

Nonlinear Dynamics of Piezoelectric Based Energy Harvester with Parametric and Galloping Instabilities

A thesis submitted in partial fulfilment of the requirements

for the degree of

Doctor of Philosophy

by

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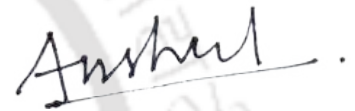
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Declaration

I declare that this thesis entitled “Nonlinear Dynamics of Piezoelectric Based Energy Harvester with Parametric and Galloping Instabilities”, submitted by me is a presentation of my original research work done under the guidance of Dr. Santosha K. Dwivedy, Professor, Department of Mechanical Engineering, Indian Institute of Technology Guwahati. This work has not been submitted elsewhere for the award of any degree, diploma, associateship, fellowship, titles in this institute or any other university or institution of higher learning.



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Certificate

This is to certify that this thesis entitled “Nonlinear Dynamics of Piezoelectric Based Energy Harvester with Parametric and Galloping Instabilities”, submitted by “Anshul Garg”, in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy, to the Indian Institute of Technology Guwahati, Assam, India, is a record of the *bona fide* research work carried out by him under my guidance and supervision at the Department of Mechanical Engineering, Indian Institute of Technology Guwahati, Assam, India. To the best of my knowledge, no part of the work reported in this thesis has been presented for the award of any degree, diploma, associateship, fellowship, titles in this institute or any other university or institution of higher learning.



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Abstract

Energy harvesting from ambient vibration is the next developmental stage for realizing the self-reliant micro-electromechanical systems. Initially, the researchers focused on the linear vibration-based transduction mechanism. But later on, the limitation of frequency bandwidth in linear systems leads to exploring the rich dynamical behaviours of nonlinear vibrations. Instabilities such as parametric excitation and galloping phenomenon are to be avoided for structural stability and integrity. However, the underlying mechanisms which trigger these unwanted instabilities can be utilized for energy scavenging purposes by using piezoelectric based energy harvesters. So, the present study is attempted to utilize the geometric and inertial nonlinearities along with modal interactions for piezoelectric based energy harvesting by extracting energy from parametric and galloping instabilities.

In this research work, four nonlinear problems related to piezoelectric energy harvester systems have been investigated. In all the cases, the harvester system consists of a vertical cantilever beam with piezoelectric patches and attached mass at an arbitrary position. In the first work, nonlinear dynamic analysis of a cantilever beam based piezoelectric energy harvester subjected to parametric base excitation is carried out. In the second problem, the harvester subjected to a combination parametric resonance case has been studied. In these problems, the second mode frequency is considered to be approximately three times the first mode frequency, giving rise to an internal resonance of 1:3. In the third problem, an experimental setup has been developed to validate the theoretically studied PEH systems with base excitation under parametric resonance conditions. In the fourth work, galloping based harvester is investigated both theoretically and experimentally. The governing equations of motion of the above-mentioned systems have been derived either by force-moment balance (Newton's method) or by energy balance approaches. These spatiotemporal governing equations of motion are then discretized to their temporal forms by using generalized Galerkin's method. The method of multiple scales is then applied to reduce the second order differential equations to a set of first order differential equations. These equations are then used to obtain the steady state response and stability either by using numerical methods for solving differential equations (Runge-Kutta method) or by solving a set of algebraic or transcendental equations

by using Newton's method. Both time and frequency responses have been plotted to obtain the output voltage and power.

In the first two studies, the response and stability of the system under principal parametric and combination parametric resonance conditions with 1:3 internal resonance has been studied. The parametric instability regions are shown for variation in different system parameters such as excitation amplitude and frequency, damping and load resistance. Bifurcations such as turning point, pitch-fork and Hopf are observed in the multi-branched non-trivial response. By tuning the attached mass, an attempt has been made to harvest ambient energy from an available wider range of frequencies.

An experimental setup with a slider-crank mechanism based shaker and a harvester consisting of a cantilever beam with PZT patch and attached mass is designed and developed. The challenges posed by parametric resonance in crack development in the PZT and the beam are reported. The theoretical and experimental output voltage and the power obtained are found to be in good agreement with the theoretical findings.

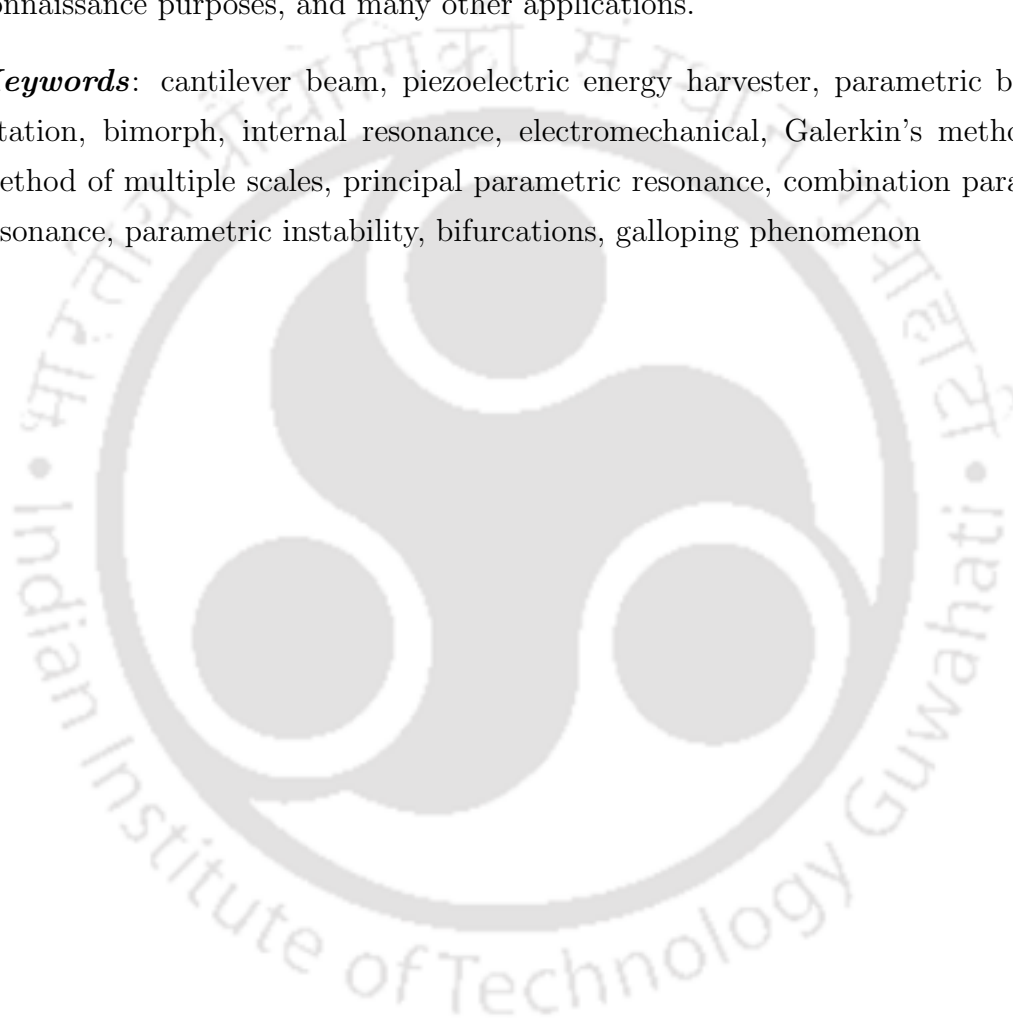
A piezoelectric energy harvester based on the galloping (a flow induced instability) phenomenon is also presented. Two different types of bluff bodies viz., D-shaped and triangular-shaped bluff bodies have been used in the experiment, which is performed in the wind tunnel to study the voltage-time response across a load resistance under uniform flow condition. The effect of the position of bluff bodies on voltage and power output are investigated. Theoretical investigations have been carried out considering both uniform and time-varying wind speed.

Extensive parametric studies have been carried out both theoretically and experimentally and the results have been presented in four major chapters in addition to a literature review chapter where more than 300 research articles have been critically reviewed. All the critical findings have been summarised in the last chapter, where the general conclusion and specific conclusions have been highlighted. Further, the scope of future work, which may be extended from the present work, is highlighted in the last chapter.

The developed analytical solutions are found to be in good agreement with the experimental findings. Hence, the developed analytical expressions for piezoelectric

based energy harvesters with the principal or combination parametric resonance conditions or the galloping based PEH system can be used effectively for determination or design and development of new PEH systems without conducting expensive experiments in facilities such as wind tunnels. These PEH systems may be used in making smart, self sufficient devices that will provide energy to low powered wireless sensor nodes or microelectromechanical systems for structural health monitoring, reconnaissance purposes, and many other applications.

Keywords: cantilever beam, piezoelectric energy harvester, parametric base excitation, bimorph, internal resonance, electromechanical, Galerkin's method, the method of multiple scales, principal parametric resonance, combination parametric resonance, parametric instability, bifurcations, galloping phenomenon





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Abbreviations

DSB	D Shape B luff B ody
EH	E nergy H arvester
FEA	F inite E lement A nalysis
FEM	F inite E lement M ethod
GPEH	G alloping based P iezoelectric E nergy H arvester
MFC	M icro- F iber- C omposite
MMS	M ethod of M ultiple S cales
PEH	P iezoelectric E nergy H arvester
PVDF	P olyvinylidene F luoride
PZT	L ead- Z irconate- T itanate
S	S table
TSB	T riangular S hape B luff B ody
US	U n S table
VEH	V ibration E nergy H arvester



Symbols

A_b	area of cross section of beam
A_n	amplitude as a complex function
A_p	area of cross section of piezoelectric element
a_n	nondimensional amplitude
b_b	width of beam
b_p	width of piezoelectric element
C_D	coefficient of drag
C_L	coefficient of lift
C_p	capacitance of piezoelectric element
c	coefficient of viscous damping
D_{bf}	width of bluff body
$\mathbf{D}$	electric displacement vector
d_{31}	piezoelectric constant
d	position of attached mass
E_b	Young's modulus of beam
E_p	Young's modulus of piezoelectric patch
E_e	electric field
$\hat{\epsilon}$	permittivity at constant strain
e_{31}	piezoelectric constant
F_D	drag force due to galloping
F_L	lift force due to galloping
F	force of excitation

g	acceleration due to gravity
H	Heaviside function
I_b	moment of inertia of the beam
I_p	moment of inertia of the piezoelectric element
I_d	moment of inertia of the arbitrary mass at a distance d
$i_p(t)$	dependent current source
J_0	moment of inertia of the arbitrary mass
$[J]$	Jacobian
J	nondimensional moment of inertia of the arbitrary mass m
K_n	nondimensional coupling coefficient
L, L_b	length of the beam
L_{bf}	length of the bluff body
L_p	length of piezoelectric element
m	attached mass
$\mathbf{n}$	outward normal vector to the surface
P	power output
$q(t)$	time modulation
R_l	load resistance
r	scaling factor for displacement
r_v	scaling factor for voltage
s	curvilinear coordinate along the beam axis
T_n	n_{th} time scale
T	Kinetic energy
t_b	thickness of beam
t_p	thickness of piezoelectric element
t	dimensional time
U	steady wind speed
$\bar{u}$	unsteady wind speed component
$u(s, t)$	longitudinal beam displacement

V	voltage across piezoelectric element electrodes
$v(s, t)$	transverse displacement of the beam
W_{nc}	non conservative work done
$x - y$	Cartesian coordinates
Z_0	amplitude of base displacement
$z(t)$	base displacement
Greek symbols	
α	nondimensional coefficient of cubic nonlinear term
β	nondimensional coefficient of inertial nonlinear term
δ	Dirac delta function
ϵ	book keeping parameter
Π	Potential energy
ϕ	nondimensional external frequency
η	electromechanical coupling term
Γ	nondimensional amplitude of base displacement
γ	nondimensional coefficient of inertial nonlinear term
κ	curvature
μ	nondimensional mass ratio
Ω	frequency of external excitation
ω_n	n_{th} nondimensional natural frequency
σ_p	axial stress in piezo
σ	detuning parameter
θ_n	n_{th} dimensional natural frequency
ρ	mass per unit length
ρ_b	mass density of the beam
ρ_p	mass density of the patch
τ	nondimensional time
ξ	dummy variable of integration
$\psi(s)$	shape function

ζ nondimensional damping coefficient



Chapter 1

Introduction

1.1 Introduction

Microelectronic devices are primarily used in sensing, actuation and data transmission purpose. These devices are basically battery powered and hence required recharging/replacement after power utilization, which may be difficult in remote locations. The recent advancement in the development of microelectronic systems [2] has reduced the power consumption of these devices significantly [3]. Now, the focus is to power these devices by harvesting energy from the ambient vibration for its energy requirements. This task has been driving researchers around the globe for the last two decades. These self-sustained systems may be capable of surviving even in remote areas on their own.

One solution to this end is to extract power from potential ambient sources of energy like solar, wind and vibration. Sources like solar, wind and water provide macro-scale energy in the range of few kW to GW. The energy requirement of low power electronic devices is less than a Watt ($< 1W$). In this regard, the micro-scale energy harvesting is required to power such devices which are in the range of μW to mW. Available kinetic energy in the ambient sources is sufficient to power such devices. Vibrational energy is available almost everywhere in the ambient, e.g., human motions, vibration of machine components, vibration of locomotives, bridges, wind induced phenomena (i.e., galloping and vortex induced vibration) and ocean waves.

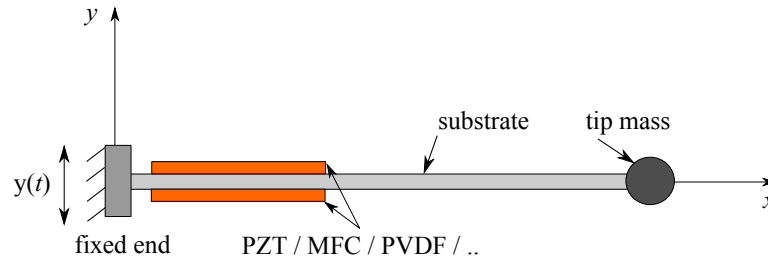


FIGURE 1.1: Conventional beam based piezoelectric energy harvesting (PEH) system

Several transduction mechanisms [4] viz., electromagnetic, electrostatic, piezoelectric and electrochemical are exploited to convert the available energy into electricity. The combination of these transduction mechanisms, e.g., electromagnetic and piezoelectric, are also explored [5, 6]. Among them, the piezoelectric based transduction mechanism has advantage over others because of its energy extracting capabilities over a wide range of available frequencies, structural simplicity, easy to fabricate with less peripheral components, easy to integrate with microelectronic circuits and having higher energy density [7, 8], which makes it most suitable for micro-scale applications.

Piezoelectric materials [9] are employed extensively in sensing, actuation and energy harvesting purposes. With these materials, the vibrational mechanical energy can be converted into electricity. These materials are compact and having quick responses within very short period of time. Vibrational energy can be converted into electrical energy by using structural elements such as beams, plates, shells or combination of these basic elements embedded with piezoelectric patches. A beam structure or a combination of several beam structures are generally used for energy harvesting purpose due to their simplicity and size. Apart from the traditional PZT (lead zirconate titanate) materials, wide ranges of flexible piezoelectric materials are also available which can be integrated with wearable devices for sensing purposes. Figure 1.1 shows a conventional beam based piezoelectric energy harvester where the ambient vibration is used to harvest the energy.

In general, linear vibration based piezoelectric energy harvester (PEH) systems work over a short range of bandwidth near the resonance frequency. Energy transduction reduces sharply for mistuned (away from resonance) linear harvesting systems. To

address this issue, researchers are focusing on tapping the rich dynamics associated with the response of the systems with inherent or induced nonlinearity [10]. Nonlinear systems display behaviours such as bifurcation, chaos, internal resonance. The multi-branched fixed point response of a nonlinear system may be periodic, quasi-periodic and chaotic. Energy harvesters based on nonlinear vibration have potential to extract energy over a wide range of available frequencies. The source of nonlinearity can be of geometric or material in nature, and it can also arise due to external excitation and constraints such as impact, friction, backlash and freeplay [11].

Nonlinear stretching or large curvature causes the geometric nonlinearity and concentrated or distributed mass cause inertial type nonlinearity in the system [12]. Nonlinear stress-strain relation leads to material nonlinearity and stretching leads to nonlinear strain-displacement relation, and large curvature leads to nonlinear curvature-displacement relation. Other sources of nonlinearity are external forces acting on a system. The external force may be gravitational, magnetic or fluid forces. Nonlinear behaviour also comes from constraints in the system. The performance of structures associated with the energy harvesting systems can be improved by exploiting some characteristics of the nonlinear dynamics. [13].

The governing equations of motion of nonlinear PEH systems are represented by coupled electromechanical partial (spatio-temporal) differential equations [14]. Apart from numerical methods, some analytical methods are also used for solving these nonlinear equations. These analytical methods include the method of harmonic balance and perturbation methods such as method of averaging, Linstedt-Poincaré technique and method of multiple scales [13]. The method of multiple scales is a commonly used technique that provides transient and steady-state responses. The reduced equation obtained in this method can be used to study the stability of the steady-state solutions. It can also give information related to the response of the harmonics of the forcing frequency [15].

The commonly used source of energy for the vibration based energy harvesters are external excitations [16–18]. These excitations may be forced, parametric or combination of both. In the case of forced vibration simple resonance occurs when the external frequency nearly equals any of the modal or natural frequencies. The parametric resonance occurs when the external frequency Ω nearly equals to the sum of the modal frequencies, i.e., $\Omega = \omega_m \pm \omega_n$. When $\omega_m = \omega_n$, the external frequency

is equal to twice the natural frequency and the resonance condition is known as principal parametric resonance condition. When the excitation frequency coincides with combination (sum or difference) of natural frequencies [19] the combination parametric occurs. The characteristic of a typical parametrically excited system is that a time-varying term appears as the coefficient of the response in the governing equation of motion [20]. Parametric resonance may lead to a large amplitude vibration. In few studies [21, 22], the PEH system is investigated under the parametric excitation.

The interaction of flexible structures with the fluid flow may cause the self excitation of the system, such as galloping [23]. In this phenomenon the aerodynamic lift forces are generated by a small transverse motion of the structure. It is self excited flow induced instability where the aerodynamic lift forces act in the transverse direction to that of the flow. The flow energy transfers to the structure by the damping-type forces. For effective energy transfer a bluff body is usually attached to the elastic structure with piezoelectric patches to extract the wind energy [24].

Another physical phenomenon that occurs in nonlinear system is internal resonance (IR) [25]. When the natural frequencies of a system are commensurable then internal resonance occurs. For example, the internal resonance of 1 : 3 may occur when the second mode frequency is 3 times the first mode frequency. The internal resonance causes mode coupling between participating modes and leads to rich dynamical responses [26] due to energy transfer between the participating modes. The system with parametric excitation and internal resonance brings many critical phenomena. The dynamic response in such cases may be fixed point, periodic, quasiperiodic or chaotic [27] which has not yet been explored for energy harvesting purposes.

Applications of PEH devices involves the condition or health monitoring of many systems such as tire pressure, bridge, building, aircraft and railway. In most of the structural health monitoring systems, wireless sensor nodes (WSN) or networks are used for data transfer which can be powered by integrated PEH systems. Energy involves in human activities [28] are also sufficient to power wearable or implantable biomedical monitoring sensors (i.e. pacemaker, pressure, tissue stimulation) [29]. In recent times, several miniature energy harvesting devices from walking [30], limb motion and elbow and knee joints have been developed.

There exist vast potential fields of application for smart and self-sustained PEH systems. Recent progress in PEHs suggests that there exist strong commercial viability apart from the usual use of these devices. Due to the above mentioned potential advantages of piezoelectric based energy harvester, in the present work, an attempt has been made to study the nonlinear dynamics of such systems to harvest energy from wide range of frequency utilizing the concept of nonlinear parametric and internal resonance conditions. A similar cantilever beam based system is also explored with galloping instability conditions for harvesting energy. The structure of the thesis is outlined in the following section.

1.2 Thesis Organization

The present thesis consists of 7 chapters which are described below.

Chapter 1 starts with the introduction to the piezoelectric energy harvester (PEH). In Chapter 2, an exhaustive literature review has been carried out, highlighting the usefulness of piezoelectric based energy harvester and its potential use and limitations. The technical gap is also highlighted which motivates to carry out the present research work and through several studies the objective of the current work is described.

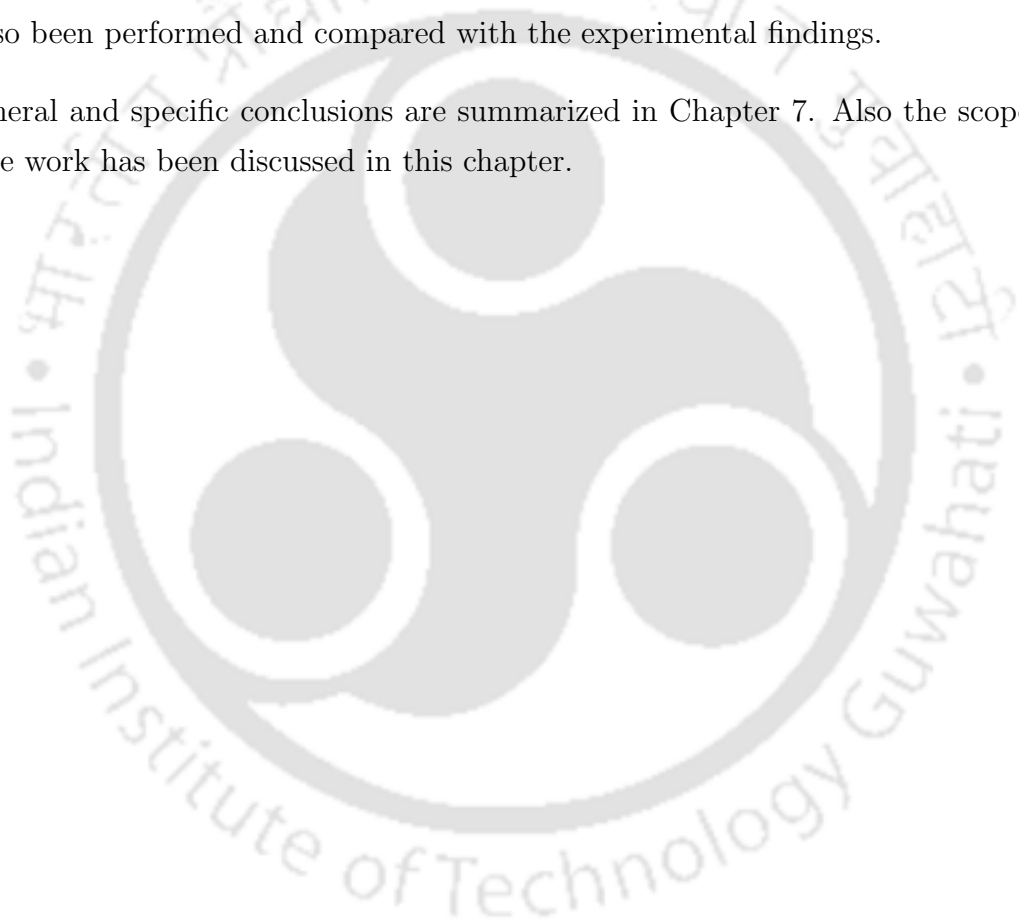
In chapter 3, the nonlinear dynamic analysis of parametrically excited cantilever beam based PEH system is studied considering cubic and inertial nonlinearities and internal resonance of 1:3. Here the effects of internal resonance, load resistance, amplitude and frequency of excitation on the performance of the PEH have been studied. The voltage and power output for different system parameters are presented.

Chapter 4, contains the study of the nonlinear dynamic behaviour of a similar system under combination parametric and 1:3 internal resonance conditions. The instability regions, time and frequency response curves, harvested output voltage and power have been investigated for different system parameters. Stability and bifurcation analysis has also been performed.

In chapter 5, the dynamic behaviour of the piezoelectric energy harvester system is experimentally investigated under the principal parametric resonance condition. An in-house shaker is designed, developed and used for harvesting the energy experimentally.

In Chapter 6, the dynamics of PEH system attached with D-section and triangular section bluff bodies under flow-induced vibration (galloping phenomenon) are studied. The effects of wind speed, load resistance and position of the bluff bodies on the harvested voltage and power have been observed experimentally. Analytical studies have also been performed and compared with the experimental findings.

The general and specific conclusions are summarized in Chapter 7. Also the scope of future work has been discussed in this chapter.



Chapter 2

Literature Review

2.1 Introduction

The objective of this chapter is to review the literature related to the piezoelectric based energy harvesters. The survey is subdivided into several aspects related to piezoelectric energy harvester (PEH) systems such as potential applications, PEH systems, review works, ambient energy sources, piezoelectric materials, mathematical modelling, linear and nonlinear energy harvesters, galloping based PEH systems, miscellaneous PEH systems and electrical circuits which are discussed in section 2.2 to 2.9. The technical gap which motivates to carry out the present research work is discussed in section 2.10. The following section briefly discusses numerous useful applications of piezoelectric based energy harvesters.

2.2 Applications of PEH

PEH devices are commonly used for health monitoring of large structures like bridge [31–33], buildings [34], aircraft [35] and railway [36] and small scale components like tire pressure [37] and powering accelerometer [38]. In most of the cases in structural health monitoring (SHM) systems, wireless sensor nodes (WSN) or networks [39] are used for data transfer, which can be powered by integrated PEH systems. Energy generated during different human activities [28, 40, 41] are found to be sufficient to

power wearable or implantable biomedical monitoring sensors such as pacemaker, pressure and tissue stimulation [29]. In recent times several miniature energy harvesting devices from walking [30], limb motion [42] and elbow and knee joints [43] have been developed. Kumar et al. [44] studied some bi-functional vibration based devices for simultaneous energy harvesting and refrigeration purpose. One may find many such uses of PEH devices that cover a vast range of applications. The next section sheds light on the basic PEH system.

2.3 PEH Systems

Figure 2.1 shows a different configurations of conventional cantilever beam based piezoelectric energy harvesters with base excitation. A basic PEH system consists of an elastic structure (substrate) with piezoelectric patches either on both the sides (bimorph, Fig. 2.1(a),(b)) or on one side (unimorph, Fig. 2.1(c),(d)) of the substrate. Usually the piezoelectric patches are kept near to the fixed end. A tip mass is also attached at its free end, which helps in large deflection at reduced frequency. The whole system is attached to a host vibrating structure. The output voltage generated across the electrodes of the piezoelectric element is in AC form, that needs to be converted to DC voltage for storage of energy into a battery. Other forms of storage are capacitors, super-capacitors and combination of batteries with capacitors. A purely resistive load element is also used to find the power output as well. In case of multiple piezoelectric elements, the series (Fig. 2.1a) or parallel connection (Fig. 2.1b) is adopted. In the following subsection base excited PEH systems have been discussed.

2.3.1 Base excited PEH systems

PEH Systems scavenge energy from the ambient vibrations for energy harvesting. Generally, the harvester is a cantilever beam with piezoelectric patch subjected to external excitation from the ambient vibration. External excitation can be broadly classified into deterministic and stochastic type [45]. Among them the deterministic type is widely explored. Within the vast deterministic excitations, harmonic type has

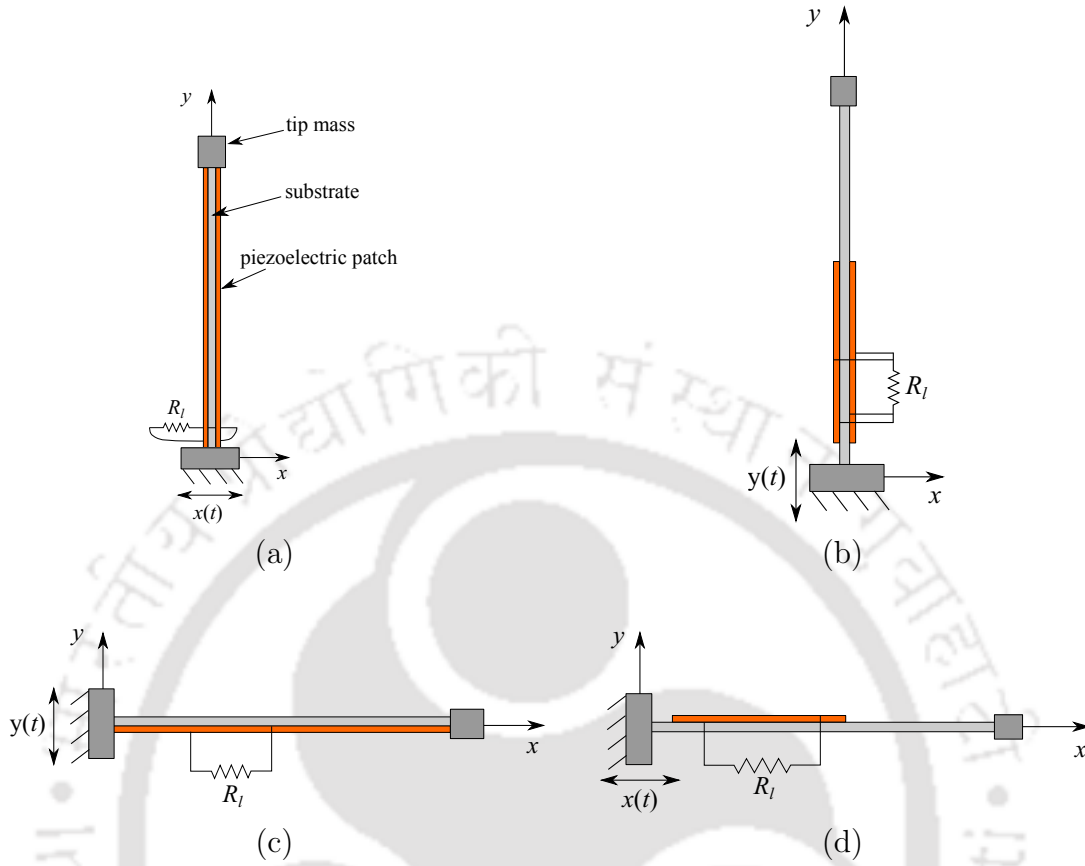


FIGURE 2.1: Different configurations of conventional cantilever beam based piezoelectric energy harvesters (PEH) with base excitation (a) PEH system I (b) PEH system II (c) PEH system III (d) PEH system IV

been mostly used. The mechanical energy associated with harmonic base excitation is expressed either in displacement or forced form. The nature of vibration may be linear if the amplitude of excitation is very small and the system is away from the resonance conditions. Further, it may be nonlinear depending on the amplitude and frequency of the external excitation and the resonance conditions. In this section the work related to linear and nonlinear energy harvesters has been discussed.

2.3.1.1 Linear PEH systems

The earlier works [28, 46–51] give the direction for the development of the vibration based energy harvesters. Later on linear vibration based energy harvesters have been

explored, where near resonance condition is essential for effective energy harvesting in harmonically base excited systems.

Roundy et al. [16] presented a basic linear electromechanical model of a conventional energy harvester with a tip mass. They have developed a prototype of size 1 cm^3 and generated a power density of $70\text{ }\mu\text{W}/\text{cm}^3$. Also, they have shown that with an optimized configuration, maximum power of $250\text{ }\mu\text{W}$ can be achieved with an acceleration of 2.5 m/s^2 and a frequency 120 Hz . Sodano et al. [52] theoretically and experimentally investigated a cantilever based linear PEH system.

Further, many researchers have attempted to increase the power and bandwidth of the linear harvester using different methods, which include strain magnification [53], geometric alteration [54, 55] and an array of harvesters having nearly spaced frequencies [56, 57]. One may find the details of early developments in the field of linear vibration based harvesters in the review work of Anton and Sodano [7] and Cook-Chennault et al. [58]. In the following section, the nonlinear PEH systems are discussed.

2.3.1.2 Nonlinear PEH systems

The limitations of linear vibration based harvesters are their limited frequency bandwidth and few resonance conditions. This leads to shift the research focus on nonlinear vibration based harvesters [10, 59–64]. The term bandwidth defined as the range of frequency that is available for energy transduction. Wider bandwidth means a higher range of ambient frequency over which energy transfer is possible. The source of nonlinearity in the dynamic systems may be of material, geometric or inertial type. Nonlinear stretching or large curvature causes the geometric nonlinearity and concentrated or distributed mass imparts inertial type nonlinearity in the system. As discussed in the introduction chapter one may observe multiple resonance conditions due to external excitation and internal resonance due to nearness of the natural frequencies. In these systems one may observe both stable and unstable fixed point and periodic responses with multiple stable equilibrium points and many bifurcation points. Also, quasiperiodic and chaotic responses [65] may be observed in this type of system. A nonlinear vibration based harvesting system brings several advantages in the power harvesting capabilities as the ambient vibration contains wide range

of frequencies. Hence these nonlinear vibration based energy harvesters outperform the linear harvesters.

Geiyer and Kauffman [66] induced chaotic nonlinear phenomena in a bistable piezo-magnetoelastic energy harvester to increase bandwidth and power output. Chaos has been observed by Cao et al. [67] in piezomagnetoelastic energy harvester under the low frequency of excitation when a nonlinear magnetic force is applied. A hybrid bistable piezoelectric beam energy harvester with internal resonance due to movable magnet is presented by Yang and Towfighian [68]. An axially loaded, tunable and clamped-clamped piezoelectric energy harvester (PEH) is presented by Masana and Daqaq [62] where an improved steady state response, power and frequency band is achieved by imparting nonlinearity by axial load. Similarly, Harne and Wang [69] developed a biologically inspired harvesting system by using a dynamic compressive axial suspension. The system comprised of a simply supported beam in horizontal position with piezoelectric patches. The axial compression helps in maximizing the beam bending and enhances the average output power harvested.

Kecik and Borowiec [70] presented an autoparametric energy harvester which consists of a pendulum and a nonlinear oscillator, where they observed that the chaotic motion of the pendulum generates the highest power as compared to swinging and rotation. A coupled nonlinear vibro-impact beam system for energy harvesting is analyzed by Vijayan et al. [71]. The power generated is found to be sensitive with beam thickness ratio and clearance between beams. A higher power is observed due to dissimilar mode shapes of the interacting modes. Borowiec [72] analysed the nonlinear dynamics of vertical beam harvester with tip mass under the harmonic excitation with an added stochastic component. It has been observed that in nonlinear piezoelectric based harvesters the power output is significantly higher than those of the linear harvesters.

A cantilever beam with a tip mass is considered as an energy harvester by Rezaei et al. [73]. An improved resonance bandwidth is observed by applying a nonlinear cubic restoring force. Scapolan et al. [74] analysed the problem of energy harvesting as a parametrically excited electromechanical oscillator with nonlinear cubic damping to extend the dynamic range of the harvester. Tang and Li [75] developed a two stage vibration energy harvester. When the second stage of higher frequency is driven by first stage of low frequency under higher acceleration, the first stage

exhibits chaotic characteristics. Friswell et al. [64] proposed a base excited vertical cantilever beam with a tip mass for energy harvesting considering cubic and inertial nonlinearity. Here an improved bandwidth is achieved as compared to linear configuration. Litak et al. [76] examined the regular and chaotic responses of a vertical beam and a tip mass based vibrational energy harvester by varying the tip mass. A similar model is evaluated under various excitation forces such as purely harmonic, purely random, and combinations of harmonic and random by Bilgen et al. [77].

Apart from the base excitation, the flow energy is also utilized as an energy source to develop PEH systems. The next subsection describes the flow induced PEH systems.

2.3.2 Flow induced PEH systems

Galloping is a common phenomenon observed in electrical transmission cable lines. Davison [78] described this phenomenon in electrical cables. Galloping is self excited vibration [79] of a flexible structure where the energy associated with the lift force (acts in the transverse direction) causes negative damping. Hartog and Pieter [80] explained the criteria for phenomenon of galloping. Apart from overhead transmission lines the galloping is also observed in tall buildings and space shuttle during launching. There exist a critical speed (onset of galloping) above which the phenomenon occurs. Galloping [81] and flutter (in air-crafts) phenomena are related to flow induced instabilities (Païdoussis et al. [82]). These instabilities have been investigated [24, 83–86] for energy harvesting purpose. Generally a prismatic structure attached with a bluff body and piezoelectric patch is employed for harvesting wind energy.

Before exploring the galloping instability for harvesting purposes researchers used to study the structural stability against galloping. For example Novak [87] studied the galloping of prismatic structures. Nayfeh and Abdel-Rohman [23] analysed a cantilever beam under galloping where a higher order lift force is considered. Mukhopadhyay and Dugundji [88] examined the response of a slender cantilever beam under galloping. In this work the response is found to be sensitive with angle of attack. Alonso and Valero [89] investigated the effect of cross sectional shape of the bluff bodies on the galloping instability.

The recent work by Allen and Smits [90], Tang et al. [91], Zhu et al. [92] and Barrero-Gil et al. [84], further inspired the research studies, where galloping instability is utilized to extract energy from wind for energy harvesting. Hobeck and Inman [93] developed a biologically inspired piezoelectric grass which consists of an array of cantilevers. Abdelkefi et al. [1] studied the performance of PEH system attached with various bluff body shapes, i.e., D section, square and isosceles triangle with 30° and 53° angle.

Li et al. [94] reviewed recent PEH systems based on flow induced vibrations from the future perspective to use existing harvesting technology in the aerospace field and discussed the drawbacks along with potential implementation challenges. Andrew and Mahmoodi [95] reviewed designs and methodologies of wind excited PEH systems. Hamlehdar et al. [96] reviewed the energy harvesting from fluid flow using piezoelectrics. Flapping foil based flexible flow energy harvesters (Xiao and Zhu [97]) also attracted the researchers. In the review work presented by Abdelkefi [17], recent energy harvesters based on aeroelastic vibration mechanisms i.e. flutter, vortex induced vibration, galloping and wake galloping are discussed. In this work, mathematical models and qualitative and quantitative comparisons are also reviewed. Another flow induced phenomenon like vortex induced vibration (VIV) is also explored for energy harvesting in the work of Akaydin et al. [98] and Sun et al. [99]. Sun et al. [99] studied PEH, utilizing flow energy of water by using VIV and galloping, here the galloping is found to be superior to the VIV for energy harvesting. Flutter [100–103] and wake galloping [104–106] are also explored for energy transduction.

Different shapes of bluff bodies (Abdelkefi et al. [1]) have been explored for better performance of the galloping based PEH (GPEH) system. The galloping of rhombic shaped bluff bodies is investigated by Ibarra et al. [107]. In another study trapezoidal shape is found to be best suited blunt body shape (Kluger et al. [108]) for energy harvesting. The regular shapes such as square and rectangular sections have been studied by many researchers. For example, in the work of Ewere and Wang [109], square and rectangular sections are compared and square is found to be better than rectangular one. The bioinspired bluff body is investigated by Ewere et al. [110], where the galloping velocity can be controlled by wavy profiled bluff body without any gain in the output power. Javed and Abdelkefi [111] analysed the effect of

inclination of bluff body on energy harvesting. Inverted piezoelectric flag (Xiao and Zhu [97], Orrego et al. [112], Shoele and Mittal [113]) is also explored for harvesting purposes with flexible piezopolymers like PVDF. Kwon [103] used a T shaped PEH system. Javed and Abdelkefi [114] found that the geometry of bluff body, angle of attack and aerodynamic force representation impacts the performance of GPEH system.

Galloping is also utilized in electromagnetic (Zhang et al. [115], Vicente-Ludlam et al. [116]) and hybrid (Javed and Abdelkefi [111]) energy harvesters. The concurrent forced and self-excited hybrid PEH systems [117–122] are also investigated for performance improvement and harnessing bridge vibration along with galloping.

Perturbation methods like the method of multiple scales [23, 123–125], Krylov-Bogoliubov [109] and method of normal form [126, 127] have been used to obtain a closed form solution.

Structural and aerodynamic nonlinearities are exploited in several studies, like, Abdelkefi et al. [128] investigated the level of harvested power from aeroelastic vibrations of a cantilever beam attached with a triangular bluff body and is supported by nonlinear torsional spring. A nonlinear restoring force is considered to improve the performance of GPEH system by Bibo et al. [125]. In most of the studies a quasi steady approximation is used to model the fluid forces while in the work of Yan and Abdelkefi [122] a GPEH system is modeled as nonlinear quasi steady approach. In this work, the GPEH is analysed under combined forces of wind and base excitation. Periodic, aperiodic, quasi periodic and chaotic responses are observed.

The effect of galloping on a tall prismatic structure is considered by Abdel-Rohman [123]. The beam of square cross section considered having two degrees of freedom (along and across the steady wind). The amplitude in the along wind direction is found to be small as compared to the across wind transverse amplitude. Generally, a steady wind condition is considered for simplicity but realistic wind conditions are mostly turbulent in nature. For more realistic modeling, in the work of Abdel-Rohman [124], a similar system is analysed under unsteady wind environment where a single harmonic component is considered as unsteady component for simplicity. Also, in the work of Zulli and Luongo [129] and Luongo and Zulli [130], unsteady component of wind with having two frequency components is considered. PEH

under turbulent air flow is also explored by Kwuimy et al. [131] and [98]. Realistic wind data maps [132] may be useful to find the average wind speeds at a particular location, which can help in deciding the design of energy harvesting system.

Many researchers have carried out a literature review on energy harvesting systems in the last twenty years. In the following section the published review papers related to energy harvesting systems are briefly discussed.

2.4 Review Works in PEH Systems

This section briefly summarises the work carried out in the review papers published since the year 2003. The very first review on power scavenging technologies and conventional energy sources have been presented in the work of Roundy et al. [16]. Here three different conversion mechanisms viz., electro-magnetic, electrostatic, and piezoelectric have been discussed. Sodano et al. [133] specifically review the vibration based PEH systems where they suggested to use rechargeable batteries instead capacitors and emphasised the key areas such as the harvesting circuitry, location of the harvesting unit, tuning of the power harvesting medium and identification of practical applications of power harvesting system such as wireless sensor and self powered damage detection unit for implementation to improve the efficiency of the system. DuToit et al. [3] developed a coupled electromechanical model to analyze the response of a PEH system and characterized a few vibrational energy sources present in the ambient. Experiments were also performed for validation purposes. Anton and Sodano [7] reviewed the piezoelectric based energy harvesters between the period of 2003 to 2006. The authors emphasize the need for the development of a complete harvesting system with useful applications. The focus is given to the improvement of the harvesting power by piezoelectric configuration, circuit implementation and storage. They also discussed the ambient source of energy for PEH from implantable and wearable power supplies, fluid flow energy and MEMS. Few self powered systems and use of materials other than piezoceramics i.e., PVDF, AFC (active fiber composite) and MFC (micro fiber composite) for harvesting purposes are also discussed. One may find a detailed study regarding PVDF material in the work of Kawai [134].

Wei and Jing [135] focused on the conversion mechanisms, basic theories and potential applications of vibration based energy harvesters (VEH). Alireza et al. [40] presented the state of the art of the kinetic energy harvesting by piezoelectric and electromagnetic transduction mechanisms. Priya [8] discussed the selection of the piezoelectric materials along with analytical models that have been reported to describe the efficiency and power of the harvesting system.

Cook-Chennault et al. [58] recognized the potential of VEHs among the regenerative sources for several applications such as powering wireless sensors, condition monitoring, medical implants and smart clothing. Bogue [136] discussed few developed harvesting modules based on electrodynamic, piezoelectric and thermoelectric effects. Here it is proposed to use the wireless sensing in the automotive industry where monitoring of tire pressure can be done by developing a micro generator by exploiting the motion and vibration of the vehicle itself. Shaikh and Zeadally [137] reviewed a comprehensive outlook of energy harvesting specifically for wireless sensor networks (WSNs) from all possible sources in the environment including mechanical and human motions.

Kim et al. [138] reviewed the VEH with piezoceramics and piezopolymers. They also discussed some key elements of the energy harvesting process such as electric circuit, storage, substrate elements (cantilever, cymbal, stack, shell) and their mathematical modelling. They highlighted the development of flexible piezoelectric material having high coupling coefficient and efficient harvesting circuit as main issues that limit the technological impact of vibration based energy harvesters. The importance of flexible piezoelectric materials is received well for further development of wearable electronic devices for biomedical purposes. One may find the recent progress in flexible and stretchable piezoelectric devices for mechanical energy harvesting, sensing and actuation, in the work of Dagdeviren et al. [139].

Twiefel and Westermann [140] briefly surveyed conversion methods, linear and nonlinear (Bistable/multistable systems, amplitude limiter and active resonance tuning) harvester systems with an improved electric circuit. The nonlinear bistable systems are specifically reviewed by Harne and Wang [141], where modeling, analysis approaches and challenges are discussed. The review work of Xiao and Wang [142] is broadly divided into linear, nonlinear systems, piezoelectric material, circuits and large scale energy scavenging.

Several studies focused on the improvement of harvesting system by tuning the resonating frequency. In this direction, Zhu et al. [143] presented a review of the mechanical and electrical tuning strategies for enhancing the capability of harvesters. Later on Yildirim et al. [144] reviewed the techniques (amplification, resonance tuning and exploiting nonlinear oscillations) for performance enhancement of the PEH. Ibrahim and Ali [145] presented a comprehensive review of the manual and autonomous frequency tuning methods for PEH systems. Here emphasis is given to the development of the integrated autonomous (self tunable) harvesters and MEMS based PEH systems. Li et al. [146] focused on the different aspects such as configurations, materials, electric circuit and optimization for the improvement of PEH.

While hybrid energy harvesting from the interaction of aerodynamic and elastic forces are reviewed by Abdelkefi [17], Tan et al. [147] focused on recent developments in VEHs based on MEMS with electromagnetic mechanism. Strategies on improving efficiency are reviewed by Uchino [148]. Wong et al. [149] summarised specifically the raindrop energy harvesting with piezoelectrics and through a brief experiment shows that the bridge type structure is suitable compared to the cantilever due to high voltage generation.

After exploring the linear regime of vibration, the research focus shifted to explore the nonlinear vibrations for further improvement in PEH systems. One may find a detailed review of the utilization of nonlinearity in PEH in the work of Daqaq et al. [10]. They presented in-depth review of the effect of nonlinearity (inherent or induced) in VEHs (vibration energy harvesters). They also felt that it would be interesting to incorporate internal resonance (IR) where modal interactions take place by energy transfer between them for an effective energy harvesting from low frequency excitations.

The design and development of PEH systems are required to prior knowledge of the available energy sources in its surroundings for energy transduction. In the next section, few ambient energy sources are discussed.

2.5 Ambient Energy Sources

The sun is the fundamental source of all renewable (flow energies of wind and water bodies) or non-renewable (coal, oil, gas and chemical) energy sources. Depends on the scale of harvesting energy, one may classify these sources at the macro or micro level. A vibration is a form of kinetic energy which is an outcome of repeated dynamic movement around the equilibrium. This vibrational energy is considered as a waste and unwanted. Hence early research was focused on mitigation of this waste energy. Since last decade the researchers are exploring the energy harvesting from vibration sources by different transduction mechanisms. In the review works of [16] several energy sources of vibration energy are discussed with approximate available acceleration values and frequency range. These sources comprise of several home appliances, car engines, shock absorbers, human activities and structures near busy roads.

The flow energy of wind is another green source of energy (both macro and micro level). Wind turbines are used to extract energy on macro scale, while micro energy is extracted through four basic mechanisms mainly flutter, vortex induced vibrations (VIV), galloping and wake galloping [17, 150]. Shaikh and Zeadally [137] found that the energy harvesters can fulfil the energy requirements of microelectronic devices and sensors such as sensor node platforms. The energy harvested is sufficient to power microelectronic devices and sensors. Yildiz [151] investigated potential ambient harvesting sources.

Human activities are also a great source of energy to power microelectronic systems [18]. One can harvest energy from walking, by inserting piezoelectric based microsystem inside the shoe (Shenck and Paradiso [152], Turkmen and Celik [30] Mateu and Moll [153]). Monsefi and Inman [154] studied the heartbeat vibration, which can also be harvested for powering pacemakers. The physical movement of systems (i.e., computer mouse) may be useful to harvest energy as well (Mikami et al. [155]) for wireless communications.

Vibration generates pressure waves, which are converted into electric signals by the human ear. Energy from these acoustic noise signals can be harvested by PVDF film (Wang et al. [156]). Also, one may harvest energy from pavements [157, 158],

bridges [159] raindrop impacts [160], friction-induced vibration [161], swirling flows [162], arterial wall deformations [163], vehicle tyres [37] from textiles [164–166] by utilizing piezoelectric or other transduction mechanism.

It is essential for a harvesting device that its natural frequencies matches or lies nearly to that of the source's frequency of excitation for better energy transfer. The fundamental frequencies of the systems considered in this work are in the range of 1 to 16 Hz. For example, in case of fundamental frequency of 4 Hz, the system can harvest energy when the excitation frequency is nearby to the frequencies 1.4 Hz (superharmonic), 8 Hz (principal parametric), 12 Hz (subharmonic) and 16 Hz (combination parametric resonance of sum type). In such cases the linear system will be active only when the excitation frequency is nearly equal to the fundamental frequency itself i.e. only at 4 Hz in this particular case. For the fundamental frequency range of 1 Hz to 16 Hz the harvester will be active from 0.2 Hz (superharmonic resonance with 1 Hz) to 70 Hz (combination parametric resonance of sum type with natural frequencies 16 Hz and 48 Hz). There are several natural processes where a frequency bandwidth of 1 to 100 Hz is available for harvesting purpose which includes vibration in bridges, buildings, moving vehicles on undulated surface, engine and several household appliances. In summary, any system or subsystem with vibrational energy can be a source of energy for harvesting units. One has to decide the transduction mechanism for harvesting such available untapped and wasted energies. In the next section the types of piezoelectric materials and their properties have been discussed.

2.6 Piezoelectric Material

Piezoelectricity is a property of certain solids (crystals, ceramics and polymers, etc.) where electricity is a by-product of the application of pressure or force (direct) and vice-versa (converse). So far the PZT (Lead zirconate titanate, $\text{Pb}(\text{Zr}, \text{Ti})\text{O}_3$) is widely and extensively used ceramic material for energy harvesting (EH) purpose. Being ceramic, it is brittle and having Young's modulus nearly 70 GPa. Flexible polymer PVDF is also useful in energy harvesting [101]. MFC (Micro fibre composite, developed by NASA) is another flexible piezoelectric material. Aluminum

nitride (AlN) as Mo/AlN/Mo is used to develop an efficient low frequency ring type harvesting system (Massaro et al. [167]). Flexible piezoelectric materials find applications in the field of biomedical [139, 168]. One can find the properties of several piezoelectric materials in the work of Bhalla et al. [169], Yang et al. [34] and Erturk and Inman [170].

$$S_{ij} = s_{ijkl}^E T_{kl} + d_{kij} E_k \quad (2.1)$$

$$D_i = d_{ikl} T_{kl} + \epsilon_{ik}^T E_k \quad (2.2)$$

Here, field variables T , S , E and D represents the stress, strain, electric field and electric displacement component respectively. Also d_{kij} , s_{ijkl}^E and ϵ_{ik}^T denotes piezoelectric constant, elastic compliance and permittivity constant respectively. The Eq. (2.1) and Eq. (2.2) represents the standard form of linear piezoelectric constitutive relations as per the IEEE standard on piezoelectricity [169–171]. The details of the above mentioned constitutive relations of piezoelectric material that governs the coupling between mechanical (stress, strain) and electrical (electric field and electric displacement) domain can be found in the books of [9] and Arnau and Soares [172]. There are three piezoelectric modes, i.e., d_{31} , d_{33} , and d_{15} which has been explored for energy harvesting applications. These modes differ in the direction of stress and field polarization (Kim et al. [173]). The direction of electric field and input strain in the PZT is perpendicular in d_{31} mode (Roundy and Wright [174]) and parallel in d_{33} mode (Jeon et al. [175]). The generated output AC voltage across the piezoceramic layers is converted into DC voltage by AC-DC converter for energy storing in capacitors or batteries.

The next section deals with the mathematical modeling and solution strategies of coupled electromechanical PEH systems.

2.7 Mathematical Modeling and Solution Strategies

A beam structure [64, 176] or the combination of several beam based PEH systems [71, 177–180] are commonly used to harvest vibrational energy. Generally, the Euler–Bernoulli beam theory is used to derive the equation of motion of the coupled electromechanical system. Theoretical modeling of the PEH devices includes the structure (substrate) with piezoelectric patches in unimorph or bimorph configurations and electrical behavior. It is the piezoelectric element that coupled the energies associated with mechanical and electrical domains. For unimorph cases, the neutral axis shifts and for bimorph patch configuration the neutral axis may (unsymmetric piezoelectric element) or may not (symmetric piezoelectric element) shift. One may use either force and moment balance method or energy principle to derive the equation of motion. In the case of force balance method, either Newton’s second law or D’Alemberts principle may be used. In the energy method, one may use the Lagrange principle or Extended Hamilton’s principle. In the case of complicated systems, one may adopt the finite element method [53] to derive the equation of motion. Gauss’s law [170] is used to find the equation governing the circuit. A typical governing equation of motion of a PEH system is presented in Eq. (2.3) and Eq. (2.4).

$$\ddot{q} + 2\zeta\dot{q} + q + \alpha q^3 + \beta q\dot{q}^2 + \gamma q^2\ddot{q} + \eta V = q f_1 \cos(\phi_1\tau) + f_2 \cos(\phi_2\tau) \quad (2.3)$$

$$\dot{V} + \chi V + \dot{q} = 0 \quad (2.4)$$

Here $q(\tau)$ and $V(\tau)$ represents the nondimensional time modulation and voltage respectively. Here ζ , α , (β, γ) , η and χ denotes the damping term, coefficient of cubic term, coefficient of nonlinear inertial terms, coupling term and time constant respectively. While Eq. (2.3) governs the dynamics of cantilever beam with piezoelectric patches, Eq. (2.4) represents the external circuit equation having only external load resistance. It may be noted that Eq. (2.3) governs the system under parametric excitation (when $f_1 \neq 0$), direct excitation (when $f_1 = 0$) and combination of both

parametric and direct excitation ($f_1 \neq 0, f_2 \neq 0$). These nondimensional electromechanical equations are coupled and nonlinear. Equation (2.4) may vary as per the electric elements such as reluctance and capacitance involves in the external electric circuit. It may be noted that in the absence of α , β and γ terms, the Eq. (2.3) becomes linear. The mathematical modeling of the PEH system can be expressed either as a lumped (discrete) parameter [3, 16, 21, 48, 174] or continuous (distributed) parameter models [14, 62, 64, 176, 181].

In the case of cantilever beam, large transverse displacement leads to a nonlinear curvature and displacement relation, which involves the cubic nonlinearity into the system. An attached mass is associated with the existing inertial nonlinearities (Friswell et al. [64], Kar and Dwivedy [12], Zavodney and Nayfeh [182]). Thus the governing equations of motion of the system become coupled, nonlinear and electromechanical. Thus a different analytical approach is required to obtain closed form solutions of the governing equations. One may find the detailed derivation approach in the work of Erturk and Inman [170], Erturk and Inman [176] and Friswell et al. [64]) for the development of governing equations by distributed parameter modelling. In most of the literature only single mode approximation is adopted for analysis. The next subsection discusses the forcing in PEH systems.

2.7.1 Forcing in PEH systems

The vibration characteristics of the host structure are mathematically modelled as forced excitation of the associated PEH system. The mechanical energy is represented in several ways, i.e., direct, parametric, stochastic, negative damping (lift force to bluff body) etc.

2.7.1.1 Direct excitation

In the case of direct excitation, the loading on the structure and the displacement of the structure takes place in the same direction. It is the most explored excitation to harvest energy so far (Roundy et al. [16], Erturk and Inman [176], Friswell et al. [64]). It may be harmonic (single or multifrequency) or stochastic in nature. The resonance plays an important role in deciding the energy transduction efficiency of

the harvesting unit. In the case of lock-in, where the external and systems frequency coincides, the amplitude becomes high. The linear vibration based systems must be tuned near to the frequency of excitation.

2.7.1.2 Parametrically excited PEH systems

Generally, in parametrically excited system, the direction of external excitation and response are orthogonal to each other. In such systems the governing differential equation of motion, the time varying excitation term, appears the coefficient of the response of the system. A small parametric excitation [11, 13, 183] can produce a large response when the frequency of excitation is away from the principal frequency of the system. When the excitation frequency is twice that of first natural frequency of the system, then principal parametric resonance occurs. In recent times, the linear and nonlinear energy harvesting systems under direct base excitation is extensively explored. Though a large quantity of literature is available on forced excitation of the system; however, very few studies focus on the investigation of the energy harvesting through parametric excitation. The parametrically excited multimodal system may exhibit the combination parametric resonance of sum and difference type [13]. Faraday [184] observed the parametric excitation while Mathieu [185] treated the phenomenon mathematically [186]. The existence of a time varying coefficient in the differential equation is the characteristic of parametrically excited systems [20]. There are several dynamical systems where parametric resonance occurs [187] and cause danger to the system. Mechanical amplification [188–190] is also one of the characteristics of such systems. Existence of such phenomenon in systems such as ships rolling [191], pipes conveying fluids [192], rail corrugation [193] and rail vibration and noise [194] causes instabilities. In some cases such as flutter suppression [195], quenching of self-excited vibration [196, 197], enhanced damping properties [198] and vibration suppression in drive systems [199], it acts as stabilizer as well. It also helps in improving the sensitivity of sensors [200, 201].

