omega)&=-\omega^2\mu(\omega)\varepsilon(\omega)\tilde{E}(r,\omega)-\varepsilon_0\chi_E^{(3)}\omega^2\mu(\omega)\left|E\right|^2\tilde{E}(r,\omega)-\mu_{NL}\omega^2\varepsilon(\omega)\tilde{E}(r,\omega)\\ \left(\frac{\partial^2}{\partial z^2}+\nabla_\perp^2\right)\tilde{H}(r,\omega)&=-\omega^2\mu(\omega)\varepsilon(\omega)\tilde{H}(r,\omega)-\mu_0\chi_M^{(3)}\omega^2\varepsilon(\omega)\left|H\right|^2\tilde{H}(r,\omega)-\varepsilon_{NL}\omega^2\mu(\omega)\tilde{H}(r,\omega)\end{aligned} \tag{2.4}$$

Our aim is to obtain a set of coupled equations for the envelopes of the electric and magnetic fields in a more general form. We write the electric and magnetic field as:

$$\begin{aligned}\mathbf{E}(r,\mathrm{t})&=\hat{x}\,\mathrm{A}(r,\mathrm{t})\exp(\mathrm{ik_0z}-\mathrm{i}\omega_0\mathrm{t})+\mathrm{c.c}\\ \mathbf{H}(r,\mathrm{t})&=\hat{y}\,\mathrm{B}(r,\mathrm{t})\exp(\mathrm{ik_0z}-\mathrm{i}\omega_0\mathrm{t})+\mathrm{c.c}\end{aligned} \tag{2.5}$$

where c.c. is the complex conjugate of the pulse. Here, $A(r,t)$ and $B(r,t)$ are the slowly varying pulse envelopes of electric and magnetic field respectively. $k_0=\omega_0 n(\omega_0)/c$ is the wave number at the central frequency ω_0 and $n(\omega_0)$ is the refractive index of the material at ω_0. Now taking the inverse Fourier transformation of Eq.(2.4) and then substituting Eq.(2.5) in it, we obtain the following coupled equations for the envelopes of the electric and magnetic fields:

$$\frac{\partial^2 A}{\partial z^2}+2ik_0\frac{\partial A}{\partial z}-k_0^2A+\nabla_\perp^2 A=-\sum_{m=0}^{\infty}D_m\frac{\partial^m A}{\partial t^m}-\varepsilon_0\chi_E^{(3)}\left(\omega_0+i\frac{\partial}{\partial t}\right)\sum_{m=0}^{\infty}F_m\frac{\partial^m}{\partial t^m}|A|^2A$$
$$-\mu_0\chi_M^{(3)}|B|^2\left(\omega_0+i\frac{\partial}{\partial t}\right)\sum_{n=0}^{\infty}G_n\frac{\partial^n A}{\partial t^n}$$

$$\frac{\partial^2 B}{\partial z^2}+2ik_0\frac{\partial B}{\partial z}-k_0^2B+\nabla_\perp^2 B=-\sum_{m=0}^{\infty}D_m\frac{\partial^m B}{\partial t^m}-\mu_0\chi_M^{(3)}\left(\omega_0+i\frac{\partial}{\partial t}\right)\sum_{n=0}^{\infty}G_n\frac{\partial^n}{\partial t^n}|B|^2B$$
$$-\varepsilon_0\chi_E^{(3)}|A|^2\left(\omega_0+i\frac{\partial}{\partial t}\right)\sum_{n=0}^{\infty}F_m\frac{\partial^m B}{\partial t^m} \tag{2.6}$$

where

$$D_m=\sum_{l=0}^{m}\frac{i^m}{l!(m-l)!}\frac{\partial^l(\omega\varepsilon)}{\partial\omega^l}\bigg|_{\omega=\omega_0}\frac{\partial^{m-l}(\omega\mu)}{\partial\omega^{m-l}}\bigg|_{\omega=\omega_0},\ F_m=\frac{i^m}{m!}\frac{\partial^m(\omega\mu)}{\partial\omega^m}\bigg|_{\omega=\omega_0}\ \text{and}\ G_n=\frac{i^n}{n!}\frac{\partial^l(\omega\varepsilon)}{\partial\omega^l}\bigg|_{\omega=\omega_0} \tag{2.7}$$

Now we introduce the travelling coordinates: ξ=z , τ=t- z/V with V as the group velocity. We keep linear dispersion terms up to the second order and nonlinear dispersion terms up to the first order. In order to make the above propagation model applicable for ultra short pulses and solvable we make some further approximations (non SVEA approximation) [139]:

$$\frac{\partial^2 A}{\partial\xi\partial\tau}=\frac{i\omega_0^2\mu(\omega_0)\chi_E^{(3)}}{2k_0}\frac{\partial|A|^2A}{\partial\tau}\ \text{and}\ \frac{\partial^2 B}{\partial\xi\partial\tau}=\frac{i\omega_0^2\varepsilon(\omega_0)\chi_M^{(3)}}{2k_0}\frac{\partial|B|^2B}{\partial\tau} \tag{2.8}$$

Under these assumptions we obtain the following coupled generalized NLSE for a nonlinear negative index material exhibiting Kerr type electric and magnetic nonlinear polarization as:

$$\frac{\partial A}{\partial\xi}=\frac{i}{2k_0}\nabla_\perp^2A-\frac{i\beta_2}{2}\frac{\partial^2 A}{\partial\tau^2}+iP_{nl}\left(1+iP_s\frac{\partial}{\partial\tau}\right)|A|^2A+iQ_{nl}|B|^2\left(A+iP_{se}\frac{\partial A}{\partial\tau}\right)$$
$$\frac{\partial B}{\partial\xi}=\frac{i}{2k_0}\nabla_\perp^2B-\frac{i\beta_2}{2}\frac{\partial^2 B}{\partial\tau^2}+iQ_{nl}\left(1+iQ_s\frac{\partial}{\partial\tau}\right)|B|^2B+iP_{nl}|A|^2\left(B+iQ_{sh}\frac{\partial B}{\partial\tau}\right) \tag{2.9}$$

where

$$P_{nl}=\frac{\omega_0^2\mu(\omega_0)\varepsilon_0\chi_E^{(3)}}{2k_0},\ P_s=\left[\frac{1}{\omega_0}\left(1+\frac{\gamma}{\mu(\omega_0)}\right)-\frac{1}{k_0V}\right],\ P_{se}=\frac{1}{\omega_0}\left(1+\frac{\alpha}{\varepsilon(\omega_0)}\right)$$
$$Q_{nl}=\frac{\omega_0^2\varepsilon(\omega_0)\mu_0\chi_M^{(3)}}{2k_0},\ Q_s=\left[\frac{1}{\omega_0}\left(1+\frac{\alpha}{\varepsilon(\omega_0)}\right)-\frac{1}{k_0V}\right],Q_{sh}=\frac{1}{\omega_0}\left(1+\frac{\gamma}{\mu(\omega_0)}\right)$$

and $\beta_2=\left[\left\{\alpha\gamma+\omega_0\mu(\omega_0)\alpha'/2+\omega_0\varepsilon(\omega_0)\gamma'/2-1/V^2\right\}/k_0\right]$ with $\gamma=\partial[\omega\mu(\omega)]/\partial\omega\big|_{\omega=\omega_0}$

$$\gamma' = \partial^2[\omega\mu(\omega)]/\partial\omega^2\big|_{\omega=\omega_0}, \alpha = \partial[\omega\varepsilon(\omega)]/\partial\omega\big|_{\omega=\omega_0}, \alpha' = \partial^2[\omega\varepsilon(\omega)]/\partial\omega^2\big|_{\omega=\omega_0} \quad \text{and}$$

$$V = 2k_0/[\omega_0\varepsilon(\omega_0)\gamma + \omega_0\mu(\omega_0)\alpha] \tag{2.10}$$

P_{nl} and P_s are the nonlinear and self-steepening coefficients for the electric field respectively. We name P_{se} as the electric coupling coefficient. Q_{nl} and Q_s are the corresponding coefficients for the magnetic field, while we call Q_{sh} as the magnetic coupling coefficient. β_2 is the GVD parameter. Eq. (2.9) is the generalized coupled NLSE for pulse propagation for a negative index material embedded into a Kerr medium. It should be noted that if $Q_{nl} = 0$ we recover exactly the same equation in Ref. [141] for the envelope of the electric field. However our model contains a few additional terms compared to previous models, especially connecting both the electric and the magnetic field envelopes in a NIM. In Fig. 2.1(a) we plot the variation of refractive index n, P_{nl}, P_s and P_{se} with the normalized frequency ω_0/ω_{pe} for $\omega_{pm}/\omega_{pe} = 0.8$ while, in Fig.2.1(b), we plot the corresponding variations for Q_{nl}, Q_s and Q_{sh}. Here ω_{pe} and ω_{pm} are the electric and magnetic plasma frequency respectively, the parameters are plotted at $\omega = \omega_0$. In the plot P_{nl} is calculated in units of $\omega_{pe}\chi_E^{(3)}/c$ while P_s and P_{se} are calculated in the units of $1/\omega_{pe}$, Q_{nl} is calculated in units of $\omega_{pe}\chi_M^{(3)}/c$ while Q_s and Q_{sh} are calculated in the units of $1/\omega_{pe}$. From the figure it is clear that at $\omega_{pm}/\omega_{pe} = 0.8$, the electric self-steepening parameter have both positive and negative values but the magnetic self-steepening parameter is positive. However if we choose $\omega_{pm}/\omega_{pe} = 1.2$, then the magnetic self-steepening parameter has both positive and negative values as shown in Fig. 2.2. We have carried out rest of the analysis by choosing $\omega_{pm}/\omega_{pe} = 0.8$.

For simplicity of calculations, it is convenient to write the coupled generalize NLSE in normalized units. The normalized variables are defined as:

$$Z = \xi/L_D, T = \tau/T_0; U = A/A_0, V = B/B_0; X = x/L_\perp, Y = y/L_\perp; u = N_E U, v = N_H V \tag{2.11}$$

where T_0 is the pulse width, $L_D = T_0^2/|\beta_2|$ is the dispersion length and A_0 and B_0 are the initial amplitude of the electric and the magnetic fields. N_E and N_H may be termed as the order of soliton for the electric and the magnetic field, defined as $N_E^2 = L_D/L_{Pnl}$, $N_H^2 = L_D/L_{Mnl}$. In this work, we take $N_E = N_H = N$. Here we also define the nonlinear polarization length, as $L_{Pnl} = 1/P_{nl}A_0^2$ and the nonlinear magnetization length as $L_{Mnl} = 1/Q_{nl}B_0^2$. A characteristic length $L_{\perp} = \sqrt{|L_d/k_0|}$ is also defined.

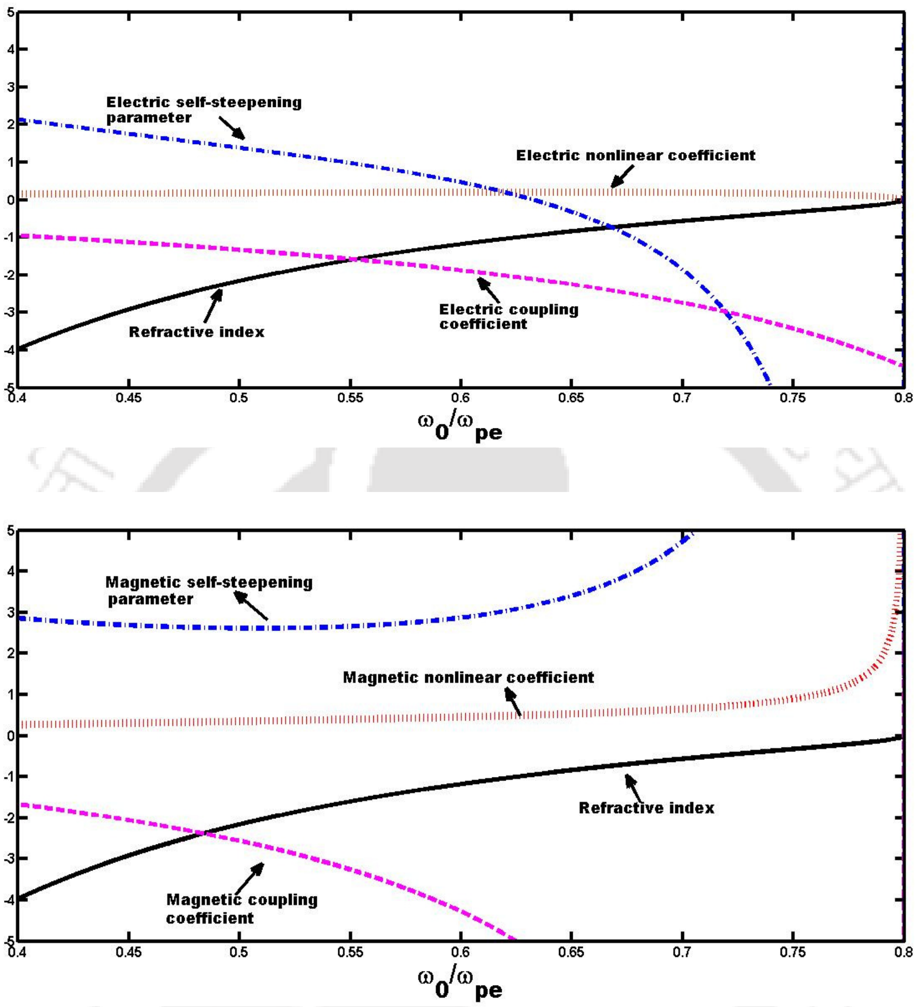

Fig.2.1: (a) Variation of n, P_{nl} , P_s and P_{se} (b) Variation of n, Q_{nl} , Q_s and Q_{sh} with ω_0/ω_{pe} with $\omega_{pm}/\omega_{pe} = 0.8$.

Eq.(2.9) is thus transformed to the following normalized form :

$$\frac{\partial u}{\partial Z} = \frac{i\,\mathrm{sgn}(k_0)}{2}\nabla_T^2 u - \frac{i\,\mathrm{sgn}(\beta_2)}{2}\frac{\partial^2 u}{\partial T^2} + i\left(1 + iS_E\frac{\partial}{\partial T}\right)|u|^2 u + i|v|^2 u - C_E|v|^2\frac{\partial u}{\partial T}$$

$$\frac{\partial v}{\partial Z} = \frac{i\,\mathrm{sgn}(k_0)}{2}\nabla_T^2 v - \frac{i\,\mathrm{sgn}(\beta_2)}{2}\frac{\partial^2 v}{\partial T^2} + i\left(1 + iS_H\frac{\partial}{\partial T}\right)|v|^2 v + i|u|^2 v - C_H|u|^2\frac{\partial v}{\partial T} \tag{2.12}$$

where $\nabla_T^2 = \partial^2/\partial X^2 + \partial^2/\partial Y^2$ is the transverse Laplacian, $S_E = |P_s|/T_0$ is the electric self-steepening parameter, $S_H = |Q_s|/T_0$ is the magnetic self-steepening parameter, $C_E = |P_{se}|/T_0$ is the electric coupling coefficient, and $C_H = |Q_{sh}|/T_0$ is the magnetic coupling coefficients in normalized units.

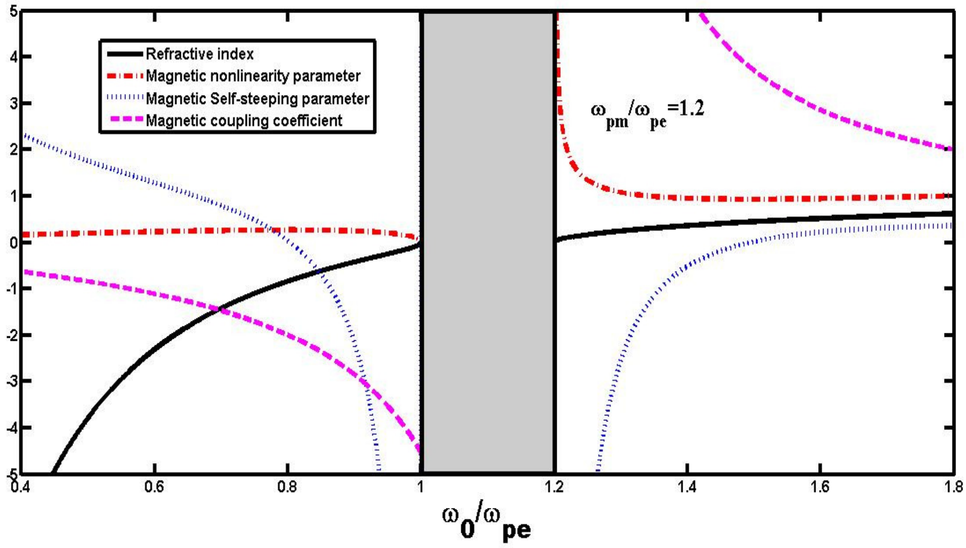

Fig.2.2: Variation of magnetic self-steepening parameter Q_s with ω_0 / ω_{pe} at $\omega_{pm} / \omega_{pe} = 1.2$.

2.3 Modulation Instability analysis

As discussed in Chapter 1, MI is a fundamental and ubiquitous process that occurs as a result of interplay between the nonlinearity and dispersion in the time domain or diffraction in spatial domain [2]. Now we would carry out an MI analysis of the above generalized coupled NLSE. We are mainly interested to know the role of both the electric and the magnetic self-steepening parameters on MI. So we neglect the diffraction and the last term of the generalized coupled NLSE. It should be noted that Wen et al.[139] have already investigated MI in NIM to understand the role of self-steepening effect. But unlike them we are considering both the electric and the magnetic self-steepening effects. Moreover this model contains a term relating both the electric and magnetic field envelopes. Hence the coupled NLSE considered is:

$$\frac{\partial u}{\partial Z} = -\frac{i\,\mathrm{sgn}(\beta_2)}{2}\frac{\partial^2 u}{\partial T^2} + i\left(1 + iS_E\frac{\partial}{\partial T}\right)|u|^2 u + i|v|^2 u$$

$$\frac{\partial v}{\partial Z} = -\frac{i\,\mathrm{sgn}(\beta_2)}{2}\frac{\partial^2 v}{\partial T^2} + i\left(1 + iS_H\frac{\partial}{\partial T}\right)|v|^2 v + i|u|^2 v \qquad (2.13)$$

Eq.(2.13) have two steady state CW solutions $u = a_0\exp(i\Omega_{0a}Z), v = b_0\exp(i\Omega_{0b}Z)$,where a_0 and b_0 are the normalized amplitudes of electric and magnetic field envelopes and Ω_{0a} and Ω_{0b} are the corresponding nonlinear phase-shifts. Here, $\Omega_{0a} = \Omega_{0b} = a_0^2 + b_0^2$.

Now, let the CW solutions be slightly perturbed from the steady state such that

$$u(Z,T) = [a_0 + a(Z,T)]\exp(i\Omega_{0a}T) \text{ and } \mathrm{v}(Z,T) = [b_0 + b(Z,T)]\exp(i\Omega_{0b}Z) \qquad (2.14)$$

where $a(Z,T)$ and $b(Z,T)$ are the perturbations such that $a, b \ll 1$. Substituting Eq.(2.14) in Eq.(2.13) we obtain the following evolution equations for the perturbations:

$$\frac{\partial a}{\partial Z} = -\frac{i\,\mathrm{sgn}(\beta_2)}{2}\frac{\partial^2 a}{\partial T^2} + ia_0^2(a + a^*) + ia_0b_0(b + b^*) - S_E a_0^2\left(2\frac{\partial a}{\partial \tau} + \frac{\partial a^*}{\partial \tau}\right)$$

$$\frac{\partial b}{\partial Z} = -\frac{i\,\mathrm{sgn}(\beta_2)}{2}\frac{\partial^2 b}{\partial T^2} + ib_0^2(b + b^*) + ia_0b_0(a + a^*) - S_H b_0^2\left(2\frac{\partial b}{\partial \tau} + \frac{\partial b^*}{\partial \tau}\right) \qquad (2.15)$$

Now, writing, $a = a_1 e^{i(KZ - \Omega T)} + a_2 e^{-i(KZ - \Omega T)}$ and similarly for b, where K and Ω are the wave number and the frequency of the perturbation in normalized units, respectively, from Eq.(2.15), we obtain the following dispersion relation:

$$K = \frac{1}{2}\left[4s\Omega \pm \left(4s^2\Omega^2 + \frac{\Omega^4}{4} + 4\delta(a_0^2 + b_0^2)\Omega^2\right)^{\frac{1}{2}}\right] \qquad (2.16)$$

where $\delta = \mathrm{sgn}(\beta_2)$. Here, we assume that $S_E a_0^2 = S_H b_0^2 = s$ and take the frequency regime to be $0.4 \le \omega/\omega_{pe} \le 0.6$. Now onwards, we term s as the reduced self-steepening parameter. Because $S_E \ne S_H$, the initial normalized amplitudes of the electric and magnetic fields would always have to be different i.e. $a_0 \ne b_0$. The steady-state solution becomes unstable only when K has an imaginary part since perturbation then grows exponentially. It can be clearly seen from Eq.(2.16) that K becomes imaginary only in the anomalous dispersion regimes i.e.

when $\delta = -1$ and, if the following condition is satisfied, $(s^2 + \Omega^2/16) < (a_0^2 + b_0^2)$. Under these MI conditions, we obtain the gain spectrum $g(\Omega)$ of the MI as

$$g(\Omega) = 2\mathrm{Im}(\mathrm{K}) = \left[4(a_0^2 + b_0^2)\Omega^2 - 4s^2\Omega^2 - \Omega^4/4\right]^{\frac{1}{2}}. \tag{2.17}$$

Clearly, the MI gain depends on the initial amplitude of electric and magnetic fields, the perturbation frequency and the electric and magnetic self steepening parameter through s.

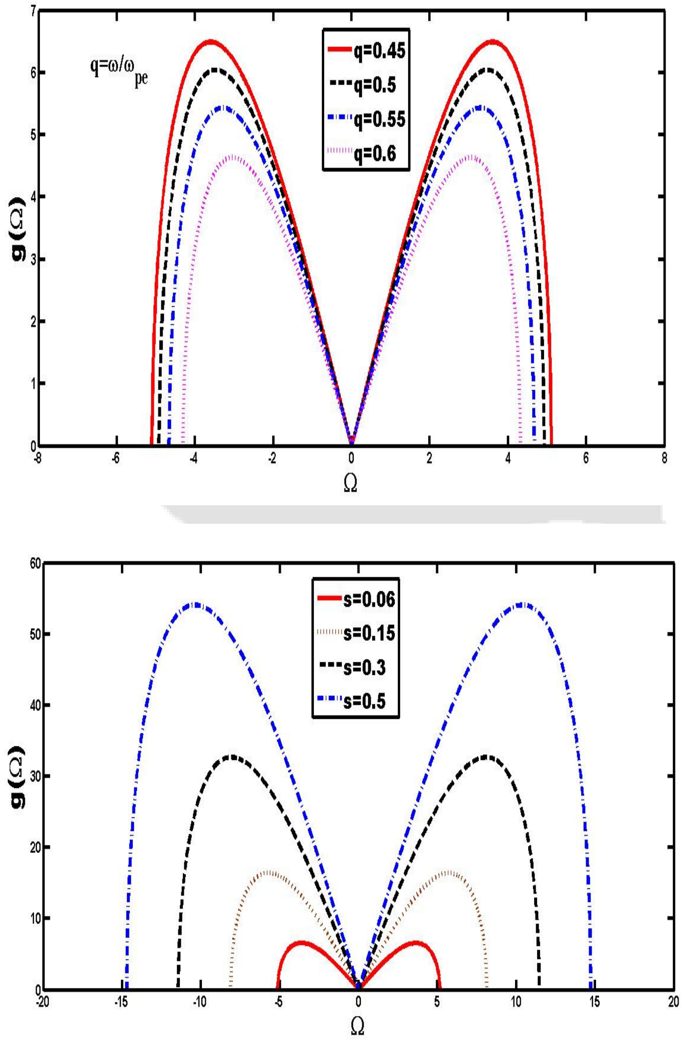

Fig.2.3: Modulation Instability gain spectrum as a function of normalized perturbation frequency (a) with normalized frequency ω_0/ω_{pe} (b) with different self-steepening parameter s.

