Two- and Three-Dimensional Analysis of Flow into Ditch Drains from a Ponded Field

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CERTIFICATE

This is to certify that the thesis entitled "**Two- and Three-Dimensional Analysis of Flow into Ditch Drains from a Ponded Field**" submitted by Mr. Ratan Sarmah, Roll No. 11610414, to the Indian Institute of Technology Guwahati, for the award of the degree of Doctor of Philosophy in Civil Engineering is a record of bonafide research work carried out by him under my supervision and guidance. Mr. Sarmah has worked on these problems for a period of about three years and the thesis is, in my opinion, worthy of consideration for the degree of Doctoral of Philosophy in accordance with the regulations of this Institute.

The results contained in this thesis have not been submitted in part or full to any other University or Institute for award of any degree or diploma.

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STATEMENT

I do hereby declare that the matter embodied in the thesis is a result of research work carried out by me in the Department of Civil Engineering, Indian Institute of Technology Guwahati, Guwahati, Assam, India.

In keeping with the general practice and reporting scientific observations, due acknowledgements have been made wherever the work described is based on the findings of other investigators.

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(Ratan Sarmah)

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LIST OF ABBREVIATIONS

Acad.	= Academy
Agri./Agric.	= Agricultural
Am.	= American
ASAE	= American Society of Agricultural Engineer
ASCE	= American Society of Civil Engineers
Bull.	= Bulletin
Can.	= Canadian
cm	= Centimeter
Co.	= Company
Cong.	= Congress
DEM	=Digital Elevation Model
Dept.	= Department
Dev.	= Development
Div.	= Division
Drain.	= Drainage
Ed.	= Edition
Eng./Engg.	= Engineering
FAO	= Food and Agriculture Organization
Fig.	= Figure
Geophys.	= Geophysical
Hydrol.	= Hydrology
ICID	= International Commission on Irrigation and Drainage
IDNP	= Indo-Dutch Network Project
IIT	= Indian Institute of Technology
Irrig.	= Irrigation
Inst.	= Institute
Int.	= International
Jour.	= Journal
kg	= Kilogram

Ltd.	= Limited
m	= Meter
Mang./Manage.	= Management
min	= Minutes
No.	= Number
pp.	= Pages
Proc.	= Proceedings
Pvt.	= Private
Res.	= Research
Resour.	= Resources
Roy.	= Royal
s	= Second
Sci.	= Science
Soc.	= Society
Tech.	= Technology
Trans.	= Transactions
US	=United States

ABSTRACT

An analytical solution is worked out for predicting steady seepage into a network of ditch drains in a multi-layered soil being underlain by an impervious barrier at a finite distance from the surface of the soil. The separation of variable method in association with appropriate applications of Fourier expansions have been utilized to obtain solution to the problem. The solution considers partial penetration, finite width and spacing of drains, anisotropy of the constituent layers and a uniform as well as a variable ponding field at the surface of the soil. The accuracy of the developed model is checked by first letting the multi-layered solution to reduce to that of a single layered soil by treating the conductivities of all the layers as same and then comparing the discharge values specific to a few drainage situations with the identical values obtained from the analytical and experimental works of others. A numerical check on the proposed solution is also carried out using the MODFLOW codes. The study shows that flow to a multi-layered ponded ditch drainage system is sensitive to the magnitude of the hydraulic conductivity as well to the anisotropy of each of the constituent layers of the soil and that considerable uniformity in leaching of a salt affected soil for such a system can be achieved, both in terms of quantum of water flow and travel time distributions of the streamlines, by providing an appropriate ponding distribution at the surface of the soil. Further, the study also corroborates with the work of others that the presence of a plow sole layer in a paddy field greatly restricts the movement of seepage water through such a soil and that for such a situation considerable time may be required by a water particle to move from the surface of the field to a recipient subsurface drain in the soil. The proposed solution, apart from being made use of in designing drains for cleaning a salt affected multi-layered soil, can also be utilized for designing ditch drains for reclaiming a waterlogged soil as well.

A transient analytical model is also worked out for predicting seepage from a ponded field of large extent to a network of equally spaced ditch drains in a homogeneous and anisotropic soil underlain by an impervious barrier at a finite distance from the surface of the soil. The solution can account for finite width and finite level of water in the ditches, finite penetration of the drains in the soil and also a constant or a variable ponding field at the surface of the soil. The validity of the developed model is tested by first reducing it to a steady state solution and then comparing discharge predictions obtained from it for a few flow situations with corresponding predic[illegible] obtained from an existing steady state analytical solution of the problem. A numerical comparison of the developed model for a flow situation is also carried out using the MODFLOW finite difference codes. The study highlights the fact that the transient state duration of a partially penetrating ponded drainage scenario may be considerable should the drains be dug in a lowly conductive soil with a high storage coefficient, particularly if the underlying impervious layer lies at a large distance from the bottom of the ditches and the separation between the adjacent ditches is also quite large at the same time. From the study, it has also come out that the exit gradients at the face of a ditch are highly responsive to the level of water in the ditch and also to the time of simulation of a ditch drainage system; the exit gradients at the face of a ditch with a low water level and at a small simulation time are found to be much higher than that in a ditch with a high water level and at a large time of simulation of the system. Thus, the breach of a river/stream/ditch bank due to sudden lowering of water level in it and the ensuing groundwater flow to it from the surrounding soil, is more likely to happen at the early stage of movement of groundwater to the river/stream/ditch than that at a later time, particularly if water level in the concerned river/stream/ditch is being lowered from its base position by an appreciable extent. The proposed solution, owing to its ability to take into account transient flow dynamics of a ditch drainage system, is expected to provide better designs of subsurface drains for reclaiming salty and waterlogged soils than designs based on drainage solutions developed with the steady state assumption.

Finally, analytical solutions are also developed for predicting time dependent three-dimensional seepage into a network of ditch drains from a ponded field of finite size, the field being assumed to be underlain by an impervious layer at a finite distance from the surface of the soil. Solutions are obtained for two variants of the three-dimensional ponded ditch drainage problem, namely, when the field is being surrounded on all its vertical faces by ditch drains and when the field is being surrounded on three of its sides by ditch drains and the fourth side is a no-flow boundary. While obtaining solutions to these problems, it is assumed that the field is homogeneous and anisotropic and the drains are dug all the way up to the underlying impervious barrier. Both of these solutions can accommodate different water level heights in the ditches and also a variable ponding distribution at the surface of the field. The separation of variable method is used to obtain general solutions for both the problems in terms of infinite series and then a careful combination of both double and triple Fourier runs are carried out to determine the requisite coefficients of these series solutions. The correctness of the proposed solutions for a few simplified situations is tested by comparing them with the relevant analytical and experimental works of others. Further, numerical checks on both the proposed solutions are also performed using the Processing MODFLOW environment. From this study, it is seen that flow in a ponded ditch drainage system is mostly governed by three dimensional flows, mainly in locations lying in the immediate vicinity of the drains. However, in a vertical plane located further away from two largely separated boundaries, flow can roughly be approximated by a two-dimensional model without introducing much error in the hydraulics on the concerned plane as measured with respect to the hydraulics on the said plane obtained through three-dimensional analysis of the problem. It is also observed that the nature of a side boundary – that is, whether a ditch or a no-flow boundary – also affects the flow lines in a noticeable way in a three-dimensional ponded ditch drainage system, again mostly in areas lying close to this boundary. From the study, it has also become quite apparent that the time taken by such systems to come to steady state may be considerable if ditches are installed in soils with low hydraulic conductivity and high specific storage. Further, for such a system, flow to the ditches is found to be sensitive to spacing, depth and water level heights in the ditches as well as to the magnitude and type of ponding distribution at the surface of the soil. The study also highlights the fact that considerable improvement on the uniformity of the flow lines in a three-dimensional ponded space specific to a flow situation can be achieved by suitably altering the ponding distribution at the surface of the soil, specific to the concerned flow situation. As the proposed models are general in nature in that they can account for three-dimensional flows, different water level heights in the ditch drains and a constant or a variable ponding distribution at the surface of the soil, it is expected that the drainage designs based on these solutions for reclaiming salt affected and waterlogged soils would prove to be more efficient and cost effective as compared to designs based on solutions developed with more restrictive assumptions.

Keywords: Analytical models; Ponded ditch drainage; Two- and three-dimensional flows; Steady and transient seepage; Multi-layered soil; Uniform and non-uniform ponding distributions; Hydraulic conductivity; Specific storage; Anisotropy.

CHAPTER 1

INTRODUCTION, LITERATURE REVIEW AND OBJECTIVES

1.1 Introduction

Global food production needs to be enhanced by about 38 percent by 2025 and more than 50 percent by the half of this century if the food and fiber requirements of the growing population are to be met satisfactorily at the current level of consumption (Wild 2003; Rengasamy 2006). By 2050, the global population is expected to reach around 9 billion (FAO 2009) and it will be quite a challenge to feed such a large population. Agriculture already accounts for about 70 percent of freshwater withdrawal (United Nations, Water for Food, 2013) from water bodies and bringing in more fresh water to the agriculture sector will be a difficult task as water demands in the municipal, industrial and environmental sectors are also bound to increase in future competing for the same pool of fresh water reserve. Further, bringing in fresh areas under agriculture may not always be easy in many places due to environmental concerns. Thus, emphasis must be placed to increase mostly the productivity per unit area of cultivable lands rather than bringing in more areas under cultivation (Rengasamy 2006) by adopting better crop varieties and water management strategies in the cultivated fields. From times long bygone, irrigation has been a key element for augmenting agricultural productivity and its role in future agricultural production is only expected to increase if the future global demands for food and fiber are to be successfully negotiated (Khan et al. 2004). The role of irrigation is still more prevalent in arid and semi-arid regions where its introduction has greatly enhanced food productivity during the latter part of the twentieth century (Smedema et al. 2000; Wichelns et al. 2002). Irrigation accounts for only about 20 percent of world's cropland but produces about 40 percent's of world's food and fiber (FAO 2003; Khan et al. 2004). Irrigation is also pivotal for producing high yielding crops as modern biotechnology inputs to such crops often require favorable soil-water ambience for optimum growth of plants (Singh and Singh 1995; Carruthers et al. 1997). The irrigation coverage of the developing world is estimated to go up to the tune of 242 million hectares by 2030 (Faures 2002). About 34 percent of the arable land of India, amounting to 57 million hectares, is currently under irrigation (ICID 2003; Ritzema et al. 2008) and this coverage is also likely to increase considerably in future for enhancing agricultural productivity of the country to feed her teeming millions.

The introduction of irrigation water to the agricultural fields, however, has resulted in the twin problems of waterlogging and salinity in many areas of the world – problems which must be tackled if sustainability of irrigated agriculture is to be maintained (Ghassemi et al. 1995, Martinez-Beltran 2002, Rhoades 1997; Wichelns et al. 2002). It is reported that about 831 million hectares of land spanning over all the continents of the world has been affected with the problems of soil salinity and alkalinity (FAO: Martinez-Beltran and Manzur 2005; Boari et al. 2012). It is estimated that salinity is costing the world's farmers about 11 billions of US dollars every year in terms of reduced income (Scheumann and Freisem 2002). In India also, vast tracts of agricultural lands have been reported to be afflicted with irrigation induced waterlogging and salinity in many parts of the country (Wolde-Kirkos and Chawla 1994; Manjunatha et al. 2004; Ritzema et al. 2008). It is expected that about 8.4 million hectares of irrigated lands of the country have been affected by salinity and alkalinity, out of which about 5.5 million hectares have also been reported to be waterlogged (IDNP 2002; Ritzema et al. 2008). Several studies (Datta et al. 2000; Smedema et al. 2000; Datta and Jong 2002; Scheumann and Freisem 2002; Ayars et al. 2003; Manjunatha et al. 2004; Sharma and Gupta 2006, Ritzema et al. 2008 – to name a few) have categorically shown that subsurface drainage can be successfully employed to bring down waterlogging and salinity of irrigated soils.

One of the most commonly used methods of controlling salinity in a soil column is to subject the soil to a ponding head of good quality irrigation water at the surface of the soil so that the water is forced though the salt affected soil and in the process washes away a part of the salt present in the soil profile, the salt enriched water then being drained with the help of a network of subsurface drains installed for the purpose (Dielman 1973, Martinez-Beltran 1978, Rao and Leeds-Harrison 1991, Youngs and Leeds-Harrison 2000; Mirjat and Rose 2009, Barua and Alam 2013). The process of cleaning a sodaic soil is similar to that of a saline soil except that it becomes first necessary to knock out the exchangeable sodium out of the soil matrix by some cheap source of calcium or magnesium salt before leaching the replaced ions out of the soil (Kirkham and Powers, 1972). Subsurface drainage is also now proving to be increasingly important for draining paddy fields so as to maintain a favorable soil-water balance at the root zone of paddies (Ogino and Murashima 1993; Tabuchi 2004, Darzi-Naftchally et al. 2013) and, most importantly, for providing a check on the emission of methane from these fields, a greenhouse gas the global warming potential of which is measured only next to carbon dioxide by mass (Shiratori et al. 2007; Qui 2009; Xiaohong et al. 2011, Zhang et al. 2011– to name a few). Subsurface drainage can be installed by laying underground tiles or by digging open ditches in the fields. Ditch drains are particularly suited for reclaiming waterlogged and salt affected soils in areas where the conductivity of the soils is low and the topography relatively flat (Abrol et al. 1988). Drainage of lowly conductive peat lands is often done with open ditches for lowering water table so as to provide optimum soil-water-air conditions at the root zone of plants of such soils (Stewart and Lance 1983). Ditches are distinctive engineered ecosystems integrating both the characteristics of streams and wetlands and play important roles in controlling the hydrologic, chemical and biological processes of a watershed; open ditches also provide unique opportunities to address problems of non-point source pollution arising out of agricultural activities in a watershed (Needelman et al. 2007). Ditch drains are also reported to have a strong influence in maintaining the biodiversity of an agriculture landscape and the replacement of open ditch drains by subsurface tiles in a watershed may have a profound influence on the ecology of the watershed (Youngs 1994; Bradbury and Kirby 2006, Marja and Herzon 2012, Marja 2013).

Thus, considering the importance of ditch drains for multifarious activities in an agricultural landscape, it is incumbent that due emphasis be given to study in detail the hydraulics associated with such a system so that efficient drainage networks specific to a purpose can be designed and implemented in fields. The hydraulics of a hydro-geological system is generally studied by mathematically modeling the same in either an analytical or a numerical platform. A mathematical model is a simplified depiction of a real flow situation where a set of differential equations (governing equations) based on some fundamental laws of nature is solved to represent the real situation (Wang and Anderson, 1982). The model must not only solve the governing equations specific to a flow situation but should also satisfy at the same time the boundary conditions of the flow domain considered for study; further, in case of transient simulations, the model should also incorporate the hydraulic status of the slow system at some reference time (initial condition).

Generally, numerical models are more realistic and adaptable than analytical models as they can tackle nonlinearities of the differential equations, heterogeneities of aquifer materials and variations of fluid properties as well as complex flow geometries with much more ease than the analytical models (Walton 1979, 1989). However, the authenticity of numerical models often depends to a great extent on the availability of a large data set which may be difficult and/or expensive to obtain. Moreover, as numerical solutions only provide approximate solutions of boundary value problems and not their exact solutions, their convergence and stability must be checked and ensured before these solutions can be actually applied for their intended purposes. Analytical models, on the other hand, provide exact solutions to problems and do not suffer from any convergence issues; also they require much less data for their use as compared to numerical models. Also, with the development and arrival of newer and powerful analytical tools like the Homotopy Analysis Method (Liao 1992) and the Adomian Decomposition Method (Adomian 1994) during the last couple of decades, analytical models are now increasingly being developed for solving complex nonlinear equations as well.

In this study, an effort is made to work out a few analytical solutions for studying steady and transient hydraulics of flow associated with a ditch drainage system receiving water from a relatively flat field being subjected to a variably ponding field at the surface of the soil. As an analytical model helps in understanding the conceptual behavior associated with a groundwater system (Haitjema 2001, 2006; Hunt et al. 2003), it is hoped that the solutions presented here would lead to having a better insight on the hydraulics of flow associated with a ditch drainage system; also, because of the general nature of these solutions, they are expected to lead to for relatively better designs of ditch drains for reclaiming waterlogged and saline soils in comparison to designs based on solutions developed with more restrictive assumptions. Further, as an analytical model is also frequently called upon to test the accuracy of complex numerical codes pertaining to a groundwater system (Elfeki et al. 1997; Kacimov 1997; Haitjema 2006; Praveena et al. 2010), it is also hoped that the models proposed here would also prove to be useful in checking the validity of complex numerical solutions related to subsurface drainage which can be reduced to the comparatively simple drainage settings for which analytical investigations have been performed in the current study.

1.2 Literature Review

As has just been mentioned, subsurface drainage through ditch drains is proving to be quite useful in reclaiming waterlogged and saline soils in areas where the conductivity of the soils is low and the topography relatively flat (Abrol et al. 1988). Leaching of a salt affected soil by forcing good quality water through it and then removing the washed salts with the help of a suitable ditch drainage system, has been a standard practice of cleaning a salt affected soil for quite some time now (Dielman 1973; Martinez Beltran 1978; Rao and Leeds-Harrison 2000). Towards this end, several ponded subsurface drainage theories have been proposed by many in the past. For an array of parallel ditch drains resting on a gravel stratum and receiving water from a relatively flat field of large extent subjected to zero or a uniform ponding depth at the surface of the soil, Kirkham (1945) developed an infinite series steady state solution to the problem. Kirkham (1950) provided a steady state solution to the ponded drainage problem for the case when the ponding depth is negligible and the drains are dug all the way up to an impervious barrier by taking suitable limits to the cylindrical coordinate solution of the fully penetrating auger hole problem for a homogeneous and isotropic soil (Kirkham and Van Bavel 1948). Fukuka (1957) presented a conformal mapping solution to the steady state partially penetrating ditch drainage problem by considering the ponded depth over the surface of the soil to be always zero and assuming the ditches to run empty all the time. Further, in his analysis, he considers the ditches to be of infinitesimal width and the spacing between the drains and depth of the impervious barrier from the surface of the soil, to be of large distance. Kirkham (1960) provided a Fourier series solution to the problem of steady seepage into an array of equally spaced ditch drains in a homogeneous and isotropic soil underlain by a gravel stratum, the drains being dug all the way up to the gravel stratum and the soil being subjected to a uniform ponding distribution at its surface. A series solution to the steady state ditch drainage problem with unequal water level height in between the adjacent ditches was provided by Kirkham (1965) using the Fourier series approach for a homogeneous and isotropic soil. This solution has the versatility of accounting for both zero as well as non-zero depth of ponding at the surface of the soil. For equally spaced ditch drains with water running on it up to the surface of the soil or higher, Warrick and Kirkham (1969) obtained conformal mapping solutions for predicting steady flow to the ditches from a uniformly ponded field for different cases of the problem, namely, when an impervious layer is present at a finite height below the ditches and the ditches are at a finite distance apart; when no impervious layer exists and the ditches are at a finite distance from each other; when an impervious layer exists but the ditches are separated from each other by a large distance; and when no impervious layer exists and the ditches are again at a large distance apart. By making use of these analytical models, Miyamoto and Warrick (1974) studied the piston type displacement of solutes into or from water-filled ditches in a homogeneous and isotropic saturated soil both for situations when there lies an impervious layer at a finite distance from the surface of the soil and when the impervious layer lies at a large distance from the bottom of the ditches. They observed that the displacement fronts advance much quicker to drains from locations close to drains as compared to their movements from locations further away from the drains. Youngs (1994) also applied the method of conformal mapping to provide analytical solutions for predicting seepage into a ditch drain partially penetrating a homogeneous and isotropic soil and receiving water from a large ponding field of zero depth of ponding over it both for situations when the ditch rests on an impervious barrier and when the impervious barrier lies at a large distance below the bottom of the ditch. In his analysis, he assumes the ditch to be of negligible width. Barua and Tiwari (1995) provided a detailed Fourier series solution to the partially penetrating ditch drainage problem for the steady state by adopting a domain discretization of the half flow space in between the ditches – the portion of the flow space below the bottom of the drain and up to the impervious barrier is named as sub-domain one and the remaining region as sub-domain two – and then solving the governing equations for both these sub-domains taking care to see that all the necessary boundary and interfacial conditions are being satisfied at the same time. This solution is fairly general in nature as it includes finite width and penetration of the drains, anisotropy of the soil and both zero as well as non-zero ponding depths at the surface of the soil. However, this solution is valid for a single-layered soil only and as such it cannot account for variations of horizontal and vertical conductivities with depth of a soil profile. Further, this solution is for a uniform ponding depth only at the surface of the soil and hence is not valid when the ponding field at the surface is a variable one. Bereslavskii (2006) utilized the Riemann-Schwartz principle of symmetry to obtain an analytical model for the fully penetrating ditch drainage problem and claims that this solution is computationally superior to that of the ones obtained using conformal mapping. Chahar and Vadodaria (2008a, 2008b, 2012) had a relook of the ditch drainage problem using conformal transformation and provided seepage equations for different variants of the problem. Their solutions, however, are also applicable only for a single-layered soil and can tackle only a constant depth (this depth can be zero also) of ponding at the surface of the soil. In order to assess the contributing role of hydraulic pressures near incised ditches in a homogeneous and isotropic aquifer in gully formation, (2009) revisited the partially penetrating ditch drainage problem using conformal mapping by considering the bottom of the ditches to be circular and neglecting the flows taking place through the seepage faces of the ditches.

Several studies (Kirkham 1950, 1960, 1965; Dielman 1973; Martinez Beltran 1978; Rao and Leeds-Harrison 1991; Youngs 1994; Barua and Tiwari 1995; Youngs and Leeds-Harrison 2000; Mirjat and Rose 2009; 2009; Chahar and Vadodaria 2008a, 2008b, 2012; Barua and Alam 2013 – to name only a few) related to ponded subsurface drainage using tiles or ditches clearly demonstrate the fact that the seepage velocity distribution at the surface of a uniformly ponded soil being drained by a network of tiles or ditches is pretty uneven with most of the flow occurring through areas lying close to the drains. Thus, leaching a contaminated soil with a uniform ponding subsurface drainage system will lead to unequal washing of the soil profile because regions close to the drains will be over-washed and the regions away from the drains under-washed. It is also worth noting here that for a uniform ponding subsurface drainage system, water particles seeping from locations far away from the drains require much longer travel times to move to the drains than those moving from areas close to the drains. An accurate estimation of the distribution of groundwater travel times of water particles in a subsurface drainage system being designed for cleaning a contaminated soil is essential as this distribution provides information about the possible time of leaching needed to bring down the level of contamination of the soil to a desired level.

The travel time of a water particle along a streamline related to a subsurface hydro-geological system can be determined by performing suitable integrals along the streamline utilizing the hydraulic theory pertaining to the studied system and the necessary hydro-geological parameters relevant to the studied location (Kirkham and Affleck 1977; Cushman and Kirkham 1978; Kirkham and Sotres 1978, 1979). The analytical evaluations of these integrals are often pretty difficult for many flow and transport situations and hence are mostly done numerically. But, many a times, numerical evaluations of these integrals are also quite tedious, particularly in locations where the streamlines take sharp detours as in such situations the integrating space needs to be divided into much finer parts for achieving acceptable accuracy of the estimated travel times. Travel times of a water particle in a subsurface flow space can also be estimated from the known velocity distributions pertinent to the flow space; the methodology for the same consists merely of summing up the time taken by the particle to move through a series of elemental distances in the flow space, the travel time to cover an elemental distance located in the neighborhood of a point being simply the ratio between the elemental distance and the pore velocity relevant to the point of consideration in the flow space (Grove et al. 1970). This method is simple to implement and does not require any complex integrals to be solved for determining the travel times. Groundwater travel times of a watershed can also be estimated from topographic information of the watershed utilizing a geographical information system (GIS) platform (Schilling and Wolter 2007). In this approach, the hydraulic gradient distribution driving groundwater flow in the watershed is derived from the digital elevation model (DEM) of the watershed, where it is inherently assumed that the water table of the watershed follows approximately its topography at the surface. However, as the hydraulic gradients at a subsurface flow space in a watershed do not mostly follow the surface topography of the watershed, Schilling and Wolter's (2007) model of estimating travel times in a watershed cannot always be used with confidence.

One way of trying to achieve a relatively even leaching of a soil is by subjecting the soil to sequential ponding at its surface of the soil rather than subjecting it to full ponding all the time. Thus, full ponding of the whole soil for some time followed by ponding of half of the soil profile for some more time and then finally ponding quarter of the soil profile in regions located halfway between the drains, may be adopted for having a relatively uniform leaching of a salt affected soil rather than carrying out the leaching of the whole soil all the time (Rao and Leeds-Harrison 1991). Another way of addressing the problem is by first ponding fraction of the area midway between the drains and then progressively increasing this area till the entire region in between the drains is covered (Youngs and Leeds-Harrison, 2000; Mirjat and Rose 2009). Still, an another way of achieving uniformity of leaching of a salt affected soil is by subjecting it to a progressively increasing ponding distribution from the centre of the drains towards the halfway distance between two adjacent drains – that way, the entire soil profile can be leached in a single stage (Barua and Alam 2013). A noticeable aspect of this type of reclamation is that the uniformity of leaching is attempted by just altering with the ponding distribution at the surface of the soil without the necessity of cleaning the soil in stages. The ponding distribution pertaining to a leaching scenario in context to reclamation of a salt affected soil can be worked out utilizing the mathematical works of Barua and Alam (2013) and, if this distribution is rightly determined for the chosen scenario, considerable saving in water as well as on time of leaching may be achieved during the leaching process. Barua and Alam's (2013) transient ditch drainage analytical model, however, is valid only for situations where the ditches fully penetrate a soil profile and rest on an impervious layer underlying the soil column. Using two-dimensional modeling studies, Zheng et al. (1988a, 1988b) showed that an interceptor drain or a stream can act as a strong hydraulic barrier to groundwater flow. Their model, however, is based on vertically averaged flow equations and as such this model may not be able to describe non-hydrostatic flow situations in the vicinity of interceptor drains with a great deal of accuracy as flow into such a system is mostly three-dimensional in nature (Brainard and Gelhar 1991; Murdoch 1994; Meigs and Bahr 1995). In places, where an aquifer cannot be treated as an infinite one in areal extent, the assumption of an infinite trench/stream while modeling such a system may lead to serious error (Murdoch 1995). Brainard and Gelhar (1991) clearly demonstrated from their finite element studies that the nature (i.e., no flow or fixed head boundaries) of the Northern and the Southern boundaries encompassing a straight stream/ditch stretch greatly influences the flow dynamics around the vicinity of the stream/ditch and that the horizontal distance travelled by a water particle parallel to the stream from its point of release up to its point of entry into the stream/ditch may be considerable if the anisotropy ratio (ratio between the horizontal and vertical hydraulic conductivity of an aquifer) of the aquifer is high. Thus, a two-dimensional analysis of a ditch drainage problem (Kirkham 1958, Nield et al. 1994) in a finitely spread aquifer with a high anisotropy ratio may yield a drainage picture which may be considerably different from that obtained by three-dimensional analysis of the same in such an aquifer.

As mentioned before, subsurface drainage is also turning out to be an important inhibitor of emission of methane from paddy fields, a greenhouse gas whose global warming potential is about 25 times than that of carbon dioxide (Qiu 2009; Yvon-Durocher et al. 2014). However, paddy fields are seldom single-layered in nature with the plow sole layer generally exhibiting a very high resistance to movement of water through it as compared to the non-puddled layer below it. Numerous numerical modeling studies (Liu et al. 2001; Chen et al. 2002; Haung et al. 2003; Liu et al. 2005 – to suggest a few) have shown that the low conductivity plow sole layer of a paddy field plays a pivotal role in controlling the movement of infiltration of water through such a soil and that the absence of this layer significantly increases the infiltration rate through a paddy field. Otherwise also, soils in nature are mostly stratified (Stephens and Heermann 1988; Huang et al. 2011) and anisotropic (Maasland 1957) and hence effort should be made, wherever possible, to include these factors while modeling flow and transport through soils.

From the above, it is clear that there is a need to study in detail the hydraulics of flow associated with a ponded ditch drainage system by considering, wherever possible, stratification of soils, partial penetration of drains as well as three-dimensional nature of the flow field in the drainage space, so that flow behavior around such a system pertaining to a drainage situation can be realistically ascertained. Currently, there does not appear to be any analytical solution to the two-dimensional partially penetrating multi-layered ponded ditch drainage problem even for the steady state situation. Also, even for a single-layered homogeneous and anisotropic soil, an analytical solution to the two-dimensional partially penetrating ponded drainage problem for the transient case appears to be missing. Further, a general analytical model for three-dimensional movement of water to a network of ditch drains fully penetrating a homogeneous and anisotropic soil receiving water from a ponded field is also found to be lacking. In view of the same and keeping in mind the importance attached to obtaining analytical solutions to hydro-geological problems, an effort is made in the current study to achieve the following set of objectives.

1.3 Objectives

(i) To develop an analytical solution for predicting steady two-dimensional seepage of water into a network of equally spaced ditch drains partially penetrating a three-layered soil of large horizontal extent, the soil being underlain by an impervious barrier and been subjected to a variably ponded distribution at its surface.

(ii) To develop an analytical solution for predicting transient two-dimensional seepage into an array of equally spaced ditch drains in a homogeneous and anisotropic soil of large horizontal extent, the soil being underlain by an impervious barrier and been subjected to a variably ponded distribution at its surface.

(iii) To obtain analytical expressions for predicting transient three-dimensional seepage of water into a network of ditch drains fully penetrating a homogeneous and anisotropic soil with an underlying impervious barrier and receiving water from a variably ponded distribution at its surface both for situations where all the four vertical sides of the soil column are ditch boundaries and where the three side boundaries of the soil column are ditch boundaries but the fourth side is a no-flow boundary.

CHAPTER 2

HYDRAULICS OF A PARTIALLY PENETRATING DITCH DRAINAGE SYSTEM IN A LAYERED SOIL RECEIVING WATER FROM A PONDED FIELD

In this chapter, an analytical solution is proposed for predicting seepage into an array of ditch drains dug in a three-layered soil underlain by an impervious barrier and receiving water from a ponded field of large extent. The solution is of a general nature and can account for partial penetration, finite width and spacing of drains, finite depth of the impervious stratum, anisotropy of individual layers and both a constant and a variable ponding field at the surface of the soil. The accuracy of the developed solution for the single-layered situation is checked by first reducing the proposed multi-layered solution to that of a single-layered one by treating the conductivity of the layers as the same and then comparing, for a few flow situations, the discharges as predicted by this reduced model with the corresponding values obtained from the analytical and experimental works of others. A MODFLOW check on the developed analytical model for a multi-layered soil is also carried out using the Processing MODFLOW (Chiang and Kinzelbach 2001) platform. The effects of hydraulic conductivity as well as anisotropy ratio (the ratio between the horizontal and vertical hydraulic conductivity of soil) of the constituent layers of a multi-layered soil on seepage to a network of partially penetrating drains under ponded conditions are also studied. Further, it is also shown how the uniformity of leaching associated with a multi-layered ponded ditch drainage system, both in terms of water flow and travel time distribution in the flow space, can be improved by adopting a suitable ponding distribution at the surface of the soil. The proposed model is also made use of to study the influence of the lowly conductive plow sole layer of a ponded paddy field on subsurface drainage in such an environment. With the help of a few examples, it is also shown how the developed analytical model can be utilized to determine the upper limit of fall of water level of a waterlogged soil when it is being drained by a network of equally spaced ditch drains having equal water level heights in them.