By using the lumped mass approach, Daqaq et al. [21] studied the parametrically excited nonlinear PEH system, which consists of a cantilever beam with a tip mass. Here the effect of electromechanical coupling and load resistance on the output voltage, power and critical excitation level are analysed. Experiments are performed to

validate the results where the beam is kept in horizontal position. A similar system is analysed by Abdelkefi et al. [181] by considering distributed parameter model. The effect of piezoelectric coefficients and quadratic air damping on harvested power are investigated. Jia et al. [202] investigated the feasibility of a parametrically excited electromagnetic based energy harvester system where a significant improvement in power and frequency bandwidth is achieved. In an investigation of the dynamics of a cubic nonlinear Mathieu type equation, Alevras et al. [203] suggested that the parametrically excited systems as an energy harvester can be very robust one for broad frequency range. The nonlinear and nonplanar cantilever beam, when subjected to parametric excitation exhibits complex response behaviours [204]. The parametrically excited systems caused by time varying damper [74] enhance bandwidth and power of the harvester as compared to the conventional directly excited systems.

Yildirim et al. [22] investigated an array of parametrically excited beam systems, and here, a significant increase in the bandwidth is reported. The system is suited for low frequency operations and each beam provides output power in the mW range. Yildirim et al. [205] designed and developed a parametrically excited, fixed-fixed nonlinear energy harvester beam system with a point mass at the center. The harvester shows the linear and nonlinear behaviour at primary and principal parametric resonances, respectively. A wideband frequency range is observed due to nonlinear and parametric effects. Karami et al. [206] investigated a parametrically excited PEH system, where the rotation of the blades induces magnetic force to the beam based harvester.

Autoparametric vibration phenomenon is also utilized for energy transduction purpose [207] where parametric and autoparametric base excitation shows the superiority over direct excitation. Yan and Hajj [208] also exploited the autoparametric base excitation to develop a cantilever based combined absorber and harvester system. Similarly, a combined autoparametric system (absorber and harvester) is examined by Kecik and Borowiec [70]. The system comprises of pendulum and oscillator and the chaotic pendulum motion gives the highest output voltage.

Higher order parametric excitation can also be explored as energy harvesters in MEMS systems at high frequencies [209]. Bobryk and Yurchenko [210] reported a significantly improved performance of energy harvester when subjected to the random parametric excitation. Li et al. [211] studied the coherence resonance of

buckled PEH under the stochastic parametric excitation. The nonlinear dynamics of a cantilever beam with parametric excitation [12, 27, 182, 212–215] is analysed extensively, where rich nonlinear behaviours, i.e., secondary resonances, bifurcations and chaos have been observed.

2.7.1.3 Stochastic excitation

The behaviour of external excitation can be classified broadly into two domains i.e. deterministic and stochastic. Since the ambient is full of random excitations, several studies focused on energy harvesting from stochastic excitation. Daqaq [45] studied the harvester with cubic geometric nonlinearity, under Gaussian white and colored excitations. It has been demonstrated that the damping and inertia nonlinearities may enhance the harvester's performance. Litak et al. [216] analysed a bistable magnetopiezoelectric energy harvester excited by Gaussian white noise. A similar system is investigated by Kumar et al. [217]. Where the corresponding Fokker-Planck (FP) equation is analyzed. It is observed that the solution approach using Fokker-Planck (FP), reduces computational time significantly [218].

The magnetoelastic harvester is observed under white Poisson noise with nonlinearity Khovanova and Khovanov [219] and it has been found that the system performance depends on nonlinearity and ambient energy. Improved performance is obtained under random impulsive excitation for monostable and bistable system. Chunbo et al. [220] investigated a cantilever beam based PEH with vertical stochastic excitation where an improvement in efficiency is reported. Bobryk and Yurchenko [210] explored random ambient excitation along with telegraphic noise as parametric excitation to enhance the harvesting capabilities of linear harvester. An inverted beam based PEH system with a tip mass is analysed by Borowiec et al. [221], which is excited by stochastic resonance. In the following section PEH systems with flow induced vibration have been described.

2.7.1.4 Flow induced excitation

The kinetic energy available in the fluid is another source of energy that has been utilized for energy harvesting purposes. Fluid phenomena such as galloping (Barrero-Gil et al. [84], Sirohi and Mahadik [24]), vortex induced vibration (VIV), wake galloping [104–106, 150] and flutter [101] have been explored in developing PEH systems.

2.7.1.5 Simultaneous excitation

Simultaneous excitation, i.e., direct with parametric excitation or direct with galloping [222], provides more opportunities to the PEH system for ambient energy harvesting. Several recent studies involve the analysis of PEH system under such simultaneous excitations. A combination of parametric and direct excitation is studied by Xia et al. [223], where direct excitation helps in overcoming the necessity of threshold amplitude of excitation to parametric excitation. In another work, rich dynamic responses are observed (Yan and Abdelkefi [122], [6]) when concurrent galloping and base excitations provide input energy to the harvester. In the next section resonance conditions used to harvest energy in PEH systems have been discussed.

2.7.2 Explored resonance conditions

This section deals with the proposed and developed PEH systems where different resonance conditions are utilized for energy harvesting. These resonance conditions include primary resonance, secondary resonances (subharmonic, superharmonic) for direct excitation and principal parametric and combination parametric resonance conditions for parametrically excited systems. The reason behind exploring different resonance conditions is because the ambient vibration contains a wide range of excitation frequencies and when any of these frequencies match, the natural frequencies of the system resonance occur. It may be noted that the harvesting of energy is possible during the resonance conditions only. The presence of more than one natural frequency and mode produces physical phenomena such as internal resonance, combinational resonance, saturation and the existence of periodic responses to a

periodic excitation. These can be utilized to harness more power. In the following subsection literature related to the primary and secondary resonance conditions has been reviewed.

2.7.2.1 Primary and secondary resonance

Primary resonance occurs when the frequency of external excitation coincides with one of the natural frequencies of the system. Away from the resonance condition, the response amplitude, and hence the output voltage reduces significantly. Hence Cheng et al. [224] used self tuning and Niri and Salamone [225] used passive tuning method to enhance the output voltage of the PEH. Available narrow bandwidth also limits the harvester functionality. Tadesse et al. [226] has proposed to separately excite the multiple modes of linear harvester for enhanced performance. Qi et al. [227] proposed that with an array of beams one can harvest energy with multiple resonance conditions simultaneously. Generally the natural frequencies are separated with a large bandwidth, but by utilizing a flexible attachment to the harvester (Seo et al. [228]) or folded configuration (Gong et al. [229]), one may bring the natural frequencies closer within a practical range. By using a magnetic forces, several authors (Leland and Wright [230], Challa et al. [231], Al-Ashtari et al. [232]) proposed a frequency tunable technique in PEH systems.

Nonlinear harvesting systems are capable of harvesting not only under primary resonance but also under secondary resonance conditions. For example in Leadenham and Erturk [178] the superharmonic resonance condition is explored. The limited literature suggests that these secondary resonances are lesser explored for harvesting applications. In the next subsection literature related to internal resonance has been covered.

2.7.2.2 Internal resonance

Nonlinearity brings an interesting dynamic phenomenon i.e., internal resonance (IR) in multimode systems. While in the case of linear systems the different modes can be excited separately by adjusting the external excitation but in nonlinear systems coupling among different modes is possible [26, 233–235]. Internal resonance arises

when the modal frequencies are in integer relationship with each other. For example if the second modal frequency is near twice the 1st mode frequency, one may observe 1:2 internal resonance condition. In other words, these commensurable or nearly commensurable frequency ratios [25] causes mode coupling phenomenon [71] and energy transfer takes place between participating modes of the system. This leads to periodic, quasi-periodic, and chaotic responses [12, 26, 27, 233, 236]. Such responses attracted scientific community from time to time and inspired them to apply internal resonance conditions in multi-dimensional interconnected systems for a large number of application fields.

Recent developments in nonlinear energy harvesting systems show that the mechanism of internal resonance can be utilized to improve the energy harvesting capabilities of harvester systems. Perhaps the consideration of internal resonance is less explored in order to harvest more energy over wide bandwidth in the field of energy harvesting from vibration. The commensurate modal frequencies can be used to broaden the steady state bandwidth Chen et al. [237] due to nonlinear energy transfer which occurs among the commensurate modes. In order to seek better performance in the field of PEH, few recently developed systems [179, 237–239] incorporated the internal resonance condition of 1:2. Similarly, Yan and Hajj [208] investigated an autoparametric vibration based PEH for enhanced frequency bandwidth. Where the frequency ratio of beam to base structure is taken as one half that causes the beam motion. A hybrid PEH system with internal resonance (1:2) is explored by Yang and Towfighian [68], where an axially movable magnet based bistable configuration is analysed. It is observed that large magnetization moment and closely placed magnets lead to wide bandwidth. Xiong et al. [240] achieved 1:2 internal resonance by incorporating an auxiliary oscillator to the main nonlinear harvester and they have shown experimentally that 130% improvement in bandwidth can be achieved. In all these internal resonance based PEH systems an auxiliary system is attached to the main system in such a way that the combined system exhibits internal resonance.

To enhance the steady state bandwidth limit, an axially loaded energy harvester with 1:2 internal resonance is studied by Jiang et al. [238]. Chen et al. [237] considered the 1:2 internal resonance with L-shaped energy harvester to improve the bandwidth. An autoparametric vibration based energy harvester is investigated by Yan and Hajj [208], where nonlinear response enhances the frequency bandwidth. Inspired from

the trees which exploit nonlinear mechanisms such as multimodal frequency distributions and nonlinearities to damp out the vibrations caused by aerodynamic forces, Harne et al. [179] studied L-shaped vibration energy harvesting system under pure harmonic, mixed harmonic and stochastic excitations which exploits the 1:2 internal resonance. The robustness and practicality of utilizing nonlinear mechanism to enhance the performance of energy harvesting systems are investigated analytically, numerically, and experimentally. Xu and Tang [239] proposed a piezoelectric cantilever beam attached with a pendulum for multi-directional energy harvesting purposes. System dynamics is analysed for 1:2 internal resonance which is due to nonlinear coupling between cantilever beam and the pendulum. L-shaped beam-mass structure with quadratic nonlinearity tuned to two-to-one internal resonance is studied by Cao et al. [241] for bandwidth enhancement. Alazmi et al. [242] examined the vibration and voltage response of an electromagnetic ferrofluid-based vibratory energy harvester with primary and a 1:2 internal resonance conditions.

Rocha et al. [243] presented a two degree of freedom frame structure for energy harvesting with nonlinear piezoelectric material which exhibits 1:2 internal resonance between symmetric and the sway modes. Chen and Jiang [244] proposed a new possibility to enhance energy harvesting by tuning harvesters at the internal resonance of 1:2. Recent experimental work of Shaw et al. [245] provides valuable insight into frequency responses of nonlinear systems consists of a cantilever beam with a nonlinear spring at the tip with deliberately induced 1:3 internal resonance. Complex dynamics of periodic and quasi periodic response have been observed. The Efficiency of PEH systems is increased by the application of internal resonance Chunbo et al. [220]. Mode coupling leads to interaction and energy transfer between higher and lower modes. Here a cantilever beam is considered under vertical stochastic excitation.

2.7.3 Solution strategies

Apart from numerical methods, some analytical methods are useful while solving nonlinear equations of motion. Some analytical methods include harmonic balance method and perturbation methods such as the method of averaging, the Lindstedt-Poincaré technique and the method of multiple scales (MMS) etc. to solve the

nonlinear governing temporal equation of motion [13, 246]. The method of multiple scale technique provides transient as well as steady state response. It helps to access the stability of steady state solution. It can also give information related to the response of the harmonics of the forcing frequency [15]. The perturbation technique of MMS [21, 181, 247] is widely used for finding closed form solutions of the harvesters. Finite element method is also adopted [248–251, 251–253] for analysing the PEH system.

2.8 Miscellaneous PEH Systems

Some miscellaneous PEH systems are also explored for energy harvesting which includes the excitation force by moving load [254, 255], impact [47, 256–259], oscillating ball [260, 261] and by magnetic plucking (Xue and Roundy [262]).

Several studies focused on the geometric alterations [54, 55, 263–270] for performance improvement of energy harvesting systems. Other unconventional shapes such as L shaped [177, 179, 237], M shaped [178], U shaped [249], E shaped [180, 271, 272], T shaped [103] and trunk shaped beam structure [273] are also explored for energy harvesting.

Few studies involve the development of bioinspired PEH system [93, 97, 273–280] as well. It has been observed that there exist enormous possibilities for energy harvesting through piezoelectric materials. Completely developed and deployable harvesting system (harvester with sensor/actuator) is not only useful but undoubtedly commercially viable as well.

2.9 Electrical Circuits

Electrical circuits are an integral part of PEH systems. The AC-DC rectifier, voltage regulator and energy storage unit are basic components of an electric circuit (Li et al. [146]). A DC-DC converter (buck, boost etc.) may also be used to provide a regulated output voltage. Also, there has been attempts to develop efficient electrical circuits as well. Like in the work of Ramadass and Chandrakasan [281], bias-flip

rectifier and shared inductor is used instead of more common full-bridge rectifiers. An improvement in the power extraction capability is reported with reduced no of passive components due to shared inductor. To reduce the voltage drop across the full wave diode bridge rectifiers, an asynchronous full-wave rectifier is explored [282] for use with multiple piezo-generators. In an another study [283], Schottky diode bridge rectifier and double voltage divider circuit is employed in a harvesting circuit for power amplification. Le et al. [284] presented several circuit designs and comparison been made, where a full wave synchronous rectification is found to perform better than others. in a recent study Adhikari et al. [285] studied a VIV based PEH system under random field where it is found that there exist an optimal value of inductor at which the mean power becomes maximum when an inductor is applied parallel to the load resistance. Similarly, electrical circuits of PEH systems has been studied for various interfacing circuits [286], for better understanding, improvement [287] and energy optimization.

Generally capacitors [56], supercapacitors [288, 289] and rechargeable batteries [289–291] have been used as an energy storage element. Leakage resistance [289] and the piezoelectric element influences the efficiency of the storage unit. Capacitors are not suitable for continuous power supply since they provide power in short burst [292]. However the requirement of certain minimum output voltage does not apply to capacitors as compared to batteries. So far the rechargeable batteries and supercapacitors [293, 294] are found to be suitable as an energy storage unit for PEH systems. Commercially available energy harvesting integrated circuits are also utilized as electric circuits in PEH systems [295, 296].

For more basic understanding, specifically related to the electrical circuits, one may go through the work of Ottman et al. [297, 298].

2.10 Summary and Identification of Research Gaps

In most of the literature base excited cantilever beam with piezoelectric patches is considered for analysis as an energy harvester. The beam is generally modelled as an Euler-Bernoulli beam. While in most cases small curvature is considered in some

cases moderately large curvature is taken which gives rise to geometric nonlinearity in the system. In some studies tip mass is considered to enhance the bending effect to generate higher output voltage and power. The governing coupled electromechanical equation of motion of such systems is either that of a forced with/or parametrically excited system with or without nonlinearity. Most of the linear harvesting system operates near the resonance condition. As the ambient vibration has a broad bandwidth of frequency so these linear harvesters are not suitable for producing energy in a wider range of frequency. So in such cases one may go for nonlinear forced vibration based harvester where multiple resonance conditions such as simple resonance, superharmonic and subharmonic resonance conditions. Further one may increase the bandwidth by using the concept of parametric excitation where principal parametric and combination parametric resonance conditions can be explored. As these phenomena are not yet explored, hence there is need to study the dynamics of such systems for enhanced performance of the harvester. While in most of the cases, only single mode approximation is taken for the analysis, few researchers also studied systems with more number of modes. In some cases these modes are having integer relations giving rise to internal resonance conditions of 1:1 and 1:2. Such internal resonance conditions can be achieved by attaching a mass at arbitrary position in the cantilever beam. While the dynamics of the cantilever beam without piezoelectric patch having attached mass either at the tip or at arbitrary position have been widely studied similar studies have not been carried out for piezoelectric embedded cantilever beam for energy harvesting purpose. Hence there is a need to carry out the nonlinear dynamic analysis of the PEH system with different types of internal resonance conditions (e.g., 1:3 internal resonance). While most of the experimental work focused on PEH with forced vibration with simple resonance conditions, very few experimental studies have been carried out on parametric excitation based PEH. So there is a need to carry out experiments to validate the theoretical findings. The position of the attached mass is chosen in such a way that the ratio between the first two frequencies is commensurable, which leads to the existence of internal resonance of 1:3 between first two frequencies of the beam system. Hence a two mode approximation is adopted here for the dynamic analysis. Due to modal interaction, energy transfer between modes takes place. The system considered in the analysis is inspired by the work of Zavodney and Nayfeh [182] and Dwivedy and Kar [27]. The present system is compact as compared to the

systems where internal resonance between multiple beam elements are considered. All these factors lead to the development of complex and coupled nonlinear equations of motion. Also the existence of geometric, inertial nonlinearity along with 1:3 internal resonance brings the possibility of multiple resonance conditions other than primary resonance, i.e., principal parametric, combination parametric, subharmonic and superharmonic resonances.

The researchers separately explored the PEH system under parametric excitation and systems involving internal resonance. However, the system with parametric excitation and internal resonance brings rich dynamical responses i.e. quasiperiodic and chaotic [12, 27, 236]. This combination has not been explored to date for energy scavenging. The parametric excitation can be of the type of principal parametric and combination parametric.

The governing spatio-temporal equation of motion is determined, which is reduced to its temporal form by using generalized Galerkin's method. The method of multiple scales (MMS) is used to solve the resulting equations of motion to obtain the steady state response and voltage. Here the system is analyzed under principal parametric and combination parametric resonance conditions. By utilizing parametric excitation and internal resonance conditions here, an attempt has been made to harvest the vibration energy for a wide range of frequencies. To perform experimental analysis, an in-house shaker is designed and developed to provide base excitation to the system. Experimental studies are performed to understand the dynamics along with challenges posed by parametrically excited system on the voltage and power output across a load resistance.

Finally, the present system configuration is also studied under the effect of flow induced instability, which is the phenomena of galloping. The energy transduction requires an attachment of the bluff body with a slender structure. In the present study, two bluff bodies having D and triangular sections are considered for dynamic analysis. In most of the literature, only steady wind conditions are explored for energy transduction purposes. However in the present work the effect of both steady and unsteady wind speeds is investigated on the dynamics of the system. The mass of the bluff body is also taken into account and since the beam is in inverted position the effect of gravity affects the dynamics of the system. In all the previous studies the bluff body is placed at the top of the slender structure. While in the present work

the mathematical modeling and the experiment, both accommodated the variation in the position of the bluff body along the beam. By this variation the frequency of the system can be adjusted. The effect on the dynamics of the bluff body position is investigated. A comparative study between D and triangular section bluff body is also presented. The parametric study involves the effect of the variations of steady wind speed, unsteady wind speed component and external load resistance. The experiments of the harvester under steady state wind conditions are also performed in the wind tunnel facility. The output voltage and power are measured for different system parameters. The unsteady wind state is modeled by a simplistic model of harmonic variations in the steady state wind condition. Such kind of model can also be employed to the systems exposed to the wake galloping phenomena. Also, it is notable that the unsteady wind component brings the parametric instability into the system caused by galloping.

The present investigation explores the piezoelectric energy transduction under two kinds of known instabilities viz., parametric excitation and galloping phenomena which, in general are to be avoided. This work may find applications in health monitoring of the machine components and civil structures like bridges and high rise buildings. The present system may be attached to vehicles under undulated roads and train in motion as well. The galloping based PEH system (GPEH) can be employed in remote locations also. Also, both excitation forces of parametric excitation and galloping are available in bridges, which can be explored for harvesting. The objective of the present work is mentioned in the following section.

2.11 Objectives of the Present Work

Taking into consideration of the gaps from the current state-of-the-art literature the objectives of the present work are as follows.

- To study the nonlinear dynamics of a base excited piezoelectric based energy harvester with parametric and internal resonance conditions to generate voltage for a wide range of frequencies.
- To study the nonlinear dynamics of galloping based piezoelectric energy harvester under flow-induced vibration with uniform and time-varying wind flow.

To achieve these objectives, the following studies are performed.

- Nonlinear dynamic analysis of parametrically excited piezoelectric energy harvester under principal parametric resonance and 1:3 internal resonance conditions.
- Nonlinear dynamic analysis of piezoelectric energy harvester under combination parametric resonance of summed type and 1:3 internal resonance conditions.
- Design and development of experimental setup to investigate the dynamics of a base excited piezoelectric energy harvester system.
- Study the dynamics of piezoelectric based energy harvester under steady and unsteady wind flow, through analytical and experimental investigations.





Chapter 3

Piezoelectric Energy Harvester under Principal Parametric and 1:3 Internal Resonance

3.1 Introduction

As discussed in the previous chapter, most of the energy harvesters comprised of a base excited cantilever beam with piezoelectric patches with or without tip mass. Inspired from the work of Zavodney and Nayfeh [182] and Dwivedy and Kar [27] where the nonlinear dynamics of the harmonically base excited cantilever beam with an attached mass at an arbitrary position is studied, the present work explores the possibility of utilizing similar system for an energy harvesting purpose by attaching bimorph piezoelectric patches. In the present work the coupled electromechanical nonlinear governing equation of motion is derived. It is reduced to its temporal form by using a generalized Galerkin principle. The method of multiple scales is used to solve the resulting equation of motion to obtain the steady state response and voltage. Here the system is considered with principal parametric and 1:3 internal resonance conditions to harvest the vibration energy for a wide range of frequencies. The results are compared with the existing literature and it is observed that the proposed harvester yields higher voltage and power.

3.2 Mathematical Modeling

A slender cantilever beam of length L with uniform rectangular cross sectional area having mass attached at a distance of d from the fixed end is subjected to a harmonic base excitation of $z(t) = Z_0 \cos(\Omega t)$ as shown in Fig. 3.1. Piezoelectric patches are attached to both the faces of the beam. Euler-Bernoulli beam equation is used for mathematical modeling. Considering an arbitrary point at a distance, s from the base, the transverse and axial displacements can be given by $v(s, t)$ and $u(s, t)$ respectively. The displacement-curvature relation of the beam is nonlinear due to large oscillation of the beam which can be represented by using geometrical and inertial nonlinearities. The bending moment at a distance, s from the fixed base can

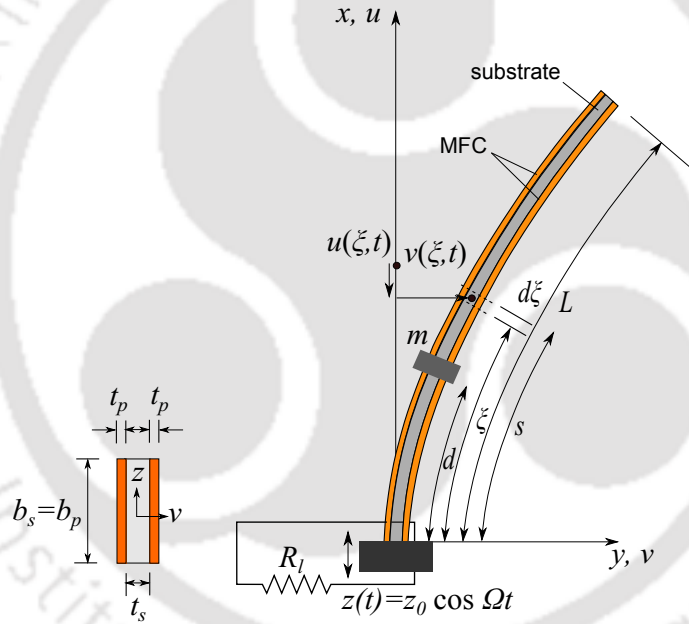


FIGURE 3.1: Schematic representation of the energy harvester

be written as follows [182].

$$M(s) = E_b I_b \kappa(s, t) \approx E_b I_b v'' \left(1 + \frac{1}{2} v'^2 \right) \quad (3.1)$$

Where the curvature $\kappa(s, t)$ is expressed in the following form.

$$\kappa(s, t) = \frac{\partial \varphi(s)}{\partial s} = \frac{v''}{\cos \varphi} = \frac{v''}{\sqrt{1 - v'^2}}$$

Here E_b , I_b and $(\cdot)'$ denotes the Young's modulus of the substrate, moment of inertia of the substrate and the differentiation with respect to s , respectively. Also $\varphi(s)$ denotes the rotation of the beam at a distance s from the base which can be expressed in terms of axial and transverse displacement as $\sin \varphi = v'$, $\cos \varphi = (1 - v'^2)^{1/2}$. By expressing the moment in the beam $M(s)$ as the sum of all external moments as follows.

$$\begin{aligned}
 M(s) = & - \underbrace{\int_s^L \{[\rho + m\delta(\xi - d)] \ddot{v} + c\dot{v}\} \left(\int_s^\xi \cos(\varphi) d\eta \right) d\xi}_{M_1} \\
 & - \underbrace{\int_s^L \{\rho + m\delta(\xi - d)\} [\ddot{u} - g] \left(\int_s^\xi \sin(\varphi) d\eta \right) d\xi}_{M_2} \\
 & + \underbrace{\int_s^L J_0 \delta(\xi - d) \ddot{\varphi} d\xi}_{M_3} \\
 & + \underbrace{\int_{t_p+t_s/2}^{-t_s/2} \left\{ E_p \left(-yv'' \left(1 + \frac{1}{2}v'^2 \right) \right) - e_{31}E_e \right\} b_p y dy + \int_{t_s/2}^{-t_p-t_s/2} \left\{ E_p \left(-yv'' \left(1 + \frac{1}{2}v'^2 \right) \right) - e_{31}E_e \right\} b_p y dy}_{M_4}
 \end{aligned} \tag{3.2}$$

By following the work of Zavodney and Nayfeh [182], Erturk and Inman [170] and Dwivedy and Kar [27] the internal moment $M(s)$ in the beam can be expressed as the sum of total external moment due to lateral (M_1) and longitudinal (M_2) inertia of beam element $d\xi$ and attached mass m and due to rotary inertia of attached mass (M_3) and stress developed in the piezoelectric patches due to bending which spanned the whole length of the beam (M_4). Here subscript 'b' and 'p' denotes the terms related to 'beam substrate' and 'piezoelectric patch' respectively. Here ρ , E_p , e_{31} and E_e denotes the mass per unit length, the Young's modulus of piezoelectric element, piezoelectric constant and Electric field respectively. The inextensibility condition $u(s, t) = \frac{1}{2} \int_0^s \{v'(\xi, t)\}^2 d\xi + z(t)$ [299] is used to express the axial displacement $u(s, t)$ in terms of the longitudinal displacement $v(s, t)$. After double differentiating the moment equation with respect to 's' one can obtain the

governing equation of motion in the following form.

$$[\rho + m\delta(s - d)] v_{tt} + cv_t + EI [v_{ssss} + \{v_s(v_s v_{ss})_s\}_s] - [J_0\delta(s - d)(v_s)_{tt}]_s - (Nv_s)_s - H_{ss}\eta\bar{V}(t) = 0 \quad (3.3)$$

The coupled electromechanical circuit equation with parallel connection for bimorph configuration can be expressed as follows [170].

$$C_p \frac{d\bar{V}(t)}{dt} + \frac{\bar{V}(t)}{2R_l} - i_p(t) = 0 \quad (3.4)$$

Here,

$$\begin{aligned} \rho &= \rho_b A_b + 2\rho_p A_p, EI = E_b I_b + E_p I_p, H = [H(s) - H(s - L)], \\ N &= \frac{1}{2}\rho \int_s^L \left[\int_0^\xi (v_s^2)_{tt} d\eta \right] d\xi + \frac{1}{2}m \int_s^L \delta(\xi - d) \left[\int_0^\xi (v_s^2)_{tt} d\eta \right] d\xi \\ &\quad + m(z_{tt} - g) \int_s^L \delta(\xi - d) d\xi + \rho L \left(1 - \frac{s}{L}\right) (z_{tt} - g) \\ &\quad - J_0 \delta(\xi - d) \left\{ \frac{1}{2} v_{stt} v_s^2 + v_s v_{st}^2 \right\}, C_p = \frac{\hat{e} b_p L_p}{t_p}, \\ i_p(t) &= - \sum_{n=1}^2 e_{31} \frac{t_p + t_b}{2} b_p \int_0^{L_p} (v_{ss})_t ds, \eta = \frac{e_{31} b_p}{t_p} \left[\left(t_p + \frac{t_b}{2} \right)^2 - \frac{t_b^2}{4} \right] \\ I_b &= \frac{b_b t_b^3}{12}, I_p = \frac{2}{3} b_p \left[\left(t_p + \frac{t_b}{2} \right)^3 - \left(\frac{t_b^3}{8} \right) \right], A_b = b_b t_b, A_p = b_p t_p \end{aligned}$$

The boundary conditions for the present cantilever beam system are expressed in the following form.

$$v(0, t) = 0, \quad v_s(0, t) = 0, \quad v_{ss}(L, t) = 0, \quad v_{sss}(L, t) = 0 \quad (3.5)$$

Here, g , c , z , ξ , δ , H and $\bar{V}$ denotes the acceleration due to gravity, the coefficient of viscous damping, the displacement of the base, the variable of integration along the beam length, the Dirac delta function, the Heaviside function and the voltage across load resistance respectively. Also b_b , b_p , t_b , t_p , A_b , A_p , I_p , $\hat{e}$, C_p and R_l are respectively, the width of beam, width of piezo-patch, thickness of beam,

thickness of piezo-patch, area of cross section of beam, area of cross section of piezo-patch, moment of inertia of piezo patch, permittivity at constant strain, piezoelectric capacitance and load resistance. As per Galerkin's method with two mode approximation, the transverse displacement $v(s, t)$ of the beam is represented by using a scaling factor r , time modulation $q(t)$ and shape function $\psi(s)$ of the beam [182] as follows.

$$v(s, t) = \sum_{n=1}^{\infty} r \psi_n(s) q_n(t) \quad (3.6)$$

The following non-dimensional parameters are used in the further study.

$$\begin{aligned} x &= \frac{s}{L}, \beta = \frac{d}{L}, \tau = \theta_1 t, \omega_n = \frac{\theta_n}{\theta_1}, \lambda = \frac{r}{L}, \mu = \frac{m}{\rho L}, \Gamma = \frac{Z_0}{Z_r} \\ J &= \frac{J_0}{\rho L r^2}, \phi = \frac{\Omega}{\theta_1}, \bar{V}(t) = r_v V(t), r_v = \frac{\bar{K}_1 r}{C_p}, K_n = \frac{\bar{K}_n r}{C_p r_v} \\ \eta_n &= \frac{-\eta r_v}{\rho \hbar_n \theta_1^2 r \epsilon L^2} \left\{ -\frac{d\psi_n}{dx} \Big|_{x=0} + \frac{d\psi_n}{dx} \Big|_{x=1} \right\}, \chi = \frac{1}{2C_p \theta_1 R_l} \\ i_p(t) &= -\sum_{n=1}^2 r \theta_1 \bar{K}_n \frac{dq_n(\tau)}{d\tau}, \bar{K}_n = e_{31} \frac{t_p + t_b}{2} \frac{b_p}{L} \int_0^1 \frac{d^2 \psi_n(x)}{dx^2} dx \end{aligned}$$

Using the above nondimensional parameters the following nonlinear coupled electromechanical temporal governing equations of motion are obtained.

$$\begin{aligned} \ddot{q}_n + 2\epsilon \zeta_n \dot{q}_n + \omega_n^2 q_n + \epsilon \sum_{k,l,m}^{\infty} \{ \alpha_{klm}^n q_l q_m + \beta_{klm}^n \dot{q}_l \dot{q}_m + \gamma_{klm}^n q_l \ddot{q}_m \} q_k + \epsilon \eta_n V(\tau) \\ - \epsilon \sum_{m=1}^{\infty} f_{nm} q_m \cos(\phi \tau) = 0 \end{aligned} \quad (3.7)$$

$$\dot{V}(\tau) + \chi V(\tau) + \sum_{n=1}^2 K_n \dot{q}_n = 0 \quad (3.8)$$

In the absence of piezoelectric patches and considering only single mode approximation, the equation of motion of the system reduces to that of the model investigated theoretically and experimentally by Zavodney and Nayfeh [182]. Also with two mode approximation the governing equation of motion without piezoelectric patches reduces to that of Kar and Dwivedy [12]. The coefficients are defined in Appendix A.

3.3 Solution of Temporal Equation

A uniform first order approximate solution of Eq. (3.7) is obtained by using the standard method of multiple scales [12, 13, 182, 300]. The time dependence can be expressed in to multiple time scales as $T_n = \epsilon^n \tau$ where ϵ is a book keeping parameter. The time derivative can be expressed as follows.

$$\frac{d}{d\tau} = D_0 + \epsilon D_1 + O(\epsilon^2), \quad \frac{d^2}{d\tau^2} = D_0^2 + 2\epsilon D_0 D_1 + O(\epsilon^2), \quad D_n = \frac{\partial}{\partial T_n} \quad (3.9)$$

The solution of the system can be expanded in the following form.

$$q_n(\tau; \epsilon) = q_{n0}(T_0, T_1) + \epsilon q_{n1}(T_0, T_1) + O(\epsilon^2) \quad (3.10)$$

$$V(\tau) = V_0(T_0, T_1) + \epsilon V_1(T_0, T_1) + O(\epsilon^2) \quad (3.11)$$

By substituting Eq. (3.9), (3.10) and (3.11) in Eq. (3.7) and further equating the coefficient of ϵ^0 and ϵ^1 equal to zero, yields the following set of equations.

$O(\epsilon^0)$:

$$D_0^2 q_{n0} + \omega_n^2 q_{n0} = 0 \quad (3.12)$$

$$D_0 V_0 + \chi V_0 = - \sum_{n=1}^{\infty} K_n D_0 q_{n0} \quad (3.13)$$

$O(\epsilon^1)$:

$$D_0^2 q_{n1} + \omega_n^2 q_{n1} = - \left[\begin{aligned} & 2\zeta_n D_0 q_{n0} + 2D_0 D_1 q_{n0} - \sum_{n,m=1}^{\infty} f_{nm} q_{m0} \cos \phi \tau \\ & + \eta_n V_0 + \sum_{klm} \left(\begin{aligned} & \alpha_{klm}^n q_{k0} q_{l0} q_{m0} \\ & + \beta_{klm}^n q_{k0} D_0 q_{l0} D_0 q_{m0} \\ & + \gamma_{klm}^n q_{k0} q_{l0} D_0^2 q_{m0} \end{aligned} \right) \end{aligned} \right] \quad (3.14)$$

$$D_0 V_1 + \chi V_1 = -D_0 V_0 - K_1 (D_0 q_{11} + D_1 q_{10}) - K_2 (D_0 q_{21} + D_1 q_{20}) \quad (3.15)$$

The solution of differential equations of $O(\epsilon^0)$ is obtained as follows.

$$q_{n0} = A_n(T_1) \exp(i\omega_n T_0) + cc \quad (3.16)$$

$$V_0 = - \sum_{n=1}^2 K_n Z_n A_n(T_1) \exp(i\omega_n T_0) + cc \quad (3.17)$$

Here cc is the complex conjugate of the preceding terms and $A_n(T_1)$ is an unknown complex function to be determined. Using Eq. (3.16) and (3.17) into Eq. (3.14) the following secular and near secular terms are obtained which should be eliminated for finite amplitude of response of the system.

$$\begin{aligned} n = 1 : \\ \frac{1}{2} (f_{12} A_2 e^{i(-\phi+\omega_2)T_0} + f_{11} \bar{A}_1 e^{i(\phi-\omega_1)T_0}) - Q_{12} A_2 \bar{A}_1^2 e^{i(-2\omega_1+\omega_2)T_0} \\ + e^{i\omega_1 T_0} \left(-2i\omega_1 (\zeta_1 A_1 + A_1') + \eta_1 K_1 Z_1 A_1 - \sum_{j=1}^2 \alpha_{e1j} A_j \bar{A}_j A_1 \right) = 0 \end{aligned} \quad (3.18)$$

$$\begin{aligned} n = 2 : \\ \frac{1}{2} f_{21} A_1 e^{i(\phi+\omega_1)T_0} - Q_{21} A_1^3 e^{3i\omega_1 T_0} \\ + e^{i\omega_2 T_0} \left(-2i\omega_2 (\zeta_2 A_2 + A_2') + \eta_2 K_2 Z_2 A_2 - \sum_j^2 \alpha_{e2j} A_j \bar{A}_j A_2 \right) = 0 \end{aligned} \quad (3.19)$$

From Eq. (3.18) and (3.19), it may be observed that by taking principal parametric case $\phi = 2\omega_1 + \epsilon\sigma_1$ and internal resonance $\omega_2 = 3\omega_1 + \epsilon\sigma_2$ and expressing $A_n(T_1) = \frac{1}{2}a_n(T_1)e^{i\vartheta_n(T_1)}$ where amplitude $a_n(T_1)$ and phase $\vartheta_n(T_1)$ are considered as slowly varying function of time scale T_1 and eliminating the secular term (the coefficient of $e^{i\omega_n T_0}$ term) for bounded solution, we obtained the following reduced equations.

$$\begin{aligned} 2\omega_1 a_1' = -2\omega_1 \zeta_1 a_1 + \frac{1}{2} \{f_{11} a_1 \sin 2\gamma_1 + f_{12} a_2 \sin(\gamma_1 - \gamma_2)\} \\ - \frac{1}{4} Q_{12} a_2 a_1^2 \sin(3\gamma_1 - \gamma_2) + \Pi_a a_1 \end{aligned} \quad (3.20)$$

$$\begin{aligned} 2\omega_1 a_1 \gamma_1' = \omega_1 a_1 \sigma_1 + \frac{1}{2} \{f_{11} a_1 \cos 2\gamma_1 + f_{12} a_2 \cos(\gamma_1 - \gamma_2)\} \\ - \frac{1}{4} \sum_{j=1}^2 \alpha_{e1j} a_j^2 a_1 - Q_{12} a_2 a_1^2 \cos(3\gamma_1 - \gamma_2) + \Pi_b a_1 \end{aligned} \quad (3.21)$$

$$2\omega_2 a_2' = -2\omega_2 \zeta_2 a_2 + \frac{1}{2} f_{21} a_1 \sin(\gamma_2 - \gamma_1) - \frac{1}{4} Q_{21} a_1^3 \sin(\gamma_2 - 3\gamma_1) + \Pi_c a_2 \quad (3.22)$$

$$\begin{aligned} 2\omega_2 a_2 \gamma_2' = -2\omega_2 a_2 (\sigma_2 - 1.5\sigma_1) + \frac{1}{2} f_{21} a_1 \cos(\gamma_2 - \gamma_1) \\ - \frac{1}{4} \sum_{j=1}^2 \alpha_{e2j} a_j^2 a_2 - 0.25 Q_{21} a_1^3 \cos(\gamma_2 - 3\gamma_1) + \Pi_d a_2 \end{aligned} \quad (3.23)$$

Where,

$$\begin{aligned}\Pi_a &= \frac{K_1 \eta_1 \chi \omega_1}{(\chi^2 + \omega_1^2)}, \quad \Pi_b = \frac{K_1 \eta_1 \omega_1^2}{(\chi^2 + \omega_1^2)}, \quad \Pi_c = \frac{K_2 \eta_2 \chi \omega_2}{(\chi^2 + \omega_2^2)}, \quad \Pi_d = \frac{K_2 \eta_2 \omega_2^2}{(\chi^2 + \omega_2^2)} \\ Z_n &= \frac{\omega_n^2 + i \chi \omega_n}{(\chi^2 + \omega_n^2)}, \quad \gamma_1 = -\vartheta_1 + 0.5 \sigma_1 T_1, \quad \gamma_2 = -\vartheta_2 + (1.5 \sigma_1 - \sigma_2) T_1\end{aligned}$$

Here the terms Π_a , Π_b , Π_c and Π_d are the functions of coupling terms η_n and K_n in Eq. (3.7) and (3.8), respectively along with time constant χ of the resistive capacitive circuit and nondimensional frequencies of the system ω_n . The time constant term η is inversely proportional to the load resistance R_l . Normalizing method is used to find the stability of trivial state by using $p_i = a_i \cos \gamma_i$; $q_i = a_i \sin \gamma_i$; $i = 1, 2$ one may get the following normalized reduced equations in Cartesian form.

$$p'_1 = \frac{-1}{2\omega_1} \left[\begin{aligned} &2\omega_1 \zeta_1 p_1 + \left(\omega_1 \sigma_1 - \frac{1}{2} f_{11} \right) q_1 + \frac{1}{2} f_{12} q_2 - \frac{1}{4} \sum_{j=1}^2 \alpha_{e1j} q_1 (p_j^2 + q_j^2) \\ &+ \frac{1}{4} Q_{12} \{ q_2 (q_1^2 - p_1^2) + 2p_1 p_2 q_1 \} - \Pi_a p_1 + \Pi_b q_1 \end{aligned} \right] \quad (3.24)$$

$$q'_1 = \frac{-1}{2\omega_1} \left[\begin{aligned} &2\omega_1 \zeta_1 q_1 - \left(\omega_1 \sigma_1 + \frac{1}{2} f_{11} \right) p_1 - \frac{1}{2} f_{12} p_2 + \frac{1}{4} \sum_{j=1}^2 \alpha_{e1j} p_1 (p_j^2 + q_j^2) \\ &+ \frac{1}{4} Q_{12} \{ p_2 (p_1^2 - q_1^2) + 2p_1 q_1 q_2 \} - \Pi_b q_1 - \Pi_a p_1 \end{aligned} \right] \quad (3.25)$$

$$p'_2 = \frac{-1}{2\omega_2} \left[\begin{aligned} &2\omega_2 \zeta_2 p_2 + \omega_2 (3\sigma_1 - 2\sigma_2) q_2 + \frac{1}{2} f_{21} q_1 - \frac{1}{4} Q_{21} \{ q_1 (3p_1^2 - q_1^2) \} \\ &- \frac{1}{4} \sum_{j=1}^2 \alpha_{e2j} q_2 (p_j^2 + q_j^2) - \Pi_c p_2 + \Pi_d q_2 \end{aligned} \right] \quad (3.26)$$

$$q'_2 = \frac{-1}{2\omega_2} \left[\begin{aligned} &2\omega_2 \zeta_2 q_2 - \omega_2 (3\sigma_1 - 2\sigma_2) p_2 - \frac{1}{2} f_{21} p_1 + \frac{1}{4} Q_{21} \{ p_1 (p_1^2 - 3q_1^2) \} \\ &+ \frac{1}{4} \sum_{j=1}^2 \alpha_{e2j} p_2 (p_j^2 + q_j^2) - \Pi_c q_2 - \Pi_d p_2 \end{aligned} \right] \quad (3.27)$$

Now after perturbing the above reduced equations Eq. (3.24) – (3.27) one can obtain the expression in the following form.

$$\{\Delta p'_1, \Delta q'_1, \Delta p'_2, \Delta q'_2\}^T = [J] \{\Delta p_1, \Delta q_1, \Delta p_2, \Delta q_2\}^T \quad (3.28)$$

Here, superscript T and matrix $[J]$ denotes the transpose and the Jacobian matrix

respectively. The eigenvalues of $[J]$ will define the stability and bifurcation of the fixed point response of the system. The stability and bifurcation of the fixed point response of the system is characterized by the eigenvalues of Jacobian matrix. Those eigenvalues which have negative real parts belong to asymptotically stable solutions and vice versa. One can obtain the steady-state beam deflection, output voltage, output power and input power by using the following dimensionalized forms.

$$v(s, \tau) = \sum_{n=1}^2 r \psi_n(s) a_n \cos(\omega_n \tau + \vartheta_n) \quad (3.29)$$

$$\bar{V}(\tau) = \sum_{n=1}^2 r_v K_n \frac{\omega_n}{\sqrt{\chi^2 + \omega_n^2}} a_n \cos(\omega_n \tau + \vartheta_n + \delta_n) \quad (3.30)$$

$$P_{out} = \frac{1}{R_l} \left(\sum_{n=1}^2 r_v K_n \frac{\omega_n}{\sqrt{\chi^2 + \omega_n^2}} a_n \right)^2 \quad (3.31)$$

$$P_{in} = \frac{1}{2} \hat{m} \ddot{u} \dot{z} \approx \frac{1}{2} \hat{m} z_0^2 \Omega^3 \quad (3.32)$$

$$\eta_h = \frac{P_{out}}{P_{in}} \quad (3.33)$$

Here, $\delta_n = \tan^{-1} \left(\frac{\chi}{\omega_n} \right)$. The efficiency of the energy harvester (η_h) can be found by dividing the resulting output power to that of the maximum input power (P_{in}) of the harvester. While the output power can be obtained by the Eq. (3.31), the input power can be obtained by the rating of the prime mover used for the base excitation. Also, $\hat{m}$, $\ddot{u}$, z_0 and Ω denotes the mass of the harvester, acceleration in axial direction, magnitude and frequency of excitation respectively.

3.4 Results and Discussion

In this section the numerical results for the parametrically excited energy harvester are discussed using the instability plot, time response, frequency response and forced response of the system for principal parametric resonance condition. The geometric and material properties considered in this work are given in Table 3.1. While the substrate properties are similar to the experimental work of Zavodney and Nayfeh [182] and theoretical work of Kar and Dwivedy [12] the piezoelectric properties are

taken similar to Chen and Jiang [244]. Using the Jacobian matrix $[J]$ (Eq.(3.28))

TABLE 3.1: Geometric and material properties of beam with piezoelectric patch

Property	Piezo Patch	Substrate
Young's modulus, GPa	$E_p = 15.86$	$E_b = 190$
Density kg m^{-3}	$\rho_p = 5440$	$\rho_b = 7800$
Length, m	$L_p = 12.54 \times 10^{-2}$	$L_b = 12.54 \times 10^{-2}$
Width, m	$b_p = 1.12 \times 10^{-2}$	$b_b = 1.22 \times 10^{-2}$
Thickness, m	$t_p = 1.25 \times 10^{-4}$	$t_b = 3.7 \times 10^{-4}$
Permittivity nFm^{-1}	$\hat{e} = 19.36$	—
Piezoelectric constant Cm^{-2}	$e_{31} = -19.84$	—

the transition or the instability curve (Fig. 3.2) has been obtained by finding the eigenvalues for the trivial state that is when, $p_1 = q_1 = p_2 = q_2 = 0$. Figure 3.2(a)

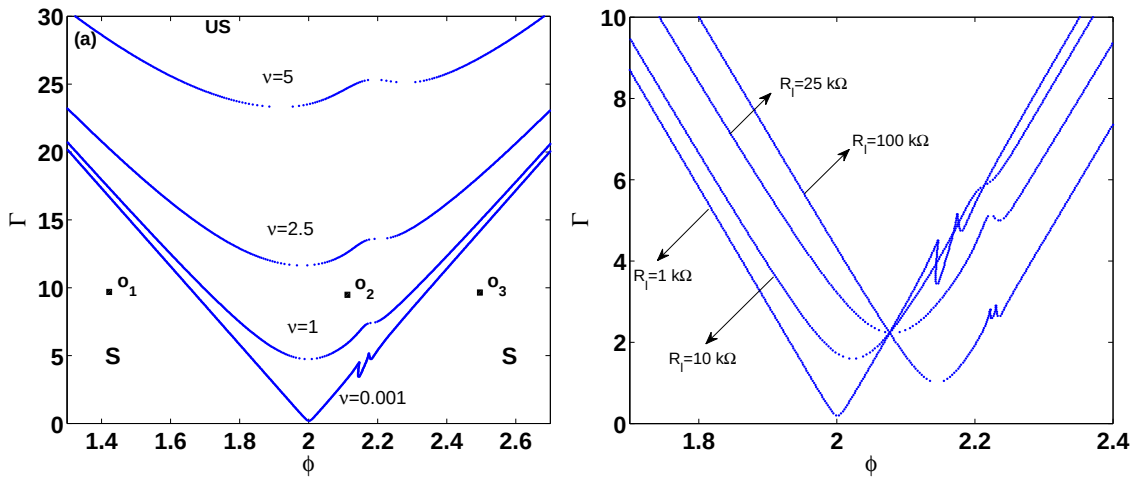


FIGURE 3.2: Trivial state instability regions for (a) $\nu = 0.001, 1, 2.5, 5$ for $R_l = 1 \text{ k}\Omega$ (b) $\nu = 0.001$ with variation of load resistance R_l .

shows the trivial state instability boundary for different nondimensional damping parameters $\nu = 0.001, 1, 2.5$ and 5 which is plotted in the nondimensional amplitude of external excitation (Γ) and nondimensional external frequency (ϕ) plane. Inside the region the system is unstable (US) and outside it is stable (S). It may be observed that as the forcing amplitude (Γ) increases the instability region also increases. It is further observed that due to presence of internal resonance alternate stable and unstable regions are observed for a particular value of (Γ). This stable and unstable zigzag zone gets diminished for higher damping parameters. It may clearly be observed that for higher values of damping parameter the system remains

stable for all frequency range below a critical value of forcing parameter (Γ). For example in case of $\nu = 1$ and 2.5, the critical values of $\phi = 2$ are $\Gamma_{crit} = 4.75$ and 11.7, respectively, below which the system is stable for all values of frequencies of external excitation, $R_l = 10 \text{ k}\Omega$. Hence for energy harvesting, the system must be operated above Γ_{crit} value so that the system may have nontrivial fixed point, periodic, quasiperiodic or chaotic responses, which will be studied in terms of the frequency and force response curves.

Figure 3.2(b) shows the trivial state boundary for damping parameter $\nu = 0.001$ with variation in load resistance from 1 k Ω to 100 k Ω . Here it may be observed that with increase in load resistance the stability boundary shifts towards right (higher values of ϕ). Initially Γ_{crit} increases with increase in R_l but for higher values of R_l it decreases. So there must be a critical value of R_l for which Γ_{crit} will be optimum. Or in other words for every load resistance the energy harvester should be operated above a critical value of Γ . For example $\nu = 0.001$ the $\Gamma_{crit} = 1.6$ (for $R_l = 10 \text{ k}\Omega$) and 1.05 (for $R_l = 100 \text{ k}\Omega$) below which there will be no vibration and hence no energy harvesting.

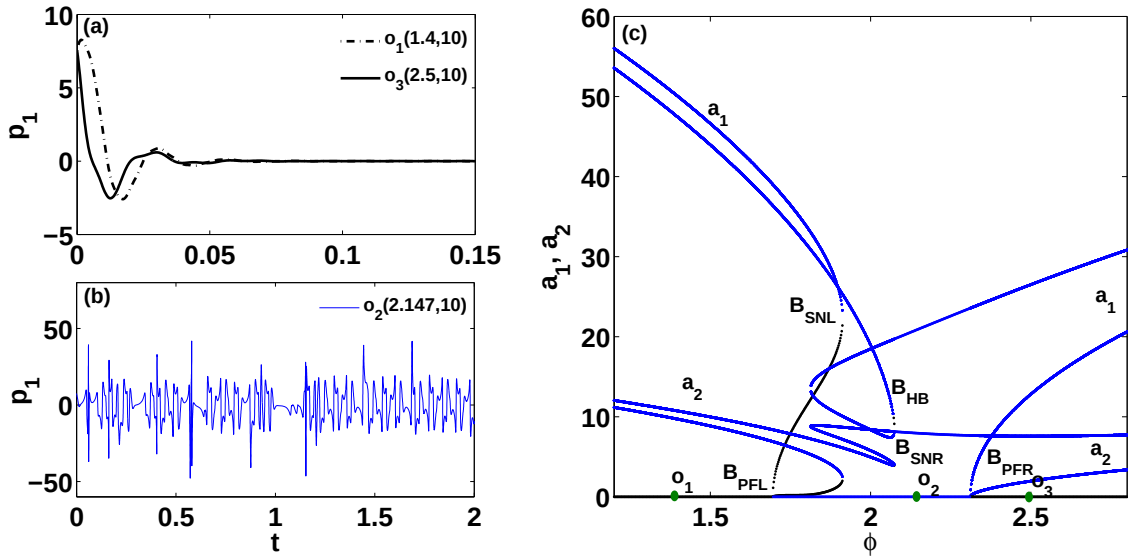


FIGURE 3.3: Time response corresponds to points (a) $o_1(1.4, 10)$, $o_3(2.5, 10)$ (b) $o_2(2.147, 10)$ (c) Frequency response for parameters $\nu = 1$, $\Gamma = 10$, $R_l = 1 \text{ k}\Omega$ as shown in Fig. 3.2.

To verify the instability region of the system the time responses have been plotted (Fig. 3.3) corresponding to 3 distinct values of ϕ (points o_1 , o_2 and o_3) as shown in

Fig. 3.2(a). Clearly, corresponding to the point o_1 ($\phi = 1.4$, $\Gamma = 10$) and o_3 ($\phi = 2.5$, $\Gamma = 10$) the system response is bounded and hence stable (Fig. 3.3(a)) and for point o_2 ($\phi = 2.147$, $\Gamma = 10$) the system is trivially unstable (Fig. 3.3(b)). Figure 3.3(c) shows the corresponding frequency response curve where point o_1 and o_3 are stable trivial fixed point response but point o_2 comes out to be unstable nontrivial response.

Further in Fig. 3.3(c), it may be observed that the trivial state instability region lies in between the super critical pitchfork bifurcation point, PFB, (B_{PFL} , $\phi = 1.694$) and sub critical pitchfork bifurcation point, PFB, (B_{PFR} , $\phi = 2.310$) from where nontrivial branches emanates. The stable nontrivial branch starting from the supercritical PFB point (B_{PFL}) becomes unstable by a left saddle node bifurcation point, SNB, (B_{SNL} , $\phi = 1.912$) hence to generate more voltage the system may be operated near to the SNB point which is denoted by B_{SNL} . Hopf bifurcation point, HBP, (B_{HB} , $\phi = 2.072$) and right SNB point, (B_{SNR} , $\phi = 2.074$) is also visible near to the positively tuned frequency. Blue and black color shows the unstable and stable fixed point response respectively.

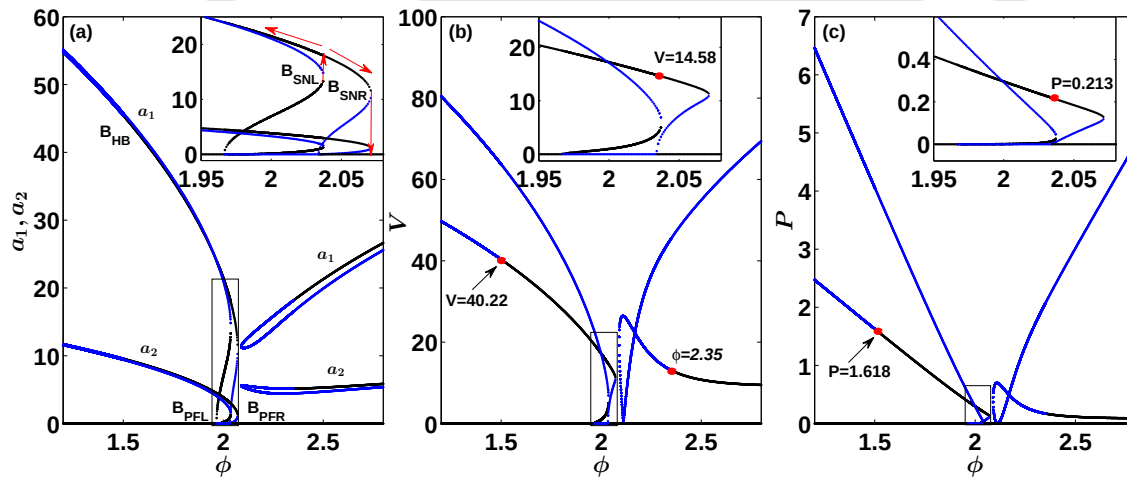


FIGURE 3.4: (a) Frequency response plot (b) variation of voltage with frequency of external excitation (c) variation of power with frequency of external excitation with parameters $\nu = 0.001$, $\Gamma = 1$, $R_l = 1$ k Ω .

