



**INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI  
SHORT ABSTRACT OF THESIS**

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During last few decades, functionally graded materials (FGMs) have been recognized as future advanced composite material, featuring thermo-mechanical loading, multi functionality and gradation at micro and nano scales. In many rotating machineries, like gas turbine, steam turbine, shafts, blades etc. are exposed to very high temperature environment. Due to their superior performances in high temperature applications, these FGMs are also recognized as potential materials for rotating machinery components like shafts, blades etc. Similar to homogeneous structures, presence of cracks in functionally graded (FG) shafts also make a serious threat to their safe performances. It is therefore important to understand the dynamic behavior of such FG shafts under loading. Therefore, the present dissertation deals with finite element (FE) formulation and analysis of a FG shaft with transverse breathing crack/cracks in a rotor-bearing system. The FG rotor system is considered to comprise a set of interconnecting components consisting of shaft elements with internal viscous and hysteretic damping, rigid discs and bearings. Hamilton's principle and Lagrangian equation have been used to derive the equations of motion for FG shaft element and rigid disc element respectively. Two noded Timoshenko beam element with four degrees of freedom (DOF) per node is used considering the effects of translational and rotary inertia, transverse shear deformations, and gyroscopic moments. The FG material is considered to be composed of aluminum oxide ( $Al_2O_3$ ) and zirconia ( $ZrO_2$ ) as ceramics and stainless steel (SS) as metal. Temperature dependent thermo-elastic material properties of the FG shaft are considered graded in the radial direction following power law of material gradation. Fracture mechanics concepts have been combined with rotor dynamical system to obtain stress intensity factors (SIFs) as well as local flexibility coefficients (LFCs) of cracked FG shafts, which change the system dynamic behavior globally. Considering breathing crack, the LFCs are determined analytically as a function of size and orientation of crack, power law gradient index and temperature using the Castigliano's theorem and Paris's equations to compute the stiffness matrix of the cracked FG shaft in the FE analysis. Based on the formulation, an FE computer code has been developed using MATLAB for dynamic analysis of rotor bearing system having a cracked FG shaft and validated with the analytical and FE solutions reported in literatures.

Numerical simulation have been performed to study the behavior of cracked FG shafts under different conditions and considering different configuration and effect of some important parameters on the dynamic behavior of FG shafts are studied. Numerical results show that for a cracked FG shaft, the magnitude of LFCs is significantly affected by material gradient index thus influencing the performance. Results also show that the gradient index has significant influence on the natural whirling frequencies and critical speed for both un-cracked and cracked FG shaft and the gradient index could be judiciously chosen in designing of such FG shafts. Numerical results show that the choice of power law gradient index has significant importance on the dynamic behavior of an FG shaft in the event of multiple transverse cracks appearing and it is possible to decide gradation parameter in damage tolerant design of such shafts. Results from stability analysis show that while the depths, orientations and locations of cracks and temperature

gradient along with damping properties of materials affect the stability threshold speed, the choice of power law gradient index has significant importance on the threshold speed in the event of multiple transverse cracks surfacing on the FG shaft. The present formulation could be useful in damage tolerant designing of FG shaft in high temperature applications where the desired range of stability threshold speed could be achieved.