The gain becomes maximum at two frequencies given by $\Omega_{\max} = \pm 2\left[2(a_0^2 + b_0^2) - 2s^2\right]^{1/2}$ with a peak value $4(a_0^2 + b_0^2 - s^2)^{1/2}$. In Fig.2.3(a) we depict the MI gain as a function of normalized perturbation frequency for various values of ω_0/ω_{pe} for a pulse width $T_0 = 10fs$

and $a_0 = 1$. We observe that MI gain spectrum is symmetric with respect to $\Omega = 0$ and the MI gain decreases with an increase in the normalized frequency. This might be attributed to the constraint $S_E a_0^2 = S_H b_0^2 = s$ and the fact that, with increases in the normalized frequency ω_0/ω_{pe}, as evident from Fig.2.1(a) and Fig.2.1(b), S_E decreases while S_H increases for given value of a_0 and pulse width T_0, resulting in the reduction of both s and b_0. The so-called reduced self-steepening parameter s could be controlled just by tuning the initial electric or magnetic field amplitudes for a given operating frequency of the MM. In order to have a clear idea about the role of s on MI, in Fig.2.3(b) we plot the variation of MI gain as a function of the normalized perturbation frequency for various values of s for a pulse width $T_0 = 10\,fs$ and $\omega_0/\omega_{pe} = 0.5$. It may be noted that the corresponding value of S_E and S_H are 0.057 and 0.10, respectively. It can be clearly seen that, with increase in s, MI gain is also increases, which should be due to the increase in the initial electric and magnetic field amplitudes. In passing, we would like to mention that our study shows that variation of the pulse width T_0 has a negligible small effect on MI gain spectrum. From the above analysis, it is clear that, by tuning the initial electric and magnetic field amplitudes or intensity, one can control the MI of electromagnetic pulses in an MM and may thereby eventually manipulate the formation of solitons in it.

2.4 Solitary wave solutions for the coupled nonlinear field equations

In this section, we would look for the solitary wave solution to Eq.(2.13), with the third order dispersion term included. The normalized coupled nonlinear field equations including the third order dispersion in the propagation model in MM which is embedded in a Kerr medium is given by

$$\frac{\partial u}{\partial Z} = -\frac{i\delta}{2}\frac{\partial^2 u}{\partial T^2} + b_3\frac{\partial^3 u}{\partial T^3} + i|u|^2 u + i|v|^2 u - S_E\frac{\partial}{\partial T}\left(|u|^2 u\right) \qquad (2.18)$$

$$\frac{\partial v}{\partial Z} = -\frac{i\delta}{2}\frac{\partial^2 v}{\partial T^2} + b_3\frac{\partial^3 v}{\partial T^3} + i|v|^2 v + i|u|^2 v - S_H\frac{\partial}{\partial T}\left(|v|^2 v\right) \qquad (2.19)$$

Here, $\delta = \mathrm{sgn}(\beta_2)$, b_3 is the normalized third order dispersion parameter. We have already discussed earlier that, in the frequency regime considered, S_E can have both positive and negative values while S_H is always positive at $\omega_{pm}/\omega_{pe}=0.8$. We take $s_{_E}=\alpha|S_E|$ and $s_{_H}=\beta|S_H|$ with $\alpha=\pm1$ and $\beta=1$.We assume the solutions of Eq.(2.18) and Eq.(2.19) in the following phase-amplitude format:

$$u(Z,T)=P_1e^{i\phi} \quad \text{and} \quad v(Z,T)=P_2e^{i\phi} \tag{2.20}$$

where P_j is the amplitude with j=1,2. $\phi=\omega T-kZ+\theta$ is the phase with ω, k and θ are the frequency, wave number and phase constant of the soliton respectively. Substituting the above Eq.(2.20) in Eq.(2.18) and Eq.(2.19), we obtain

$$\begin{aligned}\frac{\partial P_2}{\partial Z}-ikP_2=-i\frac{\delta}{2}\left[\frac{\partial^2 P_2}{\partial T^2}+2i\omega P_2-\omega^2P_2\right]+b_3\left[\frac{\partial^3 P_2}{\partial T^3}+3i\omega\frac{\partial^2 P_2}{\partial T^2}-3\omega^2\frac{\partial P_2}{\partial T}-i\omega^3P_2\right]\\+iP_2^3+iP_1^2P_2-3\beta|s_H|P_2^2\frac{\partial P_2}{\partial T}-i\beta|s_H|\omega P_2^3\end{aligned} \tag{2.21}$$

$$\begin{aligned}\frac{\partial P_2}{\partial Z}-ikP_2=-i\frac{\delta}{2}\left[\frac{\partial^2 P_2}{\partial T^2}+2i\omega P_2-\omega^2P_2\right]+b_3\left[\frac{\partial^3 P_2}{\partial T^3}+3i\omega\frac{\partial^2 P_2}{\partial T^2}-3\omega^2\frac{\partial P_2}{\partial T}-i\omega^3P_2\right]\\+iP_2^3+iP_1^2P_2-3\beta|s_H|P_2^2\frac{\partial P_2}{\partial T}-i\beta|s_H|\omega P_2^3\end{aligned} \tag{2.22}$$

Decomposing the real and imaginary part of Eq. (2.21), we obtain

$$\frac{\partial P_1}{\partial Z}+\left(3b_3\omega^2-\delta\omega\right)\frac{\partial P_1}{\partial T}-b_3\frac{\partial^3 P_1}{\partial T^3}+3\alpha|s_E|P_1^2\frac{\partial P_1}{\partial T}=0 \tag{2.23}$$

$$\left(-k-\omega^2\frac{\delta}{2}+b_3\omega^3\right)P_1+\left(\frac{\delta}{2}-3b_3\omega\right)\frac{\partial^2 P_1}{\partial T^2}+\left(\alpha|s_E|\omega-1\right)P_1^3-P_2^2P_1=0 \tag{2.24}$$

Now, we choose $P_1=A_1Sech[B(T-ZV_0)]$ and $P_2=A_2Sech[B(T-ZV_0)]$, where A_1 and A_2 are the complex amplitudes of the electric and the magnetic fields, B and V_0 are the temporal width and the inverse velocity of the soliton respectively. Substituting P_1 and P_2 in Eq.(2.23) and Eq.(2.24), we obtain

$$\begin{aligned}A_1B\left\{V_0-\left(3b_3\omega^2-\delta\omega\right)+b_3B^2\right\}sech\tau\tanh\tau-6b_3A_1B^3\sec h^3\tau\tanh\tau\\-3\alpha|s_E|A_1^3BSech^3\tau\tanh\tau=0\end{aligned} \tag{2.25}$$

$$\begin{aligned}\left[\left(-k-\omega^2\frac{\delta}{2}+b_3\omega^3\right)+\left(\frac{\delta}{2}-3b_3\omega\right)B^2\right]A_1\sec h\tau-2\left(\frac{\delta}{2}-3b_3\omega\right)A_1B^2\sec h^3\tau\\+\left(\alpha|s_E|\omega-1\right)A_1^3\sec h^3\tau-A_2^2A_1\sec h^3\tau=0\end{aligned} \tag{2.26}$$

Now, Eq.(2.25), the coefficient of the linearly independent functions $sech\tau\tanh\tau$ and $sech^3\tau\tanh\tau$ gives

$$V_0 = 3b_3\omega^2 - \delta\omega - b_3 B^2 \tag{2.27}$$

and

$$2b_3 B^2 + \alpha|s_E|A_1^2 = 0 \tag{2.28}$$

Again, setting the coefficients of the linearly independent functions of the Eq.(2.26) to zero yields

$$k = \left(\frac{\delta}{2} - 3b_3\omega\right)B^2 + b_3\omega^3 - \frac{\delta}{2}\omega^2 \tag{2.29}$$

$$2B^2\left(\frac{\delta}{2} - 3b_3\omega\right) + A_2^2 - \left(\alpha|s_E|\omega - 1\right)A_1^2 = 0 \tag{2.30}$$

Similarly from Eq.(2.22), we obtain the following equations

$$2b_3 B^2 + \beta|s_H|A_2^2 = 0 \tag{2.31}$$

$$2B^2\left(\frac{\delta}{2} - 3b_3\omega\right) + A_1^2 - \left(\beta|s_H|\omega - 1\right)A_2^2 = 0 \tag{2.32}$$

From Eq.(2.28) and Eq.(2.31) , we obtain a constraint relation as:

$$\frac{A_1}{\sqrt{\beta|s_H|}} = \frac{A_2}{\sqrt{\alpha|s_E|}} = m_0 \tag{2.33}$$

where m_0 is a free parameter and this is possible only when both α and β have the same sign. We have obtained various physical parameters related to the solitary wave solutions in terms of the medium parameters as follows

$$B^2 = -m_0^2\alpha\beta\frac{|s_E||s_H|}{2b_3} \tag{2.34}$$

$$A_1 = \left(m_0^2\beta|s_H|\right)^{1/2} \quad A_2 = \left(m_0^2\alpha|s_E|\right)^{1/2} \tag{2.35}$$

and $\omega = \dfrac{\delta}{4b_3} - \left[\dfrac{1}{\beta|s_H|} + \dfrac{1}{\alpha|s_E|}\right]$ and $k = \left(\dfrac{\delta}{2}\omega^2 + b_3\omega^3\right) - \left(\dfrac{\delta}{2} + 3b_3\omega\right)B^2$ (2.36)

The temporal width, the frequency and the wave number of the soliton is given by the Eq.(2.34) and Eq.(2.36) respectively while Eq.(2.35) represents the envelope amplitudes of the soliton. We observe from Eq.(2.33) and Eq.(2.34) that the solitary wave solutions of

Eq.(2.18) and Eq.(2.19) is possible only if we choose the third order dispersion parameter to be negative. In Fig 2.4, we have plotted the intensity profile of solitary wave envelope for both the electric field and the magnetic field for a set of given parameters. We would like to mention that in this thesis, we have not investigated the issue of stability of nonlinear waves. The study of stability of solitary waves in MM itself may turn out to be a very significant area of research.

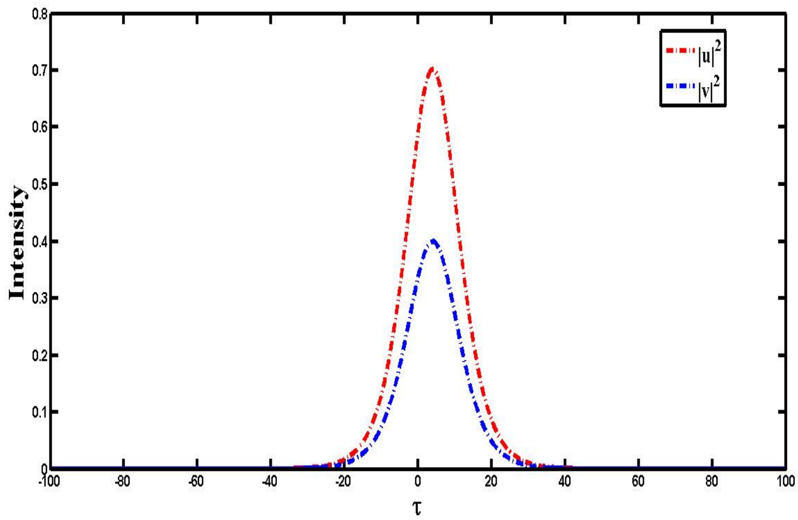

Fig 2.4: Intensity profile of electric and magnetic field envelope with $s_E = 0.057$, $s_H = 0.10$, $m_0 = 0.1$ and $b_3 = -0.01$.

2.5 Chapter Summary

In this chapter, we have derived the generalized coupled nonlinear field equations for pulse propagation in a NIM embedded into a Kerr medium. We consider the lossy Drude model to describe the permittivity and permeability in MM. Our model successfully recovers previously proposed models to describe pulse propagation in MMs exhibiting Kerr nonlinearity. Moreover, it contains a few additional terms like magnetic self-steepening effect connecting both the electric and magnetic field envelopes in an MM. We have also investigated the MI in MMs using the new model under some restrictive conditions. We have found that one could control the MI gain in an MM by tuning the initial electric or magnetic field amplitudes of the pulse. We also have reported an exact solitary wave solution to our proposed model.

Chapter 3

Modulation Instability and solitary wave solutions of the Nonlinear Schrödinger Equation in the context of a non-Kerr medium*

3.1 Introduction

In the previous chapter, solitary waves and modulation instability (MI) was studied in metamaterial (MM) in the context of Kerr nonlinearity. In this chapter we discuss MI and solitary wave solutions of nonlinear Schrödinger equation (NLSE) with reference to an usual non-Kerr medium [2,41,82,153]. Recently, it has been observed that some conventional materials exhibit higher order nonlinear susceptibilities even at moderate pulse intensity. For example, CdS_xSe_{1-x}-doped glass possesses a considerable amount of fifth-order susceptibility $\chi^{(5)}$ [84]. Again it has been demonstrated that transparent glass in intense femtosecond pulse at 620 nm shows significant nonlinear effects due to $\chi^{(5)}$ [85]. A recent experiment even shows that materials such as chalcogenide glass exhibit not only third and fifth order nonlinearities but even seventh-order nonlinearity [95]. Motivated by these developments, in this chapter, we have carried out an analysis to study solitary wave solutions in a non-Kerr medium induced by cubic-quintic nonlinearities. The pulse dynamics in a non-Kerr medium should be described by the extended NLSE, and depending on the sign of group velocity dispersion (GVD), the NLS equation has two types of solutions as bright or dark optical soliton solutions. In optical communication systems, both types of soliton have unique features during propagation. As mentioned in Chapter1, Bright solitons have potential applications in the context of pulse amplification, optical switch and pulse compression while dark solitons cannot be as useful as bright ones in high speed communication systems, but have better stability in presence of perturbation like noise, mutual interaction between two

*Part of the results presented in this chapter have been published in the paper, M. Saha and A. K. Sarma, "Solitary wave solutions and modulation instability analysis of the nonlinear Schrodinger equation with higher order dispersion and nonlinear terms" *Communications in Nonlinear Science and Numerical Simulation*, Vol. 18, p.2420 (2013).

pulses etc. [67,96, 154-157]. Many authors have studied the solitary wave solutions of the extended NLSE for femtosecond pulse propagation in a non-Kerr medium by adopting ansatz solution of Li et al. [77-79]. In this method, the combined solitary wave solutions cannot describe the properties of both bright and dark solitary wave solutions. By means of the coupled amplitude-phase formalism [108,158], we can study the bright and the dark solitary wave solutions individually. Fundamental bright and dark soliton solutions have been obtained for both anomalous and normal dispersion regime using this method in a non-Kerr medium [108,159-160].

In this chapter, we obtain the bright and dark solitary wave solution with the coupled amplitude-phase formulation in a medium with cubic–quintic nonlinearity, fourth order dispersion and self-steepening effect. We have used the higher order nonlinear Schrodinger equation to describe the ultra-short pulse propagation in this medium. An exact solitary wave solution under some restrictive conditions is worked out. We have also carried out an MI analysis both in the anomalous and the normal dispersion regime. The role of higher order dispersion, cubic-quintic nonlinear parameter and the self-steeping parameter on MI gain has been investigated analytically.

3.2 Theoretical Model

The generalized normalized nonlinear Schrodinger equation with cubic-quintic nonlinearity is written as follows [153]:

$$\frac{\partial u}{\partial \xi} = -i\frac{b_2}{2}\frac{\partial^2 u}{\partial \tau^2} + \frac{b_3}{6}\frac{\partial^3 u}{\partial \tau^3} + i\frac{b_4}{24}\frac{\partial^4 u}{\partial \tau^4} + i|u|^2 u + i\sigma|u|^4 u + s\frac{\partial}{\partial \tau}(|u|^2 u) \tag{3.1}$$

Here $u(\xi,\tau)$ represents the normalized slowly varying envelope equation of the electric field. ξ and τ are the normalized distance along the fiber and time with the frame of the reference moving along the fiber at group velocity respectively. Here, $b_2 = \mathrm{sgn}(\beta_2)$, $b_3 = \beta_3/|\beta_2|T_0$, $b_4 = \beta_4/|\beta_2|T_0^2$, $\sigma = \gamma_2 P_0/\gamma_1$ and $s = 1/\omega_0 T_0$ with β_2, β_3 and β_4 are the so-called group velocity dispersion (GVD), the third order dispersion (TOD) and the fourth order dispersion (FOD) parameter and s is the self-steepening parameter [2]. γ_1 and γ_2 are the cubic and quintic nonlinearity parameters respectively. T_0 is the initial pulse width and P_0 is the optical power of the pulse. In order to solve Eq.(3.1), we take $u(\xi,\tau) = q(\xi,\tau)\, e^{i\,\phi(\xi,\tau)}$,where $q(\xi,\tau)$ is the complex envelope function and $\phi(\xi,\tau)=k\xi-\Omega\tau$ is the linear phase shift function with k and Ω are the normalized wave vector and frequency respectively. Substituting the above ansatz in Eq. (3.1) we obtain:

$$\frac{\partial q}{\partial \xi}+a_1\frac{\partial q}{\partial \tau}+ia_2\frac{\partial^2 q}{\partial \tau^2}-a_3\frac{\partial^3 q}{\partial \tau^3}-ia_4\frac{\partial^4 q}{\partial \tau^4}+ia_5 q-i|q|^2q-i\sigma|q|^2q -s\left[-i\Omega|q|^2q+2|q|^2\frac{\partial q}{\partial \tau}+q^2\frac{\partial q^*}{\partial \tau}\right]=0 \quad (3.2)$$

with

$$a_1=b_2\Omega+\frac{b_3}{2}\Omega^2+\frac{b_4}{6}\Omega^3 \quad , \quad a_2=\frac{1}{2}\left(b_2+b_3\Omega+\frac{b_4}{2}\Omega^2\right) \quad , \quad a_3=\frac{1}{6}(b_3+b_4\Omega)$$
$$a_4=\frac{b_4}{24} \quad , \quad a_5=k-\left(\frac{b_2}{2}\Omega^2+\frac{b_3}{6}\Omega^3+\frac{b_4}{24}\Omega^4\right)$$

3.3 Solitary wave solutions

For bright soliton, we take the ansatz function as $q(\xi,\tau)=B\sec h[W(\tau-\xi/V)]$, where B, W and V are the amplitude , the pulse width and the velocity of soliton respectively in normalized units. Putting the ansatz in Eq.(3.2) and separating the real and imaginary part we obtain:

$$\left(\frac{BW}{V}-a_1BW+a_3BW^3\right)\sec h\chi\tanh\chi+\left(-6a_3BW^3+3sB^3W\right)\sec h^3\chi\tanh\chi=0 \quad (3.3)$$

$$\left(a_2BW^2 - a_4BW^4+a_5B\right)\sec h\chi+\left(-2a_2BW^2+20a_4BW^4+(s\Omega-1)B^3\right)\sec h^3\chi -\left(24a_4BW^4+\sigma B^5\right)\sec h^5\chi=0 \quad (3.4)$$

Now, equating the coefficients of the linearly independent terms to zero we get four independent equations and then imposing some constraints we obtain all the solitary wave parameters as follows:

$$\Omega=\frac{1}{b_4}\left\{-b_3+3s\left(\frac{24|a_4|}{\sigma}\right)^{1/4}\right\} \quad , \quad W^2=\frac{1}{20a_4}\left[2a_2-(s\Omega-1)\left(\frac{24|a_4|}{\sigma}\right)^{1/2}\right] \quad , \quad B^4=-\frac{24a_4}{\sigma}W^4$$
$$V=\left(a_1-4a_3W^2\right)^{-1} \quad , \quad k=a_4W^4-a_2W^2+\frac{b_2}{2}\Omega^2+\frac{b_3}{6}\Omega^3+\frac{b_4}{24}\Omega^4 \quad (3.5)$$

In order to have a physical understanding of the solution we have chosen the following parameters: $\beta_2=-50\,ps^2/km$, $\beta_3=0.5\,ps^3/km$, $\beta_4=-0.05\,ps^4/km$ and $\gamma_1=2736\,W^{-1}km^{-1}$,

$\gamma_2 = 2.63W^{-2}km^{-1}$,$T_0 = 10\,fs$ and $P_0 = 500\,W$ [162]. In Fig.3.1(a), we have plotted the intensity of the bright soliton for different normalized quintic parameter σ at s=0.02. It is observed that with the increase of the quintic parameter, the amplitude of the soliton decreases while the width of the soliton decreases slightly. In our analysis we have obtained that it is possible to have bright solitary wave solution in the presence of β_4 with the constraint that the product of β_4 and the quintic nonlinear parameter σ have to be less than zero. The expression obtained for soliton amplitude B in Eq.(3.5) imposes the constraint that in order to have bright solitary wave solution of Eq. (3.2) a_4 and σ should be opposite in sign. Since we are considering the focusing cubic-quintic nonlinear medium, we have to take the FOD parameter to be negative. In Fig.3.2(b) we have studied the effect of the SS parameter on the intensity profile of the bright soliton. It is observed that the intensity of the soliton remains constant while the width of the soliton decreases.

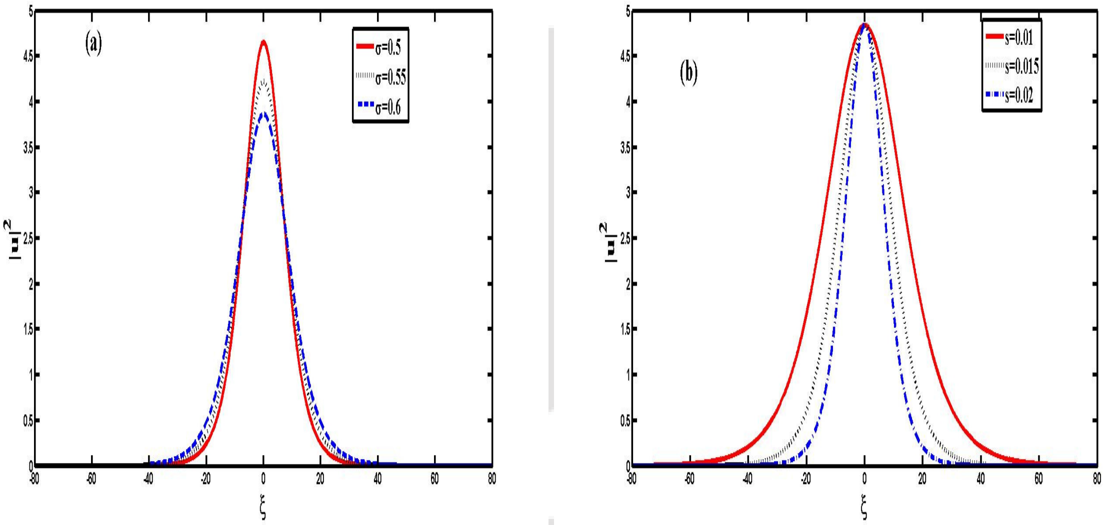

Fig.3.1: (a) Intensity profile of bright solitary waves in cubic-quintic medium for different quintic parameters with $s = 0.02$ (b) Intensity profile of bright solitary waves in cubic-quintic medium with different self-steeping parameter at $\tau = 0$.