The frequency response for modal amplitudes (a_1 , a_2), variation of voltage (V) and power (P) with frequency ratio ϕ for different values of damping parameter viz., $\nu = 0.001$ and $\nu = 0.1$ for $\Gamma = 1$, $R_l = 1$ k Ω are shown in Figs. 3.4 and 3.5, respectively. The trivial state loses stability by supercritical pitchfork bifurcation, PFB (B_{PFL} , $\phi = 1.966$) and a nontrivial branch emanates as shown in Fig. 3.4(a). At saddle node

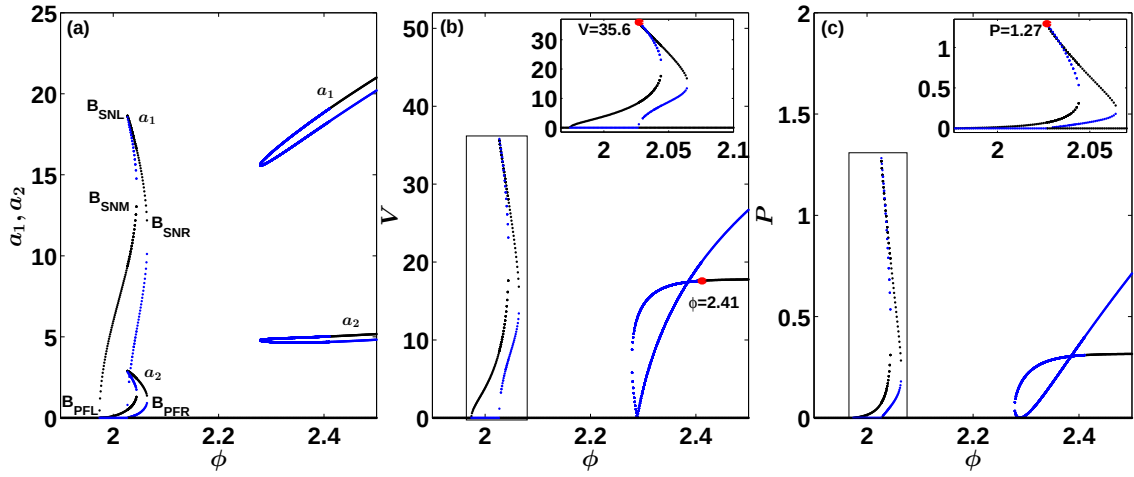


FIGURE 3.5: (a) Frequency response plot (b) variation of voltage with frequency of external excitation (c) variation of power with frequency of external excitation with parameters $\nu = 0.1$, $\Gamma = 1$, $R_l = 1$ k Ω .

bifurcation, SNB (B_{SNL} , $\phi = 2.037$) the nontrivial branch becomes unstable and response jump up to the upper stable branch for frequency sweep up. For frequency sweep down case the response follows the stable branch upto Hopf bifurcation point (B_{HB} , $\phi = 1.503$), below which there exist only stable trivial response. Frequency sweep up leads to the saddle node bifurcation point at (B_{SNR} , $\phi = 2.071$) and jump down phenomenon to trivial stable response occur hence no energy harvesting is possible between $2.071 < \phi < 2.350$. At $\phi = 1.503$ the voltage $V = 40.22$ V and power $P = 1.618$ W is achievable as shown in Fig. 3.4(b) and (c), respectively. At $\phi > 2.35$ there exist both stable trivial and nontrivial branch of solutions and as per initial conditions the system follow any of the stable branch. The trivial state is unstable between $1.966 < \phi < 2.34$ range. With increase in damping Fig. 3.5 shows the minor shift in pitchfork bifurcation points occur where supercritical and subcritical PFB points lies at (B_{PFL} , $\phi = 1.973$) and (B_{PFR} , $\phi = 2.027$) respectively. The trivial state become unstable between $1.973 < \phi < 2.027$ range. While one may generate voltage of 17.61 V near SNB point (B_{SNM} , $\phi = 2.044$) but this value may be increased to 35.6 V (as shown in Fig. 3.5(b)) after a jump up phenomenon at the SNB point (B_{SNM}) by sweeping down the frequency ϕ up to left SNB point (B_{SNL} , $\phi = 2.027$). The Hopf bifurcation in Fig. 3.4 becomes the left SNB point (B_{SNL} , $\phi = 2.027$) at higher damping. one may note that by frequency sweep up the voltage and the power will decrease up to right SNB point (B_{SNR} , $\phi = 2.064$). Hence as per the requirement of the voltage and power generation the user can operate the system

between B_{PFL} and B_{SNR} points. Similar to the previous case here also at $\phi > 2.41$ there exist both stable trivial and nontrivial branch of solutions.

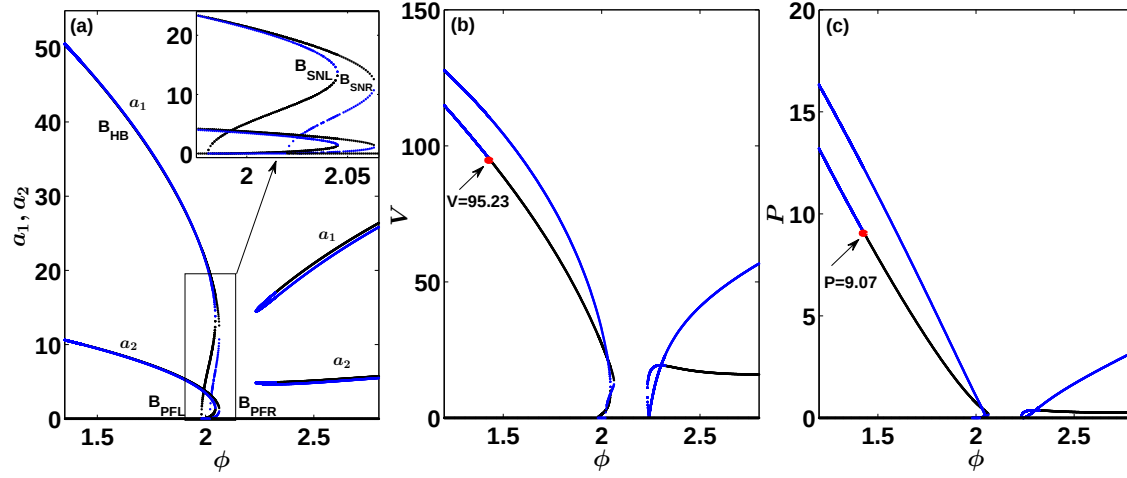


FIGURE 3.6: (a) Frequency response plot (b) variation of voltage with frequency of external excitation (c) variation of power with frequency of external excitation with parameters $\nu = 0.001$, $\Gamma = 0.6$, $R_l = 1$ k Ω .

Keeping the system parameters same as in Fig. 3.4, except the amplitude of base excitation Γ which is now reduced to $\Gamma = 0.6$, Fig. 3.6 shows the frequency response, variation of voltage and power with frequency of base excitation ϕ . It may clearly be observed that the trivial state of the multi-branched response is unstable between $1.98 < \phi < 2.02$ (Fig. 3.6(a)) and the nontrivial state of fixed point response emanates from supercritical pitchfork bifurcation points at $\phi = 1.98$ (PFB, B_{PFL}) and $\phi = 2.02$ (PFB, B_{PFR}). Multi-branch stable and unstable fixed point response with SNB (B_{SNL} , $\phi = 2.045$) and Hopf bifurcation (B_{HB} , $\phi = 1.429$) ensures the jump and hysteresis phenomena. For frequency sweep up from PFB point, $\phi = 1.98$ (B_{PFL}) the system response follows the stable branch up to SNB point (B_{SNL}) and then with further increase in ϕ leads to the jump up phenomenon and the response follows the upper stable branch. Sweeping down the frequency from SNB point (B_{SNL}) increases the response amplitude further upto the Hopf bifurcation point, (B_{HB} , $\phi = 1.429$) and slight reduction in ϕ leads to jump down phenomenon and response reaches the stable trivial state. While frequency sweep up reduces the amplitude along the stable branch up to the SNB point (B_{SNR} , $\phi = 2.063$) as shown in Fig. 3.6(a). Hence energy harvesting for $\phi < 1.429$ and $\phi > 2.063$ is not possible. From Fig. 3.6(b) and Fig. 3.6(c) it may be observed that the maximum voltage of 95.23 V and power of 9.07 W can be obtained at B_{HB} point. It has been observed that to reach the

HB point one may follow two paths viz., either by sweeping up the frequency from B_{PFL} to B_{SNL} and then sweeping down the frequency to reach HB point (B_{HB}) or by sweeping down the frequency from SNB point (B_{SNR}) to reach the HB point (B_{HB}).

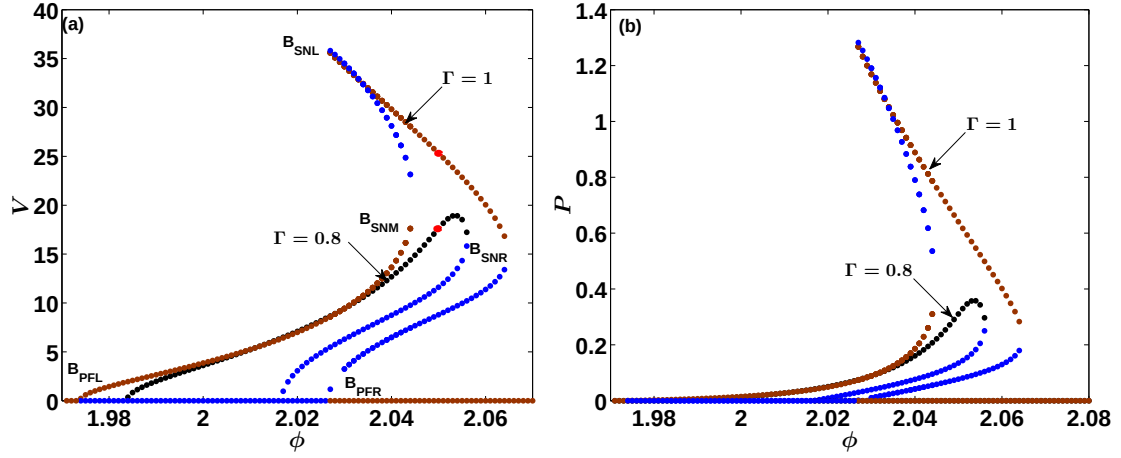


FIGURE 3.7: (a) Variation of voltage with frequency of external excitation (b) variation of power with frequency of external excitation with parameters $\nu = 0.1$, $\Gamma = 0.8$ and 1 , $R_l = 1 \text{ k}\Omega$.

Figure 3.7 shows the frequency response for variation of voltage (V) and power (P) with ϕ for higher values of damping (for example $\nu = 0.1$), amplitude of forcing ($\Gamma = 0.8, 1$) and load resistance ($R_l = 1 \text{ k}\Omega$). The topology of the response (Fig. 3.7(a)) remains similar for different amplitude of forcing. But it is observed that the trivial and non trivial instability zone of frequency increases for higher Γ value which is found to be $1.984 < \phi < 2.017$ and $1.974 < \phi < 2.026$ for $\Gamma = 0.8$ and $\Gamma = 1$, respectively. The maximum voltage of 35.6 V and power of 1.27 W can be obtained at B_{SNL} ($\phi = 2.027$) as shown in Fig. 3.7(a) and Fig. 3.7(b) respectively. At a particular value of ϕ ($\phi = 2.05$) the corresponding voltages and power are 17.62 V ($\Gamma = 0.8$), 25.32 V ($\Gamma = 1$) and 0.31 W ($\Gamma = 0.8$), 0.64 W ($\Gamma = 1$), respectively.

It may be observed that all the previous responses have been plotted by considering first two modes (a_1, a_2). To study the influence of internal resonance one may compare these results with those obtained by single mode approximation (only considering a_1). The results for single mode can be obtained by substituting $a_2 = \gamma_2 = 0$ in the obtained reduced equations which are found to be in good agreement with those in the work of Zavodney and Nayfeh [182] in absence of piezoelectric patches. Figure 3.8 shows the variation of voltage (V) with frequency of excitation, ϕ for the

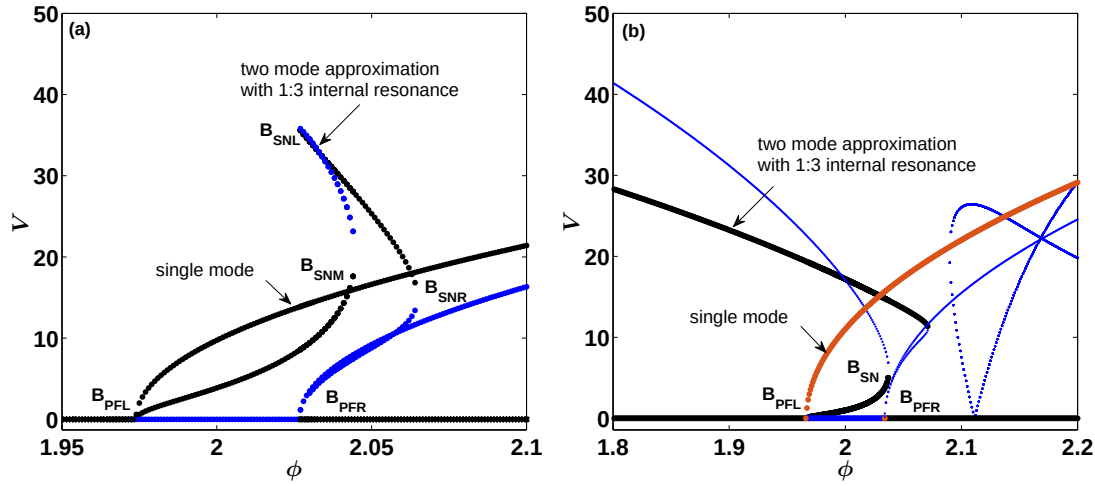


FIGURE 3.8: Variation of voltage with frequency for single mode and two modes with internal resonance for system parameters as (a) $\nu = 0.1$, $\Gamma = 1$, $R_l = 1 \text{ k}\Omega$ and (b) $\nu = 0.001$, $\Gamma = 1$, $R_l = 1 \text{ k}\Omega$.

system with single mode and two modes (with 1:3 internal resonance) approximations. It may be observed from Fig. 3.8(a) that in case of both single and two mode approximations the trivial state loses its stability at PFB point (B_{PFL} , $\phi = 1.974$), from where the nontrivial stable branch emanates. While in case of single mode approximation there is no bifurcation in the upper stable branch, in case of two mode approximation with internal resonance the system suffers three saddle node bifurcations namely B_{SNL} ($\phi = 2.027$), B_{SNM} ($\phi = 2.044$) and B_{SNR} ($\phi = 2.064$). For the range of frequency ($2.027 < \phi < 2.064$) the system with two modes (with internal resonance) gives more voltage as compared to the single mode approximation as shown in Fig. 3.8(a).

From Fig. 3.8(b) it may clearly be observed that for same loading condition ($R_l = 1 \text{ k}\Omega$) by reducing the damping and amplitude of external excitation one may generate $V = 40.22 \text{ V}$ (Fig. 3.4(b)) corresponding to the Hopf bifurcation point (B_{HB} , $\phi = 1.503$) considering two mode approximation. For $\phi > B_{SNR}$ ($\phi = 2.071$) the energy harvesting is not possible for system with two mode approximation. The single and two mode branches crosses at $\phi = 2.029$ where voltage is found to be $V = 15.06 \text{ V}$. For the frequency range of $1.503 < \phi < 2.029$ the system with two mode approximation is advantageous as it gives more voltage as compared to the system with single mode approximation. But for higher frequency (beyond $\phi > 2.029$) the system with single mode is useful as compared to the two mode. The system with

internal resonance is useful in case of frequency sweep down (Fig. 3.8(b)) while system with single mode approximation is advantageous in frequency sweep up beyond $\phi > 2.029$. Hence by adjusting the modes suitably a wideband of frequencies can be utilized for energy harvesting purpose. This can be done by adjusting the position of the attached mass.

TABLE 3.2: Bifurcation points on fixed point response curves

ν	B_{PFL}	B_{PFR}	B_{SNM}	B_{HB}	B_{SNL}	B_{SNR}	$P_{\max}(W)$	Γ
0.001	1.966	2.034	—	1.503	2.037	2.071	$1.618(\phi = 1.503)$	1
0.1	1.973	2.027	2.04	—	2.027	2.064	$1.27(\phi = 2.027)$	1
0.001	1.98	2.02	—	1.429	2.045	2.063	$9.07(\phi = 1.429)$	0.6

Table 3.2 summarizes the critical bifurcation points, pitchfork (B_{PFL} , B_{PFR}), saddle node (SNB, B_{SNL} , B_{SNM} , B_{SNR}), Hopf bifurcation points (HB, B_{HB}) for different values of damping and amplitude of forcing parameters for $R_l = 1 \text{ k}\Omega$. Here L, M and R denotes the left, middle and right, respectively. From this table it is observed that a maximum power of 9.07 W is achievable with $\nu = 0.001$, $\Gamma = 0.6$. Here, IR, Th,

TABLE 3.3: Comparison of piezoelectric energy harvesters

Ref.	IR Th Exp	Piezo	$\sim V_{\text{Peak}}$	$\sim P_{\text{Peak}}$	Dim ($L \times b \times t$)
Garg and Dwivedy [301]	N Y Y	MFC	6.5V	—	$145 \times 14 \times 0.51$
Chen and Jiang [244]	N Y Y	MFC	—	0.06mW	$304.8 \times 31.75 \times 0.51$
Vijayan et al. [71]	N Y N	MFC	—	10mW	$305 \times 16 \times 0.5/0.25$
Chen et al. [237]	Y _{1:2} Y Y	PZT	6V	—	$157.5 \times 14.6 \times 0.51$
Friswell et al. [64]	N Y Y	MFC	—	0.2mW	$300 \times 16 \times 0.254$
Harne et al. [179]	Y _{1:2} Y Y	PVDF	2.5V	—	$126.8/73.5$
Xu and Tang [239]	Y _{1:2} Y Y	PZT	5V	—	$115.25 \times 5.28 \times 0.45$
Masana and Daqaq [62]	N Y Y	PZT	—	2mW	$157.5 \times 14.6 \times 0.508$
Abdelkefi et al. [181]	N Y N	PZT	90V	0.08W	$304.8 \times 31.75 \times 0.51$
Present	Y _{1:3} Y N	MFC	95V	9W	$125.4 \times 12.2 \times 0.37$

Exp, Y, N, V_{peak} , P_{peak} denotes the internal resonance, theoretical, experimental, yes, no, peak voltage and peak power respectively.

Table 3.3 compares the performance of peak voltage and power generated of some nonlinear cantilever based piezoelectric energy harvesters. It may be observed that the present work yields better voltage and power than the other systems. References

[179, 237] used two or more beams (as subsystem) to harvest energy with 1:2 internal resonance and Xu and Tang [239] used a cantilever beam with a pendulum where 1:2 internal resonance is considered, the present system considers a single cantilever beam with an arbitrary mass and 1:3 internal resonance.

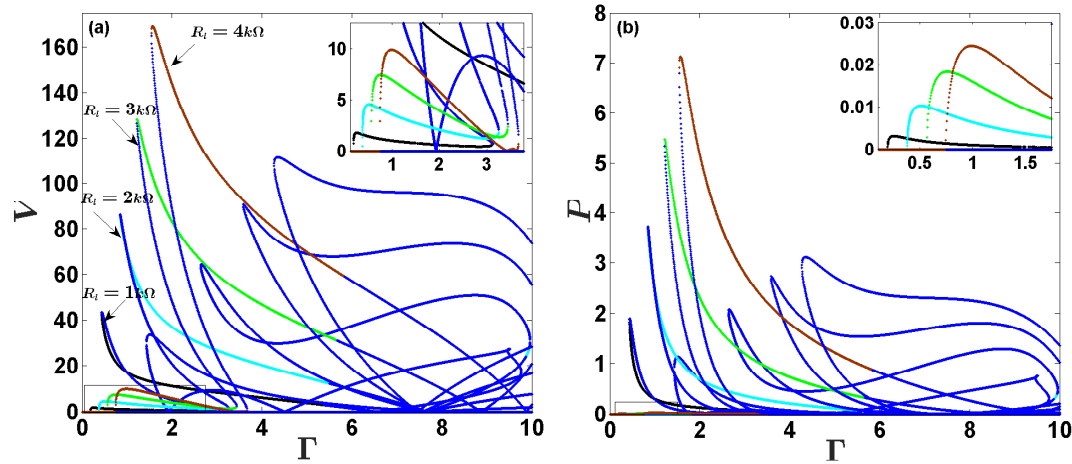


FIGURE 3.9: Variation of Voltage (V) and power (P) with amplitude of base excitation Γ for parameters $\nu = 0.001$, $\phi = 2$, with varying load resistance $R_l = 1, 2, 3, 4\text{ k}\Omega$.

To study the effect of amplitude of base excitation and operative range for energy transduction, the forced vibration response ($\phi = 2$) of the system have been plotted as shown in Figs. 3.9 and 3.10 for different load resistance and damping respectively. In Fig. 3.9 which is plotted for $\nu = 0.001$ and $R_l = 1, 2, 3, 4\text{ k}\Omega$, nontrivial thick lines and thin blue lines represent the stable and unstable branches of the response respectively. Here the multiple stable fixed point responses lose their stability either by saddle node or by Hopf bifurcation and also the pitch bifurcation point and SN bifurcation point shifts towards a higher value of Γ for increased R_l value. For example from Fig. 3.9(b), a power of 25 mW can be obtained at $\Gamma = 1$ and $R_l = 4\text{ k}\Omega$. As R_l increases the power obtained also increases although higher Γ will be required and there exist a critical load resistance R_l at which the power will be higher.

For lower value of R_l one may observe that the voltage and power decreases with slow rate between the pitchfork and saddle node bifurcation points. This observation will be useful for constant voltage and power generation. With increase in Γ beyond SN one can obtain higher power due to existence of two stable branches for a particular

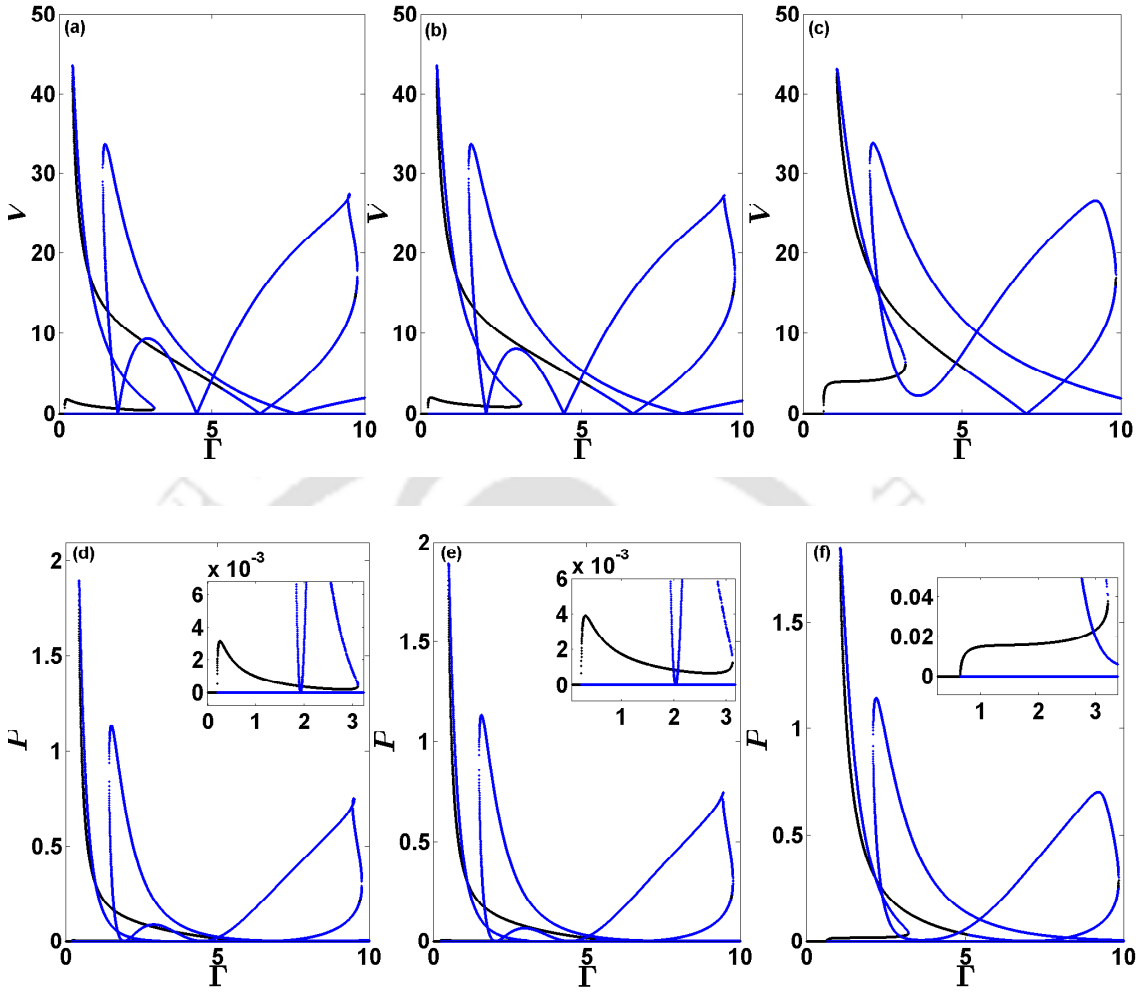


FIGURE 3.10: Variation of voltage (V) and power (P) with amplitude of base excitation Γ for parameters $\nu = 0.001, 0.01, 0.1$ at $\phi = 2$ and load resistance of $R_l = 1 \text{ k}\Omega$.

Γ value. One may further observe that with increase in R_l higher amplitude of response (Γ) is required for energy harvesting which starts at the supercritical PFB point.

Figure 3.10 shows the effect of damping parameter ν on the forced response for $R_l = 1 \text{ k}\Omega$. Higher value of Γ is required for energy harvesting as ν increases as pitchfork bifurcation point shifts towards higher Γ value. The voltage and corresponding power reduces as Γ increases for $\nu = 0.001$ and 0.01 but become stable for higher ν ($\nu = 0.1$). The occurrence of SN bifurcation disappears and nontrivial stable response loses stability at Hopf bifurcation point for higher value of ν ($\nu = 0.1$). It

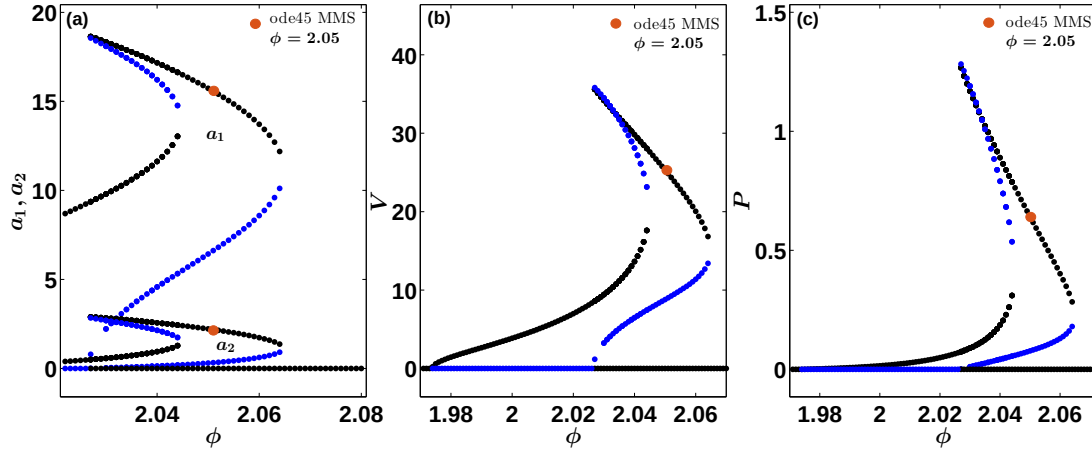


FIGURE 3.11: Time response (using ode45 in MATLAB) of reduced Eqs. (3.20) – (3.23) at $\nu = 0.1$, $\phi = 2.05$, $\Gamma = 1$ and load resistance $R_l = 1 \text{ k}\Omega$.

may be noted that in contrast to the conventional belief that higher damping reduces the amplitude of vibration thereby reducing the voltage and power generation of the harvester, here it is observed that damping helps in generating more voltage and power which is actually due to the presence of internal resonance. Figure 3.11 shows the amplitude, voltage and power response corresponds to the parameters $\nu = 0.1$, $\phi = 2.05$, $\Gamma = 1$ and load resistance $R_l = 1 \text{ k}\Omega$ with time response of reduced equations (Eqs. (3.20) – (3.23)) by MMS. The frequency response matches with the ode time response. The input power corresponds to Fig. 3.11 when $\phi = 2.05$, can be calculated using Eq.(3.32) which is found to be 0.12 W and the corresponding output power is 0.641 W (Fig. 3.11(c)). It may be noted that in case of parametrically excited system the direction of forcing and response takes place in orthogonal direction, so a small input power can be used to produce large output power. Hence the efficiency is quit large unlike the harvester where the forcing and response takes place in the same direction [302] where it is found to be approximately 8 – 12%.

To explore the possibilities of periodic, quasiperiodic and chaotic responses where the fixed point responses are unstable, state space plots (a_1 vs a_2) are shown in Figs. 3.12 – 3.14 for different values of R_l and Γ . One can clearly observe the periodic, quasiperiodic and chaotic responses with variation in load resistance R_l and amplitude of base excitation Γ . It may be noted that the frequency responses shown in Fig. 3.4 – 3.8 and 3.11 corresponds to $\Gamma \leq 1$ which is equivalent to the

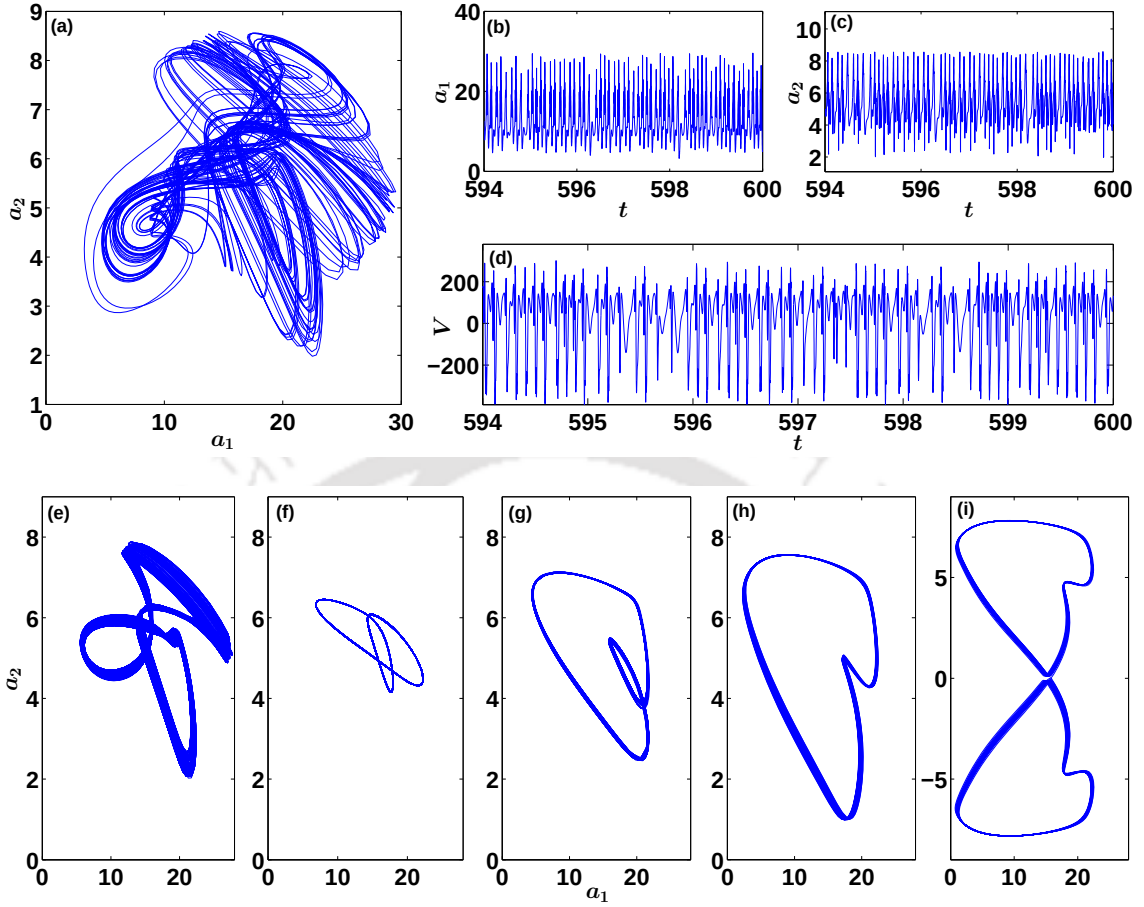


FIGURE 3.12: State space plot for $\nu = 1$, $\phi = 2.120$, $\Gamma = 12$ and variation of load resistance as (a, b, c, d) $R_l = 15 \text{ k}\Omega$ (e) $R_l = 25 \text{ k}\Omega$ (f) $R_l = 50 \text{ k}\Omega$ (g) $R_l = 65 \text{ k}\Omega$ (h) $R_l = 100 \text{ k}\Omega$ (i) $R_l = 150 \text{ k}\Omega$.

amplitude of base excitation near to $\leq 1 \text{ mm}$. This is taken considering the failure of the similar beam in the work of Zavodney and Nayfeh [182] and Garg and Dwivedy [303].

Considering high value of Γ , the time responses are shown in Fig. 3.12 – 3.14. For higher value of Γ near the SNB point, chaotic response is observed which is shown in the state space plot (Fig. 3.12(a)), the corresponding amplitude curves and variation of voltage with time are also plotted in Fig. 3.12(b, c, d). It may be observed that in certain range of time response one may obtain voltage containing several harmonics. By using suitable filter one may generate the required voltage from this chaotic response. At load resistance $R_l = 25 \text{ k}\Omega$ chaotic response disappears and response becomes quasiperiodic (Fig. 3.12(e)). With further increase in R_l the

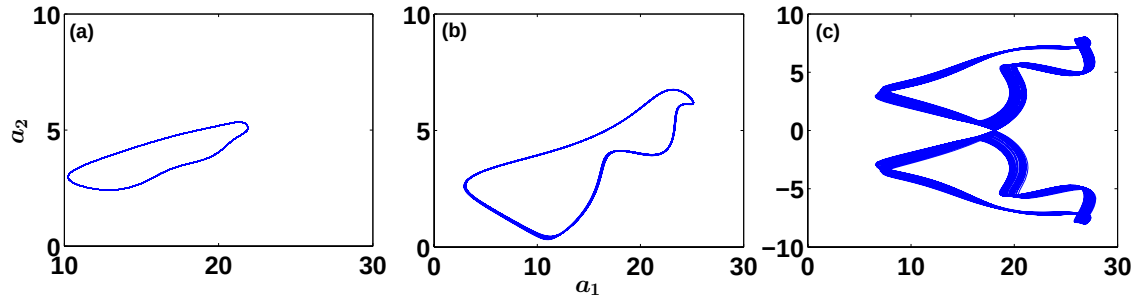


FIGURE 3.13: State space plot for $\nu = 0.001$, $\phi = 2.100$, $R_l = 25 \text{ k}\Omega$ and variation of amplitude of excitation as (a) $\Gamma = 6$ (b) $\Gamma = 8$ (c) $\Gamma = 10$.

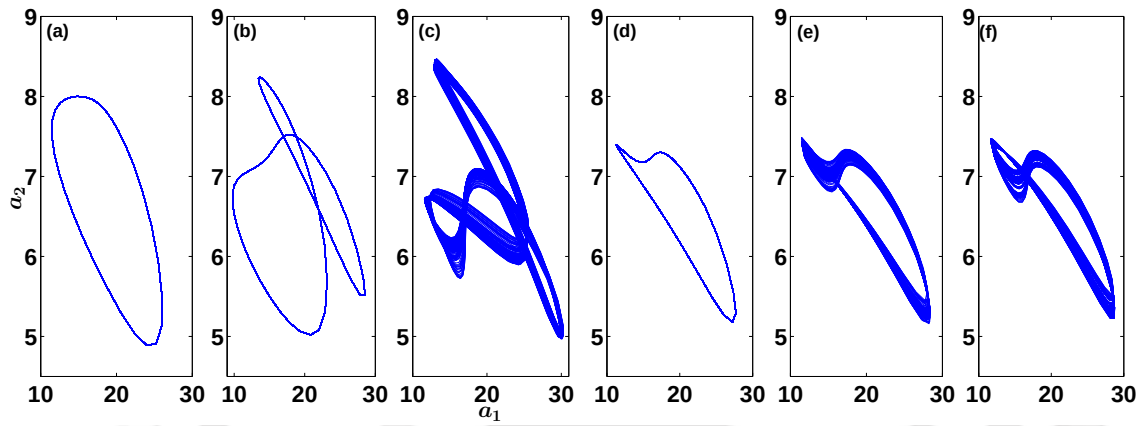


FIGURE 3.14: State space plot for $\nu = 2.5$, $\phi = 2.180$, $\Gamma = 20$ and variation of load resistance as (a) $R_l = 10 \text{ k}\Omega$ (b) $R_l = 25 \text{ k}\Omega$ (c) $R_l = 50 \text{ k}\Omega$ (d) $R_l = 75 \text{ k}\Omega$ (e) $R_l = 100 \text{ k}\Omega$ (f) $R_l = 125 \text{ k}\Omega$.

response becomes periodic containing multi harmonics. For $R_l = 45, 65 \text{ k}\Omega$ the multi harmonic responses are shown in Figs. 3.12(f, g). But with further increase in R_l the response again becomes chaotic and one observe attractor merging crisis for $R_l = 150 \text{ k}\Omega$ which is shown in Fig. 3.12(i).

The state space and corresponding voltage response for low damping (e.g., $\nu = 0.001$) is shown in Fig. 3.13 for $\phi = 2.100$, $R_l = 25 \text{ k}\Omega$ and $\Gamma = 6, 8, 10$. As Γ increases the periodic time response for ($\nu = 0.001$, $\Gamma = 4$, $R_l = 25 \text{ k}\Omega$) gets distorted due to presence of harmonics and finally chaotic and attractor merging crisis is evident as shown in Fig. 3.13(c). The state space for higher damping ($\nu = 2.5$) with variation in R_l is shown in Fig. 3.14. The response of the system changes from periodic (Fig. 3.14(a) to quasiperiodic (Fig. 3.14(b, c)) with multi harmonics and then again it becomes periodic (Fig. 3.14(d–f)) with multi harmonics as R_l increases.

3.5 Summary

In the present work dynamic analysis of an internal resonance based energy harvester has been carried out. The system is modeled as a harmonically base excited vertical cantilever beam with piezoelectric bimorph along with an attached mass at an arbitrary position such that the system exhibits 1:3 internal resonance. The electro-mechanically coupled nonlinear governing equation of motion has been derived and the steady state amplitude, voltage and power response of the system is obtained by using Method of multiple scales (MMS) for principal parametric resonance condition.

It may be noted that by adjusting the location of the attached mass the frequency of the system can be tuned and other internal resonance conditions can be utilized. From the trivial state instability plot it has been observed that due to the presence of internal resonance one may obtain a wide range of frequency for energy harvesting as the system response become stable nontrivial fixed point, periodic, quasiperiodic and chaotic.

The effect of system parameters such as damping, the amplitude of excitation and load resistance on the system response has been analyzed by plotting the time response, state space and frequency responses. Critical bifurcation points such as saddle node, pitch-fork and Hopf bifurcations have been found which will help the designer to choose the system parameters to have optimum voltage or power generation. For example in the present case one may obtain a maximum voltage of 95.23 V and power of 9.07 W (for, $\nu = 0.001$, $\Gamma = 0.6$, $R_l = 1 \text{ k}\Omega$) which are significantly higher than those obtained by researchers working in this field.

Though the analysis has been carried out for principle parametric resonance condition only, other resonance conditions such as combination parametric, subharmonic and superharmonic resonance may be investigated to get broadband of frequency for energy harvesting.



Chapter 4

Piezoelectric Energy Harvester under Combination Parametric and 1:3 Internal Resonance

4.1 Introduction

In this work, nonlinear dynamics of a cantilever beam based PEH system under combination parametric resonance is considered where the frequency of external excitation is nearly equal to the sum of the first two mode natural frequencies of the system $\phi = \omega_1 + \omega_2$. Further, the attached mass is arranged in such way that the second mode natural frequency nearly equals 3 times the first mode frequency, giving rise to 1:3 internal resonance. In section 4.2 the mathematical modeling is carried out for deriving the governing nonlinear equation of motion which is solved by using the method of multiple scales as presented in section 4.3. The results and discussion are presented in section 4.4. In section 4.5, parametric studies have been carried out to determine the system parameters for achieving 1:3 internal resonance. The stability analysis of the present system is performed by perturbing the non autonomous equations.

4.2 Modeling and Governing Equation

Figure 4.1 shows a vertical slender cantilever beam based energy harvester having length L , attached mass m at an arbitrary position d from the base with bimorph piezoelectric patches is subjected to a harmonic base excitation $z(t) = z_0 \cos(\Omega t)$. Considering an arbitrary point along the longitudinal direction of the beam, at a curvilinear distance s from the base, the transverse and axial displacements are expressed as $v(s, t)$ and $u(s, t)$, respectively. Due to large deflection, the displacement-curvature relation of the beam is considered as nonlinear. The Hamilton's principle $\int_{t_1}^{t_2} (\delta T - \delta U + \delta W_{nc}) dt$ is used to find the governing equation of motion of the beam based energy harvester. The Kinetic energy (T) and Potential energy (U) of the harvester are expressed is:

$$T = \frac{1}{2} \int_0^L [\rho + m\delta(s-d)] (\dot{u}(s, t)^2 + \dot{v}(s, t)^2) ds + \frac{1}{2} J_0 \dot{\varphi}(d)^2 \quad (4.1)$$

$$U = \frac{1}{2} \int_0^L EI \varphi'(s)^2 ds - \int_0^L [\rho + m\delta(s-d)] g u(s, t) ds \quad (4.2)$$

Similar to the work of Zavodney and Nayfeh [182] the relation between the lateral and longitudinal displacement is considered to be governed by the inextensibility condition, which can be expressed as follows.

$$u(s, t) = \frac{1}{2} \int_0^s \{v'(\xi, t)\}^2 d\xi + z(t) \quad (4.3)$$

By observing the deflected beam geometry the relation between the angle of rotation $\varphi(s)$ and transverse deflection $v(s, t)$ is defined as $\sin \varphi = v'$ and $\cos \varphi = (1 - v'^2)^{1/2}$. As per the Euler-Bernoulli beam theory the curvature $\kappa(s, t)$ is

$$\kappa(s, t) = \frac{\partial \varphi(s)}{\partial s} = \frac{v''}{\cos \varphi} = \frac{v''}{\sqrt{1 - v'^2}} \approx v'' \left(1 + \frac{1}{2} v'^2\right) \quad (4.4)$$

The non-conservative virtual work done W_{nc} involves the work done in dissipation of energy by viscous force W_d and electrical forces W_{ele} [176] due to piezoelectric patches attached with the beam system. The virtual work done W_{nc} ($W_{nc} = W_d + W_{ele}$) is

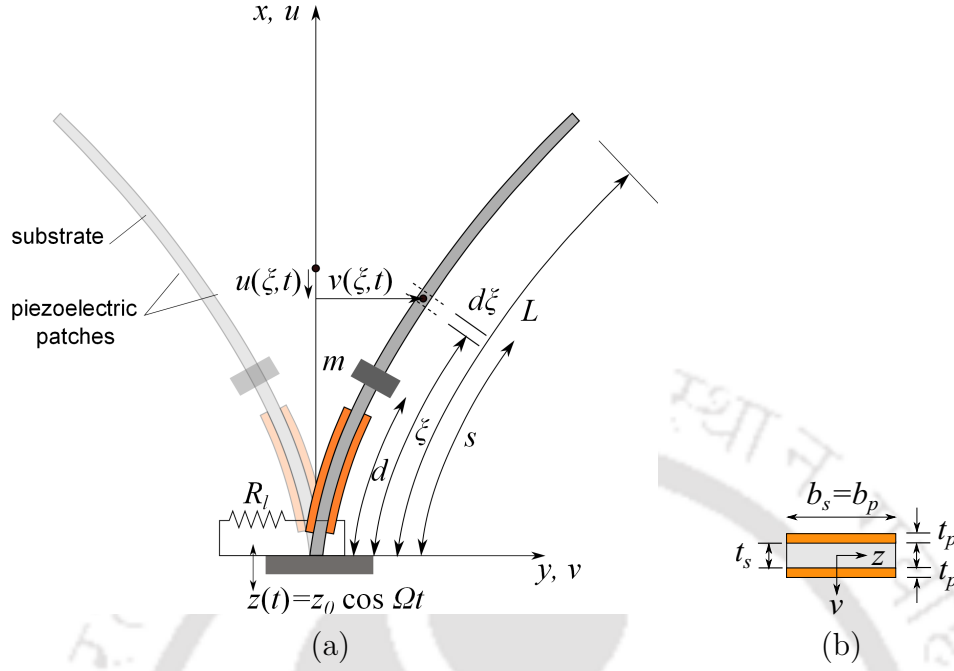


FIGURE 4.1: (a) Schematic diagram of the piezoelectric based energy harvester, (b) cross section

expressed as follows.

$$\delta W_{nc} = - \int_0^L c \dot{v}(s, t) \delta v ds - \int_0^L [H(s - L_1) - H(s - L_2)] M_{ele}(s, t) \delta \kappa(s) ds \quad (4.5)$$

The total moment M_{ele} due to piezoelectric patches is expressed as

$$M_{ele} = \int_{-h_2}^{-h_1} d_{31} E_p b_p E_e y dy + \int_{h_1}^{h_2} d_{31} E_p b_p (-E_e) y dy$$

Here

$$E_e = \frac{\bar{V}(t)}{t_p}; \quad h_1 = \frac{1}{2} h_s - \bar{y}; \quad h_2 = \frac{1}{2} h_s + h_p - \bar{y}; \quad \bar{y} = \frac{E_p t_p b_p (t_p + t_b)}{2(E_p t_p b_p + E_b t_b b_b)}$$

The Hamilton's principle $\int_{t_1}^{t_2} (\delta T - \delta \Pi + \delta W_{nc}) dt$ is used to find the governing equation of motion of the beam based energy harvester. By following the work of Zavodney and Nayfeh [182], Erturk and Inman [176] and Kar and Dwivedy [12], the

nonlinear coupled electromechanical governing equation of motions which represents the dynamics of the PEH can be expressed as follows.

$$\rho^* v_{tt} + cv_t + EI^* [v_{ssss} + \{v_s(v_s v_{ss})_s\}_s] - [J^*(v_s)_{tt}]_s - (Nv_s)_s - H_{ss}\eta\bar{V} = 0 \quad (4.6)$$

Here,

$$\begin{aligned} \rho^* &= \rho + m\delta(s-d), \quad \rho = \rho_b A_b + 2H\rho_p A_p, \quad EI^* = E_b I_b + 2HE_p I_p, \\ J^* &= J_0\delta(s-d), \quad \eta = \frac{e_{31}b_p}{t_p} \left[(t_p + \frac{1}{2}t_b)^2 - \frac{1}{4}t_b^2 \right], \quad A_b = b_b t_b, \quad A_p = b_p t_p, \\ H &= H(s-L_1) - H(s-L_2), \quad e_{31} = d_{31}E_p, \quad I_p = \frac{1}{3}b_p \left[(t_p + \frac{1}{2}t_b)^3 - \frac{1}{8}t_b^3 \right], \\ I_b &= \frac{1}{12}b_b t_b^3, \quad L_p = L_2 - L_1 \\ N &= \frac{1}{2} \int_s^L \rho^* \int_0^\xi (v_s^2)_{tt} d\eta d\xi + (z_{tt} - g) \int_s^L \rho^*(\xi) d\xi - J_0\delta(\xi-d) \left\{ \frac{1}{2}v_{stt}v_s^2 + v_s v_{st}^2 \right\} \end{aligned}$$

The subscript 'p' is used to denote terms related to PZT and $(\cdot)_s$, $(\cdot)_t$ denotes the differentiation with respect to curvilinear coordinate 's' i.e., $\frac{d(\cdot)}{ds}$ and time $\frac{d(\cdot)}{dt}$, respectively. Term N denotes the longitudinal load at 's'.

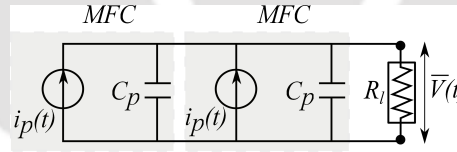


FIGURE 4.2: Electric circuit in parallel connection

The electric circuit equation in bimorph configuration (with parallel connection, Fig. 4.2) can be obtain by applying the Gauss's law [170, 176] as follows.

$$\frac{dQ(t)}{dt} = \frac{d}{dt} \int_{A_p} \mathbf{D} \cdot \mathbf{n} dA_p \quad (4.7)$$

Here $\mathbf{D}$ and $\mathbf{n}$ denote the electric displacement vector and outward normal to the surface. The electric displacement vector ($\mathbf{D}$) in terms of axial stress (σ_p) and generated electric field (E_e) in the present case is defined by the following constitutive

relation.

$$D_3 = d_{31}\sigma_p + \hat{e} E_e \quad (4.8)$$

Here $\sigma_p = E_p \epsilon_p - e_{31} E_e$, $\epsilon_p = -y\kappa(s, t)$

Using Eq. (4.8) into Eq. (4.7) the circuit equation becomes as follows.

$$C_p \bar{V}_t + \frac{1}{2R_l} \bar{V} - i(t) = 0 \quad (4.9)$$

Here $i(t) = -e_{31} t_{pc} b_p \int_{L_1}^{L_2} (v_{ss})_t ds$, $C_p = \frac{\hat{e} b_p L_p}{t_p}$. The four boundary conditions at fixed and free ends are represented as follows.

$$v(0, t) = 0, \quad v_s(0, t) = 0, \quad v_{ss}(L, t) = 0, \quad v_{sss}(L, t) = 0 \quad (4.10)$$

At the fixed end displacement and slope is zero and at the free end the bending moment and shear forces are zero. By considering the two mode approximation in the Galerkin's method, the transverse displacement $v(s, t)$ is represented by a scaling factor r , time modulation $q_n(t)$ and shape function $\psi_n(s)$ of the beam system as $v(s, t) = \sum_{n=1}^2 r \psi_n(s) q_n(t)$. After nondimensionalizing the nonlinear coupled electromechanical temporal governing equations of motion for two modes ($n = 1, 2$) are as follows.

$$\ddot{q}_n + 2\epsilon\zeta_n \dot{q}_n + \omega_n^2 q_n + \epsilon \sum_{k,l,m=1}^2 \{ \alpha_{klm}^n q_k q_l q_m + \beta_{klm}^n q_k \dot{q}_l \dot{q}_m + \gamma_{klm}^n q_k q_l \ddot{q}_m \} + \epsilon \eta_n V = \epsilon \sum_{m=1}^2 F_{nm}(\tau) q_m \quad (4.11)$$

$$\dot{V}(\tau) + \chi V(\tau) + \left(\sum_{n=1}^2 \frac{e_{31} t_{pc} b_p}{L} \frac{r}{C_p r_v} \int_0^1 \psi_{nxx}(x) dx \right) \dot{q}_n = 0 \quad (4.12)$$

Here

$$\begin{aligned}
 x &= \frac{s}{L}, \quad \beta = \frac{d}{L}, \quad \tau = \theta_1 t, \quad \omega_n = \frac{\theta_n}{\theta_1}, \quad \lambda = \frac{r}{L}, \quad \mu = \frac{m}{\rho_b A_b L}, \quad \Gamma = \frac{z_0}{z_r}, \quad J = \frac{J_0}{\rho_b A_b L r^2}, \\
 \phi &= \frac{\Omega}{\theta_1}, \quad \eta_n = \frac{-\eta r_v}{\rho_b A_b h_n \theta_1^2 r \epsilon L^2} \int_0^1 \{\delta_x(x) - \delta_x(x-1)\} \psi_n(x) dx, \quad \zeta_n = \zeta_n^* \frac{\nu}{\epsilon}, \\
 F_{nm}(\tau) &= f_{nm} \cos(\phi \tau), \quad \chi = \frac{1}{2C_p \theta_1 R_l}, \quad \bar{K}_n = \frac{e_{31} t_{pc} b_p}{L} \int_0^1 \psi_{nxx}(x) dx, \\
 K_n &= \frac{\bar{K}_n r}{C_p r_v}, \quad r_v = \frac{\bar{V}}{V}
 \end{aligned}$$

Where $(\dot{}) = \frac{d()}{d\tau}$ and ϵ is the book keeping parameter. Here the damping, geometric and inertial nonlinearities, electromechanical coupling and excitation terms are small, and, hence considered of the order of ϵ . The coefficients appear in Eqs. (4.11) and (4.12) are defined in Appendix B. By observing the last term of Eq. (4.11) the time varying forcing term appears as a coefficient of the response [13] which is a typical characteristic of parametrically excited systems. It may be noted that in the absence of piezo patches with single mode approximation, the equation of motion of the system reduces to that of the model investigated theoretically and experimentally by Zavodney and Nayfeh [182]. Also with two mode approximation the governing equation of motion without piezoelectric patches reduces to that of Kar and Dwivedy [12]. It may be noted that unlike in the previous chapter where moment based approach was adopted to find the equation of motion of the system, here, Hamilton's principle is used to find the governing equations. Also, the present mathematical model accommodates the variation in the position of piezoelectric patches. Similar approach is adopted by Yan and Hajj [208] in the development of autoparametric PEH system where tip mass is used.