2.1 A Few Solutions of the Two-Dimensional Steady State Continuity Equation of Groundwater Flow for a Homogeneous and Anisotropic Soil

Before an analytical solution to the boundary value considered here is attempted, it is essential that to determine a few general solutions of the continuity equation relevant to the problem under consideration. The governing equation describing two-dimensional groundwater flow in a saturated, homogeneous and anisotropic aquifer for an incompressible fluid can be represented as (Bear 1972)

(2.1)

where is the hydraulic head, and are the horizontal and vertical hydraulic conductivities of the medium along *x*- and *y*-axes, respectively. We now proceed to obtain a few solution of Eq. (2.1) by making use of the separation of variable method (Kirkham and Powers 1972). Thus, let us assume

(2.2)

to be the solution of Eq. (2.1), where is a function of only and is a function of only. Applying Eq. (2.2) in Eq. (2.1) and separating the variables out, we get

(2.3)

As can be seen from the above equation, the left hand side of the equality is a function of only and the right hand side is a function of only; thus, both the sides of Eq. (2.3) can naturally be equated to a constant, say This yields

(2.4)

Eq. (2.4), as may be observed, generates two equations, namely

(2.5)

and

(2.6)

The solutions of Eqs. (2.5) and (2.6) are obviously

(2.7)

and

(2.8)

respectively, where , and are any arbitrary constants, is any positive integer and is defined as the anisotropy ratio of the medium, that is

(2.9)

It should be noted here that while obtaining expressions for and in Eqs. (2.7) and (2.8), respectively, we have made use of the property that sum of solutions of a differential equation is also a solution of the differential equation.

Inserting of Eq. (2.7) and of Eq. (2.8) in Eq. (2.2) and then adding a constant to the resultant equation [this is because a constant is also a solution of Eq. (2.1)], we thus get a solution of Eq. (2.1) as

(2.10)

where is a constant.

Now, if the equated constant in Eq. (2.4) is taken as instead of that is

(2.11)

we then have the ensuing differential equations as

(2.12)

and

(2.13)

and their solutions as

(2.14)

and

(2.15)

respectively, where and are all arbitrary constants. This leads to another solution of Eq. (2.1), namely

(2.16)

We will now make use of Eqs. (2.10) and (2.16) to solve the boundary value problem considered in this chapter.

2.2 Mathematical Formulation and Solution

The geometry of the flow system considered for study is as shown in Fig. 2.1 where an array of ditch drains can be seen to be draining a three-layered soil underlain by an impervious barrier. The thickness of the layered soil upto the impervious barrier is taken as h and denotes the extent of penetration of the drains as measured from the surface of the soil. Because of symmetry about a vertical plane passing through halfway between the drains, only one half of the flow domain in between two adjacent drains is considered for analysis as the other half is simply its mirror image. Also, for mathematical convenience, this space is further divided into four zones and the soil hydraulic parameters of zones I and II are taken as the same. The depth upto the beginning of the second and third layers are taken as and respectively and represents the level of water in the ditches, all these distances being again measured from the surface of the soil as can be seen in the figure. The width of the drains is taken as $2a$ and $2S$ represents the spacing between the ditches. The horizontal and vertical hydraulic conductivities of the layers are taken as and respectively (i=1 to 4), and the width of the ditch bunds at the surface is considered as A variable and steady ponding field symmetrical with respect to the groundwater divide line of the flow domain is being imposed to the system with the help of a set of inner bunds as may be observed in the figure. Let such a field be created with ponding depths with the help of inner bunds and let denote the ponding depth corresponding to the jth inner bund, where it should be noted that the imposed ponding distribution at the surface of the soil is also a symmetrical one with respect to the centroidal plane passing through halfway between two adjacent drains. The inner bunds are also assumed to be of negligible thickness and let the distance of the end of the j^{th} inner bund from the origin O be The hydraulic heads of the layers are denoted as Further, for ease of obtaining solution to the problem, the x-axis is taken positive towards the right and the y-axis positive vertically downward, as may be observed in the figure.

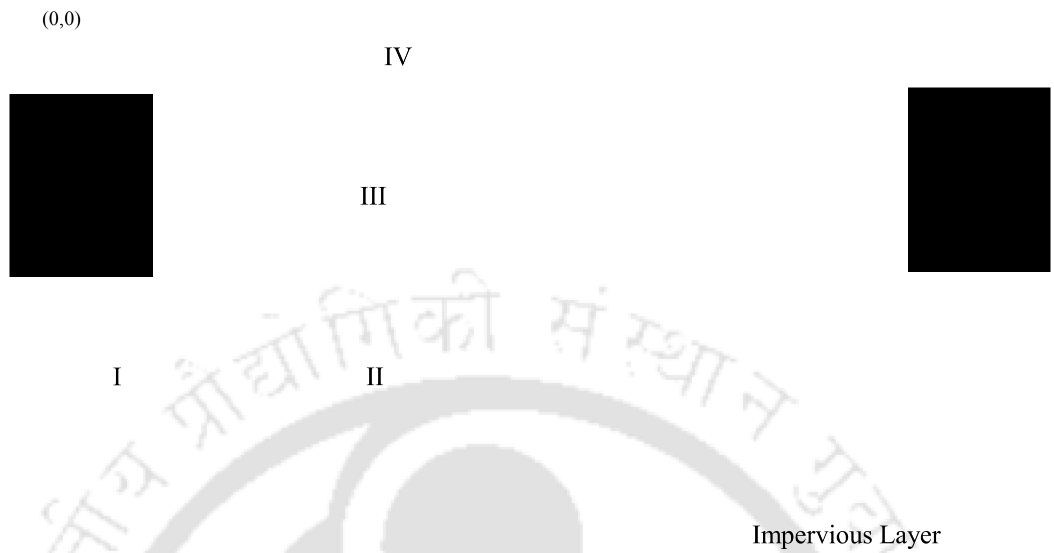


Fig. 2.1. Geometry of a partially penetrating ditch drainage system in a three-layered soil subjected to variable depths of ponding at the surface of the soil

With these definitions of the variables of the flow problem in place, we now seek to obtain an analytical solution to the boundary value problem of Fig. 2.1 by solving the following steady state continuity equations corresponding to the four zones of the flow domain

(2.17)

and incorporating at the same time the following boundary and interfacial conditions in these hydraulic head expressions.

For Zones I and II:

(I)

(II)

(III)

(IV)

(V)

If the water level in the ditch is at or below i.e., then

(VIa)

(VIb)

and if it is above i.e., then

(VIc)

(VII)

(VIII)

For Zones II and III:

(IX)

(X)

If the water level in the ditch is at or below i.e., then

(XIa)

and if it is in between and i.e., then

(XIb)

(XIc)

If the water level in the ditch is above i.e., then

(XId)

(XII)

For Zones III and IV:

(XIIIa)

(XIIIb)

(XIIIc)

If the water level in the ditch is at or below i.e., then

(XIVa)

and if it is above i.e.,

(XIVb)

(XIVc)

(XV)

(XVI)

(XVII)

where and are the hydraulic head functions in regions I, II, III and IV, respectively of the flow domain as shown in Fig. 2.1

In view of the general solutions of Eq. (2.1) as given by Eqs. (2.10) and (2.16), respectively, the hydraulic head expressions corresponding to the four sub-domains of the flow problem of Fig. 2.1 can be denoted as

(2.18)

where

(2.19)

(2.20)

(2.21)

where

(2.22)

(2.23)

and if and if

(2.24)

where

(2.25)

(2.26)

(2.27)

(2.28)

and if and if

(2.29)

where

(2.30)

(2.31)

(2.32)

and

(2.33)

It is to be noted that of Eq. (2.18) satisfies boundary conditions, (I), (II) and (III), of Eq. (2.21) satisfies boundary conditions (VII) and (VIII), of Eq. (2.24) satisfies boundary condition (XII) and of Eq. (2.29) satisfies (XV), by their very definition. We now propose to utilize the remaining boundary and interfacial conditions to evaluate the constants and appearing in the expressions of the hydraulic head functions to

Applying condition (XIVa) to Eq. (2.29), we have at for

Letting in the above expression, the constants can then be evaluated by carrying out a Fourier series expansion in the domain the relevant expression for the same can be written as

(2.34)

Eq. (2.34), upon simplification, gives

(2.35)

Again, applying conditions (XIVb) and (XIVc) to Eq. (2.29), respectively, we have now at

for

Allowing in the above expressions, can then be represented by carrying out a Fourier expansion on thus, we have

(2.36)

Simplifying the above integral, we have

(2.37)

To evaluate the constants , we next apply conditions (XIIIa), (XIIIb) and (XIIIc) to Eq. (2.29), respectively; the resultant expressions at work out to be

Now, considering in the above expressions, can then be determined by making use

of the Fourier series expansion in the range thus, we have

(2.38)

After carrying out the above integral, we get

(2.39)

To evaluate the constants for condition (XIa) can next be applied to Eq. (2.24) to

yield

Permitting in the above equation, the constants can then be evaluated by performing a Fourier series expansion in the interval thus, can be expressed as

(2.40)

Eq. (2.40), after simplification, gives

(2.41)

Now, to determine the constants for we apply conditions (XIb) and (XIc) to Eq. (2.24); we then have at

can then be evaluated by carrying out a Fourier expansion in thus, we have

(2.42)

After carrying out the above integrals, we get

(2.43)

Finally, for can be evaluated by making use of condition (XId) to Eq. (2.24); the resultant expression at then turns out to be

Carrying out a Fourier expansion now in the range can then be evaluated as

(2.44)

The above integral, as may be observed, works out to be zero, i.e.,

(2.45)

For the flow problem to be solved in totality, there still remains the constants and to be determined. To evaluate for , we apply conditions (VIc) and (III) to Eqs. (2.18) and (2.21), respectively; then we have at

Performing now a Fourier series expansion in the range by allowing in the above expressions, we have an expression for the constants as

(2.46)

After carrying out the above integration, we have for

(2.47)

and for

(2.48)

For can be determined by applying conditions (VIa), (VIb) and (III) to Eqs. (2.18) and (2.21), respectively; then we have at

Performing now a Fourier run in by again letting in the above equations, an expression for the constants for can be written as

(2.49)

Simplifying the above integrals, we get for

(2.50)

and for

(2.51)

Next to determine the constants we apply the condition (IV) to Eq. (2.18) and (2.21), respectively; the resultant expression at works out to be

(2.52)

Considering in the above expression, a Fourier run can then be performed in to yield an expression for determining the coefficients the relevant expression for the same is

(2.53)

Stating the first and second integrals of Eq. (2.53) as and respectively, we have after simplification

(2.54)

where for

(2.55)

for

(2.56)

and

(2.57)

The constants can next be evaluated by employing the condition (IX) to Eqs. (2.21) and (2.24), respectively; the relevant expression at turns out to be

(2.58)

Performing now a Fourier run on by allowing in the above expression, we have an expression for as

(2.59)

Simplifying the above integrals and naming them as and respectively, we have then

(2.60)

where

for

(2.61)

for

(2.62)

(2.63)

and

(2.64)

Applying now the condition (X) to Eqs. (2.21) and (2.24), respectively, we get at

(2.65)

Permitting in the above expression, the constants can then be evaluated by performing a Fourier series expansion on thus, can then be evaluated as

(2.66)

Carrying out the above integrals and stating them as and respectively, we have

(2.67)

where

(2.68)

for

(2.69)

for

(2.70)

for

(2.71)

for

(2.72)

We now sought to determine the constants towards this end we apply condition (XVI) to Eqs. (2.24) and (2.29), respectively; then we have at

(2.73)

Letting in the above expression, can thus be evaluated by carrying out a Fourier expansion on thus, we have

(2.74)

Solving the above integrals and calling them as and respectively, Eq. (2.74) can then be represented as

(2.75)

where

for

(2.76)

for

(2.77)

(2.78)

and

(2.79)

Finally, we evaluate the constants constant of [Eq. (2.24)] by utilizing the condition (XVII); applying the same to Eqs. (2.24) and (2.29), respectively, we get at

(2.80)

Allowing in the above expression and performing a Fourier run on we have then an expression for as

(2.81)

Simplifying the above integrals and naming them as and respectively, we have

(2.82)

where

(2.83)

for

(2.84)

for

(2.85)

for

(2.86)

for

(2.87)

for

(2.88)

for

(2.89)

The linear equations related to [Eqs. (2.47), (2.48), (2.50) and (2.51)], [Eq. (2.54)], [Eq. (2.60)], [Eq. (2.67)], [Eq. (2.75)] and [Eq. (2.82)] can now be solved using a Gauss elimination or other similar procedure (Scarborough 1966) to evaluate these constants. It should be noted that we already have explicit relations for and in the form of Eqs. (2.35), (2.37), (2.39), (2.41), (2.43) and (2.45) to evaluate these constants for various positions of the water level depth in the ditches. Thus, all the constants appearing in the hydraulic head functions and can be determined corresponding to a drainage situation and hence our boundary value stands solved.

The horizontal and vertical velocity distributions in the flow domain can be determined by making use of the Darcy's law; thus we have

(2.90)

and

(2.91)

where i=1, 2, 3, 4 and and are the horizontal and vertical velocities of the ith *layer.*

Applying Eq. (2.90) and Eq. (2.91) to Eqs. (2.18), (2.21), (2.24) and (2.29), respectively, we get

(2.92)

(2.93)

(2.94)

(2.95)

(2.96)

(2.97)

(2.98)

and

(2.99)

Now, to estimate the discharge coming from the top of the soil to a ditch drain, the Darcy's law can be applied at the surface of the soil; thus, if is the top discharge, then we have

Evaluating the above integral using Eq. (2.29), we get

(2.100)

It is to be noted that diverges if the ponding depth at the 1st strip is non-zero but the bund width is zero at the same time. To show that diverges for such a condition, we substitute the expression of from Eq. (2.39) into Eq. (2.100); the resultant expression, after some simplification, would be having a term like As reduces to -1 after being summed a couple of terms, i.e.,

we see that

reduces to an infinite series

after first three term expansion of the

series. This series, however, as we know diverges as and thus the expression for

also diverges when and

Now, discharge through the bottom as well as through the sides of a drain can also be evaluated by applying the Darcy's law at the relevant faces of the ditch drain. Thus, discharge through the bottom of a ditch, can be expressed as

Simplifying the above integral using Eq. (2.18), we get

(2.101)

Also, the discharge through the sides of a ditch, can be represented as

Evaluation of the above integrals using Eqs. (2.21), (2.24) and (2.29), yield

(2.102)

The relationship between the hydraulic head function and the stream function for a homogenous and anisotropic soil medium can be expressed as (Bear 1972)

(2.103)

and

(2.104)

Applying Eqs. (2.103) and (2.104) to Eq. (2.18) and then considering to be zero at the expression for the stream function corresponding to region I of Fig. 2.1 can be written as

(2.105)

Similarly applying Eqs. (2.103) and (2.104) to Eqs. (2.21), (2.24) and (2.29), respectively, we get after some simplifications

(2.106)

(2.107)

and

(2.108)

where the constants of integration of the relevant integrals are being evaluated utilizing the conditions and respectively.

For aid in visualizations and in order that the stream functions are being restricted between 0 and 1, they are generally first normalized before being used for plotting. For the problem here, they can be normalized as

(2.109)

(2.110)

(2.111)

and

(2.112)

where and denote the normalized stream functions corresponding to regions I, II, III and IV, respectively of Fig. 2.1.

2.3 Estimation of Travel Times

The time of travel required by a water particle to move from a point on the ponded surface to that of a recipient ditch can be estimated by adopting a simple procedure (Grove et al., 1970) utilizing the velocity functions as given by Eqs. (2.92) to (2.99). Suppose a particle is at a point

at time *t;* knowing the horizontal and vertical velocities of the particle at that point, the incremental distances traveled by the particle in the horizontal and vertical directions in an elemental time can then be calculated as

(2.113)

and

(2.114)

where

and (i=*1, 2, 3, 4) are the horizontal and velocity distributions corresponding to the layer*

on which the particle is currently traversing and is the porosity of the soil pertaining to that layer. The new position of the water particle after time can thus be written as

(2.115)

and

(2.116)

Now, knowing the new position at the position of the particle at time can again next be calculated by using the horizontal and vertical velocities of the particle at . The procedure can be repeated till the particle is traced upto the receiving ditch. It is to be noted that the travel times as shown in Figs. 2.4(a), 2.4(b), 2.5(a), 2.5(b), 2.6(a), 2.6(b), 2.7(a) and 2.7(b) have been evaluated utilizing the technique as just mentioned.

2.4 Verification of Proposed Solution

We now make comparisons of a few drain discharges obtained from our model with the corresponding values obtained from the experimental and analytical works of Fukuda (1957) and Barua and Tiwari (1995), respectively, for a few flow situations of Fig. 2.1. As stated before, Fukuda (1957) obtained an analytical expression for the flow problem of Fig. 2.1 by considering the hydraulic conductivities of all the layers to be the same (i.e., for the single-layered isotropic soil) and treating and in his model. He also performed a few laboratory experiments to check the validity of his model; the dimensions of his experimental variables, in terms of the notations adopted here, can be represented as and For the chosen dimensions, he allowed the depth of penetration of the drains to vary from 2.50 cm to 10.00 cm and noted concurrently the discharge corresponding to each depth of penetration of the drains; that way he could obtain a experimental plot for his chosen parameters of the problem, a plot which has been used to carry out a check on the correctness of the solution proposed here. It is to be noted that the flow problem considered here reduces to that of the Barua and Tiwari's 1995 ditch drainage problem if and (but may or may not be equal to thus, for such a situation, the outputs obtained from the developed model can also be compared with Barua and Tiwari's 1995 solution. Table 2.1 shows comparison of discharge among Fukuda’s experimentally observed values for his chosen dimensions with corresponding values obtained from the analytical works of Barua and Tiwari (1995) and those obtained from the solution proposed here. The computation of the discharges using the proposed model has been done by considering the hydraulic conductivity of the Fukuda’s experimental soil as a value obtained by using Eq. (2.100) to the Fukuda’s experimental discharge data corresponding to of his experimental setup (it is to be noted that the corresponding hydraulic conductivity value obtained using Barua and Tiwari’s 1995 model is turning out to be As may be observed in Table 2.1, the discharges obtained from the proposed model are matching closely with the corresponding values obtained from Fukuda’s experimental results and also with those obtained from the analytical works of Barua and Tiwari thereby showing that the analytical model developed here is a correct one.

Table 2.1. Comparison of Computed Drain Discharge Values for a few Flow Situations of Fig. 2.1 with the Corresponding Values Obtained from the Experimental and Analytical Works of Fukuda (1957) and Barua and Tiwari (1995), Respectively

Depth of penetration of ditch drains (cm)	Discharge per unit length of a ditch (cm^3/min) as obtained from		
	Fukuda’s (1957) experimental curve	Barua and Tiwari’ s (1995) analytical model	the proposed solution
2.50	1.050	1.075	1.073
4.00	1.600	1.629	1.632
5.00	2.000	1.947	1.994
7.00	2.700	2.694	2.696
8.00	3.080	3.039	3.038
9.00	3.340	3.344	3.372

A further check on the proposed solution was also carried out by drawing a numerical solution of the flow problem of Fig. 2.1 for a particular geometry of the problem using Processing MODFLOW (Chiang and Kinzelbach 2001). The geometry of the flow domain considered for numerical simulation is as shown in Fig. 2.2. Standard MODFLOW procedures were followed to set up a numerical model for the flow situation of Fig. 2.2 utilizing a grid size of 0.05 m × 0.05

m. The no flow boundaries were simulated by making the cells inactive in the concerned locations and the constant head boundaries were represented by constant value cells in the desired locations. The cells falling in the width of the ditch bunds were made inactive and the layeredness of the soil was implemented by imposing contrasting hydraulic conductivity fields at the desired domains of the drainage flow space. With the model so developed, a steady state MODFLOW run was then performed and the hydraulic head contours corresponding to a few numerical values of the hydraulic head were next plotted as shown in Fig. 2.2. For the considered flow situation, analytical heads were also obtained using the proposed model and then compared with the corresponding numerical values. As may be observed, the analytically obtained hydraulic heads for all the tested heads are matching closely with the corresponding MODFLOW generated values, thereby proving once again the rightness of the developed solution.

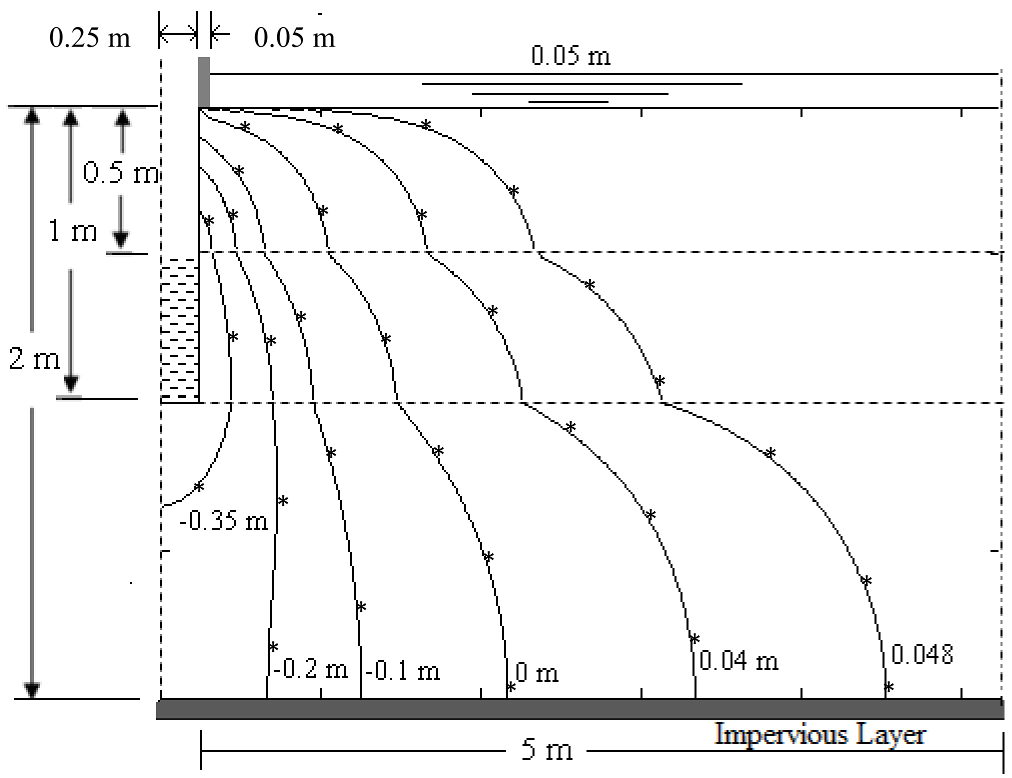
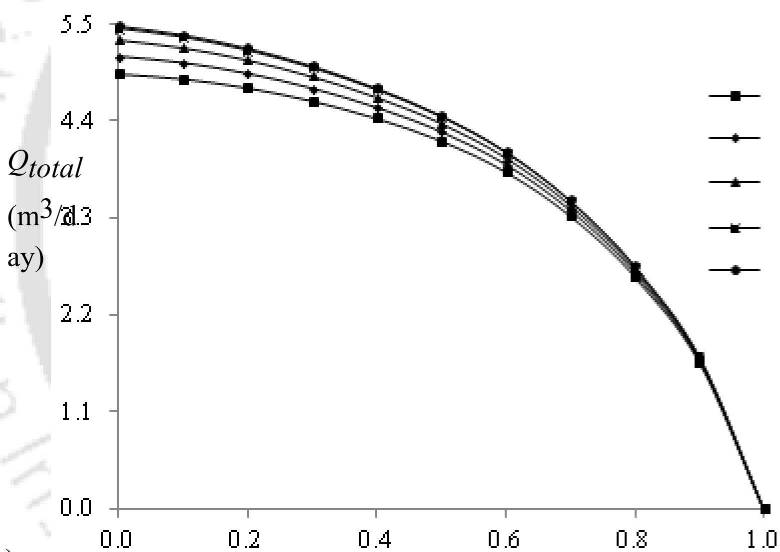


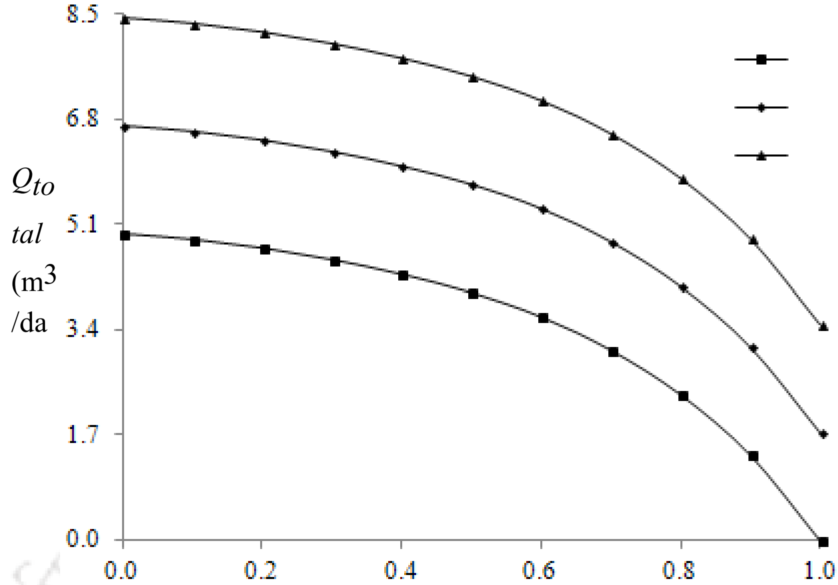
Fig. 2.2. Comparison of hydraulic head contours as obtained from the proposed analytical solution of the flow problem Fig. 2.1 with the corresponding MODFLOW generated contours when parameters of Fig. 2.1 are taken as

2.5 Discussions

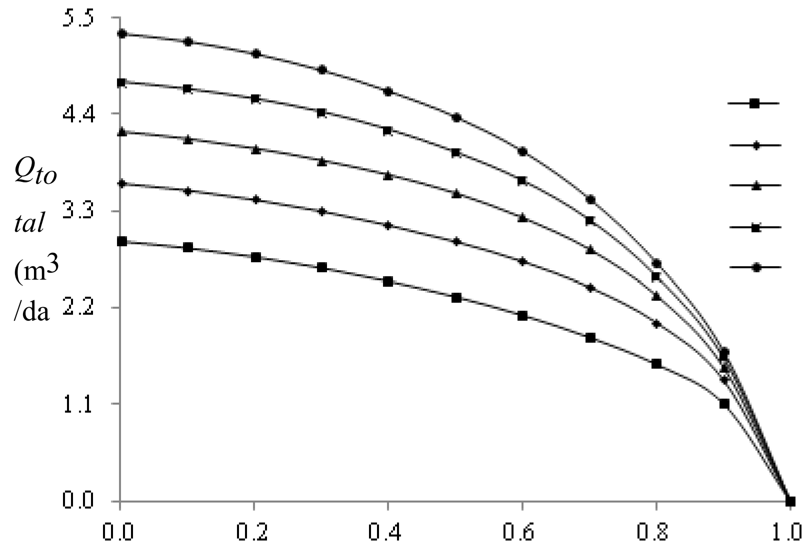
It can be seen from Figs. 2.3(a) and 2.3(b) that discharge of a ditch in a multi-layered ponded ditch drainage system is highly sensitive to the level of water in the ditch and to the ponding depth at the surface of the soil. From Fig. 2.3(c), it is observed that the versus curves show appreciable jumps when the thickness of the more conductive 3rd and the 4th layers are allowed to progressively increase, mainly again for situations where the level of water in the ditches are low. This is understandable as, for the considered flow situation, an increase of ratio (and hence also of since we have taken here actually means more regions of high conductivity soil in the subsurface drainage space as compared to a situation where the ratio is low. versus curves of Figs. 2.3(a) and 2.3(c) also show that drain discharge increases nonlinearly as well as decreasing rate with



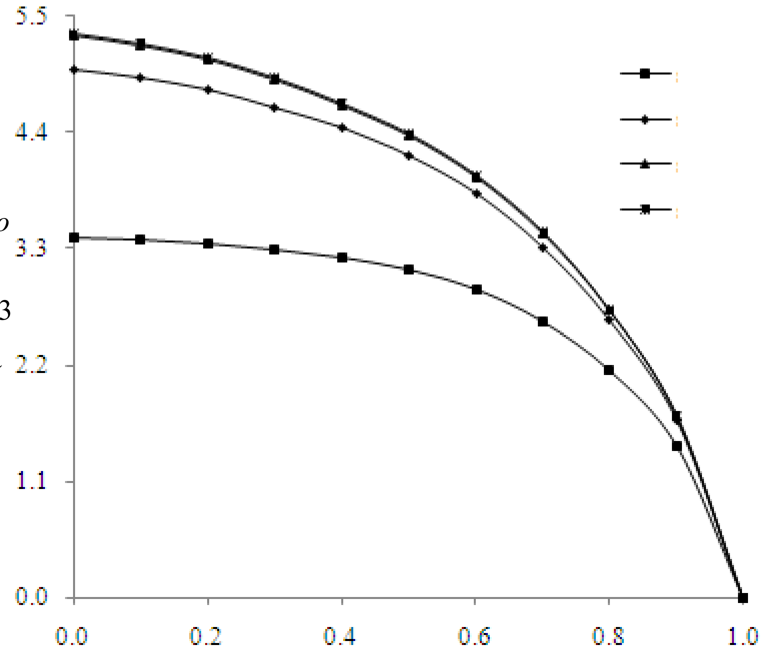
(a)



(b)



(c)



Q_{total} (m^3/da

(d)

Fig. 2.3. Variation of total discharge with ratio when the parameters of Fig. 2.1 are taken as