For dark soliton, we take the ansatz function as $q(\xi,\tau) = B\tanh[W(\tau - \xi/V)]$, where B, W and V are the amplitude, pulse width and velocity of the soliton respectively. Putting the ansatz in Eq.(3.2) and separating the real and imaginary parts we obtain:

$$\left(-\frac{BW}{V}+a_1BW-4a_3BW^3-3sB^3W\right)\sec h^2\chi+\left(6a_3BW^3+3sB^3W\right)\sec h^4\chi=0 \tag{3.6}$$

$$\begin{aligned}\left(-2a_2BW^2-16a_4BW^4+a_5B\right)\tanh\chi+\left(2a_2BW^2+40a_4BW^4+(s\Omega-1)B^3\right)\tanh^3\chi\\ -\left(24a_4BW^4+\sigma B^5\right)\tanh^5\chi=0\end{aligned} \tag{3.7}$$

In order find the soliton parameters we follow the same procedure as that of the bright soliton case. We need to impose some constraints on the parametric equations to make them compatible. We have obtained all the solitary wave parameters in terms of the model parameters, along with the constraint $\sigma a_4<0$, as follows:

$$\begin{aligned}&\Omega=\frac{1}{b_4}\left\{-b_3-3s\left(\frac{24|a_4|}{\sigma}\right)^{1/4}\right\},\quad W^2=-\frac{1}{40a_4}\left[(s\Omega-1)\left(\frac{24|a_4|}{\sigma}\right)^{1/2}+2a_2\right],\quad B=\left(\frac{24|a_4|}{\sigma}\right)^{1/4}W,\\ &V=\left(a_1-4a_3W^2-3sB^2\right)^{-1},\quad k=2a_2W^2+16a_4W^4+\frac{b_2}{2}\Omega^2+\frac{b_3}{6}\Omega^3+\frac{b_4}{24}\Omega^4\end{aligned} \tag{3.8}$$

In Fig.3.2(a) we have plotted the intensity of dark soliton for different quintic parameters at $s=0.02$. It is observed that with the increase of the quintic parameter the amplitude of the soliton decreases and the width of the soliton decreases. In Fig.3.2(b) we have studied the effect of self-steepening parameter on the intensity profile of dark soliton. It is observed that the intensity of the soliton remains constant but the width of the soliton decreases with the increase of the SS parameter. It is interesting to note that the bright and dark solitary wave solutions reported by us is valid in both normal and anomalous dispersion regime.

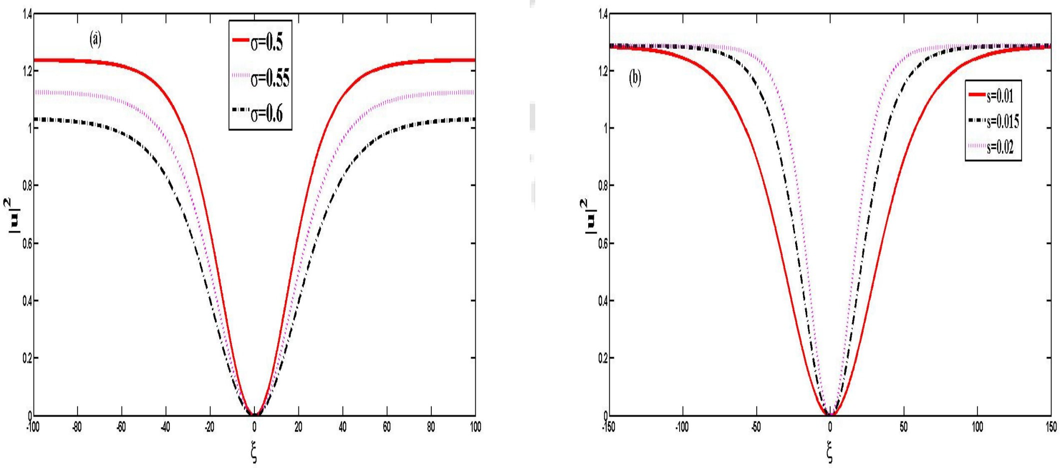

Fig.3.2: (a) Intensity profile of dark solitary waves in cubic-quintic medium with different quintic parameter with $s = 0.02$ (b) Intensity profile of dark solitary waves in cubic-quintic medium with different self-steeping parameter at $\tau = 0$.

3.4 Modulation Instability analysis in cubic-quintic medium

In the context of non-Kerr media, MI study has turned out to be an extremely exciting field of research for investigation of optical pulse propagation in higher order nonlinear medium [153, 161-162].We would now, investigate the MI of Eq.(3.1) by using the standard linear stability analysis. Eq.(3.1) has a steady state solution given by $u = \sqrt{P}\exp\left[i(P + P^2\sigma)\,\xi\right]$,where P is the normalized optical power. We introduce the perturbation $a(\xi,\tau)$, such that $a(\xi,\tau) << \sqrt{P}$, together with the steady state solution to Eq. (3.1) and linearize in $a(\xi,\tau)$ to obtain:

$$\frac{\partial a}{\partial \xi} = -\frac{i b_2}{2}\frac{\partial^2 a}{\partial \tau^2} + \frac{b_3}{6}\frac{\partial^3 a}{\partial \tau^3} + \frac{i b_4}{24}\frac{\partial^4 a}{\partial \tau^4} + i P(a + a^*) + 2 i \sigma P^2 (a + a^*) - s P\left(2\frac{\partial a}{\partial \tau} + \frac{\partial a^*}{\partial \tau}\right) \tag{3.9}$$

Now, writing $a = a_1 e^{i(K\xi - \Omega\tau)} + a_2 e^{-i(K\xi - \Omega\tau)}$, where K and Ω are the normalized wave number and the normalized frequency of the perturbation respectively. From Eq. (3.9) we obtain the following dispersion relation:

$$K = \frac{b_3}{6}\Omega^3 - 2sP\Omega \pm \sqrt{\Omega^4 A + \Omega^2 B} \tag{3.10}$$

Here, $A = \left(\frac{b_2}{2} + \frac{b_4}{24}\Omega^2\right)^2$ and $B = \left[2(\frac{b_2}{2} + \frac{b_4}{24}\Omega^2)(P + 2P^2\sigma) + s^2 P^2\right]$

The steady-state solution becomes unstable whenever K has an imaginary part since the perturbation then grows exponentially. One can easily see that for the occurrence of MI we must have $\Omega^2 A + B < 0$. Under this condition, the growth rate of MI gain spectrum $g(\Omega)$ could be expressed as:

$$g(\Omega) = 2\,\mathrm{Im}(K) = 2|\Omega|\sqrt{\Omega^2 A + B} \tag{3.11}$$

Now, we investigate the impact of the higher order dispersion effect and the nonlinear effects on the MI gain. In Fig.3.3(a) we have plotted the variation of the MI gain with perturbation frequency Ω and the FOD parameter β_4. We have observed that in the anomalous dispersion regime we obtain two sidebands. The maximum MI gain increases in $\beta_4 < 0$ region. In the $\beta_4 > 0$ region, MI gain decreases with increase of the β_4 parameter. In Fig.3.3(b) we have observed that in the normal dispersion region we also get two side bands but the side bands vanish when $\beta_4 > 0$. It also means that in the normal dispersion region, MI is not possible in $\beta_2, \beta_4 > 0$ region.

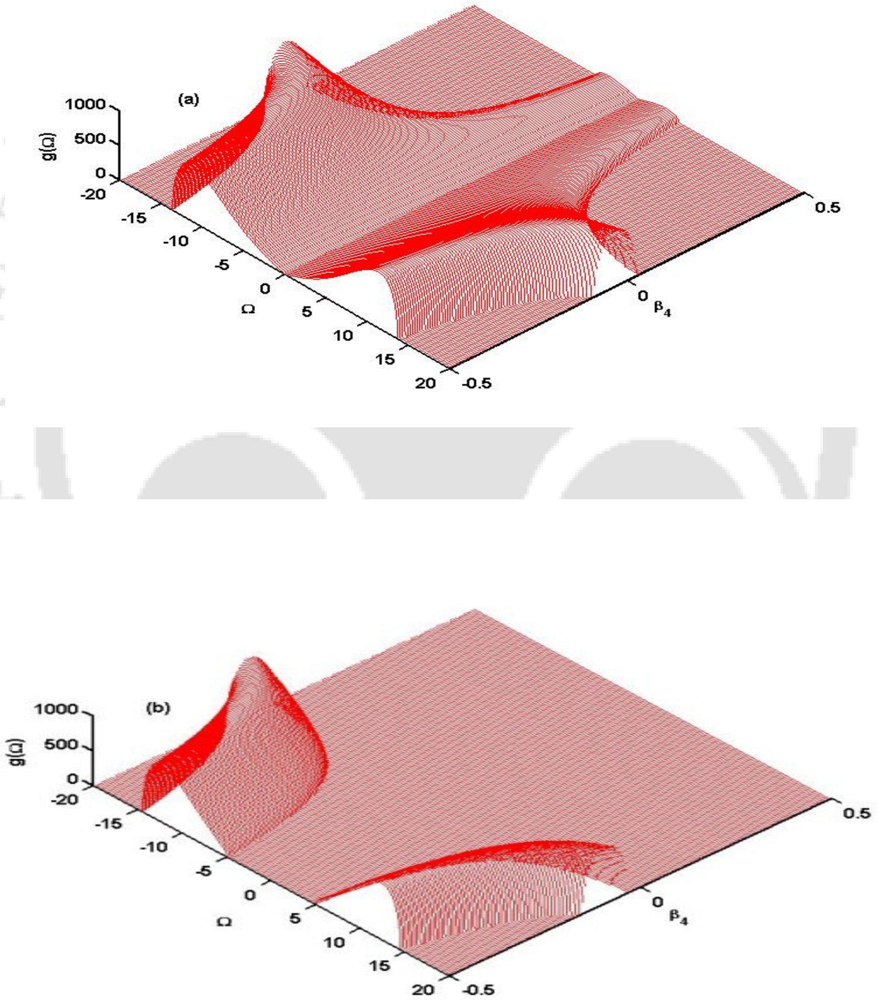

Fig.3.3: Modulation Instability gain as a function of Ω and β_4 (a) $\beta_2 = -50\ ps^2/km$ and s=0.02 (b) $\beta_2 = 50\ ps^2/km$ and s=0.02.

Finally, in Fig.3.4(a) we depict the variation of the MI gain as a function of Ω and σ. It is observed that the intensity of the MI gain increases with the increase of the cubic- quintic parameter σ and the bandwidth of the side bands also increases. Hence the higher order nonlinear effect increases the MI gain in the anomalous dispersion region as we have observed. The MI gain as a function of Ω and s is plotted in Fig. 3.4(b).We observe that two distinct side bands appear in the spectrum, however, the intensity of the MI gain remains nearly the same with increase of the self-steepening parameter. Hence, we find that in the anomalous dispersion regime both bright and dark solitary wave solutions may be possible subject to the constraint $\beta_4 \sigma < 0$. It implies that in the anomalous dispersion regime with focusing medium, the fourth order dispersion term has to be negative in order to get the exact bright or dark solitary wave solution.

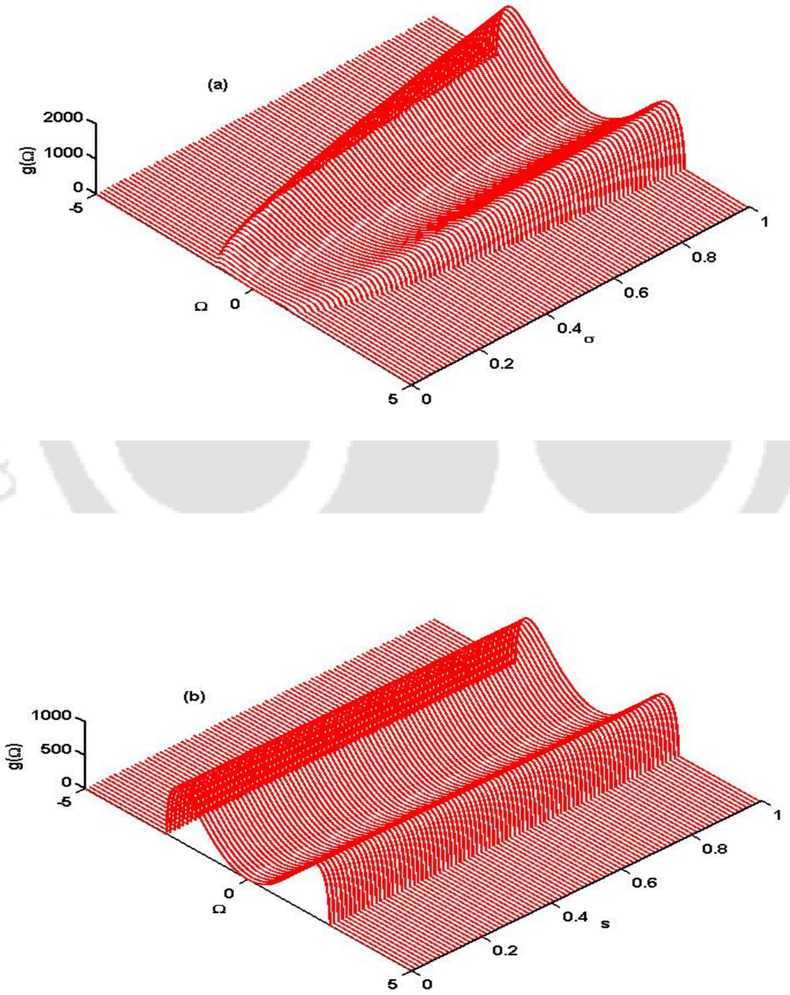

Fig.3.4: In the anomalous dispersion regime (a) MI gain as a function perturbation frequency Ω and quintic parameter σ (b) MI gain as a function perturbation frequency Ω and self-steepening parameter s.

3.5 Chapter Summary

In this chapter, we have studied the solitary wave solutions and MI analysis of the higher order NLSE that describes the ultra-short femto second pulse propagation in a non-Kerr medium. We have obtained the exact bright and dark solitary wave solutions by phase amplitude ansatz method. Further, we have also carried out an MI analysis of the higher order nonlinear Schrödinger equation and we observe that in the anomalous dispersion regime, MI is possible for both positive and negative value of β_4. On the contrary, in the normal dispersion regime, MI occurs only when $\beta_4 < 0$. The role of the cubic-quintic parameter and the self-steepening parameter on MI gain has also been discussed.

Chapter 4

Modulation instability in Metamaterials embedded into a non-Kerr medium*

4.1 Introduction

In the last chapter we discussed modulation instability (MI) in the context of a conventional non-Kerr medium. This chapter is aimed at studying MI in a metamaterial (MM) embedded into a non-Kerr medium. In conventional materials, the optical response cannot be tuned, but it is possible to do so in a MM through judicious structure design and engineering the nonlinearity appropriately [87]. We have derived an appropriate mathematical model starting with the well-known Maxwell's equations to take into account the effect of higher order dispersion upto fourth order along with nonlinear susceptibilities owing to $\chi^{(3)}$ and $\chi^{(5)}$. The model successfully reproduces previously proposed similar models under appropriate approximations from our newly derived propagation equation. It should be noted that so far many models related to the pulse propagation in nonlinear negative index material (NIM) have been proposed, including the interaction of ultrashort pulses with MMs [91-92,139-147]. Recently, a generalized nonlinear Schrodinger equation (GNLSE) suitable for few-cycle pulse propagation in the MM with delayed Raman response is reported [147]. In this work, however, we would mainly be concerned with effects arising owing to cubic-quintic nonlinearities in MMs. As stated earlier, MI study in the context of MM has turned out to be an extremely exciting field of research. One primary reason among many others is related to the fact that MI could be considered as a precursor to soliton formation and MM may offer new ways to generate and control bright and dark solitons [148-149,163-165]. MI in MM in the presence of cubic-quintic nonlinearities and higher order dispersive effects has a better controllability compared to the common nonlinear materials. Another important feature of MI

*Part of the results presented in this chapter have been published in the paper , M. Saha and A. K. Sarma, "Modulation instability in nonlinear metamaterials induced by cubic-quintic nonlinearities and higher order dispersive effects" *Optics Communications* vol. 291, p.321 (2013) in MM is that it may be triggered in both the normal and the anomalous dispersion regimes with judicious choice of the parameters.

In this chapter, we have derived an appropriate mathematical model to describe the pulse propagation with higher order nonlinearity like cubic-quintic nonlinearity and higher order dispersion effects in MM. We consider that the MM is embedded into a nonlinear material induced by cubic-quintic nonlinearities or inserting some nonlinear cubic-quintic non-Kerr type dielectric material inside the gap of the split ring resonator (SRR) path. When the unit size of the microstructures is small compare to the characteristics wavelength of the electromagnetic radiation, the effective medium approach is applicable to both linear and nonlinear MMs. In this approach, the artificial structure is treated as a homogeneous isotropic medium. We have included loss into account in our analysis and found that loss distorts the sidebands of the MI gain spectrum. We find that the combined effect of cubic-quintic nonlinearity increases the MI gain. The role of higher order nonlinear dispersive effects on MI has also been discussed.

4.2 Theoretical Model

The pulse propagation equation in nonlinear MM with both $\chi^{(3)}$ and $\chi^{(5)}$ nonlinearities are obtained by generalizing the derivation of the so called Nonlinear Schrodinger equation (NLSE) based on previous works [91-92]. It is worthwhile to mention that one of the most fundamental differences between an ordinary material and an MM is that MM exhibits strong dispersive behavior in electric permittivity and magnetic permeability, while, in ordinary materials, only one of them is dominant at a time. We consider that the MM, embedded in cubic-quintic non-Kerr medium, is characterized by the nonlinear electric polarization ($\mathbf{P}_{NL}$) as $\mathbf{P}_{NL} = \varepsilon_{NL}\mathbf{E} = \varepsilon_0\chi^{(3)} |E|^2 \mathbf{E} + \varepsilon_0\chi^{(5)} |E|^4 \mathbf{E}$; where $\mathbf{E}$ is the electric filed, ε_{NL} is the nonlinear electric permittivity and $\chi^{(n)}$ is the n-th order electric susceptibility. The dielectric permittivity (ε) and magnetic permeability (μ) are dispersive in MMs and their frequency dispersion is given by Eq.(3.1) as from the so called lossy Drude model [133]. The recent work of Stockman [152] shows that loss is an extremely important issue in MMs and in this work we are taking the electric and magnetic normalized loss parameter as $\tilde{\gamma}_e \sim \tilde{\gamma}_m \sim 0.01$, a value two orders of magnitude greater than that of the one taken by D'Aguanno et al. [133,152].

Now, starting with Maxwell's equations and adopting a procedure similar to that of Ref. [92], we obtain the following one dimensional pulse propagation equation in a nonlinear negative index metamaterial:

$$\frac{\partial A}{\partial \xi} = -\frac{i\beta_2}{2}\frac{\partial^2 A}{\partial \tau^2} + \frac{\beta_3}{6}\frac{\partial^3 A}{\partial \tau^3} + \frac{i\beta_4}{24}\frac{\partial^4 A}{\partial \tau^4} + i\sigma_0\left(|A|^2 A + iS_1\frac{\partial}{\partial \tau}\left(|A|^2 A\right) - S_2\frac{\partial^2}{\partial \tau^2}\left(|A|^2 A\right)\right) + i\eta_0'|A|^4 A - \eta_0 S_3\frac{\partial}{\partial \tau}\left(|A|^4 A\right) \tag{4.1}$$

In Eq. (4.1), A is the slowly varying envelope of the pulse, propagating along the ξ direction. β_2, β_3 and β_4 are the group velocity dispersion(GVD), the third order dispersion(TOD) and the fourth order dispersion(FOD) parameters respectively, while σ_0 and η_0' are the cubic and quintic nonlinear coefficients respectively. S_1 is the so-called self-steepening (SS) parameter due to cubic nonlinear polarization while S_3 is the SS parameter due to quintic nonlinear polarization. On the other hand, S_2 is the second order nonlinear dispersive co-efficient. The above mentioned parameters are defined as follows:

$$\beta_2 = \delta_2/2k_0 - 1/k_0V^2;\ \beta_3 = \delta_3 - 3\beta_2/k_0V;\ \beta_4 = \delta_4 - 3\beta_2^2/k_0 - 4\delta_3/k_0V;$$
$$S_1 = 1/\omega_0 - i\sigma_1/\sigma_0 - 1/k_0V;\ S_2 = -\sigma_2/\sigma_0 - i\sigma_1/\sigma_0\omega_0 - \beta/4k_0 - 1/\omega_0k_0V + i\sigma_1/k_0V\sigma_0; \tag{4.2}$$
$$S_3 = 1/\omega_0 - i\eta_1/\eta_0 - 1/k_0V$$

with

$$\delta_m = \frac{m!}{2k_0}\sum_{l=0}^{m}\frac{1}{l!m-l!}\frac{\partial^l(\omega\varepsilon)}{\partial\omega^l}\bigg|_{\omega=\omega_0}\frac{\partial^{m-l}(\omega\mu)}{\partial\omega^{m-l}}\bigg|_{\omega=\omega_0};\ V = 2k_0/\omega_0(\varepsilon(\omega)\gamma + \mu(\omega)\alpha);$$

$$\sigma_m = m!\omega_0\varepsilon_0\chi^{(3)}F_m/2k_0;\ \eta_m = m!\omega_0\varepsilon_0\chi_E^{(5)}F_m/2k_0, \tag{4.3}$$

where $F_m = \frac{i^m}{m!}\frac{\partial^m(\omega\mu)}{\partial\omega^m}\bigg|_{\omega=\omega_0}$, $k_0 = n\omega_0/c$, $\alpha = \partial\{\omega\varepsilon(\omega)\}/\partial\omega|_{\omega=\omega_0}$ and $\gamma = \partial\{\omega\mu(\omega)\}/\partial\omega|_{\omega=\omega_0}$

The non-SVEA correction terms to first order is approximated as:

$$\frac{\partial^2 A}{\partial\xi^2} \approx -\frac{\beta_2^2}{4}\frac{\partial^4 A}{\partial\tau^4} - \sigma_0^2|A|^4 A + \frac{\sigma_0\beta_2}{2}\frac{\partial^2}{\partial\tau^2}|A|^2 A$$
$$\frac{\partial^2 A}{\partial\xi\partial\tau} \approx -\frac{i\beta_2}{2}\frac{\partial^3 A}{\partial\tau^3} + \frac{\delta_3}{6}\frac{\partial^4 A}{\partial\tau^4} + i\sigma_0\frac{\partial}{\partial\tau}|A|^2 A + i\eta_0\frac{\partial}{\partial\tau}|A|^4 A - \frac{\sigma_0}{\omega_0}\frac{\partial^2}{\partial\tau^2}|A|^2 A + i\sigma_1\frac{\partial^2}{\partial\tau^2}|A|^2 A \tag{4.4}$$

It is clear that S_2 term arises, due to the dispersive nature of electric permittivity and magnetic permeability, which is not present in conventional materials. It is worthwhile to mention that the FOD parameter β_4 contains one extra term while the second order nonlinear dispersive parameter S_2 has two additional terms as compared to the ones considered in an earlier work [144]. This difference could be attributed to the fact that, in this work, we are considering both the cubic and the quintic nonlinearities. In the expression for β_4, we are getting an additional term due to the TOD effect. In our model equation, we are getting one additional term associated with S_3 which represents the self-steepening effect arising due to quintic nonlinearity. In Fig.4.1, we plot the variation of the refractive index n, β_2, β_3 and β_4 with the normalized frequency ω_0 / ω_{pe} for $\omega_{pm} / \omega_{pe} = 0.8$. Similar plots are depicted for σ_0 and η_0' in Fig. 4.2(a), and for S_1 and S_2 in Fig. 4.2(b). All the parameters are calculated at

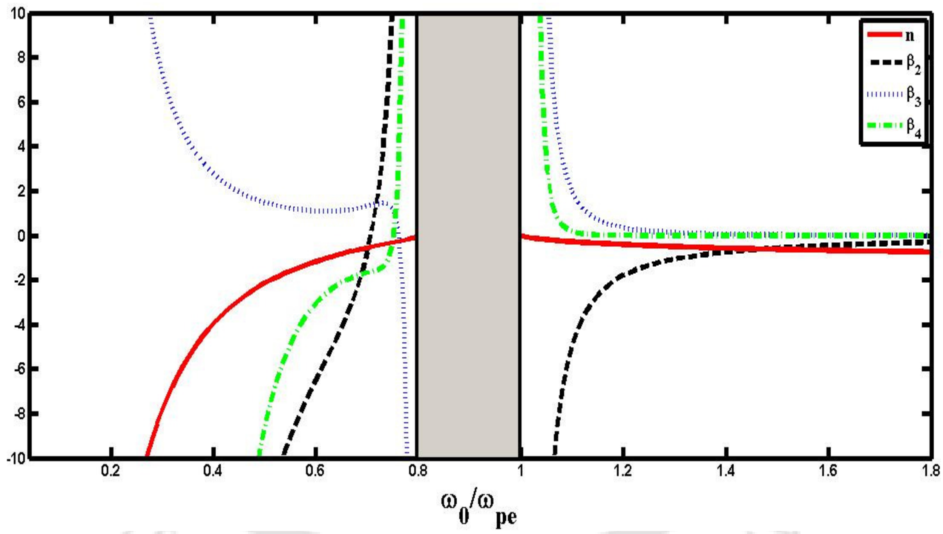

Fig.4.1: Plot of the refraction index (n), group velocity dispersion (β_2), the third order dispersion parameter (β_3) and the fourth order dispersion parameter (β_4) versus the normalized frequency ω/ω_{pe} with $\tilde{\omega}_{pm} = 0.8$.