4.3 Solution of the Temporal Equation

A uniform first order approximate analytical solution of Eqs. (4.11) and (4.12) is obtained by using the standard method of multiple scales (MMS). It may be noted that in the present case combination parametric resonance is studied, while in different from the work presented in Chapter 3 and in the work of Zavodney and Nayfeh [182], and Kar and Dwivedy [12], principal parametric resonance condition

were discussed. In the work by Dwivedy and Kar [27, 304], the combinational resonance cases were discussed where the dynamic analysis of the beam is studied. However, in the present work, the nonlinear electromechanical system is explored as an energy harvester. Similar to previous chapter the time dependence is expressed into multiple time scales as $T_n = \epsilon^n \tau$; $n = 0, 1, 2, \dots$ and the time derivatives can be $\frac{d}{d\tau} = D_0 + \epsilon D_1 + O(\epsilon^2)$, $\frac{d^2}{d\tau^2} = D_0^2 + 2\epsilon D_0 D_1 + O(\epsilon^2)$ where $D_i = \frac{\partial}{\partial T_i}$. The solution of the system (q_n, V) can be expanded in the following form.

$$q_n(\tau; \epsilon) = q_{n0}(T_0, T_1) + \epsilon q_{n1}(T_0, T_1) + O(\epsilon^2) \quad (4.13)$$

$$V(\tau; \epsilon) = V_0(T_0, T_1) + \epsilon V_1(T_0, T_1) + O(\epsilon^2) \quad (4.14)$$

By substituting Eqs. (4.13) and (4.14) into Eq. (4.11) and further equating the coefficient involving the terms of ϵ^0 and ϵ^1 to zero, yields the following set of equations. $O(\epsilon^0)$:

$$\begin{aligned} D_0^2 q_{n0} + \omega_n^2 q_{n0} &= 0; \\ D_0 V_0 + \chi V_0 &= -K_1 D_0 q_{10} - K_2 D_0 q_{20} \end{aligned} \quad (4.15)$$

$O(\epsilon^1)$:

$$D_0^2 q_{n1} + \omega_n^2 q_{n1} = -2\zeta_n D_0 q_{n0} - 2D_0 D_1 q_{n0} + \sum_{n,m=1}^2 f_{nm} q_{m0} \cos(\phi\tau) - \eta_n V_0 \quad (4.16a)$$

$$- \sum_{klm} (\alpha_{klm}^n q_{k0} q_{l0} q_{m0} + \beta_{klm}^n q_{k0} D_0 q_{l0} D_0 q_{m0} + \gamma_{klm}^n q_{k0} q_{l0} D_0^2 q_{m0})$$

$$D_0 V_1 + \chi V_1 = -D_0 V_0 - K_1 (D_0 q_{11} + D_1 q_{10}) - K_2 (D_0 q_{21} + D_1 q_{20}) \quad (4.16b)$$

The solution of differential equations Eq. (4.15) is expressed as follows.

$$\begin{aligned} q_{n0} &= A_n(T_1) \exp(i\omega_n T_0) + cc, \\ V_0 &= - \sum_{n=1}^2 K_n Z_n A_n(T_1) \exp(i\omega_n T_0) + cc \end{aligned} \quad (4.17)$$

Here cc is the complex conjugate of the preceding terms and A_n is an unknown complex function to be determined later. Using Eq. (4.17) into Eq. (4.16a) the following secular and near secular terms for first two modes ($n = 1, 2$) are obtained

which should be eliminated for finite amplitude response of the system.

$$n = 1 : \quad \frac{1}{2}e^{i(\phi-\omega_2)T_0}f_{12}\bar{A}_2 - Q_1^*A_2\bar{A}_1^2e^{i(\omega_2-2\omega_1)T_0} \\ + e^{i\omega_1T_0} \left[-2i\omega_1(\zeta_1A_1 + A_1') + \eta_1K_1Z_1A_1 - \sum_{j=1}^2\alpha_{e1j}A_j\bar{A}_jA_1 \right] = 0 \quad (4.18)$$

$$n = 2 : \quad \frac{1}{2}e^{i(\phi-\omega_1)T_0}f_{21}\bar{A}_1 - Q_2^*e^{3i\omega_1T_0}A_1^3 \\ + e^{i\omega_2T_0} \left[-2i\omega_2(\zeta_2A_2 + A_2') + \eta_2K_2Z_2A_2 - \sum_{j=1}^2\alpha_{e2j}A_j\bar{A}_jA_2 \right] = 0 \quad (4.19)$$

The above equations Eq.(4.18), (4.19) are obtained by considering a particular case of combination resonance of sum type when the frequency of external excitation is nearly commensurate to the sum of first two natural frequencies of the system i.e., $\phi \approx \omega_1 + \omega_2$. Also 1:3 internal resonance condition is considered here by taking $\omega_2 \approx 3\omega_1$. To express the nearness of ϕ to that of $\omega_1 + \omega_2$ and ω_2 to that of ω_1 the detuning parameters σ_1 and σ_2 are introduced as follows.

$$\omega_2 = 3\omega_1 + \epsilon\sigma_2, \\ \phi = 4\omega_1 + \epsilon\sigma_1 = \omega_1 + \omega_2 + \epsilon(\sigma_1 - \sigma_2) \quad (4.20)$$

Expressing the complex function $A_n(T_1)$ into its polar form as $A_n(T_1) = \frac{1}{2}a_n(T_1)e^{i\vartheta_n(T_1)}$, where amplitude $a_n(T_1)$ and phase $\vartheta_n(T_1)$ are slowly varying function of time scale T_1 . Now by using this polar form in Eq. (4.18) and (4.19) and eliminating the secular term (the coefficient of $e^{i\omega_nT_0}$) to find the bounded solution one obtains the following reduced equations.

$$2\omega_1a_1' = -2\omega_1\zeta_1a_1 + \frac{1}{2}\{f_{12}a_2\sin(\gamma_1 + \gamma_2)\} - \frac{1}{4}Q_1^*a_2a_1^2\sin(3\gamma_1 - \gamma_2) + \Pi_a a_1 \quad (4.21)$$

$$2\omega_1a_1\gamma_1' = 0.5\omega_1a_1\sigma_1 + \frac{1}{2}\{f_{12}a_2\cos(\gamma_1 + \gamma_2)\} \\ - \frac{1}{4}\sum_{j=1}^2\alpha_{e1j}a_j^2a_1 - \frac{1}{4}Q_1^*a_2a_1^2\cos(3\gamma_1 - \gamma_2) + \Pi_b a_1 \quad (4.22)$$

$$2\omega_2a_2' = -2\omega_2\zeta_2a_2 + \frac{1}{2}f_{21}a_1\sin(\gamma_2 + \gamma_1) - \frac{1}{4}Q_2^*a_1^3\sin(\gamma_2 - 3\gamma_1) + \Pi_c a_2 \quad (4.23)$$

$$\begin{aligned}
2\omega_2 a_2 \gamma_2' &= \omega_2 a_2 (1.5\sigma_1 - 2\sigma_2) + \frac{1}{2} f_{21} a_1 \cos(\gamma_2 + \gamma_1) - \frac{1}{4} \sum_{j=1}^2 \alpha_{e2j} a_j^2 a_2 \\
&\quad - \frac{1}{4} Q_2^* a_1^3 \cos(\gamma_2 - 3\gamma_1) + \Pi_d a_2
\end{aligned} \tag{4.24}$$

Here

$$\begin{aligned}
\Pi_a &= \frac{K_1 \eta_1 \chi \omega_1}{(\chi^2 + \omega_1^2)}, \quad \Pi_b = \frac{K_1 \eta_1 \omega_1^2}{(\chi^2 + \omega_1^2)}, \quad \Pi_c = \frac{K_2 \eta_2 \chi \omega_2}{(\chi^2 + \omega_2^2)}, \quad \Pi_d = \frac{K_2 \eta_2 \omega_2^2}{(\chi^2 + \omega_2^2)} \\
Z_n &= \frac{\omega_n^2 + i\chi\omega_n}{(\chi^2 + \omega_n^2)}, \quad \gamma_1 = -\vartheta_1 + 0.25\sigma_1 T_1, \quad \gamma_2 = -\vartheta_2 + 0.75\sigma_1 T_1 - \sigma_2 T_1
\end{aligned}$$

The stability of nontrivial state can be obtained by perturbing the above Eqs. (4.21) – (4.24). Normalizing method is required to find the stability of trivial state, as for the trivial state the perturbed equations will not contain the perturbations γ_n' . By using the transformation $p_i = a_i \cos \gamma_i$; $q_i = a_i \sin \gamma_i$; $i = 1, 2$, the following normalized reduced equations in Cartesian form is obtained.

$$p_1' = \frac{-1}{2\omega_1} \left[\begin{aligned} &2\omega_1 \zeta_1 p_1 + \frac{1}{2} \omega_1 \sigma_1 q_1 - \frac{1}{2} f_{12} q_2 + \frac{1}{4} Q_1^* \{q_2 (q_1^2 - p_1^2) + 2p_1 p_2 q_1\} \\ & - \frac{1}{4} \sum_{j=1}^2 \alpha_{e1j} q_1 (p_j^2 + q_j^2) - \Pi_a p_1 + \Pi_b q_1 \end{aligned} \right] \tag{4.25}$$

$$q_1' = \frac{-1}{2\omega_1} \left[\begin{aligned} &2\omega_1 \zeta_1 q_1 + \frac{1}{2} \omega_1 \sigma_1 p_1 - \frac{1}{2} f_{12} p_2 + \frac{1}{4} Q_1^* \{p_2 (p_1^2 - q_1^2) + 2p_1 q_1 q_2\} \\ & + \frac{1}{4} \sum_{j=1}^2 \alpha_{e1j} p_1 (p_j^2 + q_j^2) - \Pi_b q_1 - \Pi_a p_1 \end{aligned} \right] \tag{4.26}$$

$$p_2' = \frac{-1}{2\omega_2} \left[\begin{aligned} &2\omega_2 \zeta_2 p_2 - \omega_2 (2\sigma_2 - 1.5\sigma_1) q_2 - \frac{1}{2} f_{21} q_1 - \frac{1}{4} Q_2^* \{q_1 (3p_1^2 - q_1^2)\} \\ & - \frac{1}{4} \sum_{j=1}^2 \alpha_{e2j} q_2 (p_j^2 + q_j^2) - \Pi_c p_2 + \Pi_d q_2 \end{aligned} \right] \tag{4.27}$$

$$q_2' = \frac{-1}{2\omega_2} \left[\begin{aligned} &2\omega_2 \zeta_2 q_2 + \omega_2 (2\sigma_2 - 1.5\sigma_1) p_2 - \frac{1}{2} f_{21} p_1 + \frac{1}{4} Q_2^* \{p_1 (p_1^2 - 3q_1^2)\} \\ & + \frac{1}{4} \sum_{j=1}^2 \alpha_{e2j} p_2 (p_j^2 + q_j^2) - \Pi_c q_2 - \Pi_d p_2 \end{aligned} \right] \tag{4.28}$$

Here, Newton's method is used to find the nontrivial fixed point responses of the system. Further to find the stability of the obtained fixed point responses, the above reduced modulation equations Eqs. (4.25) – (4.28) are then perturbed to obtain

the following equations.

$$\{\Delta p'_1, \Delta q'_1, \Delta p'_2, \Delta q'_2\}^T = [J]\{\Delta p_1, \Delta q_1, \Delta p_2, \Delta q_2\}^T \quad (4.29)$$

Where Δ , superscript T and matrix $[J]$ denotes the perturbation to the fixed-point, the transpose and the Jacobian matrix respectively. The stability and bifurcation of the fixed-point response of the system is can be obtained by finding the eigenvalue value of the Jacobian $[J]$. It may be noted that, the solutions having eigenvalues with negative real part are stable solutions and those with positive real parts are unstable solutions. One can obtain the steady-state deflection of the beam v , output voltage V and power P , by using the following expressions.

$$v(s, \tau) = \sum_{n=1}^2 r \psi_n(s) a_{n0} \cos(\omega_n \tau + \vartheta_n) \quad (4.30)$$

$$V(\tau) = \sum_{n=1}^2 r_v K_n \frac{\omega_n}{\sqrt{\chi^2 + \omega_n^2}} a_{n0} \cos(\omega_n \tau + \vartheta_n + \delta_n) \quad (4.31)$$

$$P = \frac{1}{R_l} \left(\sum_{n=1}^2 r_v K_n \frac{\omega_n}{\sqrt{\chi^2 + \omega_n^2}} a_{n0} \right)^2 \quad (4.32)$$

Here $\delta_n = \tan^{-1} \left(\frac{\chi}{\omega_n} \right)$

4.4 Results and Discussions

This section deals with the parametric study of energy harvester for combination parametric and 1:3 internal resonance condition using the reduced equations developed in previous section. The material and geometric properties of substrate and piezoelectric patch considered in this work are given in Table 4.1. The trivial state instability regions are obtained by finding the eigenvalues of the Jacobian matrix $[J]$ [Eq. (4.25) – (4.28)] by substituting $p_1 = p_2 = q_1 = q_2 = 0$. Figure 4.3(a) shows the trivial state instability regions with variation in load resistance (R_l) for damping parameter $\nu = 0.0005$. In this figure, the abbreviation S and US denote the stable and unstable trivial state regions. It may be noted that there exists a critical value of nondimensional amplitude of excitation Γ_{crit} below which for all the frequency range

the trivial state remains stable. Hence, there will be no transverse vibration of the beam system and hence no power generation is possible. Further, it may be observed latter that, in the US region, branches of nontrivial fixed point response and periodic responses are possible and hence this trivial state unstable region can be used for the energy harvesting purpose. As the load resistance (R_l) increases instability regions shifted slightly towards the higher frequency side. For example, the critical value of Γ above which trivial state loses its stability gets increased from $\Gamma = 0.55$ ($\phi = 4.026$ for $R_l = 0.5 \text{ k}\Omega$) to $\Gamma = 3.55$ ($\phi = 4.033$ for $R_l = 10 \text{ k}\Omega$). In Fig. 4.4 the time response for three data points $O_1(\phi = 3.95, \Gamma = 6)$, $O_2(\phi = 4.00, \Gamma = 6)$ and $O_3(\phi = 4.10, \Gamma = 6)$ shows that there is no voltage generation in the region marked with S and only the region marked with US is available for harvesting. Further, it may be noted that for a particular value of Γ , as the unstable range increases with increase in R_l , hence suitable value of R_l may be taken depending on the requirement of the available amplitude and frequency range of the ambient vibration source for the PEH applications. Figure 4.3(b) shows the instability region plot in $\Gamma - \phi$

TABLE 4.1: Material and geometric properties of substrate and piezoelectric patches

Property	Piezo Patch	Substrate
Young's modulus, GPa	$E_p = 15.86$	$E_b = 130$
Density, kg m^{-3}	$\rho_p = 5440$	$\rho_b = 1794$
Length, m	$L_p = 7.6 \times 10^{-2}$	$L_b = 32 \times 10^{-2}$
Width, m	$b_p = 1.8 \times 10^{-2}$	$b_b = 1.8 \times 10^{-2}$
Height, m	$t_p = 1.25 \times 10^{-4}$	$t_b = 2.5 \times 10^{-4}$
Permittivity, nFm^{-1}	$\hat{e} = 19.36$	—
Piezoelectric constant, Cm^{-2}	$e_{31} = -19.84$	—
Subscript	p:piezo	b:beam

plane, with variation in damping parameter $\nu = 0.0005, 0.001, 0.005, 0.01$ for a load resistance of $R_l = 0.5 \text{ k}\Omega$. As the nondimensional damping parameter ν increases the stability region shifts towards higher values of Γ . Hence, similar to the previous case, here with increase in ν , the critical value of Γ above which the PEH can operate becomes higher. In Fig. 4.5, frequency response curves are plotted for the system parameters $\nu = 0.0005, \Gamma = 4, R_l = 0.5 \text{ k}\Omega$ along with variation of voltage and power with respect to the frequency of the excitation ϕ . Trivial ($a_1 = a_2 = 0$) and nontrivial branches of the fixed-point solutions with the stable and unstable points

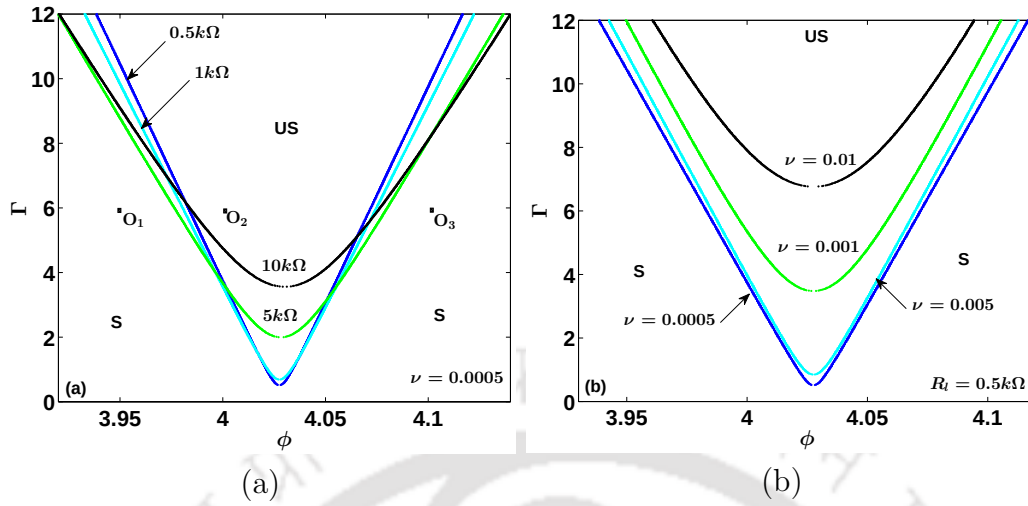


FIGURE 4.3: Instability plot with (a) variation in load resistance $R_l = 0.5, 1, 5, 10 \text{ k}\Omega$ for $\nu = 0.0005$ and (b) variation in damping parameter ν with $R_l = 0.5 \text{ k}\Omega$.

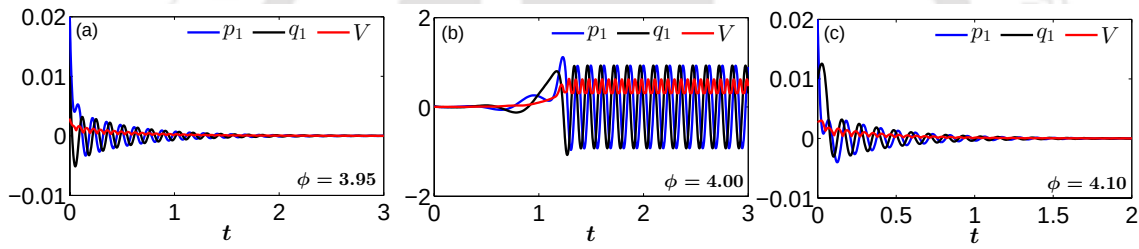


FIGURE 4.4: Time response corresponds to the points $O_1(3.95,6)$, $O_2(4.00,6)$, $O_3(4.10,6)$

(black-stable, blue-unstable) are shown in the figure. It may clearly be observed from Figs. 4.3– 4.5 that the trivial state loses its stability by Hopf bifurcation (left HB_T^L) at $\phi = 3.99$ and remain unstable up to $\phi = 4.057$ (right HB_T^R). Among all the nontrivial branches only two are stable (as shown by black color) and loses its stability by Hopf bifurcation (HB) at $\phi = 3.970$ (left HB_{NT}^L) and $\phi = 4.007$ (right HB_{NT}^R) respectively. If one sweeps down the frequency from $\phi > 4.057$, as the stable trivial state loses its stability at right HB point ($\phi = 4.057$) the response will jump to the nontrivial stable branch. For further sweeping down the frequency ϕ , the nontrivial response remain stable in the considered range and one may observe decrease in amplitude, voltage and power. Further, sweeping up the frequency ϕ , the nontrivial response remains stable and increase in amplitude, voltage and power is observed. For example, for a positively tuned system (i.e., $\sigma_1 > 0$) when $\phi = 4.1$, the voltage and power are 1.8 V and 6.3 mW, respectively. Similarly, for $\phi = 4.2$,

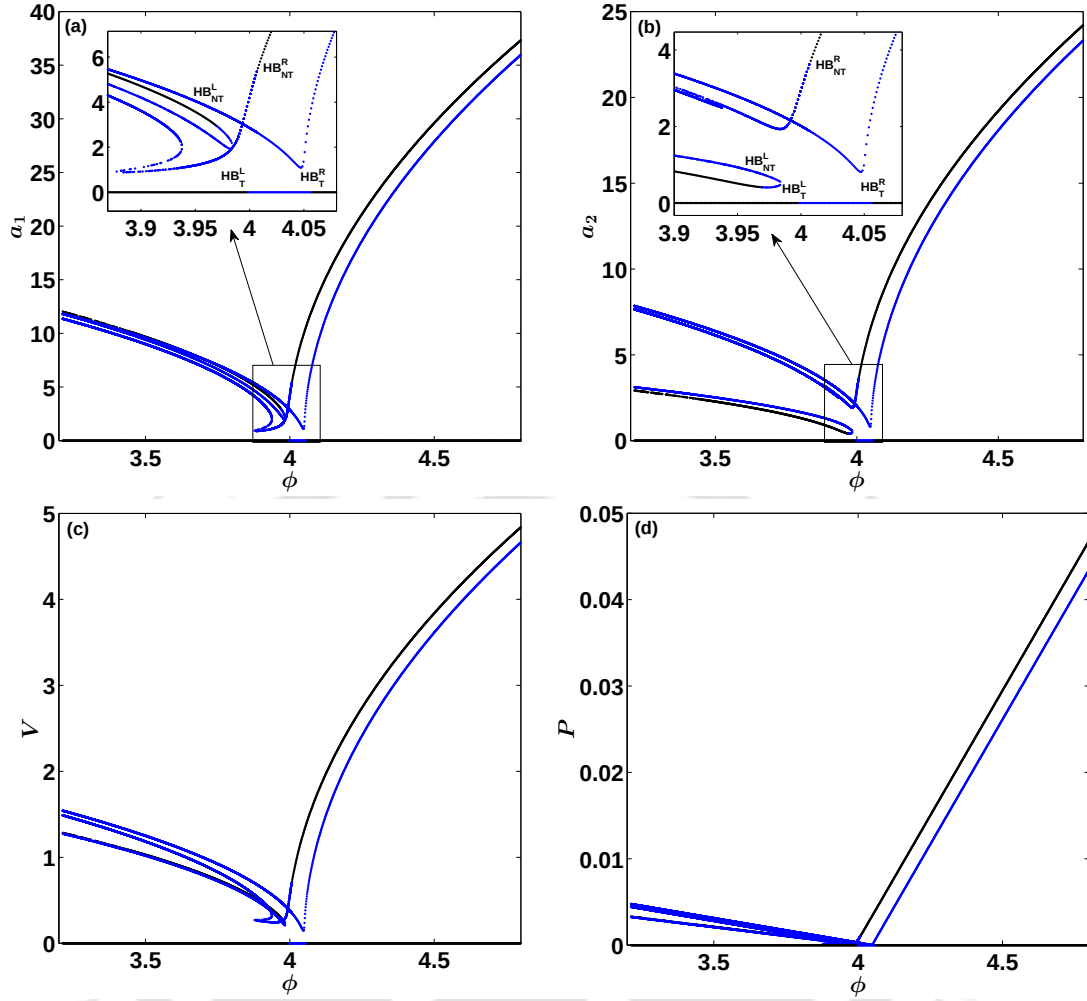


FIGURE 4.5: (a, b) Frequency response plot, variation of (c) voltage and (d) power with frequency; for parameters $\nu = 0.0005$, $\Gamma = 4$ and $R_l = 0.5 \text{ k}\Omega$

the voltage and power increases to 2.46 V and 12.1 mW and for negatively tuned cases such as $\phi = 3.5$, the voltage and power are found to be 1.03 V and 2.08 mW, respectively. So, depending on the applications, the harvester may be operated at a particular frequency to obtain the required voltage and power.

Figure 4.6 shows the variation of voltage (V) and power (P) with frequency ϕ for amplitude of excitation $\Gamma = 2$ and 4. One may observe that when Γ is decreased from 4 to 2, keeping the other parameters (ν, R_l) constant, the stable nontrivial (N-T) branch for $\phi < 4$ disappears. Also, the trivial unstable zone broadens for the higher amplitude of excitation and HB point gets shifted towards negative detuning from $\phi = 4.014$ to $\phi = 4.007$. The voltage output increases for higher Γ value. For

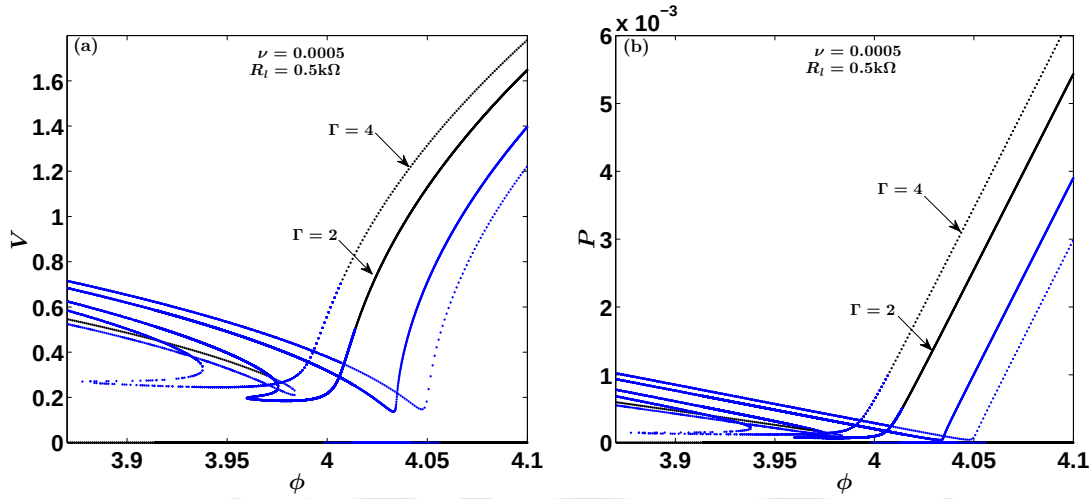


FIGURE 4.6: Variation of (a) Voltage (b) power with frequency of excitation (ϕ) for system parameter $\nu = 0.0005$, $\Gamma = 2, 4$ and $R_l = 0.5 \text{ k}\Omega$

example, the output voltage increases from 1.13 V to 1.32 V and power increases from 2.55 mW to 3.45 mW at a particular value of $\phi = 4.050$. For $\phi < 3.969$, the energy harvesting is possible in case of $\Gamma = 4$, which depends on the initial conditions since the trivial branch is also stable for the mentioned zone of frequency. The N-T stable and unstable branches follow similar trends for positively tuned cases and voltage increases with increase in ϕ .

Figure 4.7 shows the frequency response curve for parameters $\nu = 0.0005$ and $\Gamma = 4$ with the variation in load resistance $R_l = 0.1, 0.5, 1, 2 \text{ k}\Omega$. It is observed that the voltage and power increase significantly as R_l increases which shows that the load resistance plays an important role in achieving an optimum power output. Here at $\phi = 4.5$, a voltage of 0.77, 3.8, 7.77 and 15.3 V with corresponding power of 5.9, 29.5, 58.9 and 117 mW is achievable as the load resistance increases from $R_l = 0.1 \text{ k}\Omega$ to $R_l = 2 \text{ k}\Omega$.

In this case, the unstable trivial frequency zone remain fixed ($4.001 < \phi < 4.054$, $\Delta\phi = 0.053$) and stable nontrivial branches for frequencies $\phi < 4.001$ disappears (as shown in insight of Fig. 4.7(a), only showing stable trivial and stable nontrivial branches) for higher load resistance values $R_l > 1 \text{ k}\Omega$.

Figure 4.8 shows the variation of voltage and power with frequency when damping parameter (ν) of the system increases from 0.0005 to 0.005 (for $\Gamma = 10$ and $R_l = 0.5$

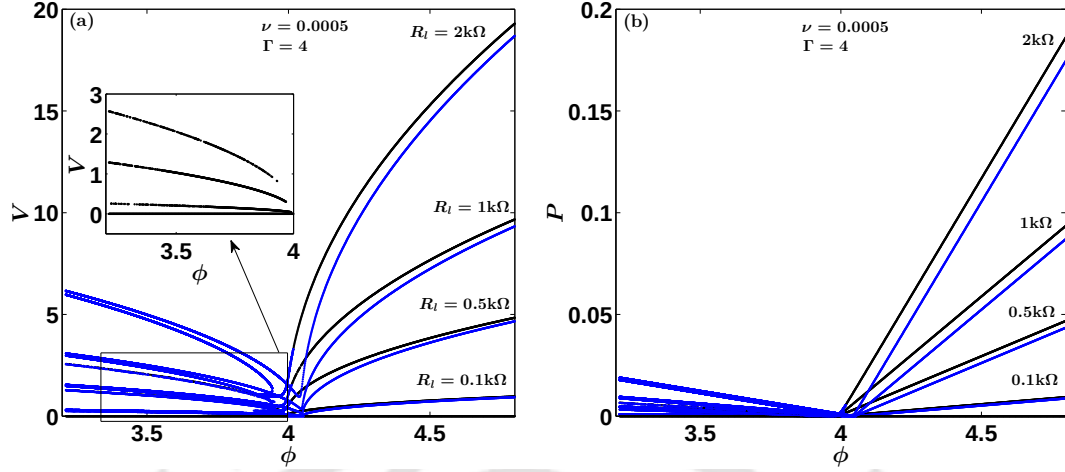


FIGURE 4.7: Variation of (a) Voltage and (b) power with frequency for $\nu = 0.0005$, $\Gamma = 4$ and load resistance $R_l = 0.1, 0.5, 1, 2$ kΩ.

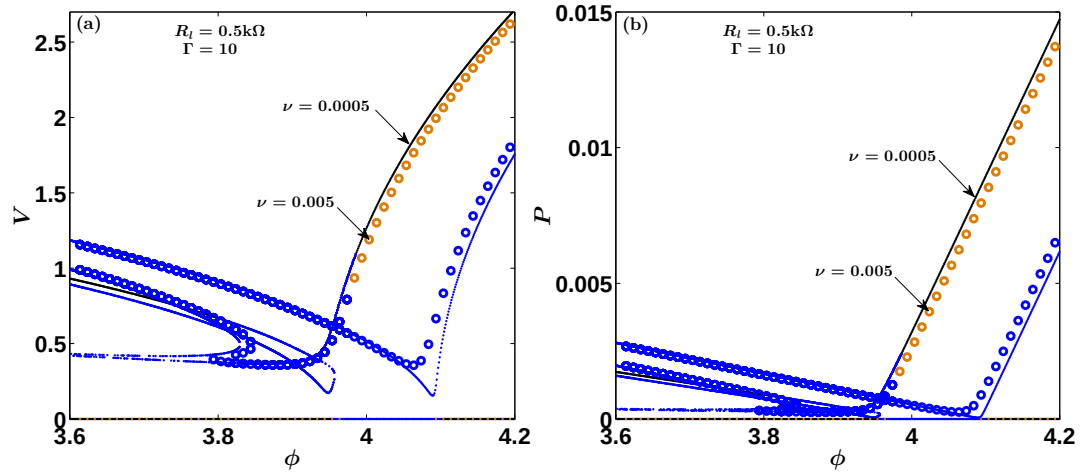


FIGURE 4.8: The effect of damping parameter ν on (a) voltage and (b) power response for $\nu = 0.0005$ and $\nu = 0.005$ for $\Gamma = 10$, $R_l = 0.5$ kΩ.

kΩ). The unstable trivial zone decreases slightly ($\Delta\phi = 0.017$) and the nontrivial branch which lies in the negative detuning zone ($\phi < 3.813$, $\nu = 0.0005$) disappears as ν increases. However, the voltage and power reduction are not significant for positively tuned system and both increases as the frequency of excitation increases. On the contrary, a significant change in the topology of N-T branch of solution is observed when damping parameter changes from 0.001 to 0.01 as shown in Fig. 4.9. The N-T stable and unstable branches in the positively tuned zone form a closed

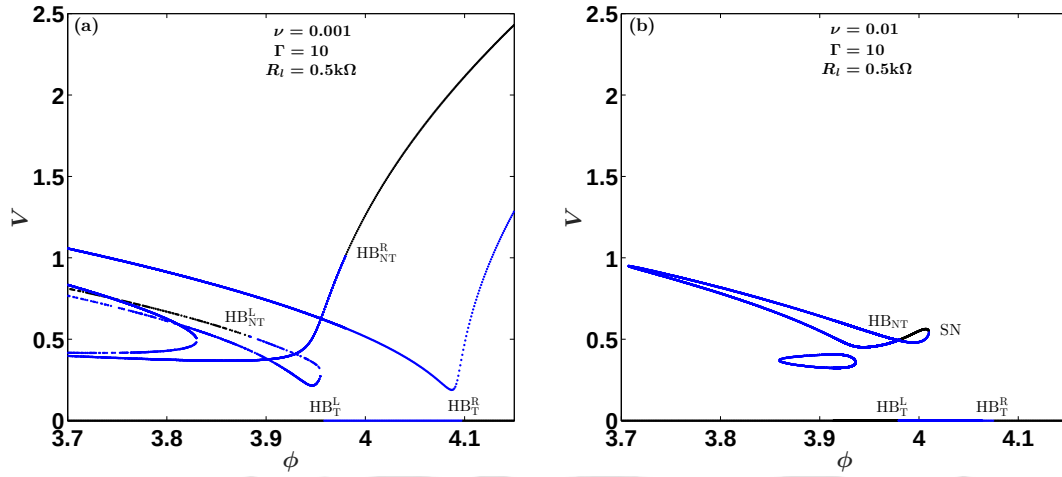


FIGURE 4.9: The effect of damping on voltage response for (a) $\nu = 0.001$ (b) $\nu = 0.01$ for $\Gamma = 10$, $R_l = 0.5 \text{ k}\Omega$.

loop and stable branch loses stability at saddle node bifurcation point ($\phi = 4.01$, $\nu = 0.01$) where voltage output found to be 0.54 V which is much reduced from 2.118 V for $\nu = 0.001$. Similar to the previous case here also the nontrivial branch, which lies in the negative detuning zone ($\phi < 3.883$, $\nu = 0.001$) disappears as ν increases and form a close unstable N-T branch of solution. The trivial unstable zone gets reduced.

Variation of voltage (V) and power (P) with amplitude of harmonic excitation Γ are shown in the Fig. 4.10 for $\phi=4$ (tuned case) for different damping ratios $\nu = 0.001$ and 0.005 respectively along with change in R_l . Both voltage and power increases and the critical value of Γ shifts towards higher side with R_l . The topology of the nontrivial branches of solution marginally changes as ν increases and the nature of stable nontrivial critical point changes from HB to Saddle node bifurcation point. For example in Fig. 4.10(a) the $\Gamma_{cr} = 2.96$ ($V = 1.67 \text{ V}$, at HB point) for nontrivial branch where $R_l = 1.5 \text{ k}\Omega$ while for higher damping $\nu = 0.005$ in Fig. 4.10(c) the $\Gamma_{cr} = 5.96$ ($V = 1.43 \text{ V}$ at SN point) for nontrivial branch where $R_l = 1.5 \text{ k}\Omega$. It is also observed that voltage and power is slightly reduces as damping increases and higher damping ratio requires higher critical value of Γ for harvesting purpose.

Considering $\nu = 0.001$, $\Gamma = 0-10$ and $R_l = 0.5 \text{ k}\Omega$ for perfectly tuned ($\phi = 4$, $\sigma_1 = 0$), negatively tuned ($\phi = 3.9$) and positively tuned ($\phi = 4.1$) cases, variation of voltage

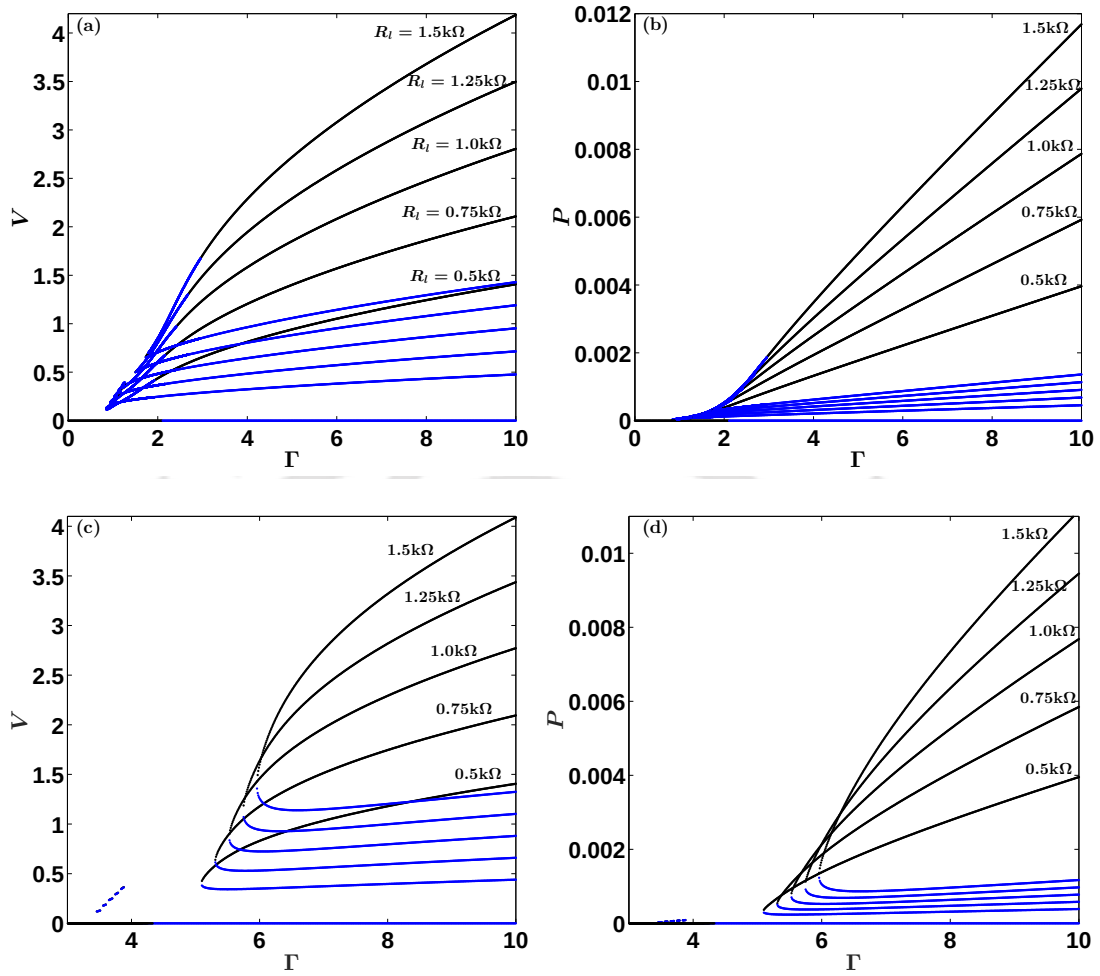


FIGURE 4.10: Voltage and power variation with amplitude of excitation (Γ) for the system parameters $R_l = 0.5, 0.75, 1, 1.25, 1.5$ k Ω , $\phi = 4$ (a, b) $\nu = 0.001$ (c, d) $\nu = 0.005$

and power with Γ is shown in Fig. 4.11. By Fig. 4.11(a,b) it has been observed that the voltage and power remains constant as Γ increases. The trivial state is stable in negatively and positively tuned cases while it loses stability by Hopf bifurcation for perfectly tuned case as shown in Fig. 4.11(c). The voltage and power increases as Γ increases for perfectly and positively tuned cases however the values are higher for positively tuned case. The unstable nontrivial branch disappears as frequency is tuned to perfect or kept to the positive side.

Voltage and power variation with load resistance (R_l) is shown in Fig. 4.12 for perfectly and positively tuned case and the effect of amplitude of excitation Γ on output

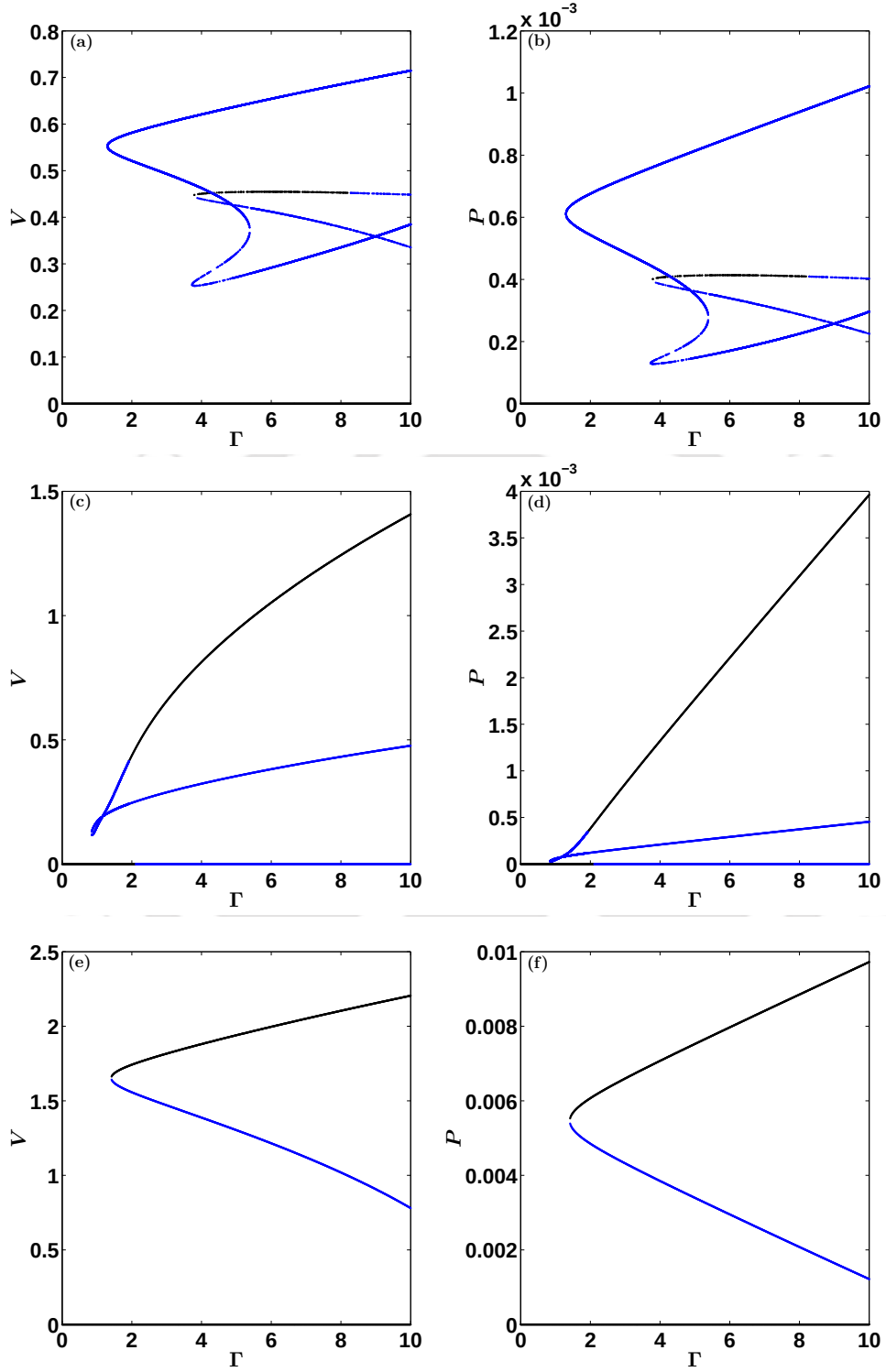


FIGURE 4.11: Variation of voltage and power with amplitude of excitation (Γ) for the system parameters $\nu = 0.001$, $\Gamma = 0-10$, $R_l = 0.5 \text{ k}\Omega$ (a, b) negatively tuned $\phi = 3.9$ (c, d) perfectly tuned $\phi = 4.0$ and (e, f) positively tuned $\phi = 4.1$ case

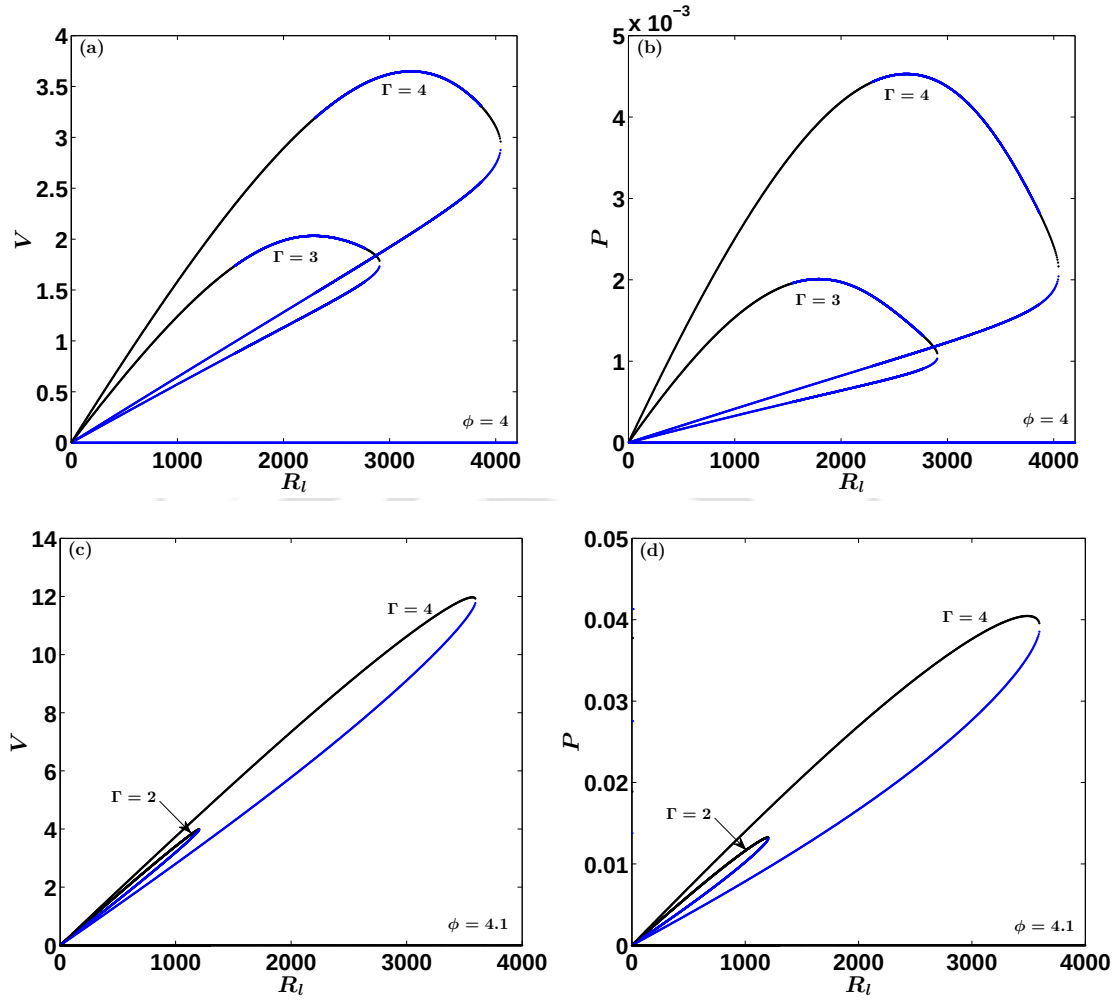


FIGURE 4.12: Voltage and power variation with load resistance (R_l) for the system parameter $\nu = 0.001$ and (a, b) $\phi = 4$, $\Gamma = 3, 4$ (c, d) $\phi = 4.1$, $\Gamma = 2, 4$

voltage and power is also observed. The trivial branch is unstable for the whole range of resistance variation in perfectly tuned case ($\phi = 4$) while it is completely stable for positively tuned case ($\phi > 4$). Output voltage increases with increase in R_l up to a certain value and nontrivial stable branch loses stability by Hopf bifurcation at 1.545 k Ω ($\Gamma = 3$) and 2.295 k Ω ($\Gamma = 4$) as shown in Fig. 4.12(a). Again for a range of $2.775 < R_l < 2.905$ and $3.870 < R_l < 4.045$ for $\Gamma = 3$ and $\Gamma = 4$, respectively the stable branch appears and loses stability by SN bifurcation point. Similar trend is observed for positively tuned case ($\phi = 4.1$) except that there is continuous stable nontrivial branch exist and loses stability at SN bifurcation as R_l increases and voltage and power values are significantly higher than that of tuned case. The

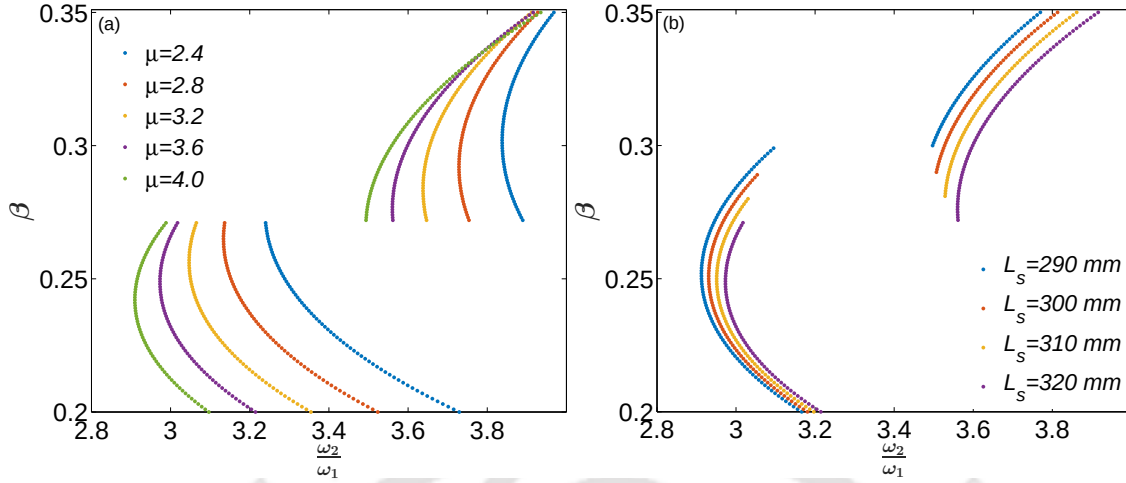


FIGURE 4.13: The variation of distance of arbitrary mass (β) with frequency ratio ($\frac{\omega_2}{\omega_1}$) for different value of (a) mass ratio μ and (b) beam length L_s while other geometric and material system parameters are same as mentioned in Table 4.1.

maximum value of power is found to be 40.46 mW at $R_l = 3.485$ k Ω and $\Gamma=4$ as shown in Fig. 4.12(d).

4.5 On Achieving the Internal Resonance of 1:3

In the present energy harvester, the material properties of the substrate are considered from the work of Zavodney and Nayfeh [182]. However, the possibility of achieving the required commensurate frequency ratio of 1:3 by considering different values of system parameters i.e., geometric (L_s, t_s, β, μ) and material properties (E_s) are explored in this section. The effect of the distance of arbitrary mass (β) from the fixed end on frequency ratio $\frac{\omega_2}{\omega_1}$ is shown in the Fig. 4.13, with change in mass ratio (μ) and beam length (L_s). As the mass ratio increases the frequency ratio becomes closer to the required value 3. A discontinuity is observed near $\beta = 2.72$ which remain invariant with respect to μ but changes with change in L_s value. This causes the frequency ratio $\frac{\omega_2}{\omega_1} > 3.4$ away from internal resonance phenomenon. For larger beam length, the discontinuity occurs at lower value of β and vice versa. These curves are very much useful while considering the geometry and location of arbitrary mass and its distance from the base in order to achieve or avoid the commensurate frequency ratio, which depends on the application.

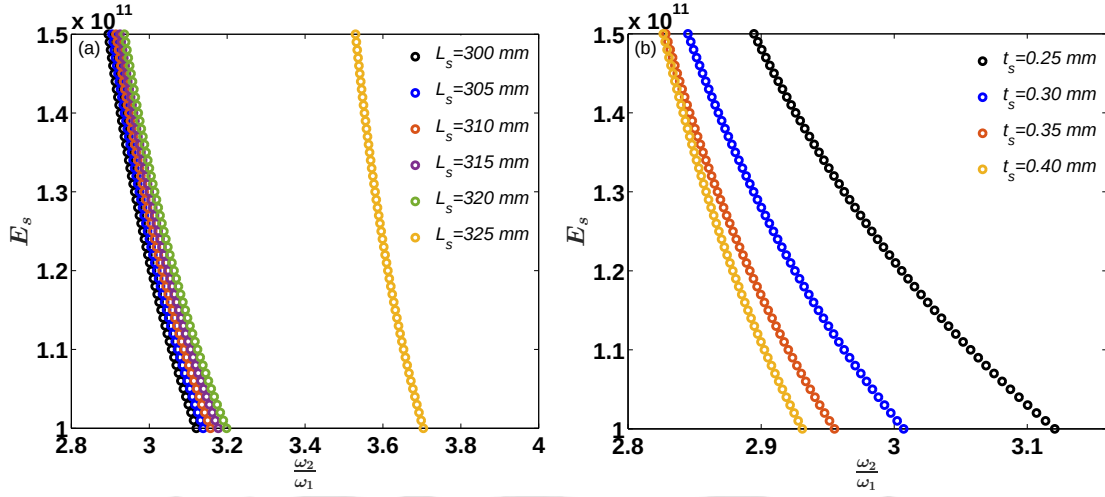


FIGURE 4.14: The variation of Young's modulus of beam (E_s) with frequency ratio $\left(\frac{\omega_2}{\omega_1}\right)$ for different value of (a) beam length and L_s (b) substrate thickness t_s while other parameters are same as in Table 4.1.

In Fig. 4.14 variation in material property (Young's modulus of the beam, E_s) is plotted with frequency ratio $\left(\frac{\omega_2}{\omega_1}\right)$ with change in substrate length (L_s) and thickness (t_s). As E_s increases, the ratio of frequencies gets reduce and increasing the length of the beam causes the curve to shift at higher frequency ratio side as shown in the Fig. 4.14(a). However, as the thickness of the substrate (t_s) increases the curve shifts towards lower frequency ratio side. One may chose suitable values of system parameters to get internal resonance of 1:3, for the present system, which is associated with the cubic and inertial nonlinearities due to the large deformation and an attached arbitrary mass.

In Fig. 4.15 the variation of mass ratio (μ) with frequency ratio $\left(\frac{\omega_2}{\omega_1}\right)$ is observed for different values of β , E_s , L_s and t_s . As the mass ratio increases the frequency ratio reduces. As the distance of attached mass (β) from the fixed end increases the curve shifts towards lower frequency ratio side and around $\beta \approx 0.27$ the frequency ratio switch to the higher ratio side as shown in the Fig. 4.15(a). Young's modulus in the range of $150 < E_s < 210$ is varied in Fig. 4.15(b) and in every case one can get a frequency ratio of nearly 1:3. Higher E_s brings the frequency ratio at lower side and this trend is similar for thickness t_s but opposite for larger beam length L_s values as shown in Figs. 4.15(c) and 4.15(d), respectively.

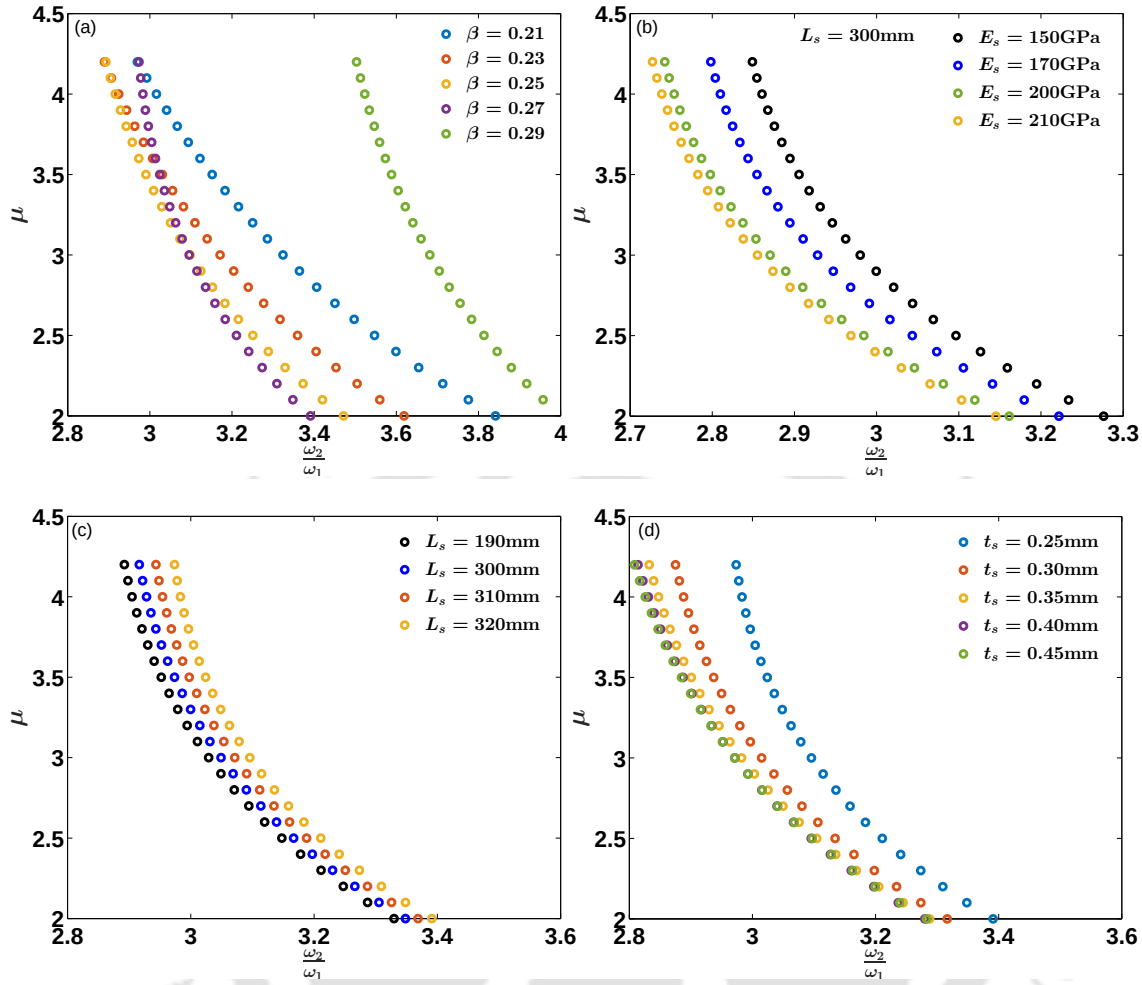


FIGURE 4.15: The variation of mass ratio (μ) with frequency ratio ($\frac{\omega_2}{\omega_1}$) for different value of (a) distance of arbitrary mass, β (b) Young's modulus of beam, E_s (c) beam length, L_s and (d) substrate thickness, t_s while other parameters are same as in Table 4.1).

4.6 Summary

In this work a harmonically base excited vertical cantilever beam with piezoelectric path and attached mass is used as an energy harvester. The mass is attached at a position such that the second modal frequency is thrice the fundamental frequency and hence the system exhibits 1:3 internal resonance. The system is excited at a frequency nearly equal to the combination of the first and second modal frequencies. The nonlinear governing equation of motion which is similar to that of a

parametrically excited system with cubic nonlinearities is derived and solved using the method of multiple scales. Initially the instability regions have been plotted to obtain the range of frequencies for which the harvester can be effectively used for various system parameters such as the load resistance, excitation amplitude, excitation frequency and damping. From these instability regions it has been observed that there exist critical value of amplitude of base excitation for different value of load resistance and damping below which the harvester can't be used as it leads to trivial state fixed point response. Then the frequency response curves have been plotted for the same parameters to obtain the voltage and power of the harvester. Time responses have been plotted to verify the instability regions and the frequency response curves which are found to be in good agreement. From the frequency vs power curves, it has been observed that there exist critical values of load resistance for optimum power generation. It has been observed that the present harvester will not only useful in energy extraction in the primary resonance zone but also in the higher frequency zone of combination type ($\omega_1 + \omega_2$). Also, parametric excitation means a high deformation at a cost of very low excitation magnitude which provides high conversion rate of power with only limitation of crossing the threshold limit of excitation magnitude necessary for transverse deflection of the vertical cantilever beam based harvester.

