INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

This thesis considers risk-sensitive stochastic control and game problems on countable/Borel state space for discrete/continuous-time Markov decision processes (MDPs) under certain Lyapunov conditions. Here, infinite horizon control/game problems are analyzed with various cost criteria. The controllers can take action in discrete/continuous-time from their admissible strategies.

In the single-player setup, we investigate infinite-horizon discounted cost criterion for continuous-time pure jump-controlled Markov processes. The controller tries to minimize his/her payoff through a Markov decision process and finds an optimal risk-sensitive control in the class of Markov control.

When there is more than one controller, the stochastic control problem is referred to as a stochastic game problem, we study zero/nonzero-sum game problems. For zero-sum game problems, we consider infinite-horizon discounted/ergodic cost criteria for discrete/continuous-time Markov decision processes on countable/Borel state space. Here, player 1 is a maximizing player and player 2 is a minimizing player. So, player 1 tries to maximize his/her reward while player 2 always tries to minimize his/her costs. For these problems, a saddle-point equilibrium point is achieved. To study nonzero-sum game problems, we consider infinite-horizon ergodic cost criteria for continuous-time Markov decision processes (CTMDPs) on a countable state space. Here, each player tries to minimize his/her ergodic payoff criterion. We establish the existence of a Nash-equilibrium in the class of stationary strategies for nonzero sum ergodic game problems. For each model, suitable real-life examples are provided to support the models.

First, we investigate risk-sensitive continuous-time discounted control problem for pure jump Markov processes on general Borel state space. The transition and the cost rates are possibly unbounded. We establish the existence and uniqueness of the solution to the Hamilton-Jacobi-Bellman (HJB) equation under certain Lyapunov conditions. Also, we provide proof of the existence of optimal risk-sensitive control in the class of Markov control and completely characterize the optimal control. Moreover, we consider an illustrative example to support our results and assumptions.

After that, a continuous-time risk-sensitive zero-sum stochastic game for controlled Markov decision processes with discounted cost criterion on countable state space is analyzed. Here, the transition and cost rates are possibly unbounded. Under a Foster-Lyapunov condition, we prove the existence of the value of the game and saddle-point equilibrium in the class of admissible strategies by studying the corresponding Hamilton-Jacobi-Isaacs (HJI) equation. Also, an illustrative example is used to support our results.

Next, we consider risk-sensitive zero-sum stochastic games for controlled continuous-time Markov decision processes on a general state space with discounted cost criteria. The transition and cost rates are allowed to be unbounded. Under a stability assumption, we prove the existence of the value of the game and saddle-point equilibrium in the class of Markov strategies and give a characterization in terms of the corresponding Hamilton-Jacobi-Isaacs equation. Moreover, we illustrate our results and assumptions by example.

After that, we analyze risk-sensitive zero-sum stochastic games for controlled discrete-time Markov decision processes with ergodic cost criteria on countable/compact state space and Borel action spaces. For countable state space case, the payoff function is nonnegative and possibly unbounded and it is a real-valued and bounded function for compact state space case. Under a certain Lyapunov type stability assumption on the dynamic, we establish the existence of the value and a saddle-point equilibrium, for countable state space case. But for compact state space case, we establish these results without any Lyapunov type stability assumptions. Using the stochastic representation of the principal eigenfunction of the associated optimality equation, we completely characterize all possible saddle point strategies in the class of stationary Markov strategies. Also, we present and analyze an illustrative example.

Subsequently, a nonzero-sum stochastic game for continuous-time Markov decision processes on a denumerable state space with risk-sensitive ergodic cost criterion is considered. We allow the transition rates and cost rates to be unbounded. Under a certain stability assumption, we show the existence of a solution of the corresponding system of coupled HJB equations which leads to the existence of a Nash equilibrium in stationary strategies. We establish this using an approach involving principal eigenvalues associated with the HJB equations. Furthermore, we completely characterize Nash equilibrium in the space of stationary Markov strategies by exploiting appropriate stochastic representation of principal eigenfunctions. Also, a controlled population system is considered to illustrate our results.

Finally, we investigate risk-sensitive continuous-time stochastic zero-sum games for controlled Markov decision processes with ergodic cost criteria. Here, the transition and the cost rates may be unbounded. Under a Lyapunov stability condition, we provide proof of the existence of the value of the game and a saddle-point equilibrium in the class of all stationary strategies. This is accomplished by establishing the existence of a principal eigenpair for the corresponding Hamilton-Jacobi-Isaacs (HJI) equation. This, in turn, is established by using a nonlinear version of Krein-Rutman theorem. We also give a characterization of the saddle-point equilibrium in terms of the corresponding HJI equation. Lastly, we use a controlled population system to illustrate our results.