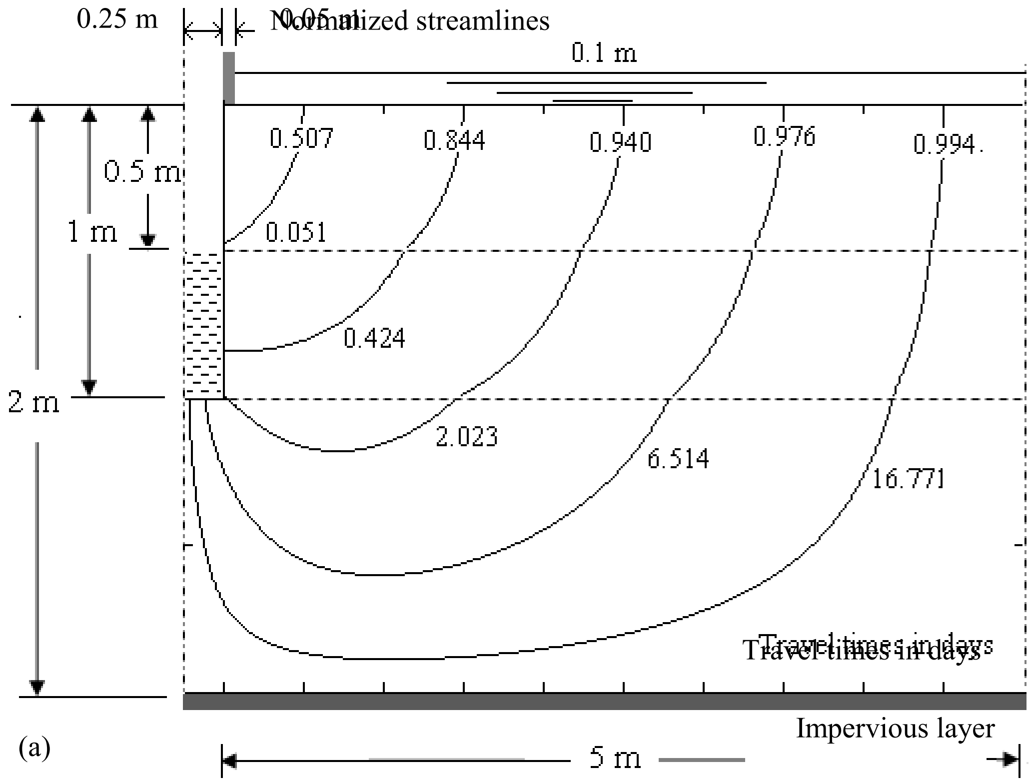
and (a)

and varies from 1/1.25 to 1/4 (b)

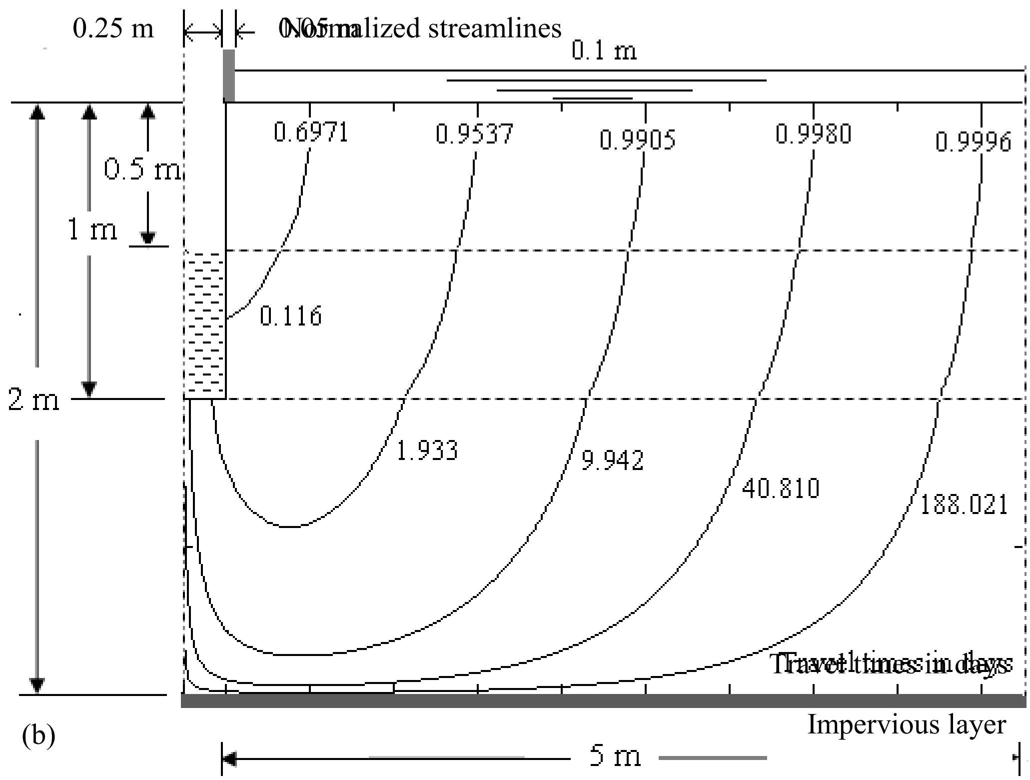
and varies from 0 m to 0.2 m (c)

the increase of depth of a multi-layered soil profile and that increasing *h* beyond a certain depth has virtually no impact on the discharge rate; however, within a certain range of the *h* values, depending on the parameters of the problem, discernible increase in drain discharge may occur for situations when the level of water in the ditches is low. The flow to a drain does not seem to be much influenced by the spacing of the drains when the drains are spread at relatively large distance from each other, as has been shown in Fig. 2.3(d); however, at small spacing distances between the adjacent drains, drain discharges do show noticeable variations with the change in spacing between the drains.

From Figs. 2.4(a) and 2.4(b), it is clear that the streamline distribution and time of travel of water particles in a multi-layered ponded drainage space is highly responsive to the magnitude and directions of conductivity of the constituent layers; high vertical conductivities of the layers have a tendency to push the streamlines downward and high horizontal conductivities of the layers have a tendency to flatten them. Further, it may also be observed from these figures that high anisotropy ratios (ratio between horizontal and vertical conductivities of soil) of the layers have a


0.25 m
0.05 m
Normalized streamlines
0.1 m
0.507
0.844
0.940
0.976
0.994
0.5 m
1 m
2 m
0.051
0.424
2.023
6.514
16.771
Travel times in days
Impervious layer
5 m

(a)



(b)

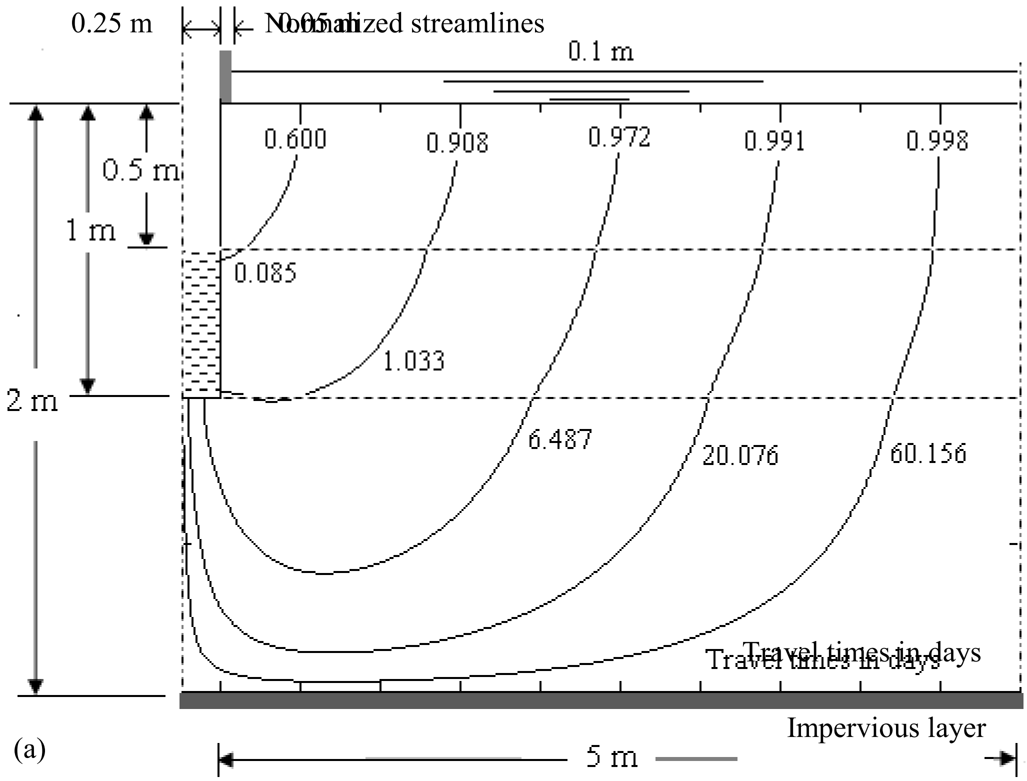
Fig. 2.4. Travel times of water particles (in days) starting from the surface of the soil to the ditches when the parameters of Fig. 2.1 are taken as

and (a)

and (b)

tendency to provide a more equitable distribution of the streamlines in the flow domain as compared to situations where the anisotropy ratios of the layers are low. This is an advantage since in most of the natural deposits, the directional conductivities of soil along the bedding is generally more than that across the bedding (Maasland 1957; Schafer 1996) and as such the very nature of the conductivity variations in soil, in general, assist in a relatively uniform cleaning of a salt affected soil when such a soil is being leached of its salts through a ponded ditch drainage system. It is also interesting to note here that high anisotropy ratio of the layers have a tendency to decrease the contribution of bottom flow [about 6% of the total flow for the flow situation of Fig. 2.4(a)] and a low anisotropy ratio to increase it [about 7.6 % for the flow situation of Fig. 2.4(b) – not mentioned in the figure]. Also, for the tested flow situations, considerable differences in time of travel of water particles moving along streamlines can be observed, mainly when seen with respect to travel times of water particles traversing in streamlines located further away from the drains.

From Figs. 2.5(a) and 2.6(a) it is clear that the distribution of the conductivity in the layers play an important role in deciding flow behavior around the ditches – keeping other factors remaining the same, a gradually decreasing conductivity variation with depth results in a relatively non-uniform distribution of the streamlines as compared to a situation where the conductivity increases with depth of the soil profile. Further, for the latter situation, the fraction of the bottom flow through the ditches may also be quite substantial as compared to the former situation. Thus, for the flow condition of Fig. 2.6(a), as may be observed, bottom flow accounts for about 39% of the total flow to the ditches; the corresponding figure for the flow situation of Fig. 2.5(a) is only about 9%. It can also be seen from Fig. 2.5(a) that, for the considered flow situation, about 91% of the flow is being contributed to the ditches from an area extending to about 1.5 m from the edge of the ditches as measured on the surface of the soil and that for the same distance, about 62% of the flow to the ditches is being accounted for the flow situation of Fig. 2.6(a). Thus, it is


0.25 m
0.05 m
Normalized streamlines
0.1 m
0.5 m
1 m
2 m
0.600
0.908
0.972
0.991
0.998
0.085
1.033
6.487
20.076
60.156
Travel times in days
Impervious layer
5 m

(a)

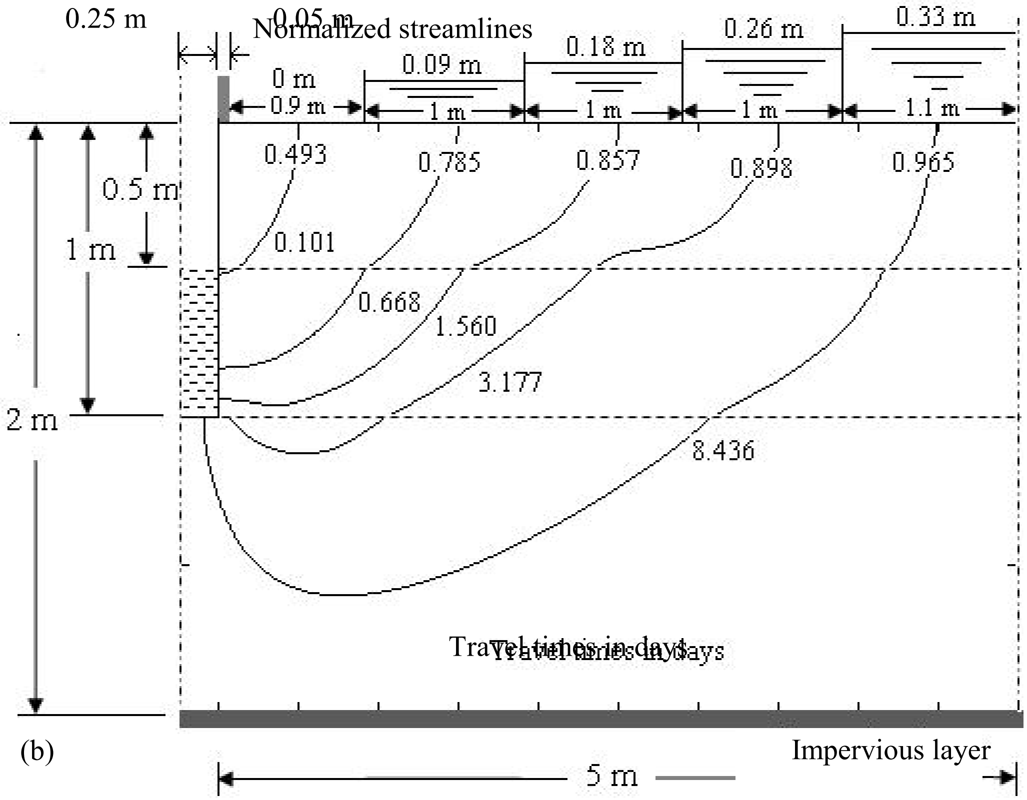


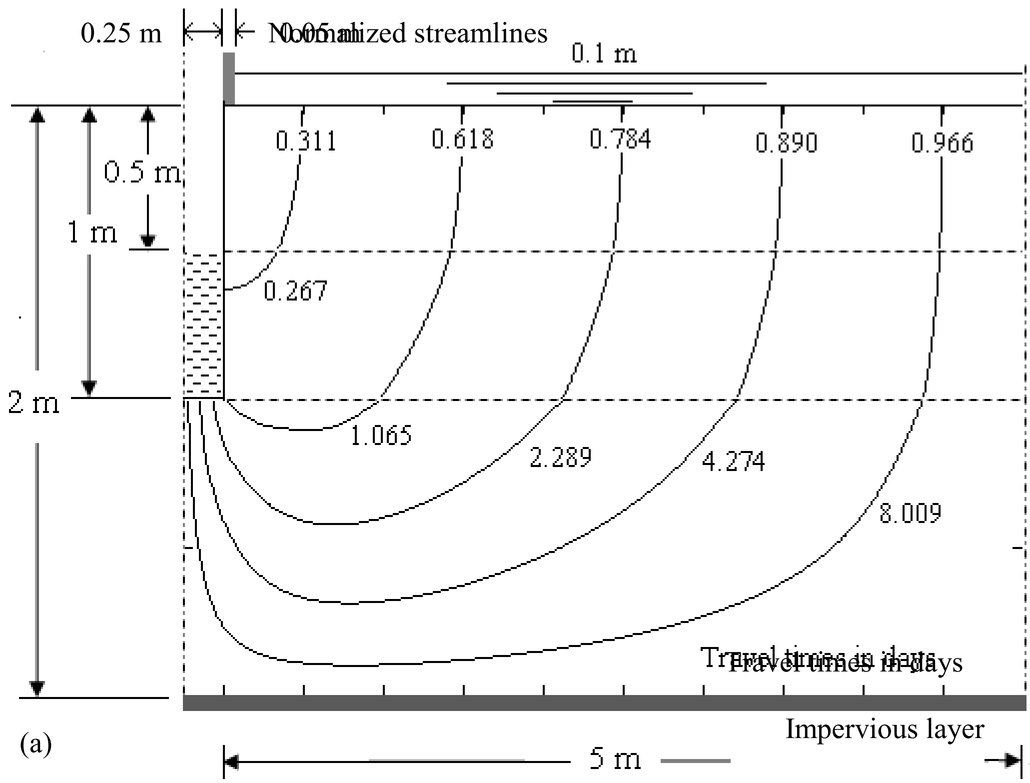
Fig. 2.5. Travel times of water particles (in days) starting from the surface of the soil to the ditches when the parameters of Fig. 2.1 are taken as

and

(a) and (b)

clear that a mere variation of the conductivity distribution may prove to be enough to bring about a sizeable change in the distribution of the streamlines around the drains of a multi-layered ponded ditch drainage system. From Fig 2.5(a), it is also very clear that the travel times of water particles along the streamlines are also pretty unevenly distributed along the flow domain – whereas the particles travelling on the 0.998 streamline is taking about 60 days to complete their journey from their originating point at the surface of the soil to the ditch, the particles moving on the 0.600 streamline from the surface is taking only 0.085 days (about 2 hours) to do so.

For the flow situation of Fig. 2.6(a) where the conductivity increases with depth, the travel times along the streamlines are relatively more uniformly distributed as compared to the flow situation of Fig. 2.5(a), where, as may be observed, the conductivity is decreasing with depth. However, for both these situations, both the streamline as well travel time distributions can be considerably improved by imposing a variable ponding fields of the type as shown in Figs. 2.5(b) and 2.6(b), respectively. The effect of the variable ponding fields, however, can be seen to be much more prominent on the travel time distributions corresponding to the flow situation of Fig. 2.5(a) than that of the flow situation corresponding to Fig. 2.6(a) – as may be observed, the travel times of the particles travelling on the streamline originating at a surficial distance of 4.5 m from the edge of the ditches has been drastically reduced from about 60 days for the flow situation of Fig. 2.5(a) to about 8.5 days for the variable ponded situation of Fig. 2.5(b), the corresponding figures for the flow situations of Figs. 2.6(a) and 2.6(b) are only 8 and 4 days, respectively. We would like to point out here that the discharge per unit width for the flow situations corresponding to Figs. 2.5(a) and 2.5(b) are 5.55 m^3/day and 4.26 m^3/day, respectively and that corresponding to Figs. 2.6(a) and 2.6(b), 3.09 m^3/day and 2.93 m^3/day, respectively. Thus, it can be seen that the introduction of the variable ponding fields in the concerned flow situations have actually resulted in less discharges to the ditch drains as compared to uniform ponding scenarios; however, as


0.25 m
0.05 m
Normalized streamlines
0.1 m
0.311
0.618
0.784
0.890
0.966
0.5 m
1 m
0.267
2 m
1.065
2.289
4.274
8.009
Travel times in days
Impervious layer
5 m

(a)

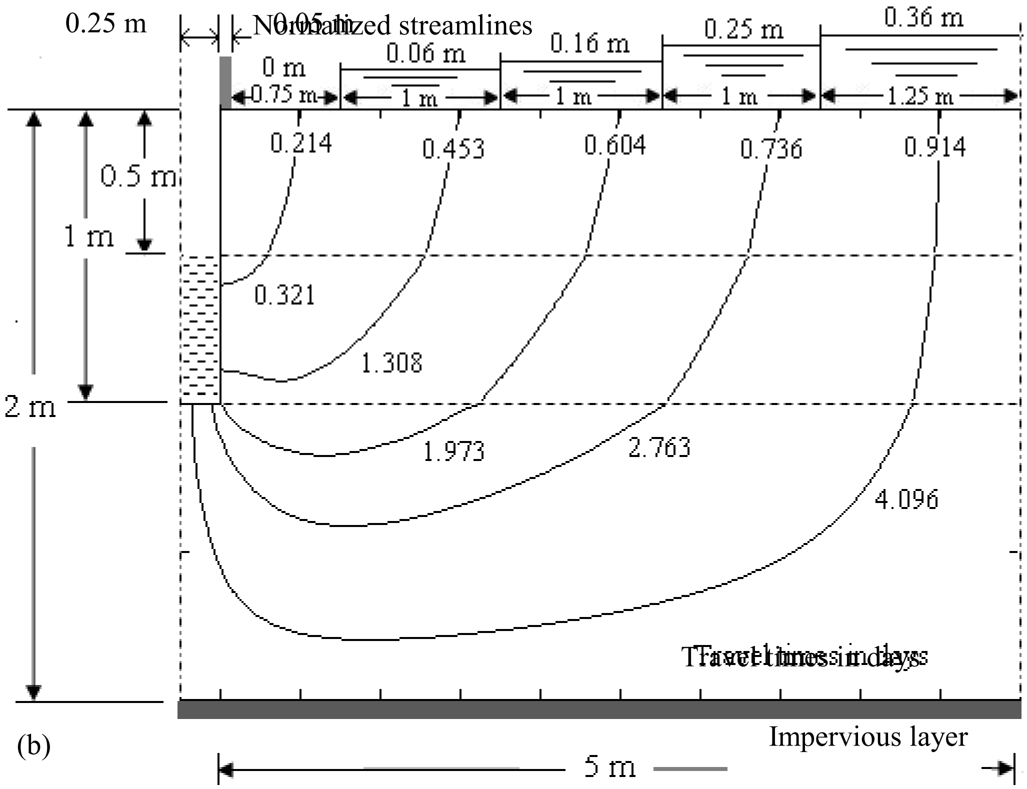
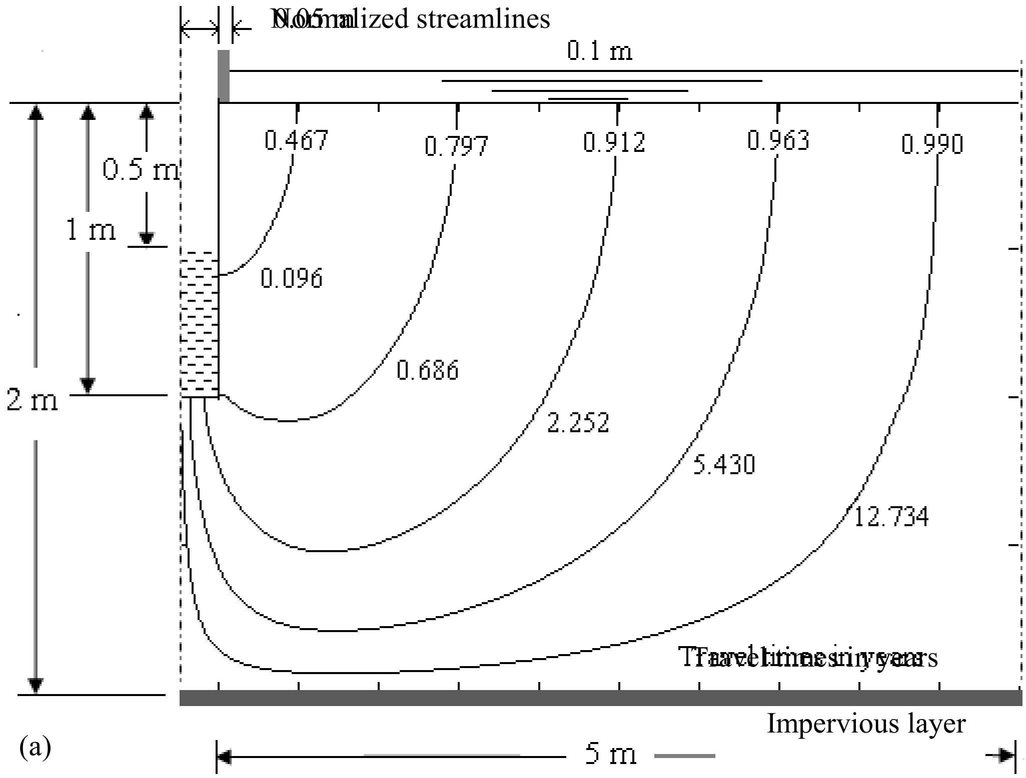


Fig. 2.6. Travel times of water particles (in days) starting from the surface of the soil to the ditches when the parameters of Fig. 2.1 are taken as

mentioned before, the variable ponding distributions have lead to much more uniform distribution of the streamlines and the travel times in the considered flow domains in comparison to uniform ponding scenarios. Thus, it may be concluded that the uniformity of leaching, both in terms of distribution of the streamlines and in distribution of the travel times, of a salt affected soil by a ponded ditch drainage system can be greatly improved by subjecting the soil to a suitable variable ponding field at the surface of the soil in comparison to the case when leaching is being carried out by imposing the soil with only a constant depth of ponding over the surface of the soil.

Figs. 2.7(a) and 2.7(b) show ditch drainage scenarios for ponded paddy fields without and with the muddy and plow sole layers. As may be observed, the presence of the muddy and plow sole layers in a paddy field greatly inhibit water flow to drains in comparison to situations where these layers are not present. This finding is in tune to the observations of Liu et al. (2001), Chen et al. (2002), Huang et al. (2003) and others who obtained their results through numerical experiments. The presence of the plow pan, however, causes the streamlines to be more evenly distributed in a ponded drainage domain in comparison to drainage scenarios where this layer is not present. It is also observed that for the flow situations of Figs. 2.7(a) and 2.7(b), the discharges per unit width of the trenches are 0.017 m^3/day and 0.004 m^3/day, respectively, showing that the presence of a plow pan in a paddy field also greatly decreases the total discharge to the ditches. Further, it may also be observed that, in general, inordinately large travel times are required for the water particle to move to the drains travelling along streamlines located away from the drains in paddy fields; however, should a pan layer be present, the travel times of water particle moving even along travel paths located close to the drains, may also be quite high as well. These are important observations and must be taken into consideration while designing drainage networks for paddy fields. It is necessary to reiterate here again that paddy

0.25 m


0.05 m
Normalized streamlines
0.1 m
0.467
0.797
0.912
0.963
0.990
0.5 m
1 m
2 m
0.096
0.686
2.252
5.430
12.734
Travel times in years
Impervious layer
5 m
(a)

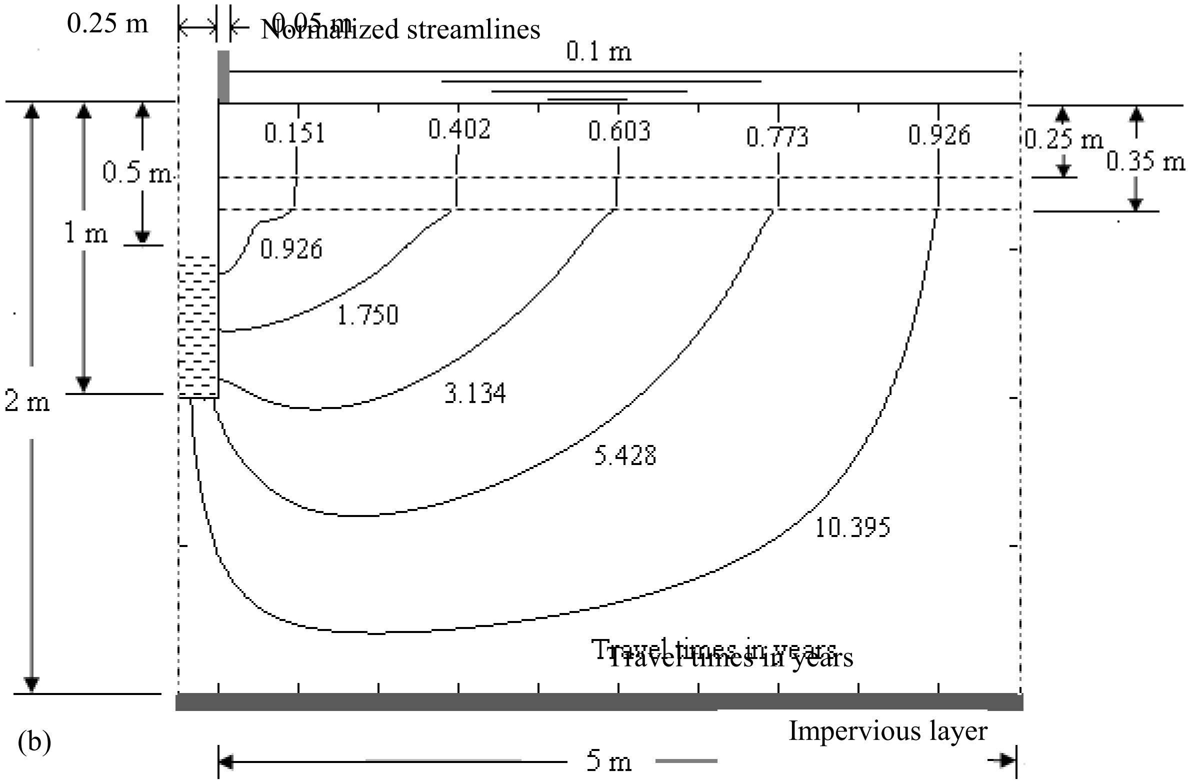


Fig. 2.7. Travel times of water particles (in years) starting from the surface of the soil to the ditches when the parameters of Fig. 2.1 are taken as

and (a)

and (b)

fields are now increasingly drained to have a control on the emission of methane from these environments, a green house gas whose contribution to global warming, as mentioned before, is quite substantial (Qiu 2009; Yvon-Durocher et al. 2014).

The model here can also be utilized to work out the upper limit of fall of water level from a waterlogged area should such an area be proposed to be reclaimed by a ponded ditch drainage system. Thus, for the flow situations of Figs. 2.5(a) and 2.5(b), the upper limits of fall of water level from the ponded fields will be about 2.30 cm and 1.30 cm respectively in 1 hour [where 5.55 m^3/day and 3.09 m^3/day are the discharges per unit width corresponding to drainage situations of Figs. 2.4(a) and 2.4(b), respectively]. These are called upper limits since, in reality, the ponded depth of 0.1 m being assumed constant for the discharge calculations for the flow situations of Figs. 2.4(a) and 2.4(b) will not remain constant during the draining process and will actually decrease with time. As an upper limit of fall of water from a flooded field through a ditch drainage system in a stipulated time provides insight about the efficacy of the system in reclaiming the waterlogged field by a desired amount within that time, the estimation of these limits are certainly helpful so far as reclamation of waterlogged soils utilizing subsurface drainage is concerned. A plus point about the proposed solution is that, as has been just shown, it can also be utilized to estimate the upper limit of fall water of a multi-layered waterlogged soil in a desired time interval when the same is being drained by a network of ditch drains of the type as shown in Fig. 2.1.

2.6 Conclusions

Equations of the hydraulic head functions and from them the stream functions have been derived for each layer of a three-layered ponded ditch drainage being underlain by an impervious layer at a finite depth from the bottom of the ditches. The solution can accommodate both a uniform as well as a variable ponding distribution at the top of the field. The separation of variable technique in association with Fourier series have been used to obtain solution to the problem. The accuracy of the developed solution has been checked with the experimental and analytical works of Fukuda (1957) and Barua and Tiwari (1995) for a few simplified situations of the problem. A MODFLOW verification of the solution has also been performed.

The study shows that flow to the ditch drains from a uniformly ponded field is mostly confined to locations close to the drains and that an increase in the anisotropy ratio of the layers increases and a decrease in the anisotropy of the layers decreases the uniformity of distribution of the streamlines in a ponded drainage flow space. Considering all other factors to remain the same, the fraction of the total flow through the bottom of the ditches increases if the anisotropy ratio of the constituent layers is made to decrease and reverse is the case if the anisotropy ratio of the layers is made to increase.