By suitably using different dimensions of the harvester, the load resistance, different types of piezoelectric patch and base material and attached mass one can extend the present analysis using the developed equations to find the range of voltage and power of the harvester which can be used for a wide range of applications. This analysis of combination parametric resonance may be coupled with other resonance conditions for harvesting energy for a wider frequency range as the ambient response contain both low and high frequencies.



Chapter 5

Piezoelectric Energy Harvester under Parametric Excitation: A Theoretical and Experimental Investigation

5.1 Introduction

In the present work, both theoretical and experimental investigation of a vertical cantilever beam based piezoelectric energy harvester (PEH) is carried out under principal parametric resonance condition. A piezoelectric patch is attached near the fixed end of the cantilever beam along with an attached mass positioned at an arbitrary location. The extended Hamilton's principle is used to derive the spatio-temporal equation of motion and generalized Galerkin's approximation is used to obtain the temporal nonlinear electromechanical governing equation of motion. The method of multiple scales (MMS) is used to find the reduced modulation equations. Due to the large transverse deflection and effect of rotary inertia of the attached mass, the system exhibits cubic and inertial nonlinearities. An experimental setup with slider crank mechanism based shaker and a harvester consisting of a cantilever beam with piezoelectric patch and attached mass is designed and developed. The challenges posed by parametric resonance in crack development in the PZT and the

beam are reported. The theoretical and experimental output voltage and the power obtained are found to be in good agreement. Further, a qualitative and quantitative comparative study of 17 energy harvesters has been carried out. The normalized power densities have also been compared.

5.2 Design and Development of the Experimental Setup

The objective of this work is to develop a low-cost shaker to provide parametric excitation to the energy harvester. The requirement of the frequency of excitation, amplitude (stroke length), specimen dimensions and its weight are the basic parameters on which the design and development of the harvester is carried out. The basic specimen considered in this work is a vertical steel cantilever beam having dimensions of $300 \text{ mm} \times 25.5 \text{ mm} \times 0.51 \text{ mm}$ attached with a unimorph piezoelectric patch of dimensions $50 \text{ mm} \times 25 \text{ mm} \times 0.5 \text{ mm}$, near to its fixed end. A mass (m) is attached at an arbitrary distance d from the fixed end. The neodymium magnets (rectangular plate form, $25 \text{ mm} \times 10 \text{ mm} \times 1 \text{ mm}$, the weight of 4.5 gm each) are used as the attached mass here so that the mass and its position are adjustable along the beam. The PZT patch (PZT-5H) by Sparkler Ceramics Pvt. Ltd., is attached near the fixed end of the cantilever beam. The PZT patch is glued on to the beam surface by using Araldite epoxy-resin (Grade AV138M/HV 998) with a 60 – 40 ratio.

5.3 Shaker Design

A slider-crank mechanism (Fig. 5.1a) is used to convert the rotational motion of the DC motor to the translational motion of the slider. A base plate is taken to mount the DC motor with the slider-crank mechanism and the electronic components. The CAD model of the setup developed using ProE is shown in Fig. 5.1b. To design the components initially kinematic and dynamic analysis of the slider crank mechanism has been carried out.

5.4 Kinematic Analysis

The schematic diagram of the mechanism is shown in Fig. 5.1a. The mechanism is assumed to have stroke lengths l_s between 13 mm to 22 mm with a maximum frequency of 25 Hz (1500 rpm) to excite the harvester near twice the principal parametric resonance condition that is twice the natural frequency of the system. As the stroke length is considered to be in the range of 13 mm to 22 mm, so a crank is made out of a circular disk having radius 12 mm having slots to connect the connecting rod at 6.5, 8, 9 and 11 mm. The crank is attached to the shaft of the DC motor. In Fig. 5.1a, OP is the radius of the crank of length r , PQ is the connecting rod of length l_c . A guided extension rod (QR) of length l_r is bolted to the connecting rod at point Q in the vertical position. A holder is placed over the rod QR to attach a cantilever beam and it can accommodate a beam up to 35 mm width. As it is required to move the point Q in a straight line, a rod QR is attached to the connecting rod and it is made to move inside a guide supported by ball bearings. The crank radius r decides the stroke length (displacement of point R, l_s) by the relation $l_s = 2r$. The length QR is decided based on kinematic constraints.

5.5 Dynamic Analysis

A dynamic analysis is performed to decide the required motor torque to develop the PEH system. The components are made up of mild steel. The components, which are supported by the DC motor with their weights, are mentioned in Table 5.1. The total load of the basic supporting structure is 251 gm. The required torque is

TABLE 5.1: Components supported by DC motor with weights.

Components	weight (g)
crank with connecting rod	48
slider with holder	129
specimen	62
others (screws)	12
total	251

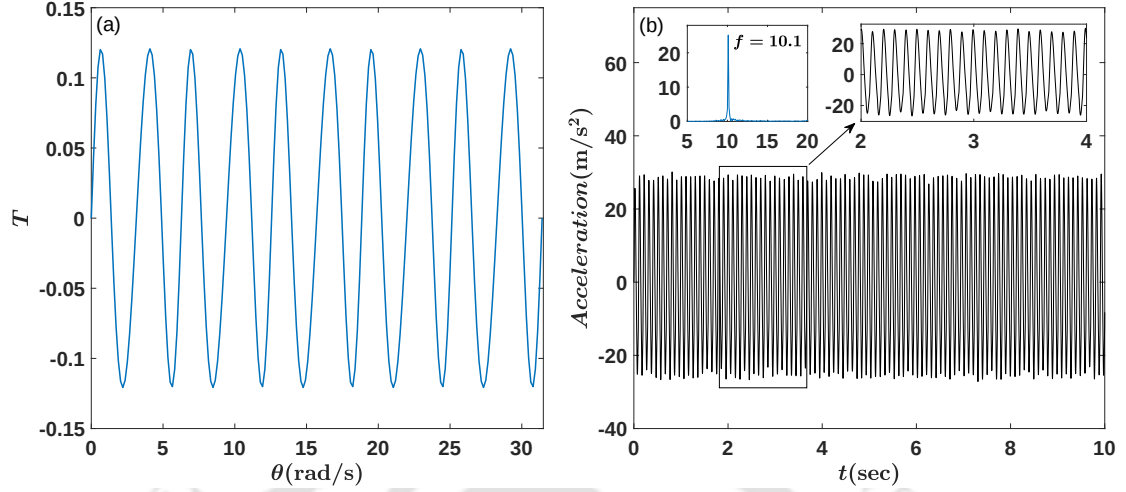


FIGURE 5.2: (a) Variation of torque with crank angle (b) Experimental acceleration time response (after filtering) of the holder without beam system attachment at a particular speed of DC motor by using the accelerometer.

minute. Microcontroller (mbed NXP LPC1768) is programed in such a way that one can regulate the rotational speed of the DC motor by just rotating the knob and display the output in display unit (16×2 LCD). The microcontroller required 5V DC power, which is provided by USB cable. Power supply converts AC into DC and is connected to the motor driver. The motor driver provides 12V or 24V power to the motor.

5.6 Analytical Model

The schematic diagram of parametrically excited PEH system consists of a cantilever beam of length L , mass m at a distance d from its fixed end and a PZT patch of length L_p near its base is shown in Fig. 5.4. The electrodes of PZT patch are connected to an external load resistance through a circuit to find out the power output of the harvester. The longitudinal and transverse motion of the beam is denoted by $u(\xi, t)$ and $v(\xi, t)$ respectively, where ξ is the curvilinear coordinate along the beam length.

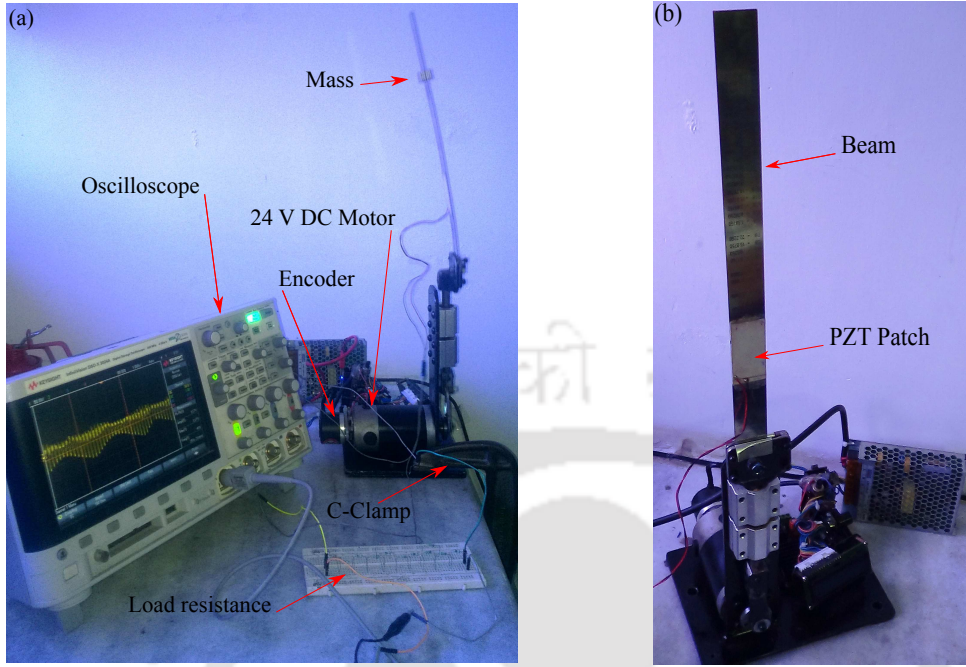


FIGURE 5.3: (a) Experimental setup of PEH system with electronic circuit and oscilloscope (b) Clear view of the harvester with PZT patch.

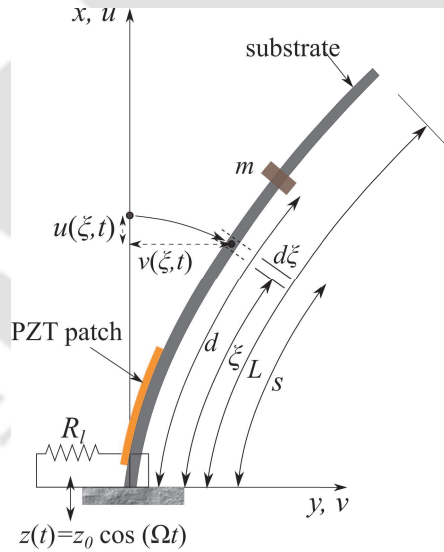


FIGURE 5.4: Schematic diagram of piezoelectric energy harvester (PEH) system with parametric excitation

The nondimensional nonlinear coupled electromechanical temporal governing equations of motion of the system is expressed in the following equations. The detailed

derivation can be found in the Appendix C.

$$\ddot{q} + 2\epsilon\zeta\dot{q} + q + \epsilon\alpha q^3 + \epsilon\beta q\dot{q}^2 + \epsilon\gamma q^2\ddot{q} + \epsilon\eta V = \epsilon q \hat{f} \cos(\phi\tau) \quad (5.2)$$

$$\dot{V} + \chi V + \dot{q} = 0 \quad (5.3)$$

Here,

$$\begin{aligned} x &= \frac{s}{L}, \quad \beta = \frac{d}{L}, \quad \tau = \omega t, \quad \lambda = \frac{r}{L}, \quad \mu = \frac{m}{\rho_b A_b L}, \quad \Gamma = \frac{z_0}{z_r}, \\ J &= \frac{J_0}{\rho_b A_b L r^2}, \quad \phi = \frac{\Omega}{\omega}, \quad \zeta = \zeta^* \frac{\nu}{\epsilon}, \quad \chi = \frac{1}{C_p \omega R_l}, \\ \eta &= \frac{-\bar{\eta} r_v}{\rho_b A_b \hbar \omega^2 r \epsilon L^2} \int_0^1 \{\delta_x(x) - \delta_x(x-1)\} \psi(x) dx, \\ \bar{K} &= \frac{e_{31} t_{pc} b_p}{L} \int_0^1 \psi_{xx}(x) dx, \quad t_{pc} = \frac{t_p + t_s}{2} - \bar{y}, \quad r_v = \frac{\bar{V}}{V} = \frac{\bar{K} r}{C_p} \end{aligned}$$

Where $(\dot{}) = \frac{d()}{d\tau}$. Here the damping, geometric and inertial nonlinearities, electromechanical coupling and excitation terms are small, and hence considered of the order of ϵ (book keeping parameter). Here α , β , γ and η are the coefficients of cubic, inertial ($q\dot{q}^2$, $q^2\ddot{q}$) and coupling terms, respectively. The coefficients appear in Eqs. (5.2) and (5.3) are defined in Appendix C2. By observing the R.H.S. term of Eq. (5.2) the time-varying forcing term appears as a parameter, which is a typical characteristic of parametrically excited systems that provides energy input to the system. In the absence of PZT patch, the Eq. (5.2) reduced to that of [182].

5.7 Perturbation Analysis

A uniform first-order approximate analytical solution of Eqs. (5.2) and (5.3) is obtained by using the standard method of multiple scales. In the present work, the nonlinear electromechanical system is explored as an energy harvester for the principal parametric resonance condition. The time dependence is expressed into multiple time scales as $T_n = \epsilon^n \tau$; $n = 0, 1, 2, \dots$ and the time derivatives expressed as: $\frac{d}{d\tau} = D_0 + \epsilon D_1 + O(\epsilon^2)$, $\frac{d^2}{d\tau^2} = D_0^2 + 2\epsilon D_0 D_1 + O(\epsilon^2)$ where $D_i = \frac{\partial}{\partial T_i}$. The system

solution (q, V) expanded in a series form as

$$q(\tau; \epsilon) = q_0(T_0, T_1) + \epsilon q_1(T_0, T_1) + O(\epsilon^2) \quad (5.4)$$

$$V(\tau; \epsilon) = V_0(T_0, T_1) + \epsilon V_1(T_0, T_1) + O(\epsilon^2) \quad (5.5)$$

By substituting Eqs. (5.4) and (5.5) into Eq. (5.2), (5.3) and further equating the coefficients involving the terms of ϵ^0 and ϵ^1 to zero, yields the set of equations in different order of ϵ

$$D_0^2 q_0 + q_0 = 0 \quad (5.6a)$$

$$D_0 V_0 + \chi V_0 = -D_0 q_0 \quad (5.6b)$$

$$\begin{aligned} D_0^2 q_1 + q_1 = & -2\zeta D_0 q_0 - 2D_0 D_1 q_0 + q_0 \hat{f} \cos \phi \tau \\ & - \eta V_0 - \alpha q_0^3 - \beta q_0 (D_0 q_0)^2 - \gamma q_0^2 D_0^2 q_0 \end{aligned} \quad (5.7a)$$

$$D_0 V_1 + \chi V_1 = -D_0 V_0 - (D_0 q_1 + D_1 q_0) \quad (5.7b)$$

The solution of differential equations (Eq. (5.6a) and (5.6b)) of the order $O(\epsilon^0)$ are expressed as

$$q_0 = A(T_1)e^{iT_0} + cc; \quad V_0 = -ZA(T_1)e^{iT_0} + cc \quad (5.8)$$

Here cc is the complex conjugate of the preceding terms and $A(T_1)$ is an unknown complex function to be determined later. Using Eq. (5.8) into Eq. (5.7a) the following secular and near secular terms are obtained which should be eliminated for finite amplitude response of the system

$$\frac{\hat{f}}{2} \bar{A} e^{iT_0 \phi - iT_0} - (3\alpha + \beta - 3\gamma) \bar{A} A^2 e^{iT_0} - 2i(A' + \zeta A) e^{iT_0} + \eta Z A e^{iT_0} = 0 \quad (5.9)$$

One may obtain similar expression for single mode approximation without considering 1:3 internal resonance case from [301]. To express the nearness of ϕ to that of 2 (principal parametric resonance) the detuning parameters σ is introduced as $\phi = 2 + \epsilon\sigma$. Expressing the complex function $A(T_1)$ into polar form as $A(T_1) = \frac{1}{2}a(T_1)e^{i\vartheta(T_1)}$ where amplitude $a(T_1)$ and phase $\vartheta(T_1)$ are slowly varying function of time scale T_1 . Now by using this polar form in Eq. (5.9) and eliminating

the secular term (the coefficient of e^{iT_0}) to find the bounded solution one obtains the following reduced modulation equations.

$$a' = \zeta_{eff}a + aF_{eff} \sin(\delta) \quad (5.10)$$

$$\frac{1}{2}a\delta' = \frac{1}{2}\sigma_{eff}a - N_{eff}a^3 + aF_{eff} \cos(\delta) \quad (5.11)$$

Here

$$N_{eff} = \frac{3\alpha}{8} + \frac{\beta}{8} - \frac{3\gamma}{8}; \quad \zeta_{eff} = -\zeta + \zeta_e - \zeta_a a; \quad \sigma_{eff} = \sigma + \sigma_e; \quad F_{eff} = \frac{\hat{f}}{4};$$

$$\zeta_e = \frac{\eta\chi}{2(\chi^2 + 1)}; \quad \sigma_e = \frac{\eta}{(\chi^2 + 1)}; \quad \delta = \sigma T_1 - 2\vartheta; \quad \tan \delta_0 = \frac{-\zeta_{eff}}{-0.5\sigma_{eff} + N_{eff}a^2}$$

Here ζ, ζ_e, ζ_a and σ_e represents the internal material damping, the damping term caused by electric circuit, aerodynamic damping [21, 181, 306] and the shift in systems frequency due to electromechanical coupling respectively which depends on parameters η, ξ . Also, N_{eff} denotes the total effect due to cubic (α) and inertial (β, γ) nonlinear terms in the equation of motion of the beam. The nontrivial ($a \neq 0$) frequency response of the reduced equations can be calculated by the following frequency response expression.

$$\sigma = 2N_{eff}a^2 - \sigma_e \pm 2\sqrt{F_{eff}^2 - (\zeta_{eff})^2} \quad (5.12)$$

The above expression is parabolic in nature with respect to the nondimensional amplitude a . Steady state displacement, output voltage and power are obtained from the following expressions.

$$v(s, t) = r \psi(s) a \cos(\omega t + \vartheta) \quad (5.13)$$

$$V(t) = r_v \frac{1}{\sqrt{\chi^2 + 1}} a \cos(\omega t + \vartheta + \varphi) \quad (5.14)$$

$$P = \frac{a^2}{R_l} \left(r_v \frac{1}{\sqrt{\chi^2 + 1}} \right)^2 \quad (5.15)$$

Here $\varphi = \tan^{-1}(\chi)$.

5.8 Results and Discussion

For result and discussion purpose, the geometric and material properties of the specimen (cantilever beam and piezoelectric patch of PZT-5H) as shown in Fig. 5.1b are given in Table 5.2. It may be noted that a mass of 27 gm is attached at a

TABLE 5.2: Geometric and material properties of beam with piezoelectric patch.

Property	PZT Patch	Sym.	Substrate-S1	Substrate-S2	Substrate-S3
Young's moduli, GPa	$E_p = 66.7$	E_b	190	190	190
Density, kgm^{-3}	$\rho_p = 7500$	ρ_b	7800	7800	7800
Length, m	$L_p = 0.05$	L_b	30×10^{-2}	29.1×10^{-2}	30×10^{-2}
Width, m	$b_p = 0.025$	b_b	2.55×10^{-2}	2.55×10^{-2}	2.55×10^{-2}
Height, m	$t_p = 0.0005$	t_b	5.1×10^{-4}	5.1×10^{-4}	6.5×10^{-4}
Capacitance, nF	$C_p = 71.30$	—	—	—	—
Piezoconstant, pCN^{-1}	$d_{31} = -285$	—	—	—	—

distance of 213 mm from the base of the system. Further, the PZT is placed at a distance of 5.7 mm from the fixed base. Initially the free vibration response of the system is found out to obtain the natural frequency and the damping ratio of the system. Figure 5.5(a) shows the voltage-time response of the harvester, which is obtained experimentally by simply tapping the end of the harvester in open circuit condition. As the voltage is proportional to the displacement of the system, using the logarithmic decrement method, the damping ratio of the system is found to be 0.0134.

The corresponding FFT of the voltage-time signal is plotted in Fig. 5.5(b), where the fundamental frequency of the PEH system is found to be 3.6 Hz. By using the expression given in the Appendix C2, the natural frequency of the system is found to be 3.53 Hz. Figure 5.6(a) shows the frequency response plot obtained by using Eq. 5.12. The external load resistance is kept at 100 k Ω . It may be noted that by sweeping up the frequency up to 6.76 Hz, the beam has a rigid-body motion and does not undergo transverse vibration, which is one of the characteristics of the parametrically excited systems.

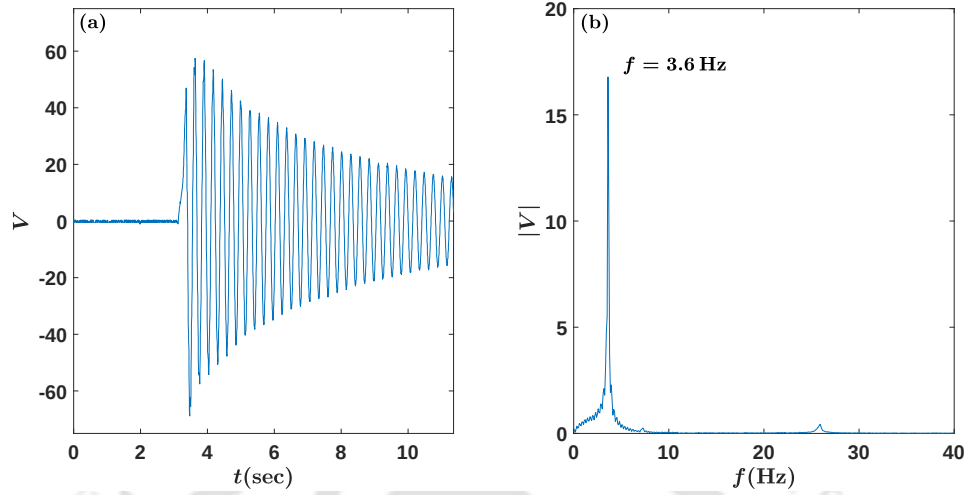


FIGURE 5.5: (a) Open circuit, the voltage-time response of the system when just tapping the top edge of the beam for damping coefficient calculation by logarithmic decrement method (b) corresponding FFT of the voltage-time signal

Therefore the piezoelectric patch does not undergo any strain and hence, no voltage is produced (Fig. 5.6(b)). Further, sweeping up this frequency, after this critical value where it undergoes supercritical pitchfork bifurcation (PF_L), the system has a nontrivial response and the corresponding output voltage starts to increase with increase in the external frequency. This continues until the saddle-node bifurcation (marked SN in Fig. 5.6(b)) and a further increase in frequency leads to the jump down phenomenon where the beam response may again become trivial. Here the beam may develop cracks near the fixed end. On sweeping down the frequency, the jump up phenomenon occurs near the subcritical pitchfork bifurcation point (PF_R) and the trivial response jumps to the nontrivial branch. The acceleration level for parametric onset is found to be $(|\ddot{z}| = z_0(2\pi f)^2 = 11.7 \text{ m/s}^2$, where $z_0 = 6.5 \text{ mm}$ and $f = 6.76 \text{ Hz}$).

To compare, the analytical result with the experimental result, the system is excited near to twice its fundamental frequency (2ω) to find out the voltage response by sweeping up the frequency. Figure 5.6(b) shows the plot for the variation of the voltage with frequency. Here both theoretical and experimental results are plotted, and both results are found to be almost in good agreement up to 7.33 Hz. The difference between experimental and analytical findings may be due to the parameter taken for theoretical analysis and minor imperfection in the experimental setup. The theoretical prediction of critical threshold frequency and the highest output voltage

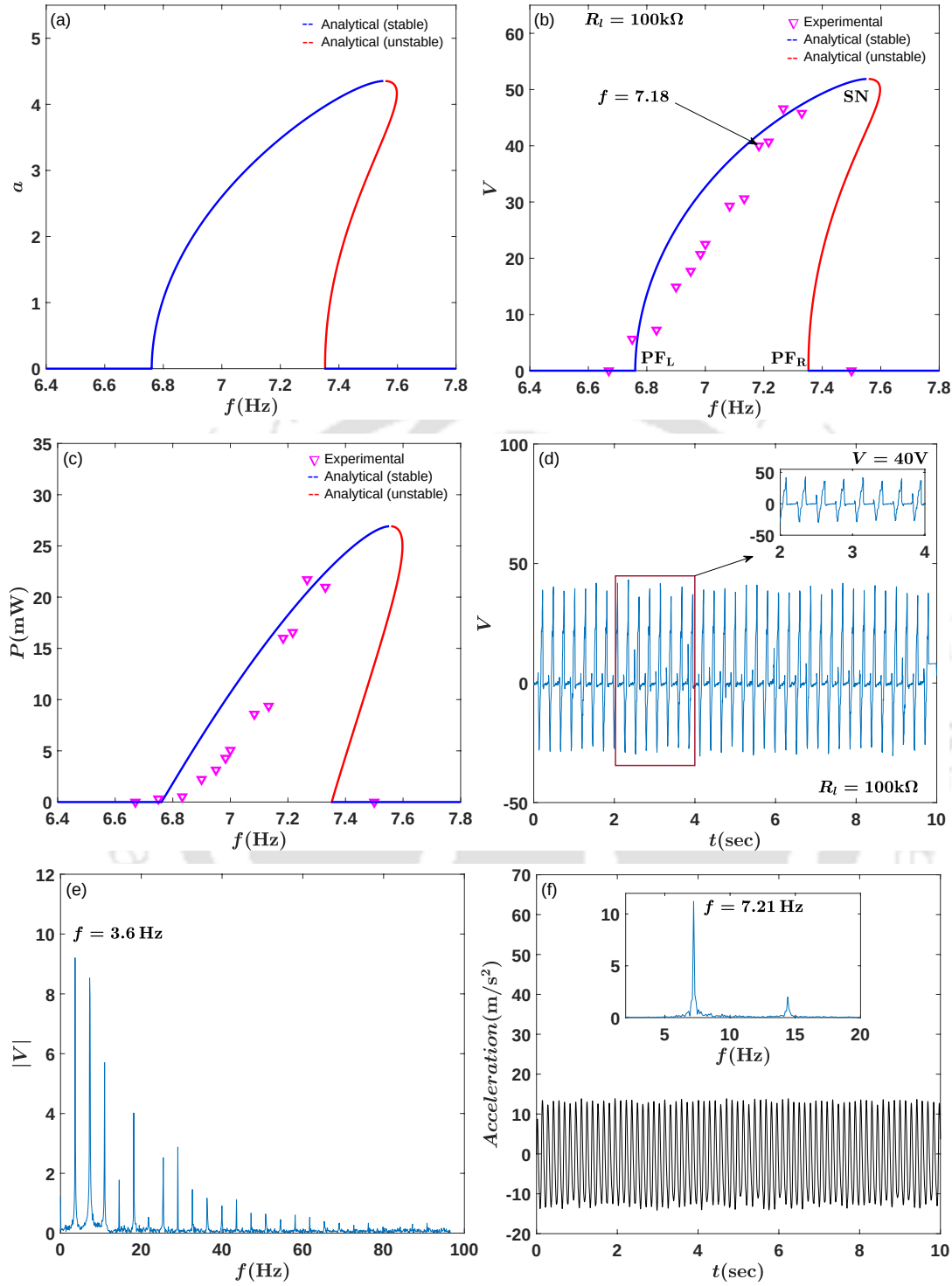


FIGURE 5.6: (a) Amplitude response with frequency of excitation at $R_l = 100 \text{ k}\Omega$ and (b) Comparison of analytical and experimental voltage response (c) Comparison of analytical and experimentally obtained power with frequency of excitation (d) Time response of the output voltage at a particular frequency of excitation of $f = 7.18 \text{ Hz}$ (with piezo-patch with hairline cracks) (e) FFT of the voltage-time response shown in Fig. 5.6(d), (f) Input acceleration time plot with FFT of the filtered signal.

are closely resembles the experimental findings. It may be noted that the error is found to be around 2% near the supercritical pitchfork bifurcation point and saddle-node bifurcation point. In between these points the error is coming to be around 10-15%. The error is calculated as the ratio of the total deviation in output voltage by total theoretical voltage. Also one may note that the analytically predicted response consists of stable (solid blue lines) and unstable (solid red lines) nontrivial branches of fixed points, which coincides with experiment findings.

The steady state power is then calculated by $\frac{V_{\max}^2}{R_l}$ as shown in Fig. 5.6(c). One may achieve a maximum power of nearly 22.7 mW around a frequency of excitation of 7.25 Hz for external load resistance of $R_l = 100 \text{ k}\Omega$. Figure 5.6(d) shows the experimental voltage-time response for a particular frequency of excitation $f = 7.18$ and load resistance of $R_l = 100 \text{ k}\Omega$ where the average voltage is found to be $V = 40.21 \text{ V}$. The FFT of the output voltage vs time response shown in Fig. 5.6(d) is plotted in Fig 5.6(e) and the time response of the input acceleration along with FFT of the filtered signal is shown in Fig. 5.6(f).

When the harvester system is excited near the parametric resonance condition, few hairline cracks are observed transverse to the length of the piezo-patch (Fig. 5.7(c)). This is because the patch is placed near to the base (spanned from 5.7 mm to 10.7 mm length from the base) where the bending moment is high and large amplitude of vibration causes PZT patch to break transversely and visible hairline cracks appear due to the brittle nature (low Young's Modulus) of patches. Due to these cracks, the time response (shown in Fig. 5.6(d)) differs from well-expected continuous periodic profile and sharp peaks rather than smooth variations are visible. Here, near the SN bifurcation point the beam may develop cracks close to the fixed end due to high transverse deformation, which produces maximum bending moment at the base and after few cycles the beam may break as shown in Fig. 5.7(a). The corresponding close up figure is shown in Fig. 5.7(b), where the direction of the crack is transverse to the length of the beam. This could be further quantified in terms of the amplitude of the base acceleration, which is responsible for the onset of a crack in the beam. From the expression of the base excitation ($z = z_0 \cos(\Omega t)$) one can find the amplitude of the base acceleration ($|\ddot{z}| = z_0(2\pi f)^2 = 13.78 \text{ m/s}^2$, where $z_0 = 6.5 \text{ mm}$ and $f = 7.33 \text{ Hz}$) at which the crack in the beam is possible due to high amplitude transverse displacement. The length of the beam after disintegration (Fig. 5.7(c)) from near its fixed end is reduced to $L_b = 291 \text{ mm}$, here further it used as next specimen (S2).

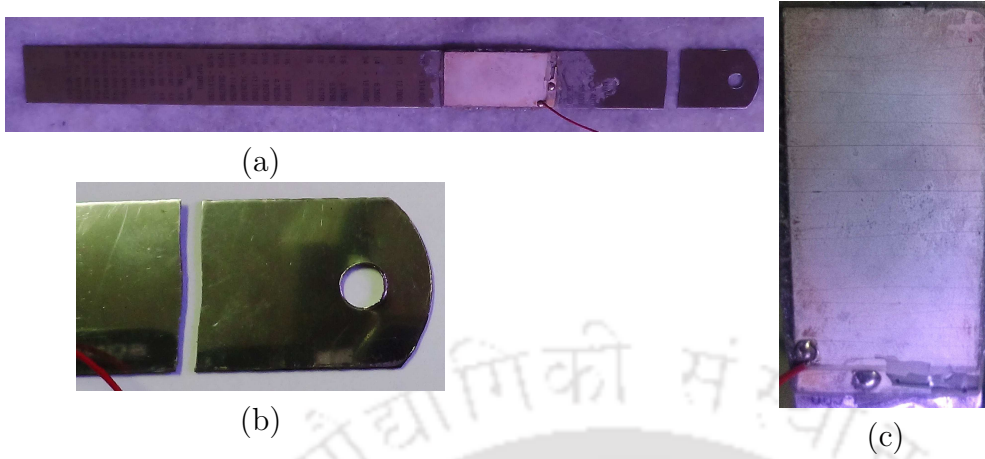


FIGURE 5.7: (a) Cracked beam with piezo-patch (b) cracked beam (zoomed view) (c) Piezoelectric patch (PZT-5H) with visible hairline cracks in transverse direction and disintegration near the electrode separation.

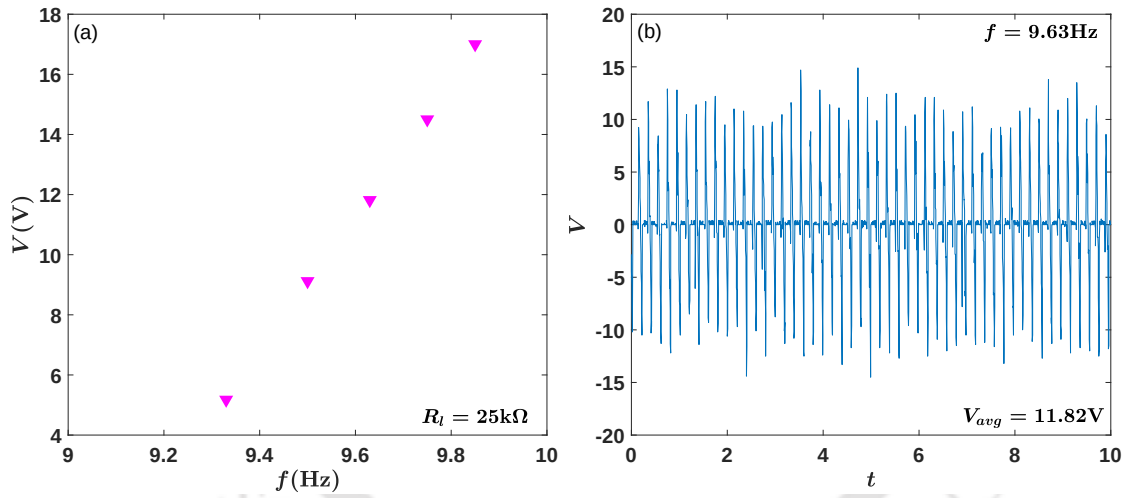


FIGURE 5.8: (a) Experimental voltage response with frequency of excitation at $R_l = 25\text{ k}\Omega$ and (b) voltage time response at a particular frequency of excitation of $f = 9.63\text{ Hz}$ (when its length reduces to $L_b = 291\text{ mm}$ from previous length of $L_b = 300\text{ mm}$)

Figure 5.8 shows the frequency and time response of output voltage for external load resistance of $25\text{ k}\Omega$. The natural frequency of the system increases due to reduced length of the harvester, hence to trigger the parametric excitation the frequency of excitation shifts towards higher frequency side as shown in Fig. 5.8(a). The voltage time response corresponds to $f = 9.63$ is shown in Fig. 5.8(b) where an average voltage of 11.82 V is achievable. The amplitude and voltage response grows as the frequency of excitation f and the right side bend of the response shows the hardening

behaviour, which observed earlier as well. In Fig. 5.9(a) the time response of the voltage is shown for $R_l = 25 \text{ k}\Omega$, one can achieve an average voltage of 17 V in this case for the frequency of excitation $f = 9.85 \text{ Hz}$. The corresponding point is from voltage response in Fig. 5.8(a).

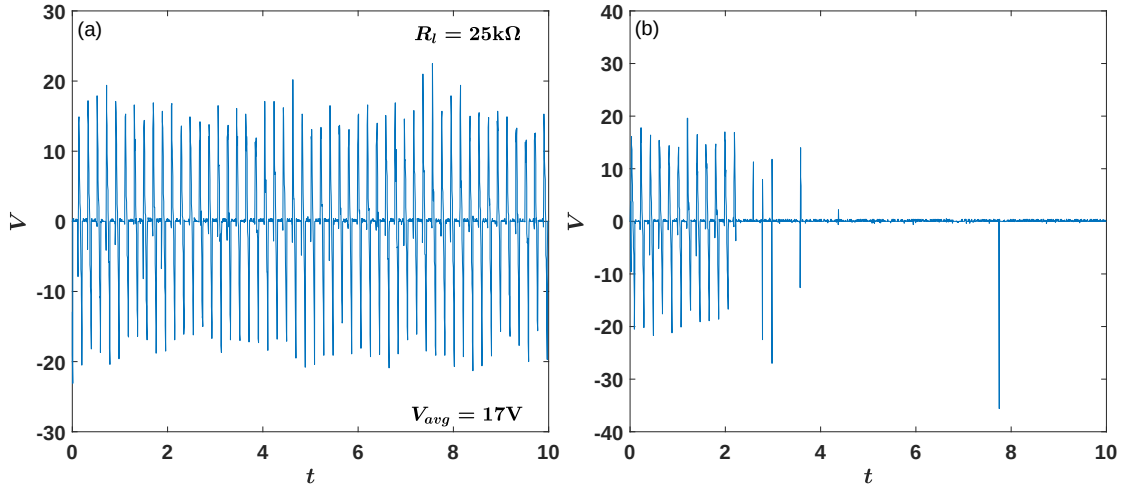


FIGURE 5.9: (a) Experimental time response of output voltage at $R_l = 25 \text{ k}\Omega$ and $f = 9.85 \text{ Hz}$ (b) when at very high bending of the beam the piezoelectric patch disintegrate at the frequency of excitation of $f > 9.85 \text{ Hz}$

Figure 5.9(b) captures the voltage time response of pre and post-failure of piezoelectric patch attached to the beam due to high rate of deformation (a violent transverse movement of the beam) when external frequency is nearly $f > 9.85$. The time response becomes trivial from nontrivial as time progresses and some high peaks are observed which shows that the electric circuit sometimes gets completed, depends on the nature of the disintegration of the PZT patch. Since the PZT patch disintegrated in the previous experiment, a new specimen is fabricated for further analysis. The new specimen (with substrate S3) is having all the material and geometric properties similar to previous specimens with substrates S1 and S2 (as mentioned in Table 5.2), except the thickness. The beam thickness ($t_b = 0.65 \text{ mm}$) of the new specimen is more than that of previous ones and hence its natural frequency is slightly higher than the previous specimen. Figure 5.10 shows the variation of voltage and power with external load resistance when the frequency of excitation kept at a constant value of $f = 8.28 \text{ Hz}$. Here few readings of output voltages are taken at a particular value of load resistance to plot the variation. This is achieved by keeping the rpm of DC motor at a constant level. The quadratic curve fit depicts the average

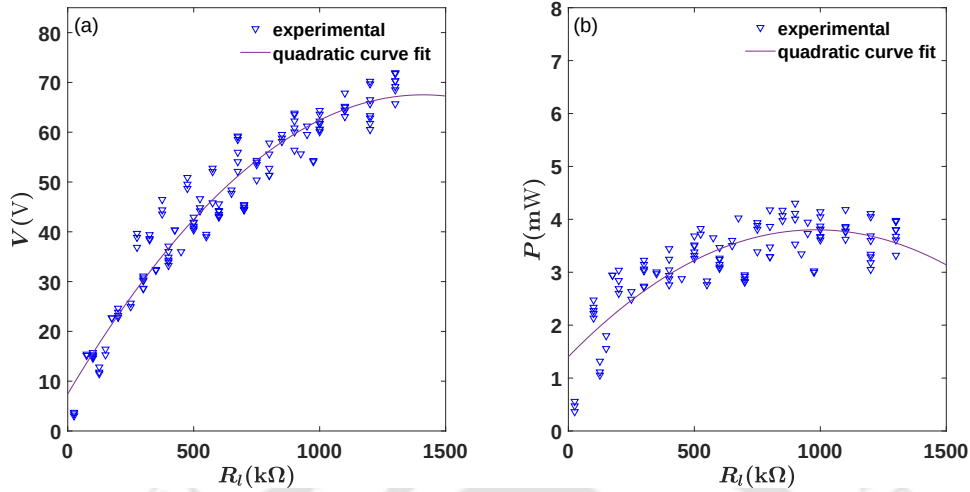


FIGURE 5.10: Experimental voltage and power with the variation of load resistance R_l (new beam system with similar geometric (except thickness) and material properties without any crack) keeping μ , β constant and $f = 8.28$ Hz

variation of voltage and power. It is observed that as the load resistance increases, the voltage and corresponding power (Fig. 5.10(b)) also increases and then stabilizes for certain resistance values. It shows that the load resistance influences the power output and at a certain value of R_l , the output power gets maximum. One may obtain the optimal resistance by performing power limit analysis as given in [307]. The voltage time response at a particular load resistance $R_l = 800$ kΩ is shown in Fig. 5.11(a) and corresponding FFT is plotted in Fig. 5.11(b). The voltage-time response is amplitude-modulated response with frequency of oscillation around 4.3 Hz.

The continuous and smooth variation of the voltage-time response means the PZT patch is working well and there is no evident hairline crack is developed until now, at this frequency level. The average value of voltage amplitude is nearly 54.72 V. The participation of other higher harmonics of fundamental frequency is shown in an insight zoomed figure in Fig. 5.11(b) where their participation is weak as compared to the fundamental harmonic. The amplitude modulation may be due to certain imbalances, which causes minor irregularities in energy transfer.

In Fig. 5.12, the variation of output voltage with nondimensional distance (β) and frequency of excitation (f) along with variation of f with β is shown while keeping $m = 27$ gm and load resistance $R_l = 100$ kΩ invariant. The average voltage is

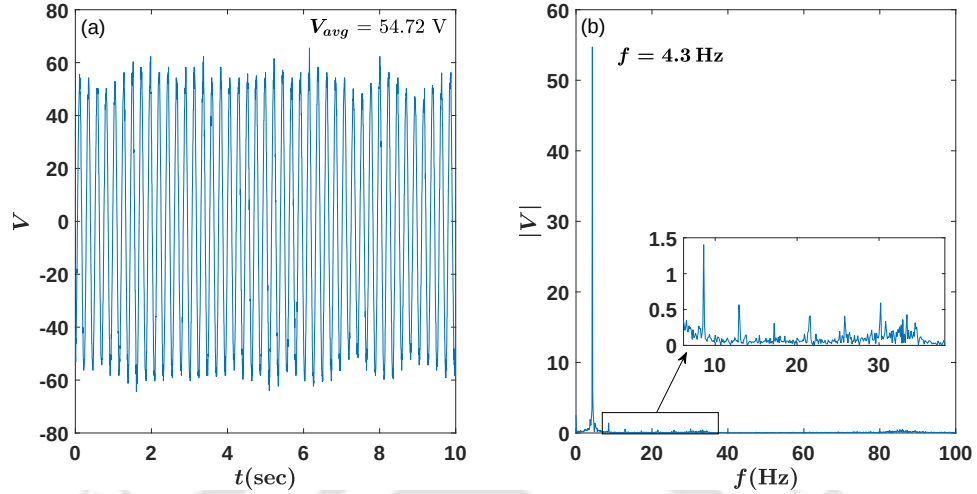


FIGURE 5.11: Experimental (a) voltage-time response for load resistance $R_l = 800 \text{ k}\Omega$ and (b) FFT of the voltage-time response

kept stable within a certain range by keeping the transverse displacement near the triggering onset frequency of excitation as β and f increases. In Fig. 5.12(c) as β increases the threshold frequency at which beam starts transverse vibration reduces linearly. By a suitably position of the attached mass m , one may adjust the natural frequency of the system and critical threshold excitation value can be reduced further. Voltage time response at specific point P_1 located in Fig. 5.12(a) is shown in Fig. 5.12(d) where at $\beta = 0.533$ the output voltage is found to be $V = 13.58 \text{ V}$.

The voltage-time response at point P_2 as mentioned in Fig. 5.12(a), are shown in Fig. 5.13(a). By observing Fig. 5.13(a), it is found that the pattern of the response changes (ellipsoidal zone), which denotes the patch abnormalities caused by sudden high deformation. This pattern carries forward in the responses shown in Fig. 5.13(b, c, d) for $\beta = 0.833$. High voltage peaks are also observed in Fig. 5.13(b,c) and 5.13(d), which further provide concrete proof that PZT patch has suffered some permanent structural damage. One conclusion can be drawn from here is that beyond a certain transverse deflection of beam system the PZT patch may get damages. The energy extraction from a broken piezo patch poses challenges related to the circuitry section.

This is probably the first time when time response from a broken patch is analysed. However to avoid such kind of disruption in energy transduction two possible solutions are suggested. First, the large amplitude vibration may be restricted by using

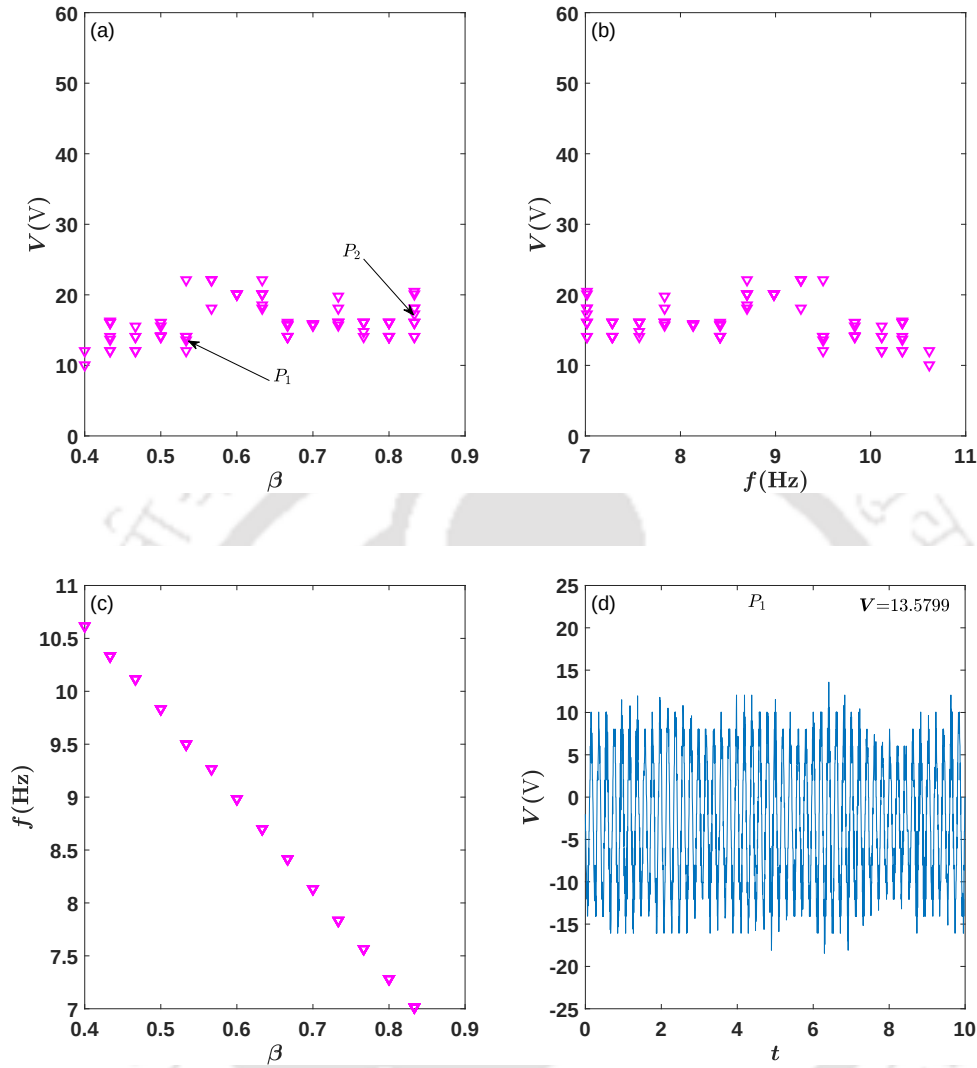


FIGURE 5.12: Experimental findings of (a) voltage with nondimensional distance of mass from the fixed end of the beam (β) (b) corresponding voltage response (c) frequency of excitation with (β) and (d) voltage-time response at point P_1 located in Fig. 5.11(a) while keeping attached mass $m = 27$ gm and $R_l = 100$ k Ω constant.

stoppers and second, the more flexible MFC (micro-fiber-composites) may be used instead of using brittle PZT patches. Apart from MFC's, a wide range of flexible piezoelectric materials [139] are available, which can be integrated into the structures to not only harvest energy but in wearable devices for sensing purpose as well. It also allows using post buckle structures and curved surfaces. However, use of MFC slightly increases the cost of the system. Another challenge is to avoid the cracks

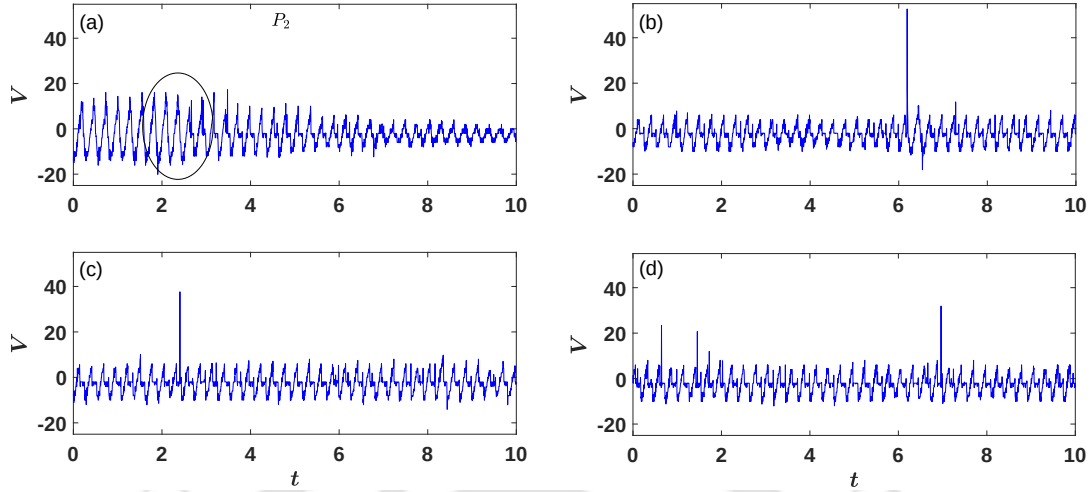


FIGURE 5.13: Experimental findings of voltage-time response, where mass $m = 27$ gm, $\beta = 0.833$ and $R_t = 100$ k Ω

near the fixed end of the cantilever beam system for long-lasting use and keeping the structural integrity intact. The best way to avoid this failure is by providing concave shaped supporting structure of suitable radius [308] to the base attachment element (holder) or using stoppers [309–311] to prevent large transverse displacement. The provision of mass at an arbitrary position and its value adjustment is helpful in order to tune the frequency of the harvester as per the available ambient frequency of excitation.

A qualitative comparison of several energy harvesters is presented in Table 5.3. Comparison has been made based on the type of used piezoelectric patch (viz., PZT, MFC or PVDF), its configuration (unimorph or bimorph), type of external excitation (direct, parametric), orientation of the specimen (horizontal or vertical), nature of study (analytical or experimental), length covered by the piezoelectric patch (partial or full) and transduction mechanism utilized (piezoelectric or electromagnetic). One sample case [21] is described here and the interpretation in a similar manner for the other cases can be drawn from Table 5.3. Daqaq et al. [21] were the first to analyze energy harvesting via parametric excitation. In this work, a lumped parameter model of the cantilever beam with a tip mass and MFC patch is used. The beam is kept in the horizontal position in this experimental study. It may be noted that in the present work, the cantilever beam is placed in the vertical position where the effect of gravity is accommodated in the dynamics of the system. Also, the mass can be attached to any arbitrary position along the beam and PZT-5H

patch is used in the present work. Abdelkefi et al. [181] studied a similar system where a nonlinear distributed parameter model is developed and the effect of nonlinear piezoelectric coupling coefficients on the output voltage and power are analyzed analytically. The unimorph piezoelectric patch spans the whole length of the beam and any issues in using PZTs under parametric excitation are not reported. Few systems utilized the parametrically excited energy harvesters based on electromagnetic transduction mechanism [202, 205, 312] as well. In such systems, the volume of the system is generally high as compared to the harvesters based on piezoelectric transduction.

TABLE 5.3: Qualitative comparison of the present work with previous literature

References	PZT/ MFC	Type: (U,B) (P,D,AP) (H,V)	A/Exp	Length covered	PE/ EM
Daqaq et al. [21]	MFC	U P H	A+Exp	Partial	PE
Abdelkefi et al. [181]	PZT	U P H	A	Full	PE
Friswell et al. [64]	MFC	U D V	A+Exp	Partial	PE
Zhu et al. [313]	PVDF	U P V	A+Exp	Partial	PE
Jia et al. [202]	—	—	A+Exp	—	EM
Jia and Seshia [207]	PZT	B P-AP V	A+Exp	Partial	PE
Yildirim et al. [205]	—	—	A+Exp	—	EM
Yan and Hajj [208]	PZT	B P V	A	Full	PE
Yildirim et al. [22]	MFC	U P H	A+Exp	Partial	PE
Mam et al. [314]	PZT	B D-P V	A+Exp	Partial	PE
Panyam et al. [315]	MFC	U P H	A+Exp	Partial	PE
Searle et al. [309]	MFC	U D-P H	A+Exp	Partial	PE
Bahareh et al. [316]	PZT	U P H	A+Exp	Partial	PE
Garg and Dwivedy [301]	MFC	B P V	A	Full	PE
Kuang and Zhu [312]	—	—	A+Exp	—	EM
Xia et al. [223]	PZT	B D-P H	A	Full	PE
Present work	PZT	U P V	A+Exp	Partial	PE

U $\rightarrow$ Unimorph, B $\rightarrow$ Bimorph, P $\rightarrow$ Parametric, D $\rightarrow$ Direct,

AP $\rightarrow$ AutoParametric, H $\rightarrow$ Horizontal V $\rightarrow$ Vertical, A $\rightarrow$ Analytical,

Exp $\rightarrow$ Experimental, PE $\rightarrow$ Piezoelectric, EM $\rightarrow$ Electromagnetic

A combination of direct excitation along with parametric excitation [223, 309, 314] or autparametric with parametric [207] is also explored for performance enhancement. The emphasis is given on harvesting from low frequency direct excitation and post-buckled beam system in the work of [64]. Zhu et al. [313] and Panyam et al. [315] explore the effect of axial load in PEH system where the former used piezofilm. Bahareh et al. [316] analyzed the PEH system with time varying stiffness. In the previous work of Garg and Dwivedy [301] a system similar to the present one is studied where it undergoes parametric and 1:3 internal resonance. Here due to the internal resonance, the first two modes of the system interacts and the energy transfer takes place between them which helps in extracting more output power and enhances the bandwidth of the harvester. Without considering internal resonance and using unimorph PZT in the limited span instead of bimorph MFC, the system can be reduced to the present system.

The studies of PEH systems mentioned in Table 5.3 are extended in Table 5.4 for the quantitative comparison of the normalized power density. Here, the parameters considered for the characterization of the harvesting system are the input acceleration, volume, frequency, maximum power output and normalized power density, (NPD). NPD is determined using the expression ($\text{NPD} = \text{power}/(\text{volume} \times \text{acceleration}^2)$), which is taken from the work of Beeby et al. [317]). The present system yields a maximum power output and NPD (in $\mu\text{W cm}^{-3}\text{m}^{-2}\text{s}^4$) value of 22.7 mW and of 15.9 respectively. It may be observed from the Table 5.4 that the maximum power obtained in the present case is better than most of the harvesters presented in that table. It is less than from the work of Abdelkefi et al. [181] where full-length patch was taken. Also it is found to be less than that of Yan and Hajj [208] where very high acceleration was applied and the volume is higher than the present case. Now comparing NPD, it is found to be better than most of the cases. It is less than that of Friswell et al. [64] where the acceleration is very less, but it may be noted that the output power is higher in the present case than that of Friswell et al. [64].

In comparison with the previous studies where nonlinear (geometric and inertial type) parametric excited harvesters are analysed the present work differs mainly in placing the attached mass in arbitrary position along the beam length and keeping the beam in vertical position instead horizontal in order to let gravity assist in large transverse displacement of the beam system. Also, the present work is the first in reporting the challenges in using PZT and cracks in the beam system due to large

deformations where previous works failed to do so. The future work involves the experimental investigation on how in such kind of parametrically excited systems one may bring more stability.

