INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

In this thesis, an attempt has been made to study the higher order of convergence for time-dependent problems. This thesis aims to design and analyze higher-order convergence of weak Galerkin finite element approximations to the true solutions for time-dependent problems on polygonal meshes. The mathematical analysis of higher-order convergence for time-dependent problems with polygonal meshes adds more challenges than one could imagine. First, describe a systematic numerical study on WG-FEMs for second-order linear parabolic problems by allowing polynomial approximations with various degrees for each local element. Convergence of both semidiscrete and fully discrete WG solutions is established. Here, we assume that the true solution satisfies full regularity assumptions. Next, we proceed to discuss the WG algorithm for the parabolic problems, when the solution u in $L^2(0, T; H^{k+1}(\Omega))\cap H^1(0, T; H^{k-1}(\Omega))$. Such regularity holds where forcing function f in $L^2(0, T; H^{k-1}(\Omega))$ and initial function u^0 in $H^k(\Omega)$. for some $k\geq1$. Optimal order error estimates in $L^2(L^2)$ and $L^2(H^1)$ norms are shown to hold for the spatially discrete-continuous time and the discrete-time weak Galerkin finite element schemes. Further, we explore the L^2 error convergence of weak Galerkin finite element approximations for a homogeneous parabolic equation with non-smooth initial data using polygonal meshes. Our next focus is to describe WG-FEMs for solving the wave equation. We propose both semidiscrete and fully discrete schemes to solve the second-order linear wave equation numerically. In our last problem, we designed and analyzed the WG-FEMs to approximate a general linear second-order hyperbolic equation with variable coefficients on polygonal meshes. The convergence analysis is carried out for the semidiscrete and fully discrete weak Galerkin approximations. The fully discrete scheme can be reinterpreted as an implicit second-order accurate Newmark scheme that is unconditionally stable.