It is clearly reflected in the study that ponded drainage with a constant ponding depth at the surface of the soil is mostly limited to areas close to the ditches, unless the sub-soils close to the surface possess very low conductivities like the muddy and plow sole layers of a paddy field. The study clearly highlights the fact that considerable improvement in the distribution of streamlines and in the travel times of water particle in a multi-layered ponded drainage space can be brought about by imposing a variable ponding field at the surface of the soil. This is important since such a system, if installed for leaching a salt affected soil, will result in a better, faster and uniform cleaning of the drainage space in comparison to the case when the leaching is being carried out using only a constant depth at the surface of a ponded ditch drainage system. Further, a variable ponding field may actually result in a lesser rate of water input to the system as compared to a constant ponding situation. The study also shows that subsurface drainage in a paddy field is a slow process, particularly in presence of the muddy and plow sole layers, and water particles may require large time intervals to reach the drains from the surface of the soil. The presence or absence of the plow sole layer strongly influences the hydraulics of flow to a ditch drainage system in a paddy field and must be accounted for while studying flow dynamics in such a system. It should be noted that drainage in paddy fields is getting increasing important in recent times owing to its ability to reduce the emission of the greenhouse gas methane from such environments in a substantial way.

As the proposed solution for the ponded ditch drainage problem is for a multi-layered soil where, apart from the directional conductivities of the layers, partial penetration of the ditches as well as variable ponding fields on the surface of the soil, have also been included in the model, it is hoped that the solution provided here will lead to better design of drainage networks for reclaiming salt affected soils and in draining paddy fields than designs based on solutions developed using more stringent assumptions. This is because, as mentioned before, most of the soils in nature including paddy fields are mostly layered in nature. The developed analytical model may also be used to estimate the upper limit of water to be pumped from a ditch drainage system to drain a waterlogged soil by a desired amount within a specific time. Thus, the model developed here should also be useful from the point of reclamation of a waterlogged soil as well.

2.7 List of Notations

The following notation are used in this chapter

= constants with *m* = 1,2,3,...., p = *1,2,3,....,* q = 1,2,3,...., r = *1,2,3,....,* n = 1,2,3,...., *t* = 1,2,3,...., w = *1,2,3,....,* k = 1,2,3,...., *and* *l* = 1,2,3,...., ;

semiwidth of ditch of Fig. 2.1 [L];

depth up to the beginning of the third layer of Fig. 2.1 as measured from the water table [L];

depth up to the beginning of the second layer of Fig. 2.1 as measured from the water table [L];

depth up to impermeable layer of Fig. 2.1 as measured from the water table [L];

depth of water in the ditch of Fig. 2.1 as measured from the water table [L];

depth of penetration of the partially penetrating ditch of Fig. 2.1 as measured from the water table [L];

horizontal hydraulic conductivity of the layer of Fig. 2.1 [LT^{-1}];

vertical hydraulic conductivity of the layer of Fig. 2.1 [LT^{-1}];

anisotropy ratio (dimensionless);

number of terms to be summed in the infinite series solutions, 1, 2, 3, …

with *m* = 1,2,3,...., ;

with *n* = 1,2,3,...., ;

with p = 1,2,3,...., ;

with q = 1,2,3,...., ;

with r = 1,2,3,...., ;

with t = 1,2,3,...., ;

with w = 1,2,3,...., ;

with k = 1,2,3,...., ;

with l = 1,2,3,...., ;

number of divisions of the ponding surface at the top of the soil of Fig 2.1;

discharge through the bottom of the partially penetrating ditch of Fig. 2.1

discharge through the side face of the partially penetrating ditch of Fig. 2.1

discharge through the top surface of the partially penetrating ditch of Fig. 2.1

semispacing between any two adjacent ditches of Fig. 2.1

horizontal velocity distribution for the layer of Fig. 2.1

vertical velocity distribution for the layer of Fig. 2.1

horizontal coordinate of Fig. 2.1 [L];

vertical coordinate of Fig. 2.1 [L];

hydraulic head distribution for the layer of Fig. 2.1 [L];

stream function for the layer of Fig. 2.1 [L];

normalized stream function for the layer of Fig. 2.1 (dimensionless);

ponding depth for the strip at the soil surface of Fig. 2.1 [L];

width of the ditch banks of Fig. 2.1

CHAPTER 3

AN ANALYTICAL SOLUTION FOR PREDICTING TRANSIENT SEEPAGE INTO PARTIALLY PENITRATING DITCH DRAINS RECEIVING WATER FROM A PONDED FIELD

This chapter deals with the development of an analytical model for predicting time dependent seepage into a network of ditch drains partially penetrating a homogeneous and anisotropic soil underlain by an impervious barrier at a finite distance from the bottom of the ditches, the drains being fed by a variably ponded distribution at the surface of the soil. The model is a versatile one in that it accounts for finite width and finite level of water in the ditches, finite penetration of the drains in the soil and, as mentioned before, also a variable ponding field at the surface of the soil. The accuracy of the proposed model is checked for some simplified situations by comparing discharge and travel times of water particles specific to a few simplified situations with the identical values obtained from the analytical and experimental works of others. A numerical check on the accuracy of the proposed model is also performed utilizing the PMWIN (Chiang and Kinzelbach 2001) platform. Using the proposed solution, the transient state durations corresponding to different hydro-geological settings of a ponded ditch drainage system are also studied. Efforts are also directed to see how the hydraulic gradients at the face of a ditch vary with time and water level heights in a ponded ditch drainage system. Further, it is also demonstrated with the help of a few examples how the streamline distribution in a ponded drainage space can be modulated to a desired pattern by imposing an appropriate ponding distribution at the surface of the soil.

3.1 A Few Solutions of the Two-Dimensional Continuity Equation of Groundwater Flow for a Homogeneous and Anisotropic Soil

As was done while solving the multi-layered ditch drainage problem considered in Chapter 2, here also a similar procedure can be adopted to solve the problem considered in this chapter by first finding a few general solutions to the two-dimensional continuity equation describing transient groundwater flow for a compressible, homogeneous, anisotropic and saturated soil column. For such a flow system, the governing equation can be expressed as (Bear 1972)

(3.1)

where is the hydraulic head, and are the horizontal and vertical hydraulic conductivities and specific storage of the medium, and are coordinate variables along the horizontal and vertical directions, respectively and is the time variable.

Dividing Eq. (3.1) by on both of its sides and then introducing a transformation

(3.2)

on the resultant equation, we find, after some simplification, Eq. (3.1) to be

(3.3)

where

(3.4)

is the anisotropy ratio of the soil and

(3.5)

We now find a few solutions of Eq. (3.3); towards this end, let us consider to be a solution of Eq. (3.3) and to be a solution of its steady part, that is

(3.6)

Then, naturally, we also have as a solution of Eq. (3.3); this is because

(3.7)

We now find a solution of Eq. (3.3) using the separation of variable method (Kirkham and Powers 1972); for that, let us take

(3.8)

and apply the same on Eq. (3.3); the resultant expression, after some simplification, gives

(3.9)

Equating and – where and are constants – and solving these equations, we get

(3.10)

and

(3.11)

where and are any arbitrary constants. Also, Eq. (3.9) then becomes

(3.12)

where The solution of Eq. (3.12), obviously, is

(3.13)

where is any arbitrary constant. Substituting now Eq. (3.10), (3.11) and (3.13) in Eq. (3.8), we get a solution of Eq. (3.3) as

(3.14)

where is any arbitrary constant. Since, adding solutions of a differential equation also results in an another solution of the equation, a solution of Eq. (3.3), using Eq. (3.14), can also be expressed as

(3.15)

where are all arbitrary constants, and are summation indices and and are any positive integers. In the same way, if we assume a solution of Eq. (3.6) as

(3.16)

– where and are functions of only and respectively – and then apply the same to Eq. (3.6), we get, after adjusting the variables

(3.17)

Here also, Eq. (3.17) can be equated to a constant, the left and right hand sides of it being a function of and only; let this constant be then we have

(3.18)

From the above, we get two equations, namely

(3.19)

and

(3.20)

Now, solving the above two equations, we get

(3.21)

and

(3.22)

where , and are any arbitrary constants and is any positive integer. Incorporating functions of Eq. (3.21) and of Eq. (3.22) into Eq. (3.16), we thus find a solution of Eq. (3.6) as

(3.23)

If Eq. (3.17) is being equated to a positive constant, say, instead of as in Eq. (3.18), we will then have

(3.24)

(3.25)

and

(3.26)

Thus, from Eqs. (3.15), (3.23) and (3.26) and noting the fact that a constant is also a solution of Eq. (3.3), we see that

+

(3.27)

and

(3.28)

are solutions of Eq. (3.3). It should be noted that in Eqs. (3.27) and (3.28), is any arbitrary constant. The transient drainage problem considered here will now be solved by considering solutions of Eq. (3.3) of the forms as given by Eqs. (3.27) and (3.28).

3.2 Mathematical Formulation and Solution

Fig. 3.1 shows the geometry of the flow problem under consideration. An array of equally spaced partially penetrating ditch drains are draining a homogeneous and anisotropic soil underlain by an impervious layer, the soil being subjected to a variable ponding field as can be seen in the figure. We take the origin O on the vertical line passing through the centre of a ditch and measure the x-axis to be positive towards the right of the origin and the y-axis to be positive vertically downward of the origin. The width of the drains is taken as $2a$ and the semi-spacing of the adjacent drains as The thickness of the impervious barrier is considered as h *and* and denote the water level depth and depth of penetration of the drains, respectively, all these distances being measured with respect to the origin *O*. The directional conductivities of the soil in the horizontal and vertical directions are taken as and respectively and is the specific storage of the soil. The symmetrical ponding distribution at the surface of the soil with respect to the groundwater divide line passing though halfway between the drains, is being imposed like in the ponded ditch drainage problem considered in Chapter 2 in between two adjacent drains with the help of number of bunds with representing the ponding depth corresponding to the jth bund.

Fig. 3.1. Geometry of a partially penetrating ditch drainage system in a homogeneous and anisotropic soil subjected to a variable depth of ponding at the surface of the soil

The inner bunds are all assumed to be of infinitesimal thickness and the distance of the j^{th} inner bund from the edge of a ditch to the end of the bund is taken as The ponded water at the surface of the soil is being prevented from directly flowing to a ditch with the help of two ditch banks of width placed on either side of the ditch. In our analysis, we assume the soil to be fully saturated and the ponding depths and the height of water in the ditches to be non-changing with time. Further, we also assume that the imposition of the ponding field at the surface of the soil and the lowering of water level in the ditches have been done instantaneously, the soil being previously assumed to be saturated all the way up to the surface of the soil with the level of water in the drains flush with the horizontal soil surface. As the flow is symmetrical with respect to the groundwater divide line passing through midway between the drains, it sufficient to consider only half of the flow space for analysis as the flow in one half will be the mirror image of the other half. Further, for mathematical convenience, we perform a domain discretization of the flow domain of a nature as shown in Fig. 3.1 and name the hydraulic heads in the three sub-domains as and , respectively.

With the above definitions of the symbols in place, we now sought an analytical solution satisfying Eq. (3.1) in all the three sub-domains of the flow problem of Fig. 3.1 together with the following initial, boundary and interfacial conditions

(I)

(II)

(III)

(IV)

(V)

(VI)

(VII)

(VIII)

(IX)

(X)

(XI)

(XII)

(XIII)

(XIV)

(XV)

(XVII)

Taking in view the nature of the initial, boundary and interfacial conditions of the problem, the hydraulic head functions corresponding to the three regions of the considered flow space can be expressed, by suitably adjusting the constants of Eqs. (3.27) and (3.28), as

(3.29)

(3.30)

and

(3.31)

where

(3.32a)

(3.32b)

(3.32c)

(3.32d)

(3.32e)

(3.32f)

(3.32g)

(3.32h)

(3.32i)

(3.32j)

(3.32k)

(3.32l)

(3.32m)

and

(3.32n)

In order that the coefficients and associated with the double summation series of the hydraulic head functions as given by Eqs. (3.29), (3.30) and (3.31) be suitably evaluated satisfying the initial and the interfacial conditions of the problem at the same time, we now add the term in and the term

in with the understanding that the first term is zero in and the second term is also zero in It is also worth noting at this stage that the derivative of the double summation term involving above is not zero at for all and the derivative of the double summation term involving above is also not zero at for all Including these terms in the original expressions for and we thus have

(3.33)

and

(3.34)

where it should be clearly understood that the double series term involving in Eq. (3.33) is zero in the domain and the double series term involving in Eq. (3.34) is zero in the domain It is to be stated here that the expressions to be used for and are as given by Eqs. (3.29) and (3.31), respectively; the alternate expressions of and as given by Eqs. (3.33) and (3.34) above, are being provided for mathematical purpose only so that the constants and can be adequately determined satisfying the initial condition of the problem together with the interfacial conditions in between the sub-domains.

We first obtain a steady state solution to the problem by letting in the expressions for the hydraulic head functions; it would naturally make the exponential terms in these equations to disappear leaving only the steady state terms to deal with. It can be seen that Eq. (3.29) satisfies (II), (III) and (VI), Eq. (3.30) satisfies (VIII) and (IX) and Eq. (3.31) satisfies (XV) as a consequence of their very definitions. To evaluate we apply conditions (XIII) and (XIV) to Eq. (3.31); thus, we have at

Allowing to increase indefinitely in the above expression, can then be evaluated by running a Fourier series on the interval thus, we have

(3.35)

Eq. (3.35), upon simplification, yields

(3.36)

For evaluating the constants , we apply conditions (XVI), (XVII) and (XVIII) to Eq. (3.31) to get at

where

(3.37)

Now making use of a Fourier expansion in the range we have then an expression for as

(3.38)

Simplifying the above integral, we get

(3.39)

Now, applying condition (IV) to Eqs. (3.29) and (3.30), respectively, we have at

(3.40)

Thus, can be evaluated using a Fourier series expansion in if it is assumed that R in the above expression goes to infinity; considering such is the case, we then have

(3.41)

The above integral survives only when otherwise, for all other cases, it is zero. Thus, for

and for

We now apply condition (V) to Eq. (3.29) and Eq. (3.30), respectively to get at

(3.42)

The constants can then be evaluated, considering as a Fourier expansion in thus, we have

(3.43)

Naming the first integral of Eq. (3.43) as and the second integral as the above equation can then be expressed as

(3.44)

Simplifying the above integrals, we have for

(3.45)

and for

(3.46)

and

(3.47)

Applying now the equality of the hydraulic head condition (X) at the interface of regions II and III to Eqs. (3.30) and (3.31), respectively, we get at

(3.48)

By letting the constants hence, can be evaluated by a Fourier series expansion in the range thus, we have

(3.49)

If the three integrals of Eq. (3.49) are named as and respectively, then Eq. (3.49) can be written as

(3.50)

Simplification of the first integral gives, for

(3.51)

and for

(3.52)

further, the other two integrals, upon simplification, reduce to

(3.53)

and

(3.54)

Also, application of the flux equality condition (XI) at the interface of regions II and III to Eq. (3.30) and (3.31), respectively, yields at

(3.55)

Thus, can be evaluated, considering by performing a Fourier run in hence, can be expressed as

(3.56)

Simplifying the above integrals, we get

(3.57)

where

(3.58)

for

(3.59)

and for

(3.60)

for

(3.61)

and for

(3.62)

Now, the linear equations resulting from Eqs. (3.41), (3.44), (3.50) and (3.57) can be solved using Gauss elimination or some other suitable method (Scarborough 1966) to obtain the constants and with the evaluation of these constants, all the constants involved in the steady state forms of the hydraulic head functions are thus determined and the boundary value problem for such a situation is thus solved.

It is now proposed to tackle the transient part of the problem by incorporating the initial condition into the solution domain. As may be observed, the expressions for and with the steady state coefficients and just been determined satisfy the interfacial conditions for all time to incorporate the initial condition and hence determine the coefficients and of the hydraulic heads, conditions (XII), (VII) and (I) can next be employed to [Eq.(3.34)], [Eq.(3.30)] and [Eq. (3.33)], respectively.

Thus, we have at as

(3.63)

It can be observed in Eq. (3.63) that the *y*-bases of and are not the same – whereas the former is defined for the latter is for the whole vertical length, namely In order that a Fourier series may be run on the whole space comprising of the second and the third domains, we now express each term of

as a Fourier series in thus, we have for each and

(3.64)

Performing a Fourier double run on by letting and to increase to infinity in Eq. (3.64), we have

(3.65)

The first double integral, obviously, is zero; naming the second double integral of Eq. (3.65) as and expressing it as a product of two single integrals, and (one in X and the other in *y*), we get

(3.66)

and

(3.67)

The integral of Eq. (3.66), upon simplification, yields for

(3.68)

and for

(3.69)

Also, for works out to be

(3.70)

and for

(3.71)

Thus, from Eqs. (3.66) and (3.67), Eq. (3.65) can be expressed as

(3.72)

Plugging Eq. (3.64) in Eq. (3.63) and simplifying, we get

(3.73)

Applying now condition (VII) to Eq. (3.30), we get

(3.74)

Like before, adjusting the bases of the double summation terms of the above equation, we get

(3.75)

Now, from Eqs. (3.73) and (3.75), an expression for

can be worked out

by undertaking a double Fourier expansion in the range by letting

and in these equations; the ensuing relation works out to be

(3.76)

Calling the first, second, third, fourth, fifth and sixth double integrals of Eq. (3.76) as

and respectively and simplifying these integrals, we have

(3.77)

where for

(3.78)

and for

(3.79)

and

(3.80)

(3.81)

where

(3.82)

and for

(3.83)

and for

(3.84)

(3.85)

where

(3.86)

and for

(3.87)

and for

(3.88)

(3.89)

(3.90)

where for

(3.91)

and for

(3.92)

and

(3.93)

(3.94)

where

(3.95)

and for

(3.96)

and for

(3.97)

Thus, from Eq. (3.76),

can be expressed as

(3.98)

Now, applying conditions (I) to Eq. (3.33) and (VII) to Eq. (3.30), respectively, we get

(3.99)

and

(3.100)

Now, adjusting again the bases of the double summation terms of Eqs. (3.99) and (3.100), we get for each and

(3.101)

Considering and in the above expression, can then be determined by performing a Fourier run on thus, we have

(3.102)

Simplifying the above integrals, we get

(3.103)

where for

(3.104)

and for

(3.105)

for

(3.106)

and for

(3.107)

Substituting Eq. (3.101) in Eqs. (3.99) and (3.100), respectively, we obtain

(3.108)

and

(3.109)

Thus, letting and in Eqs. (3.108) and (3.109), a Fourier run can be executed

in this yields an expression for as

(3.110)

Identifying the first, second, third and the fourth integrals as and

respectively, and solving these integrals, we have

(3.111)

(3.112)

where for

(3.113)

and for

(3.114)

and

(3.115)

(3.116)

where for

(3.117)

and for

(3.118)

and

(3.119)

(3.120)

where

(3.121)

and for

(3.122)

and for

(3.123)

Thus, from Eq. (3.110),

can be expressed as

(3.124)

Now, Eqs. (3.98) and (3.124) can be utilized to work out the necessary linear equations to evaluate the required double Fourier coefficients and by considering suitable integral values of In all the drainage examples studied here, we have taken while carrying out the computations. It should be noted that of Eq. (3.98) is being linked with the through Eq. (3.72) and that of the above equation is being lined with the through Eq. (3.103). Once the linear equations corresponding to a flow situation are in place, they can then be solved by resorting to a suitable iterative procedure (Saad 2003) to evaluate these constants. Thus, all the unknown coefficients associated with the hydraulic heads function of Eqs. (3.29), (3.30) and (3.31) corresponding to a drainage scenario are evaluated and hence our boundary value problem stands solved.

The horizontal and the vertical velocity distribution functions, and for the sub-domains I, II and III of Fig. 3.1 in the real space can be determined by applying the Darcy's law to the relevant hydraulic head functions related to these sub-domains; thus we have

(3.125)

and

(3.126)

Carrying out the above differentiations after first converting the hydraulic head functions to the real space using the relations as given by Eq. (3.2), (3.32m) and (3.32n), we get

(3.127)

(3.128)

(3.129)

(3.130)

(3.131)

and

(3.132)

To determine the transient discharge function at the top of the soil, we apply the Darcy's law at the surface of the soil using the hydraulic head function (Eq. 3.31) of sub-domain III; thus, we have

(3.133)

where is the discharge rate at the surface of the soil per unit length of the ditches and

(3.134)

Evaluation of the integral of Eq. (3.133), using Eq. (3.31), yields

(3.135)

We now show that diverges if the depth of ponding in the 1st strip, is non-zero and at the same time the width of the ditch bunds is zero. To do that, we substitute of Eq. (3.39) in Eq. (3.135); this will result, after some simplifications, an expression which will be having a series of the type and Remembering noting that tends to -1 when m *is allowed to increase incessantly, that is*

we see that the series

reduces to is assumed to reach somewhere very close if

to -1 after carrying out the summation of the first three terms of the series. This infinite series, however, is a diverging series and thus, diverges if and

Now, to estimate the discharge per unit length through the sides of a ditch, we apply the Darcy's law at the sides, namely

(3.136)

Simplification of the above integral using Eq. (3.31), gives

(3.137)

In the same way, the discharge through the bottom of a ditch per unit length, can also determined by making use of the Darcy's law at the bottom face of a ditch; thus, we have

(3.138)

Simplification of the above integral, using Eq. (3.29), leads to

(3.139)

We would like to point out here that all the expressions related to the hydraulic heads and discharge functions will reduce to the steady state situation when the time variable t in them is given a very large value (theoretically infinite) – as may be noted, letting in these expressions make the exponential terms in these equations to disappear leaving only the steady state terms behind.

Now, the volume of water seeping through the surface of the soil in time T can be expressed by integrating Eq. (3.135); thus, we have

(3.140)

Evaluation of the above integral gives

(3.141)

Similarly, expressions for the volume of water seeping though the sides and bottom of a ditch can be represented as

(3.142)

and

(3.143)

Simplifying the above integrals using Eqs. (3.137) and (3.139), respectively, we get

(3.144)

and

(3.145)

Now, expressions for the steady state stream functions, and corresponding to the three domains of Fig. 3.1 can be evaluated by making use of the following relations (Bear 1972)

(3.146)

and

(3.147)

where is the hydraulic conductivity of an isotropic soil. We have here, in the computational domain thus, applying Eqs. (3.146) and (3.147) (after letting in them) to Eq. (3.29), (3.30), (3.31), respectively, we get

(3.148)

(3.149)

and

(3.150)

where the constants of integration in the stream functions are being conveniently chosen to be zero by assuming and For clarity of presentations, the stream functions are generally first normalized before being used for plotting. In order that the magnitude of the normalized streamlines increase as one moves away towards the halfway distance between the ditches, we carry out the normalization of the streamline as under

(3.151)

(3.152)

and

(3.153)

where and are the normalized stream functions corresponding to sub-domains I, II and III, respectively of Fig. 3.1.

The travel time of a water particle from any arbitrary point in the flow domain to a recipient ditch can also be determined from the known velocity functions (Eqs. (3.127), (3.128), (3.129), (3.130), (3.131) and (3.132)) by adopting the same Grove et al.'s (1970) approach as has been employed for estimating the travel times for different drainage situations considered in Chapter 2. In this context, we would like to mention that the travel times as portrayed in Fig. 3.8 have been estimated by making use of this approach only.

3.3 Model Verification and Discussions

3.3.1 Verification of the proposed solution

As mentioned before, the ponded ditch drainage problem solved here can be considered as an extension of the problem taken by Barua and Alam (2013) from that of a fully penetrating ditch drainage system to that of a partially penetrating one. Thus, if in Fig. 3.1 is allowed to approach h, the solution obtained here then should yield results comparable to the ones obtained from the solution of the fully penetrating version of the problem. Thinking in this line, we carried out a comparison of the time dependent top discharges as predicted by our model for increasing values of with the ones obtained using the fully penetrating analytical model of Barua and Alam (2013) when the flow parameters of Fig. 3.1 are taken as shown in Table 3.1. As can be seen, for the considered flow situation, with the increase of values, the discharge predictions are getting increasing closer to the full penetrating discharge values of 1.1091 and 1.4580 corresponding to the tested times **of 25 and 100 seconds, respectively. As our** model could successfully reproduce the discharge values of the fully penetrating ditch drainage scenarios when of Fig. 3.1 is being extended close to h*, the way it should be, we hence* conclude that the proposed model is being correctly developed.

Table 3.1. Comparison of computed drain discharge values as obtained from the proposed model for a few flow situations of Fig. 3.1 with the corresponding values obtained from the analytical work of Barua and Alam (2013) when the parameters of the flow problem of Fig. 3.1 are taken as

Depth of penetration of ditch drains (m)	Discharge per unit length of a ditch (m^3/day/m) as obtained from			
	Barua and Alam's solution (m)	Proposed solution	Barua and Alam's solution (m)	Proposed solution
	t = 25 s		t = 100 s	
0.45	1.1091	0.5619	1.4580	0.9673
0.55		0.6875		1.1118
0.65		0.8766		1.2473
0.75		1.0305		1.3531
0.85		1.0983		1.4094

We would like to mention now that Zhang et al. (1983) performed modeling and sand tank studies to study the movements of dispersive and non-dispersive solutes to subsurface drains from ponded and uniformly recharged fields. We find that one of their experimental results can be made use of to perform an experimental verification of the solution proposed here. Using one of their sand tank experimental data and the dimensions of the concerned sand tank (Fig. 3.2), a comparison of the depth-time variations, as predicted by our model, of an inert solute moving along a specific streamline in the concerned drainage space was compared with the identical results as obtained from their experimental work. Fig. 3.2 shows such a comparison. As may be observed from this figure, the analytically predicted time dependent movements of the solute from the ponded surface in the concerned streamline are matching very closely with the corresponding values obtained by experimental means, thereby showing, once again, that the developed solution is a correct one. It is to be noted here that, in Zhang et al.'s (1983) sand tank experiment, the width of the ditch is not specified; for comparison purpose, we have taken the same as 0.1 cm, a value which can be treated as almost negligible compared to the semi-spacing distance of 50 cm between the drains taken by them in their sand tank model.

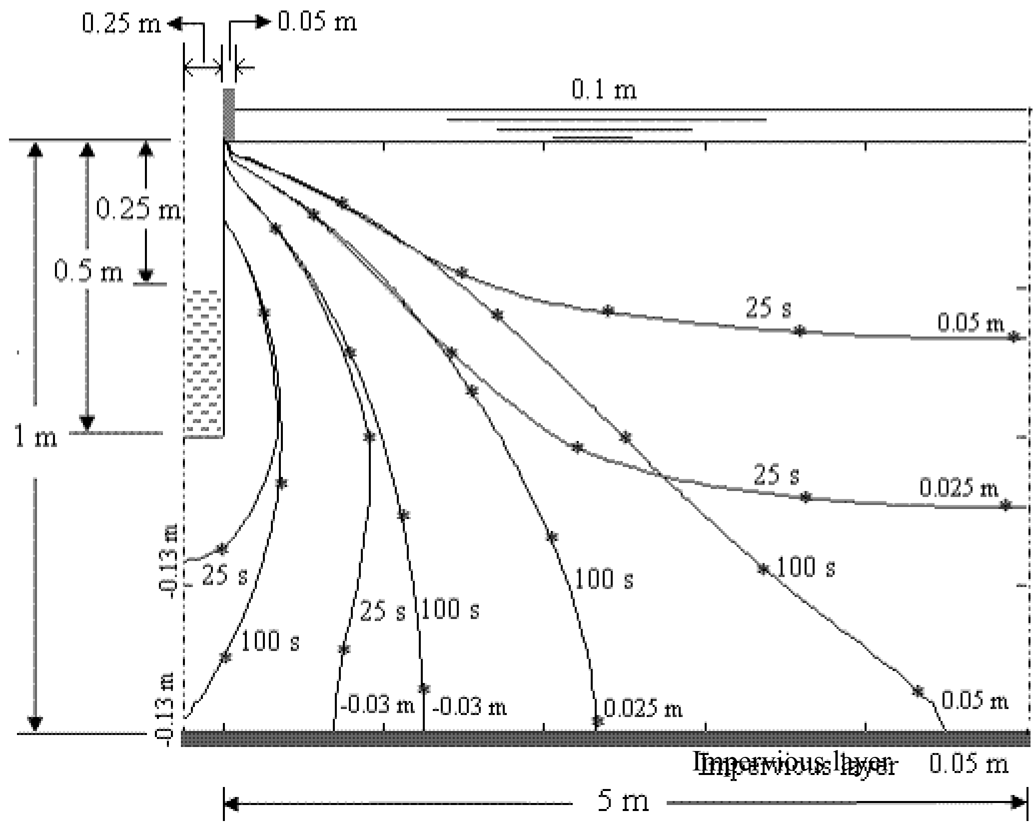
*   *Time dependent movements of solute particle as obtained from Zhang et al.'s experimental results*
Time dependent movements of solute particle as predicted by the proposed analytical model

Fig. 3.2. Comparison of time dependent movements of an inert solute particle from the surface of a ponded field as obtained from the proposed solution with the corresponding values obtained from the experimental results of Zhang et al. (1983) when the parameters of Fig. 3.1 are taken as

and the solute particle being moving on the vertical streamline passing

To have a further check on our analytical model, we also performed a MODFLOW (Chiang and Kinzelbach 2001) verification of the same for a specific flow situation of Fig. 3.1, where we have considered

and To formulate such a ponded drainage model in the MODFLOW platform, a hypothetical ponded soil of surface area 15 m by 5 m and thickness 1 m (i.e., half of the soil space in between two adjacent drains) was first considered and then simulated with the help of a grid network comprising of 152 rows, 107 columns and 22 layers. Thus, the grid cells considered for modeling were having the row spacing as 0.1 m, the column spacing as 0.05 m and the thickness of each cell as 0.05 m. All the cells of the last layer (i.e., the 22nd layer) were made inactive so as to represent the impervious layer underlying the soil column and all the cells of the 1st layer starting from the 8th column to the 106th column and the 2nd row to the 151th row were given a constant value of 0.1 m so as to represent a uniform ponding depth of 0.1 cm on the surface of the soil. The no-flow boundary on the centroidal plane in between the ditches was introduced by making all the cells falling in the 1st layer and continuing up to the 21st layer of the 107th column inactive. Also, the no-flow Northern and Southern boundaries of the model were represented by making all the cells falling in the vertical planes of the 1st and the 152th rows and starting from the 7th column to the 106th column inactive. The ditch bank of width 0.05 m was simulated by making all the cells falling in the 7th column of the 1st layer and running from the 2nd row to the 151th row in that column inactive. The semi-bottom width of 0.25 m was modeled by assigning all the cells falling in the 2nd to the 6th columns in the 11th layer with a constant value of -0.25 m. The side of the half-filled ditch having a water level of 0.25 m, as measured from the surface of the soil, was simulated by assigning a constant cell value of 0 m for all the cells falling in the 6th column of the second layer, -.05 m for all the cells falling in the 6th column of the 3rd layer and so on up to the 6th layer after which a constant value of -0.25 was being imposed to all the cells of the 6th column up to the 10th layer. The no-flow boundary condition on the plane passing through the centre and below the bottom of the ditch, was being modeled by making all the cells starting from the 12th layer and moving up to the 21st layers of the 1st column inactive. The horizontal and vertical hydraulic conductivities of and respectively and the specific storage of of the soil were then inputted for all the active cells of the model. With the above definition of the model in place, a transient MODFLOW run was then carried out and the hydraulic head contours obtained from the run for two time steps were next compared with the corresponding analytically obtained values from our proposed model. Fig. 3.3 above shows such a comparison. As may be seen, our analytically obtained values for the considered flow situation for both the time steps are matching very closely with the corresponding MODFLOW predicted values thereby verifying, once again, the rightness of the developed solution. We would like to point out that we have considered here the plane passing through the 76th row for extracting the numerically obtained hydraulic heads for comparison with our analytical model. This is because this zone is located halfway between the Northern and Southern boundaries of the numerical



* *Transient hydraulic head contours as generated by MODFLOW*
Transient hydraulic head contours as predicted by the proposed analytical solution for two different times
Depth of ponding and height of the ditch bund are not in scale; all other dimensions are in scale

Fig. 3.3. Comparison of hydraulic head contours as obtained from the proposed analytical solution with the corresponding MODFLOW generated contours for two different time steps when the parameters of the flow problem of Fig. 3.1 are taken as

and

model and hence at the farthest distance between these two boundaries. It is to be noted that the analytical model developed here for the partially penetrating ponded ditch drainage problem is based on the assumption of two-dimensional flow, a condition which will ideally be satisfied if the ponded field is assumed to be of infinite size. The MODFLOW model considered here for comparison with our analytical solution, however, is a finite area model with the distance between the Northern and the Southern sides of the model being restricted to only 15 m (152 rows). But it was observed that, even with a lesser separation of about 12 m between the Northern and Southern boundaries of the model instead of the 15 m separation taken in this model in between these boundaries, for the considered drainage situation, a sufficiently good approximation of two-dimensional flow could be achieved in the vertical plane passing midway in between these boundaries.