TABLE 5.4: Quantitative comparison of the present work with previous literature

References	Acc. (ms^{-2})	Vol. (cm^3)	Freq. (Hz)	Max Power Output	Normalized Power Density (NPD) $\mu\text{W cm}^{-3}\text{m}^{-2}\text{s}^4$
Daqaq et al. [21]	3.14	9.51	5.37	0.06mW	6.42E - 01
Abdelkefi et al. [181]	12	11.7	—	80mW	4.75E + 01
Friswell et al. [64]	1.19	1.23	2.45	0.2mW	1.15E + 02
Zhu et al. [313]	6.37	2.66	9	—	3.30E - 03
Jia et al. [202]	0.57	1800	3.57	171.5mW	2.93E + 02
Jia and Seshia [207]	20	1.21	20	0.15mW	3.10E - 01
Yildirim et al. [205]	9	—	54.4	—	4.14E - 04
Yan and Hajj [208]	98.3	170.5	8.44	1.4W	8.50E - 01
Yildirim et al. [22]	20	1.12	12.1	2.15mW	1.87E + 00
Mam et al. [314]	57.13	10.92	29.8	0.96mW	2.70E - 02
Panyam et al. [315]	60	1.51	224	0.04mW	1.66E - 02
Searle et al. [309]	—	4.1	10.8	0.068mW	—
Bahareh et al. [316]	47	15.44	10.9	0.0087mW	2.55E - 04
Garg and Dwivedy [301]	36.9	3.94	15.3	9W	1.67E + 03
Kuang and Zhu [312]	4.9	—	18	3.6mW	—
Xia et al. [223]	—	0.6	—	0.36W	—
Present work	13.38	7.98	7.22	22.7mW	1.59E + 01

5.9 Experimental Findings with MFC Patch

Unlike in the previous section where PZT-5H patch has been used for harvesting energy, in this section micro fibre composite (MFC) patch has been used for experimentally finding the output voltage and power. The material and geometric properties of substrate and piezoelectric patch (MFC) are mentioned in Table 5.5. The beam attached with MFC patch and arbitrary mass is shown in Fig. 5.14.

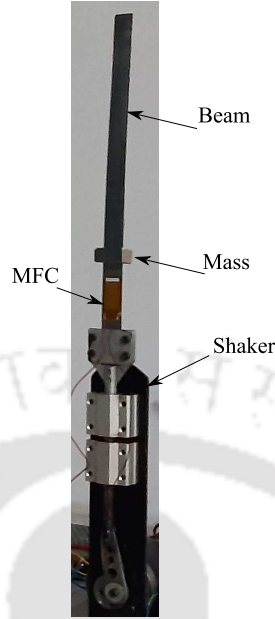


FIGURE 5.14: A vertical beam with unimorph MFC patch and an attached mass

Similar to the previous section here also the beam is attached to a slider crank mechanism based shaker. The MFC patch is attached using 3M 465 film tape. Unlike the PZT ceramics, the MFC patch can be detached from the substrate for reuse purpose. Using the Oscilloscope the rms voltage V_{rms} , maximum voltage

TABLE 5.5: Material and geometric properties of substrate and piezoelectric patch

Property	Piezo Patch	Sym	Substrate-S1	Substrate-S2
Young's modulus, GPa	$E_p = 15.86$	E_b	190	190
Density, kg m^{-3}	$\rho_p = 5440$	ρ_b	7800	7800
Length, m	$L_p = 28 \times 10^{-3}$	L_b	270×10^{-3}	190×10^{-3}
Width, m	$b_p = 14 \times 10^{-3}$	b_b	14×10^{-3}	14×10^{-3}
Height, m	$t_p = 0.30 \times 10^{-3}$	t_b	0.25×10^{-3}	0.25×10^{-3}
Permittivity, nFm^{-1}	$\hat{e} = 19.36$	—	—	—
Piezoelectric constant, Cm^{-2}	$e_{31} = -19.84$	—	—	—
L_1 , m	—	—	20×10^{-3}	10×10^{-3}
L_2 , m	—	—	48×10^{-3}	38×10^{-3}
Subscript	$p:\text{piezo}$	$b:\text{beam}$		

V_{max} are determined and plotted in Fig. 5.15 and Fig. 5.16 for various values of attached mass and load resistance, respectively. The experimentally obtained data are shown using the symbols either triangle or the diamond. Using the

curve fitting method the variation is also shown by solid lines. The effect of variation of attached mass m on the output voltage and power is observed in Fig. 5.15 by keeping the load resistance $R_l = 500 \text{ k}\Omega$ and position $\beta = 0.233$. It is observed that the maximum power achieved when $m = 36 \text{ gm}$. It may be noted from the same figure that the frequency of excitation decreases as m increases.

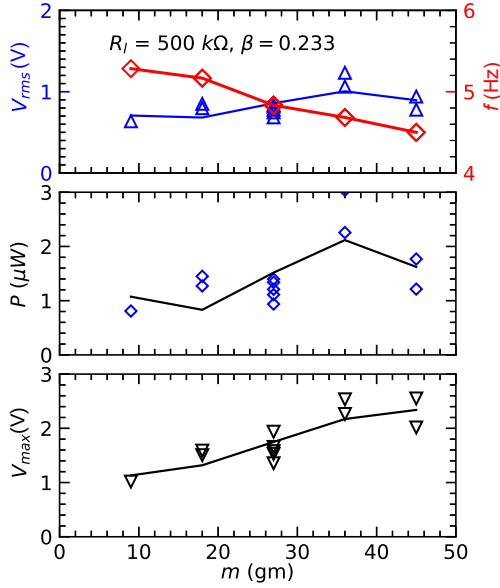


FIGURE 5.15: Variation of V_{rms} , P and V_{max} with attached mass m

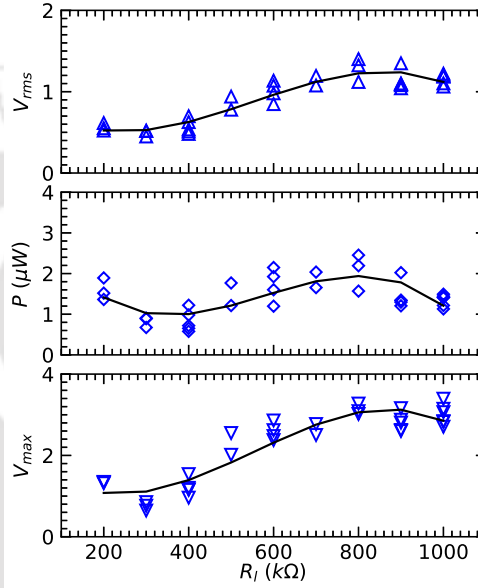


FIGURE 5.16: Variation of V_{rms} , P and V_{max} with load resistance R_l

The output voltage and power is plotted against external load resistance R_l in Fig. 5.16. The voltage increases with R_l while power P achieves its maximum value when $R_l = 800 \text{ k}\Omega$. Here the frequency of excitation is kept around $f = 4.5 \text{ Hz}$. The voltage time response and FFT, corresponds to the specific points in Fig. 5.15 and Fig. 5.16 respectively are shown in Fig. 5.17. Figure 5.17(a) and (b) shows the time response and FFT related to mass value $m = 36 \text{ gm}$. Two peaks are observed in FFT, showing that the higher peak is near the excitation frequency (synchronous frequency) and the other one is observed at 2.5 Hz which is similar to the sub-synchronous frequency that are generally observed in rotor bearing systems. It may be noted that the 2.5 Hz frequency is same as the first mode frequency of the system. Similarly the time response and FFT for $R_l = 800 \text{ k}\Omega$ in Fig. 5.16 is shown in Fig. 5.17(c) and (d) where a maximum power of $2 \text{ }\mu\text{W}$ is observed. It is

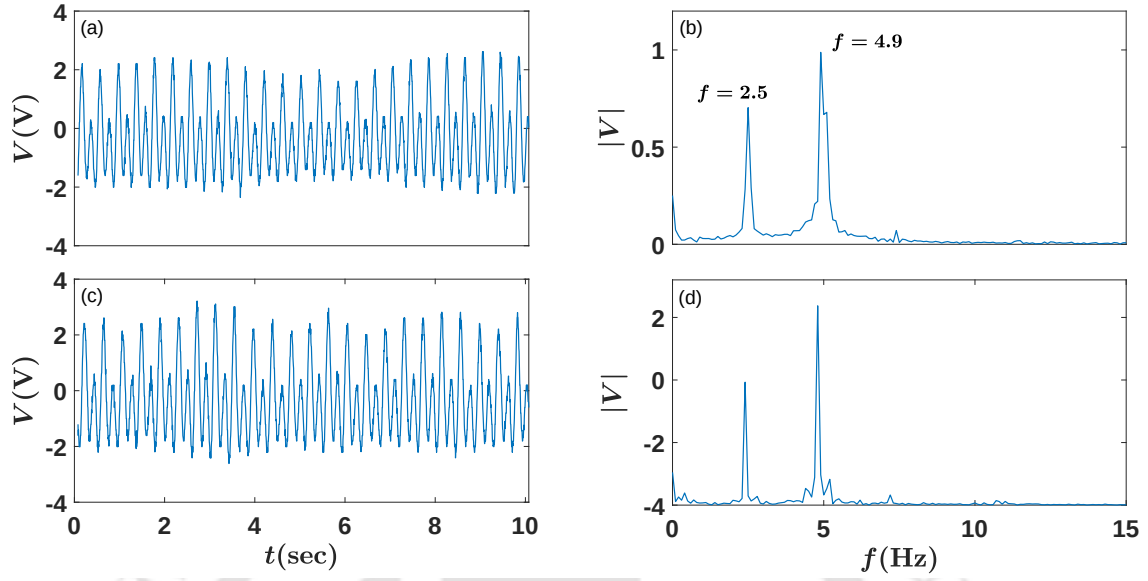


FIGURE 5.17: Voltage time response and FFT for (a) mass value $m = 36gm$ in Fig. 5.15 (b) FFT of (a) (c) at $R_l = 800 k\Omega$ in Fig. 5.16 (d) FFT of (c)

to be noted that the MFC patch covers only 5.2% of the total surface area of the substrate.

Figure 5.18 shows the effect of the position of attached mass β on output voltage and power by keeping the load resistance $R_l = 500 k\Omega$ and mass $m = 18 gm$. The voltage and power output increases initially then almost remain constant with increase in β . One may get an output power of $1.45 \mu W$ at $\beta = 0.296$. Also as the position of the mass along the beam goes towards the free end, the excitation frequency decreases. Figure 5.19 shows the variation output voltage and power with external load resistance R_l . The voltage increases with R_l while power P achieves its maximum value of $0.6 \mu W$ when $R_l = 700 k\Omega$. The frequency of excitation in this case is kept nearly 10 Hz.

Figure 5.20(a) and (b) shows the voltage time response and FFT for $\beta = 0.296$ with other system parameters as in Fig. 5.18. Similar to the previous case, two frequency peaks at 2.5 Hz and 4.9 Hz are obtained. Further, in Fig. 5.20(c) and (d), the voltage time response and FFT corresponds to $R_l = 1000 k\Omega$ is plotted, where other system parameters as in Fig. 5.19. Here several frequency peaks are observed in the FFT, which is due to the energy interaction between participating modes.

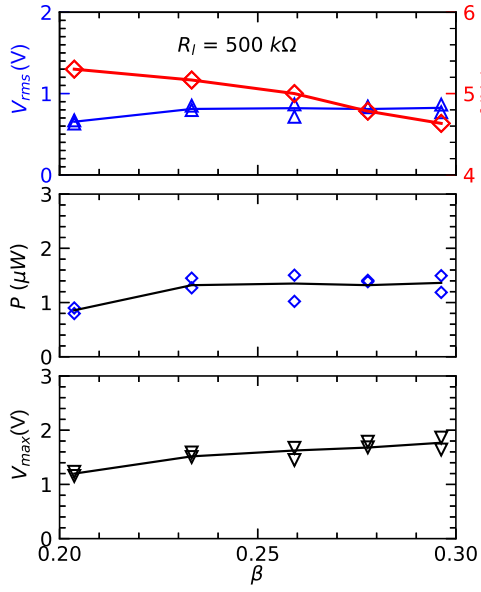


FIGURE 5.18: Variation of V_{rms} , P and V_{max} with the position of attached mass β

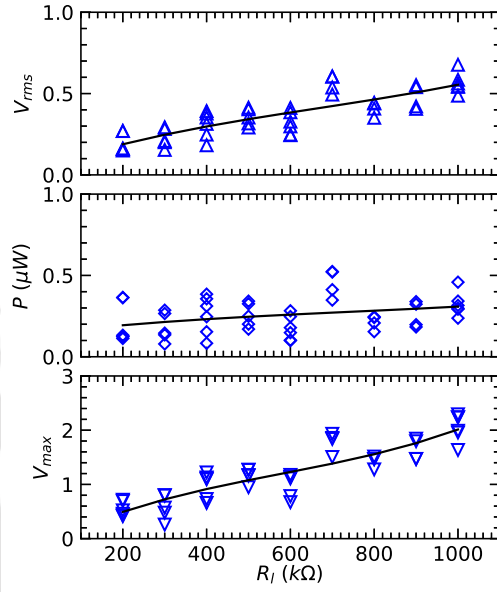


FIGURE 5.19: Variation of V_{rms} , P and V_{max} with load resistance R_l .

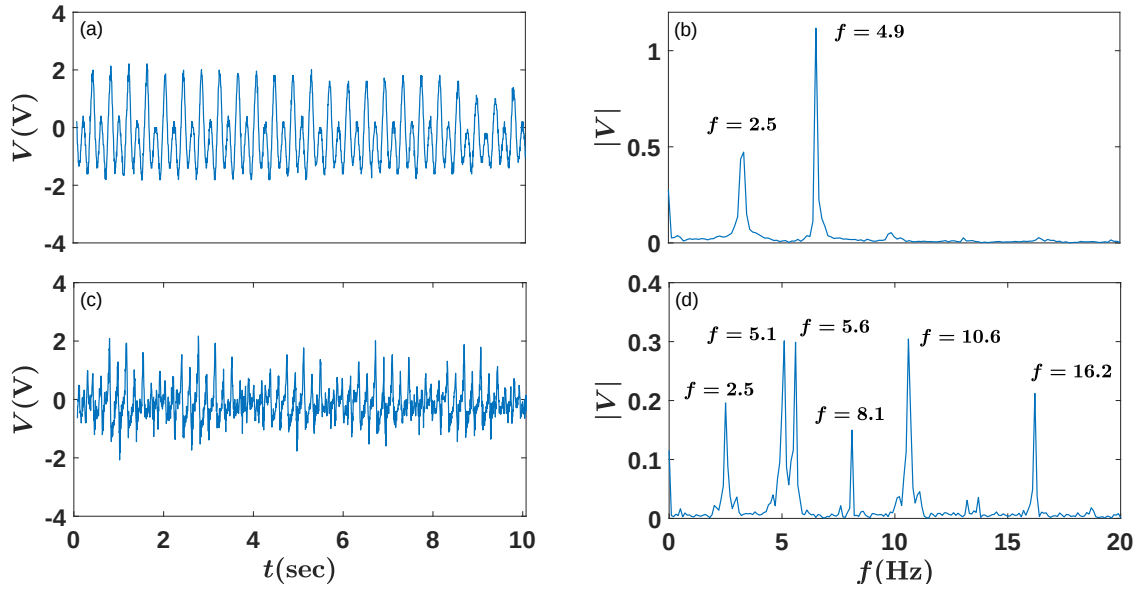


FIGURE 5.20: Voltage time response (a) for $\beta = 0.296$ (Fig. 5.18) (b) corresponding FFT (c) voltage time response at $R_l = 1000 \text{ k}\Omega$ (Fig. 5.19) (d) corresponding FFT plot.

The effect of length of the substrate and position of the MFC patch are also studied and plotted in Figs. 5.21 to 5.23. As given in Table 5.5 length of the substrate (S2) is lesser than that of substrate (S1). With decrease in the length of the beam the natural frequency of the system increases from 2.5 Hz to 4.3 Hz. Here the mass, its position along the beam and load resistance values are taken as $m = 36$ gm, $\beta = 0.263$ and $R_l = 1000$ k Ω .

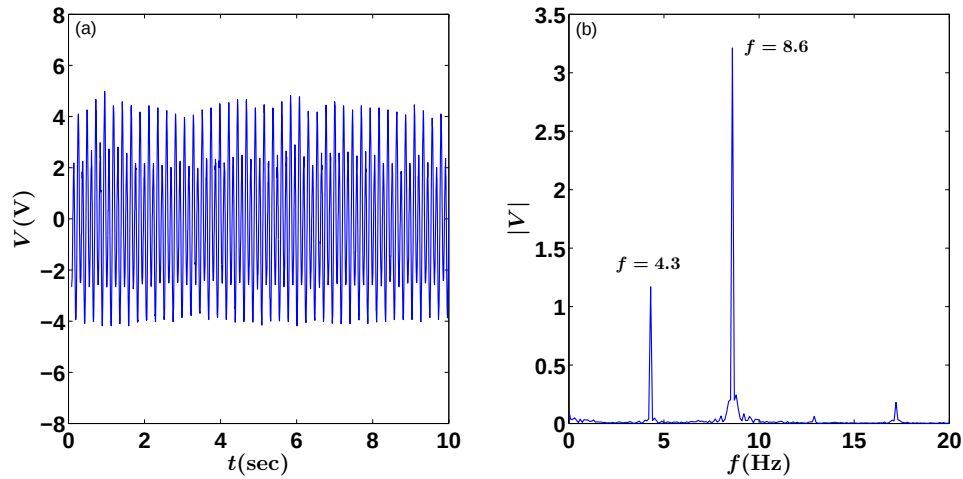


FIGURE 5.21: (a) Voltage time response and (b) corresponding FFT, for parameters $m = 36$ gm, $\beta = 0.263$, $R_l = 1000$ k Ω and frequency of excitation around 8 Hz

Figure 5.21 shows the voltage time response and corresponding FFT when the frequency of excitation is kept nearly 8 Hz, which is close to twice of the first natural frequency of the system with substrate S2. As shown in Fig. 5.21(b), which are similar to other figures, here also two peaks are observed with frequency 4.3 Hz and 8.6 Hz and they are in 1:2 ratio. In S2 the MFC patch is close to the fixed end of the beam (see Table 5.5) as compared to that of the system with S1. This leads to higher strain and, consequently more output voltage around 5 V and power of 25 μ W, which is significantly higher as compared to the previous case where S1 is considered with longer beam length and position of the MFC patch is slightly far away from the fixed end.

When the excitation frequency increases to that of combination of first two natural frequencies of the system the voltage time response changes (shown in Fig. 5.22a) and higher frequencies dominates the time response which is visible in FFT (in

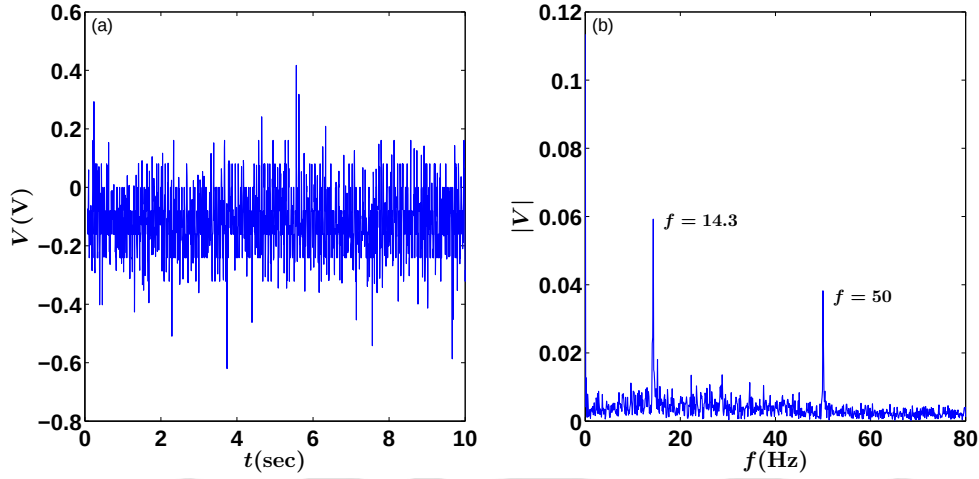


FIGURE 5.22: (a) Voltage time response and (b) corresponding FFT, for parameters $m = 36$ gm, $\beta = 0.263$, $R_l = 1000$ k Ω . The frequency of excitation is 13 Hz which is around the combination of first two modal frequencies.

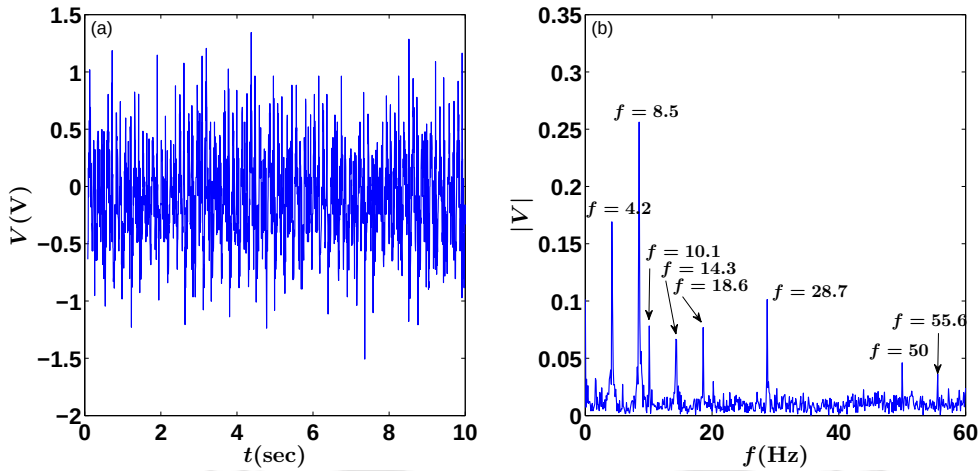


FIGURE 5.23: (a) Voltage time response and (b) corresponding FFT, for parameters $m = 36$ gm, $\beta = 0.263$, $R_l = 1000$ k Ω and frequency of excitation is positively detuned to that of 13 Hz

5.22b). Further a positive detuning leads to multimodal voltage time response (in Fig. 5.23a) with multiple frequency components in FFT (in Fig. 5.23b).

5.10 Summary

In the present analysis, the vertical cantilever beam based piezoelectric energy harvester under parametric excitation is investigated theoretically and experimentally. The system consists of an arbitrary positioned mass, and piezoelectric patch near its fixed end. The nonlinear coupled electromechanical governing equations of motions are derived by using Hamilton's principle and method of multiple scales is used to reduce the second order nonlinear governing equations to a set of first order reduced equations that are used for finding the steady state response and the stability of the system. The results from these equations are compared with those available in the literature and by developing an experimental setup.

A shaker with piezoelectric energy harvester is developed to experimentally validate the theoretical findings of the vertical beam based harvester. The limiting supercritical bifurcation point at which the nontrivial state is observed theoretically is found to be in good agreement from the experimental findings. Further the theoretical and experimental responses are found to be in good agreement. The maximum output voltage obtained are also found to be in close agreement. The output voltage and power are compared with those found in the literature. It is observed that the present vertical cantilever beam based harvester nearly produced more output voltage and power than that of a horizontally placed cantilever beam with same configurations. Hence one may use the developed reduced equations for finding the response, voltage and power for similar piezoelectric energy harvester. A broadband range of frequency between the subcritical and supercritical pitchfork bifurcation points may be used to harvest the energy. The amount of harvested voltage and power depends on the system parameter, which may suitably be adjusted by using the developed reduced equations to harvest more energy. The challenges of cracking in beam and breaking of piezo-patch during experiments are discussed in this work. In addition, some suggestion on how the challenges can be overcome is also discussed for system stability and durability. A qualitative and quantitative comparison of 17 energy harvesters have been carried out and the present harvester is found to be better than most of the harvesters in terms of the normalized power density. The present experimental and theoretical investigation will help to overcome the challenges posed by the parametrically excited energy harvester.

The experimental findings suggests that modal interaction takes place when there

exist a commensurable ratio between the frequencies of the system. The variation in position of mass and its value can be altered as per available excitations. The position of piezoelectric patch and length of the substrate affects the power output. More closer the patch to the fixed end more power output is observed. Here MFC patch is found to be stable under large strain due to parametric excitation.



Chapter 6

Dynamics of Galloping Based Piezoelectric Energy Harvester

6.1 Introduction

While in the previous chapters harmonically base excited piezoelectric energy harvester has been considered, in the present chapter wind based energy harvester is studied. Here a similar structure, i.e., a cantilever beam with piezoelectric patches having a bluff body near its free end subjected to wind force, is considered. The system dynamics under uniform and harmonically varying wind flow conditions are investigated. Experimental investigations have been carried out to validate the theoretical findings. In section 6.2 the mathematical modelling for galloping based PEH has been discussed. In section 6.3 the solution of the temporal equations is solved by using the method of multiple scales. In the next section 6.4 the theoretical and experimental results are presented and discussed.

6.2 Mathematical Modelling

A vertical cantilever beam of length L , having an unimorph piezoelectric patch (PZT-5H) (length L_p) near its fixed end, subjected to the wind speed U^* is considered for analysis. A bluff body of mass m near its free end is attached at a distance d from its fixed end as shown in the Fig. 6.1. The bluff body is subjected to an aerodynamic lift force in the direction transverse to the wind flow. Similar to the previous chapters the beam can be modelled as an Euler Bernoulli beam and the

equations of motion for the system can be derived by using extended Hamilton's principle [318].

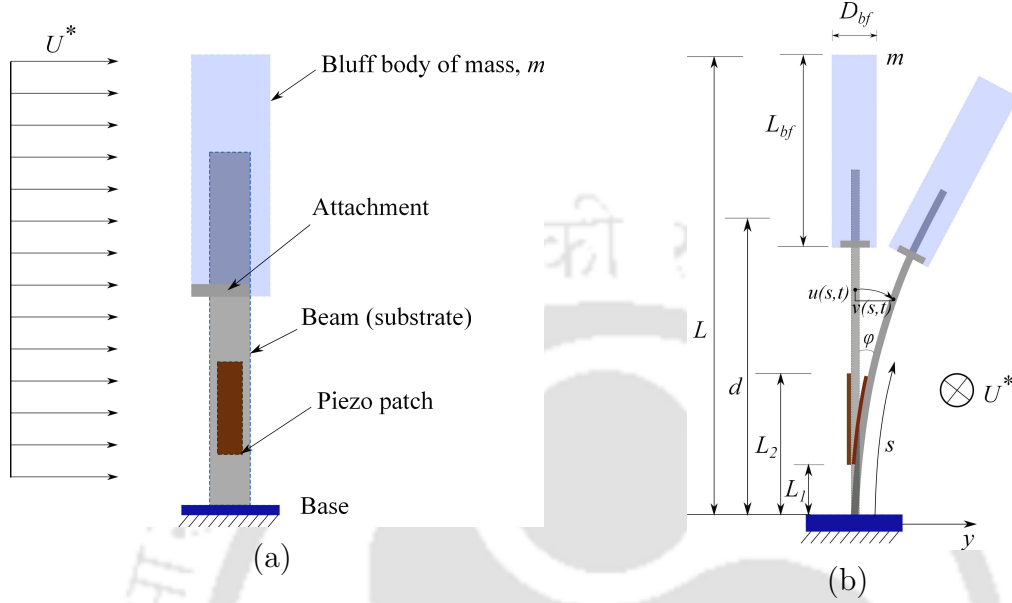


FIGURE 6.1: Schematic diagram of the galloping based PEH system (a) view transverse to the wind flow (b) view along the wind flow

The kinetic energy T and the potential energy Π of the system in terms of curvature $\kappa(s, t)$ longitudinal displacement $u(s, t)$ and transverse displacement $v(s, t)$ are given by the following equations.

$$T = \frac{1}{2} \int_0^L [\rho_b A_b + (H_{L_1} - H_{L_2}) \rho_p A_p + m \delta(s - d)] [\dot{v}(s, t)^2 + \dot{u}(s, t)^2] ds + \frac{1}{2} I_d \dot{\varphi}^2 \quad (6.1)$$

$$\Pi = \frac{1}{2} \int_0^L E_b I_b \kappa(s, t)^2 ds - \int_0^L [\rho_b A_b + (H_{L_1} - H_{L_2}) \rho_p A_p + m \delta(s - d)] g u(s, t) ds \quad (6.2)$$

Here the subscript 'b', 'p' and 'd' are used for beam, piezoelectric patch and distance d , respectively. Also, $(\dot{})$, ρ_b , ρ_p , A_b , A_p , I_d , E_b , I_b , g , φ and $\delta(\cdot)$ denotes the time derivative, mass density of beam per unit length, mass density of patch per unit length, cross sectional area of beam, cross sectional area of patch, cross sectional moment of inertia of mass m at a distance d , modulus of elasticity of beam, cross sectional moment of inertia of beam, gravity constant, angle of rotation of the beam in the transverse direction and the Dirac delta respectively. The external virtual

work done W_{nc} on the system due to the aerodynamic lift force and piezoelectric patch is given by the following expression.

$$\begin{aligned} \delta W_{nc} = & \int_0^L F_g \delta(s-d) \delta v(s,t) ds + \int_0^L M_g \delta(s-d) \delta v_s(s,t) ds \\ & + \int_0^L (H_{L_1} - H_{L_2}) M_p(s,t) \delta \kappa(s,t) ds \end{aligned} \quad (6.3)$$

Here subscript g is used for galloping related terms. Also F_g , M_g , $(\cdot)_s$ and M_p denote the aerodynamic force due to lift force, moment due to lift force (caused by galloping phenomenon), the derivative with respect to the curvilinear coordinate 's' and the moment in the piezoelectric patch due to the generated voltage across the patch [319]. The expression for F_g , M_g and M_p are as follows (details can be found in the Appendix D).

$$F_g(t) = -F_a \int_0^{L_{bf}} \left[a_1 \left(\frac{\dot{v}(d,t) + s\dot{v}'(d,t)}{U} \right) + a_3 \left(\frac{\dot{v}(d,t) + s\dot{v}'(d,t)}{U} \right)^3 \right] ds \quad (6.4)$$

$$M_g(t) = -F_a \int_0^{L_{bf}} s \left[a_1 \left(\frac{\dot{v}(d,t) + s\dot{v}'(d,t)}{U} \right) + a_3 \left(\frac{\dot{v}(d,t) + s\dot{v}'(d,t)}{U} \right)^3 \right] ds \quad (6.5)$$

$$M_p(s,t) = \gamma_c \bar{V}(t); \quad \gamma_c = E_p d_{31} b_p (t_b + t_p) \quad (6.6)$$

Here,

$$F_a = \frac{1}{2} \rho_a D_{bf} [U^2 + 2U \bar{u}(t)]$$

Also, ρ_a , D_{bf} and U and $\bar{u}$ denote the air density, width of the bluff body, uniform wind speed and the time varying wind speed respectively. Also a_1 and a_3 are the empirical coefficients [1] which depends on the cross sectional geometries of the bluff bodies. Using the extended Hamilton's principle $\int_{t_1}^{t_2} (\delta T - \delta \Pi + \delta W_{nc}) dt$, the nonlinear coupled electromechanical governing equation of motion of the GPEH

system can be expressed as follows.

$$\begin{aligned} & \{\rho_b A_b + (H_{L_1} - H_{L_2}) \rho_p A_p + m\delta(s - d)\} v_{tt} + cv_t \\ & + \{E_b I_b + (H_{L_1} - H_{L_2}) E_p I_p\} [v_{ssss} + \{v_s(v_s v_{ss})_s\}_s] - [I_d \delta(s - d)(v_s)_{tt}]_s \\ & - (N v_s)_s - H_{ss} \bar{\eta} \bar{V} = F(t) \end{aligned} \quad (6.7)$$

$$\begin{aligned} H &= H_{L_1} - H_{L_2} = H(s - L_1) - H(s - L_2), \quad \bar{\eta} = \frac{e_{31} b_p}{t_p} \left[(\bar{t} - \bar{y})^2 - \left(\frac{t_b}{2} - \bar{y}\right)^2 \right], \\ C_p &= \frac{\hat{e} b_p L_p}{t_p}; \quad A_b = b_b t_b, \quad A_p = b_p t_p, \quad \bar{t} = \left(t_p + \frac{t_b}{2}\right), \quad t_{pc} = \frac{1}{2}(t_p + t_b) - \bar{y}; \quad e_{31} = E_p d_{31}, \\ F(t) &= \delta(s - d) F_g - \delta_s(s - d) M_g, \quad L_p = L_2 - L_1, \\ I_p &= \frac{1}{3} b_p \left(\bar{t}^3 - \frac{t_b^3}{8} \right), \quad I_b = \frac{b_b t_b^3}{12}, \quad \bar{y} = \frac{E_p t_p b_p (t_p + t_b)}{2(E_p t_p b_p + E_b t_b b_b)}, \\ N &= \frac{1}{2} \int_s^L [\rho_b A_b + (H_{d_1} - H_{d_2}) \rho_p A_p + m\delta(\xi - d)] \left(\int_0^\xi (v_\eta^2)_{tt} d\eta \right) d\xi \\ &- \int_s^L [\rho_b A_b + (H_{d_1} - H_{d_2}) \rho_p A_p + m\delta(\xi - d)] g d\xi - I_d \delta(s - d) \left\{ \frac{1}{2} v_{stt} v_s^2 + v_s v_{st}^2 \right\}. \end{aligned}$$

Here H , $\bar{\eta}$, N , C_p , $\hat{e}$ and c denotes the Heaviside function, coupling term, the longitudinal load at 's', capacitance of piezoelectric patch, permittivity component and viscous damping coefficient respectively. Also, d_{31} , e_{31} , E_p , I_p , L_p , b_b , b_p , t_b , t_p and $\bar{y}$ are the piezoelectric strain constant, piezoelectric stress constant, Young's modulus of the piezoelectric patch, moment of inertia of the patch, length of the patch, width of beam, width of patch, thickness of beam, thickness of the patch and shift in the neutral axis (for, $L_1 \leq s \leq L_2$) respectively. Also, $F(t)$ denotes the total galloping force on the system. The expression for transverse force F_g and the moment M_g due to aerodynamic wind load is expressed as follows. One may find the detailed modelling of the wind load in the Appendix D. The electric circuit equation can be obtain by applying the Gauss's law [14, 23, 170].

$$\frac{dQ(t)}{dt} = \frac{d}{dt} \int_{A_p} \mathbf{D} \cdot \mathbf{n} dA_p \quad (6.8)$$

Where the electric displacement vector ($\mathbf{D}$) is defined by the constitutive equation $D_3 = d_{31}\sigma_p + \hat{e} E_e$

$$C_p \bar{V}_t + \frac{1}{R_l} \bar{V} + e_{31} t_{pc} b_p \int_{L_1}^{L_2} (v_{ss})_t ds = 0 \quad (6.9)$$

Here Q , n , σ_p and E_e , R_l and $\bar{V}$ are defined as electric charge, unit normal vector, stress induced in piezoelectric element, electric field, load resistance and the voltage across the external load R_l respectively.

It may be noted that the governing Eq. (6.2) is similar to the equations studied in the previous chapters without considering the wind force and the term related to the parametric excitation. While in theoretical work of Abdelkefi et al. [1] a horizontal cantilever beam attached with bluff body at the tip is considered under uniform wind flow where cubic and inertial nonlinearities are not taken into account, in the present system the beam is vertically positioned and the model accommodate both nonlinearities due to large transverse deformation. Also the bluff body can be positioned along the beam length arbitrarily and the system is investigated under time varying flow speeds as well. Sirohi and Mahadik [85] analysed the similar system (as that of Abdelkefi et al. [1]) experimentally and analytically. Here a D shape bluff body is considered and a power output of nearly 1 mW is achieved at a wind speed of around 4.5 m/s. Also, it may be noted that the time varying wind model with single frequency component which is considered in the present study is adopted from the work of Abdel-Rohman [123, 124].

6.2.1 Steady and unsteady wind flow

Figure 6.2 shows the aerodynamic lift force (F_L) and drag force (F_D) on the D-shape bluff body. Here U^* , dv/dt , U_{rel}^* and α denote the wind speed, transverse velocity of the beam, the relative wind speed and the angle of attack respectively. The wind speed U^* is considered of having both steady U and unsteady $\bar{u}$ components.

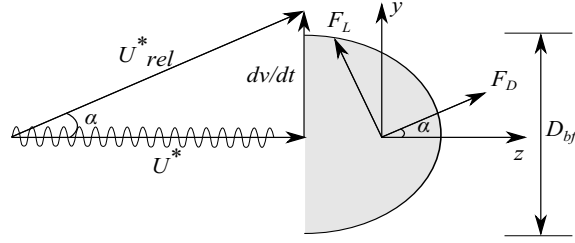


FIGURE 6.2: Aerodynamic lift force (F_L) and drag force (F_D) on the D-shape bluff body

$$U^* = U(s) + \bar{u}(s, t) = U_{rel} \cos \alpha \quad (6.10)$$

$$U_{rel}^2 \approx [U^2(s) + 2U(s)\bar{u}(s, t)] \sec^2 \alpha \quad (6.11)$$

Here, the unsteady component is considered as harmonic. The present study is limited to the effect of harmonically varying unsteady wind component on the voltage and power output. This wind model is similar to that considered in the work of Abdel-Rohman [124]. Similar wind model may be applicable for the wake galloping based energy harvesters as well. The fluctuations in the wind speed due to vortex shedding may also be modelled as harmonic variation where the frequency of oscillation is vortex-shedding frequency. Here, in the present study the unsteady effect in the transverse direction is neglected due to its negligible effect. In general the wind speed varies with the altitude, however in the present case it is assumed to be invariant to the spatial coordinate 's', due to having limited dimensions.

$$\bar{u}(s, t) = u_0 \cos(\Omega t) \quad (6.12a)$$

$$\bar{v}(s, t) \approx 0 \quad (6.12b)$$

Here u_0 and Ω denotes the amplitude and frequency of assumed unsteady harmonic component. By considering only single mode approximation in the Galerkin's method the transverse displacement of the GPEH system $v(s, t) = r \psi(s) q(t)$, where r is the scaling factor, $q(t)$ is the time modulation and $\psi(s)$ is the shape function of the system, the nondimensionalized nonlinear coupled electromechanical temporal governing equations of motion can be expressed as follows.

$$\ddot{q} + 2\epsilon\zeta\dot{q} + q + \epsilon \{ \alpha q^3 + \beta q\dot{q}^2 + \gamma q^2\ddot{q} \} + \epsilon\eta V = \epsilon [k_1\dot{q} + k_2\dot{q}^3 + k_3 \cos(\phi\tau)\dot{q} + k_4 \cos(\phi\tau)\dot{q}^3] \quad (6.13)$$

$$\dot{V}(\tau) + \chi V(\tau) + \dot{q} = 0 \quad (6.14)$$

Here the nondimensionalized parameters have been used.

$$\begin{aligned} x &= \frac{s}{L}, \quad \beta = \frac{d}{L}, \quad \beta_1 = \frac{L_1}{L}, \quad \beta_2 = \frac{L_2}{L}, \quad \tau = \omega t, \quad \lambda = \frac{r}{L}, \quad \mu = \frac{m}{\rho_b A_b L}, \quad J = \frac{I_d}{\rho_b A_b L r^2}, \\ \phi &= \frac{\Omega}{\omega}, \quad \eta = \frac{-\bar{\eta} r_v}{\rho_b A_b \hbar \omega^2 r \epsilon L^2} \int_0^1 \{\delta_x(x - \beta_1) - \delta_x(x - \beta_2)\} \psi(x) dx, \quad \zeta = \zeta^* \frac{\nu}{\epsilon}, \\ \chi &= \frac{1}{C_p \omega R_l}, \quad \bar{K} = \frac{e_{31} t_{pc} b_p}{L} \int_{\beta_1}^{\beta_2} \psi_{xx}(x) dx, \quad K = \frac{\bar{K} r}{C_p r_v}, \quad r_v = \frac{\bar{V}}{V} = \frac{\bar{K} r}{C_p} \\ k_1 &= \frac{\frac{1}{2} \rho_a U D_{bf} \hat{k}_1}{\epsilon \rho_b A_b r L \hbar \omega}, \quad k_2 = \frac{\frac{1}{2} \rho_a D_{bf} \omega \hat{k}_2}{\epsilon \rho_b A_b r L \hbar U}, \quad k_3 = \frac{\rho_a u_0 D_{bf} \hat{k}_1}{\epsilon \rho_b A_b r L \hbar \omega}, \quad k_4 = \frac{\rho_a u_0 D_{bf} \omega \hat{k}_2}{\epsilon \rho_b A_b r L \hbar U^2} \end{aligned}$$

The equations of motion Eq. (6.13) and (6.14) contains inertial, cubic type and coupled nonlinear terms. Here the damping, geometric and inertial nonlinearities, electromechanical coupling and excitation terms are small, and, hence considered of the order of ϵ (book keeping parameter). The expression for nondimensional parameters ($\alpha, \beta, \gamma, \zeta^*$ etc.) are defined in Appendix C. Similarly the expressions for the parameters related to the wind driven force i.e., $\hat{k}_1$ and $\hat{k}_2$ are defined in Appendix D. For steady wind speed, neglecting the unsteady component from Eq. (6.10) and (6.11) as $\bar{u}(s, t) = \bar{v}(s, t) = 0$

$$U^* = U(s) = U_{rel} \cos \alpha \quad (6.15)$$

$$U_{rel}^2 = U^2(s) \sec^2 \alpha \quad (6.16)$$

The net galloping force due to lift and drag forces under steady wind flow becomes

$$F(t) = \frac{1}{2} \rho_a U^2 D_{bf} \left[\hat{k}_1 \left(\frac{\dot{q}}{U} \right) + \hat{k}_2 \left(\frac{\dot{q}}{U} \right)^3 \right] \quad (6.17)$$

The detailed derivation is given in the Appendix D. After nondimensionalization the nonlinear coupled electromechanical temporal governing equations of motion for single mode in case of steady wind flow is expressed as follows.

$$\ddot{q} + 2\epsilon \zeta \dot{q} + q + \epsilon \{ \alpha q^3 + \beta q \dot{q}^2 + \gamma q^2 \ddot{q} \} + \epsilon \eta V = \epsilon [k_1 \dot{q} + k_2 \dot{q}^3] \quad (6.18)$$

$$\dot{V}(\tau) + \chi V(\tau) + \dot{q} = 0 \quad (6.19)$$

Here, $\dot{(\cdot)} = \frac{d(\cdot)}{d\tau}$, τ is the nondimensional time and V denotes the nondimensional open circuit voltage. The coefficients appear in Eqs. (6.18) and (6.19) are defined in Appendix C. It may be noted that in the absence of piezoelectric patches, galloping force and inclusion of base excitation with single mode approximation, the equation of motion of the system is similar to that of the model investigated theoretically and experimentally by Zavodney and Nayfeh [182].

6.2.2 Solution using by the method of multiple scales

By using the method of multiple scales, a uniform first order approximate solution of Eqs. (6.18) and (6.19) can be obtained. Similar to the derivation to the previous Chapters, the time dependence is expressed in to the multiple time scales [13] as

$$T_n = \epsilon^n \tau, \quad n = 0, 1, 2 \quad (6.20)$$

$$\frac{d}{d\tau} = D_0 + \epsilon D_1 + O(\epsilon^2); \quad \frac{d^2}{d\tau^2} = D_0 + 2\epsilon D_0 D_1 + O(\epsilon^2) \quad (6.21)$$

where $D_n = \frac{\partial}{\partial T_n}$. According to this method, the solution of the system $q(\tau)$ and $V(\tau)$ can be expanded in the following series form.

$$q(\tau) = q_0(T_0, T_1) + \epsilon q_1(T_0, T_1) + O(\epsilon^2) \quad (6.22a)$$

$$V(\tau) = V_0(T_0, T_1) + \epsilon V_1(T_0, T_1) + O(\epsilon^2) \quad (6.22b)$$

After substituting Eq. (6.22a) and (6.22b) into Eq. (6.18) and (6.19), then equating the coefficient of ϵ^0 and ϵ^1 equal to zero gives the following differential equations.

$$D_0^2 q_0 + q_0 = 0 \quad (6.23a)$$

$$D_0 V_0 + \chi V_0 = -D_0 q_0 \quad (6.23b)$$

$$\begin{aligned} D_0^2 q_1 + q_1 = & -\alpha q_0^3 - \eta V_0 - 2\zeta D_0 q_0 + k_1 D_0 q_0 + k_3 D_0 q_0 \cos(\phi t) - \beta q_0 D_0 q_0^2 \\ & + k_2 D_0 q_0^3 + k_4 D_0 q_0^3 \cos(\phi t) - 2D_0 D_1 q_0 - \gamma q_0^2 D_0^2 q_0 \end{aligned} \quad (6.24)$$

The solution of differential equations Eq. (6.23) of $O(\epsilon^0)$ is expressed as follows.

$$q_0 = A(T_1) \exp(iT_0) + cc, \quad (6.25a)$$

$$V_0 = -ZA(T_1) \exp(iT_0) + cc \quad (6.25b)$$

where, $Z = Z_r + iZ_i$; $Z_r = \frac{1}{\chi^2 + 1}$; $Z_i = \frac{\chi}{\chi^2 + 1}$.

Here cc is the complex conjugate of the preceding terms and $A(T_1)$ is an unknown complex function to be determined later. Using Eqs. (6.25) into Eq. (6.24) the following secular terms are obtained which should be eliminated for finite amplitude response of the system. Considering only principal parametric resonance condition i.e., when the frequency of the external excitation Ω is nearly twice that of the natural frequency of the system ω , the nearness of external excitation is expressed as $\phi = 2 + \epsilon\sigma$ and the following secular terms are obtained.

$$\begin{aligned} & -2i\zeta A + iAk_1 - \frac{1}{2}ie^{-i\sigma T_0}A^3k_4 + \eta AZ - 3\alpha A^2\bar{A} - \beta A^2\bar{A} + \\ & 3\gamma A^2\bar{A} + 3iA^2k_2\bar{A} - \frac{1}{2}ie^{i\sigma T_0}k_3\bar{A} - \frac{3}{2}ie^{i\sigma T_0}Ak_4\bar{A}^2 - 2iA' = 0 \end{aligned} \quad (6.26)$$

Expressing A in its polar form as $A = \frac{1}{2}ae^{i\lambda}$, Eq. (6.26) reduces to the following equation.

$$\begin{aligned} & -i\zeta a - \frac{3}{8}\alpha a^3 - \frac{1}{8}\beta a^3 + \frac{3}{8}\gamma a^3 + \frac{1}{2}iak_1 + \frac{3}{8}ia^3k_2 - \frac{1}{4}ie^{i\sigma T_0-2i\lambda}ak_3 - \\ & \frac{3}{16}ie^{i\sigma T_0-2i\lambda}a^3k_4 - \frac{1}{16}ie^{-i\sigma T_0+2i\lambda}a^3k_4 + \frac{1}{2}\eta aZ - ia' + a\lambda' = 0 \end{aligned} \quad (6.27)$$

Further, separating the real and imaginary parts and equating them to zero lead to the following modulation equations.

$$a' = -\zeta a + \frac{1}{2}ak_1 + \frac{3}{8}a^3k_2 - \frac{1}{4}ak_3 \cos(\sigma T_0 - 2\lambda) - \frac{1}{4}a^3k_4 \cos(\sigma T_0 - 2\lambda) + \frac{1}{2}\eta aZ_i \quad (6.28)$$

$$a\lambda' = \frac{3}{8}\alpha a^3 + \frac{1}{8}\beta a^3 - \frac{3}{8}\gamma a^3 - \frac{1}{4}ak_3 \sin(\sigma T_0 - 2\lambda) - \frac{1}{8}a^3k_4 \sin(\sigma T_0 - 2\lambda) - \frac{1}{2}\eta aZ_r \quad (6.29)$$

These equations can be written in a concise way as follows.

$$a' = \zeta_{eff}a + \frac{3}{8}a^3k_2 - \left(\frac{1}{4}ak_3 + \frac{1}{4}a^3k_4\right)\cos(\delta) \quad (6.30)$$

$$\frac{1}{2}a\delta' = \frac{1}{2}a\sigma_{eff} - N_{eff}a^3 + \left(\frac{1}{4}ak_3 + \frac{1}{8}a^3k_4\right)\sin(\delta) \quad (6.31)$$

Here,

$$\begin{aligned} N_{eff} &= \frac{3}{8}\alpha + \frac{1}{8}\beta - \frac{3}{8}\gamma; \quad \zeta_{eff} = -\zeta + \frac{1}{2}k_1 + \frac{1}{2}\eta Z_i - \zeta_a a; \quad \sigma_{eff} = \sigma + \eta Z_r \\ \delta &= \epsilon\sigma T_0 - 2\lambda; \quad \delta' = \sigma - 2\lambda'; \quad \lambda' = \frac{1}{2}(\sigma - \delta'), \\ \tan(\delta) &= \left(\frac{a^2 N_{eff} - 0.5\sigma_{eff}}{\zeta_{eff} + 0.375 a^2 k_2} \times \frac{k_3 + a^2 k_4}{k_3 + 0.5 a^2 k_4} \right) \end{aligned}$$

At steady state $a' = \delta' = 0$ leads to the following equation between the detuning parameter σ and amplitude a which will be required to plot the frequency response curves.

$$\sigma = 2N_{eff}a^2 - \eta Z_r \pm (0.5k_3 + 0.25k_4a^2) \sqrt{1 - \left(\frac{\zeta_{eff} + \frac{3}{8}k_2a^2}{0.25(k_3 + k_4a^2)} \right)^2} \quad (6.32)$$

At steady flow condition $k_3 = k_4 = 0$. From Eq. (6.30), the response amplitude a can be given by the following expression.

$$a^2 - \left(\frac{8}{3k_2}\right)a + \frac{4}{3k_2}(k_1 + \eta Z_i - 2\zeta) = 0 \quad (6.33)$$

Hence,

$$a = \left(\frac{4}{3k_2}\right) \pm \sqrt{\left(\frac{4}{3k_2}\right)^2 - \left\{ \frac{4}{3k_2}(k_1 + \eta Z_i - 2\zeta) \right\}} \quad (6.34)$$

One can obtain the steady-state response $v(s, t)$, output voltage $V(t)$ and output power P over the load resistance R_l by using the following expressions.

$$v(s, t) = r \psi(s)a \cos(\omega t + \lambda) \quad (6.35)$$

$$V(t) = \frac{r_v a}{\sqrt{\chi^2 + 1}} \cos(\omega t + \lambda + \vartheta) \quad (6.36)$$

$$P = \frac{1}{R_l} \left(\frac{r_v a}{\sqrt{\chi^2 + 1}} \right)^2 \quad (6.37)$$

Here, $\vartheta = \tan^{-1}(\chi)$. In the next section the experimental and theoretical results and discussions are presented.

6.3 Results and Discussion

A galloping based piezoelectric energy harvester (GPEH) system setup is fabricated which is shown in Fig. 6.3. The setup consists of a vertical cantilever beam with piezoelectric patch (PZT-5H) in unimorph configuration near its fixed end and a bluff body attached near its free end. Two different bluff bodies of D-shape (Fig. 6.3(a)) and triangular shape (Fig. 6.3(b)) are considered for this analysis. The geometric and material properties of the beam and piezoelectric patch are mentioned in Table 6.1. Similar properties of the bluff bodies (D and isosceles triangular shape) are given in Table 6.2. Experiments have been carried out to determine the free vibration

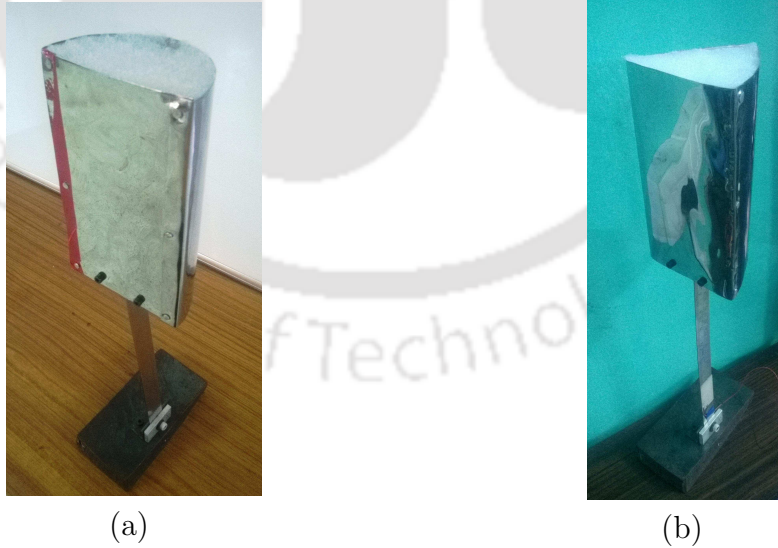


FIGURE 6.3: GPEH with (a) D shape bluff body (b) Triangular shape bluff body

characteristics viz., damping ratio and natural frequency of the GPEH system as given in the next subsection.

TABLE 6.1: Material and geometric properties of substrate and piezo patch (PZT-5H)

Property	Piezo Patch	Substrate
Young's modulus, GPa	$E_p = 66.7$	$E_b = 190$
Density, kg m ⁻³	$\rho_p = 7500$	$\rho_b = 7800$
Length, m	$L_p = 50 \times 10^{-3}$	$L_b = 304 \times 10^{-3}$
Width, m	$b_p = 23.4 \times 10^{-3}$	$b_b = 23.4 \times 10^{-3}$
Height, m	$t_p = 0.61 \times 10^{-3}$	$t_b = 0.735 \times 10^{-3}$
Capacitance, nF	$C_p = 71.30$	—
Piezoelectric constant, CN ⁻¹	$d_{31} = -285 \times 10^{-12}$	—

TABLE 6.2: Geometric properties, mass and coefficients a_1, a_2 [1] of the Bluff bodies

Bluff-Body	Length (L_{bf} , m)	Width (D_{bf} , m)	Mass (m , kg)	Angle	a_1	a_2
D-Shape	182×10^{-3}	107×10^{-3}	140×10^{-3}	—	0.79	-0.19
Triangular	182×10^{-3}	107×10^{-3}	140×10^{-3}	53°	1.90	-6.7

6.3.1 The determination of the free vibration characteristics

One may study the free vibration response by tapping the free end of the cantilever beam. The generated open circuit voltage due to this free vibration is measured by using the Oscilloscope (InfinitiVision, DSO-X, 3024A, Fig. 5.3(a)). The voltage time response of the PEH without bluff body is shown in Fig. 6.4(a) and the corresponding FFT is plotted in Fig. 6.4(b). Using the logarithmic decrement method, one may find the damping ratio of the GPEH system. In the present case, the damping ratio is found to be 0.0069 and the natural frequency of the beam without the bluff body is found to be 6.8 Hz.

The experiment is repeated to find the free vibration responses for three different position of the bluff body along the beam length. These responses are shown in Fig. 6.5(a,c,e) for $\beta = 0.88, 0.82$ and 0.75 , respectively. The corresponding FFTs are shown in Fig. 6.5(b,d,f). It is observed that, the natural frequency of the system increases from $f = 1.04$ Hz to $f = 1.34$ Hz as β decreases from 0.88 to 0.75. By calculating the natural frequencies from Fig. 6.4 and Fig. 6.5, it is observed that the mass of the bluff body reduces the natural frequency of the beam from $f = 6.8$ Hz to nearly $f \approx 1$ Hz. Now, after calculating damping and natural frequency of

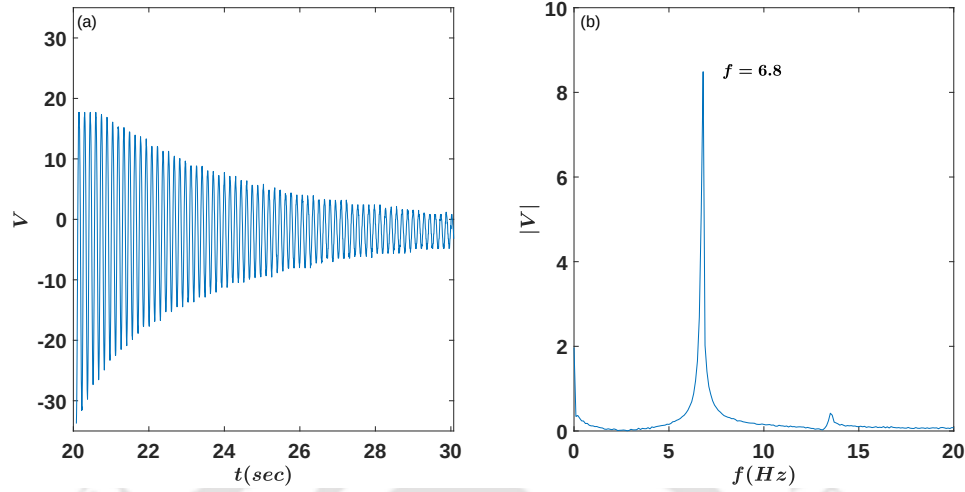


FIGURE 6.4: (a) Voltage time response of the beam without bluff body attachment
(b) Corresponding FFT

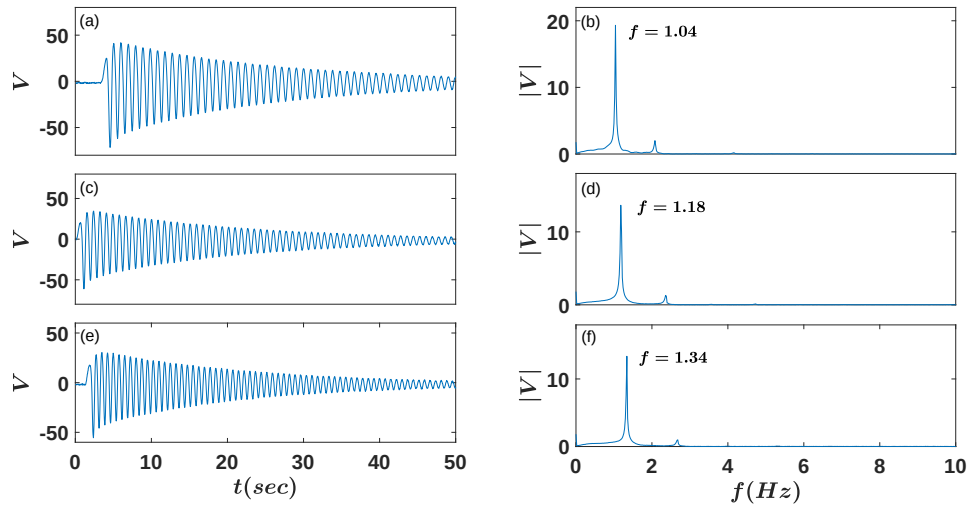


FIGURE 6.5: (a, c, e) Voltage time response for $\beta = 0.88, 0.82$ and 0.75 respectively (b, d, f) Corresponding FFT

the GPEH system, the next subsection deals with the experimental findings of the output voltage and power by performing experiments in the wind tunnel.

