Trace Formulas and Finite Dimensional Approximations

by

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Abstract

The dissertation gives a new proof of some existing second-order trace formulas, namely the Koplienko-Neidhardt trace formula for pair of unitaries in the multiplicative path, the Koplienko-Neidhardt trace formula for pair of contractions via linear path with one of them being normal. Our proofs are based on the idea of the finite-dimensional approximation method introduced by Voiculescu. As a consequence of our results and the Schäffer matrix unitary dilation, we obtained second-order trace formula for a class of pairs of contractions via linear path. Using a different setup of finite dimensional approximations, we extend the Koplienko-Neidhardt trace formula for a class of pairs of contractions via multiplicative path.

Moreover, in this thesis, using the dilation theory and the existing higher-order trace formula for pair of unitaries via multiplicative path, we obtain a higher-order trace formulae for a class of pairs of contractions and a class of pairs of maximal dissipative operators via multiplicative path.

Finally, this dissertation establishes estimates and an integral representation for the higher order Taylor remainder of the spectral action functional $V \mapsto \mathrm{Tr}(f(H_0 + V))$ on bounded self-adjoint perturbations, where H_0 is a self-adjoint operator with compact resolvent and f belongs to a 'nice' class of scalar functions.