INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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Programme of Study	:	Ph.D.
Thesis Title	:	High-Order Weak Galerkin Finite Element Methods for Some Hyperbolic PDEs
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Thesis Submitted to the Department/ Center	:	Department of Mathematics
Date of completion of Thesis Viva-Voce Exam	:	January 2, 2025
Key words for description of Thesis Work	:	Weak Galerkin Methods, Wave Equation, Non Linear Problems, Interface Problems, Hyperbolic PDEs.

SHORT ABSTRACT

This thesis is devoted to developing an advanced high-order numerical method for some hyperbolic PDEs including linear, semilinear and quasilinear hyperbolic problems. High-order convergence of weak Galerkin finite element approximations to the true solutions for hyperbolic problems have been designed and analyzed. First, a systematic numerical study for second-order linear wave equations using weak Galerkin finite element method (WG-FEM) has been described for both semidiscrete and fully discrete cases. Different degrees of polynomials are used to construct weak Galerkin finite element spaces. Error estimates for both semidiscrete and fully discrete cases are carried out in suitable norms. Next, weak Galerkin finite element methods has been proposed for the semilinear Klein-Gordon equation. Optimal order error estimates in suitable norms have been carried out for both semidiscrete and fully discrete cases. For fully discrete case, a second-order time Newmark scheme has been employed for temporal discretization. Further, weak Galerkin finite element method has been proposed for Westervelt's model of ultrasound waves. The spatial discretization of Westervelt's quasilinear strongly damped wave equation is investigated using high-order weak Galerkin discretization. Convergence analysis in suitable norms for linearized Westervelt's equation has been performed with variable coefficients. Then the results have been extended for Westervelt's quasilinear acoustic wave equation relying on the Banach fixed-point theorem for sufficiently small data and mesh size, given an appropriate choice of initial data. In the final problem, WG-FEM for general linear hyperbolic interface problem is described for non-homogeneous jumps of the exact solution along the interfaces. Convergence analysis is carried out for both semidiscrete and fully discrete schemes. Optimal error estimates in suitable norms have been obtained for locally smooth solutions. The Crank-Nicolson scheme has been employed for fully discrete scheme,. Finally, several numerical experiments have been performed to justify the accuracy, efficiency, flexibility and robustness of each proposed algorithm.