6.3.2 Experimental study of GPEH in wind tunnel

Figure 6.6 shows the wind tunnel which is used to test the performance of the GPEH system. The dimensions of the wind tunnel are $600 \text{ mm} \times 600 \text{ mm} \times 2000 \text{ mm}$ with a maximum wind speed of 50 m/s . Figure 6.7 shows the GPEH system inside the wind



FIGURE 6.6: Wind tunnel facility

tunnel which undergoes the galloping phenomenon. The voltage time response of the GPEH system is obtained by varying the system parameters such as types of bluff body (Fig. 6.8(a)), bluff body position (β), external load resistance (Fig. 6.8(b)) and wind speed. In the next subsection the voltage generated using D-shaped bluff body in the GPEH system is discussed.



FIGURE 6.7: Galloping of GPEH system inside the wind tunnel



FIGURE 6.8: (a) D and Triangular shape bluff bodies (b) External electric circuit with load resistances

6.3.2.1 GPEH with D-shaped bluff body (GPEH-DSB)

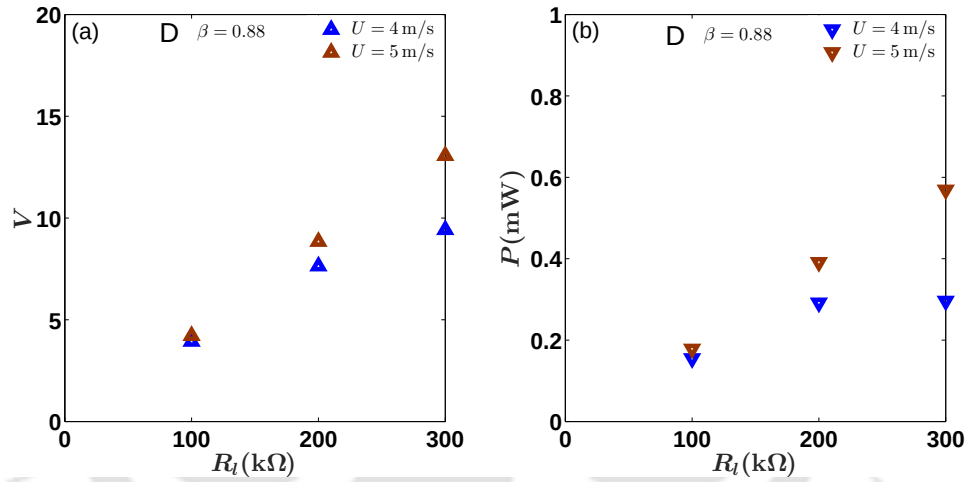


FIGURE 6.9: (a) Voltage variation with load resistance (b) Corresponding power output with load resistance for GPEH-DSB at $\beta = 0.88$, $U = 4$ m/s and 5 m/s

The present subsection deals with the experimental findings of the GPEH system attached with D-shape bluff body. The output voltage and the power with variation of load resistance is observed for two different wind speeds $U = 4$ m/s and 5 m/s as shown in Fig. 6.9. Here the nondimensional distance of the bluff body from the fixed end of the beam is kept at $\beta = 0.88$. It is observed from Fig. 6.9 that as the load resistance increases the output voltage and corresponding power increases. Also, as the wind speed increases the output voltage and power increases. At $R_l = 300$ $k\Omega$ the voltage is found to be 9.4 V and 13.07 V at wind speed of $U = 4$ m/s and 5 m/s respectively. The corresponding output power at the same load resistance is found to be 0.3 mW and 0.57 mW.

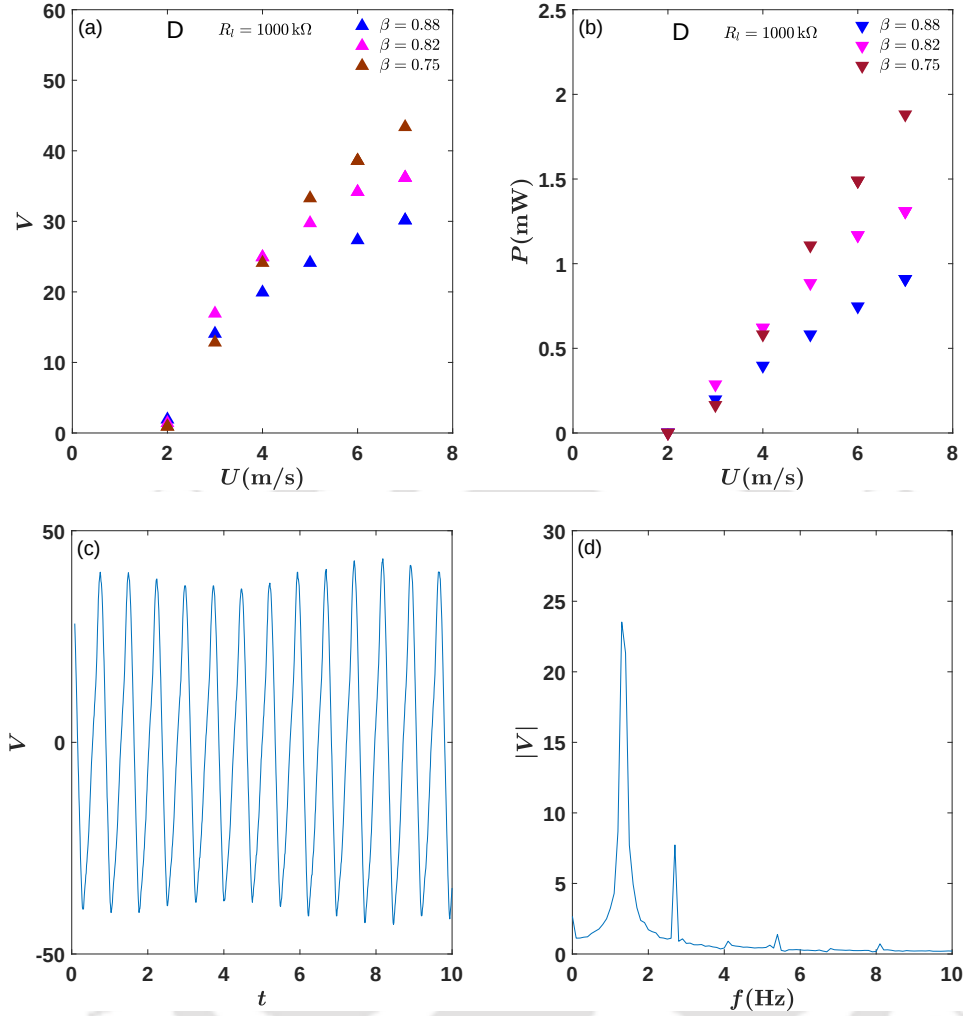


FIGURE 6.10: (a) Variation of voltage with wind speed (U) (b) Corresponding output power with wind speed for GPEH-DSB at different values of β (c) Voltage time response when $\beta = 0.75$ and $U = 7$ m/s (d) Corresponding FFT plot

The output voltage and power with the variation of wind speed is observed for three different positions of D shape bluff body along the cantilever beam, and the load resistance is kept invariant at $R_l = 1000 \text{ k}\Omega$ as shown in Fig. 6.10. Here, β is taken as 0.88, 0.82 and 0.75. As the wind speed increases the output voltage and corresponding power increase for all β values. It may be observed from this that below certain wind speed the beam does not vibrate and hence voltage is not generated. The minimum speed at which the beam starts to vibrate which causes generation of voltage is known as the onset speed of galloping. This is the pitchfork bifurcation in V vs U plane and Hopf bifurcation in the plane of q vs U or V vs U plane. This observation is similar to the finding of Hopf bifurcation in the work of

Abdelkefi et al. [127]. For all β values the onset of galloping is invariant around 2 m/s wind speed. As β decreases the output voltage at a certain wind speed follows an increasing trend. For example the voltages at wind speed of 7 m/s are found to be 43.37 V, 36.18 V and 30.15 V for $\beta = 0.75$, 0.82 and 0.88, respectively and the corresponding power output is found to be 1.9, 1.3 and 0.91 mW. The voltage time response and corresponding FFT is shown in Fig. 6.10(c) and (d) respectively, at $\beta = 0.75$ ($U = 7$ m/s). The major frequency component is having frequency of $f = 1.3$ Hz, which is same as the experimental value of the natural frequency as shown in Fig. 6.5(f). Further, increase in the wind speed from 7 m/s causes high transverse displacement of the beam system.

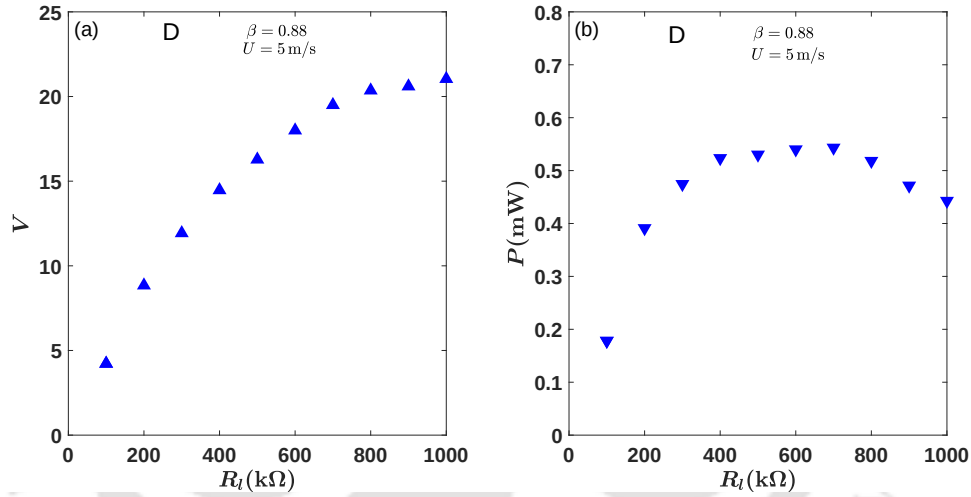


FIGURE 6.11: (a) Variation of voltage with load resistance (b) Corresponding power output with load resistance for GPEH-DSB at $\beta = 0.88$, $U = 5$ m/s

Figure 6.11 shows the variation of the output voltage and power with load resistance for wind speed $U = 5$ m/s and $\beta = 0.88$. It is observed that as the load resistance increases the output voltage and corresponding power increases. At $R_l = 1000$ k Ω the voltage is found to be 21.04 V. The corresponding output power at the same load resistance is found to be 0.44 mW. However, the maximum power output is 0.54 mW at a load resistance of 500 k Ω . This shows that there exist an optimal load resistance at which the maximum power is obtained.

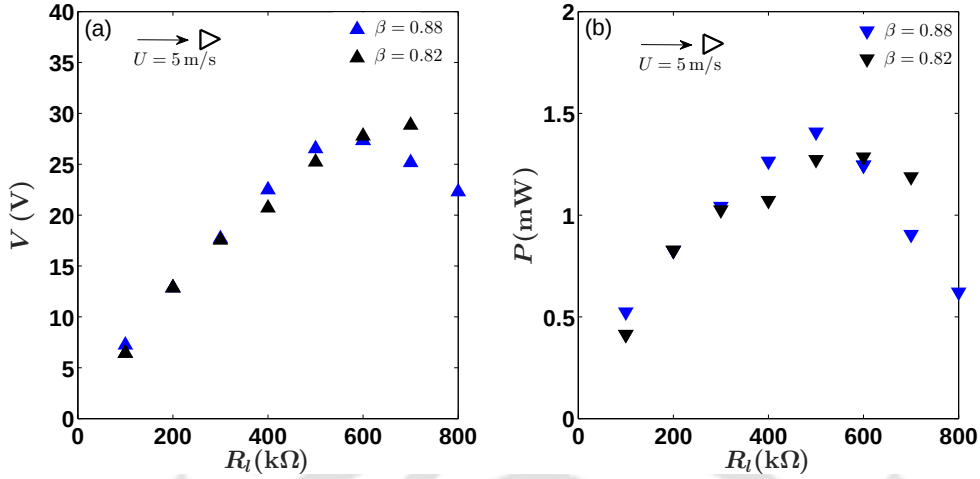


FIGURE 6.12: (a) Variation of voltage with load resistance (b) Corresponding power output with load resistance for GPEH-TSB; $\beta = 0.88$ and 0.82 , $U = 5$ m/s

6.3.2.2 GPEH with Triangular-shape bluff body (GPEH-TSB)

Figure 6.12 shows the variation of output voltage and power with variation of load resistance for GPEH-TSB at $U = 5$ m/s, $\beta = 0.88$ and 0.82 . The dimension of the bluff body is mentioned in Table 6.2 and the dimensions and material properties of the whole system are previously mentioned in Table 6.1. It is observed that for $\beta = 0.88$ the output voltage increases with R_l up to 600 kΩ and then decreases with further increase in R_l . In this case, at $R_l = 600$ kΩ the voltage and output power are found to be 27.34 V and 1.41 mW, respectively which is observed to be the maximum power output. Similar variation is observed for $\beta = 0.82$, however V and P become higher in this case when R_l exceeds the 600 kΩ mark. In this case the maximum output power obtained is 1.29 mW, found at $R_l = 600$ kΩ and the corresponding voltage is found to be 27.77 V.

For triangular shaped bluff body the variation of output voltage and power with wind speed for $\beta = 0.88, 0.82$ and 0.75 , $R_l = 1000$ kΩ is shown in Fig. 6.13. It is observed that with increases in wind speed the output voltage and power increase for all values of β . One may clearly observe that the onset speed of galloping increases as β decreases. Since maintaining the steady wind speed below 2 m/s in the wind tunnel is not possible, hence the exact onset of galloping speed for higher value of β could not be determined below $U = 2$ m/s. However, for lower β value as in case of $\beta = 0.75$, the onset of galloping is clearly observable. One may obtained the maximum output power of 2.95 mW (for $\beta = 0.88$) at a wind speed of $U = 7$ m/s

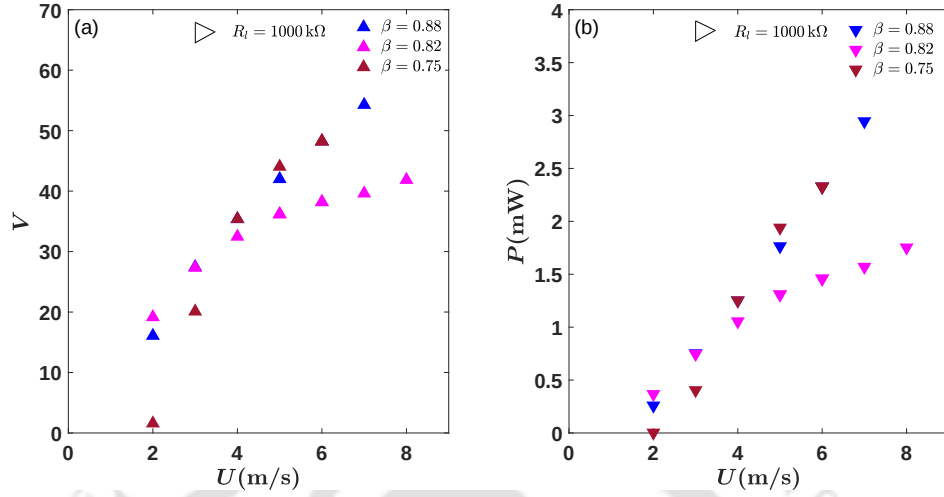


FIGURE 6.13: (a) Variation of voltage with wind speed (U) (b) Corresponding output power with wind speed for GPEH-TSB at different values of β and $R_l = 1000 \text{ k}\Omega$

and load resistance of $R_l = 1000 \text{ k}\Omega$. The maximum output power for $\beta = 0.82$ and 0.75 are 1.75 mW (at $U = 8 \text{ m/s}$) and 2.33 mW (at $U = 6 \text{ m/s}$), respectively.

6.3.2.3 Comparison of GPEH with D and triangular shaped bluff bodies

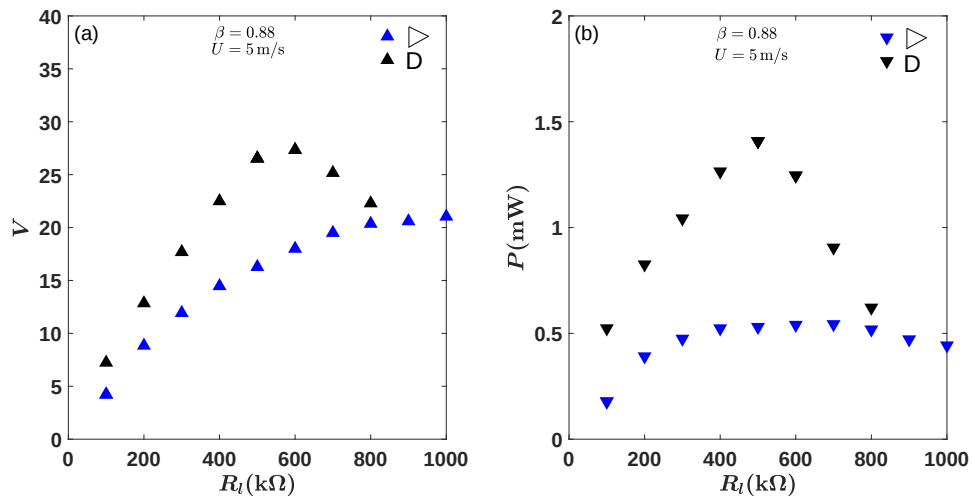


FIGURE 6.14: (a) Variation of voltage with load resistance (b) Corresponding power output with R_l for GPEH-DSB and GPEH-TSB at $\beta = 0.88$; $U = 5 \text{ m/s}$

In this section the comparison has been made for the output voltage and power with D-shaped and triangular shaped bluff bodies. Figure 6.14 shows the effect of R_l on

output voltage and power for $U = 5$ m/s and $\beta = 0.88$. Clearly the GPEH-DSB yields higher voltage and power than that of GPEH-TSB upto a load resistance of $800 \text{ k}\Omega$. The output power variation is shown in Fig. 6.14(b). The maximum power output for D shaped GPEH system is 1.41 mW at $R_l = 500 \text{ k}\Omega$ and for GPEH-TSB it is 0.54 mW at $R_l = 700 \text{ k}\Omega$. Further it may be observed that same values of voltage (or power) can be obtained for two different values of R_l . Hence the designer may choose the required load resistance for proper design of the GPEH system.

The variation of output voltage and power with wind speed for $\beta = 0.75$ and $R_l = 1000 \text{ k}\Omega$ considering D-shaped and triangular shaped bluff bodies are shown in Fig. 6.15. One may observe that at $U = 2$ m/s the output voltage and power are same for the GPEH with both D and triangular shaped the bluff bodies. The output voltage and power are found to be more in case of GPEH with triangular shape (TS) bluff body. For example the voltage is found to be 48.24 V (at $U = 6$ m/s) and 38.6 V (at $U = 6$ m/s) for GPEH-TSB and GPEH-DSB, respectively. The corresponding output powers are 2.33 mW and 1.49 mW as shown in Fig. 6.15(b).

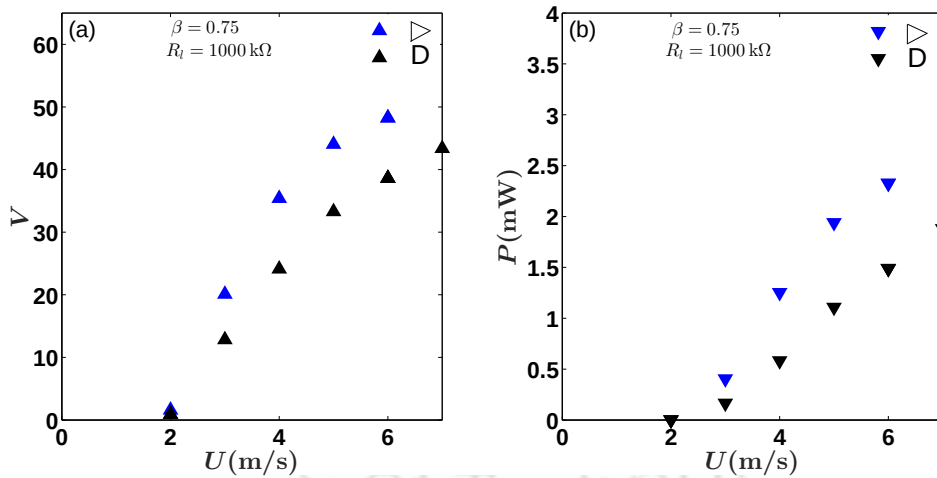


FIGURE 6.15: (a) Voltage variation with wind speed (b) Corresponding power output with wind speed for GPEH-DSB and GPEH-TSB; at $\beta = 0.75$, $R_l = 1000 \text{ k}\Omega$

For $\beta = 0.82$ and $R_l = 1000 \text{ k}\Omega$ the variation of voltage and output power with wind speed is shown in Fig. 6.16. Fitting a cubic order polynomial the onset of galloping is observed at $U = 0.5$ m/s for GPEH-TSB and $U = 2$ m/s for GPEH-DSB. The maximum power obtained are found to be 1.31 (at $U = 7$ m/s) and 1.57 mW (at $U = 7$ m/s) for GPEH-DSB and GPEH-TSB, respectively are shown in Fig. 6.16(b).

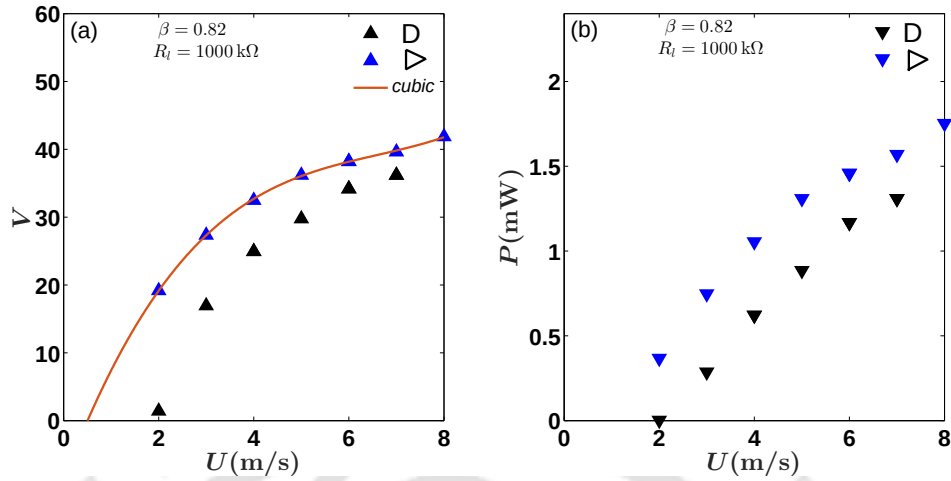


FIGURE 6.16: (a) Variation of voltage with wind speed (b) Corresponding power output with wind speed for GPEH-DSB and GPEH-TSB; at $\beta = 0.82$, $R_l = 1000 \text{ k}\Omega$.

Here also GPEH-TSB perform better than that of GPEH-DSB not only in output power extracted but also in terms of having lesser value of onset of galloping as well. Further increasing β to 0.88, Fig. 6.17 shows the variation of V and P with U for the same R_l .

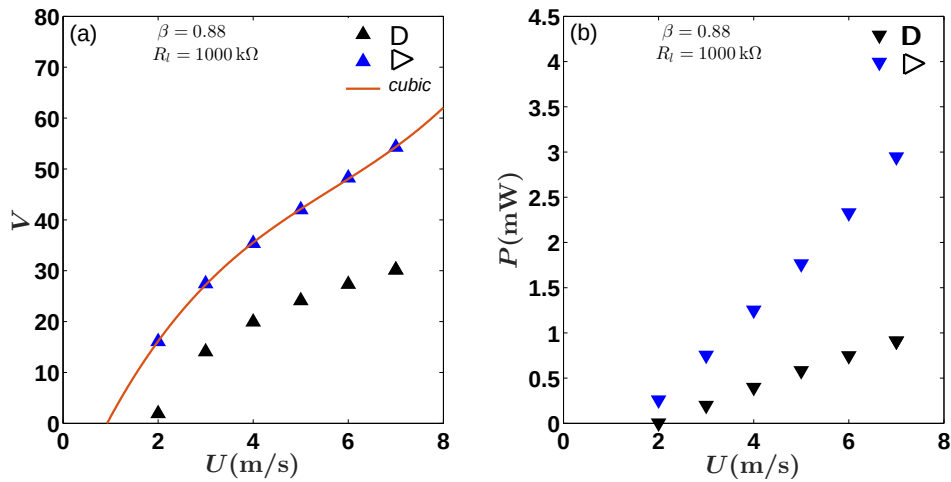


FIGURE 6.17: (a) Voltage variation with wind speed (b) Corresponding power output with wind speed for GPEH-DSB and GPEH-TSB; at $\beta = 0.88$, $R_l = 1000 \text{ k}\Omega$.

Figure 6.17 shows a comparative study between D-shaped (DS) and triangular shaped (TS) bluff bodies for parameters $\beta = 0.88$ and load resistance $R_l = 1000 \text{ k}\Omega$. The voltage and corresponding output power variation with wind speed is observed

in Fig. 6.17(a) and 6.17(b) respectively. Here the onset of galloping for GPEH-TSB is at $U = 0.94$ m/s (by cubic fit), and that for GPEH-DSB is $U = 2$ m/s. As wind speed increases the voltage and corresponding power also increases for both bluff bodies (BB); however, GPEH with the triangular shaped bluff body, one may extract higher output voltage and power. The output power is found to be 2.95 mW (at $U = 7$ m/s) and 0.91 mW (at $U = 7$ m/s) for TS and DS bluff bodies, respectively. From the last three figures it may be observed that there is not much variation in the onset of galloping for GPEH-DSB. However, for GPEH-TSB system the onset of galloping depends on β and takes place at a lower value of U than that of GPEH-DSB. The GPEH-TSB yields higher voltage and power for a constant β , R_l and U . In the next section parametric studies have been carried out by taking the expressions given in Eq.(6.34) and Eq.(6.32).

6.3.3 Theoretical study

This section deals with the theoretical findings for the galloping based PEH system with the constant wind flow. Initially, the theoretical results have been compared with those of experimental findings obtained using the wind tunnel test.

6.3.3.1 GPEH with constant wind flow

Taking the system parameters as given in Table 6.1 and equations Eq. 6.34 and (6.36), the variation of voltage with β , U and R_l are determined. Figure 6.18 shows both experimental and analytically obtained (using the method of multiple scales) values of output voltage and power with variation in wind speed for $\beta = 0.88$ and $R_l = 1000$ k Ω . While Fig. 6.18(a) is plotted for GPEH-DSB Fig. 6.18(b) is plotted for GPEH-TSB.

It may be noted that the experimentally obtained curve is for a beam of a thickness of 0.71 mm. For the GPEH with the same dimension, the experimentally observed onset of galloping, voltage and power is not found theoretically. However, by reducing the thickness of the beam to 0.470505660 mm the simulated theoretical results are found to be very close to that of the experimental one. Similarly, for a GPEH with triangular shape bluff body the comparison between the experimental and analytical results is shown in Fig 6.18(b). It is observed that the experimental results for GPEH with 0.71 mm thick beam is found to be lies between those of a beam

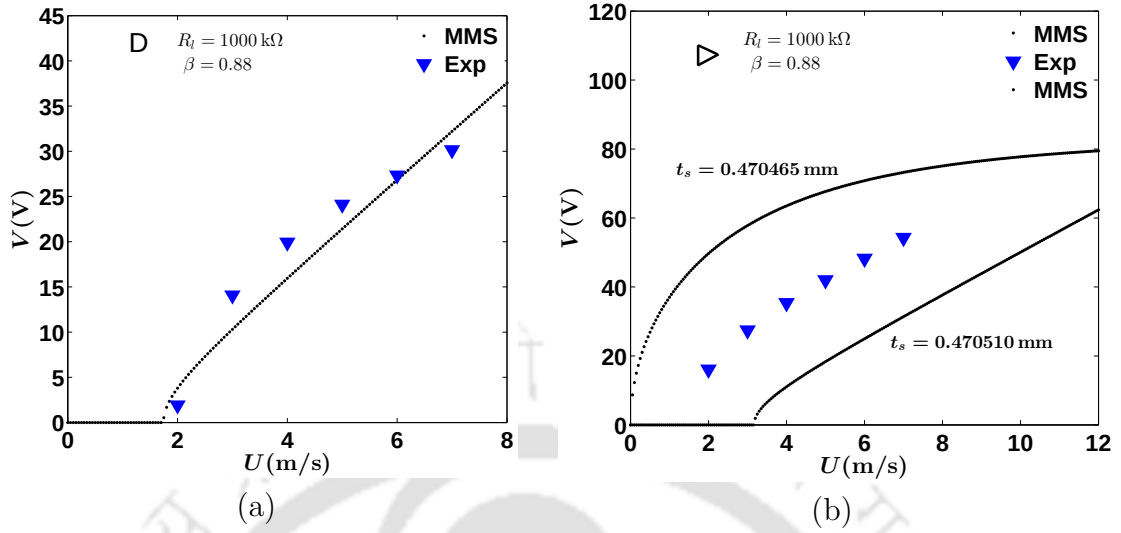


FIGURE 6.18: A comparison of experimental results with the analytical findings with (a) D shape bluff body (b) Triangular shape bluff body

with thickness 0.470510 mm and 0.470465 mm, respectively for the lower and the upper curve. Further it has been observed that with minor changes in parameters such as geometric, material and electric properties of beam, piezoelectric patch and the bluff body of the GPEH system one can obtain similar voltage and power as obtained in the experiment. Also it may be noted that the exact environment in experiment can not be modelled in theoretical modeling and hence one may use a thickness correction factor theoretically to obtain results very close to that of the experimental findings.

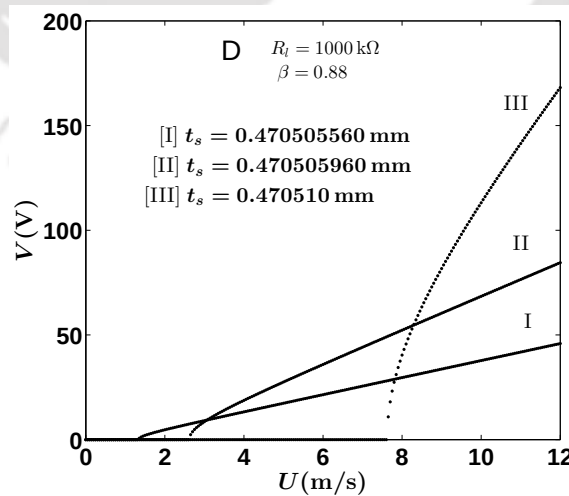


FIGURE 6.19: The variation of output voltage with the thickness of the beam

In Fig. 6.19 the variation of output voltage with the thickness of the beam is shown for $\beta = 0.88$ and $R_l = 1000 \text{ k}\Omega$. It is observed that as the thickness of the beam increases, the onset of galloping wind speed and the output voltage increases. From curve I and III, it may be noted that these variations are very sensitive to the value of thickness of the beam.

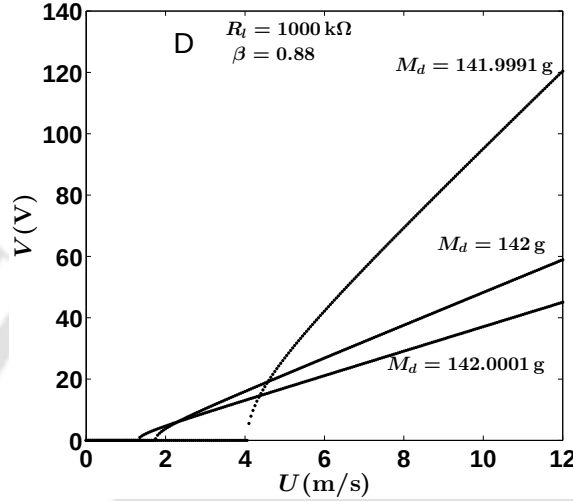


FIGURE 6.20: The variation of output voltage with the mass of the bluff body

Figure 6.20 shows the variation of output voltage with the mass of the D shaped bluff body for $\beta = 0.88$ and $R_l = 1000 \text{ k}\Omega$. It is observed that as the mass of the bluff body increases from $M_d = 141.9991 \text{ gm}$ to $M_d = 142.0001 \text{ gm}$, the onset of galloping wind speed reduces and simultaneously the voltage gets reduced. Figure 6.21 shows the the variation of output voltage with the external load resistance R_l . It is observed that as R_l increases from $R_l = 600 \text{ k}\Omega$ to $R_l = 1000 \text{ k}\Omega$, the required onset galloping wind speed for the oscillation of the GPEH system increases. However the required speed is within the range of 2 m/s in this case.

One may note that the obtained analytical solution is very sensitive with respect to the variation in thickness of the beam and the mass of the bluff body as compared to the load resistance. One may observe that the obtained trends in the analytical results are similar to those of the experimental findings. In the next subsection the theoretical study is presented for the GPEH with time varying wind flow.

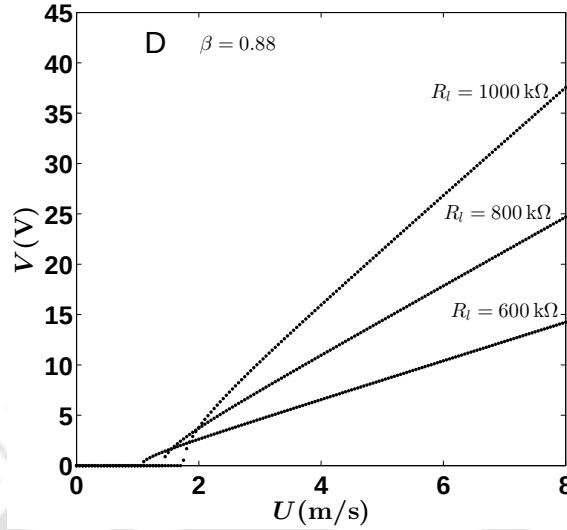


FIGURE 6.21: The variation of output voltage with the external load resistance R_l

6.3.3.2 GPEH with time varying wind flow

In this section considering the wind speed as a sum of a constant U and a harmonically varying speed $u_0 \cos(\Omega t)$, the voltage and power of the GPEH has been determined for different system parameter. The close form expression (Eq. (6.32)) obtained using method of multiple scale is used to plot the variation of V and P with different system parameters such as amplitude of steady and unsteady wind component, position of bluff body along the beam, type of bluff body attached and load resistance. These results are qualitatively compared with a theoretical findings available in the literature.

Figure 6.22 shows the frequency response plot of the GPEH-DSB with different values of u_0 for $\beta = 0.88$, $U = 7 \text{ m/s}$ and $R_l = 500 \text{ k}\Omega$. The system parameters are same as given in Table 6.1 and Table 6.2 unless otherwise mentioned. The stable and unstable response amplitudes are shown by black color and red color, respectively. Using the expression Eq. (6.35), Eq. (6.36) and Eq. (6.37) one may obtained the response amplitude v , voltage V and power P . The response amplitude are in the form of isolated closed curve with stable and unstable parts. At a particular frequency of excitation there exist two solutions where the upper half is stable and the remaining is unstable. As u_0 increase, the size of the concentric response curve grows and hence the galloping amplitude increases as shown in Fig. 6.22(a). Similar effect is also observed in the tip displacement and voltage response due to proportionality

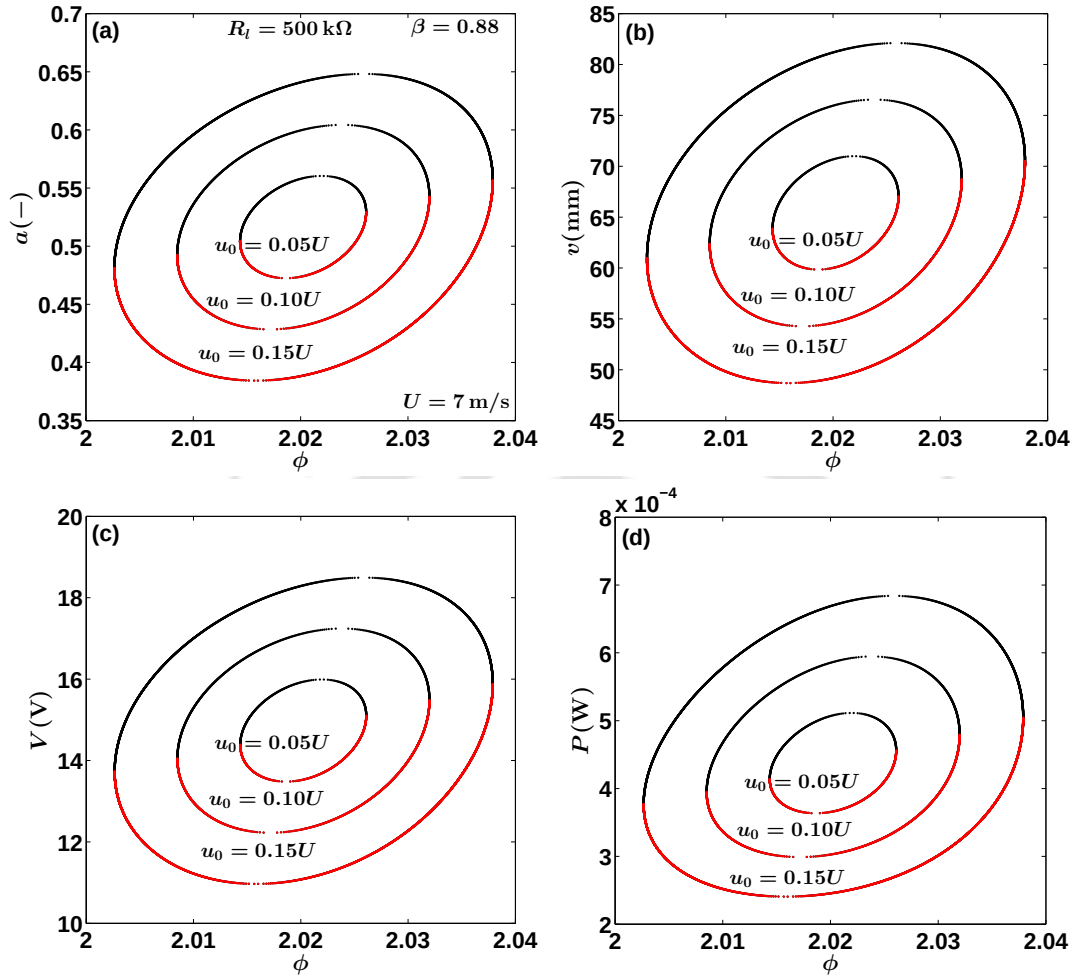


FIGURE 6.22: Frequency response with the variation of u_0 for (a) Galloping amplitude a (b) Tip displacement (c) Voltage (d) Power output, for D-shape bluff body with system parameters $\beta = 0.88$, $U = 7$ m/s and $R_l = 500$ k Ω

with galloping amplitude which are plotted in Fig. 6.22(b) and (c) respectively. The effect of load resistance R_l on the frequency response of GPEH-DSB is shown in Fig. 6.23. As external load resistance R_l increases from $R_l = 500$ k Ω to $R_l = 800$ k Ω , the nondimensional galloping amplitude a decreases as shown in Fig. 6.23(a). The response curve shifts towards higher values of external excitation frequency ϕ . Similar trend is observed in the tip displacement as shown in Fig. 6.23(b). However the output voltage amplitude (Fig. 6.23(c)) increases with increase in R_l . Higher power is obtained with $R_l = 800$ k Ω , shown in Fig. 6.23(d).

The effect of steady wind speed U is studied by using the frequency response curves as given in Fig. 6.24. Here the system parameters considered here are load resistance

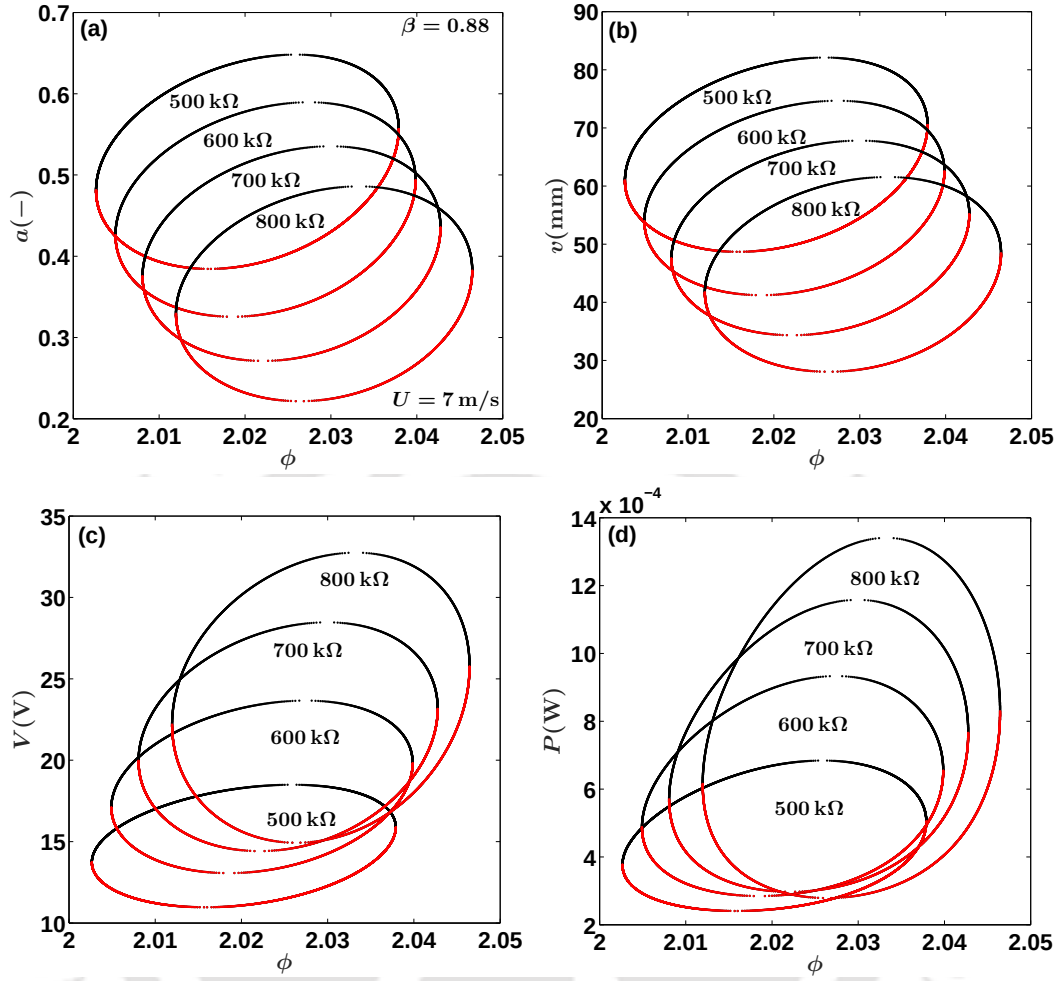


FIGURE 6.23: Frequency response with the variation of load resistance R_l for (a) Galloping amplitude a (b) Tip displacement (c) Voltage (d) Power output, for D-shape bluff body with system parameters $\beta = 0.88$, $U = 7$ m/s and $u_0 = 0.15U$

$R_l = 500$ k Ω , $\beta = 0.88$ and $u_0 = 0.15U$. As the wind speed increases from $U = 4$ m/s to $U = 7$ m/s, the output voltage also increases as shown in Fig. 6.24(a). The galloping phenomenon is possible for broad frequency range as well. The corresponding power is shown in Fig. 6.24(b) with trends similar to the voltage.

The effect of the position (β) of the D-shaped bluff body is shown in Fig. 6.25. The system parameters considered here are $U = 7$ m/s, $R_l = 700$ k Ω and $u_0 = 0.15U$. As β increases from $\beta = 0.75$ to $\beta = 0.88$, the output voltage increases as shown in Fig. 6.25(a). It may be noted that the higher value of β indicates that the bluff body is nearer to the free end of the beam. The corresponding power is shown in Fig. 6.25(b). A maximum power of 1.16 mW at $\phi = 2.03$ is obtained for $\beta = 0.88$.

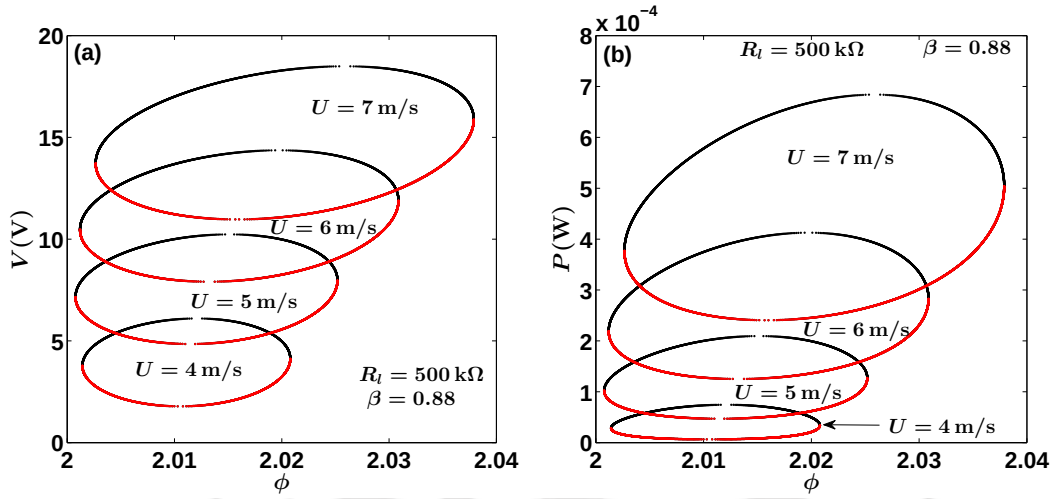


FIGURE 6.24: The effect of steady wind speed, U on (a) Voltage (b) Corresponding output power, with system parameters $R_l = 500$ kΩ, $\beta = 0.88$ and $u_0 = 0.15U$

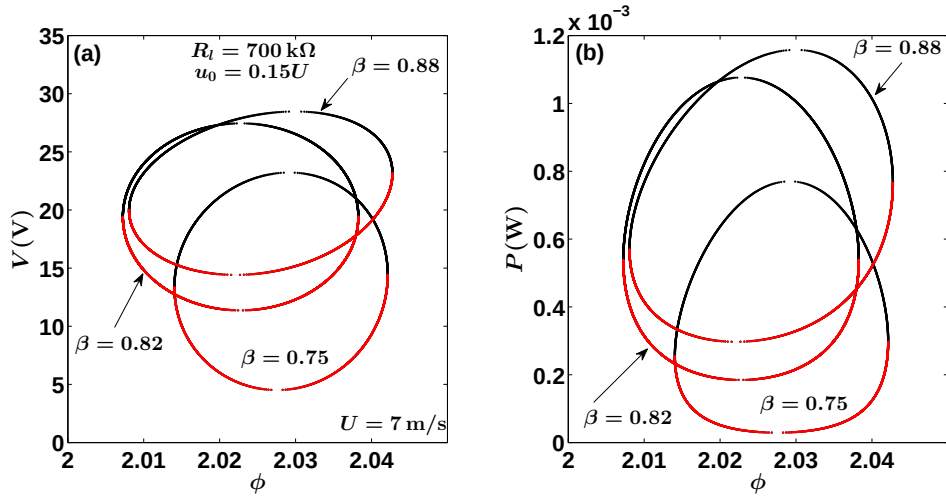


FIGURE 6.25: The effect of the position of D-shape bluff body, β on (a) Voltage (b) Corresponding output power, with system parameters $U = 7$ m/s, $R_l = 700$ kΩ and $u_0 = 0.15U$

The effect of steady wind speed U on output voltage and power for a triangular shape bluff body is shown in Fig. 6.26 with system parameters $u_0 = 0.15U$, $\beta = 0.88$ and $R_l = 500$ kΩ. As the steady wind speed U increases from 6 m/s to 8 m/s, the output voltage increases. The response curve shifts towards the higher value of ϕ and the GPEH is effective for broader frequency range as well. The maximum power of 5.58 mW is obtained at $\phi = 2.136$ as shown in Fig. 6.26(b). The effect of load resistance R_l is shown in Fig. 6.27 for triangular shape bluff body. The voltage and

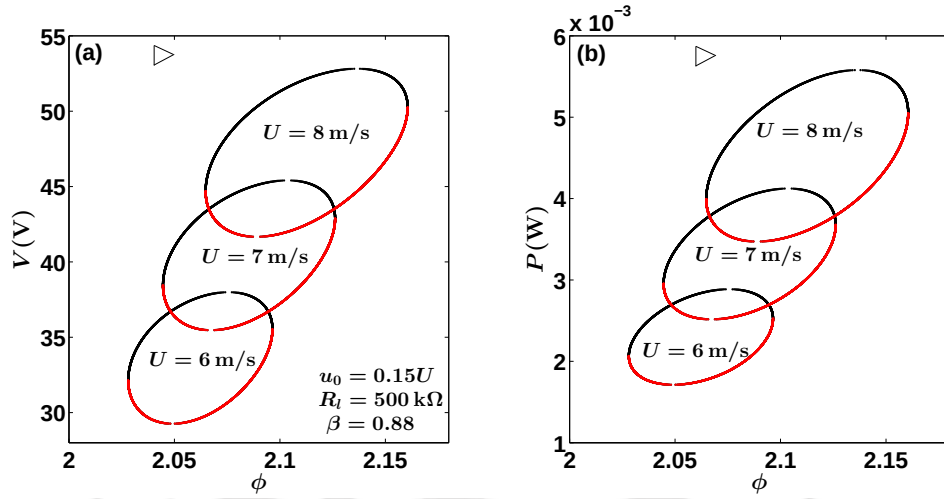


FIGURE 6.26: The effect of steady wind speed U on (a) Voltage (b) Corresponding power output for triangular shape bluff body with system parameters $u_0 = 0.15U$, $\beta = 0.88$ with load resistance $R_l = 500 \text{ k}\Omega$

power output increases with R_l . The response curve shifts towards higher voltage range without change in the frequency range. The maximum power of 9.33 mW is obtained at $\phi = 2.1$ as shown in Fig. 6.27(b).

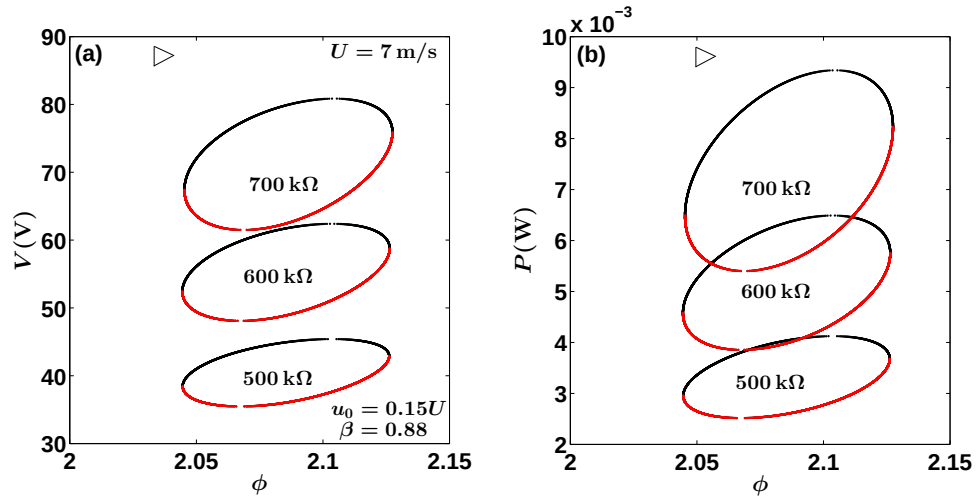


FIGURE 6.27: The effect of load resistance R_l on (a) Voltage (b) Corresponding power output for triangular shape bluff body with system parameters $u_0 = 0.15U$, $\beta = 0.88$ and $U = 7 \text{ m/s}$

The effect of variation of u_0 and the position of triangular shape bluff body β is observed in Fig. 6.28 and Fig. 6.29 respectively. While concentric response grows in case of higher values of u_0 , the response curve shifts towards left and rotate counter clockwise direction as well with increase in β value. From the voltage response

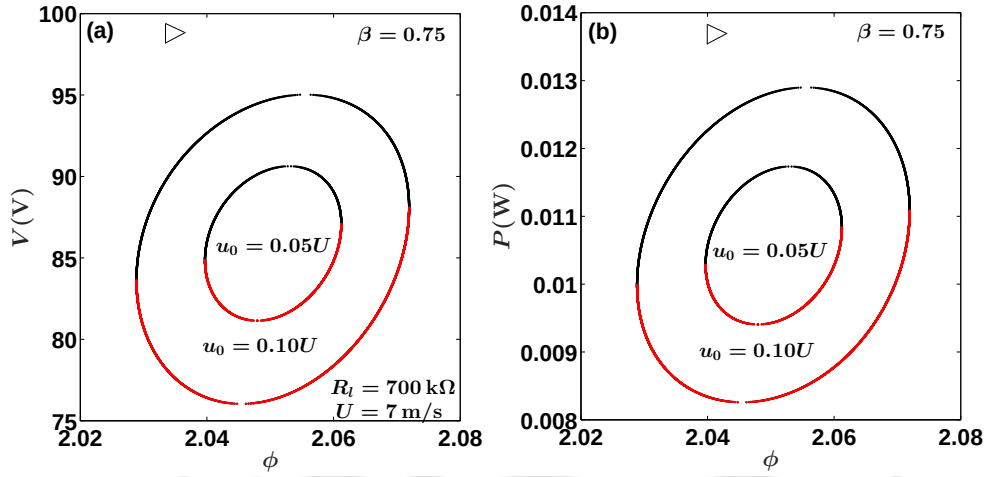


FIGURE 6.28: The effect of variation of u_0 on (a) Voltage (b) Corresponding power output for triangular shape bluff body with system parameters $R_l = 700$ k Ω , $\beta = 0.75$ and $U = 7$ m/s

curve in Fig. 6.29(a) one may observe that there exist an optimal position of bluff body where the output voltage and power become maximum. Here for example, for $\beta = 0.82$ the maximum voltage and power values obtained are 58 V and 6.76 mW, respectively for $\phi = 2.05$.

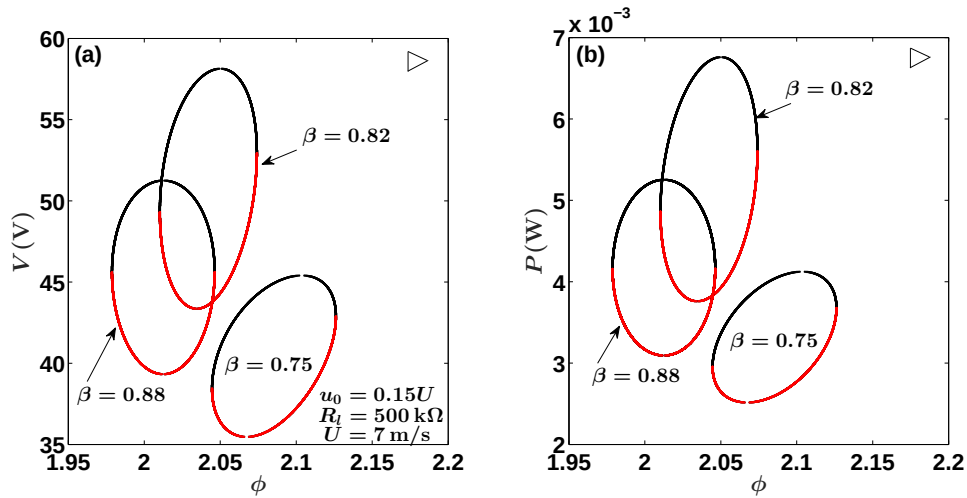


FIGURE 6.29: The effect of the position of triangular shape bluff body, β on (a) Voltage (b) Corresponding output power, with system parameters $U = 7$ m/s, $R_l = 500$ k Ω and $u_0 = 0.15U$

6.4 Summary

In this chapter, the galloping based piezoelectric energy harvester (GPEH), which consist of a vertical cantilever beam with a piezoelectric patch and attached with a bluff body at near its free end, is investigated theoretically and experimentally. The response of the GPEH system is studied by varying the system parameters viz., wind speed, load resistance and the position of the attached bluff body. Triangular and D shaped bluff bodies are used in this investigation.

