INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

Given a subset A of the set of integers, let R(A) denote the ratio set or quotient set of A. It is an open problem to study the denseness of R(A) in the field of p-adic numbers Q_p. We study the problem for the cases when A is the set of nonzero values attained by an integral form, polynomial and when A is the set of integers formed by a linear recurrence relation. We study the denseness of ratio sets of images of cubic forms and extend the results to diagonal forms of any degree greater than or equal to 3. More general forms are also considered. We also give some necessary conditions for the denseness of values attained by polynomials.

We find a sufficient condition for the denseness of the quotient set of certain kth-order linear recurrences. We show that if the characteristic polynomial of the recurrence sequence has a root which is plus or minus a Pisot number and if p is a prime such that the characteristic polynomial of the recurrence sequence is irreducible in the field of p-adic numbers, then the quotient set of the recurrence sequence is dense in Q_p. Further, we answer the problem for certain linear recurrence sequences whose characteristic polynomials are reducible over Q.

In a recent paper, Bilu et al. studied a conjecture of Marques and Lengyel on the p-adic valuation of the Tribonacci sequence. We study the p-adic valuation of third order linear recurrence sequences by considering a generalisation of the conjecture of Marques and Lengyel for third order linear recurrence sequences. We also demonstrate how p-adic valuations of a given linear recurrence sequence can be used to solve certain Diophantine equations involving the terms of the recurrence sequence. Finally, we consider a generalisation of ratio sets in the set of positive real numbers, known as Direction sets.