SHORT ABSTRACT

The aim of this thesis is to study *a priori* and *a posteriori* error analysis of finite element methods for elliptic and parabolic optimal control problems with measure data in a bounded convex domain in $\mathbb{R}^d (d = 2 \text{ or } 3)$. The control problems governed by elliptic partial differential equation with measure data are used to model the potential of an electric field with an electric charge distribution, whereas parabolic optimal control problems with measure data in space are used in the design and management of waste water treatment systems, mainly the disposal of sea outfalls discharging polluting effluent from a sewerage systems. On the other hand, parabolic optimal control problems with measure data in time appear in the optimality conditions of some optimal control problems with pointwise state constraints in time. The solution of the state equation of such type of problems exhibits low regularity due to the presence of measure data which introduces some difficulties for both theory and numerics of the finite element method. An effort has been made in this thesis to investigate both *a priori* and *a posteriori* error analysis of finite element method for these control problems. The strategy *optimize-then-discretize* is employed for the approximations of these control problems.

We first analyze elliptic optimal control problem with measure data and prove the existence, uniqueness and regularity of the solution to the optimal control problem. To discretize the control problem we use piecewise linear and continuous finite elements for the approximation of the state and co-state variables, whereas piecewise constant functions are used for the approximation of the control variable. We derive *a priori* error estimates for the state, co-state and control variables in the L^2-norm with an order of convergence $\mathcal{O}(h^{2-\frac{d}{2}})$. Further, global *a posteriori* upper bounds for the state, co-state and control variables in the L^2-norm are established. Moreover, local lower bounds for the errors in the state and co-state variables, and global lower bound for the error in the control variable are demonstrated in case of two space dimension ($d = 2$).

We next consider parabolic optimal control problems with measure data. Two kinds of problems, namely measure data in space and measure data in time are considered and analyzed. The existence, uniqueness and regularity of the solutions of both type of control problems are proved. The continuous piecewise linear functions are used for the approximations of the state and co-state variables, and piecewise constant functions for the approximation of the control variable. Both spatially discrete and fully discrete finite element approximations of the control problems with measure data in space and time are analyzed. *A priori* error estimates of order $\mathcal{O}(h^{2-\frac{d}{2}})$ is demonstrated for the spatially discrete control problem with measure data in space whereas error estimate of order $\mathcal{O}(h^{2-\frac{d}{2}} + k^{1/2})$ is established for the fully discrete backward Euler time discretization. For parabolic optimal control problem with measure data in time, we have obtained error estimates of order $\mathcal{O}(h)$ for the state, costate and control variables for the spatially discrete problem. A time discretization scheme based on implicit backward-Euler method is analyzed and error estimates of order $\mathcal{O}(h + k^{1/2})$ are derived for the state, co-state and control variables. Further, we study the *a posteriori* error analysis for the space-time finite element discretization for both type of control problems. We derive global upper bounds for the errors in the state, co-state and control variables in the $L^2(0, T; L^2(\Omega))$-norm.

Finally, numerical results for two dimensional test problems are presented to illustrate our theoretical findings.

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