It is observed that as the wind speed increases, the amplitude of transverse vibration increases due to high lift forces in the bluff body and hence the output voltage also increases. Further, it has been observed that there exist an optimal external load resistance and position of the bluff body at which the power output will be maximum. The onset of galloping speed has been determined and they are found to be in good agreement with the experimental findings. While for D shaped bluff body the onset of galloping is found to be invariant with position of the bluff body, for triangular shaped bluff body the onset of galloping is found to vary with the position of the bluff body.

The GPEH system with triangular shaped bluff body performance better than that of the D-shaped bluff body, where one may harvest more kinetic energy available in the wind. Keeping the uniform wind speed and for a range of load resistance, the GPEH with D shaped bluff body yields higher voltage than the triangular shaped bluff body when the position of the bluff body is very close to the free end. However GPEH with triangular shaped bluff body yields better output voltage for a higher value of load resistance. One may achieve power in the range of mW. The analytical solution presented here accommodates the variation in the position of the bluff body along the beam, which helps adjust the onset of galloping and the natural frequency of the harvesting system. Here the unsteady wind component is considered as harmonically varying with having a single frequency, which makes the system parametrically excited and hence multi resonance states of the system are possible. The present system is capable of harvesting in low wind speeds. The ceramic PZT-5H piezoelectric patch considered for analysis is found to be safe for higher wind speeds, where large transverse deflection of the beam takes place. The parametrically excited GPEH system can provide voltage for a wide range of frequencies even with low amplitude of excitation.



Chapter 7

Conclusions and Scope of Future Work

7.1 Introduction

In this Chapter, the conclusions related to two different types of piezoelectric based energy harvesters (PEH) have been presented. In the first type base excited piezoelectric based energy harvester subjected to principal parametric and combination parametric resonance conditions along with 1:3 internal resonance conditions have been investigated. In the second type galloping based piezoelectric energy harvester due to flow induced vibration have been studied. Both theoretical and experimental investigations have been carried out and the general conclusions and the specific conclusions are given in the next sections. Further, the scope of future work also highlighted in this chapter.

7.2 General Conclusions

In this research work, four nonlinear problems related to piezoelectric energy harvester systems have been investigated. In the first work, a PEH system that consists of a vertical cantilever beam with an attached mass at an arbitrary position and piezoelectric patches covering the whole length of the beam is studied. The attached mass is placed in such a way that the second mode frequency is three times the first mode frequency giving rise to 1:3 internal resonance condition. Due to large transverse deformation and rotary inertia of the attached mass, the system exhibits cubic

and inertial nonlinearities. Here, the base excitation is given in such a way that the system undergoes principal parametric resonance conditions, i.e., the frequency of external excitation is twice the natural frequency of the system. The steady state output voltage and power of the system are obtained for various system parameters such as load resistance, amplitude and frequency of excitation and damping ratio. The parametric instability regions with different load resistance and damping ratios are also studied. The critical system parameters below which the system will always remain stable are determined. So the system has to be operated above this critical value for harvesting energy. The time and frequency response plots are obtained for various system parameters which will help the designer to obtain the amplitude and range of frequencies of external excitation where the PEH system can operate.

In the second work, a similar system is studied under combination parametric resonance of sum type. Here, the excitation frequency is nearly equal to the sum of the first two modal frequencies of the system. The fixed point response and the stability of the system are investigated. The instability regions for different system parameters are analysed. The nonlinear frequency responses of the system are studied for different system parameters like load resistance, amplitude and frequency of excitation and damping ratio. Unlike the operating range of frequencies for the harvesters available in the literature here, by employing 1:3 internal resonance and principal parametric and combination parametric resonance, it has been shown that the harvester can operate in a wide range of frequency which is available in ambient condition.

It has been demonstrated that there exists a range of system parameters such as length and thickness of the beam, position of the attached mass, mass ratio and Young's modulus of the substrate and piezoelectric patch, which can be varied to obtain 1:3 internal resonance.

In the third problem, an experimental investigation has been attempted to understand the system dynamics and challenges posed by the system due to large transverse vibrations. A slider-crank mechanism based shaker is designed and developed in-house, to provide harmonic base excitation. The system is excited at principal parametric resonance condition and the output voltage has been determined using oscilloscope. These experimental results have been compared with the theoretical findings and they are found to be in good agreement.

In the fourth work galloping based harvester is investigated both theoretically and

experimentally. Here, a vertical cantilever beam with unimorph PZT patch and an attached bluff body near its free end is considered as the PEH system. Taking both D type and triangular bluff bodies, experiments are performed in the wind tunnel to study voltage time response across a load resistance under uniform flow condition. The effect of the position of bluff bodies on voltage and power output are also studied. Theoretical investigation have been carried out considering both uniform and time varying wind speed.

For the first two problems, two mode approximations have been used as internal resonance conditions of 1:3 is considered. In the last two work single mode approximation has been used. The temporal equations are further reduced by using the method of multiple scales to obtain the approximate solution. The steady state responses and their stability and bifurcations have also been studied to know the operating range of system parameters for the PEH.

In the present work the nonlinear dynamics of PEH system is studied under the principal parametric and combination parametric resonance conditions along with internal resonance condition. Due to the existence of cubic nonlinearity, the harvester system is capable of harvesting energy not only in principal parametric resonance condition (external excitation (Ω) twice that of fundamental frequency (ω) of the system, i.e. $\Omega \approx 2\omega$) but also at subharmonic ($\Omega \approx 3\omega$), superharmonic ($\Omega \approx \omega/3$) and combination parametric resonance of sum and difference type ($\Omega \approx \omega_m \pm \omega_n$, here $m = 1, 2$ and $n = 1, 2$) as well.

Based on the theoretical and experimental findings the parameters affecting the power output are the position of the piezoelectric patch, number of patches and optimum value of external load resistance. Patch position nearer to the fixed end of the beam is advantageous and also more patches will help to increase the power. Proper selection of load resistance by impedance matching procedure can be utilised to maximize the output power as well.

As the developed analytical solutions are found to be in good agreement with the experimental findings hence these analytical solutions can be used for the parametric studies for a wide range of system parameters for newly developed base excited or PEH systems without conducting expensive experiments. In the next section, the specific conclusions related to different PEH systems are highlighted.

7.3 Specific Conclusions

7.3.1 PEH under principal parametric and 1:3 internal resonance condition

In this section specific conclusions related to harmonically base excited vertical cantilever beam with piezoelectric bimorph and an attached mass under principal parametric and 1:3 internal resonance condition have been presented.

- The instability region increases with an increase in the amplitude of base excitation Γ . For example with increase in Γ from 2 to 3, the unstable frequency range increases from $1.932 < \phi < 2.069$ to $1.869 < \phi < 2.104$ which is found to be an increase of 51.8 %. Hence the PEH system can operate for a wider frequency range by increasing the amplitude of base excitation.
- The internal resonance condition increases the trivial state of instability. Here, alternate stable and unstable zones with supercritical and subcritical pitchfork bifurcations have been observed which significantly increase the operating range of the PEH system. As an example it is found that for $\Gamma = 4$ and $\nu = 0.001$, the instability region is observed between $1.863 < \phi < 2.133$ with an addition unstable range of $2.144 < \phi < 2.157$.
- With an increase in damping the regions of instability decreases and the critical amplitude of base excitation (Γ_{crit}) increases. Hence one requires to operate the PEH at a higher amplitude of base excitation to generate voltage. For example, at $\phi = 2$, the critical value of amplitude is found to be 4.75 and 11.7, for $\nu = 1$ and $\nu = 2.5$, respectively. The overall damping is varied in the range of 0.007 % to 10 %.
- With an increase in load resistance R_l , the stability boundary shifts towards higher frequency range. Initially Γ_{crit} increases with R_l , but for higher values of R_l it decreases. For example, the Γ_{crit} is found to be 1.6 at $R_l = 10 \text{ k}\Omega$ and $\Gamma_{crit} = 1.05$ at $R_l = 100 \text{ k}\Omega$.
- The critical bifurcation points in the trivial and nontrivial branches of the nonlinear response are very critical to decide the operating range of the harvester. These bifurcation points include saddle node, pitch-fork and Hopf bifurcations.
- While sweeping up the frequency for $\phi > 2.029$, the PEH system without internal resonance performs better. While sweeping down the frequency for $\phi < 2.029$, the

PEH system with internal resonance perform better for the system parameters $\nu = 0.001$, $\Gamma = 1$ at $R_l = 1 \text{ k}\Omega$.

- It may be further noted that by adjusting the location of the attached mass the frequency of the system can be tuned and other internal resonance conditions can be utilized for harvesting more energy.

7.3.2 PEH under combination parametric and 1:3 internal resonance

In this section, specific conclusions related to an energy harvester consisting of a harmonically base excited vertical cantilever beam with piezoelectric path and attached mass excited at a frequency equal to sum of first two modal frequencies is highlighted. The first modal frequencies are nearly in the ratio of 1:3.

- The instability region is found to vary between the nondimensional frequency ϕ range of 3.95 to 4.1, with increase in damping and decrease in load resistance. The critical amplitude of base excitation below which the harvester is ineffective is determined and found to increase with damping and load resistance. Hence to harvest energy in a wide range of frequency one should use low damping and low load resistance. It may be noted that the trivial state loses its stability due to Hopf bifurcation. But in case of principal parametric resonance it loses its stability by pitchfork bifurcation.
- From the frequency response plots it is evident that more voltage can be generated by operating the harvester at a frequency greater than sum of the first two mode frequencies (that is when the system is positively tuned). In the studied system a maximum voltage of 20 V and power of 200 mW are obtained. One may increase these values by suitably changing the system parameters.
- Both saddle node and Hopf bifurcations are observed in the multibranched nontrivial state.
- It is observed that the voltage and power increase significantly as R_l increases which shows that the load resistance plays an important role in achieving an optimum power output. Here at $\phi = 4.5$, a voltage of 0.77, 3.8, 7.77 and 15.3 V with corresponding power of 5.9, 29.5, 58.9 and 117 mW is achievable as the load resistance increases from $R_l = 0.1 \text{ k}\Omega$ to $R_l = 2 \text{ k}\Omega$.

- With increase in damping the nontrivial states at a frequency below the sum of two modal frequencies (negatively detuned) disappears. Hence no energy can be generated in the region of frequency.
- With decrease in amplitude of base excitation the PEH system becomes ineffective for $\phi < 4$. It is operative only for $\phi > 4$. Hence to generate voltage for a wider range of frequency the amplitude of base excitation is moderately high.
- A significant change in the topology of the nontrivial branch of solution is observed when the amplitude of base excitation and the damping are very high ($\Gamma = 10$, $\nu = 0.01$). Power is generated only for narrow bandwidth of frequency. Hence one should properly choose the system parameters to obtain the required voltage and power.
- It has been shown that by suitably changing the mass ratio μ , length of the beam, position of the attached mass β , Young's modulus of the beam and thickness of the beam, one may obtain 1:3 internal resonance condition. It is clearly observed that for $0.2 < \beta < 0.27$ and $0.3 < \mu < 0.4$ one may find the first two modal frequencies in the ratio of 1:3 giving rise to internal resonance.

7.3.3 PEH under parametric excitation: A theoretical and experimental investigation

In this section, the specific conclusions obtained for the experimentally studied base excited PEH system is presented.

- A slider-crank mechanism is developed for providing base excitation to the PEH system which has a variable stroke length l_s between 13 mm to 22 mm. Here a maximum frequency of 25 Hz can be obtained to excite the harvester near twice the principal parametric resonance condition.
- The experimentally obtained damping ratio and the fundamental frequency of the system are found to be 0.0134 and 3.6 Hz, respectively. These values are found to be in good agreement with the analytical results. Hence one may use the developed analytical formulation for determining these parameters for any other sets of system parameters.
- The supercritical pitchfork bifurcation is observed at 6.76 Hz, which is below twice the natural frequency of the system. At this point the onset of harvesting takes place. With increase in the external frequency of excitation the output voltage

increases up to the saddle node bifurcation point, beyond which one may expect a catastrophic failure due to jump down phenomenon. Hence one may operate the PEH system between the supercritical and subcritical pitchfork bifurcation point which is well below the saddle node bifurcation point.

- One may achieve a maximum power of nearly 22.7 mW around a frequency of excitation of 7.25 Hz (near saddle node bifurcation point) for external load resistance of 100 k Ω . This experimental results closely matches with the analytical findings.
- Near the theoretically observed saddle node bifurcation point, few hairline cracks are observed in the piezoelectric patch (PZT-5H) after few cycles of operation. This is because the patch is placed near to the base (spanned from 5.7 mm to 10.7 mm length from the base), where the bending moment is maximum.
- It is observed that cracks are developed in the beam near to its fixed end due to maximum stress, which leads to the breaking of the beam. Hence to improve the life of the PEH system, it should be operated at a frequency lower than the saddle node frequency. Also design modification may be carried out to strengthen the base part by using torsional springs and stoppers.
- It is observed that the load resistance influences the power output and at a certain value of R_l , which is 800 k Ω in the present case, the output power gets maximum. The average value of voltage amplitude is found to be 54.72 V.
- As the position of attached mass increases, the threshold frequency at which beam starts transverse vibration reduces. By suitably positioning the mass, one may adjust the natural frequency of the system. Hence the resonance condition can also be tuned to regulate the output voltage. Here it may be noted that magnetic materials are used as attached mass, so they can be shifted to suitable position easily.
- The normalized power density (power/(volume $\times$ acceleration²)) for the present PEH system is found to be 15.9.
- The PEH system with the MFC patch is also investigated experimentally under the parametric excitation. It is observed that the maximum power of 2.25 μ W is achieved for system parameters $m = 36$ gm, $\beta = 0.233$ and load resistance of 500 k Ω . Also the frequency of excitation decreases as m increases.
- As the attached mass varies from 9 gm to 45 gm while keeping its position and load resistance constant, the voltage increases from 1 V to 2.6 V (160% increase). Also

the threshold frequency at which supercritical pitchfork vibration occurs decreases from 5.2 Hz to 4.5 Hz (13.5% decrease).

- An internal resonance of 1:2 is observed in FFT of voltage time response, Which enhances the output voltage and power of the PEH system.
- As the position of the attached mass β increases, a percentage increase of 58.3 % is observed in the output voltage.
- As the MFC patch positions bring nearer to the fixed end of the beam the output power increases to nearly 25 μ W.
- When the PEH system is excited at the frequency near to the sum of the first two natural frequencies (4.3 Hz + 8.3 Hz $\approx$ 13 Hz) two major frequency peaks are observed with having nearly 1:3 frequency ratio.
- For an external frequency more than 13 Hz, the FFT of voltage time response for the PEH system shows multiple frequency components. Here, due to modal interactions between several participating modes, synchronous and sub-synchronous frequencies are observed which are generally observed in rotor bearing system.

7.3.4 Dynamics of galloping based piezoelectric energy harvester

Here the specific conclusions related to the galloping based PEH system (GPEH) is described.

- Reduction of 84.6% is observed when the position of the bluff body β is kept at 0.88 as compared to the beam without a bluff body.
- PEH with D-shaped bluff body as the wind speed increases from 4 m/s to 5 m/s, a 39% increase in voltage, and a 90% increase in output power is observed at load resistance of 300 k Ω . Hence by adjusting the position of the bluff body, one may control the resonance conditions and hence the output voltage.
- As the bluff body placed close to the fixed end, the output voltage increases for PEH with D-shaped bluff body. The critical speed at which the PEH system starts to vibrate and generate power is known as the onset speed of galloping. It is observed that the galloping speed does not vary much by changing the position of the bluff body. At higher speed ($U = 7$ m/s), a variation of 43.8% in voltage is observed.

Hence though the onset of galloping does not depend on the positioning of the bluff body, the voltage can be controlled by varying the position of the bluff body at a higher speed.

- An optimal load resistance is observed at which the output power is found to be maximum.
- GPEH with the triangular-shaped bluff body (GPEH-TSB), as β decreases, the voltage variation with load resistance follows a similar trajectory up to 600 k Ω . Beyond $R_l > 600$ k Ω , the GPEH with lower β performs better.
- For GPEH-TSB, the onset of galloping changes for different positions of the bluff body.
- The onset speed of galloping is invariant for GPEH with D-shaped bluff body but varies for GPEH with the triangular shaped bluff body with respect to the bluff body position along the axis of the vertical cantilever beam.
- GPEH-DSB performs better than that of GPEH-TSB before 800 k Ω . The maximum voltage variation of 38.6% at 500 k Ω is observed.
- It is observed that the onset of galloping wind speed is nearly similar at higher β value for both GPEH-DSB and GPEH-TSB systems, but it is found to be lower for GPEH-TSB as compared to GPEH-DSB for lower β values.
- The ceramic PZT-5H piezoelectric patch, which is considered for the analysis, is found to be safe in the wind tunnel experiments, even in higher wind speeds beyond the 12 m/s. Here the piezoelectric patch (of length 5 cm) is placed at a distance of 3.3 cm from the fixed end.
- The maximum power of 2.95 mW is achieved with the triangular-shaped bluff body (GPEH-TSB) for parameters $\beta = 0.88$, $U = 7$ m/s and $R_l = 1000$ k Ω . Also in case of GPEH with time varying wind flow condition a maximum power of 13 mW can be achieved with system parameters $\beta = 0.75$, $U = 7$ m/s, $R_l = 700$ k Ω and $u_0 = 0.1U$.

7.4 Scope of Future Work

In the present study, initially parametrically excited cantilever beam based PEH systems are considered and later galloping based PEH system has been considered.

There are many scopes to extend the present work in order to enhance the frequency bandwidth and output power of the harvester. Some of them are given below.

- Considering hard excitation, the present system can be extended to study super-harmonic, subharmonic, simultaneous direct and parametric resonance conditions which will further improve the bandwidth and output voltage and power of the harvester. One may include the internal resonance conditions also to further improve the bandwidth.
- In the present work, the beam is in a vertical position. The beam may be considered in horizontal or in inclined positions as well.
- The attached mass is placed symmetrically in the present case. However it may be placed unsymmetrically, which may bring more nonlinearities to the system and investigation is required to know about the improvement in the power output and frequency bandwidth.
- The dynamics of galloping based harvester may be studied under unsteady flow conditions experimentally by developing new facilities in the wind tunnel.
- In the present system the internal resonance conditions is not considered in the galloping based energy harvester. So, this may be considered for further analysis.
- In the present work only two mode interactions have been considered. Hence, this analysis can be extended to study three mode interactions such as $1 : 3 : 5$ or $1 : 3 : 9$ as observed in the work of Dwivedy and Kar [215, 236].
- Here only D-shaped and triangular shaped bluff bodies have been taken. More different bluff bodies with an optimized shapes may be considered both experimentally and analytically for further improvement of the harvester.
- Here only a single harvester is used for the analysis. An array of such systems along with passive elements like springs and dampers may be used in the vibrating structures such as moving vehicles, bridges, tall buildings etc., to generate more voltage and power.
- Similar PEH systems may be explored for harvesting energy for biomedical applications.
- The PEH systems presented in this work can be further studied under more realistic loading conditions such as random parametric excitation. Also, in case of Galloping

based PEH system, a periodic variation in wind speed is considered which may be extended to stochastic variation as well.

- There is a need to develop the electric circuit for regulated output voltage supply for the present energy harvesters.
- The two degree of freedom beam system [320] may be investigated under the parametric excitation condition as well.





Appendix A

Coefficients and expressions

$$\begin{aligned}
 h_{11} &= \int_0^1 \psi_n^2 dx, \\
 h_{12} &= \int_0^1 \delta(x - \beta) \psi_n^2 dx, \\
 h_{13} &= \int_0^1 \psi_{nx}^2(\beta) dx, \\
 h_{21} &= h_{31} = h_{11}, \quad h_{32} = h_{12}, \\
 h_{33} &= \int_0^1 (1 - x) \psi_{nx}^2 dx, \\
 h_{34} &= \int_0^1 \int_x^1 \delta(\xi - \beta) d\xi \psi_{nx}^2 dx, \\
 h_{41} &= \frac{1}{2} \int_0^1 \psi_k \psi_{lx} \psi_{mx} \psi_n dx, \\
 h_{42} &= \frac{1}{2} \int_0^1 \delta(x - \beta) \psi_k \psi_{lx} \psi_{mx} \psi_n dx, \\
 h_{43} &= 3 \int_0^1 \psi_{kx} \psi_{lxx} \psi_{mxxx} \psi_n dx + \int_0^1 \psi_{kxx} \psi_{lxx} \psi_{mxx} \psi_n dx, \\
 h_{51} &= \int_0^1 \left\{ \int_x^1 \left(\int_0^\xi \psi_{l\eta} \psi_{mx} d\eta \right) d\xi \right\} \psi_{kx} \psi_{nx} dx, \\
 h_{52} &= \left(\int_0^\beta \psi_{lx} \psi_{mx} dx \right) \left(\int_0^\beta \psi_{kx} \psi_{nx} dx \right), \\
 h_{53} &= \{ \psi_{kx} \psi_{lx} \psi_{mx} \psi_{nx} \}_{x=\beta}, \\
 h_{61} &= h_{51}, \\
 h_{62} &= \frac{1}{2} \int_0^1 \psi_{kx} \psi_{lx} \psi_{mx} \psi_n dx - \int_0^1 \psi_{kx} \psi_{lxx} \left(\int_x^1 \psi_m d\xi \right) dx, \\
 h_{63} &= h_{52},
 \end{aligned}$$

$$h_{64} = \frac{1}{2} \{ \psi_{kx} \psi_{lx} \psi_m \psi_{nx} \}_{x=\beta} - \psi_m(\beta) \int_0^\beta \psi_{kx} \psi_{lx} \psi_n dx,$$

$$h_{65} = \frac{1}{2} h_{53},$$

$$\hbar_n = h_{11} + \mu h_{12} + J \lambda^2 h_{13},$$

$$\theta_n^2 = \frac{EI \kappa_n^4}{\rho L^4 \hbar_n} (h_{31} + \mu h_{32}) - \frac{g}{L \hbar_n} (h_{33} + \mu h_{34}),$$

$$f_{nm} = \hat{f}_{nm} \frac{\Gamma}{\epsilon} = \frac{\Omega^2 Z_0}{\epsilon \theta_1^2 \hbar_n L} (h_{33} + \mu h_{34}),$$

$$\alpha_{klm}^n = \frac{EI \lambda^2}{\epsilon \rho L^4 \hbar_n \theta_1^2} \{ \kappa_k^4 (h_{41} + \mu h_{42}) + h_{43} \},$$

$$\beta_{klm}^n = \frac{\lambda^2}{\epsilon \hbar_n} \{ h_{51} + \mu h_{52} + J \lambda^2 h_{53} \},$$

$$\gamma_{klm}^n = \frac{\lambda^2}{\epsilon \hbar_n} \{ h_{61} - h_{62} + \mu (h_{63} - h_{64}) + J \lambda^2 h_{65} \},$$

$$\alpha_{enj} = \alpha_{nj} + \beta_{nj} + \gamma_{nj},$$

$$\alpha_{nj} = \begin{cases} 3\alpha_{nnn}^n & \text{for } j = n \\ 2(\alpha_{njj}^n + \alpha_{jjn}^n + \alpha_{jnn}^n) & \text{for } j \neq n \end{cases}$$

$$\beta_{nj} = \begin{cases} \omega_n^2 \beta_{nnn}^n & \text{for } j = n \\ 2\omega_j^2 \beta_{njj}^n & \text{for } j \neq n \end{cases},$$

$$\gamma_{nj} = \begin{cases} -3\omega_n^2 \gamma_{nnn}^n & \text{for } j = n \\ -2 \{ \omega_j^2 (\gamma_{jnn}^n + \gamma_{njj}^n) + \omega_n^2 \gamma_{jjn}^n \} & \text{for } j \neq n \end{cases},$$

$$Q_{12} = \alpha_{121}^1 + \alpha_{211}^1 + \alpha_{112}^1 + \omega_1 \omega_2 (\beta_{121}^1 + \beta_{112}^1) \\ - \{ \omega_1^2 (\beta_{211}^1 + \gamma_{211}^1 + \gamma_{121}^1) + \omega_2^2 \gamma_{112}^1 \},$$

$$Q_{21} = \alpha_{111}^1 - \omega_1^2 (\beta_{111}^1 + \gamma_{111}^1),$$

The transcendental equation is

$$\frac{4(h_1 h_4 - h_2 h_3)}{\lambda^2 \mu J} + 2\kappa^4 \{1 - \cos \kappa \beta \cosh \kappa \beta\} \\ + \frac{2\kappa^3}{\mu} \left[\begin{matrix} h_1(\sin \kappa \beta + \sinh \kappa \beta) \\ + h_2(\cos \kappa \beta - \cosh \kappa \beta) \end{matrix} \right] + \frac{2\kappa}{J \lambda^2} \left[\begin{matrix} h_4(\sin \kappa \beta - \sinh \kappa \beta) \\ - h_2(\cos \kappa \beta - \cosh \kappa \beta) \end{matrix} \right] = 0 \quad (\text{A.1})$$

where, κ denotes the eigenvalue and n^{th} linear natural frequency of the system is given as follows

$$\Theta_n^2 = \frac{EI}{\rho} \left(\frac{\kappa_n}{L} \right)^4 \quad (\text{A.2})$$

The shape function in nondimensional form is given as

$$\begin{aligned} \psi_n(s) = & [(\sin \kappa_n x - \sinh \kappa_n x) - \Lambda (\cos \kappa_n x - \cosh \kappa_n x)] \\ & + U(\bar{x}) [(h_1 - \Lambda h_2) \{\sin \kappa_n \bar{x} \sinh \kappa_n \bar{x}\} + (h_3 - \Lambda h_4) \{\cos \kappa_n \bar{x} \cosh \kappa_n \bar{x}\}] \end{aligned} \quad (\text{A.3})$$

Here, $\bar{x} = x - \beta$, and U is the unit step function and the values of h_1 , h_2 , h_3 , h_4 and Λ are specified in Ref. [182]. Also the following parameter values are considered for simulation.

$$\begin{aligned} Z_r = 1 \text{ mm}, c = 1 \text{ Ns/mm}^2, \mu = 3.9524, J = 1.0473, \beta = 0.28, \kappa_1 = 1.761541, \\ \kappa_2 = 3.158918, \omega_1 = 1, \omega_2 = 3.25, \epsilon = 0.001, \lambda = 0.1 \\ \alpha_{e11} = 3.1, \alpha_{e12} = -28.70, \alpha_{e21} = 23.05, \alpha_{e22} = -245.927, \\ \hat{f}_{11} = 0.068713, \hat{f}_{12} = 0.01480294, \hat{f}_{21} = 0.0359777, \hat{f}_{22} = 0.147767, \\ Q_{12} = 13.5708, Q_{21} = 5.2148, \zeta_1 = 0.0780 \frac{\nu}{\epsilon}, \zeta_2 = 0.0378 \frac{\nu}{\epsilon} \\ K_1 = 1.0000, K_2 = -1.2261, \eta_{1s} = -152.9231, \eta_{2s} = 455.7053 \end{aligned}$$

TABLE A1: Few beam configurations to achieve 1:3 internal resonance with $b_p = 11.2$ mm; $t_p = 0.125$ mm; $L_p = 107$ mm; $\rho_s = 8100$ kg m⁻³; $\rho_p = 5440$ kg m⁻³; $E_s = 190$ GPa; $E_p = 15.86$ GPa

m (g)	β	L_s (mm)	b_s (mm)	t_s (mm)	$\omega_1 : \omega_2$
50	0.30	295	20	0.51	1 : 3.2731
60	0.28	180	20	0.51	1 : 3.3503
60	0.28	180	17	0.51	1 : 3.2601
60	0.28	180	16	0.51	1 : 3.2299
40	0.29	150	14	0.51	1 : 3.2998
60	0.28	160	24	0.51	1 : 3.3724
60	0.28	150	24	0.51	1 : 3.3250
60	0.28	140	24	0.51	1 : 3.2793
30	0.28	140	15	0.30	1 : 3.2972
30	0.28	140	16	0.30	1 : 3.2758
30	0.28	140	15	0.30	1 : 3.2499
30	0.28	135	15	0.30	1 : 3.2277



Appendix B

$$\begin{aligned}
 h_{11} &= \int_0^1 \frac{\rho(x)}{\rho_b A_b} \psi_n^2 dx, \\
 h_{12} &= \int_0^1 \delta(x - \beta) \psi_n^2 dx, \\
 h_{13} &= \int_0^1 \delta(x - \beta) \psi_{nx}^2 dx, \\
 h_{21} &= h_{11}, \\
 h_{31} &= \int_0^1 EI^*(x) \psi_n^2 dx, \\
 h_{32} &= \int_0^1 \frac{\rho_b A_b EI^*(x)}{\rho(x)} \psi_n^2 dx, \\
 h_{33} &= \int_0^1 \frac{\rho}{\rho_b A_b} (1 - x) \psi_{nx}^2 dx, \\
 h_{34} &= \int_0^1 \int_x^1 \delta(\xi - \beta) d\xi \psi_n^2 dx, \\
 h_{41} &= \frac{1}{2} \int_0^1 EI^*(x) \psi_k \psi_{lx} \psi_{mx} \psi_n dx, \\
 h_{42} &= \frac{1}{2} \int_0^1 \delta(x - \beta) EI^*(x) \psi_k \psi_{lx} \psi_{mx} \psi_n dx, \\
 h_{43} &= 3 \int_0^1 EI^*(x) \psi_{kx} \psi_{lxx} \psi_{mxxx} \psi_n dx + \int_0^1 EI^*(x) \psi_{kxx} \psi_{lxx} \psi_{mxx} \psi_n dx, \\
 h_{51} &= \int_0^1 \left\{ \int_x^1 \left(\int_0^\xi \psi_{l\eta} \psi_{m\eta} d\eta \right) d\xi \right\} \frac{\rho(x)}{\rho_b A_b} \psi_{kx} \psi_{nx} dx, \\
 h_{52} &= \left(\int_0^\beta \psi_{lx} \psi_{mx} dx \right) \left(\int_0^\beta \psi_{kx} \psi_{nx} dx \right), \\
 h_{53} &= \{ \psi_{kx} \psi_{lx} \psi_{mx} \psi_{nx} \}_{x=\beta}, \quad h_{61} = h_{51}
 \end{aligned}$$

$$\begin{aligned}
h_{62} &= \int_0^1 \frac{1}{2} \frac{\rho(x)}{\rho_b A_b} \psi_{kx} \psi_{lx} \psi_{mx} \psi_n dx - \int_0^1 \frac{\rho(x)}{\rho_b A_b} \psi_{kx} \psi_{lxx} \left(\int_x^1 \psi_m d\xi \right) dx \\
h_{63} &= h_{52}, \quad h_{65} = 0.5h_{53}, \\
h_{64} &= \frac{1}{2} \{ \psi_{kx} \psi_{lx} \psi_m \psi_{nx} \}_{x=\beta} - \psi_m(\beta) \int_0^\beta \psi_{kx} \psi_{lxx} \psi_n dx \\
\zeta_n^* &= \frac{ch_{21}}{2\epsilon \bar{h}_n \rho_b A_b \theta_1}; \quad \bar{h}_n = h_{11} + \mu h_{12} + J\lambda^2 h_{13}, \\
\theta_n^2 &= \frac{\kappa_n^4 (h_{31} + \mu h_{32})}{\rho_b A_b L^4 \bar{h}_n} - \frac{g(h_{33} + \mu h_{34})}{L \bar{h}_n}, \\
f_{nm} &= \frac{\Omega^2 z_0}{\epsilon \theta_1^2 \bar{h}_n L} (h_{33} + \mu h_{34}), \\
\alpha_{klm}^n &= \frac{\lambda^2}{\epsilon \rho_b A_b L^4 \bar{h}_n \theta_1^2} \{ \kappa_k^4 (h_{41} + \mu h_{42}) + h_{43} \}, \\
\beta_{klm}^n &= \frac{\lambda^2}{\epsilon \bar{h}_n} \{ h_{51} + \mu h_{52} + J\lambda^2 h_{53} \}, \\
\gamma_{klm}^n &= \frac{\lambda^2}{\epsilon \bar{h}_n} \{ h_{61} - h_{62} + \mu (h_{63} - h_{64}) + J\lambda^2 h_{65} \} \\
\alpha_{enj} &= \alpha_{nj} + \beta_{en} + \gamma_{en}, \\
\alpha_{nj} &= \begin{cases} 3\alpha_{nnn}^n & \text{for } j = n \\ 2(\alpha_{njj}^n + \alpha_{jjn}^n + \alpha_{jnj}^n) & \text{for } j \neq n \end{cases}, \\
\beta_{nj} &= \begin{cases} \omega_n^2 \beta_{nnn}^n & \text{for } j = n \\ 2\omega_j^2 \beta_{njj}^n & \text{for } j \neq n \end{cases}, \\
\gamma_{nj} &= \begin{cases} -3\omega_n^2 \gamma_{nnn}^n & , j = n \\ -2 \{ \omega_j^2 (\gamma_{jnj}^n + \gamma_{njj}^n) + \omega_n^2 \gamma_{jjn}^n \} & , j \neq n \end{cases}, \\
Q_1^* &= \alpha_{121}^1 + \alpha_{211}^1 + \alpha_{112}^1 + \omega_1 \omega_2 (\beta_{121}^1 + \beta_{112}^1) - \omega_1^2 \beta_{211}^1 - \{ \omega_1^2 (\gamma_{211}^1 + \gamma_{121}^1) + \omega_2^2 \gamma_{112}^1 \}, \\
Q_2^* &= \alpha_{111}^1 - \omega_1^2 (\beta_{111}^1 + \gamma_{111}^1)
\end{aligned}$$

Other parameters are defined as follows

$$z_r = 1 \text{ mm}, \quad c = 1 \text{ Ns} - \text{mm}^{-2}, \quad \mu = 3.6, \quad J = 0.0366, \quad \beta = 0.27, \quad \lambda = 0.1$$

$$\kappa_1 = 1.791, \quad \kappa_2 = 3.244, \quad \epsilon = 0.001, \quad \omega_1 = 1, \quad \omega_2 = 3.013$$

$$\alpha_{e11} = -9.306, \quad \alpha_{e12} = 9.312, \quad \alpha_{e21} = 60.12, \quad \alpha_{e22} = -255.7$$

$$f_{11} = 0.033, \quad f_{12} = 0.0124, \quad f_{21} = 0.0358, \quad f_{22} = 0.0852$$

$$Q_1^* = -10.12, \quad Q_2^* = -37.4, \quad \zeta_1 = 3.126 \frac{\nu}{\epsilon}, \quad \zeta_2 = 1.267 \frac{\nu}{\epsilon}$$

Appendix C

In the present work, the extended Hamilton's principle $\int_{t_1}^{t_2} (\delta T - \delta \Pi + \delta W_{nc}) dt$ is used to derive the governing equation of motion of the system. The kinetic energy (T) and potential energy (Π) of the harvester under parametric excitation are expressed by the following expression.

$$T = \frac{1}{2} \int_0^L [\rho(s) + m\delta(s-d)] (\dot{u}(s,t)^2 + \dot{v}(s,t)^2) ds + \frac{1}{2} J_0 \dot{\varphi}(d)^2 \quad (C.1)$$

$$\Pi = \frac{1}{2} \int_0^L E_b I_b \varphi'(s)^2 ds - \int_0^L [\rho(s) + m\delta(s-d)] g u(s,t) ds \quad (C.2)$$

Here ρ , δ , J_0 , φ , $E_b I_b$ and g denotes the unit mass density of beam system, Dirac delta function, rotary inertia of mass m , angle of rotation, the flexural rigidity of beam system and gravity constant, respectively. The lateral and longitudinal dynamics is governed by inextensibility condition [12] as $u(s,t) = \frac{1}{2} \int_0^s \{v'(\xi,t)\}^2 d\xi + z(t)$. Here $z(t)$ denotes the harmonic base excitation. On observing the deflected beam geometry the relation between the angle of rotation $\varphi(s)$ and transverse deflection $v(s,t)$ is defined as $\sin \varphi = v'$ and $\cos \varphi = (1 - v'^2)^{1/2}$. As per the Euler-Bernoulli beam theory the curvature $\kappa(s,t)$ is defined as follows.

$$\kappa(s,t) = \varphi' = \frac{\partial \varphi(s)}{\partial s} = \frac{v''}{\cos \varphi} = \frac{v''}{\sqrt{1 - v'^2}} \approx v'' \left(1 + \frac{1}{2} v'^2 \right) \quad (C.3)$$

The non-conservative virtual work done W_{nc} involves the work done in dissipation of energy by viscous force effect and electrical forces in moving charges by piezoelectric

elements [176].

$$\delta W_{nc} = - \int_0^L c\dot{v}(s, t)\delta v ds - \int_0^L [H(s - L_1) - H(s - L_2)] M_{ele}(s, t)\delta\kappa(s)ds \quad (C.4)$$

The total moment due to piezoelectric patches M_{ele} is expressed as

$$M_{ele} = \int_{h_1}^{h_2} d_{31}E_p b_p (-E_e) y dy \quad (C.5)$$

Here,

$$E_e = \frac{\bar{V}(t)}{t_p}; \quad h_1 = \frac{1}{2}t_b - \bar{y}; \quad h_2 = \frac{1}{2}t_b + t_p - \bar{y}; \quad \bar{y} = \frac{E_p t_p b_p (t_p + t_b)}{2(E_p t_p b_p + E_b t_b b_b)}$$

The governing nonlinear equation of motion becomes

$$\rho^* v_{tt} + cv_t + EI^* [v_{ssss} + \{v_s(v_s v_{ss})_s\}_s] - [J^*(v_s)_{tt}]_s - (Nv_s)_s - H_{ss}\bar{\eta}\bar{V} = 0 \quad (C.6)$$

Here,

$$\rho^* = \rho(s) + m\delta(s - d), \quad \rho(s) = \rho_b A_b + H\rho_p A_p, \quad EI^* = E_b I_b + H E_p I_p, \quad J^* = J_0 \delta(s - d),$$

$$\bar{\eta} = \frac{e_{31} b_p}{t_p} \left[(t_p + \frac{1}{2}t_b - \bar{y})^2 - (\frac{1}{2}t_b - \bar{y})^2 \right], \quad A_b = b_b t_b, \quad A_p = b_p t_p,$$

$$H = H(s - L_1) - H(s - L_2), \quad e_{31} = d_{31} E_p,$$

$$I_p = \frac{1}{3}b_p \left[(t_p + \frac{1}{2}t_b)^3 - \frac{1}{8}t_b^3 \right], \quad I_b = \frac{1}{12}b_b t_b^3, \quad L_p = L_2 - L_1$$

$$N = \frac{1}{2} \int_s^L \rho^* \int_0^\xi (v_s^2)_{tt} d\eta d\xi + (z_{tt} - g) \int_s^L \rho^* \xi d\xi - J_0 \delta(\xi - d) \left\{ \frac{1}{2} v_{stt} v_s^2 + v_s v_{st}^2 \right\}$$

where subscript 'p' is used to denote terms related to PZT and $()_s, ()_t$ denotes the differentiation *w.r.t* curvilinear coordinate 's' i.e. $\frac{d()}{ds}$ and time $\frac{d()}{dt}$ respectively. The term N denotes the longitudinal load at 's'. The electric circuit equation in unimorph configuration can be obtain by applying the Gauss's law [176]

$$\frac{dQ(t)}{dt} = \frac{d}{dt} \int_{A_p} \mathbf{D} \cdot \mathbf{n} dA_p \quad (C.7)$$

Here $\mathbf{D}$ and $\mathbf{n}$ denotes the electric displacement vector and outward normal to the surface. The electric displacement vector ($\mathbf{D}$) in terms of axial stress (σ_p) and generated electric field (E_e) in the present case is defined by constitutive relation $D_3 = d_{31}\sigma_p + \hat{e}E_e$ where, $\sigma_p = E_p\epsilon_p - e_{31}E_e$ and $\epsilon_p = -y\kappa(s, t)$. Using these expressions in to Eq. (C.7) the circuit equation becomes

$$C_p \bar{V}_t + \frac{1}{R_l} \bar{V} - i(t) = 0 \quad (\text{C.8})$$

Here,

$$i(t) = -e_{31}t_{pc}b_p \int_{L_1}^{L_2} (v_{ss})_t ds, \quad C_p = \frac{\hat{e}b_p L_p}{t_p}$$

The four boundary conditions at fixed and free ends are represented as follows

$$v(0, t) = 0, \quad v_s(0, t) = 0, \quad v_{ss}(L, t) = 0, \quad v_{sss}(L, t) = 0 \quad (\text{C.9})$$

At the fixed end, displacement and slope are zero and at the free end, the bending moment and shear forces are zero. By considering single-mode approximation in the Galerkin's method, the transverse displacement $v(s, t)$ is represented by a scaling factor r , time modulation $q_i(t)$ and shape function $\psi(s)$ [182] of the beam system as $v(s, t) = r \psi(s) q(t)$. After nondimensionalizing the nonlinear coupled electromechanical temporal governing equations of motion for single mode, are given as follows.

$$\ddot{q} + 2\epsilon\zeta\dot{q} + q + \epsilon\alpha q^3 + \epsilon\beta q\dot{q}^2 + \epsilon\gamma q^2\ddot{q} + \epsilon\eta V = \epsilon q \hat{f} \cos(\phi\tau) \quad (\text{C.10})$$

$$\dot{V} + \chi V + \dot{q} = 0 \quad (\text{C.11})$$

Appendix C2

Coefficients are as follows

$$\begin{aligned}
 h_{11} &= \int_0^1 \frac{\rho(x)}{\rho_b A_b} \psi^2 dx, \\
 h_{12} &= \int_0^1 \delta(x - \beta) \psi^2 dx, \\
 h_{13} &= \int_0^1 \delta(x - \beta) \psi_x^2 dx, \\
 h_{21} &= h_{11}, \quad h_{22} = \int_0^1 \psi^3 dx \\
 h_{31} &= \int_0^1 EI^*(x) \psi^2 dx, \\
 h_{32} &= \int_0^1 \frac{\rho_b A_b EI^*(x)}{\rho(x)} \psi^2 dx, \\
 h_{33} &= \int_0^1 \frac{\rho(x)}{\rho_b A_b} (1 - x) \psi_x^2 dx, \\
 h_{34} &= \int_0^1 \int_x^1 \delta(\xi - \beta) d\xi \psi^2 dx \\
 h_{41} &= \frac{1}{2} \int_0^1 EI^*(x) \psi^2 \psi_x^2 dx, \\
 h_{42} &= \frac{1}{2} \int_0^1 \delta(x - \beta) EI^*(x) \psi^2 \psi_x^2 dx, \\
 h_{43} &= 3 \int_0^1 EI^*(x) \psi \psi_x \psi_{xx} \psi_{xxx} dx + \int_0^1 EI^*(x) \psi \psi_{xx}^3 dx, \\
 h_{51} &= \int_0^1 \left\{ \int_x^1 \left(\int_0^\xi \psi^2 d\eta \right) d\xi \right\} \frac{\rho(x)}{\rho_b A_b} \psi_x^2 dx, \\
 h_{52} &= \left(\int_0^\beta \psi_x^2 dx \right); \quad h_{53} = \{ \psi_x^4 \}_{x=\beta}, \quad h_{61} = h_{51}, \\
 h_{62} &= \int_0^1 \frac{1}{2} \frac{\rho(x)}{\rho_b A_b} \psi \psi_x^3 dx - \int_0^1 \frac{\rho(x)}{\rho_b A_b} \psi_x \psi_{xx} \left(\int_x^1 \psi d\xi \right) dx \\
 h_{63} &= h_{52}, \quad h_{65} = 0.5 h_{53}
 \end{aligned}$$

$$\begin{aligned}
h_{64} &= \frac{1}{2} \{ \psi \psi_x^3 \}_{x=\beta} - \psi(\beta) \int_0^\beta \psi \psi_x \psi_{xx} dx, \\
\zeta^* &= \frac{ch_{21}}{2\hbar\rho_b A_b \omega}; \quad \zeta_a = \frac{4}{3\pi} \frac{\lambda L \mu_a h_{22}}{\rho_b A_b \hbar}, \quad \hbar = h_{11} + \mu h_{12} + J\lambda^2 h_{13}, \\
\omega^2 &= \frac{\kappa^4}{\rho_b A_b \hbar L^4} (h_{31} + \mu h_{32}) - \frac{g}{L\hbar} (h_{33} + \mu h_{34}), \\
\hat{f} &= \frac{\Omega^2 z_0}{\epsilon \omega^2 \hbar L \lambda^2} (h_{33} + \mu h_{34}), \\
\alpha &= \frac{\epsilon \rho_b A_b \hbar L^4 \omega^2}{\lambda^2} \{ \kappa^4 (h_{41} + \mu h_{42}) + h_{43} \}, \\
\beta &= \frac{\lambda^2}{\epsilon \hbar} \{ h_{51} + \mu h_{52} + J\lambda^2 h_{53} \}, \\
\gamma &= \frac{\lambda^2}{\epsilon \hbar} \{ h_{61} - h_{62} + \mu (h_{63} - h_{64}) + J\lambda^2 h_{65} \}
\end{aligned}$$

The characteristic equation to find the Eigenvalue (κ_n) of n^{th} mode vibration is

$$\begin{aligned}
&\frac{4(h_1 h_4 - h_2 h_3)}{\lambda^2 \mu J} + 2\kappa^4 \{1 - \cos \kappa \beta \cosh \kappa \beta\} \\
&+ \frac{2\kappa^3}{\mu} \left[\begin{array}{c} h_1 (\sin \kappa \beta + \sinh \kappa \beta) \\ + h_2 (\cos \kappa \beta - \cosh \kappa \beta) \end{array} \right] + \frac{2\kappa}{J\lambda^2} \left[\begin{array}{c} h_4 (\sin \kappa \beta - \sinh \kappa \beta) \\ - h_2 (\cos \kappa \beta - \cosh \kappa \beta) \end{array} \right] = 0
\end{aligned}$$

Here h_i ($i = 1, 2, \dots$) and Λ are defined in [182]. The nondimensional mode shape $\psi(x)$ is considered as

$$\begin{aligned}
\psi_n(x) &= \left\{ (\sin \kappa_n x - \sinh \kappa_n x) - \Lambda (\cos \kappa_n x - \cosh \kappa_n x) \right\} + \\
&\quad \left\{ h_{12} (\sin \kappa_n \bar{x} - \sinh \kappa_n \bar{x}) + h_{34} (\cos \kappa_n \bar{x} - \cosh \kappa_n \bar{x}) \right\} U(\bar{x})
\end{aligned}$$

where, $\bar{x} = x - \beta$, $h_{12} = h_1 - \Lambda h_2$, $h_{34} = h_3 - \Lambda h_4$ and U is the unit step function. Other parameters are defined as follows

$$\begin{aligned}
z_r &= 1 \text{ mm}; \quad c = 1 \text{ Ns} - \text{mm}^{-2}; \quad \mu = 0.92; \quad J = 0.0107 \\
\beta &= 0.71; \quad \lambda = 0.1; \quad \kappa = 1.469; \quad \epsilon = 0.01
\end{aligned}$$



Appendix D

Displacement-curvature relation

By considering an arbitrary point along the longitudinal direction of the beam, at a curvilinear distance 's' from the base, the transverse and axial displacements are expressed as $v(s, t)$ and $u(s, t)$, respectively. Due to large deflection, the displacement-curvature relation of the beam is considered to be nonlinear. By observing the deflected beam geometry the relation between the angle of rotation $\varphi(s)$ and transverse deflection $v(s, t)$ is defined as $\sin \varphi = v'$ and $\cos \varphi = (1 - v'^2)^{1/2}$. As per the Euler-Bernoulli beam theory the curvature can be expressed as follows.

$$\kappa(s, t) = \frac{\partial \varphi(s)}{\partial s} = \frac{v_{ss}}{\cos \varphi} = \frac{v_{ss}}{\sqrt{1 - v_s'^2}} \approx v_{ss} \left(1 + \frac{1}{2} v_s'^2 \right) \quad (\text{D.1})$$

Here $\kappa(s, t)$, $\varphi(s)$ denotes the curvature and the angle of deflection of the cantilever beam. Here $()_s$ denotes the differentiation with respect to 's'. The relation between lateral and longitudinal displacement is assumed to be governed by the inextensibility condition which can be expressed by the following expression.

$$u_s = 1 - \sqrt{1 - v_s'^2} \approx \frac{1}{2} v_s'^2 \quad (\text{D.2})$$

The corresponding longitudinal displacement and its time derivative can be expressed as follows.

$$u(s, t) = \frac{1}{2} \int_0^s \{v_\xi(\xi, t)\}^2 d\xi \quad (\text{D.3})$$

$$\dot{u}(s, t) = \frac{1}{2} \frac{d}{dt} \int_0^s \{v_\xi(\xi, t)\}^2 d\xi = \int_0^s v_\xi(\xi, t) v_{\xi t}(\xi, t) d\xi \quad (\text{D.4})$$

The boundary conditions at fixed and free ends of the cantilever beam system are represented as follows.

$$v(0, t) = 0, \quad v_s(0, t) = 0, \quad v_{ss}(L, t) = 0, \quad v_{sss}(L, t) = 0 \quad (D.5)$$

Moment due to piezoelectric patch

In the present work the PZT-5H piezoelectric patch is added to the vertical beam in unimorph configuration. The moment M_p , which is generated due to the voltage $\bar{V}$ across the PZT layer electrodes can be written as [64, 319]

$$M_p(s, t) = \gamma_c \bar{V}(t); \quad \gamma_c = E_p d_{31} b_p (t_b + t_p) \quad (D.6)$$

where the constant γ_c depends on the dimensions and material properties of the components of the system, configuration and piezoelectric material. Also, d_{31} , E_p , b_p , t_b and t_p denotes piezoelectric constant, Young's modulus of piezoelectric material, width, beam thickness and piezo-patch thickness respectively.

Modelling of wind load on the structure

Figure D.1 shows the lift and drag forces on the D-shape bluff body in a top view. The

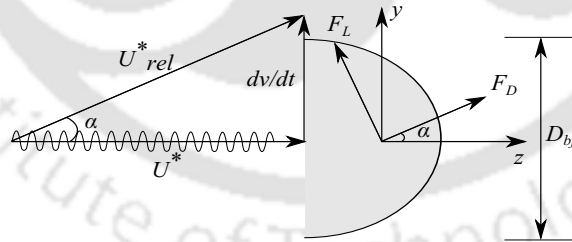


FIGURE D.1: Lift and drag forces on bluff body with D section

The aerodynamic lift and drag forces, denoted respectively by F_L and F_D , acting across and along the relative wind speed direction are as follows.

$$F_L = \frac{1}{2} \rho_a U_{rel}^2 D_{bf} C_L = \frac{1}{2} \rho_a D_{bf} C_L [U^2 + 2U \bar{u}(t)] \sec^2 \alpha \quad (D.7)$$

$$F_D = \frac{1}{2} \rho_a U_{rel}^2 D_{bf} C_D = \frac{1}{2} \rho_a D_{bf} C_D [U^2 + 2U \bar{u}(t)] \sec^2 \alpha \quad (D.8)$$

The total fluid force per unit length in the transverse direction to the incident flow is

$$F_y = F_L \cos \alpha + F_D \sin \alpha = \frac{1}{2} \rho_a D_{bf} [U^2 + 2U \bar{u}(t)] (C_L \cos \alpha + C_D \sin \alpha) \sec^2 \alpha \quad (\text{D.9})$$

$$F_y = F_a C_a \quad (\text{D.10})$$

Here,

$$F_a = \frac{1}{2} \rho_a D_{bf} [U^2 + 2U \bar{u}(t)] ; C_a = (C_L \cos \alpha + C_D \sin \alpha) \sec^2 \alpha$$

The excitation lift force $F_g(t)$ and moment $M_g(t)$ at near the free end of the harvester is due to the galloping effect on the attached bluff body which is modelled as force and moment imparts to the beam structure. The total force and moment component acting in the transverse direction of the system is expresses as follows. One may find the similar expressions in the work of Abdelkefi et al. [1], where only steady state wind component is taken into consideration.

$$F_g(t) = - \int_0^{L_{bf}} [F_L \cos \alpha + F_D \sin \alpha] ds \quad (\text{D.11})$$

$$M_g(t) = - \int_0^{L_{bf}} s [F_L \cos \alpha + F_D \sin \alpha] ds \quad (\text{D.12})$$

The total aerodynamic force coefficient (for higher Reynolds number) can be expressed by a polynomial function [1, 84] as

$$C_a = a_1 \tan \alpha + a_3 (\tan \alpha)^3 \quad (\text{D.13})$$

where C_a is the total aerodynamic force coefficient in the direction normal to the incoming flow. Also, the coefficients a_1 and a_2 depends on the geometry of the attached bluff body. Aerodynamic lift and drag forces are across and along the

relative wind speed direction is (as shown in the Fig. D.1). The angle of attack is expressed as

$$\alpha = \tan^{-1} \left(\frac{\dot{v}(d, t) + s\dot{v}'(d, t)}{U} \right) \quad (\text{D.14})$$

Respective force and moment at near the free end (a distance d from the fixed end, where $d < L$) are expressed as follows.

$$F_g(t) = -F_a \int_0^{L_{bf}} \left[a_1 \left(\frac{\dot{v}(d, t) + s\dot{v}'(d, t)}{U} \right) + a_3 \left(\frac{\dot{v}(d, t) + s\dot{v}'(d, t)}{U} \right)^3 \right] ds \quad (\text{D.15})$$

$$M_g(t) = -F_a \int_0^{L_{bf}} s \left[a_1 \left(\frac{\dot{v}(d, t) + s\dot{v}'(d, t)}{U} \right) + a_3 \left(\frac{\dot{v}(d, t) + s\dot{v}'(d, t)}{U} \right)^3 \right] ds \quad (\text{D.16})$$

By considering only single mode approximation ($i = 1$) in the Galerkin's method, the transverse displacement of the GPEH system is represented by a scaling factor r , time modulation $q_i(t)$ and shape function $\psi_i(s)$ of the beam system as follows.

$$v(s, t) = \sum_{i=1}^n r \psi_i(s) q_i(t) \quad (\text{D.17})$$

Now the galloping force becomes

$$F(t) = \frac{1}{L} \int_0^1 F_g \delta(x - \beta) \psi(x) L dx - \frac{1}{L^2} \int_0^1 M_g \delta_x(x - \beta) \psi(x) L dx \quad (\text{D.18})$$

$$F(t) = F_g \psi(\beta) + \frac{1}{L} M_g \psi_x(\beta) \quad (\text{D.19})$$

$$F(t) = \frac{1}{2} \rho_a [U^2 + 2U \bar{u}(t)] D_{bf} \left[\hat{k}_1 \left(\frac{\dot{q}}{U} \right) + \hat{k}_2 \left(\frac{\dot{q}}{U} \right)^3 \right] \quad (\text{D.20})$$

On further simplification the galloping force $F(t)$ expressed by the following expression.

$$F(t) = \frac{1}{2}\rho_a D_{bf} \left(U \hat{k}_1 \dot{q} + \frac{\hat{k}_2}{U} \dot{q}^3 + 2u_0 \cos(\Omega t) \hat{k}_1 \dot{q} + 2u_0 \cos(\Omega t) \frac{\hat{k}_2}{U^2} \dot{q}^3 \right) \quad (D.21)$$

Here,

$$\begin{aligned} \hat{k}_1 &= a_1 r \left[\psi^2(\beta) L_{bf} + \frac{1}{L} \psi(\beta) \psi_x(\beta) L_{bf}^2 + \frac{0.33}{L^2} \psi_x^2(\beta) L_{bf}^3 \right] \\ \hat{k}_2 &= a_3 r^3 \left[\psi^4(\beta) L_{bf} + \frac{2}{L} \psi^3(\beta) \psi_x(\beta) L_{bf}^2 + \frac{2}{L^2} \psi^2(\beta) \psi_x^2(\beta) L_{bf}^3 \right. \\ &\quad \left. + \frac{1}{L^3} \psi(\beta) \psi_x^3(\beta) L_{bf}^4 + \frac{0.2}{L^4} \psi_x^4(\beta) L_{bf}^5 \right] \end{aligned}$$



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