INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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Programme of Study : Ph.D.

Thesis Title: Analysis of robust computational techniques for singularly perturbed system of parabolic partial differential equations

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Thesis Submitted to the Department/ Center : Department of Mathematics

Date of completion of Thesis Viva-Voce Exam : 22nd November 2018

Key words for description of Thesis Work : Singularly perturbed system of parabolic PDEs, Finite difference scheme, Uniformly convergent numerical method, Shishkin mesh

SHORT ABSTRACT

In this thesis, our primary interest is to provide some uniformly convergent computational techniques for solving singularly perturbed system of parabolic initial-boundary-value problems (IBVPs) of convection-diffusion and reaction-diffusion types with boundary and interior layers in one- and two-dimensions. These kinds of problems are identified by system of partial differential equations in which the highest order spatial derivatives are multiplied by small parameters, known as singular perturbation parameters. The solution of such kind of problems exhibits boundary or/and interior layers where the solution varies rapidly, while away from these layers the solution behaves smoothly. Due to appearance of the layer phenomena, it is an interesting task to develop parameter-uniform numerical methods. The purpose of this thesis is to apply and analyze parameter-uniform fitted mesh methods (FMMs) for solving singularly perturbed system of parabolic PDEs in one- and two-dimensions.

We begin this thesis with an introduction followed by a section describing the objectives and the motivation for solving singularly perturbed system of parabolic PDEs. Then, we discuss preliminaries which are used throughout the thesis. Next, we move forward to the main work of the thesis. First, we analyze a uniformly convergent numerical scheme for system of 1D parabolic convection-diffusion IBVPs exhibiting overlapping boundary layers. Then, to improve the accuracy of the proposed numerical scheme, a post-processing technique is discussed. The hybrid difference numerical scheme is proposed for system of 1D parabolic PDE on the piecewise-uniform Shishkin mesh. Later, we have considered uniformly convergent upwind based numerical scheme for singularly perturbed system of 1D parabolic convection-diffusion problems with overlapping interior layers. Then we discuss singularly perturbed system of 2D parabolic convection-diffusion and reaction-diffusion problems. First, we analyze a fractional-step method to discretize the time-derivative of the singularly perturbed system of 2D convection-diffusion PDEs. The resulting one-dimensional equations are solved by using the classical upwind scheme. Next, we consider singularly perturbed system of two-dimensional reaction-diffusion problems with one parameter. Here, we discretize the time-derivative by the fraction-step method and spatial derivatives by the central difference scheme for the reduced stationary problem. At the end, we have considered system of 2D reaction-diffusion problem containing different diffusion parameters. The spatial derivatives are discretized by the central difference scheme on piecewise-uniform Shishkin mesh. Then, the time derivative is discretized by implicit-Euler scheme on uniform mesh in the resulting problem. Numerical results are produced to validate the theoretical error estimates. Finally, we summarize the results obtained in this thesis. At the end of this thesis, possible future works are discussed based on the work carried out in this thesis.