$\omega = \omega_0$ and we have taken $\tilde{\gamma}_e \sim \tilde{\gamma}_m \sim 0.01$. It should be noted that β_2, β_3 and β_4 are plotted in the units of $1/c\omega_{pe}$, $10^2/c\omega_{pe}^2$ and $10^3/c\omega_{pe}^3$ respectively while σ_0 is calculated in units of $\omega_{pe}\chi^{(3)} / c$ and η_0' is calculated in units of $\omega_{pe}(\chi^{(3)})^2 / c$. We have chosen $\left(\chi^{(3)}\right)^2 \approx \chi^{(5)}$ in Fig.4.2(a). S_1, S_2 and S_3 are calculated in the units of $1/\omega_{pe}$, $1/\omega_{pe}^2$ and $1/\omega_{pe}$ respectively.

We have studied the parameters both in the negative and the positive index regime. In the negative index region, $0 < \omega_0/\omega_{pe} < 0.8$, $\beta_2, \beta_3, \beta_4, S_1$ and S_2 can be either positive or negative, but in the positive index region, $\omega_0/\omega_{pe} > 1$, β_2, β_4, S_1 and S_2 are always negative and β_3 is always positive. Both σ_0 and η_0' are positive in the negative and the positive index region. The behavior of S_3 is similar to that of the self-steepening parameter S_1, as could be seen from Eq.(4.2). However, the only difference is that S_3 arises due to quintic nonlinear polarization, while S_1 results due to the cubic nonlinearity.

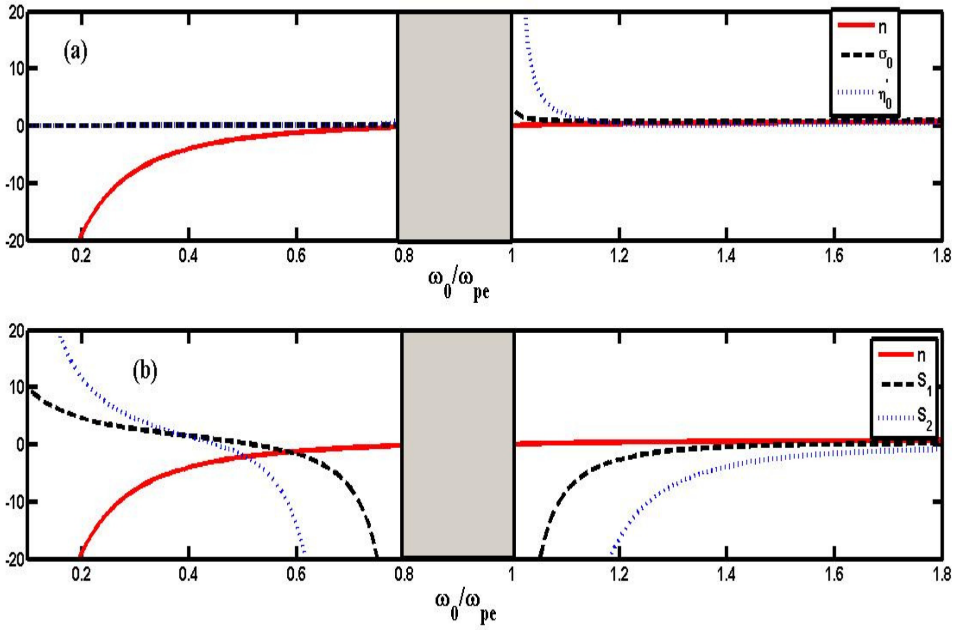

Fig.4.2: (a) Plot of the refraction index (n),the third order nonlinear polarization parameter(σ_0) and the fifth order nonlinear polarization parameter (η_0') versus the normalized frequency ω/ω_{pe} with $\tilde{\omega}_{pm} = 0.8$ b) Plot of refraction index (n),the first order SS parameter(S_1) and second order nonlinear parameter(S_2) versus the normalized frequency ω/ω_{pe} with $\tilde{\omega}_{pm} = 0.8$.

In our analysis the second order nonlinear dispersion term S_2 contains two additional terms compare with the earlier results [144] which arises due to the proper approximation of the non-SVEA correction terms. As a result of this as the normalized frequency increases S_2 parameter decreases from positive to negative value in the negative index region as shown in Fig.4.2(b).

For the purpose of simplification it is convenient to write Eq. (4.1) in the normalized form. We assume the normalized variables as : $Z = \xi / L_D$, $T = \tau / T_0$, $A = \sqrt{P_0} U$, $u = N_0 U$. Here $L_D = T_0^2 / |\beta_2|$ is the so-called second order dispersion length, T_0 is the pulse width. We can similarly define the third and the fourth order dispersion length as $L_D' = T_0^3 / |\beta_3|$ and $L_D' = T_0^4 / |\beta_4|$ respectively. N_0 is termed as the order of the soliton, defined as $N_0^2 = L_D / L_{NL}$ with $L_{NL} = 1/\sigma_0 P_0$, the so-called nonlinear length.

Eq. (4.1) can thus be written in the dimensionless units as follows:

$$\frac{\partial u}{\partial Z} = -\frac{i b_2}{2}\frac{\partial^2 u}{\partial T^2} + \frac{b_3}{6}\frac{\partial^3 u}{\partial T^3} + \frac{i b_4}{24}\frac{\partial^4 u}{\partial T^4} + i |u|^2 u + i p |u|^4 u - s_1 \frac{\partial}{\partial T}\left(|u|^2 u\right) - i s_2 \frac{\partial^2}{\partial T^2}\left(|u|^2 u\right) - s_3 p' \frac{\partial}{\partial T}\left(|u|^2 u\right) \tag{4.5}$$

Here $b_2 = \text{sgn}(\beta_2)$,$b_3 = L_D / L_D'$ and $b_4 = L_D / L_D''$. $s_1 = |S_1| / T_0$ and $s_2 = |S_2| / T_0^2$ are the normalized SS parameter and second order normalized nonlinear dispersion parameter respectively, while $s_3 = |S_3| / T_0$ is the normalized SS parameter due to quintic nonlinearity. $p = \eta_0' / \sigma_0^2 L_D - 1/2k_0 L_D$ is the normalized quintic nonlinear parameter and $p' = \eta_0' / \sigma_0^2 L_D$.

4.3 Modulation Instability analysis

MI in MM in the presence of cubic-quintic nonlinearities and higher order dispersive effects has a better controllability compared to the common nonlinear materials. Another important feature of MI in MM is that it may be triggered in both the normal and the anomalous dispersion regimes with judicious choice of the parameters. We would now, on the basis of Eq. (4.5), investigate the MI by using the standard linear stability analysis [2]. Eq. (4.5) has a steady state solution given by $u = \sqrt{P} \exp\left[i(P + P^2 p)Z\right]$, where P is the normalized optical power. We introduce perturbation $a(Z,T)$, such that $a(Z,T) << \sqrt{P}$, together with the steady state solution to Eq. (4.5) and linearize in $a(Z,T)$ to obtain:

$$\frac{\partial a}{\partial Z} = -\left(\frac{b_2}{2} + 2 s_2 P\right)\frac{\partial^2 a}{\partial T^2} + \frac{b_3}{6}\frac{\partial^3 a}{\partial T^3} + \frac{b_4}{24}\frac{\partial^4 a}{\partial T^4} + iP(a + a^*) + 2i p P^2 (a + a^*) - (2 s_1 P + 3 s_3 p' P^2)\frac{\partial a}{\partial T} + (s_1 P + 2 s_3 p' P^2)\frac{\partial a^*}{\partial T} - i s_2 P \frac{\partial^2 a^*}{\partial T^2} \tag{4.6}$$

Now, writing $a = a_1 e^{i(KZ-\Omega T)} + a_2 e^{-i(KZ-\Omega T)}$, where K and Ω are the normalized wavenumber and the frequency of perturbation respectively, from Eq. (4.6) we obtain the following dispersion relation:

$$K = \frac{b_3}{6}\Omega^3 + (2s_1 P + 3s_3 p' P^2) \pm \sqrt{\Omega^4 M + \Omega^2 N}. \tag{4.7}$$

Here, $M = \left(\frac{b_2}{2} + 2s_2 P + \frac{b_4}{24}\Omega^2\right)^2 - s_2^2 P^2$ and

$$N = \left[2(\frac{b_2}{2} + 2s_2 P + \frac{b_4}{24}\Omega^2)(P + 2P^2 p) - 2s_2 P(P + 2P^2 p) + (s_1 P + 2s_3 p' P^2)^2\right]$$

One can easily see that for the occurrence of MI we must have $\Omega^2 M + N < 0$ since perturbation then grows exponentially. Under this condition, the gain spectrum $g(\Omega)$ of the MI could be expressed as:

$$g(\Omega) = 2\,\mathrm{Im}(K) = 2|\Omega|\sqrt{N + \Omega^2 M} \tag{4.8}$$

The MI gain spectrum depends on the parameters M and N, which in turn depend on parameters like P, s_2, s_3, p etc. Hence, MI in MM in the presence of cubic-quintic nonlinearities and higher order dispersive effects has a better controllability compared to the common nonlinear materials.

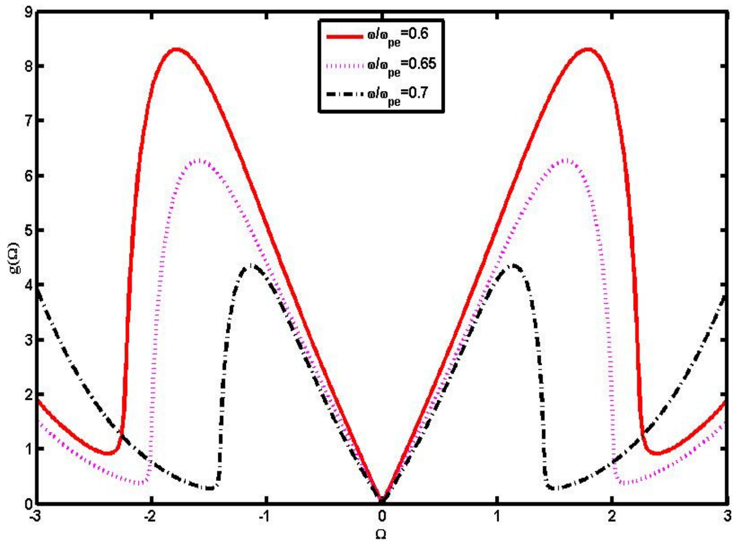

Fig.4.3: MI gain as a function of perturbation frequency Ω for different values of ω/ω_{pe} with $\gamma_e = 0.01$.

In Fig.4.3 we show the variation of the MI gain with normalized perturbation frequency for different values of the normalized frequency ω/ω_{pe} with $\gamma_e = 0.01$. We observe that the MI gain amplitude as well as the gain bandwidth is reduced with increase of the normalized frequency ω/ω_{pe}. The appearance of distortions in the sidebands could be attributed to the presence of linear loss. The influence of loss on MI is briefly discussed in a separate section.

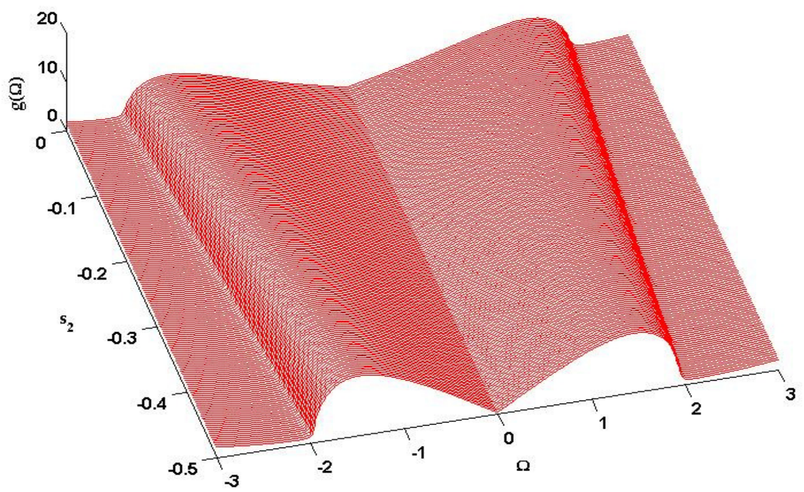

Fig.4.4: MI gain spectrum with normalized second order self-steepening parameter s_2 and perturbation frequency Ω for P=2 and s_3=0 in the anomalous dispersion region.

The role of the SS parameter s_1 on MI has been studied earlier and it was shown that MI gain decreases and the gain band width also shrinks owing to self-steepening effect and it does not depend on the sign of the SS parameter s_1 [144]. In Fig. 4.4 we depict the role of the second order nonlinear dispersion parameter s_2 on MI gain with $P = 2$, $s_1 = -0.168$ and $s_3 = 0$ for a pulse width $T_0 = 10 fs$ and $\omega_0 / \omega_{pe} = 0.6$. We notice that as s_2 parameter increases, the MI gain decreases while the bandwidth enlarges. We have also investigated the role of the nonlinear parameter p, which results due to the combined effect of cubic-quintic nonlinearities, on the MI gain spectrum. From Fig.4.5 (a) we observe that MI gain as well as the gain band width increases with the increase of the nonlinear parameter p. This could also be seen clearly from Fig. 4.5 (b). The effect of s_3, which arises due to quintic nonlinearity, on

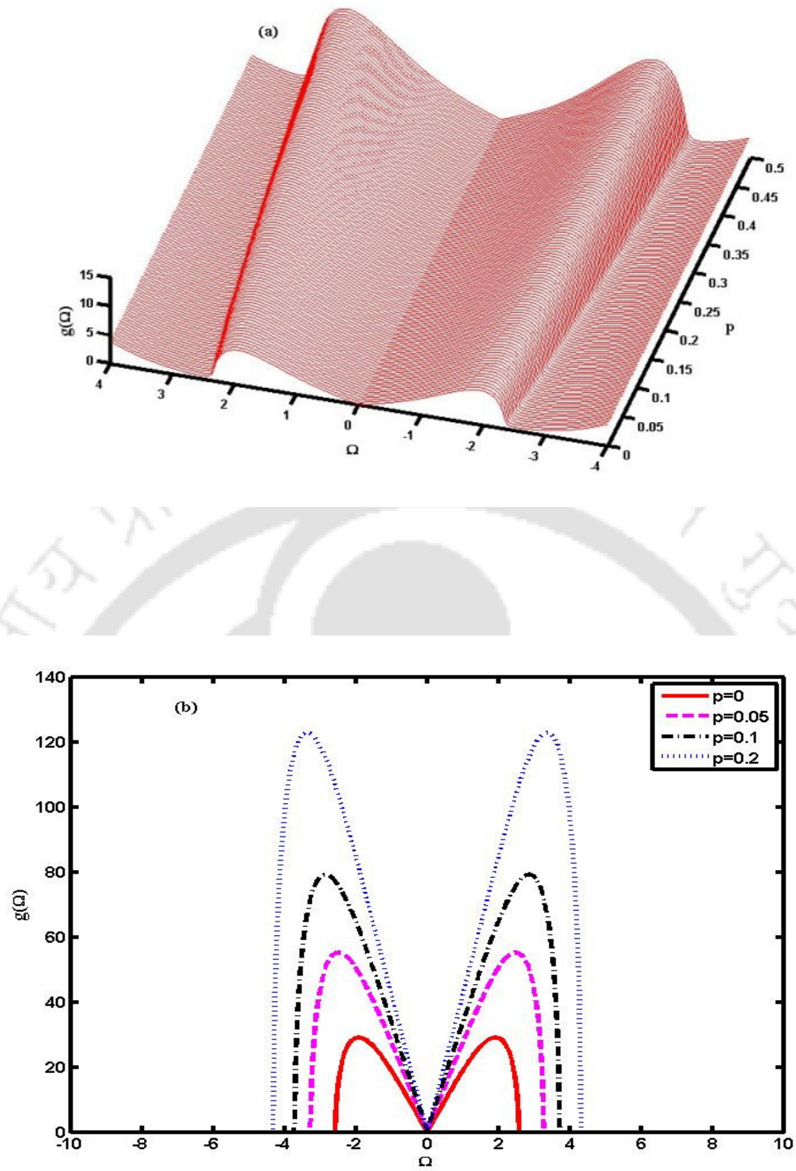

Fig.4.5 (a) MI gain as a function of perturbation frequency Ω and higher order nonlinear parameter p with fixed value of s_2. (b) MI gain as a function of higher order nonlinear parameter p.

the MI gain is shown in Fig.4.6. It could be seen that s_3 suppresses the MI gain and reduces the bandwidth, a feature exhibited also by the self-steepening parameter s_1 [144]. We find that MI gain is also independent of the sign of s_3.

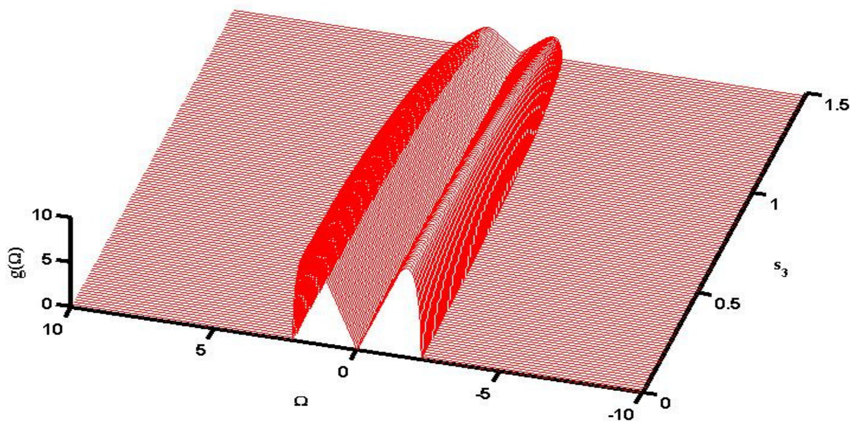

Fig.4.6: MI gain as a function of perturbation frequency Ω and higher order self-steepening parameter s_3 with fixed value of s_2.

4.4 Influence of loss on Modulations Instability

Linear loss is an important and very relevant issue in MMs [132,152,166]. At optical wavelengths MM suffer from high dissipative losses due to the metallic nature of their constituent meta-molecules. Hence there is a great thrust in research in overcoming the loss restrictions in MM. However this issue of loss is very complex in the so called MM or NIM. For example, based on a causality study, it has been shown theoretically that if the losses in MM are eliminated or significantly reduced by any means, including the compensation by gain media, then the negative refraction will disappear [152]. This clearly shows that one has to be careful in addressing any issue related to loss in MM. As regards, MI is concerned, the impact of loss on MI have been studied in conventional materials, such as an optical fiber, is studied by some authors. They have shown that in the presence of loss, the cw background decays during propagation, and the associated gain spectrum changes. It has been shown that the effectiveness of MI processes in high index optical fibers is not merely influenced by the strength of the nonlinearity, but is also strongly determined by the linear attenuation of waves in the fiber material [166-167]. In those high-index glass fibers, this attenuation acts as a strong perturbation, causing a frequency drift of the MI sidebands. However, techniques to suppress this frequency drift are completely developed. In contrast to the ordinary materials, in MM, the impact of loss on MI is generally avoided, may be for the reasons mentioned above. The issue of loss is generally included in the mathematical model through the so called Drude model. In order to see the role of loss on MI, in Fig.4.7, we depict the MI gain $g(\Omega)$ as a function of the perturbation frequency for different loss parameters with normalized peak power at $P = 2$.

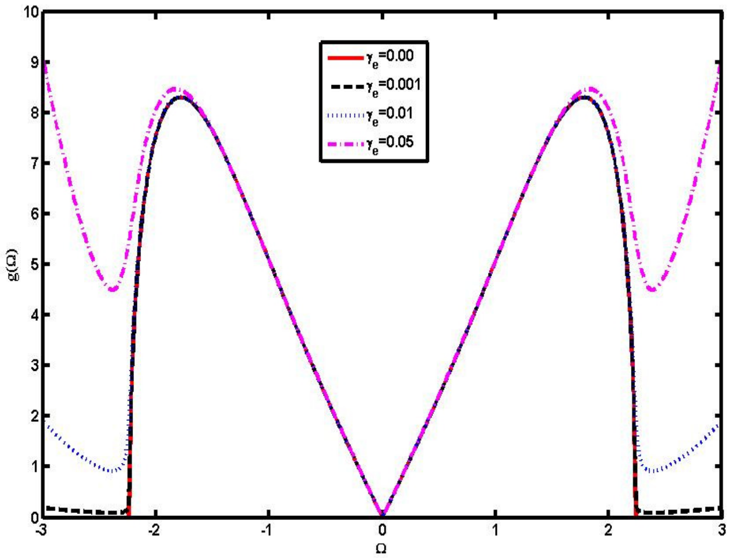

Fig.4.7: Modulation Instability gain Vs perturbation frequency with different loss parameter with peak power P=2.