3.3.2 Discussions

Figs. 3.4(a), 3.4(b) and 3.4(c) clearly show that the time taken by a partially penetrating ponded ditch drainage system to reach steady state may be considerable if the ditches are being installed in a soil with low values of directional conductivities and a high value of specific storage, particularly for situations where the separation between the adjacent ditches is quite large and the underlying impervious layer lies at a sufficiently large distance from the bottom of the ditches. It is worth noting here that the hydraulic conductivity of many soils like silty clay loam, dense clay, glacial tills and clayey paddy soils (Hendry 1982, Chen *et al*. 2002, Tabuchi 2004, Stibinger 2009, MacDonald *et al*. 2012) may be quite low; also, the specific storage of many unconfined aquifers may be quite high as well (Neuman 1975), particularly for aquifers in glacial tills (Grisak and Cherry 1975, Sharp 1984, Jones *et al*. 1992, Shaver 1998, Chen and Chang 2003).

Q_{top}/2K

Q_{ditch}/2K

(a)

Q_{top}/2K

Q_{ditch}/2K

(b)

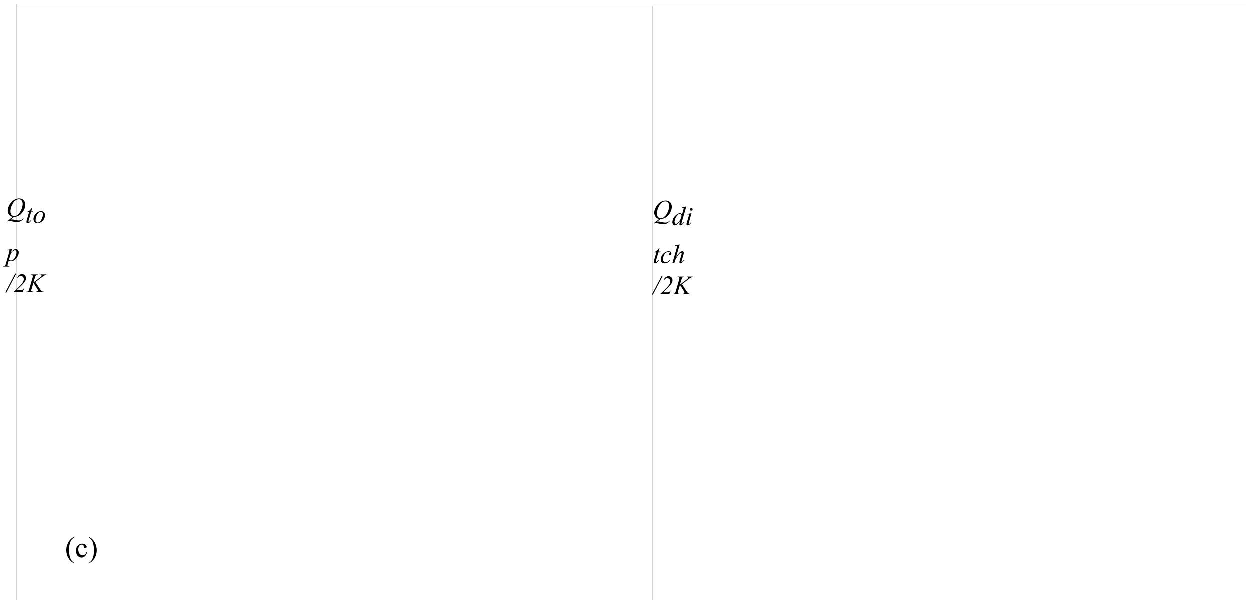
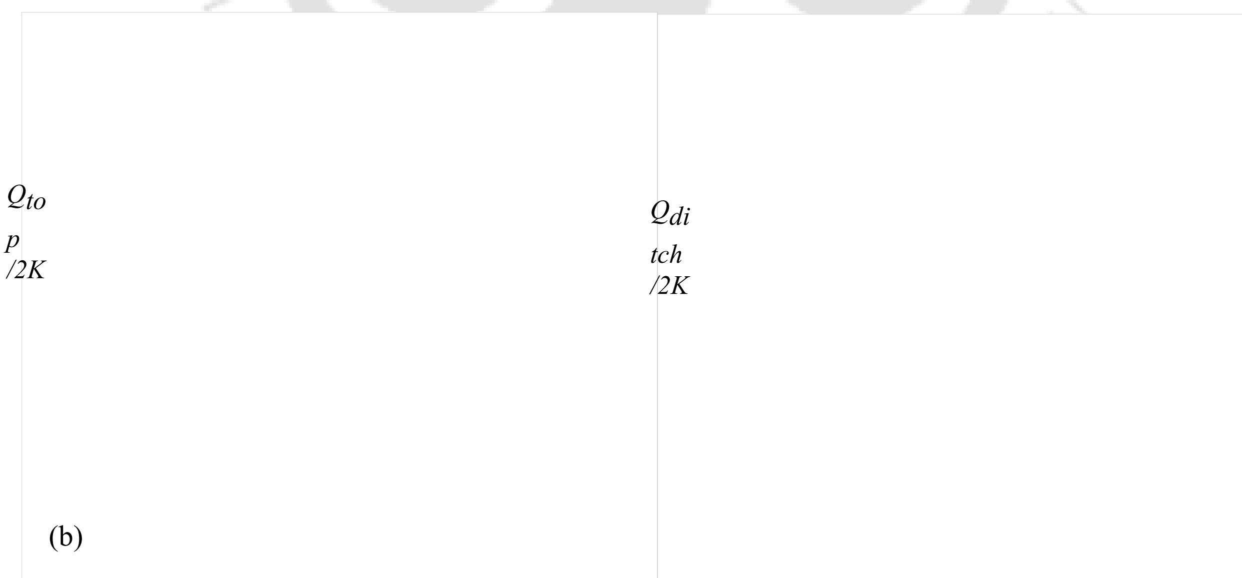


Fig. 3.4. Variations of and ratios with time for different ratios when the parameters of the flow problem of Fig. 3.1 are taken as and (a) (b)

Further, as mentioned before, since soils with high anisotropy ratios also do frequently occur in nature, we see that the transient duration of a partially penetrating ditch drainage system in these soils may be quite high. From Figs. 3.4(b) and 3.4(c), we also observe that the specific storage has a strong influence on the transient duration of a ponded drainage system – decreasing the specific storage alone from 0.001 m^{-1} to 0.0001 m^{-1} for the concerned flow situations has caused the time for the top discharge to attain steady state to decrease considerably for these situations. It can also be inferred from Figs. 3.4(a), 3.4(b) and 3.4(c) that both top discharge as well as discharge through the sides and bottom of a ditch are pretty sensitive to the level of water in the ditch with the discharge values for an empty ditch far outweighing those of a quarterly or a half-filled ditch at all times of simulation of a partially ponded ditch drainage system.

Figs. 3.5(a) and 3.5(b) also corroborate the fact that the time to reach steady state for a partially penetrating ditch drainage system draining a ponded field comprising of a low conductivity soil (0.0254 m/day, i.e., one inch per day – glencoe silty soil, Kirkham 1949) may be quite high, particularly if the ditches are being dug to a relatively deeper depth into the soil as measured with respect to the thickness of the soil. Thus, as may be observed, the top discharge for the flow situation Fig. 3.5(b) is requiring about 155100 seconds for it to stabilize to the steady state whereas for the flow situation of Fig. 3.5(a), this figure is turning out to be about 125100 seconds only. We also see from these figures that the width of the ditch drains may not be having much of an impact on the surface discharge as well as on the bottom and side discharges of the ditches, specifically for ponded drainage scenarios where the spacing between the adjacent ditches is much higher than that of the width of the ditches.

(a)

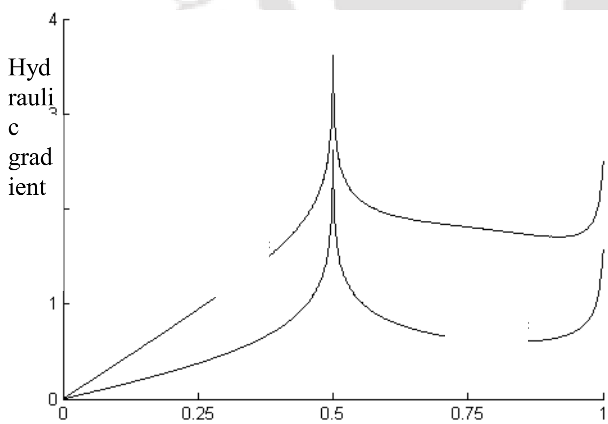


(b)

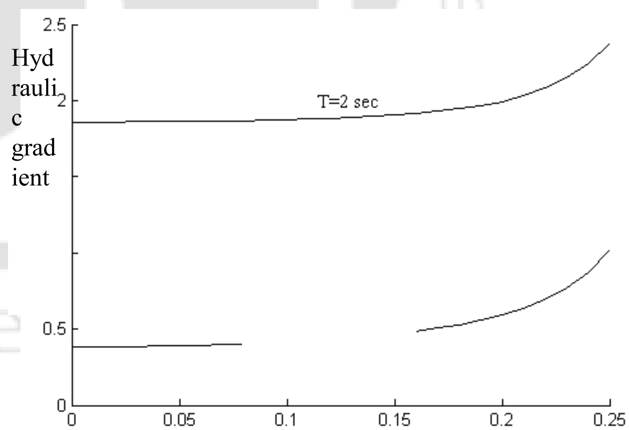
Fig. 3.5. Variations of and ratios with time for different ratios when the parameters of the flow problem of Fig. 3.1 are taken as

and (a)

Figs. 3.6(a), 3.6(b) and 3.6(c) show the distribution of hydraulic gradient on the side and bottom of a ditch receiving water from a ponded field for a few flow situations of Fig. 3.1. It should be noted that the hydraulic gradients at the boundary of a stream/ditch can be taken as a measure of the effect of subsurface drainage on incipient gully formation at the banks of a stream/ditch when the stream/ditch is being running in bottomland areas having water levels lower than that of the surrounding water table (It is interesting to note that the exit gradients at a ditch are quite sensitive to time in the transient phase of simulation of a ditch drainage system and that the exit gradients at the sides as well as on the bottom face of a ditch keep on increasing with the decrease of water level in the ditch, attaining maximum values when the water level goes flush up to the bottom face of the ditch (i.e., when the ditch has become empty). It is also worth noting here that the maximum exit gradients occur at points where the level of water in a ditch meets the sides of the ditch.



(a) Vertical distance at the face of ditch (m)



Horizontal distance at the bottom of ditch (m)

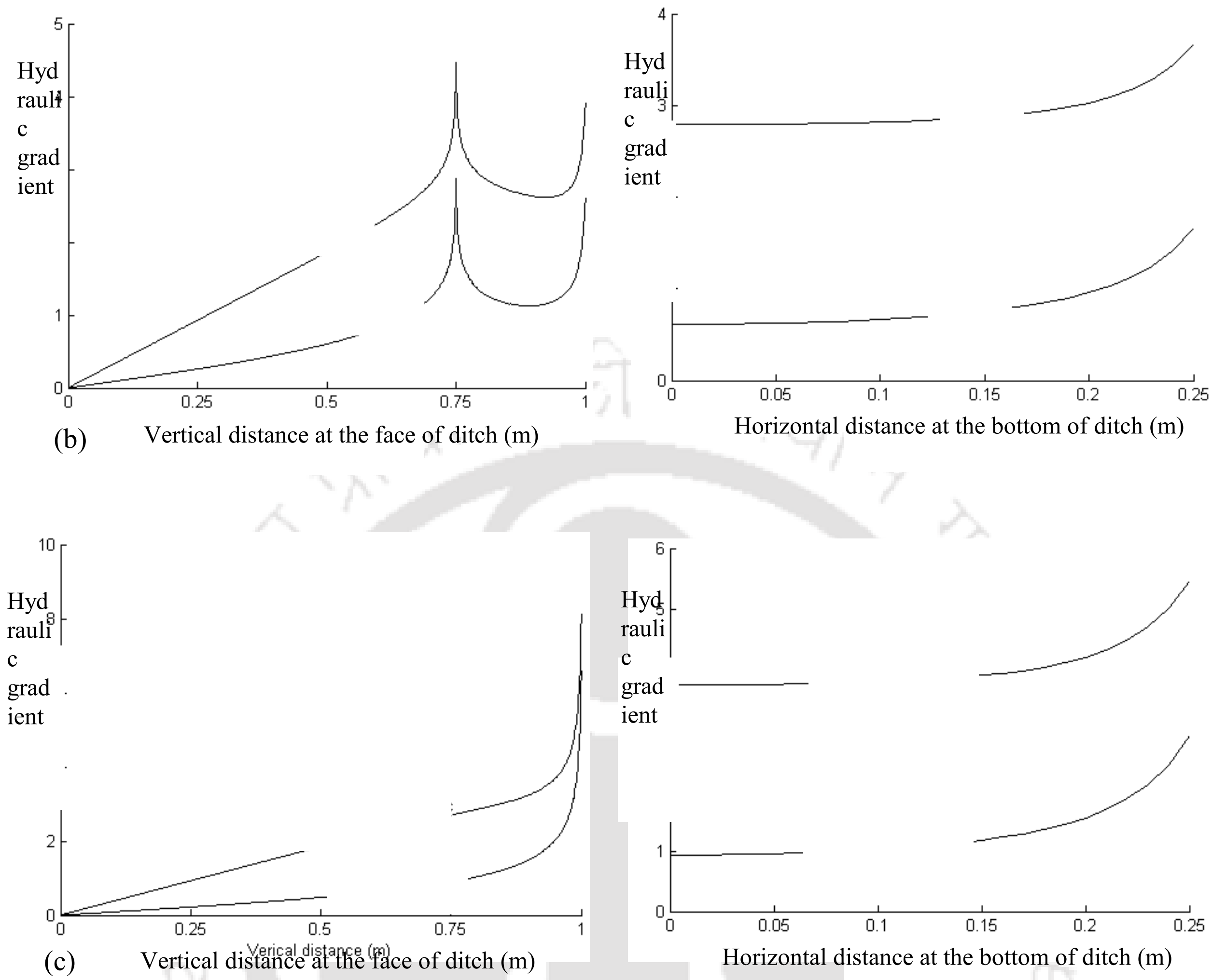
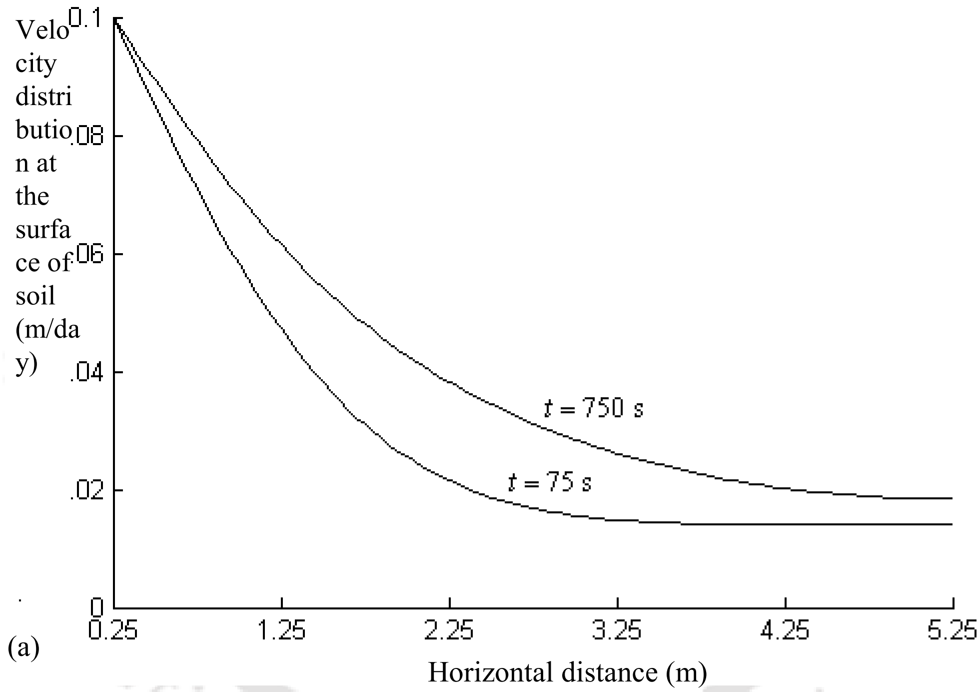


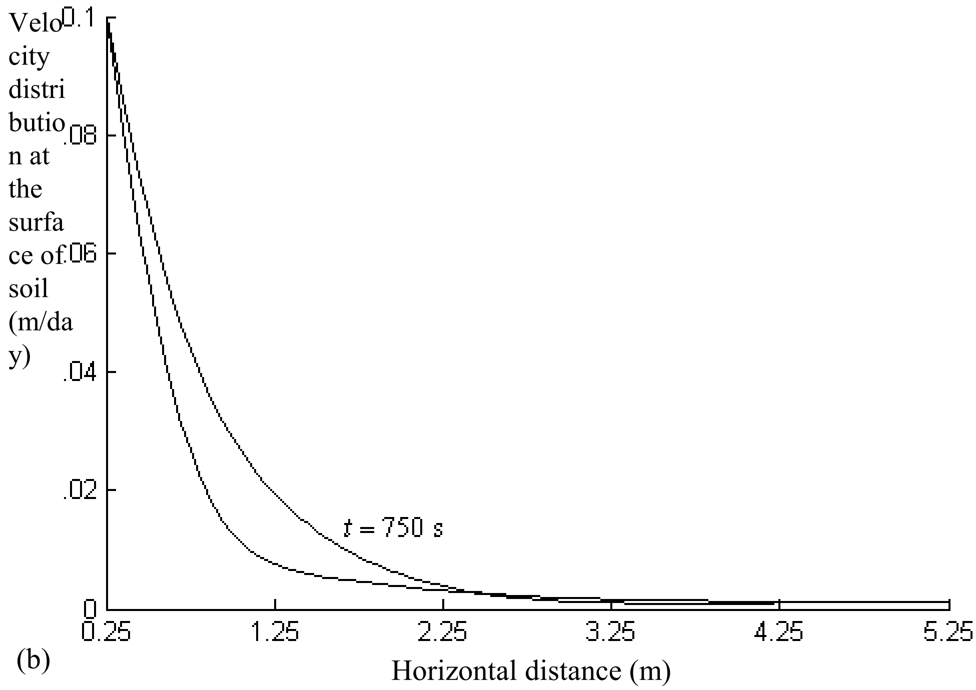
Fig. 3.6. Variation of hydraulic gradient at two different times at the face and bottom of a ditch when the parameters of Fig. 3.1 are taken as

and (a) (b) and (c)

From Figs. 3.7(a), 3.7(b) and 3.7(c), we observe that the velocity distribution at the surface of a uniformly ponded ditch drainage scenario is pretty uneven with the velocity field falling rapidly as one moves away from the ditches. This has been observed to be true for both isotropic as well as anisotropic soils and also for both small as well as large times of simulation of a uniformly ponded drainage system. This finding is in concordant with the observations of Kirkham (1965), Youngs (1995), Barua and Tiwari (1995), Youngs and Leeds-Harrison (2000), Mirjat and Rose (2009), (2009), Chahar and Vadodaria (2008a, 2008b, 2012), Barua and Alam (2013) and others who also observed that the velocity field at the surface of a ponded ditch drainage system rapidly decreases with the increase of distance from the centre of the ditches. However, from Figs. 3.7(a) and 3.7(b), we also see that, considering all other factors to remain the same, an increase in the anisotropy ratio of a soil column has an improving effect and a decrease of anisotropy ratio of a soil has a deteriorating affect, on the uniformity of surface velocity distribution of a uniformly ponded ditch drainage system. As most of the natural soils have a tendency to exhibit a higher water transmitting capacity along the bedding planes in comparison to that across the planes, the intrinsic anisotropy ratio of most soils thus, in general, has a positive influence on the uniformity of distribution of the vertical velocities at the surface of a

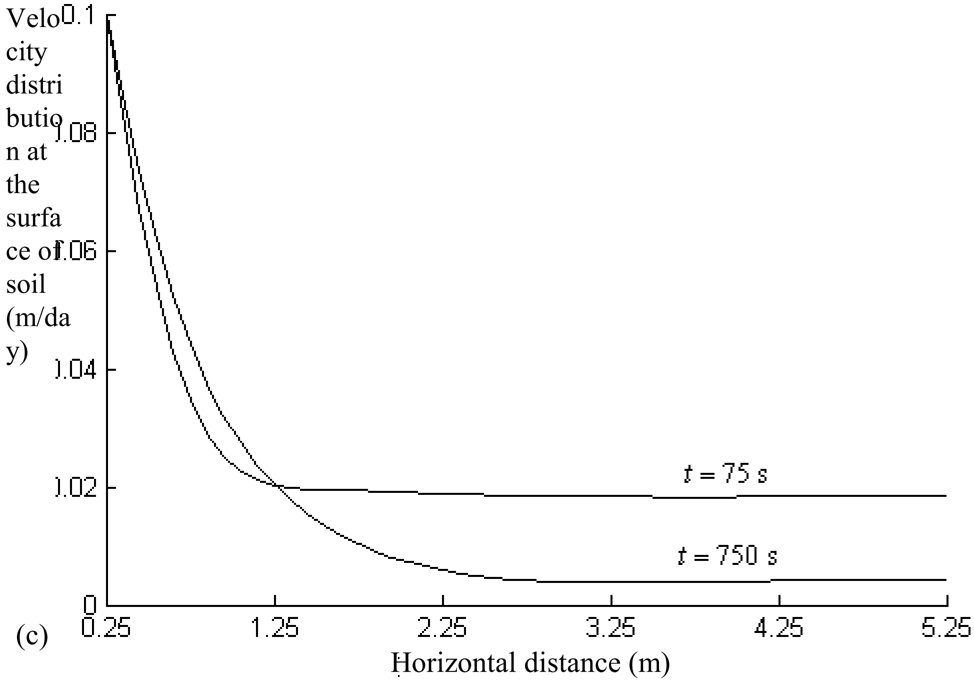


(a)



Velocity distribution at the surface of soil (m/day)
0.1
.08
.06
.04
.02
0
t = 750 s
0.25
1.25
2.25
3.25
4.25
5.25
Horizontal distance (m)

(b)



(c)

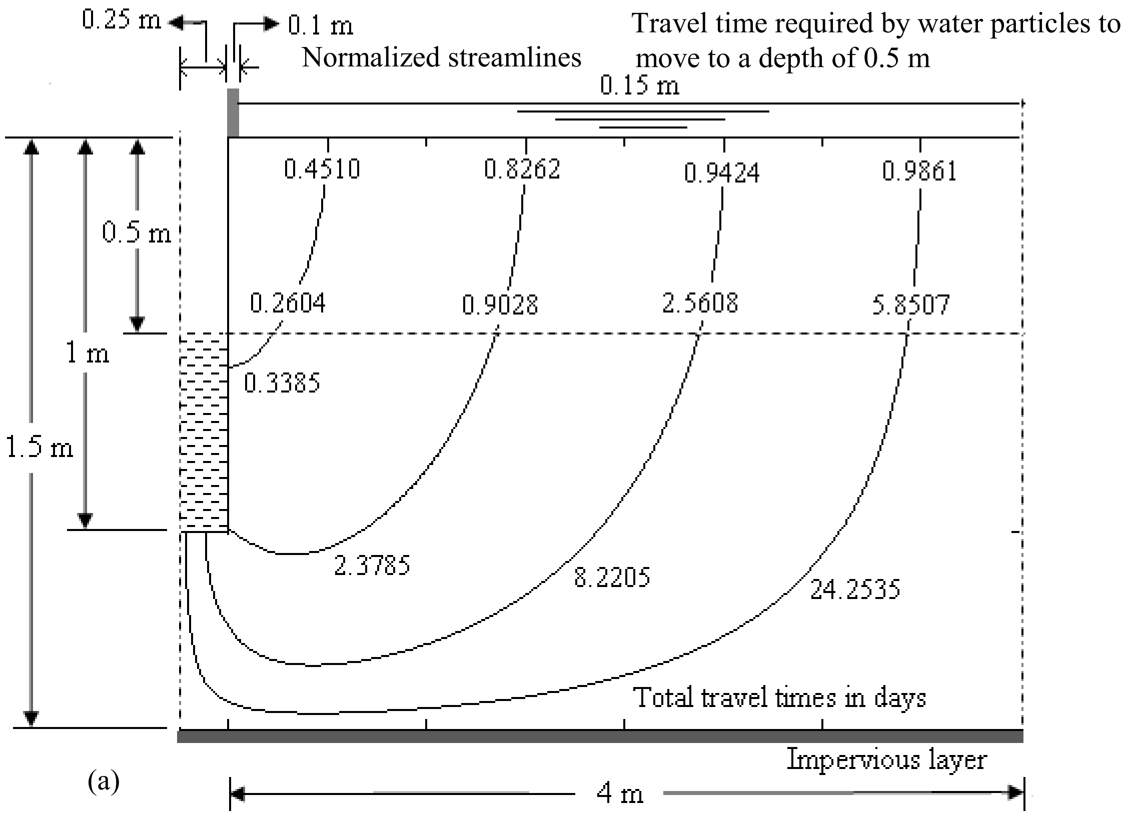
Fig. 3.7. Vertical velocity distribution at the surface of the soil at two different times when the parameters of the flow problem of Fig. 3.1 are taken as

and (a) (b)

and (c)

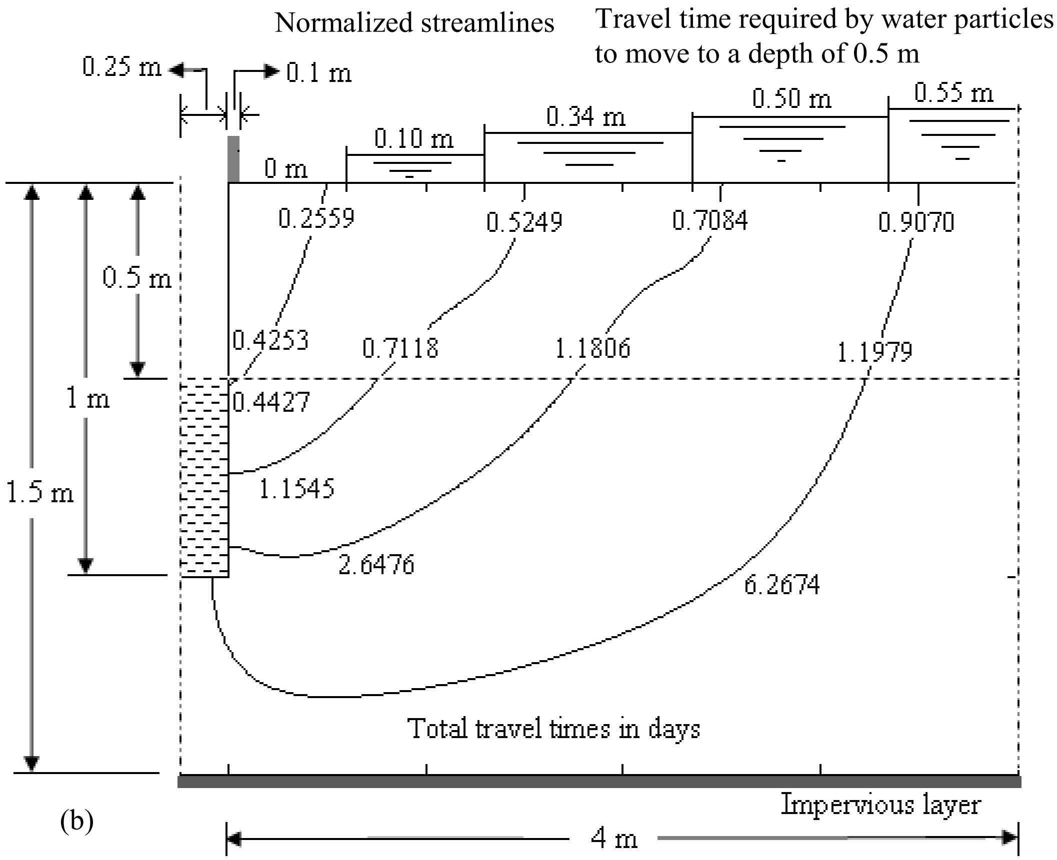
uniformly ponded ditch drainage system. From Figs. 3.7(a) and 3.7(c), it can further be observed that magnitude of the ponding depth at the surface of the soil of a ponded ditch drainage scenario also plays an important part in deciding the velocity distribution (and hence the time varying top discharge function as well) at the surface of the soil; a change of the ponding depth from 0 m in the flow situation of Fig. 3.7(a) to that of 0.10 m by keeping all the other parameters of the flow problem same as before (Fig. 3.7(c)), has resulted in shifting upward the velocity profiles corresponding to both 75 and 750 seconds of simulation of the system with the 75 second profile showing relatively higher velocities at distances away from the ditches as compared to the 750 second profile. This means that, considering all the other variables to remain the same, an increase in the depth of ponding in a partially ponded drainage system may result not only in increase of the top discharges corresponding to different times of running of the system but may also result in considerable lifting of the tails of the velocity profiles, particularly for velocity profiles being traced for early times of operation of the system.