It can be observed from Fig. 4.7 that when we consider the Drude model without any loss we get two prominent sidebands. But as loss is incorporated, with increase in the value of the loss coefficient, the peak of the MI gain remains nearly the same but the sidebands get distorted. However at $\tilde{\gamma}_e \sim \tilde{\gamma}_m \sim 0.01$, a value two orders of magnitude greater than that of the one taken by D'Aguanno et al. [35], peak of the side bands are distinct and distortions in the sideband could be fairly neglected. Finally, we would like to conclude that our study of the impact of loss on MI in nonlinear MMs is not a detailed or rigorous one. However, this study may enhance some interest in this very critical issue related to MMs.

4.5 Chapter Summary

In this chapter, we have discussed a MI analysis based on a model appropriate for pulse propagation in MMs embedded into a non-Kerr medium with cubic-quintic nonlinearities and higher order dispersive effects. We have included loss into account in our analysis as loss is a crucial issue in MM. We have found that loss distorts the sidebands of the MI gain spectrum. However, in this work we are taking the loss parameters to be $\tilde{\gamma}_e \approx \tilde{\gamma}_m = 0.01$, at which the peak of the side bands are distinct and distortions in the sideband could be fairly neglected. We find that the combined effect of cubic-quintic nonlinearity increases the MI gain. The role of higher order nonlinear dispersive effects on MI has been also discussed. It is shown that self-steepening parameter s_3, which arises due to $\chi^{(5)}$ nonlinear polarization, exhibits similar MI features with that of the self-steepening parameter s_1. It is clear that with so many controllable parameters, MM or NIM embedded in cubic quintic nonlinear medium may provide us more enriching features to manipulate MI and there by soliton generation or control in MMs, which are not possible in the case of conventional materials.

Chapter 5

Solitary wave solutions of Nonlinear Schrödinger equation with evolution parameter dependent coefficients*

5.1 Introduction

Optical solitons is one of the major areas of research in the field of Nonlinear Optics with non-Kerr nonlinearity [169-175]. We would like to emphasize that one of the important aspects of this area is the issue of integrability in presence of perturbation terms especially for non-Kerr law nonlinearity [82,173]. Although many numerical simulations are done, the exact soliton solution always is a very useful piece of information that supplements the numerics [96-97]. In this chapter, one such non-Kerr law nonlinearity namely, the power law nonliearity is considered. We are interested to study the dynamics of bright and dark optical solitons in a power law media with distributed co-efficients i.e. the co-efficient associated with the dispersion and nonlinear effects are time or space dependent. The nonlinear Schrödinger equation (NLSE) with evolution parameter dependent co-efficient has caught attention recently owing to the phenomena of Bose-Einstein condensate (BEC), dispersion management systems and similaritons [176-180]. In dispersion management systems, the group velocity dispersion (GVD) coefficient is not constant but function of propagation distance [177]. In the context of BEC, the nonlinear potential term can be made time dependent by Feshbach resonance [178-179]. On the other hand, NLSE with space dependent GVD, nonlinearity and gain/loss parameters have been studied in the context of self-similar solutions [180].

In this chapter, we are going to study the dynamics of optical solitons in a power law media

*Part of the results presented in this chapter have been published in the papers, M. Saha, A. K. Sarma and A. Biswas, "Dark Optical solitons in power law media with time-dependent coefficients" *Physics Letters A*, vol. 373 p.4438 (2009) and A. K. Sarma, M. Saha and A. Biswas, "Optical Solitons with Power Law Nonlinearity and Hamiltonian Perturbations: An Exact Solution" *Journal of Infrared, Millimeter and Terahertz waves* vol. 31, p.1048 (2010).

and in presence of linear attenuation, self-steepening term as well as third order dispersion (TOD), with time or space dependent coefficients. In addition, the GVD term and the nonlinear term also have time or space dependent coefficients. This study is general in the sense that the NLSE with constant coefficient could be considered as a special case of it. As a special case, we have also studied the solitary wave solutions of the NLSE with constant higher order dispersion and nonlinear co-efficients.

5.2 Evolution parameter dependent nonlinear Schrodinger equation

The evolution parameter dependent NLSE with power law nonlinearity, in dimensionless form, is given by [96,181]:

$$i\, q_t + a(t)\, q_{xx} + b(t)\,|\,q\,|^{2m}\, q = i\,\alpha(t)\, q + i\,\lambda(t)\,(|\,q\,|^{2m}\, q)_x - i\,\gamma(t)\, q_{xxx} \tag{5.1}$$

Here the variables t and x are interchangeable. We would name t as time and x as space variable. In Eq.(5.1), the complex valued function q(x, t) represents the wave profile where the independent variables or evolution parameters are the spatial x and time t. On the left-hand side of the equation, the first term represents the evolution term, the second term is the GVD and the third term is the nonlinear term. Here m is the index of the power law nonlinearity. It needs to be noted here that, it is necessary to have $0 < m < 2$ for solitons to exist. In particular, $m \neq 2$ as this case leads to the issue of self-focusing singularity [83]. Physically, power law nonlinearity arises in various materials, including semiconductors. Moreover, this law of nonlinearity arises in nonlinear plasmas that solve the problem of small *K*-condensation in weak turbulence theory [82]. The respective coefficients of GVD and nonlinearity are the real valued functions a(t) and b(t).On the right-hand side, α(t) represents the time-dependent coefficient of linear attenuation, while *λ(t)* is the time-dependent coefficient of self-steepening term (SST) and finally γ (t) is the time dependent coefficient of the TOD term. In dispersion management system, GVD parameter can be made space depended. In the context of BEC, a(t) is a parameter related to kinetic energy, b(t) is a scattering length of attractive interaction for b(t) > 0. b(t) can be made time-dependent and can be used to manage the condensate, the so-called, Feshbach resonance management [178]. The above equation is more general one as the nonlinear and dispersion co-efficient may be time or space dependent. The coefficients α(t), λ(t) and γ(t) are all real valued functions. In the context of nonlinear optics, the SST arises for the study of short pulses that are typically

<100 femto seconds. For pulses of widths smaller than 1 picosecond., the approximation for quasi-monochromaticity is no longer valid and so higher order dispersion terms come in. Also, for short pulse widths where group velocity dispersion changes, within the spectral bandwidth of the signal cannot be neglected, one have to consider the effect of higher order dispersion terms [83].

5.3 Dark solitary wave solution of evolution parameter dependent NLSE

Eq.(5.1) with $\alpha = 0$ is known as the generalized Radhakrishnan-Kundu-Lakshmanan (RKL) equation[174,181].The bright soliton solution of the RKL equation has been reported[174]. In this section we are going to look for dark solitary wave solutions of Eq.(5.1). In section (5.4) we would report both bright and dark solitary wave solution of the NLSE with constant co-efficients.

We assume the solution of Eq.(5.1) in the following phase amplitude form:

$$q = P\, e^{i\phi} \tag{5.2}$$

where P is the amplitude portion while ϕ is the phase portion of the solution. It is also assumed that,

$$\phi = -\kappa(t)\, x + \omega(t)\, t + \theta(t) \tag{5.3}$$

Because of the time-dependent coefficients in Eq.(5.1), the soliton frequency κ, wave number ω and the phase constant θ are also, in general, time dependent. Therefore,

$$\frac{\partial\phi}{\partial t} = -\frac{d\kappa}{dt}x + \omega(t) + t\frac{d\omega}{dt} + \frac{d\theta}{dt} \tag{5.4}$$

Putting Eq.(5.2) and Eq.(5.3) in Eq.(5.1) and simplifying the algebra, after decomposing the real and imaginary parts, we obtain:

$$P\left(\frac{d\kappa}{dt}x - \omega - t\frac{d\omega}{dt} - \frac{d\theta}{dt}\right) + a(t)\left(\frac{\partial^2 P}{\partial x^2} - \kappa^2 P\right) + b(t)P^{2m+1} = \kappa\lambda(t)P^{2m+1} + \kappa^3\gamma(t) - 3\kappa\gamma(t)\frac{\partial^2 P}{\partial x^2} \tag{5.5}$$

$$\frac{\partial P}{\partial t} - 2\kappa a(t)\frac{\partial P}{\partial x} = \alpha(t)\,P - \gamma(t)\frac{\partial^3 P}{\partial x^3} + 3\kappa^3\gamma(t)\frac{\partial P}{\partial x} + \lambda(t)\,(2m+1)P^{2m}\frac{\partial P}{\partial x} \tag{5.6}$$

For dark soliton, the assumption is

$$P = A\tanh^p \tau \tag{5.7}$$

where $\tau = B(x - \upsilon t)$ and the free parameters A and B as well as the velocity υ are functions of time. Thus $A = A(t)$, $B = B(t)$ and $\upsilon = \upsilon(t)$. Thus Eq.(5.5) and Eq.(5.6) respectively reduces to

$$A\tanh^{p}\tau\left(x\frac{d\kappa}{dt}-\omega-t\frac{d\omega}{dt}-\frac{d\theta}{dt}\right)-a(t)\kappa^{2}A\tanh^{p}\tau+b(t)A^{2m+1}\tanh^{(2m+1)p}\tau$$

$$+a(t)pAB\left\{(p-1)\tanh^{p-2}\tau-2p\tanh^{p}\tau+(p+1)\tanh^{p+2}\tau\right\}=\kappa\lambda(t)A^{2m+1}\tanh^{(2m+1)p}\tau \qquad (5.8)$$

$$+\kappa^{3}\gamma(t)A\tanh^{p}\tau-3\kappa\gamma(t)pA^{2}B\{(p-1)\tanh^{p-2}\tau-2p\tanh^{p}\tau+(p+1)\tanh^{p+2}\tau$$

and

$$\frac{dA}{dt}\tanh^{p}\tau+pA(\tanh^{p-1}\tau-\tanh^{p+1}\tau)\times\left\{\frac{\tau}{B}\frac{dB}{dt}-B\left(\upsilon+t\frac{d\upsilon}{dt}\right)\right\}-2\kappa(t)pAB(\tanh^{p-1}\tau$$

$$-\tanh^{p+1}\tau)=\alpha(t)A\tanh^{p}\tau+3\kappa^{2}\gamma(t)pAB(\tanh^{p-1}\tau-\tanh^{p+1}\tau)+\lambda(t)(2m+1)$$

$$pA^{2m+1}B(\tanh^{2mp+p-1}\tau-\tanh^{2mp+p+1}\tau)-\gamma(t)pAB^{3}\{(p-1)(p-2)\tanh^{p-3}\tau \qquad (5.9)$$

$$-(3p^{2}-3p+2)\tanh^{p-1}\tau+(3p^{2}-3p+2)\tanh^{p+1}\tau-(p+1)(p+2)\tanh^{p+3}\tau\}$$

Now from Eq.(5.8), equating the exponents $(2m+1)p$ and $p+2$ yields $(2m+1)p = p+2$ which gives

$$p=\frac{1}{m} \qquad (5.10)$$

Again, from Eq.(5.8) the linearly independent functions are $\tanh^{p}\tau$ where j = −2, 0, 2. However, $\tanh^{p-2}\tau$ is a stand-alone linearly independent function. Therefore, its coefficient must be zero. This gives $p=1$ and hence from Eq.(5.10) $m=1$. Thus, dark solitons exist only for Kerr law nonlinearity. Now, setting the coefficients of the other two linearly independent functions in Eq.(5.8) to zero yields

$$\omega=-a(t)\left(\kappa^{2}+2B^{2}\right)-\kappa\gamma(t)\left(\kappa^{2}+6B^{2}\right) \qquad (5.11)$$

and

$$B=A\left[\{\kappa\lambda(t)-b(t)\}/2\{a(t)+3\kappa\gamma(t)\}\right]^{1/2} \qquad (5.12)$$

which forces the conditions of existence of dark solitons to be

$$a(t)+3\kappa\gamma(t)\neq 0 \quad \text{and} \quad \{\kappa\lambda(t)-b(t)\}\{a(t)+3\kappa\gamma(t)\}>0 \tag{5.13}$$

Also, from Eq.(5.8) it is easy to see that $d\kappa/dt=0$, $d\omega/dt=0$, $d\theta/dt=0$, so that κ,ω,θ are all constants. Now, from Eq.(5.9), equating the exponents $2mp+p-1$ and $p+1$ and the pair $2mp+p+1$ and $p+3$ yields the same results as in Eq.(5.10). Here in Eq.(5.9) the linearly independent functions are $\tau(\tanh^{p-1}\tau-\tanh^{p+1}\tau)$ and $\tanh^{p+j}\tau$ where j =–3,–1, 0, 1, 3.The coefficients of $\tanh^{p}\tau$ gives

$$dA/dt=\alpha(t)A \tag{5.14}$$

which leads to

$$A(t)=A_0\, e^{\int\alpha(t)dt} \tag{5.15}$$

where A_0 is the initial value of the free parameter A. Now, the coefficient of $\tau(\tanh^{p-1}\tau-\tanh^{p+1}\tau)$, $\tanh^{p-1}\tau$, $\tanh^{p+1}\tau$ and $\tanh^{p+3}\tau$ gives

$$B(t)=\text{constant} \tag{5.16}$$

Also, the coefficients of $\tanh^{p-1}\tau$ and $\tanh^{p+1}\tau$ respectively yields

$$\upsilon=-\frac{1}{t}\left[2\kappa\int a(t)\,dt+(2B^2+3\kappa^2)\int\gamma(t)\,dt\right] \tag{5.17}$$

and

$$\upsilon=-\frac{1}{t}\left[2\kappa\int a(t)\,dt+(8B^2+3\kappa^2)\int\gamma(t)\,dt-3A^2\int\lambda(t)\,dt\right] \tag{5.18}$$

Again, the coefficients of $\tanh^{p-3}\tau$ is zero as $p=1$. Finally, the coefficients of $\tanh^{p+3}\tau$ gives

$$B=A\sqrt{\frac{\lambda(t)}{2\gamma(t)}} \tag{5.19}$$

Equating the two values of B from Eq.(5.12) and Eq.(5.19) gives

$$\gamma(t)\{\kappa\lambda(t)-b(t)\}=\lambda(t)\{a(t)+3\kappa\gamma(t)\} \tag{5.20}$$

which is constraint condition for dark solitons to exist. Again from Eq.(5.15), Eq.(5.16) and Eq.(5.19)

$$[\lambda(t)/\gamma(t)]e^{2\int\alpha(t)dt}=c_1 \tag{5.21}$$

And similarly from Eq.(5.12), Eq.(5.14) and Eq.(5.16)

$$[\{\kappa\lambda(t)-b(t)\}/\{a(t)+3\kappa\gamma(t)\}]e^{2\int\alpha(t)dt}=c_2 \tag{5.22}$$

where c_1 and c_2 are constants. Eq.(5.21) and Eq.(5.22) are two more constraint relations for dark solitons to exist for Eq.(5.1). From Eq.(5.21) and Eq.(5.22)

$$\lambda(t)\{a(t)+3\kappa\gamma(t)\}/\gamma(t)\{\kappa\lambda(t)-b(t)\}=c_1/c_2=c_3 \tag{5.23}$$

with c_3 is a new constant. For $c_3 = 1$, we get the exact same constraints as Eq.(5.20). Thus it is possible to conclude that Eq.(5.21) and Eq.(5.22), when combined is equivalent to the constraint Eq.(5.20). Finally, equating the two values of the velocity υ of the soliton given by Eq.(5.17) and Eq.(5.18) yields relation (5.19) and this closes the case and also shows that the method is consistent. Also, the velocity of the soliton given by Eq.(5.18) can be simplified by using Eq.(5.19) to

$$\upsilon=-\frac{1}{t}\left[2\kappa\int a(t)\,dt+B^2\left\{8\int\gamma(t)\,dt-6\frac{\gamma(t)}{\lambda(t)}\int\lambda(t)\,dt\right\}+3\kappa^2\int\gamma(t)\,dt\right] \tag{5.24}$$

Thus, the dark 1-soliton solution to (5.1) is given by

$$q(x,t)=A\tanh\tau\, e^{i(-\kappa x+\omega t+\theta)} \tag{5.25}$$

where the relation between the free parameters *A* and *B* are given by Eq.(5.15) and Eq.(5.16), the velocity (υ) is given by Eq.(5.17) and Eq.(5.18) simultaneously and the wave number (ω) is given by Eq.(5.11). These lead to the constraint conditions given by Eq.(5.13), Eq.(5.20)-Eq.(5.23). Finally, all what is necessary is that these time-dependent coefficients must be Riemann integrable as is evident from Eq. (5.17), Eq.(5.18) and Eq.(5.21) or Eq.(5.22).

In order to have a physical understanding of the soliton solution, given by Eq.(5.25), the soliton intensity $|q|^2$ is plotted at t = 0, 5 and 10 for the constant parameters given by $A_0 = 1$, $\alpha(t) = -0.001$, $\lambda(t) = 0.03$, and $\gamma(t) = 0.03$. It can be seen from Fig.5.1 that with an increase in t, the soliton intensity gets reduced as expected from Eq.(5.15). Also the soliton is shifted along the negative x-direction with t. This happens due to the decrease in velocity of the soliton as evident from Eq.(5.18) through its dependence on A by virtue of Eq.(5.15).

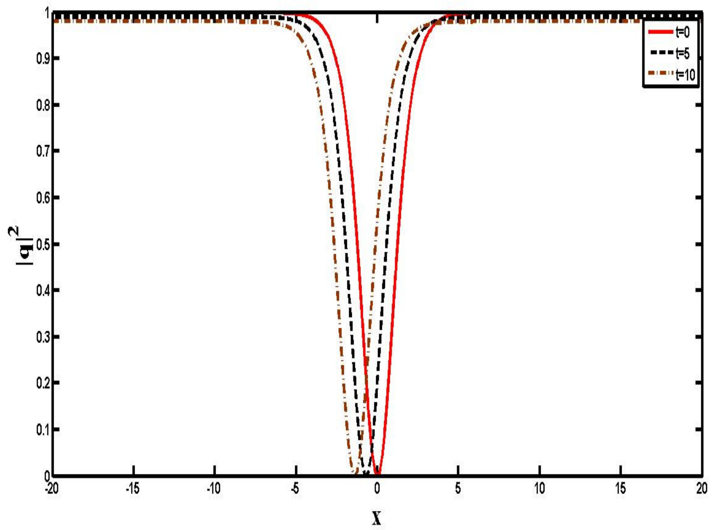

Fig.5.1: Plot of normalized soliton intensity $|q|^2$ against x

5.4 Bright and Dark solitary wave solution with constant co-efficients of generalized NLSE

In this section, bright and dark solitary wave solution to the NLSE with power law nonlinearity is discussed. We assume that the propagation medium to be a typical optical fiber. The governing NLSE, in dimensionless form, for the propagation of solitons through an optical fiber is given by [82]

$$iq_t + aq_{xx} + b|q|^{2m} q = i\alpha q_x - i\gamma q_{xxx} + i\lambda\left(|q|^{2m} q\right)_x + i\nu\left(|q|^{2m}\right)_x q \tag{5.26}$$

The terms on the right hand side of Eq.(5.1) are also known as Hamiltonian perturbations terms. Here, *a, b* are real numbers representing the coefficients of GVD and nonlinearity respectively. α , γ and λ are the coefficient of inter-modal dispersion, third order dispersion and nonlinear dispersion respectively and they are constant numbers. The last term on the right hand side represents the intra-pulse raman effect with ν as the nonlinear dispersion coefficient.

In order to solve Eq.(5.26) , we assume the solution as Eq.(5.2). Now substituting Eq.(5.2) in Eq.(5.26) and equating the real and imaginary parts respectively yields

$$\left\{\omega + \alpha\frac{\partial\phi}{\partial x} + a\left(\frac{\partial\phi}{\partial x}\right)^2 + \gamma\left(\frac{\partial\phi}{\partial x}\right)^3\right\}P - \left(b - \lambda\frac{\partial\phi}{\partial x}\right)P^{2m+1} - \left(a - 3\gamma\frac{\partial\phi}{\partial x}\right)\frac{\partial^2 P}{\partial x^2} = 0 \tag{5.27}$$

$$\frac{\partial P}{\partial t}-\left\{\alpha+2a\frac{\partial\phi}{\partial x}+3\gamma\left(\frac{\partial\phi}{\partial x}\right)^2\right\}-\{\lambda(2m+1)+2m\nu\}P^{2m}\frac{\partial P}{\partial x}-\gamma\frac{\partial^3 P}{\partial x^3}=0 \quad (5.28)$$

This pair of relations given by Eq.(5.27) and Eq.(5.28) will be further analyzed in the following two subsections, based on the type of soliton that is being studied, namely bright or dark. In each of these two cases, an exact closed form soliton solution will be obtained along with a few parameter constraints that will be seen.

5.4.1 Exact Bright soliton solutions

For bright optical solitons, a judicious choice would be

$$P=A\,\mathrm{sech}^p\,\tau \quad (5.29)$$

with $\tau=B(x-\upsilon t)$ and $\phi=-\kappa x+\omega t+\theta$ where the frequency of the soliton (ω), the wave number (κ), the phase (θ) all are constant parameters. Also, A is the amplitude, B is the inverse width of the bright soliton and υ is the soliton velocity while the exponent p is unknown at this stage. This unknown exponent will be determined in the course of derivation of the soliton solution to Eq.(5.26). Thus, from Eq.(5.29), Eq.(5.27) and Eq.(5.28) respectively reduce to

$$(\omega A+a p^2AB^2+a\kappa^2A)\frac{1}{\cosh^p\tau}-a\,p(p+1)AB^2\frac{1}{\cosh^{p+2}\tau}+\frac{bA^{2m+1}}{\cosh^{(2m+1)p}\tau}$$
$$=(\alpha\kappa A-3\gamma\kappa p^2AB^2+\gamma\kappa^3A)\frac{1}{\cosh^p\tau}+\lambda\kappa A^{(2m+1)}\frac{1}{\cosh^{(2m+1)p}\tau} \quad (5.30)$$

$$(\upsilon pAB+2a\kappa pAB)\frac{\tanh\tau}{\cosh^p\tau}=-(\alpha pAB-\gamma p^3AB^3+3\gamma\kappa^2AB)\frac{\tanh\tau}{\cosh^{p+2}\tau}$$
$$-\gamma p(p+1)(p+2)AB^3\frac{\tanh\tau}{\cosh^{p+2}\tau}-\{2m\nu+(2m+1)\lambda\}pA^{(2m+1)}B\frac{\tanh\tau}{\cosh^{(2m+1)p}\tau} \quad (5.31)$$

Now, from (5.30) or (5.31), equating the exponents (2m + 1) p and (p + 2) gives (2m + 1)p = p + 2 that gives

$$p=\frac{1}{m} \quad (5.32)$$

Now, from Eq.(5.30), the two linearly independent functions are $1/\cosh^{p+j}\tau$, for j = 0, 2. Setting their respective coefficients to zero yields

$$\omega=\frac{1}{m}\left[a(m^2\kappa^2-B^2)+\kappa\{m^2\alpha+\gamma(m^2\kappa^2-3B^2)\}\right] \quad (5.33)$$

and

$$B = mA^m \sqrt{\frac{b-\lambda\kappa}{(m+1)(a+3\gamma\kappa)}} \tag{5.34}$$

From Eq.(5.34), the restriction that is clearly imposed is

$$(a+3\gamma\kappa)(b-\lambda\kappa) > 0 \tag{5.35}$$

Similarly, from Eq.(5.31), it is possible to recover

$$\upsilon = -2a\kappa \tag{5.36}$$

$$\text{and } B = m\left[\frac{(a+3\gamma\kappa^2)(\lambda\kappa-b)}{m(2m+3)\gamma(b-\lambda\kappa)+(m+1)(a+3\gamma\kappa)\{2m\nu+(2m+1)\lambda}\right]^{\frac{1}{2}} . \tag{5.37}$$

Eq.(5.36) gives the velocity of the soliton, Eq.(5.37) gives the width '*B*' of the soliton and the constraint condition is given by Eq.(5.35). The amplitude '*A*' can be retrieved from Eq.(5.34). Thus, the bright 1-soliton solution of the NLSE with power law nonlinearity is given by

$$q = \frac{A}{\cos^{\frac{1}{m}}[B(x-\upsilon t)]} e^{i(-\kappa x+\omega t+\theta)} , \tag{5.38}$$

where the amplitude-width relation of the soliton, the velocity υ of the soliton and the frequency ω of the soliton are respectively given by Eq.(5.34), Eq.(5.36) and Eq.(5.33). The width is given by (5.37) and the amplitude is available from Eq.(5.34).