From Figs. 3.8(a) and 3.8(b), it is obvious that the introduction of a variable ponding field of the nature as shown for the chosen ditch drainage scenario has improved considerably the uniformity of the streamline distribution at the surface of the soil as compared to the situation when the surface of the soil for such a flow situation is being subjected to only a uniform ponding depth. Thus, whereas about 83% of the total flow to a ditch for the uniform ponding scenario of Fig. 3.8(a) is being contributed from a surficial distance of only 1.5 m on either sides of the ditch, the corresponding figure, when the flow situation of Fig. 3.8(a) is subjected to a variable ponding field of the type as shown Fig 3.8(b), is about 52% only. Further, as can be observed in these figures, the imposition of this variable ponding field to the considered flow geometry is also making the travel times of water particles originating from the surface of the soil to the ditches relatively more uniform as compared to the situation when the considered drainage situation is being subjected to a uniform ponding depth of 0.15 m on the surface of the soil. Thus, whereas water particles starting from the surface of the soil at distances of 0.5 m, 1.5 m, 2.5 m and 3.5 m, respectively from the vertical faces of the ditch and traversing on the normalized streamlines 0.4510, 0.8262, 0.9424 and 0.9861 are taking 0.2604, 0.9028, 2.5608 and 5.8507 days, respectively to move to a vertical distance of 0.5 m from the surface of the soil for the uniformly ponded flow situation of Fig. 3.8(a), the corresponding figures for the water particles moving on the normalized streamlines 0.2559, 0.5249, 0.7084 and 0.9070 are now only 0.4253, 0.7118, 1.1806 and 1.1979 days, respectively, when the flow geometry of Fig. 3.8(b) is being imposed with a variable ponding field of ponding depth variations on the surface as shown. We would also like to point out here that the steady discharge rate for the uniformly ponded flow situation of Fig. 3.8(a) is turning out to be 1.8038 m^3/day/m and that for the variable ponded situation of Fig. 3.8(b) to be 1.5804 m^3/day/m, a value which is actually less than that of the uniform ponded situation. Hence, it can be concluded that, in comparison to a uniform ponded ditch drainage system, a properly worked out variable ditch ponding drainage system for a concerned drainage situation may result in considerable improvement not only on the distribution of the streamlines on the drainage space but on the uniformity of travel times of water particles as well from their sources of origin at the surface of the soil to a desired horizontal plane below the surface of the soil or to the recipient ditches; further, the water needed for reclaiming a salt affected soil vide a variable ponding ditch drainage system may actually turn out to be less than that needed for reclaiming the same soil vide a uniform ponding ditch drainage system even though the former


0.25 m
0.1 m
Normalized streamlines
Travel time required by water particles to move to a depth of 0.5 m
0.15 m
0.4510
0.8262
0.9424
0.9861
0.5 m
1 m
1.5 m
0.2604
0.9028
2.5608
5.8507
0.3385
2.3785
8.2205
24.2535
Total travel times in days
Impervious layer
4 m

(a)

(b)



(b)

Fig. 3.8. Travel times for water particles (in days) starting from the surface of the soil to the ditches when the parameters of Fig. 3.1 are taken as

and (a) and (b)

system, as has been just shown, may lead to a much better and cheaper cleaning of the salt profile as compared to the latter system of reclaiming the soil utilizing a uniform ponded ditch drainage system.

The analytical model proposed here can also be used to work out the upper limit of fall of water level of a waterlogged soil in a desired duration due to a partially penetrating subsurface ditch drainage system. For example, if we consider the flow situation of Fig. 3.8(a), we find the volume of water seeping from the surface of the soil in between two adjacent ditches at the end of the first 1 hour to be 0.0774 m^3 per unit length of the ditches and at the end of the first 5 hours to be 0.3811 m^3 per unit length of the ditches. If we now allow the water level to fall, the depth of fall then within this time interval would be 9.918 mm at the end of 1 hour and 48.863 mm at the end of 5 hours. It is to be noted that these are upper limits of fall of the water level since, the volumes which are being used to determine these falls have been estimated based on the assumption that the ponded water level of 0.15 is a constant at all times of simulation of the ditch drainage system. In reality, if the waterlogged field is not being fed by any external source of water, the ponded water level on the surface of the soil would go on decreasing with time and as such the head would then be not a constant one but would keep on decreasing with time.

3.4 Conclusions

An analytical solution has been proposed for predicting transient seepage into an array of equally spaced ditch drains partially penetrating a homogeneous and anisotropic soil underlain by an impervious barrier, the drains being fed by a variable ponding field at the surface of the soil. The soil has been assumed to be of finite thickness, the surface of the soil to be horizontal and of infinite extent and the drains have been assumed to be separated from each other by a finite distance. The solution can account for anisotropy of the soil, partial penetration of the drains and finite bottom width of the ditches as well; thus, it is of a pretty general nature. The assumption of infinitely long parallel drains in an infinitely large horizontal field is transforming what actually is a three-dimensional problem to that of a two-dimensional one. The solution to the boundary value problem has been attempted by carrying out an appropriate domain discretization of the flow domain and then solving the governing equation in each of the sub-domains, taking due care, at the same time, to see that the necessary boundary and interfacial conditions pertaining to each of the sub-domains have also been concurrently satisfied. The separation of variable method in association with a right mix of single and double Fourier series have been utilized to obtain the necessary expressions of the hydraulic head function in each of the sub-domains. The validity of the proposed solution has been confirmed by comparing the discharge values obtained from it for a specific configuration of the problem at two time steps for large penetration depths with the discharge values obtained from the existing fully penetrating analytical solution of the problem. A MODFLOW check of the proposed solution has also been performed for a particular configuration of the problem.

The study reflects that the time taken by a partially penetrating ponded ditch drainage system in a homogenous and anisotropic soil to go to steady state may be considerable if the directional conductivities of the soil is low and the specific storage and anisotropy ratio of the soil is high. This is all the more true if the soil is of a large thickness and the depth of penetration of the drains is relatively high as scaled with respect to the thickness of the soil. The width of the ditch drains receiving water from a ponded field does not seem to have much of an influence on the drain discharge as well as on the discharge taking place through the surface of the ponded field but the level of water in the ditches has been observed to have a strong influence on the surface as well as on the side and bottom discharges of the ditches. The study clearly demonstrates that flow to a ditch drainage system from a uniformly ponded field is mostly confined to areas close to the drains and that considerable uniformity in the movement of water in the drainage space, both in terms of quantity of flow and in water particle travel times from the surface of the soil to a recipient ditch or a desired horizontal plane below the surface of the soil (say, up to the root zone of a crop), can be brought about by introducing a proper ponding field over the surface of the soil specific to a drainage situation. Thus, leaching of a salt affected soil vide a variable ponding field ditch drainage system may result in considerable saving of both water and time of leaching of a soil profile as compared to a uniform ponded ditch drainage system.

The study also clearly shows that the exit gradients at the boundaries of a ditch are very sensitive to the time of simulation of a ditch drainage system as well as to the position of the water level in the ditch. The exit gradients are found to be higher for early times of simulation of the system and are also observed to vary inversely with the level of water in the ditches. Thus, the possibility of breaching of the banks of a stream/ditch due to sudden lowering of water level in the stream/ditch than the surrounding water table and the ensuing subsurface seepage to the stream/ditch because of it, is more likely to occur at the early transient stage of movement of the groundwater to the stream/ditch, particularly at points where the water level in the stream/ditch touches the soil surface. The analytical model developed here can also be utilized to design a network of subsurface ditch drains for lowering the level of flood water in an area by a desired amount in a specific time. Thus, the ditch drainage model presented here, apart from being made use of in designing ditch drains for reclaiming a salt affected soil, can also be used to design drains for reclaiming a flooded field as well within a desired time.

3.5 List of Notations

The following notation are used in this chapter

= constants with p = 1,2,3,...., q = *1,2,3,....,* r = 1,2,3,...., n = *1,2,3,....,* w = 1,2,3,...., = 1,2,3,...., = 1,2,3,...., = 1,2,3,...., and = 1,2,3,...., ;

semiwidth of ditch of Fig. 3.1 [L];

semiwidth of ditch in transform domain [L];

depth up to impermeable layer of Fig. 3.1 as measured from the water table [L];

depth of water in the ditch of Fig. 3.1 as measured from the water table [L];

depth of penetration of the partially penetrating ditch of Fig. 3.1 as measured from the water table [L];

horizontal hydraulic conductivity of soil [LT^{-1}];

vertical hydraulic conductivity of soil [LT^{-1}];

anisotropy ratio of the soil (dimensionless);

number of terms to be summed in the infinite series solutions, 1, 2, 3, …

with n = 1,2,3,...., ;

with p = 1,2,3,...., ;

with q = 1,2,3,...., ;

with r = 1,2,3,...., ;

with w = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

number of divisions of the ponding surface at the top of the soil of Fig 3.1;

discharge through the bottom of the partially penetrating ditch of Fig. 3.1

discharge through the side face of the partially penetrating ditch of Fig. 3.1

discharge through the top soil surface of the partially penetrating ditch of Fig. 3.1

semispacing between any two adjacent ditches of Fig. 3.1

semispacing between any two adjacent ditches in transform domain

specific storage of soil

time variable of the flow problem of Fig. 3.1 [T];

horizontal velocity distribution for the layer of Fig. 3.1

vertical velocity distribution for the layer of Fig. 3.1

volume of water seeping through the bottom of the partially penetrating ditch of Fig. 3.1

volume of water seeping through the side face of the partially penetrating ditch of Fig. 3.1

volume of water seeping through the top soil surafce of the partially penetrating ditch of Fig. 3.1

horizontal coordinate of Fig. 3.1 [L];

horizontal coordinate in transform domain [L];

vertical coordinate of Fig. 3.1 [L];

hydraulic head distribution for the layer of Fig. 3.1 [L];

stream function for the layer of Fig. 3.1 [L];

normalized stream function for the layer of Fig. 3.1 (dimensionless);

porosity of flow domain of Fig. 3.1 (dimensionless);

ponding depth for the strip at the soil surface of Fig. 3.1 [L];

width of the ditch banks of Fig. 3.1

width of the ditch banks in the transform domain of Fig. 3.1

;

;

CHAPTER 4

ANALYSIS OF THREE-DIMENSIONAL TRANSIENT SEEPAGE INTO DITCH DRAINS FROM A PONDED FIELD

This chapter is concerned with the development of a few analytical solutions for predicting three-dimensional seepage into a network of ditch drains receiving water from a variably ponded field underlain by an impervious barrier. Two cases are considered for model development, namely, when the ponded soil profile is being surrounded on all its vertical faces by ditch drains having equal or unequal level water level heights in them and when the soil profile is being flanked on three of its sides by ditch drains with again, equal or unequal water level heights in them and the fourth vertical face is a no-flow (Neumann) boundary. The accuracy of the derived solutions for a few simplified situations is checked by comparing them with the relevant analytical and experimental works of others. MODFLOW checks on both the proposed models are also carried out using the PMWIN (Chiang and Kinzelbach 2001) platform. The proposed models are used to study how the water particle travel times to the drains from the surface of the soil are being influenced by the various parameters of the problems like the hydraulic conductivity, length, width and thickness of a soil profile, water level heights in the ditches, ponding depths and the presence or absence of a no-flow boundary on a vertical face of the flow domain. The variation of discharge from the surface of the soil with time for different combinations of hydraulic conductivity, specific storage, thickness of soil profile and the presence or absence of a no-flow boundary on the flow space, is also explored using these models. Further, the derived solutions are also utilized to study the effects of variations of water level heights and surface ponding distributions on dispersion of the stream surfaces corresponding to a few three-dimensional ponded ditch drainage scenarios.

4.1 A Few Solutions of the Three-Dimensional Continuity Equation of Groundwater Flow for a Homogeneous and Anisotropic Soil

For the purpose of having solutions to the three-dimensional ditch drainage problems considered in the chapter, here also, like in the previous two chapters, it is necessary that we first obtain a few general solutions to the three-dimensional continuity equation describing saturated groundwater flow in a homogeneous, anisotropic and compressible aquifer under transient situations. For such a hydro-geological setup, the concerned governing differential equation can be expressed as (Bear 1972)

(4.1)

where is the hydraulic head, and are the hydraulic conductivities of the medium along *x*-, *y*- and *z*- directions, respectively and is the specific storage of the medium. Dividing both sides of Eq. (4.1) by we find that the continuity equation then reduces to

(4.2)

In order that the solutions of Eq. (4.2) become readily amenable for solving the flow problems considered here, we now introduce two transformations along the *x*- and *y*-axes, namely

(4.3)

and

(4.4)

where

(4.5)

and

(4.6)

Applying Eqs. (4.3) and (4.4) to Eq. (4.2) and simplifying, we get

(4.7)

where

(4.8)

Let us take to be a solution of Eq. (4.7) and to be a solution of the steady part of Eq. (4.7), that is

(4.9)

Then, clearly will also be a solution of Eq. (4.7) since

(4.10)

We now obtain a solution of Eq. (4.7) invoking the separation of variable method (Kirkham and Powers 1972). Towards this end, we assume

(4.11)

to be a solution of Eq. (4.7), where and are functions of only and respectively. Applying Eq. (4.11) to Eq. (4.7), we get, after separating the similar variables out

(4.12)

Equating and in Eq. (4.12) – where and are constants – and then solving these equations, we obtain

(4.13)

(4.14)

and

(4.15)

where and are any arbitrary constants. Equating each term on the left hand side of Eq. (4.12) to the constants and will also result in an another equation, namely

(4.16)

where

Naturally, the solution of Eq. (4.16) is

(4.17)

where is any arbitrary constant. Thus, an expression for can be worked out by substituting Eqs. (4.13), (4.14), (4.15) and (4.17) in (4.11); the resultant expression turns out to be

(4.18)

where is any arbitrary constant. Since the sum of solutions of a differential equation is also its solution, an another solution of Eq. (4.7) can be eked out of Eq. (4.18) by simply summing the constants appearing in it; thus, we find

(4.19)

to be also a solution of Eq. (4.7), where are any constants, and are summation indices and and are any positive integers.

If we now assume the solution of Eq. (4.9) as

(4.20)

– where and are functions of and only – and apply the same to Eq. (4.9), we get, after separating the like variables out

(4.21)

If the second and third terms on the left hand side of Eq. (4.21) are equated to and – where and are the arbitrator constants – and solve the differential equations originating from them, we then get and as

(4.22)

and

(4.23)

where and are any arbitrary constants. Also, when the last two terms of Eq. (4.21) are equated to the constants and Eq. (4.21) will then become

(4.24)

the solution of which can be expressed as

(4.25)

Applying now Eqs. (4.22), (4.23) and (4.25) to Eq. (4.20), we get

(4.26)

where is any arbitrary constants. Also, by summing the constants appearing in Eq. (4.26), an another solution of Eq. (4.9) can be worked out as

(4.27)

where is any constants, and are summation indices and and are any positive integers.

In a similar way, by equating and and and next by we find the alternate solutions of as

(4.28)

and

(4.29)

respectively. Thus, we have from Eqs. (4.19), (4.27), (4.28) and (4.29), a few general solutions of Eq. (4.7) as

(4.30)

(4.31)

and

(4.32)

Equations (4.30), (4.31) and (4.32) will now be utilized to solve the flow problems considered in this chapter.

4.2 Mathematical Formulation and Solution

4.2.1 Case 1: Ponded field flanked on four of its vertical faces by ditch drains

The geometry of the ponded drainage problem considered for study is as shown in Fig. 4.1, where, as can be seen, a finitely sized horizontal ponded field of surface area is being drained by four ditch drains placed on the four vertical sides of the soil column. The thickness of the soil is taken as *h* and the soil is assumed to be overlying an impermeable barrier. The water level heights on the North, South, East and West boundary drains are taken as and respectively, all these distances being measured from the origin *O*, as can be observed in the sectional views of the flow problem. The ponding pyramid of depths at the surface of the soil is being introduced by constructing ponding strips with the help of bunds (Fig. 4.1) and the ponding depth at the i^{th} strip is denoted as A coordinate system as shown in

Northern
Boundary

Western
Boundary

(0,0,0)

Eastern
Boundary

Southern
Boundary

Cross-section along

Western
Boundary

Eastern
Boundary

Impervious Layer

Cross-section along

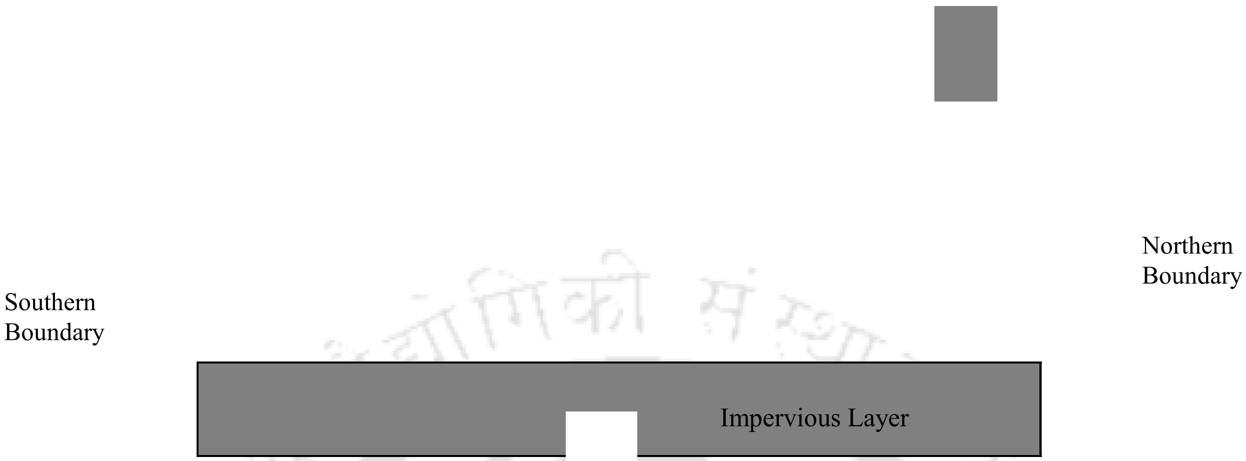


Fig. 4.1. Geometry and sectional views of seep[illegible]om a variably ponded field to ditches flanked on all the four of sides of the field

Fig. 4.1 is being introduced into the flow system with the *x*-axis positive towards the Eastern boundary, *y*-axis positive towards the Northern boundary and *z*-axis positive vertically downward towards the impervious barrier. Ditch bunds of width and along the edges of the ditches are being provided so as to prevent the ponding water from flowing directly to the ditches. The distance of the i^{th} bund, as measured from the origin *O*, in the *x*- and *y*- directions are taken as and respectively. It should be noted that for there will be no inner bunds on the surface of the soil and the ponding depth will then be a uniform one. The directional conductivities of the soil column along the *x*-, *y*- and *z*-directions are taken as and respectively and denotes the specific storage of the soil column. The ponding depths in between the bunds are assumed to be non-changing with time and the soil is assumed to be fully saturated. The constancy of the ponding depths in the ponding strips can be maintained by constantly feeding the surface of the soil with irrigation water; that way, the loss of water to the soil due to infiltration can be balanced at all times. Further, by calling the hydraulic head function as and the time variable as t*, the initial and boundary conditions for the* three-dimensional flow problem as shown in Fig. 4.1 can be expressed as

(I)

(IIa)

(IIb)

(IIIa)

(IIIb)

(IVa)

(IVb)

(Va)

(Vb)

(VI)

(VIIa)

(VIIb)

(VIIc)

(VIId)

(VIIe)

(VIIf)

(VIIg)

(VIIh)

(VIIi)

where

We now propose to obtain an analytical solution of Eq. (4.1) subject to the above initial and boundary conditions. Considering the nature of the initial and boundary conditions above for the chosen flow problem, a solution of Eq. (4.1) in the transformed space, in view of Eq. (4.30), (4.31) and (4.32), can be expressed as

(4.33)

where

(4.34)

(4.35)

(4.36)

(4.37)

(4.38)

(4.39)

(4.40)

(4.41)

(4.42)

(4.43)

(4.44)

(4.45)

(4.46)

(4.47)

(4.48)

(4.49)

and are all integers tending to infinity. A glance on Eq. (4.33) shows that it is satisfying the condition (VI) directly by its very definition; we will now evaluate the coefficients and of Eq. (4.33) using the remaining boundary conditions and the coefficients utilizing the initial condition (I).

Applying conditions (IIIa) and (IIIb) to Eq. (4.33) at we get

Now, running a double Fourier series in the domain covered by and we have

(4.50)

Simplifying the above integrals, we find

(4.51)

Similarly, an application of conditions (IIa) and (IIb) to Eq. (4.33) yields as

(4.52)

Likewise, conditions (Va) and (Vb) and (IVa) and (IVb) can be utilized to evaluate the constants and of Eq. (4.33); the relevant expressions for the same can be expressed as

(4.53)

and

(4.54)

Next, to work out the constants of Eq. (4.33), we apply conditions (VIIa) to (VIIj) to it; thus, we have

where

(4.55)

and

[ ]. (4.56)

Now, running a double Fourier series in the space defined by the intervals and we get an expression for the constants as

(4.57)

Performing the above integrals, we get

(4.58)

Finally, we utilize the initial condition (I) to Eq. (4.33) to determine the constants the pertinent expression for evaluating these constants can then be expressed as

(4.59)

Now, performing a triple Fourier run in the space defined by the intervals

and the constants can then be expressed as

(4.60)

Identifying the first, second, third, fourth and fifth triple integrals of Eq. (4.60) as

and respectively, we get, after simplifying these integrals

(4.61)

where

(4.62)

for

(4.63)

and for

(4.64)

(4.65)

for

(4.66)

and for

(4.67)

(4.68)

for

(4.69)

and for

(4.70)

(4.71)

for

(4.72)

and for

(4.73)

(4.74)

where

(4.75)

for

(4.76)

and for

(4.77)

for

(4.76)

and for

(4.79)

(4.80)

where

(4.81)

for

(4.82)

and for

(4.83)

for

(4.84)

and for

(4.85)

(4.86)

for

(4.87)

and for

(4.88)

for

(4.89)

and for

(4.90)

and

(4.91)

Thus, we have determined all the coefficients of Eq. (4.33) and the considered flow problem of Fig. 4.1 is, hence, solved.

The velocity distributions in the *x*-, *y*- and *z*-directions, and in the drainage space can be next determined by applying the Darcy's law to the hydraulic head expression of Eq. (4.33) after first converting it to the real space by making use of Eqs. (4.3) and (4.4); thus, we have

(4.92)

(4.93)

and

(4.94)

In the same way, the Darcy's law can be applied to evaluate the top discharge function, at the surface of the soil; thus, can be expressed as

(4.95)

Naturally, to obtain the total discharge, infiltrating through the surface of the soil at any instant of time *t*, we need to simply put and in the above expression; thus, we have

(4.96)

For better clarity of presentation, the top discharge function can also be normalized in percentage form by dividing by and then multiplying the resultant by 100; denoting such a function as we thus have

(4.97)

We will now establish that diverges if the ponding depth in the first annular strip (Fig. 4.1) is non-zero but and/or are zero at the same time. For that, we substitute of Eq. (4.58) in Eq. (4.96); this gives us an expression which will be having a term like, for as where, and Now, for a particular value of we see that

and is allowed to of the above series tend to 1 when

increase continually, that is

and

If we assume and to approximately reach 1 after

summing up to four terms of the inside summation of the double summation series of above for a particular then, we find

to reduce to a series of the nature

an infinite series, which, as we know, diverges. Since this is true for any chosen value of we,

thus, see that

and, hence, always diverges should is not zero but is zero at the same time. In a similar way, we can also show that and at the same time, also makes to diverge.

Further, the Darcy's law can also be applied to evaluate the time dependent discharges being received through the Northern, Southern, Eastern and Western faces of the ditches; naming these discharges as and respectively, their expressions, thus, can be represented as

(4.98)

(4.99)

(4.100)

and

(4.101)

To get the hydraulic head and discharge expressions for the steady state situation for the considered flow problem, we need to simply allow in their transient expressions to go to infinity; this, will make the exponential terms in these expressions to go to zero, leaving behind only the steady terms. Thus, for the steady state situation, the transient hydraulic head expression of Eq. (4.33) reduces to

(4.102)

where the subscript '*st*' under the parenthesis sign signifies that this hydraulic head expression is valid only for the steady state situation. In the same way, steady state discharge expressions for the top and side discharges can easily be determined from their transient expressions as given by Eqs. (4.96), (4.98), (4.99), (4.100) and (4.101), respectively.

It is worth noting at this point that the volume of water seeping though the surface of the soil or being fed to a drainage ditch during a specific time *T*, can also be easily determined by carrying out a time integral on the discharge expression of interest for the duration Thus, the volume of water seeping through the surface of the soil in time can be calculated as

(4.103)

Evaluating the above integral using Eq. (4.96), we get

(4.104)

Similarly, the volume of water seeping into the North, South, East and West ditches in time can be determined by carrying out time integrals of the relevant discharge expressions for the concerned duration, namely Eqs. (4.98), (4.99), (4.100) and (4.101); the resultant expressions for the same work out as

(4.105)

(4.106)

(4.107)

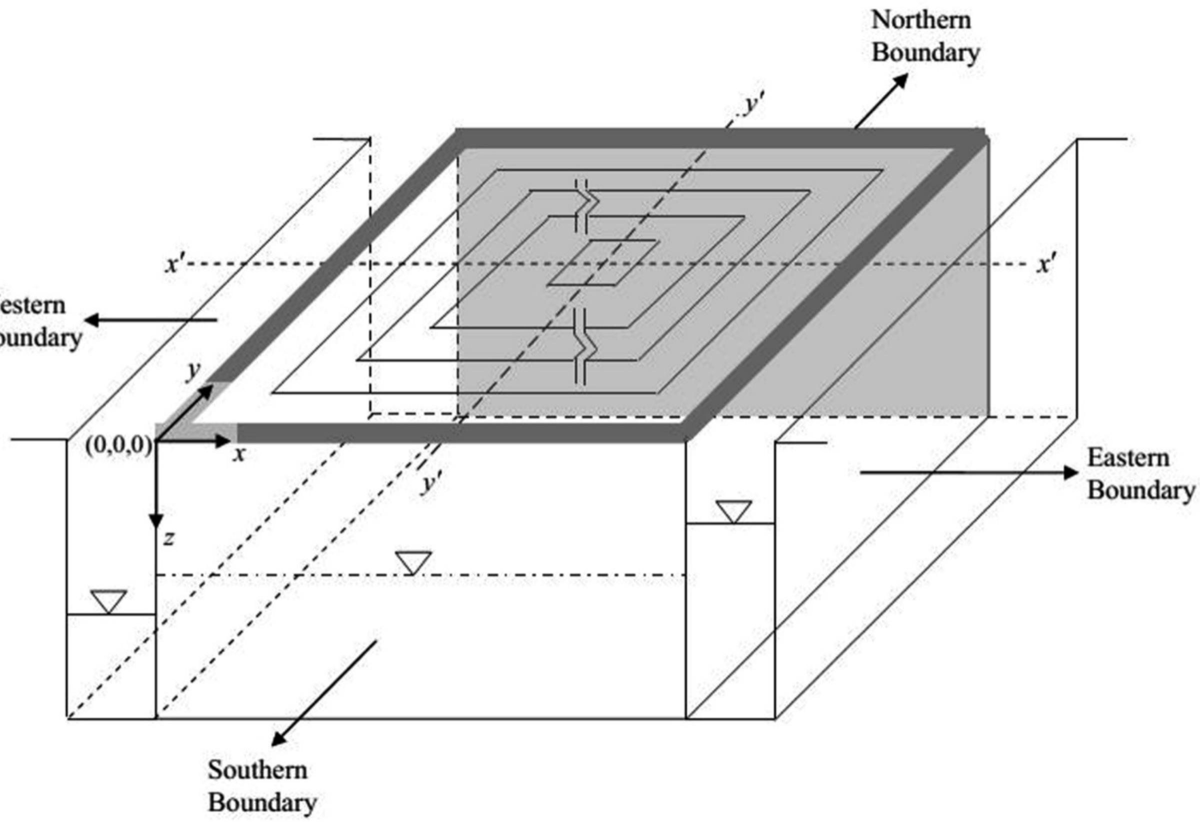
and

(4.108)

We finally end this section by saying that, as has been shown for the top discharge function, all the side discharge and volume expressions as derived above can be shown to diverge for situations where but and/or are zero at the same time.

4.2.2 Case 2: Ponded field flanked on three of its vertical faces by ditch drains and an impervious barrier on the fourth face

The geometry of the ditch drainage problem considered now is as shown below (Fig. 4.2). As may be observed, this problem is similar to the one considered in Case 1 except that the Northern boundary is now not a ditch boundary but an impervious barrier. Hence, except boundary conditions (IIa) and (IIb), all the other conditions of the flow problem of Fig. 4.1 will be applicable for this problem as well.



Cross-section along

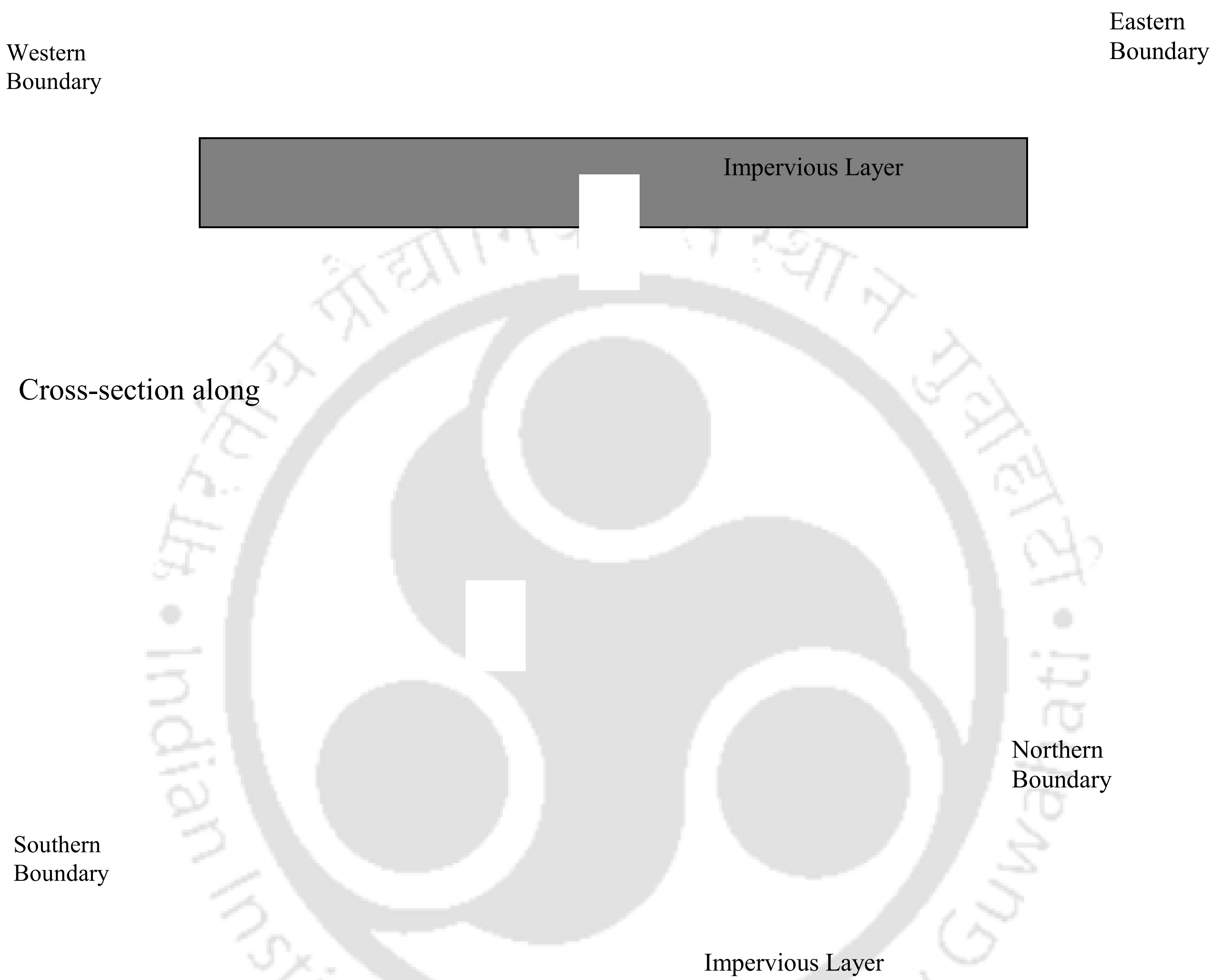


Fig. 4.2. Geometry and sectional views of se from a variably ponded field to ditches flanked on three of its sides, the fourth side being a no-flow (Neumann) boundary

Thus, we have now an impervious barrier at instead of a ditch drain, as can be observed in Fig. 4.2. Hence, boundary conditions (IIa) and (IIb) of the first problem should now be replaced with

(II)

for the problem as shown in Fig. 4.2.

Using exactly the same approach, the hydraulic head function for the flow problem of Fig. (4.2) can be expressed in the computational space as

(4.109)

where the hydraulic head function is now being represented with a bar symbol (i.e., as so as to distinguish it from the head function as taken for the previous problem and

(1.110)

(1.111)

(1.112)

(1.113)

and

(1.114)

and

and carry the same meaning as defined before while obtaining solution to the first problem.

Again, like in the previous problem, the coefficients of the series of Eq. (4.109) can be determined by making use of the conditions (IIIa), (IIIb), (IVa), (IVb), (Va), (Vb), (VIIa), (VIIb), (VIIc), (VIId), (VIIe), (VIIf), (VIIg), (VIIh) and (VIIi), respectively on the hydraulic head expression given by Eq. (4.109); the relevant expressions are now working out as

(1.115)

(1.116)

(1.117)

(1.118)

Further, to determine of Eq. (4.109), the initial condition (I) can next be applied; the required expression for the same then turns out to be

(4.119)

where

(4.120)

for

(4.121)

and for

(4.122)

(4.123)

for

(4.124)

and for

(4.125)

(4.126)

where

(4.127)

for

(4.128)

and for

(4.129)

for

(4.130)

and for

(4.131)

(4.132)

where

(4.133)

for

(4.134)

and for

(4.135)

for

(4.136)

and for

(4.137)

(4.138)

for

(4.139)

and for

(4.140)

for

(4.141)

and for

(4.142)

(4.143)

Also, the relevant velocity and discharge expressions, namely, and

for this case, like in the previous problem, can be worked out by making use of the Darcy's law to the hydraulic head expression as given by Eq. (4.109); by doing so, we get

(4.144)

(4.145)

(4.146)

(4.147)

(1.148)

(1.149)

and

(1.150)

Further, by performing time integrals on the discharge expressions for a desired time interval the volume of water seeping through the surface of the soil as well seeping into the South, East and West ditches within that time can next be easily evaluated; calling these volumes as

and respectively, we thus, have

(4.151)

(1.152)

(1.153)

and

(1.154)

It should be noted that no flow is taking place for the present situation along the Northern boundary as we have now an impervious barrier in it instead of a ditch drainage boundary as considered earlier for the flow problem of Fig. 4.1. We would also like to point out here that, like in the flow problem of Fig. 4.1, all the discharge and volume expressions derived for the flow situation of Fig. 4.2 can also be shown to diverge should the ponding depth, of the first strip is taken as non-zero and and/or are taken as zero at the same time.

4.3 Estimation of Travel Times

Just as we have done in Chapters 2 and 3, a water particle from any point in a three-dimensional ponded drainage space pertaining to a drainage situation can be traced to a recipient ditch by taking resort to the Grove et al.'s (1970) approach as mentioned in Chapter 1, utilizing again the velocity information relevant to the concerned drainage scenario. This procedure also gives the time of travel of a water particle along a traced pathline from its point of start to its point of exit in a ditch. It should, however, be noted that, unlike the two-dimensional flow problems considered in the earlier chapters, the ones considered here deal with three-dimensional flows and hence the displacement of a water particle at a point in the flow spaces of Figs. (4.1) and (4.2) will be governed by three components of the velocity field at the point rather than only the *x*- and *y*- components of the velocity distribution. However, Grove et al.'s (1970) approach can easily be extended to accommodate three-dimensional flow situations as well. For convenience, we are now again providing a brief description of the same for the three-dimensional flow situations. Suppose a particle occupies a position given by the coordinate at time t.

Now, knowing the velocity distribution of the particle at this point at the considered instant, the displacements of the particle at the end of an incremental time interval along the x-, y- *and* *z*-directions can then be calculated by the expressions

(4.155)

(4.156)

and

(4.157)

where is the porosity of the soil. Of course, for Eqs. (4.155), (4.156) and (4.157) to give reasonably accurate results, must be given a small value – smaller the value better will be the accuracy of the mapped pathlines (which will also be streamlines in case of steady state flow) – since it is being assumed that the velocity field is a constant within the neighborhood of the point during the considered time interval. Thus, the new coordinates of the particle at the end of the time interval can be represented as

(4.158)

(4.159)

and

(4.160)

In the same way, the particle can also be traced from to during the next incremental the procedure can be repeated till the particle is fully traced up to the recipient ditch. The methodology not only traces the path of a water particle in a drainage space from its point of start to its point of exit in a drain but can also be used to estimate the time of travel of the water particle in the concerned path. Further, the procedure can also be utilized to trace a stream surface, a stream surface being the locus of infinite number of streamlines being drawn from a continuous line segment (Hultquist 1992; Steward 1998). It is to be noted that the pathlines and the stream surfaces shown in Figs (4.6), (4.7), (4.8), (4.9), (4.10), (4.11), (4.12), (4.13), (4.14), (4.15), (4.19), (4.20), (4.21) and (4.22) have been determined using the method as just mentioned.

4.4 Verification of Proposed Solutions

We now carry out a few checks to determine the validity of our proposed solutions. Towards this end, we first make a comparison of the hydraulic heads as predicted by our models with the corresponding values obtained from an earlier analytical solution to the fully penetrating ponded ditch drainage problem developed utilizing the two-dimensional flow assumption. It is to be noted that the three-dimensional ponded drainage problems considered here should approximately reduce to that of a two-dimensional one in a vertical plane located further away from the Northern and Southern boundaries of Figs 4.1 and 4.2, if *B* in these problems is given a very large value. As mentioned earlier, the ponded ditch drainage problem with the two-dimensional flow situation has been solved by Barua and Alam (2013) and hence for drainage situations where *B* is relatively much larger than that of *L*, the hydraulic heads for a ponded drainage situation in a plane where the three-dimensional affect can be approximately neglected should match closely with the identical values obtained vide Barua and Alam's analytical model. Thinking in this line, a comparison study was being carried out between the hydraulic head predictions as obtained from the proposed solutions with the corresponding values obtained from Barua and Alam's solution for a specific drainage situation of Figs. 4.1 and 4.2 (see Fig. 4.3) in a vertical plane passing midway between the Northern and Southern boundaries of the flow domain (i.e., at a plane at Fig. 4.3 shows the comparison of these models. As can be seen, the hydraulic heads as obtained from the models proposed here are matching closely with the corresponding values as obtained from Barua and Alam's solution for the chosen flow situation, thereby showing that the proposed solutions are correctly developed.

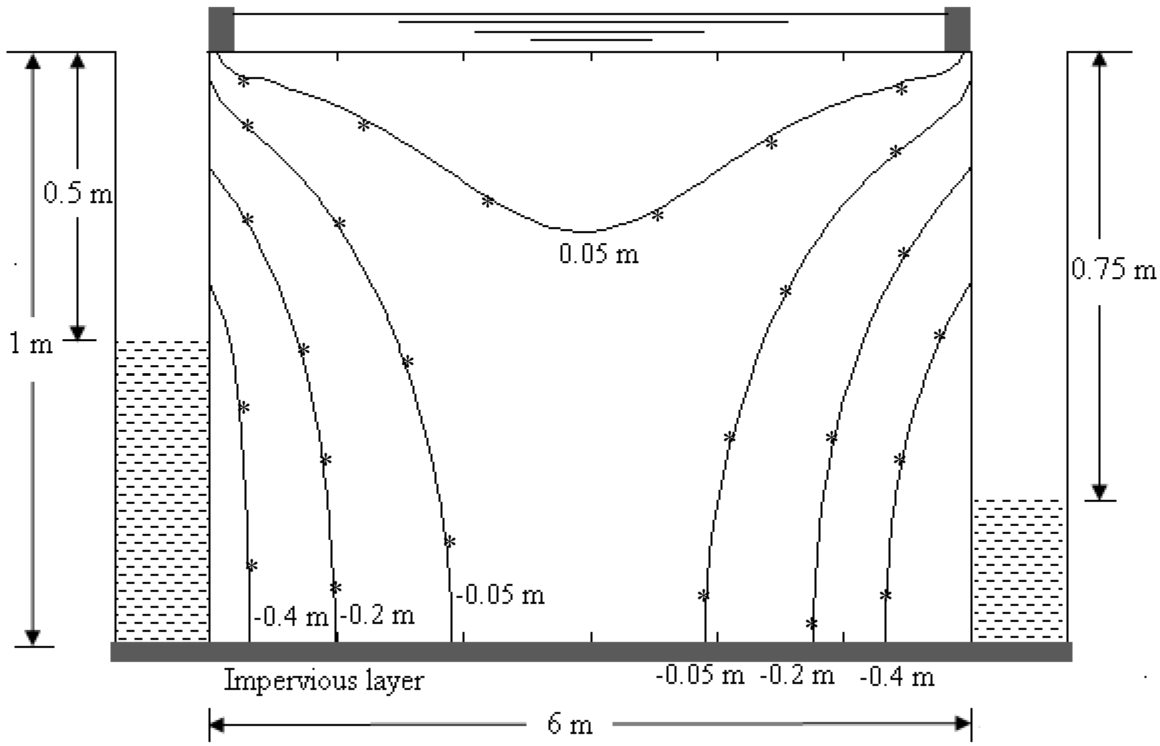
Also, if we now take the parameters of Figs 4.1 as

and and

then determine the ratio for the same, we find it to be

0.723. If now, the same parameters are being applied for the flow situation of Fig. 4.2 (except that will now not be applicable for this situation since the Northern boundary is now not a ditch boundary but a no-flow zone), we then find the

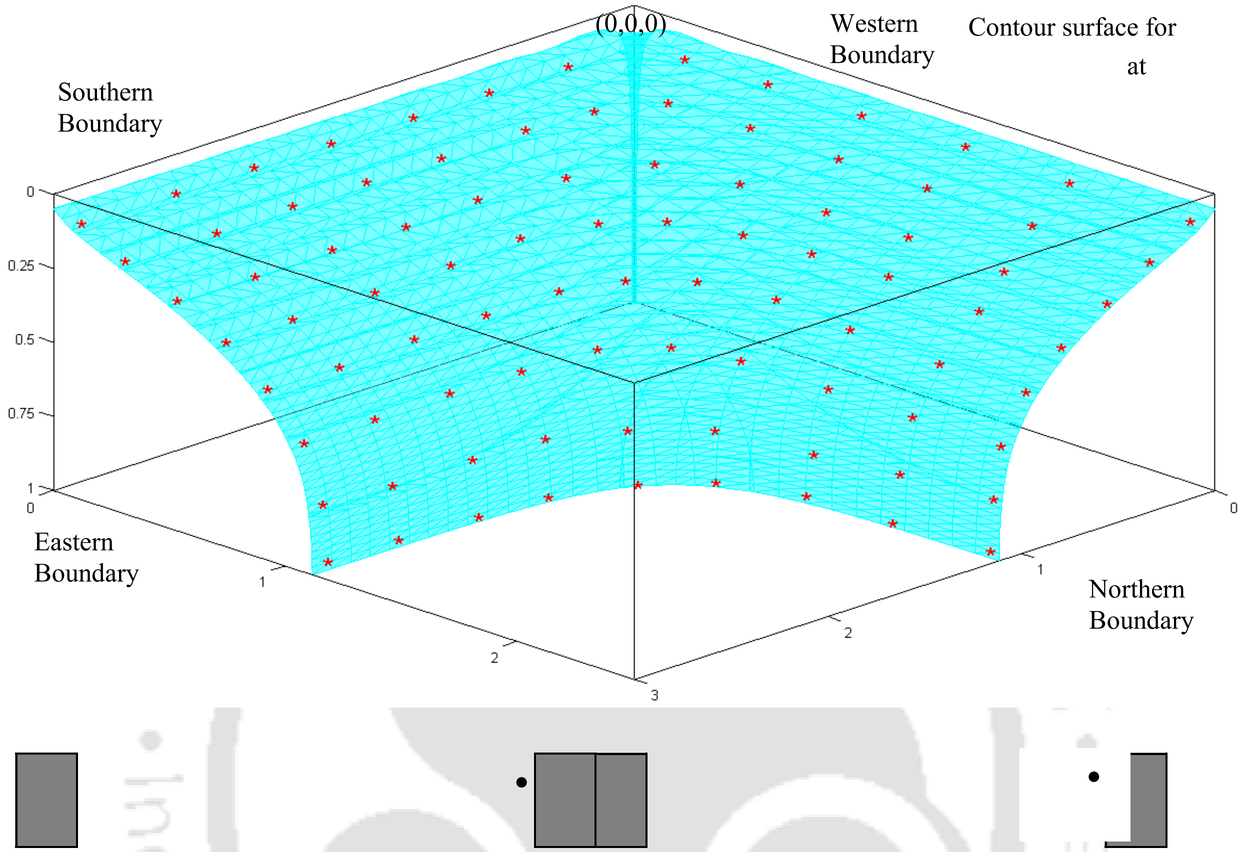
ratio as 0.728.This ratio works out to be 0.743 and 0.742, respectively when calculated using Fukuda's (1957) and Youngs' (1994) analytical solutions, thereby providing us with an another check on the validity of the proposed solutions. Further, Fukuda also found this ratio as 0.720 from his experimental observations. Thus, the close matching of this ratio as obtained from the proposed models with the identical ratio obtained utilizing Fukuda's experimental results can also be treated as an experimental verification of the analytical solutions proposed here.



Transient hydraulic head contour as generated by the proposed analytical solution of Fig. 4.1
Transient hydraulic head contour as generated by the proposed analytical solution of Fig. 4.2
* *Transient hydraulic head contour as generated by the analytical solution of Barua and Alam (2013)*

Fig. 4.3. Comparison of hydraulic head contours as obtained from the proposed analytical solutions of Figs. 4.1 and 4.2 with the corresponding values as obtained from the analytical solution of Barua and Alam (2013) at time and at (i.e., at the mid-plane between the Northern and \Southern boundaries) when the flow parameters of Figs. 4.1 and 4.2 are taken as

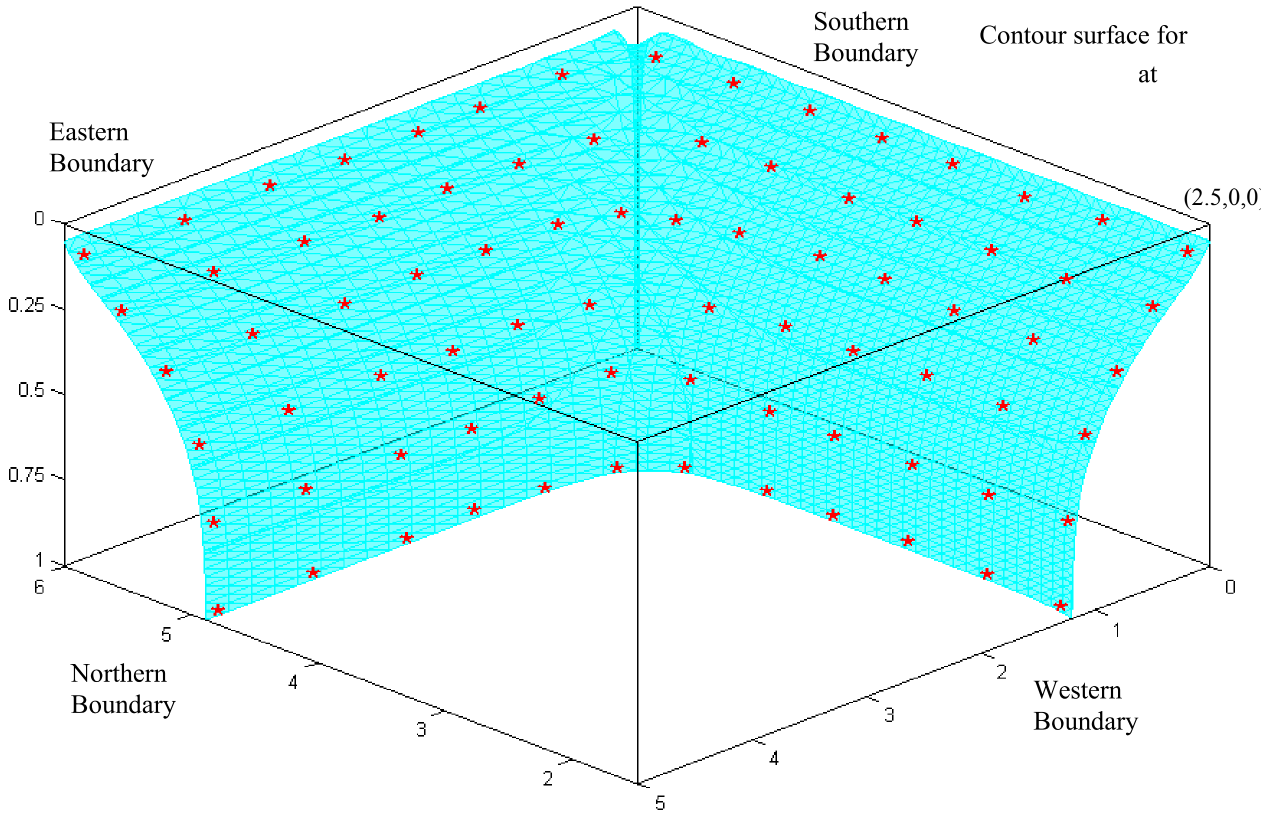
In order to ascertain once again the correctness of the developed analytical models, numerical checks on both of these models were also performed by drawing appropriate numerical models utilizing the Processing MODFLOW (Chiang and Kinzelbach 2001) codes. To simulate first a ponded drainage scenario in the MODFLOW platform conforming to the ditch drainage model considered here under Case 1, a hypothetical drainage situation as shown in Fig. 4.4 was taken for the modeling purpose. The concerned drainage situation where, as may be observed, the separations between the Northern and the Southern and the Eastern and the Western boundaries are 5 m and 6 m, respectively and also where the thickness of the soil is 1 m, was modeled with the help of 102 rows, 122 columns and 22 layers in the MODFLOW environment. Thus, the size of each grid cell taken for modeling was 0.05 m×0.05 m×0.05 m. The bottom impervious layer was simulated by making all the cells of the 22nd layer inactive. A uniform ponding depth of 0.2 m was introduced in the model by assigning a constant value of this magnitude to all the cells of the first layer, except the first and the last two rows and columns of the layer, which were used to represent the ditch banks and the first layer of the ditches. The ditch banks of width 0.05 m each running at the edges of the ditches along the Northern and the Southern boundaries of the flow domain were modeled by treating all the cells falling in the 2nd and the 101th rows in between the 1st and the 122 columns of the first layer as ineffective; similarly, the ditch banks having 0.05 m width each running at the edge of the ditches along the Eastern and the Western boundaries of the flow space were modeled by making all the cells falling in the 2nd and the 121th columns in between the 1st and the 102th rows of the first layer as ineffective. The ditch drain having a water level height of 0.5 m, as measured from the origin *O*, was simulated by assigning a fixed cell value of 0 m to all the cells belonging to the rows of the first column of the first layer, -.05 m to all the cells belonging to the rows of the first column of the second layer and so on up to the 11th layer, after which a constant cell value of -0.5 m was assigned to all the row cells of the first column up to the 21st layer. In the same way, ditch levels of 0.5 m in all the other ditches running on the Western, Northern and Southern boundaries, respectively were modeled. All the active cells of the flow domain were then inputted with the soil properties of and and a transient MODFLOW simulation was performed and the numerically observed hydraulic heads corresponding to a time step for the considered drainage situation compared with the corresponding analytical values obtained from the proposed analytical model of the flow problem of Fig. 4.1. Fig. 4.4 shows such a comparison. From this figure, it is clear that the analytically predicted hydraulic heads are in close conformity with the numerically obtained values, thereby showing once again that the proposed analytical model for the drainage problem of Fig. 4.1 has been correctly developed.



* *Transient hydraulic heads as generated by MODFLOW*
Depth of ponding and height of the ditch bunds are not in scale; all other dimensions are in scale
\# *For better visibility, contour surface is shown for only a portion of the flow space*

Fig. 4.4. Comparison of hydraulic head contour surface as obtained from the proposed analytical solution of the flow problem of Fig. 4.1 with the corresponding MODFLOW generated contours at time when the flow parameters of Fig. 4.1 are taken as

and



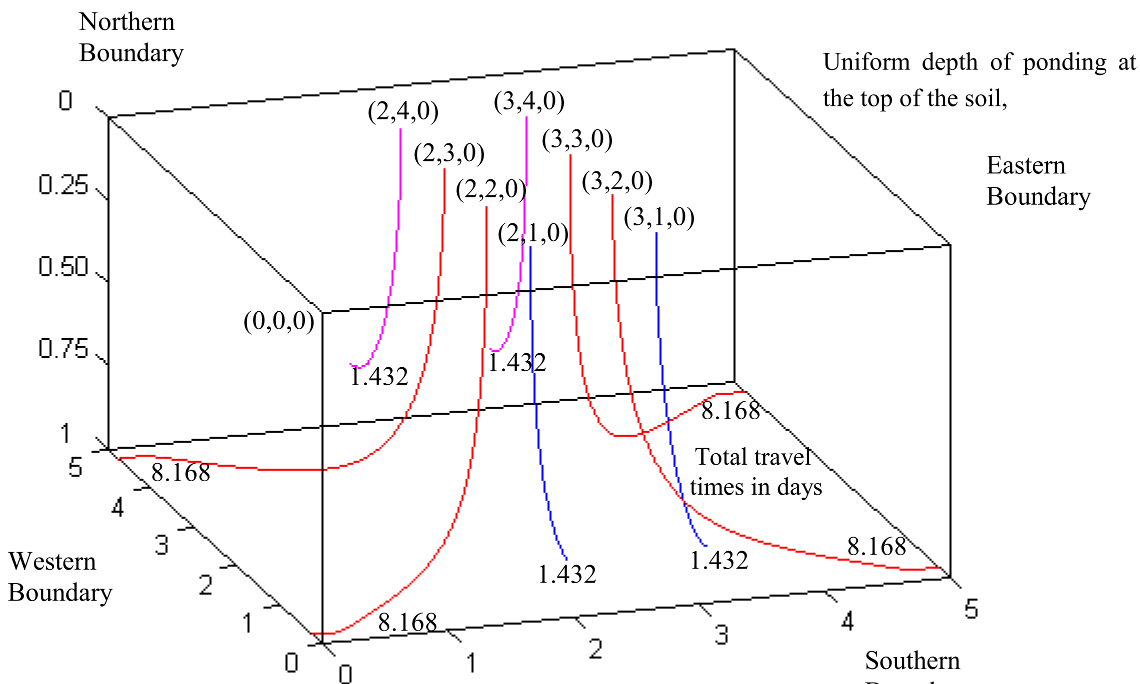
* *Transient hydraulic heads as generated by MODFLOW*
Depth of ponding and height of the ditch bunds are not in scale; all other dimensions are in scale
\# *For better visibility, contour surface is shown for only a portion of the flow space*

Fig. 4.5. Comparison of hydraulic head contour surface as obtained from the proposed analytical solution of the flow problem of Fig. 4.2 with the corresponding MODFLOW generated contours at time when the flow parameters of Fig. 4.2 are taken as

Following exactly a similar procedure, a MODFLOW model for the flow problem of Fig. 4.2 was also developed for the same drainage situation as considered above except that the Northern boundary modeled now is an impervious boundary (i.e., a Neumann boundary) rather than a ditch boundary. The imperviousness of the Northern boundary was realized by making all the cells of the 102th row in between the 1st and the 122th columns and from the 1st to the 21st layers as inactive. Fig. 4.4 shows the comparison of the analytically obtained hydraulic heads at a time step for the considered drainage situation with the corresponding values being generated by MODFLOW; as may be seen, here also a very good agreement is being obtained between the analytical and numerical outputs, thereby proving once again the accuracy of the proposed analytical model for the flow situation of Fig. 4.2.

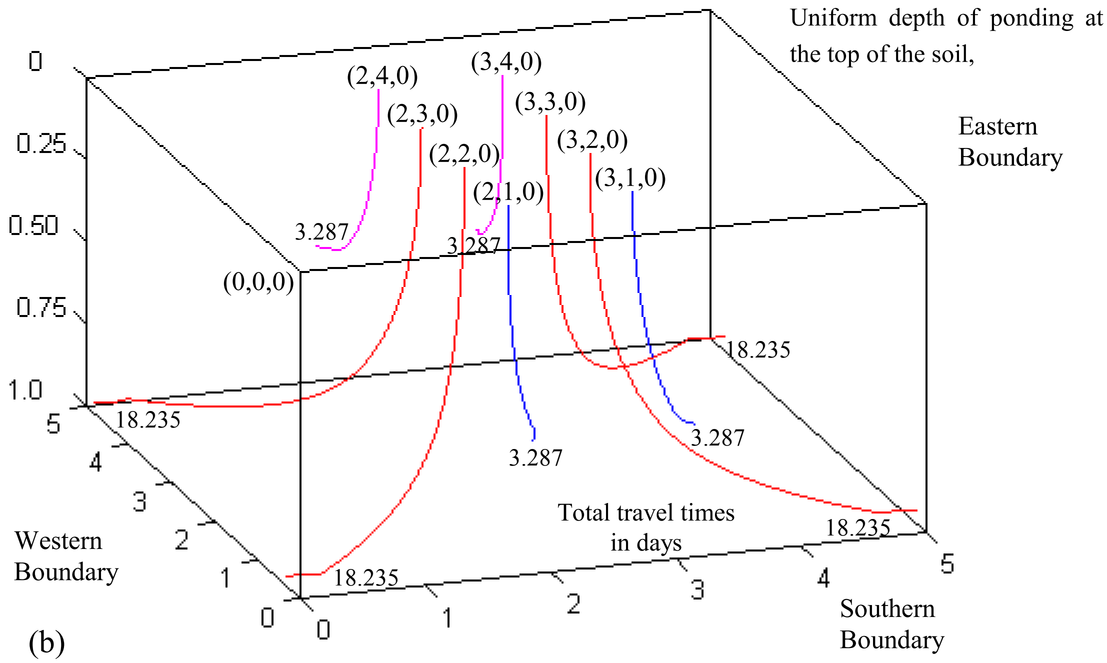
4.5 Discussions

From Figs. 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12, 4.13, 4.14 and 4.15, it is clear that flow to a ditch drainage system from a ponded field of finite and limited size is mostly of a three-dimensional nature, particularly in areas close to the drains. This is observed to be true both for situations where the ponded field is being surrounded on all its vertical sides by ditch drains and where one of its side boundaries is a no-flow zone. It can also be observed from Figs. 4.10 and 4.15 that, even for drainage situations of Figs. 4.1 and 4.2 when two parallel vertical faces of the flow domain is separated from each other by a relatively large distance as compared to the separation distance between the other two faces, three dimensional nature of the pathlines still prevail, mainly again in locations close to the drains. However, from these figures (i.e., Figs. 4.10 and 4.15) we also observe that, in a vertical plane located further away from the longer side boundaries of the drainage situations of Figs. 4.1 and 4.2, flow can roughly be approximated as a two-dimensional one without introducing any appreciable error when measured vis-à-vis a three-dimensional model. It is also interesting to note from Figs. 4.6 and 4.11 that the nature of the Northern boundary alone – i.e., whether a ditch or a no-flow boundary – can lead to a visible difference on the distribution of the pathlines and water particle travel times in a three-dimensional ponded drainage space, particularly in areas located in the immediate vicinity of this boundary. For example, for the drainage situation of Fig. 4.6(a), where the Northern boundary is a ditch drain boundary with a water level of 0.5 m in it (the other drainage parameters being explained in the figure itself), the pathline originating from the coordinate (2,4,0) is exiting the drainage space in the Northern boundary itself with a water particle travel time of 1.432 days in it; however, for the flow situation of Fig. 4.11(a), where the Northern boundary is now a no-flow zone (and hence, of course, no flow line can exit through this boundary) the pathline originating from (2,4,0) is now exiting the Eastern ditch boundary with a water particle travel time of 7.665 days. We would like point out here that the flow situation as shown in Fig. 4.1 can also very well represent subsurface flow to a straight river reach from a flooded field of negligible depth of flood water over the surface of the soil. In fact, Brainard and Gelhar (1991) also performed similar studies using the finite element method for predicting three-dimensional seepage of water into a straight river reach of finite length from a horizontal field receiving a uniform recharge at the water table. Their numerical studies, however, as just mentioned, assume a uniform recharge input at the water table whereas the analytical models proposed here assume a known ponding distribution at the surface of the soil. It can also be seen in Figs. 4.6(a), 4.6(b), 4.11(a) and 4.11(b) that an increase in the vertical conductivity causes not only the pathlines to penetrate relatively much deeper distances in ponded drainage spaces of the types as shown in Figs. 4.1 and 4.2 as compared to situations where the vertical conductivity is relatively less but also brings about a considerable reduction on the water particle travel times along the pathlines as well. The travel times of water particles in a ponded drainage space are also, expectedly, found to decrease with the increase of the ponding head at the surface of the soil, as can be clearly seen in Figs. 4.6(b) and 4.8 and in Figs. 4.11(b) and 4.13. Also, from Figs. 4.6(b) and 4.7, it can be seen that by merely changing the level of water in the ditches, extensive changes in the pathline distribution as well as on the travel times of water particles can be bought about; whereas the pathline originating from the coordinate (2,4,0) is exiting in the Northern boundary for the flow situation of Fig. 4.6(b), mere change of ditch water levels in the North, South, East and West drains of the flow situation of Fig. 4.6(b) from 0.5 m to 0.25 m, 0.5 m, 0.75 m and 0.75 m, respectively has now caused the (2,4,0) pathline to exit through the Eastern boundary (Fig. 4.7). Further, the travel times of water particles on the (2,2,0), (3,2,0), (2,3,0), (3,3,0), (2,4,0) and (3,4,0) pathlines in the flow situation of Fig. 4.7 are also significantly different from that of corresponding pathlines in the flow situation of Fig. 4.6(b). For the drainage situation of Fig. 4.2 also, as can be seen in Figs. 4.11 and 4.12, the water levels of the ditches are found to have a noticeable impact on the distribution of the pathlines and the travel times of water particles on it.