5.4.2 Exact Dark soliton solutions

For dark optical solitons the choice for the function P in Eq.(5.2) is [2]

$$P(x,t) = A\tanh^p \tau \tag{5.39}$$

where τ is same as defined earlier . It needs to be noted that for the case of dark optical solitons the parameters A and B are known as free parameters. In this case, Eq.(5.27) and Eq.(5.28) respectively reduce to

$$-\omega A\tanh^p\tau + apAB^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}$$

$$-\kappa^2 A\tanh^p\tau + bA^{2m+1}\tanh^{(2m+1)p}\tau = \alpha\kappa A\tanh^p\tau + \lambda\kappa A^{(2m+1)p}\tanh^{(2m+1)p}\tau \tag{5.40}$$

$$+\gamma\kappa^3 A\tanh^p\tau - 3\gamma pAB^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\}$$

and

$$-p\upsilon AB(\tanh^{p-1}\tau-\tanh^{p+1}\tau)-2a\kappa pAB(\tanh^{p-1}\tau-\tanh^{p+1}\tau)=\alpha pAB(\tanh^{p-1}\tau$$

$$-\tanh^{p+1}\tau)+3\gamma\kappa^2 pAB(\tanh^{p-1}\tau-\tanh^{p+1}\tau)-\gamma pAB^3\{(p-1)(p-2)\tanh^{p-3}\tau \tag{5.41}$$

$$-(3p^2-3p+2)\tanh^{p-1}\tau\}+\{(2m+1)\lambda+2m\nu\}pA^{2m+1}B(\tanh^{(2m+1)p-1}\tau-\tanh^{(2m+1)p+1}\tau)$$

Now, from Eq.(5.40) equating the exponents (2m +1) p and p + 2 gives the same value of p as in Eq.(5.32). The linearly independent functions in Eq.(5.40) are $\tanh^{p+j}\tau$ where j= −2, 0, 2. First, setting the coefficient of $\tanh^{p-2}\tau$ to zero yields

$$p=1 \tag{5.42}$$

This together with Eq.(5.32) gives

$$m = 1 \tag{5.43}$$

which shows that the power law of nonlinearity reduces to the Kerr law nonlinearity. This implies that dark optical solitons will exist when the power law nonlinearity reduces to Kerr law nonlinearity. Again, setting the coefficients of $\tanh^{p+j}\tau$ where j = 0, 2 to zero respectively yields

$$\omega=-[a(B^2+\kappa^2)+\alpha\kappa+\gamma\kappa(6B^2+\kappa^2)] \tag{5.44}$$

and

$$B=A\sqrt{\frac{b+\lambda\kappa}{2(a+3\gamma\kappa)}} \tag{5.45}$$

Eq.(5.45) introduces the constraint given by

$$(a+3\gamma\kappa)(b+\lambda\kappa)<0 \tag{5.46}$$

Also, since A and B are constants, Eq.(5.45) implies

$$\frac{a+3\gamma\kappa}{b+\lambda\kappa}=\text{constant} \tag{5.47}$$

From Eq.(5.41), the linearly independent functions are $\tanh^{p+j}\tau$ for j= −3,−1,1, 3. The coefficient of $\tanh^{p-3}\tau$ is zero by Eq.(5.43). Setting the coefficients of the remaining linearly independent functions to zero yields

$$\upsilon=-2a\kappa-\alpha-\gamma(3\kappa^2+2B^2) \tag{5.48}$$

$$\upsilon = -2a\kappa - \alpha - \gamma(3\kappa^2 + 8B^2) + (3\lambda + 2\upsilon)A^2 \quad (5.49)$$

And

$$6\gamma B^2 - (3\lambda + 2\nu)A^2 + 3\gamma(b + \lambda\kappa) = 0 \quad (5.50)$$

Now, equating the two values of the velocity υ given by Eq.(5.48) and Eq.(5.49) also yields Eq.(5.50) which show the consistency of the method. Finally, Eq. (5.50) simplifies to

$$(a + 3\gamma\kappa)\,(3\lambda + 2\nu)A^2 + 3\gamma(b + \lambda\kappa) = 0 \quad (5.51)$$

on using Eq.(5.45) and it serves as a constraint relation between the soliton parameters and the perturbation coefficients for the dark solitons to exist. Thus, the dark 1-soliton solution to Eq.(5.26) is given by

$$q(x,t) = A\tanh[B(x - \upsilon t)]e^{i(-\kappa x + \omega t + \theta)} \quad (5.52)$$

where the free parameters A and B are related as in Eq.(5.45) and the wave number is given by Eq.(5.44), while the velocity of the soliton is given by Eq.(5.48) or Eq.(5.49), which introduces the constraint relation given by Eq.(5.51). The two additional constraints are Eq.(5.50) and Eq.(5.51) must also be valid for dark solitons to exist.

5.5 Chapter Summary

In this chapter, we have derived the exact dark one solitary wave solutions of generalised nonlinear Schrodinger equation with evolution (time or space) parameter dependent co-efficients in a power law medium. In the presence of the perturbations terms, we obtained the various physical parameters with solitary wave ansatz method. In presence of the perturbation terms, it is observed that the dark solitons can only exist when power law nonlinearity reduces to Kerr law nonlinearity. Also, the time dependent co-efficients must be Reimann Integrable for the solitons to exist. Also, a few other additional constraints, to these co-efficients, fall out during the course of calculation for the soliton to exist. We have also reported the exact bright and dark one solitary wave solutions of the generalised NLSE with constant co-efficients. The parameter domain restrictions have been identified in the process of obtaining these solutions. In future, this study can be extended to others laws of non-Kerr nonlinearity, namely in the case of parabolic and dual power laws.

Chapter 6

Soliton propagation and soliton-soliton interaction in a silicon waveguide*

6.1 Introduction

This thesis is mainly devoted towards studying modulation instability (MI) and solitary wave solutions by simple analytical methods. However, there are situations where these methods could not be applied and one needs to rely on numerical methods. One such example would be found in the context of silicon photonics. Silicon photonics has attracted considerable attention recently because of its excellent transmission properties at near to mid infrared region. The mature processing technology of Si and the sustainability of Silicon-on-Insulator (SOI) makes it possible to design compact and low loss (<0.1dB/cm) waveguide devices which have important applications of IC integration [98-107,182-189]. Several key properties of silicon make it an ideal material for photonic devices. In silicon waveguide, silicon is cladded with low index medium like SiO_2 or air. Silicon has a very large refractive index n=3.5 compare to the low index cladding SiO_2 with n=1.45 and this results very tight confinement of the optical modes inside the waveguide. Such strong tight confinement leads Si waveguide to scale down to ultra-small cross-section ($< 0.1\mu m^2$). Moreover, Silicon has very large Kerr nonlinear parameter which is 10,000 times larger than silica fiber [107-182]. This large cubic nonlinearity along with tight mode confinement enhances the effective nonlinear property in silicon waveguide as a result low optical power is required for achieving strong nonlinear effects. This feature makes it possible to form solitons within a very short length of waveguide [183]. Optical solitons in a Si waveguide results from a delicate balance of group velocity dispersion (GVD) and self-phase modulation (SPM). Because of the ultrashort dimension of silicon waveguide, GVD can be controlled through the

*Part of the results presented in this chapter have been published in the paper, A. K. Sarma, M. Saha and A. Biswas, "Effect of two-photon absorption on soliton propagation and soliton-soliton interaction in a silicon waveguide" *SPIE Journal of Optical Engineering* , Vol.49, p.035001-1 (2010).

geometry of their cross-sectional area. The GVD parameter β_2 ranges from -1 to -11 ps^2/m and the dispersion length can be made ~1 cm for pulse 100fs wide or less. Similarly, the nonlinear length can also be made ~1 cm at moderate peak powers <10 mW. Thus soliton can propagate inside a silicon waveguide with length few mms. Recently, the formation of optical soliton inside a 5mm long SOI waveguide is reported by launching femto-second pulses under some appropriate device design and pulse launch condition [103]. This may lead to new applications of SOI waveguides in high-speed optical switching [184-185]. Unlike silica, in silicon based waveguides, the so-called two photon absorption (TPA) cannot be neglected [106-186]. In fact the presence of TPA parameter in nonlinear Schrödinger equation (NLSE) makes the equation non-integrable and we need to use numerical methods to investigate solitons in a silicon-based waveguide. In this chapter, we primarily focus on studying the effect of TPA on soliton propagation in a Si waveguide.

6.2 TPA effect on soliton propagation in silicon waveguide

The two-photon absorption (TPA) is a nonlinear absorption process where two photons are absorbed simultaneously to excite an atom to a higher energy state. In silicon waveguide this occurs when two photons combine to overcome silicon's indirect bandgap energy ~1.12eV. This effect is insignificant in silica fiber because it has large band gap energy ~ 9eV. TPA is related to the third order nonlinear susceptibility $\chi^{(3)}$ as [107]

$$\frac{\omega}{c}n_2 + \frac{i}{2}\beta_T = \frac{3\omega}{4\varepsilon_0 c^2 n^2}\chi^{(3)} \tag{6.1}$$

where n_2 is the Kerr nonlinear parameter and β_T is the two photon absorption co-efficient and it has a value between $3\times10^{-12}\, m/W$ and $11\times10^{-12}\, m/W$, depending on the waveguide structure. Recently it is reported that in Si waveguide, the TPA limits the extent of SPM through nonlinear absorption and generates free carrier absorption causing nonlinear absorption loss [106].The equation governing the pulse propagation in a silicon waveguide with TPA effect is given by [107]:

$$\frac{\partial A}{\partial z} + \frac{1}{2}\alpha A + i\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} = i\gamma_0 |A|^2 A - \frac{c\beta_T}{2n_2\omega_0}\gamma_0 |A|^2 A \quad , \tag{6.2}$$

where $A, \alpha, \beta_2, \gamma_0$ and β_T represent the slowly varying field amplitude, linear loss parameter, second order dispersion coefficient, nonlinear Kerr coefficient and TPA coefficient respectively.

Using the dimensionless variable [2]

$U = A / \sqrt{P_0}$, $\xi = z / L_D$, $\tau = t / T_0$, $N^2 = \gamma_0 P_0 T_0^2 / |\beta_2|$, $u = NU$

Eq. (6.1) can again be written in the normalized form:

$$\frac{\partial u}{\partial \xi} = \frac{i}{2}\frac{\partial^2 u}{\partial \tau^2} + (1 + i r)|u|^2 u - \alpha_0 u \quad (6.3)$$

where $r = c\beta_T / 2n_2\omega_0$ is the dimensionless TPA parameter and $\alpha_0 = \alpha T_0^2 / 2|\beta_2|$ is the dimensionless loss parameter which includes both the linear scattering loss and free-carrier absorption. In writing Eq. (6.2) we are considering the anomalous dispersion regime. We define a parameter called transmission T as follows:

$$T = \int_{-\infty}^{\infty} |u(\xi,\tau)|^2 d\tau \Bigg/ \left[\int_{-\infty}^{\infty} |u(0,\tau)|^2 + |u(\xi,\tau)|^2 \right] d\tau \quad (6.4)$$

The transmission may give us an estimate of the effect of TPA on energy or power of the pulse propagating through the silicon waveguide.

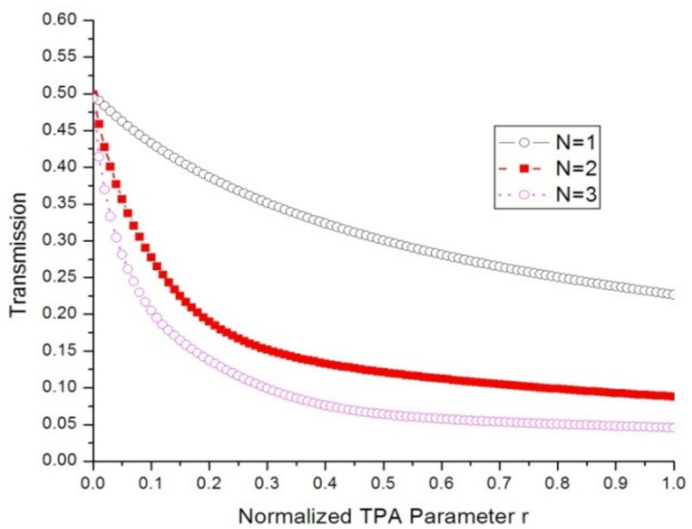

Fig.6.1: Transmission vs. normalized TPA parameter for N=1, 2 and 3 soliton

We solve Eq. (6.2) by using the well-known split-step Fourier method with the initial condition: $u(0,\tau) = N\,Sech(\tau)$ where N is the order of the soliton. In Fig.6.1 we depict the effect of TPA on the transmission of the waveguide for N=1, N=2 and N=3 soliton for a silicon waveguide with $\beta_2 = -2.15\,ps^2/m$, $T_0 = 30\,fs$, $\alpha = 2.30\,\mathrm{m}^{-1}$ [103]and length 1.1 m. Quite expectedly, the transmission of the waveguide decreases with increase in the TPA coefficient due to nonlinear absorption occurring in the waveguide. The higher the order of the soliton greater is the effect of TPA on transmission. However, we may exploit this result to our advantage.

6.3 Effect of TPA on N=2 Soliton

In Fig.6.2 we plot the intensity of a second order soliton at the output of the given silicon waveguide for various values of TPA parameter. It can be seen very clearly that for r=0.006 the second order soliton is taking the shape of a fundamental soliton. However if the TPA parameter is increased further, it is no longer a soliton. This may be explained as follows: the

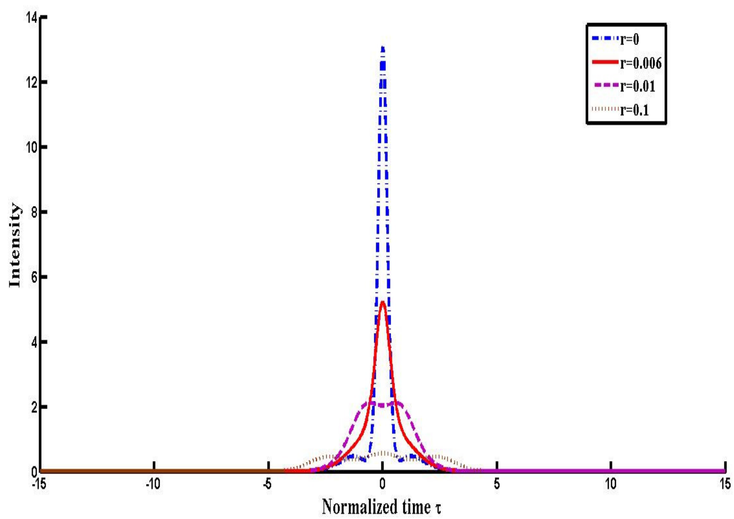

Fig.6.2: Intensity vs. normalized time for different TPA parameter

formation of a fundamental soliton requires $L_N = L_D$, i.e. the peak power of the soliton must be $P_0 = |\beta_2| / \gamma_0 T_0^2$. Now as we have seen, the TPA effect decreases the power of the pulse, so at a particular value of the TPA coefficient it so happens that the fundamental soliton formation criteria is nearly met and that is when we obtain a N=1 soliton from a N=2 soliton.

The spatio-temporal evolution of the N=2 soliton with r=0.006, through the silicon waveguide and its transformation to an N=1 soliton at the output of the waveguide may be clearly seen in Fig.6.3.

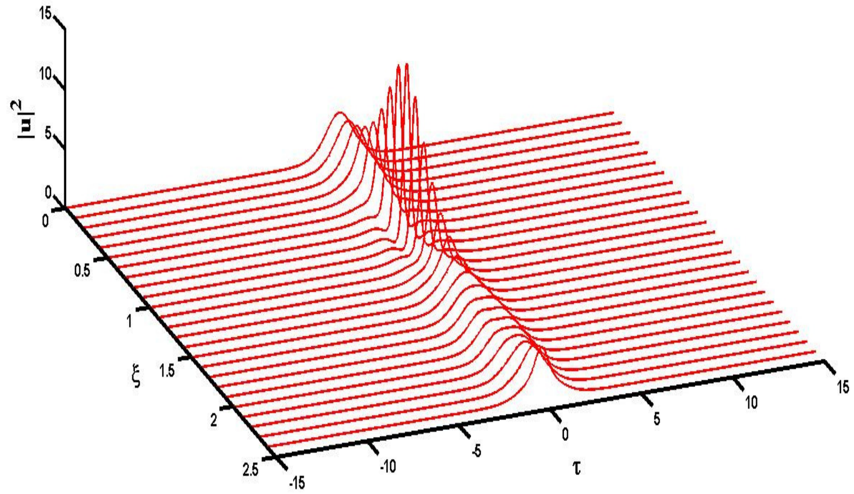

Fig.6.3: Spatio-temporal evolution of an N=2 soliton in silicon waveguide for r=0.006

6.4 Soliton-soliton interaction

In a usual silica fiber based soliton communication system, the interaction of solitons is one of the most pressing issues that needs to be addressed [2,190]. Soliton interaction may result in pulse distortion, deterioration in the transmission characteristics, transmission rate decrease etc. thereby affecting the performance of the soliton transmission system. In this spirit we study the soliton interaction in a silicon waveguide. We use the following input form at the input end of the silicon waveguide [2]:

$$u(0,\tau) = N\left[\sec h(\tau - q_0) + r_0 \sec h\left[r_0(\tau + q_0)\right]e^{\theta}\right] \tag{6.5}$$

Here r_0 is the relative amplitude, θ is the initial phase difference and $2q_0$ is the initial separation between the two solitons. In our analysis, we are taking $q_0 = 2$ and consider the solitons to be in phase, i.e. $\theta = 0$.

In Fig. 6.4 we depict the evolution of an N=1 soliton pair through a distance of 4.1 km for different values of the TPA parameter. It is observed that as long as the TPA parameter is small the soliton pair attracts each other and collide periodically along the waveguide length as is the case with a normal silica fiber. But if the TPA parameter 'r' is larger than 0.1, it evolves into a single soliton at the output end of the silicon waveguide. We find that it indeed evolves as a single soliton, without showing any further periodic evolution except for an obvious decrease in soliton amplitude, if the numerical study is carried for longer distances, say 6 km, with $r = 0.2$.

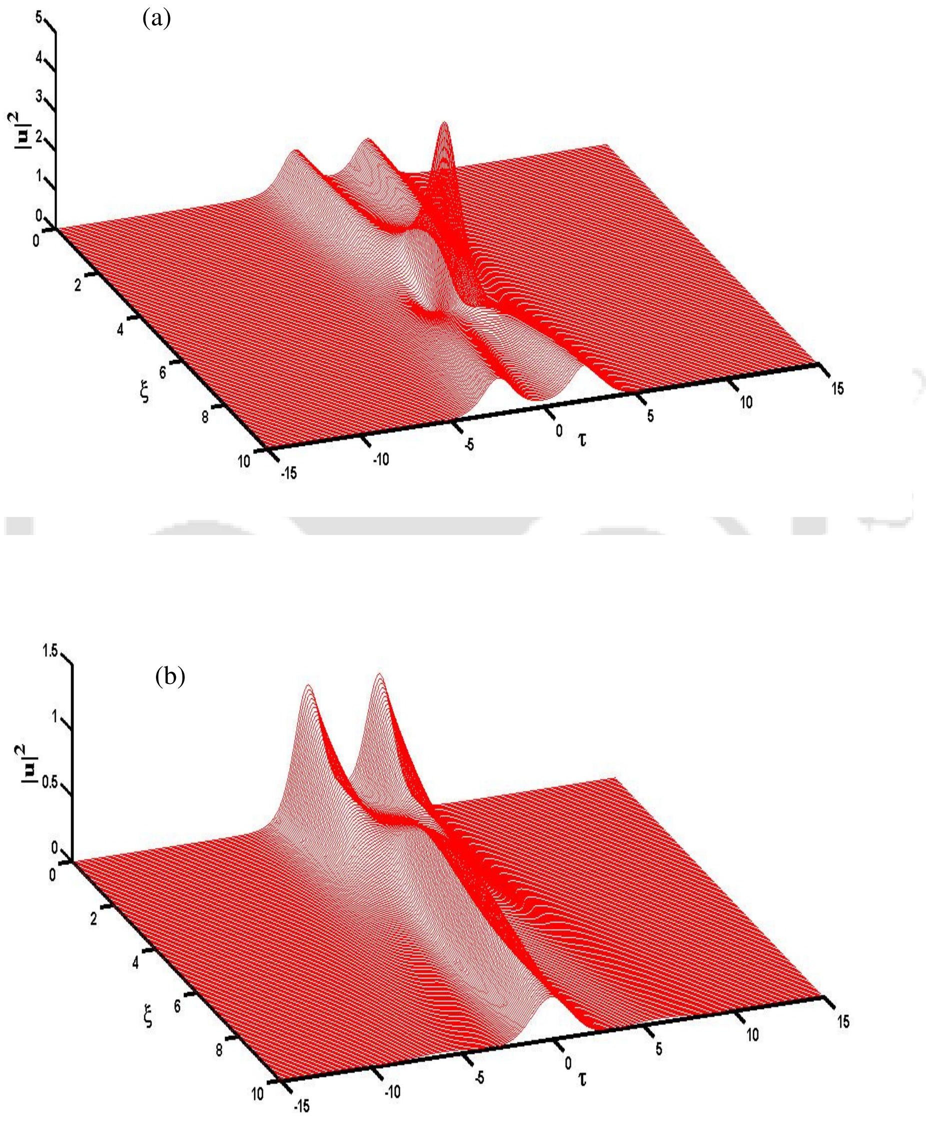

Fig.6.4: Spatio-temporal evolution of a N=1 soliton pair with normalized TPA parameter (a) r=0.006 (b) r=0.2

Similar study is carried out for a pair of N=2 solitons. In Fig 6.5 we plot the output intensity profiles vs. time (in normalized units) for various TPA parameters for a pair of N=2 soliton.

We find that when the TPA parameter is small, with increase in the TPA parameter the soliton pair is evolving into a number of smaller pulses with decreasing amplitude. But if the TPA parameter is increased beyond r=0.2, it evolves into a single soliton. In Fig.6.6 we plot the spatio-temporal evolution of an N=2 soliton pair with TPA parameter r=0.5.

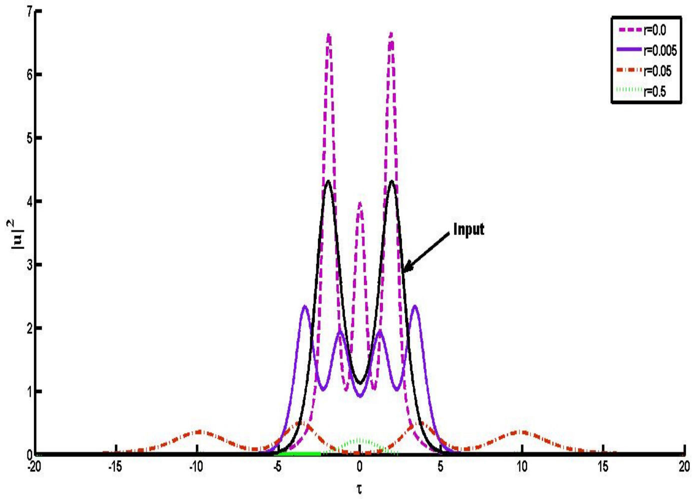

Fig.6.5 Intensity vs. time for a pair of $N = 2$ solitons at the output of a silicon waveguide for different TPA parameter $r = 0, 0.005, 0.05, 0.5$.

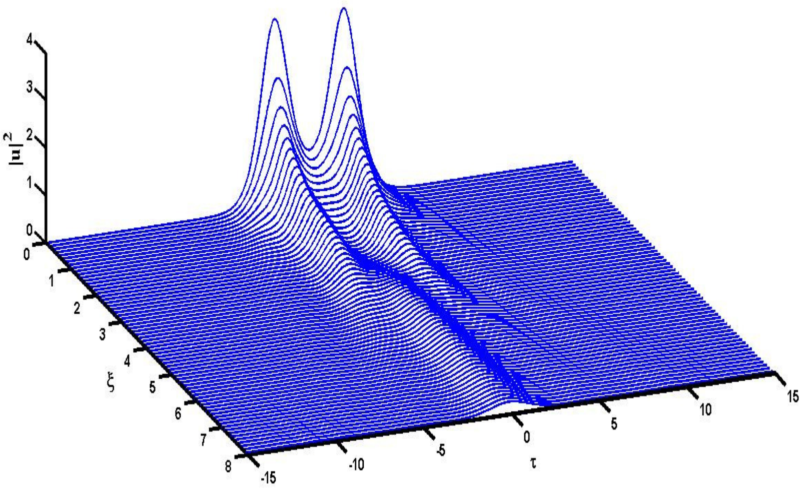

Fig.6.6 Spatio-temporal evolution of a N=2 soliton pair with normalized TPA parameter $r = 0.5$.

6.5 Chapter Summary

In this chapter, a numerical investigation is carried out to find the effect of TPA on soliton propagation and soliton-soliton interaction in a silicon waveguide. We demonstrated the formation of fundamental soliton from a higher order soliton if the TPA parameter is judiciously chosen. In addition to this, the effect of TPA parameter on the soliton-soliton interaction in a silicon waveguide is discussed.

Chapter 7

Summary and Future Aspects

7.1 Summary of the thesis work

This thesis reports a study of modulational instability (MI) and solitary waves in a few important nonlinear optical systems. The thesis focuses mainly on the following nonlinear optical systems: nonlinear negative index metamaterials, non-Kerr media exhibiting power law nonlinearity and very briefly, silicon waveguides.

A new mathematical model is proposed to study pulse propagation in negative refractive index metamaterial (NIM) embedded in a Kerr medium. The model is generalized in the sense that it is capable of reproducing previous models under appropriate approximations and also the proposed model predicts new phenomena like magnetic self-steepening effect. MI analysis is carried out using the new model in Chapter 2. This study is extended to the case of a metamaterial (MM) embedded into a medium with cubic-quintic nonlinearity. The importance of the above two studies lies in the fact that, many key parameters, some of which are controllable and need specific attention from the experimentalist, is pointed out. Apart from that, attempts are made to obtain exact solitary wave solutions to the model equations by simple methods. In the later part of the thesis, the generalized nonlinear Schrodinger equation, appropriate to model nonlinear pulse propagation in non-Kerr cubic-quintic media is integrated to find the exact solitary wave solutions. The parameter domain restrictions have also been identified in the process of obtaining these solutions. The MI analysis is also carried out in this media and the effect of higher order dispersion and higher order nonlinearity on MI gain spectrum is also discussed. The exact dark soliton solutions to the generalized nonlinear Schrödinger equation (NLSE) with coefficients dependent on the evolution parameters are obtained. This study is very general in the sense that the NLSE with constant coefficient could be considered as a special case of it. Finally, in Chapter 6, it is stressed that there are situations where the analytical methods adopted in this thesis could not be applied and one needs to rely on numerical methods. Because of the presence of the two-photon absorption (TPA) term, the ansatz method adopted in this thesis cannot be applied to get the exact one-soliton solution of the generalized NLSE governing the pulse propagation in a silicon waveguide. Solitary wave propagation in a silicon waveguide is studied numerically. The formation of a fundamental soliton from a higher order soliton is demonstrated in a silicon waveguide; soliton-soliton interaction is also studied.

7.2 Future Direction

Recently, the NLSE with coefficients dependent on the evolution parameter have been proposed as a more realistic model in nonlinear optics and other areas of physics. The method applied in this thesis to obtain the solitary wave solutions for the generalized nonlinear Schrodinger equation with evolution parameter dependent co-efficient could find attention in other areas of science and engineering as well, e.g. Bose Einstein condensate (BEC) [191-196] and dispersion managed optical communication systems. In the context of negative index materials, in this thesis , the coupled nonlinear field equations is derived considering that the MM is embedded in Kerr as well as non-Kerr nonlinear medium. However, if the nonlinear dielectric embedded in the MM has extremely high nonlinear refractive index and is liable to exhibit the saturable nonlinear effect, even for a moderate incident light intensity, then MMs may possess saturable nonlinearity. On the contrary, the local field enhancements in the split ring resonator arrays can be very intense, which allows for enhanced nonlinear effects, and makes the MM to exhibit saturable magnetic nonlinearity as well. These issues are not addressed in this thesis. Very recently, MI with saturation nonlinearity in MM is reported considering only the electric saturation nonlinearity [197-199]. The works reported in this thesis could be extended to incorporate both the electric and magnetic saturation nonlinearity.

This thesis does not address the important issue of the stability of solitary waves in the nonlinear optical systems considered. It could be one of the extremely potential areas of investigation in future. Another area which needs lots of attention is few-cycle pulse propagation in the nonlinear optical systems considered in this thesis. The so-called slowly varying approximation which is routinely used to derive many of the pulse propagation models needs to be revisited or abandoned. Investigating nonlinear waves in the few-cycle pulse regime could be very insightful and useful and may provide good amount of new physics.

Bibliography

1. V. E. Zakharov and L. A. Ostrovsky, "Modulations instability: The beginning", Physica D **238**, 540 (2009).
2. G. P. Agrawal, *Nonlinear Fiber Optics*, 4th ed. Academic, New York 2007.
3. H. C. Yuen, B. M. Lake, "Instabilities of waves on deep water", Annual Review of Fluid Mechanics **12**, 303 (1980).
4. A.P. Misra and P. K. Shukla, "Stability and evolution of wave packets in strongly coupled degenerate plasmas", Phy. Rev. E **85**, 026409 (2012).
5. S. Chandra & B.Ghosh, "Modulational instability of electron-acoustic waves in relativistically degenerate quantum plasma", Astrophys. Space. Sci. **342**, 417 (2012).
6. T. B. Benjamin, J. E. Feir, "The disintegration of wave trains on deep water. Part 1. Theory", Journal of Fluid Mechanics **27**, 417 (1967).
7. T. B. Benjamin, "Instability of periodic wavetrains in nonlinear dispersive systems", Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences **299**, 59 (1967).
8. L.I. Zagryadskaya, L.A. Ostrovskii, "Observed self-influence of modulated waves in a nonlinear line", Transl. Radiophys. and Quantum Electronics **11**, 548 (1969).
9. L. A. Ostrovskii, "Propagation of wave packets and space-time self-focusing in a nonlinear medium", Transl. Sov. Phys. JETP **24**, 797 (1967).
10. A. I. Lomtev "Modulation instability of electromagnetic excitations in the nonlocal Josephson electrodynamics of thin magnetic superconducting films", Technical Physics, **48**, 1424 (2003).
11. L. A. Ostrovskii, L.V. Soustov, "Self-modulation of electromagnetic waves in nonlinear transmission lines", Transl. Radiophys. and Quantum Electronics **15** , 182 (1973).
12. J-S. Huang, Z-W. Xie, M. Zhang and L-F. Wei, "Modulational instability of two component Bose–Einstein condensates in an optical lattice formed inside a cavity", J. Phys. B: At. Mol. Opt. Phys. **43,** 065305 (2010).
13. K. Dysthe, E. H. Krogstad, P. Müller, "Oceanic rogue waves", Annu. Rev. Fluid Mech. **40**, 287 (2008).

14. M. Erkintalo, K. Hammani, B. Kibler, C. Finot, N. Akhmediev, J. M. Dudley and G. Genty , " Higher-order modulation instability in nonlinear fiber optics", Phy. Rev. Let. **107**, 253901(2011).
15. S. Sudo, H. Itoh, K. Okamoto, K. Kubodera, "Generation of 5 THz repetition optical pulses by modulation instability in optical fibers", Appl. Phys. Lett. **54** , 993 (1989).
16. R. V. J. Raja, K. Porsezian, K. Nithyanandan, "Modulational-instability-induced supercontinuum generation with saturable nonlinear response", Phy. Rev. A **82**, 013825 (2010).
17. J. M. Cha´vez Boggio, S. Tenenbaum, H.L. Fragnito, "Amplification of broadband noise pumped by two lasers in optical fibers", J. Opt. Soc. Am. B **18**, 1428 (2001).
18. J. M. Dudley, G. Genty and S. Coen, "Supercontinuum generation in photonic crystal fiber", Rev. Mod. Phys. **78**, 1135 (2006).
19. C.R. Phillips, M.M. Fejer, "Stability of the singly resonant optical parametric oscillator", J. Opt. Soc. Am. B **27**, 2687(2010).
20. A. Hasegawa, "Generation of a train of soliton pulses by induced modulational instability in optical fibers", Opt. let.**9**, 288 (1984).
21. K. Tai, A. Hasegawa and A. Tomita, "Observation of modulational instability in optical fibers", Phy.Rev. Lett **56**,135 (1986).
22. K. Tai, A. Hasegawa, J. L. Jeweell, and A. Tomita, "Generation of subpicosecond solitonlike optical pulses at 0.3 THz repetition rate by induced modulational instability", Appl. Phys. Lett **49**, 236 (1986).
23. A. Höök and M. Karlsson, "Ultrashort solitons at the minimum-dispersion wavelength: effects of fourth-order dispersion", Opt. Lett. **18**, 1388 (1993).
24. F. Kh. Abdullaev, S. A. Darmanyan, S. Bischoff, P. L. Christiansen, and M. P. Sørensen, "Modulational instability in optical fibers near the zero dispersion point", Opt. Commun. **108**, 60 (1994).
25. J. D. Harvey, R. Leonhardt, S. Coen, G. K. L. Wong, J. Knight, W. J. Wadsworth, and P. St. J. Russell, "Scalar modulation instability in the normal dispersion regime by use of a photonic crystal fiber", Opt. Lett. **28**, 2225 (2003).
26. G. P. Agrawal, "Modulation instability induced by cross phase modulation", Phys. Rev. Lett. **59**, 880(1987).
27. S. Coen and M. Haelterman, "Modulational instability induced by cavity boundary conditions in a normally dispersive optical fiber", Phys. Rev. Lett. **79**, 4139 (1997).

28. A. R. Osborne, *Nonlinear Ocean Waves and the Inverse Scattering Transform*, Academic Press, New York 2010.

29. M. S. Ruderman, "Freak wave in laboratory and space plasmas", Eur. Phys. J. Special Topics **185**, 57 (2010).

30. A. Chabchoub, N. Hoffmann, M. Onorato, A. Slunyaev, A. Sergeeva, E. Pelinovsky, and N. Akhmediev, "Observation of a hierarchy of up to fifth-order rogue waves in a water tank", Phys. Rev. E **86**, 056601 (2012).

31. D. R. Solli, C. Ropers, P. Koonath and B. Jalali, "Optical Rouge waves", Nature letters **450**, 1054 (2007).

32. M. Remoissenet, *Waves Called Solitons*, third ed., Springer–Verlag, Berlin, 2003.

33. S. Lui, "Four-wave mixing and modulation instability of continuous optical waves in single-mode optical fibers", Appl. Phys. Lett. **89**, 171118 (2006).

34. P. G. Drazin, *Solitons: An Introduction*, Cambridge University Press, 1993.

35. L. Lam, *Introduction to nonlinear Physics*, Springer, New York, 1996.

36. J. Wu, J. Wheatley, S. Putterman and I. Rudnick, "Observation of envelope solitons in solids", Phys. Rev. Lett. **59**, 2744 (1987).

37. M. Savkovic and Z. Yoshida, "Spiral soliton solution for disk galaxies", Serb. Astron. J. N. **167**,11 (2003).

38. J. R. Apel, L. A. Ostrovsky, Y. A. Stepanyants, J. F. Lynch, "Internal solitons in the ocean and their effect on underwater sound", J. Acoust. Soc. Am. **121**,695 (2007).

39. A. Piccardi, U. Bortolozzo, S. Residori and G. Assanto1, "Spatial solitons in liquid-crystal light valves", Opt. Lett. **34** , 737(2009).

40. N. S Manton, "Solitons as elementary particles: a paradigm scrutinized", Nonlinearity **21**, T221 (2008).

41. Y.S. Kivshar, G.P. Agrwawal, *Optical Solitons: From Fibers to Photonics*, Academic Press, 2003.

42. Jr. G.L. Lamb, *Elements of Soliton Theory*, Dover New York, 1994.

43. T. Miwa, *Mathematics of Solitons,* Cambridge University Press, New York, 1999.

44. N. J. Zabusky and M. D. Kruskal, "Interaction of "Solitons" in a collisionless plasma and the recurrence of initial states", Phys. Rev. Lett. **15**, 240 (1965).

45. A. Hasegawa, A. Kodama, *Solitons in Optical Communications*, Oxford University Press, New York 1995.

46. J. T. Taylor, *Optical Solitons—Theory and Experiment*, Cambridge University Press, New York, 1992.

47. F. K. Abdullaev, *Optical Solitons*, Springer-Varlag, New York, 1993.
48. J. Scott Russell, Report of 14th Meeting of the British Association for Advancement of Science, NewYork, September 311 (1844).
49. G. B. Airy, "Tides and waves", In Hugh James Rose, et al. Encyclopaedia Metropolitana. Mixed Sciences **3**, 1817 (1841).
50. G. G. Stokes, "On the theory of oscillatory waves", Transactions of the Cambridge Philosophical Society **8**, 441 (1847).
51. H. Bazin, Recherches experimentales relatives aux remoux et a la propagation des ondes, in Recherches Hydrauliques, ed. By H. Darcy and H. Bazin.(1865).
52. J. Boussinesq, "Théorie de l'intumescence liquide, applelée onde solitaire ou de translation, se propageant dans un canal rectangulaire", Comptes Rendus de l'Academie des Sciences **72**, 755 (1871).
53. L. Rayleigh , "On waves", Phil. Mag. **1**, 257 (1876).
54. D. J. Korteweg, G. de Vries, (1895), "On the change of form of long waves advancing in a eectangular canal, and on a new type of long stationary waves", Philosophical Magazine **39**, 422(1895).
55. E. Fermi, J. Pasta, S. Ulam, "Studies of Nonlinear Problems", Los Alamos Sci. Lab. Rep., LA-1940 (1955).
56. C. S. Gardner, J. M. Greene, M. D. Kruskal, and R. M. Miura, "Method for solving the Korteweg-de Vries equation", Phys. Rev. Lett.**19**, 1095 (1967).
57. P. Lax, "Integrals of nonlinear equations of evolution and solitary waves", Comm. Pure Applied Math. **21**, 467 (1968).
58. V.E. Zakharov and A. B. Shabat, "Exact theory of two-dimensional self-focusing and one dimensional self-modulation of waves in nonlinear media", Sov. Phys. JETP **37**, 823 (1972).
59. A. Hasegawa and F. Tappert, "Transmission of stationary nonlinear optical pulses in dispersive dielectric fibers. I. Anomalous dispersion", Appl. Phys. Lett. **23**, 142 (1973).
60. L. F. Mollenauer, R. H. Stolen, and J. P. Gordon, "Experimental observation of picosecond pulse narrowing and solitons in optical fibers", Phys. Rev. Lett. **45**, 1095 (1980).
61. R. H. Stolen, L. F. Mollenauer, and W. J. Tomlinson, "Observation of pulse restoration at the soliton period in optical fibers", Opt. Lett. **8**, 186 (1983).

62. L. F. Mollenauer, R. H. Stolen, J. P. Gordon, and W. J. Tomlinson, “Extreme picosecond pulse narrowing by means of soliton effect in single-mode optical fibers”, Opt. Lett. **8**, 289(1983).

63. M. J. Ablowitz, G. Biondini, and L. A. Ostrovsky, “Optical solitons: Perspectives and applications”, Chaos **10**, 471 (2000).

64. L. F. Mollenauer, R. H. Stolen ,“The soliton laser”, Opt. Lett. **9**, 13(1984).

65. S. V. Chernikov, E. M. Dianov, D. J. Richardson and D. N. Payne, “Soliton pulse compression in dispersion-decreasing fibers”, Opt. Lett. **18**, 1 (1993).

66. N. J. Doran and D. Wood, “ Soliton processing element for all-optical switching”, J. opt. Soc. Am. B **4**, 1843(1987).

67. A. M. Weiner, J. P. Heritage, R. J. Hawkins, R. N. Thurston, E. M. Krischner, D. E. Leaird, and W. J. Tomlinson, “Experimental observation of the fundamental dark soliton in optical fibers”, Phys. Rev. Lett. **61,** 2445 (1988).

68. A. Chabchoub, O. Kimmoun, H. Branger, N. Hoffmann, D. Proment, M. Onorato and N. Akhmediev, “Experimental observation of dark dolitons on the surface of water”, Phys. Rev. Lett. **110**, 124101 (2013).

69. G. P. Agrawal, *Fiber-Optic Communication Systems*, 2nd ed. Wiley, New York, 1997.

70. E. Bourkoff,W. Zhao, R. L. Joseph, and D. N. Christodulides, “Evolution of femtosecond pulses in single-mode fibers having higher-order nonlinearity and dispersion”, Opt. Lett. **12**, 272(1987).

71. W. Zhao and E. Bourkoff, “Femtosecond pulse propagation in optical fibers: higher order effects”, IEEE J. Quantum Electron. **24**, 365 (1988).

72. V. Yanosky and F. Wise, “Nonlinear propagation of high-power, sub-100-fs pulses near the zero-dispersion wavelength of an optical fiber”, Opt. Lett. **19**, 1547 (1994).

73. C. Headley and G. P. Agrawal, “Unified description of ultrafast stimulated Raman scattering in optical fibers”, J. Opt. Soc. Am. B **13**, 2170 (1996).

74. F. M. Mitschke and L. F. Mollenauer, “Discovery of the soliton self-frequency shift”, Opt. Lett. **11**, 659 (1986).

75. K Hizanidis, D J Frantzeskakis and C Polymilis, “Exact travelling wave solutions for a generalized nonlinear Schrödinger equation”, J. Phys. A. Math. Gen. **29** ,7687 (1996).

76. R. Yang, L. Li, R. Hao, Z.Li, G. Zhou, “Combined solitary wave solutions for the inhomogeneous higher-order nonlinear Schrodinger equation”, Phys. Rev. E Stat. Nonlin. Soft. Matter. Phys. **71**, 036616 (2005).

77. W-P Hong, "Optical solitary wave solutions for the higher order nonlinear Schrödinger equation with cubic-quintic non-Kerr terms", Opt. Commun. **194**, 217 (2001).
78. . Z. Li, L. Li, "New types of solitary wave solutions with the higher-order nonlinear Schrodinger equations", Phys. Rev. Lett. **84**, 4096 (2000).
79. A. K. Sarma, "Solitary wave solutions of higher-order NLSE with Raman and self-steepening effect in a cubic-quintic-septic medium", Commun. Nonlinear. Sci. Nummer. Simulat. **14** , 3215 (2009).
80. A. V. Buryak and Y. S. Kivshar, "Spatial optical solitons governed by quadratic non-linearity", Opt. Lett. **19**, 1612(1994).
81. Y. R. Shen and N. Bloembergen, "Theory of stimulated Brillouin and Raman Scattering", Phys. Rev. E **137**, A 1787 (1965).
82. A. Biswas and S. Konar, *Introduction to non-Kerr optical solitons* , CRC Press, Boca Raton, FL, USA 2006.
83. R. Kohl, A. Biswas, D. Milovic & E. Zerrad. "Optical soliton perturbation in a non-Kerr law media", Opt. Las. Techno. **40,** 647 (2008).
84. J. L. Coutaz and M. Kull, "Saturation of the nonlinear index of refraction in semiconductor-doped glass", J. Opt. Soc. Am. B **8**, 95 (1991).
85. Z. Jovanoski and D. R. Roland, "Variational analysis of solitary waves in a homogeneous cubic-quintic nonlinear medium", J. Mod. Opt. **48**, 1179 (2001).
86. B. Lawrence, W.E. Torruellas, M. Cha, M. L. Sundheimer, G.I. Stegeman, J. Meth, S. Eteman and G. Baker, "Identification and role of two-photon excited states in a π-conjugated polymer" , Phys. Rev. Lett. **73**, 597(1994).
87. S. A. Ramakrishna and T. M. Grzegorczyk, *Physics and Applications of Negative Refractive Index Materials*, CRC Press, 2009.
88. V. M. Shalaev, W. Cai, U. K. Chettiar, H. Yuan, A. K. Sarychev, V. P. Drachev, and A. V. Kildishev, "Negative index of refraction in optical metamaterials", Opt. Lett. **30**, 3356 (2005).
89. G. Dolling, M. Wegener, C. M. Soukoulis, and S. Linden, "Negative-index metamaterial at780 nm wavelength", Opt. Lett. **32**, 53 (2007).
90. A. A.Zharov, I. V. Shadrivov, and Y. S. Kivshar, "Nonlinear properties of left-handed meta-materials", Phys. Rev. Lett. **91**, 037401 (2003).
91. M. Scalora, M. S. Syrchin, N. Akozbek, E. Y. Poliakov, G. D'Aguanno, N. Mattiucci, M. J. Bloemer, and A. M. Zheltikov, "Generalized nonlinear Schrödinger equation for

dispersive susceptibility and permeability: Application to negative index materials", Phys. Rev. Lett. **95**, 013902 (2005).

92. S. Wen, Y. Xiang, X. Dai, Z. Tang, W. Su, and D. Fan, "Theoretical models for ultrashort electromagnetic pulse propagation in nonlinear Metamaterials", Phys. Rev. A **75**, 033815 (2007).
93. J-G Ma and Zh. Chen, "Solitary electromagnetic waves in non-Kerr like nonlinear media", IEEE Transaction on magnetic. **33**, 2 (1997).
94. X. Liu,Y. Adachi, Y. Tomita,1, J Oshima, T. Nakashima and T. Kawai, " High-order nonlinear optical response of a polymer nanocomposite film incorporating semiconducotor CdSe quantum dots", Opt. Exp.**20**,13457 (2012).
95. Y. Chen, K. Beckwitt, F. Wise, B. Aitken, J. Sanghera, and I.D. Aggarwal, "Measurement of fifth- and seventh-order nonlinearities of glasses", J. Opt. Soc. Am. B **23**, 347 (2006).
96. A. K. Sarma. "Dark soliton switching in an NLDC in the presence of higher-order perturbative effects", Optics and Laser Technology. **41**, 247 (2009).
97. A. K. Sarma. "A comparitive study of soliton switching in a two- and three-core coupler with TOD and IMD", Optik. **120**, 390 (2009).
98. R. Soref, "The past, present, and future of silicon photonics", IEEE J. Sel. Top. Quantum Electron. **12**, 1678 (2006).
99. B. Jalai, S. Fathpour, " Silicon Photonics", J. Lightwave Technol. **24** ,4600 (2006).
100. V. R. Almeida, C. A. Barrios, R. R. Panepucci, and M. Lipson, "All-optical control of light on a silicon chip", Nature **431**, 1081 (2004).
101. L. Pavesi and D.J. Lockwood, Eds., *Silicon Photonics* ,Springer, New York, 2004.
102. G.T. Reed and A.P.Knights, *Silicon Photonics : An Introduction* , Willey Hoboken, NJ, 2004.
103. J. Jhang, Q. Lin, G. Piredda, R. W. Boyd, G. P. Agrawal, and P. M. Fauchet, "Optical solitons in a silicon waveguide", Opt. Express **15**, 7682 (2007).
104. E. Bulgan, Y. kanamori and K. Hane, "Submicron silicon waveguide optical switch driven by micro electromechanical actuator", Appl. Phys. Lett. **92**, 101110 (2008).
105. Q. Xu, B. Schmidt, J. Shakya, and M. Lipson, "Cascaded silicon micro-ring modulators for WDM optical interconnection", Opt. Exp. **14**, 9430 (2006).
106. L. Yin and G. P. Agrawal, "Impact of two-photon absorption on self-phase modulation in silicon waveguides", Opt. Lett. **32**, 2031 (2007).
107. Q. Lin, O. J. Painter and G. P. Agrawal, "Nonlinear Optical phenomena in silicon waveguides: Modeling and applications", Opt. Exp. **15**, 16604 (2007).

108. M. Du, A. K. Chan, C. K. Charles, "A novel approach to solving the nonlinear Schrodinger equation by the coupled amplitude-phase formulation", IEEE J. Quantum Elect**. 31,** 177 (1995)

109. W. Cai and V. Shalaev, *Optical Metamaterials*, Springer, 2010.

110. H. Lamb, "On group velocity", Proc. London Math. Soc. **S2–1**, 473 (1904).

111. L. I. Mandelshtam, "Lectures given on February 26, 1940 and on May 5, 1944" in Complete Issue of Works (Izdatelstvo Trudov AN SSSR, 1950) (in Russian), Vol. 5

112. D. V. Sivuhin, "About the energy of electromagnetic field in dispersing media", Opt. Spektrosk. **3**, 308 (1957).

113.V. G. Veselago, "Electrodynamics of substances with simultaneously negative electrical and magnetic permeabilities", Sov. Phys. Usp. **10**, 509 (1968).

114. D. R. Smith, W.J. Padilla, D. C. Vier, S.C. Nemat-Nasser , S. Schultz , "Composite medium with simultaneously negative permeability and permittivity", Phys. Rev. Lett. **84**, 4184 (2000).

115. J. B. Pendry, "Negative refraction makes a perfect lens", Phys. Rev. Lett. **85**, 3966 (2000).

116.V. M. Shalaev, "Optical negative-index metamaterials", Nature Photonics **1**, 41(2007).

117. R. A. Shelby, D. R. Smith, S. C. Nemat-Nasser, and S. Schultz, "Microwave transmission through a two-dimensional, isotropic, left-handed Metamaterial", Apl. Phys. Lett. **78**,489(2001).

118. C. Caloz and T. Itoh, *Electromagnetic Metamaterials: Transmission line theory and Microwave applications,* Wiley Interscience, 2006.

119. S. H. Lee, C. M. Park, Y. Mun Seo and C. K. Kim, "Reversed Doppler effect in double negative Metamaterials", Phys. Rev. B **81**, 241102 (2010).

120. S. Xi, H. Chen, T. Jiang, L. Ran, J. Huangfu,, B-I Wu , J. A. Kong, and M. Chen, "Experimental verification of reversed Cherenkov Radiation in Left-Handed Metamaterial", Phy. Rev.Lett. **103,** 194801 (2009).

121. A. Lakhtakia, "On Planewave Remittances and Goos–Hänchen Shifts of Planar Slabs with Negative Real Permittivity and Permeability", Electromagnetics **23**, 71(2005).

122.J. B. Pendry, A. J. Holden and W. J. Stewart, "Extremely low frequency plasmons in metallic mesostructures", Phys. Rev. Lett. **76**, 4773(1996).

123. J. B. Pendry, A. J. Holden, D. J. Robbins, and W. J. Stewart, "Magnetism from conductors, and enhanced non-linear phenomena", IEEE Trans. Microw. Theory. Tech. **47**, 2075(1999).

124. R. A. Shelby, D. R. Smith and S. Schultz, "Experimental verification of a negative index of refraction", Science **292**, 77(2001).

125. A. Berrier, M. Mulot, M. Swillo, M. Qiu, L. Thylen, A. Talneau, and S. Anand, "Negative refraction at infrared-wavelengths in a two-dimensional photonic crystal", Phys. Rev. Lett. **93**, 073902 (2004).

126.S. Zhang et al, "Experimental demonstration of near-infrared negative index Metamaterials", Phys. Rev. Lett. **95**, 137404 (2005).

127.E. Schonbrun, M. Tinker, W. Park, and J.-B. Lee, "Negative re-fraction in a Si-polymer photonic crystal membrane," IEEE Photon. Technol. Lett.**17**, 1196 (2005).

128.S. Zhang et. al, "Demonstration of metal-dielectric-negative-index Metamaterials with improved performance at optical frequencies", J. Opt. Soc. Am. B **23**,434(2006).

129.G. Dolling, "Low-loss negative-index metamaterial at telecommunication wavelength", Opt. Lett. **31**,1800 (2006).

130. V. M. Shalaev, "Loss free and active optical negative-index metamaterials", Nature Letters **465**, 735 (2010).

131. W. T. Lu, J. B. Sokoloff, and S. Sridhar, "Refraction of electromagnetic energy for wave packets incident on a negative-index medium is always negative", Phys. Rev. E **69**, 026604 (2004).

132. X. Huang and W. L. Schaich "Wavepacket propagation into a negative index medium", Am. J. Phys.**72**, 1232 (2004).

133. G. D'Aguanno, N. Akozbek, N. Mattiucci, M. Scalora, M. J.Bloemer, and A. M. Zheltikov, " Dispersion-free pulse propagation in a negative-index material", Opt. Lett. **30**, 1998 (2005).

134. R. W. Ziolkowski, "Superluminal transmission of information through an electromagnetic metamaterial", Phys. Rev. E **63**, 046604 (2001).

135. G. Doling,C. Enrich, M. Wegener, C. M. Soukoulis and S. Linden, "Simultaneous negative phase and group velocity in metamaterial", Science **312**, 5775 (2006).

136. M. Lapine, M. Gorkunov, and K. H. Ringhofer, "Nonlinearity of a metamaterial arising from diode insertions into resonant conductive elements", Phys. Rev. E **67**, 065601 (2003).

137. D. Schurig, J. J. Mock, B. J. justice, S.a. Cummer, J. B. pendry, A. F. Starr and D. R. Smith, "Metamaterials electromagnetic cloak at microwave frequencies", Science **314**, 977 (2006).

138. N. Landy and D. R. Smith, "A full parameter unidirectional metamaterial cloak for microwaves", Nature materials **12**, 25 (2013).

139. S. Wen, Y. Wang, W. Su, Y. Xiang, X. Fu and D. Fan, "Modulation instability in nonlinear negative-index material", Phys. Rev. E **73**, 036617-1 (2006).

140. S. Wen, Y. Xiang, W. Su, Y. Hu, X. Fu, and D. Fan, "Role of the anomalous self-steepening effect in modulation instability in negative-index material", Opt. Express **14**, 1568 (2006).

141. N. Lazarides, and G. P. Tsironis, "Coupled nonlinear Schrodinger field equations for electro-magnetic wave propagation in nonlinear left-handed materials", Phys. Rev. E **71**, 036614(2005).

142. I. Kourakis and P. K. Shukla, "Nonlinear propagation of electromagnetic waves in negative-refraction-index composite materials", Phys. Rev. E **72**, 016626 (2005).

143. Y. Xiang, X. Dai , S. Wen and D. Fan, "Review of nonlinear optics in metamaterials", PIERS proceedings China 952 (2008).

144. Y. Xiang, S. Wen, X. Dai, Z. Tang, W. Su, and D. Fan, " Modulation instability induced by nonlinear dispersion in nonlinear Metamaterials", J. Opt. Soc. Am. B **24**, 3058 (2007).

145. I. Kourakis, N. Lazarides, and G. P. Tsironis, "Self-focusing and envelope pulse generation in nonlinear magnetic Metamaterials", Phys. Rev. E **75**, 067601 (2007).

146. J. Zhang, S. Wen, Y. Xiang, Y. Wang and H. Luo, "Spatiotemporal electromagnetic soliton and spatial ring formation in nonlinear Metamaterials", Phys. Rev. A **81**, 023829 (2010).

147. Y. Xiang, X. Dai, S. Wen, J. Guo and D. Fan, "Controllable Raman soliton self-frequency shift in nonlinear Metamaterials", Phys. Rev. E **84**, 033815 (2011).

148. G. D'Aguanno, N. Mattiucci, M. Scalora, and M. J. Bloemer, "Bright anad dark gap solitons in a negative index fabry-Perot Etalon", Phys. Rev. Lett. **93**, 213902 (2004).

149. Y. M. Liu and G. Bartal, "Subwavelength discrete soliton soliton in nonlinear Metamaterial", Phys. Rev. Lett. **99**, 153901(2007).

150. P. Kinsler, "Optical pulse propagation with minimal approximations", Phys. Rev. A **81**, 013819 (2010).

151. P. Kinsler, "Unidirectional optical pulse propagation equation for materials with both electric and magnetic responses", Phys. Rev. A **81**, 023808 (2010).

152. M. Stockman, "Criteria for negative refraction with low optical losses from a fundamental principle of causality", Phys. Rev. Lett. **98**, 177404 (2007).

153.M. J. Potasek, "Modulation instability in an extended nonlinear Schrodinger equation", Opt. Lett. **12**, 921(1987).

154. R. A. Vicencio, M.I. Molin, Y. S. Kivsher, "All-optical switching and amplification of discrete vector solitons in nonlinear cubic birefringement waveguide arrays", Opt. Lett. **29**, 2905(2004).

155. L. Y. Wang, L. Li , Z. H. Li, G. S. Zhou, D. Mihalache, "Generation, compression, and propagation of pulse trains in the nonlinear Schrodinger equation with distributed coefficients", Phys. Rev. E **72**,036614(2005).

156. R. A. Vicencio, M. I. Molina, Y. S. Kivsher, "Polarization instability,steering, and switching of discrete vector solitons", Phys. Rev. E **71**, 056613(2005).

157.W. Krolikowski, N. Akhmediev , B. Luther-Davies, "Darker-than-black solitons: Dark solitons with total phase shift greater than π", Phys. Rev. E **48**,13980 (1993).

158. F. Azzouzi, H. Triki, K. Mezghiche, A. El. Akrmi, "Solitary wave solutions for high dispersive cubic-quintic nonlinear Schrodinger equation", Chaos, Solitons and Fractals **39**,1304 (2009).

159. A. Biswas, "Optical solitons with time-dependent dispersion, nonlinearity and attenuation in a power law medium", Commun. Nonlinear. Sci. Numer. Simulat. **14,** 1078 (2009).

160. M. Saha, A. K. Sarma , A. Biswas, "Dark optical solitons in power law media with time-dependent coefficients", Phys Lett A **373**, 4438(2009).

161. P. K. Shukla and J. J. Rasmussen, "Modulational instability of short pulses in long optical fibers", Opt. Lett. **11**,171(1986).

162. A. Mohamadou, C. G. LatchioTiofack and T. C. Kofane , " Wave train generation of solitons in systems with higher order nonlinearities", Phy. Rev. E **82,** 016601 (2010).

163. M. Scalora, D. D'Ceglia, G. D'Aguanno, N. Mattiucci, N. Akozbek, M. Centini and M. J. Bloemer, "Gap solitons in a nonlinear quadratic negative-index cavity", Phys. Rev. E **75**, 066606 (2007).

164. A. K. Sarma, "Solitary waves in a negative index material with dispersive permittivity and permeability", Eur. Phys. J. D. **62**, 421 (2011).

165. A. K. Sarma and M. Saha, "Modulational instability of coupled nonlinear field equations for pulse propagation in a negative index material embedded into a Kerr medium", J. Opt. Soc. Am. B **28**, 944 (2011).

166. S. Wuestner, A. Pusch, K. L. Tsakmakidis, J. M. Hamm, and O. Hess, "Overcoming losses with gain in a negative refractive index Metamaterial", Phys. Rev. Lett. **105**, 127401 (2010).

167. M. Karlsson, "Modulational instability in lossy optical fibers", J. Opt. Soc. Am. B, **12**, 2071 (1995).

168. M. N. Z. Abou'ou, P. T. Dinda, C. M. Ngabireng, B. Kibler, and F. Smektala , "Impact of the material absorption on the modulational instability spectra of wave propagation in high-index glass fibers", J. Opt. Soc. Am. B **28**, 1518 (2011).

169. A. Biswas, D. Milovic & M. E. Edwards. *Mathematical Theory of Dispersion-Managed Optical Solitons*. Springer Verlag, New York, NY, USA 2010.

170. A. Biswas. "1-soliton solution of the generalized Radhakrishnan, Kundu, Lakshmanan equation", Phys. Lett. A **373**, 2456 (2009).

171. C. M. Khalique & A. Biswas, "Optical solitons with parabolic and dual-power law nonlinearity via Lie group analysis", J. Electromagn. Waves Appl. **23**, 963 (2009).

172. S. Konar, S. Jana & M. Mishra, "Induced focusing and all optical switching in cubic-quintic nonlinear media", Opt. Coms. **255**, 114 (2005).

173. X. Lü, J. Li, H-Q Zhang, T. Xu, L-L. Li and B. Tian, "Integrability aspects with optical solitons of a generalized variable coefficient *N*-coupled higher order nonlinear Schrödinger system from inhomogeneous optical fibers", J. Mat. Phys. **51**, 043511(2010).

174. D.D. Ganji, A. Asgari, Z.Z. Ganji, "Exp-Function Based Solution of Nonlinear Radhakrishnan, Kundu and Laskshmanan (RKL) Equation", Acta Appl. Math. **104**, 201 (2008).

175. K. Porsezian, R. Ganapathy, A. Hasegawa & V. N. Serkin. "Non autonomous soliton dispersion management", IEEE J. Quant. Elects.**45**, 1577(2009).

176. R. Kohl, A. Biswas, D. Milovic and E. Zerrad. "Soliton perturbation theory for dispersion managed optical fibers", J. Non. Opt. Phys. Mats. **18**, 227 (2009).

177. K. Porsezian, A. Hasegawa, V. N. Serkin, T. L. Belyaeva and R. Ganapathy. "Dispersion and nonlinear management for femtosecond optical solitons", Phys. Lett. A. **361**, 504 (2007).

178. J. He and Y. Li , "Designable Integrability of the Variable Coefficient Nonlinear Schrödinger Equations", Std. Appl. Maths. **126**, 1 (2011).

179. L. P. Pitaevskii and S. Stringari, *Bose–Einstein Condensation*, Oxford, 2003.

180. A. C. Peacock, "Applications of Light propagation in novel photonic devices", 2004.

181. J-L. Zhang, M-L. Wang, "Various exact solutions for two special type RKL models", Chaos Solitons Fractals **37,** 215 (2008)

182. M. Dinu, F.Quochi and H. Garica, "Third-order nonlinearities in silicon at telecom wavelengths", Appl. Phys. Lett. **82**, 2954 (2003).

183.L. Yin, Q. Lin and G. P. Agrawal, "Dispersion tailoring and soliton propagation in silicon waveguides," Opt. Lett.**31**, 1295–1297 (2006).

184. A. K. Sarma, "Silicon waveguide based nonlinear directional coupler as a soliton switch", Opt. Eng.**47**, 120503-1 (2008).

185.J. Wu and A. K. Sarma, "Ultrafast all-optical XOR logic gate based on a symmetrical Mach-Zehnder interferometer employing SOI waveguides", Opt. Comm. **283**, 2914 (2010).

186. H. K. Tsang, C. S. Wong, T. K. Liang, I. E. Day, S. W. Roberts, A.Harpin, J. Drake and M. Asghari, "Optical dispersion, two-photon absorption and self-phase modulation in silicon waveguides at 1.5 mm wavelength", Appl. Phys. Lett.**80**, 416–418 (2002).

187. O. Boyraz, T. Indukuri, and B. Jalali, "Self-phase-modulation induced spectral broadening in silicon waveguides", Opt. Exp. **12**, 829 (2004).

188. G. W. Rieger, K. S. Virk, and J. F. Young, "Nonlinear propagation of ultrafast 1.5m pulses in high-index contrast silicon-on-insulator waveguide", Appl. Phys. Lett.**84**, 900 (2004).

189. L. Yin, Q. Lin, and G. P. Agrawal, "Soliton fission and supercontinuum generation in silicon waveguides", Opt. Lett.**32**, 391 (2007).

190. J. P. Gordon, "Interaction forces among solitons in optical fibers", Opt. Lett.**8**, 596 (1983).

191. F. Dalfovo , S. Giorgini , L. P. Pitaevskii and S. Stringari, "Theory of Bose-Einstein condensation in trapped gases", Rev. Mod. Phys. **71**, 463(1999).

192. L. Khaykovich, F. Schreck, G. Ferrari, T. Bourdel, J. Cubizolles, L. D. Carr, Y.Castin and C. Salomon " Formation of a Matter-Wave Bright Soliton", Science **296**,1290 (2002).

193.K. E. Strecker, G. B. Partridge, A. G. Truscott and R. G. Hulet, "Formation and propagation of matter-wave soliton trains", Nature **417**, 150 (2002).

194.S. L.Cornish, S. T. Thompson and C. E. Wieman, "Formation of bright matter-wave solitons during the collapse of attractive Bose-Einstein Condensates", Phys. Rev. Lett. **96**, 170401 (2006).

195. Z. Dutton, M. Budde, C. Slowe and L. V. Hau , “Observation of quantum shock waves created with ultra-compressed slow light pulses in a Bose-Einstein Condensate”, Science **293**, 663 (2001).

196. B. Eiermann, Th. Anker, M. Albiez, M. Taglieber, P. Treutlein, K.-P. Marzlin and M. K. Oberthaler, “Bright Bose-Einstein gap solitons of atoms with repulsive interaction”, Phys. Rev. Lett. **92,** 230401 (2004).

197. Y. Xiang, X. Dai, S. Wen, and D. Fan, “Modulation instability in metamaterials with saturable nonlinearity”, J. Opt. Soc. Am. B **28**, 908 (2011).

198. C.G. L. Tiofack, A. Mohamadou, Alim , K. Porsezian and T. C. Kofane ,“Modulational instability in metamaterials with saturable nonlinearity and higher-order dispersion”, J. Mod. Opt. **59**, 972 (2012).

199. Z. X. Qiong, Cheng K and X. A. Ping “Impacts of higher-order dispersions and saturable nonlinearities on modulation instability in negative-refractive Metamaterials”, Chin. Phys. B **22**, 034205(2013).

List of Publications

A. International Journals

1. **Manirupa Saha** and Amarendra K. Sarma, "Solitary wave solutions and modulation instability analysis of the nonlinear Schrodinger equation with higher order dispersion and nonlinear terms", *Communications in Nonlinear Science and Numerical Simulation* Vol. 18, p.2420(2013).

2. **Manirupa Saha** and Amarendra K. Sarma, "Modulation instability in nonlinear metamaterials induced by cubic-quintic nonlinearities and higher order dispersive effects" *Optics Communications* Vol. 291, p.321 (2013).

3. Amarendra K. Sarma and **Manirupa Saha**, "Modulational instability of coupled nonlinear field equations for pulse propagation in a negative index material embedded into a Kerr medium", *Journal of Optical Society of America B* (JOSA-B) Vol. 28, p.944 (2011).

4. Amarendra K. Sarma, **Manirupa Saha** and Anjan Biswas, "Optical Solitons with Power Law Nonlinearity and Hamiltonian Perturbations: An Exact Solution", *Journal of Infrared, Millimeter and Terahertz waves* Vol. 31, p.1048 (2010) (Springer).

5. Amarendra K. Sarma, **Manirupa Saha** and Anjan Biswas, "Effect of two-photon absorption on soliton propagation and soliton-soliton interaction in a silicon waveguide", *SPIE Journal of Optical Engineering* Vol.49, p.035001-1 (2010).

6. **Manirupa Saha**, Amarendra K. Sarma and Anjan Biswas, "Dark Optical solitons in power law media with time-dependent coefficients", *Physics Letters A* Vol. 373 p.4438 (2009).

B. Conferences (International/National)

1. **Manirupa Saha** and Amarendra K. Sarma, "Modulation Instability analysis in cubic-quintic non-Kerr media", XXXVII National symposium of Optical Society of India, Pondicherry University, Jan. 22-25, 2013.

2. **Manirupa Saha** and Amarendra K. Sarma, "Modulation Instability with cubic-quintic nonlinearities and higher order dispersive effects in nonlinear metamaterials", Frontiers in Optics and Laser science 2012, Frontiers in Optics/Laser Science XXVIII meeting, Rochester, New York, Oct.14-18, 2012.

3. **Manirupa Saha** and Amarendra K. Sarma, "Modulation Instability for coupled field equations in nonlinear metamaterials", International Conference on Fiber Optics and Photonics, IIT Guwahati, Dec. 12-15, 2010.

4. **Manirupa Saha**, Amarendra K. Sarma and Anjan Biswas, "Role of two-photon absorption on solitary wave propagation in a silicon waveguide", International Conference on Optics and Photonics , Central Scientific Instruments Organization, (CSIR), Chandigarh Oct. 30-Nov.1, 2009

Vita

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