(a)

Northern
Boundary



(b)

Fig. 4.6. Travel times of water particles (in days) on a few pathlines starting from the surface of the soil to the recipient ditches under steady state condition when the parameters of Fig. 4.1 are taken as

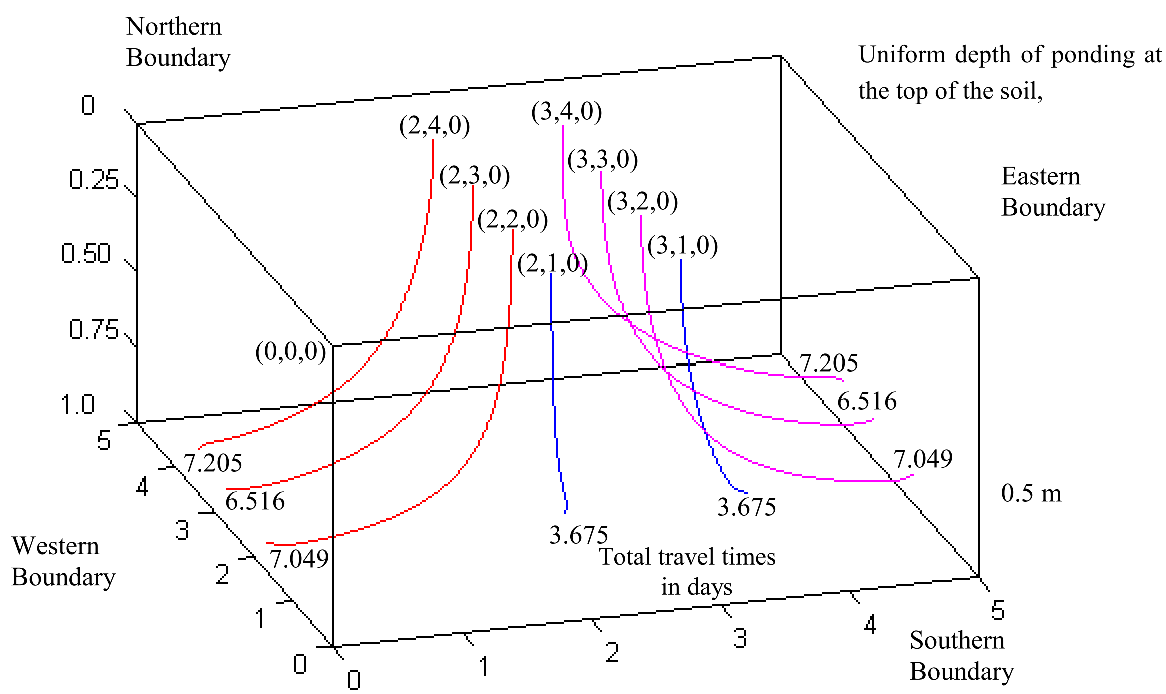


Fig. 4.7. Travel times of water particles (in days) on a few pathlines starting from the surface of the soil to the recipient ditches under steady state condition when the parameters of Fig. 4.1 are taken as

Northern
Boundary

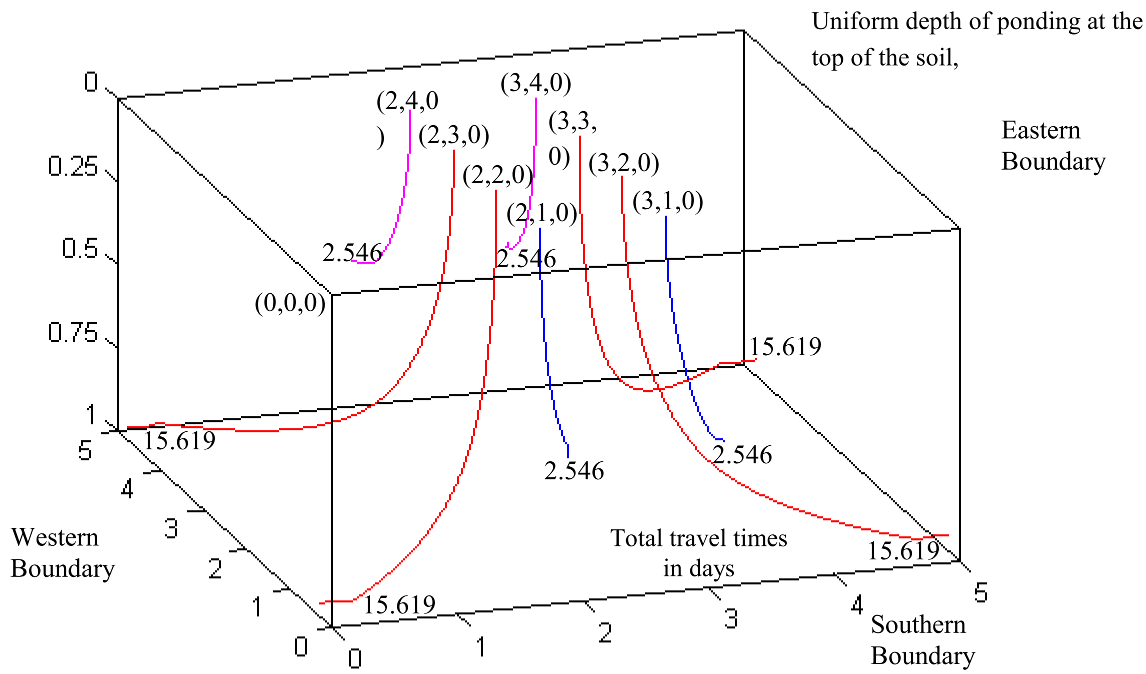


Fig. 4.8. Travel times of water particles (in days) on a few pathlones starting from the surface of the soil to the recipient ditches under the steady state condition when the parameters of Fig. 4.1 are taken as

and

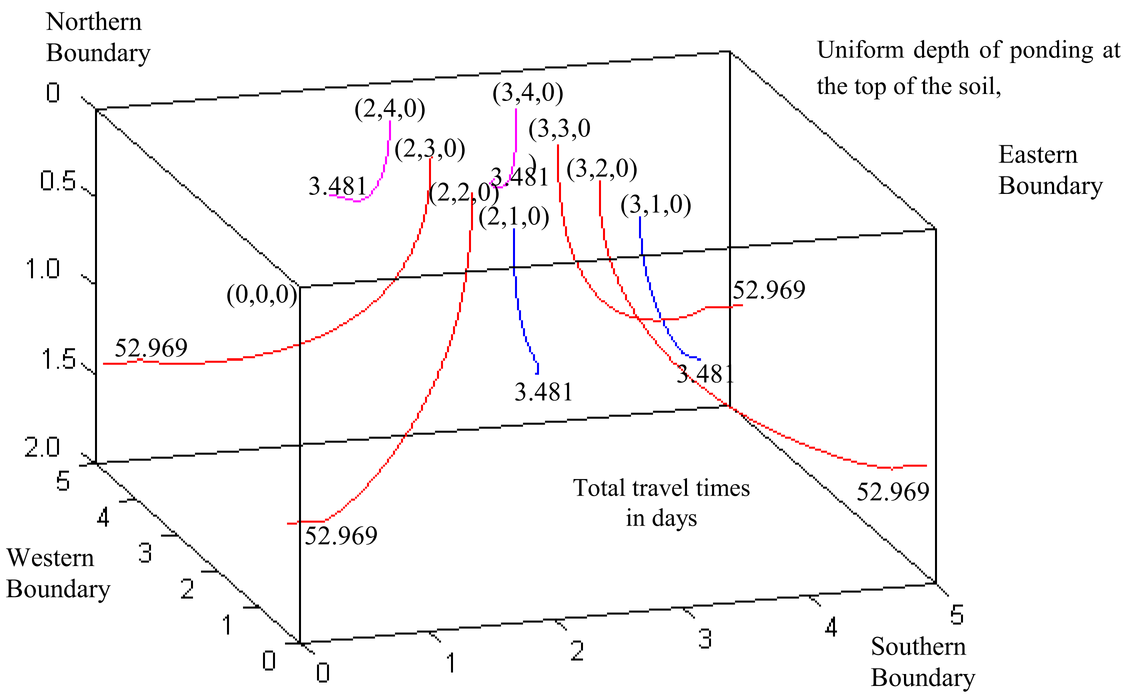


Fig. 4.9. Travel times of water particles (in days) on a few pathlines starting from the surface of the soil to the recipient ditches under steady state condition when the parameters of Fig. 4.1 are taken as

Northern
Boundary

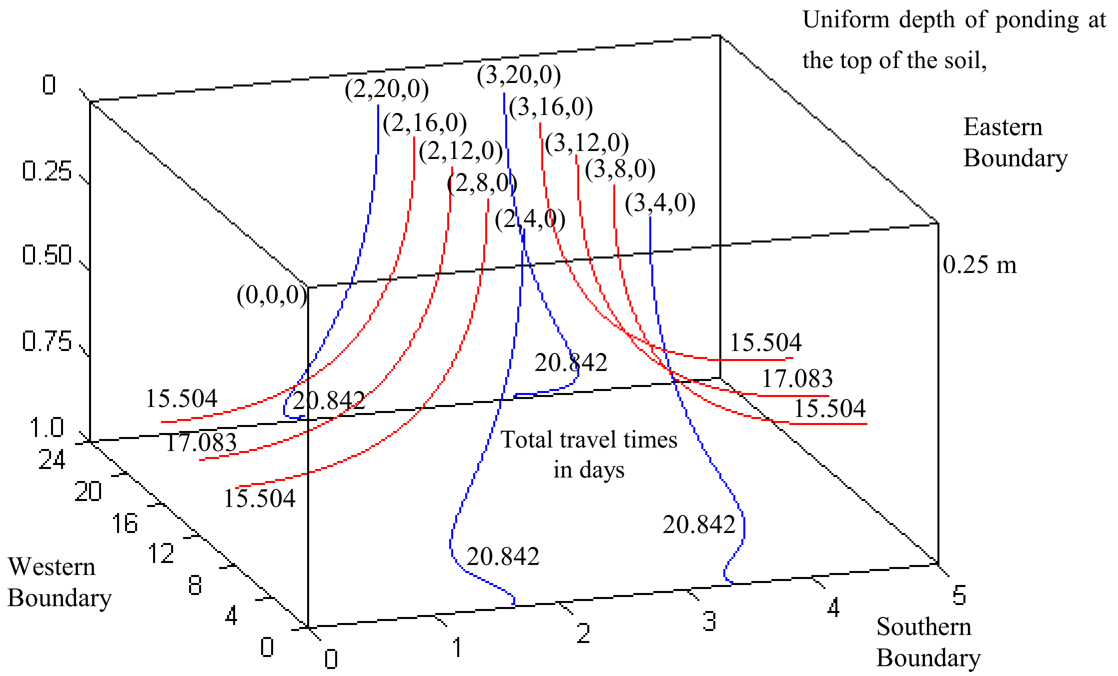
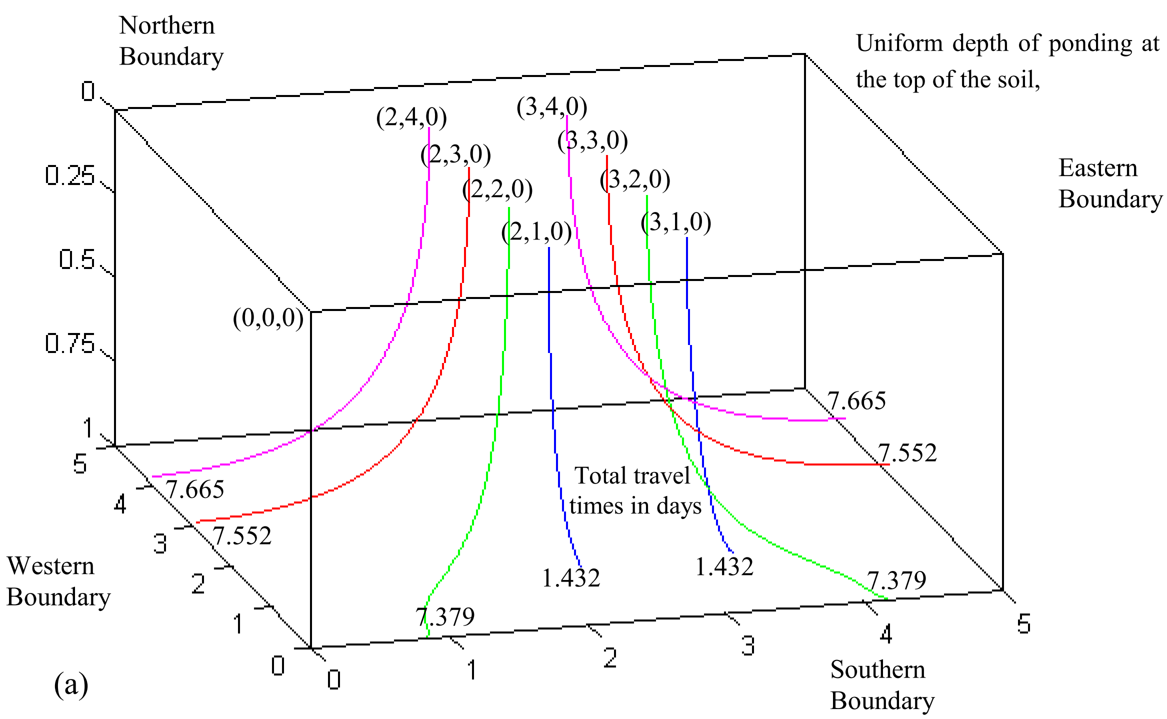
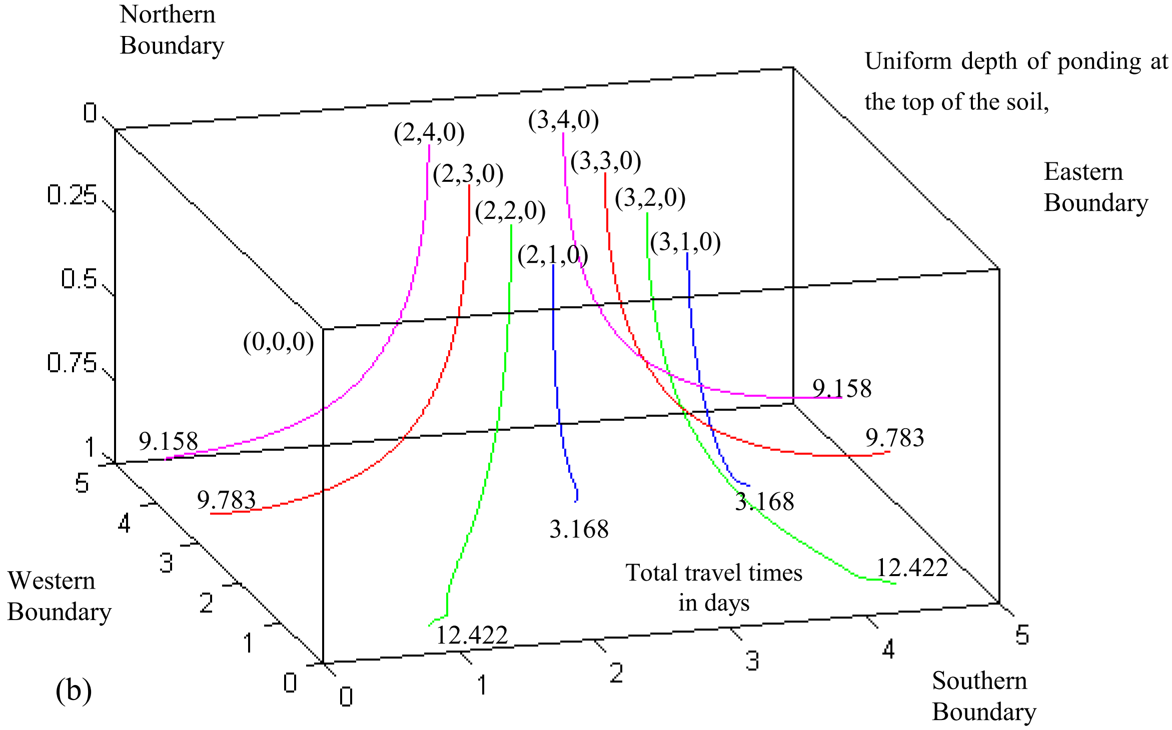


Fig. 4.10. Travel times of water particles (in days) on a few pathlines starting from the surface of the soil to the recipient ditches under steady state condition when the parameters of Fig. 4.1 are taken as



(a)



(b)

Fig. 4.11. Travel times of water particles (in days) on a few pathlines starting from the surface of the soil to the recipient ditches under steady state condition when the parameters of Fig. 4.2 are taken as

(a)

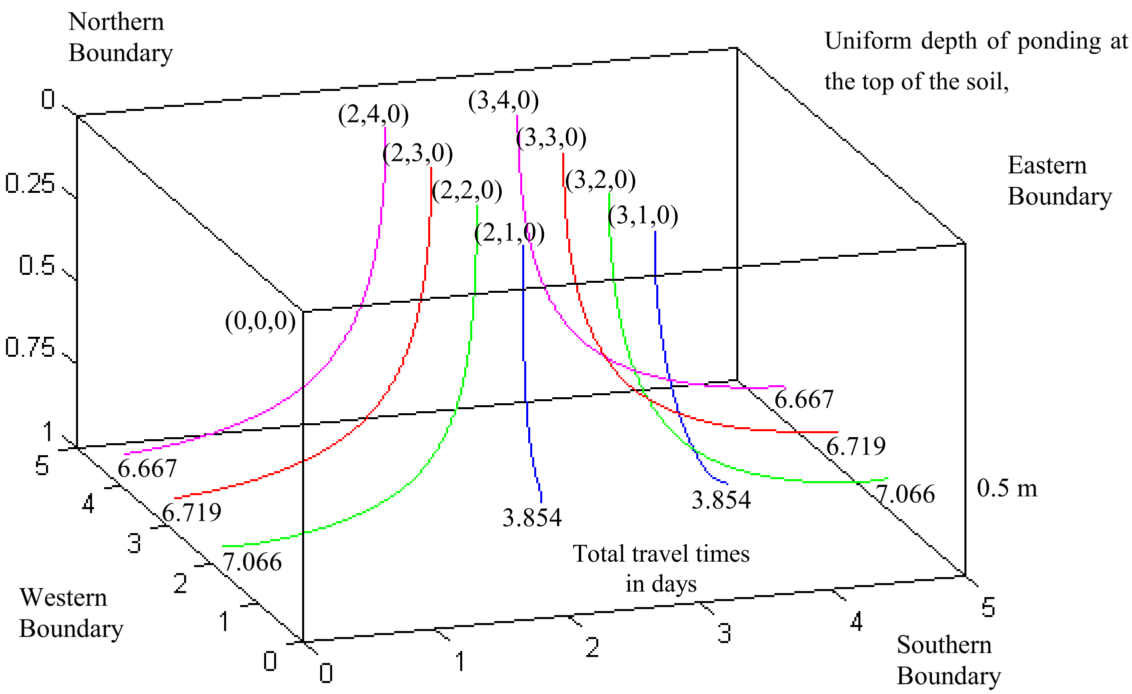


Fig. 4.12. Travel times of water particles (in days) along a few pathlines starting from the surface of the soil to the recipient ditches under steady state condition when the parameters of Fig. 4.2 are taken as

Northern
Boundary

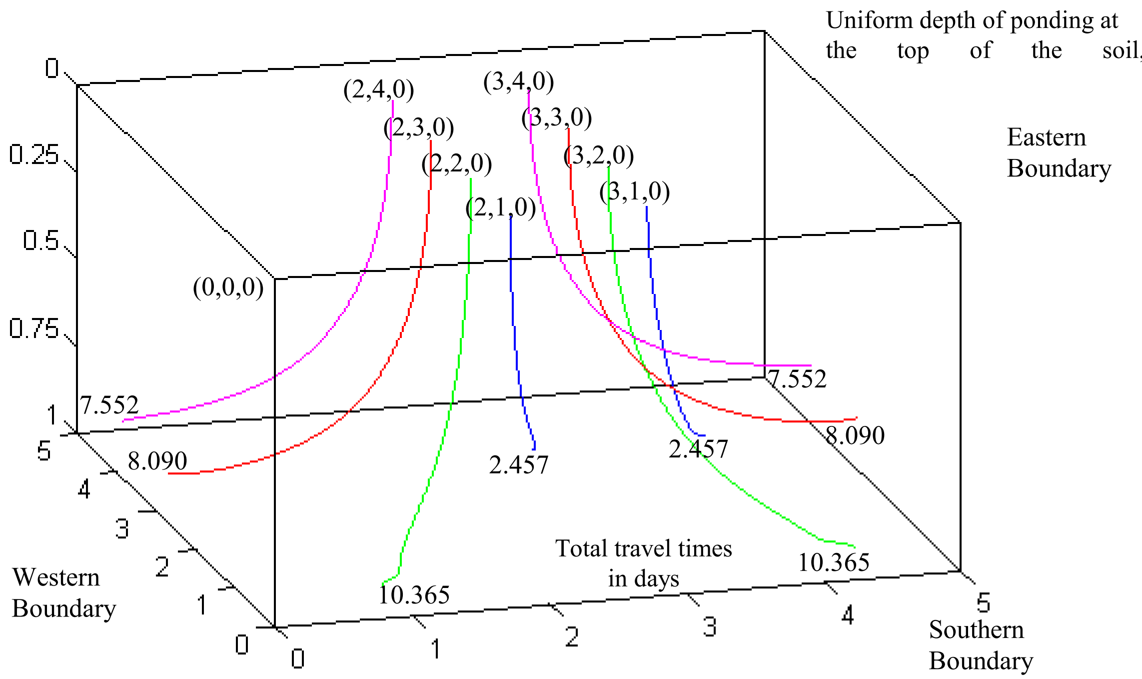


Fig. 4.13. Travel times of water particles (in days) along a few pathlines starting from the surface of the soil to the recipient ditches under steady state condition when the parameters of Fig. 4.2 are taken as

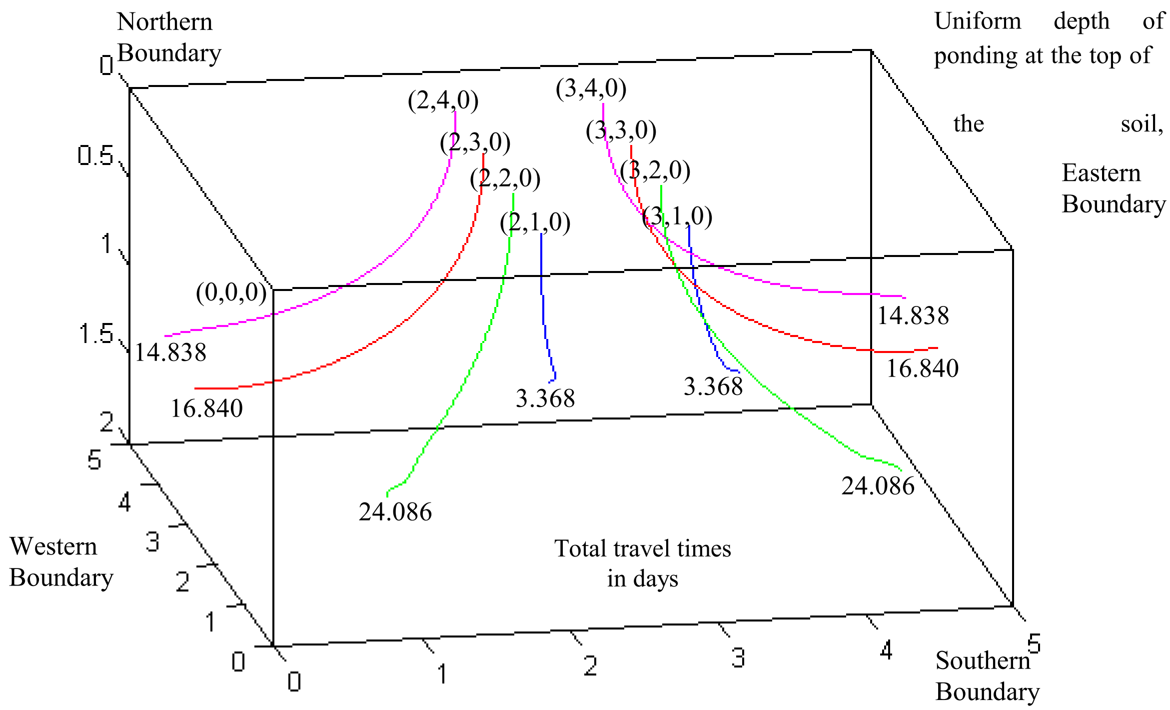


Fig. 4.14. Travel times of water particles (in days) along a few pathlines starting from the surface of the soil to the recipient ditches under steady state condition when the parameters of Fig. 4.2 are taken as

Northern
Boundary

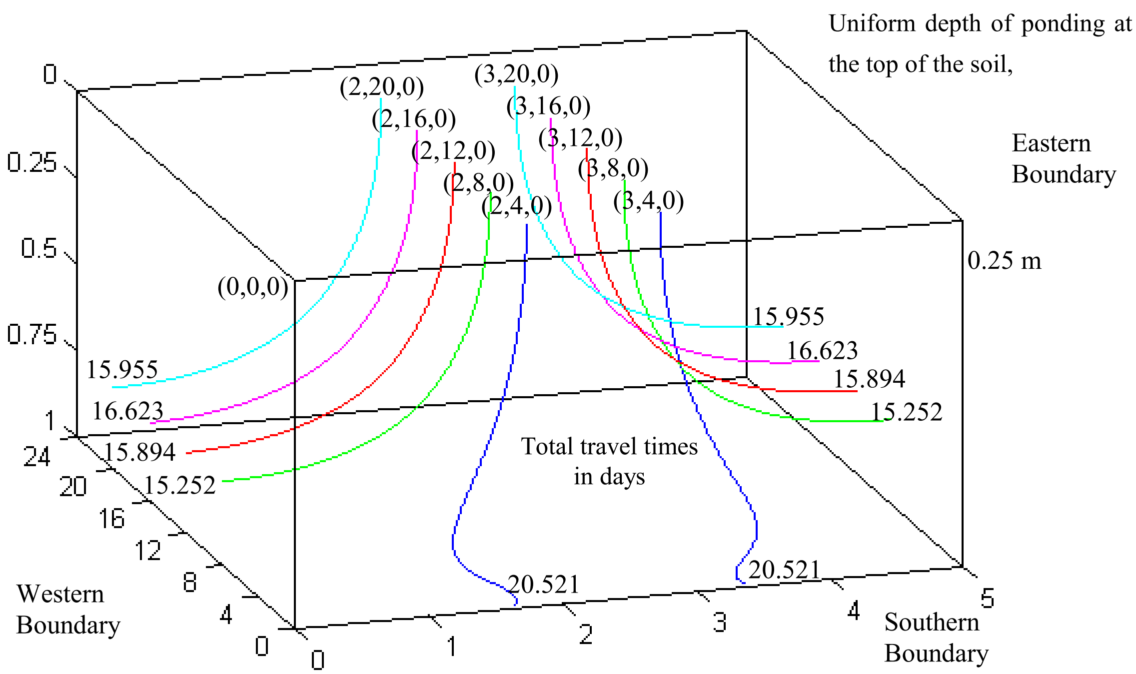


Fig. 4.15. Travel times of water particles (in days) along a few pathlines starting from the surface of the soil to the recipient ditches under steady state condition when the parameters of Fig. 4.2 are taken as

Another important variable, so far as the travel times of water particles in a ponded drainage is concerned, is the thickness of the soil column overlying the impervious barrier. Among other factors remaining the same, an increase in this thickness may result in a substantial increase of the water particle travel times along the pathlines, as has been aptly demonstrated in the drainage situations of Figs. 4.6(b) and 4.9. A similar observation, albeit to a lesser degree, is also noticed in the drainage examples of Figs. 4.11 and 4.14, where the Northern boundary is a now a no-flow zone and not a ditch boundary.

Figs. 4.16 and 4.17 show variations of with time for a few flow situations of Figs. 4.1 and 4.2. From these figures, it is clear that the time taken by a three-dimensional ponded ditch drainage system to attain steady state may be considerable if the directional conductivities of the soil are low and the specific storage high. This is all the more true for situations where the ditches are dug relatively deeper into the ground. From Figs. 4.16 and 4.17, it can also be observed that, considering all the other factors to remain the same, an increase in of over that of by only 10 times brings about a noticeable increase in the transient state durations of the considered flow situations, mainly again for situations where the thickness of the soil is large with drains being dug all the way through it. As the hydraulic conductivities of most of the natural deposits along the bedding planes are generally higher than that across the bedding planes (Maasland 1957, Schafer 1996) and lowly conductive soils like glacial tills, dense clays and clayey paddy soils are also quite common in nature (Chen and Liu 2002, Tabuchi 2004, Stibinger 2009, Macdonald et al. 2012) and further since the specific storage of soils like glacial tills and lacustrine clays can also be quite high (Grisak and Cherry 1975, Neuman 1975, Sharp 1984, Shaver 1998, Chen and Chang 2003), the transient state duration of a three-dimensional ponded drainage situation may turn out to be quite large for many a ponded drainage situations. Thus, in these types of drainage scenarios, drainage designs based on transient drainage models probably will lead to better and realistic designs of subsurface drains in comparison to drain designs based on steady state drainage models.

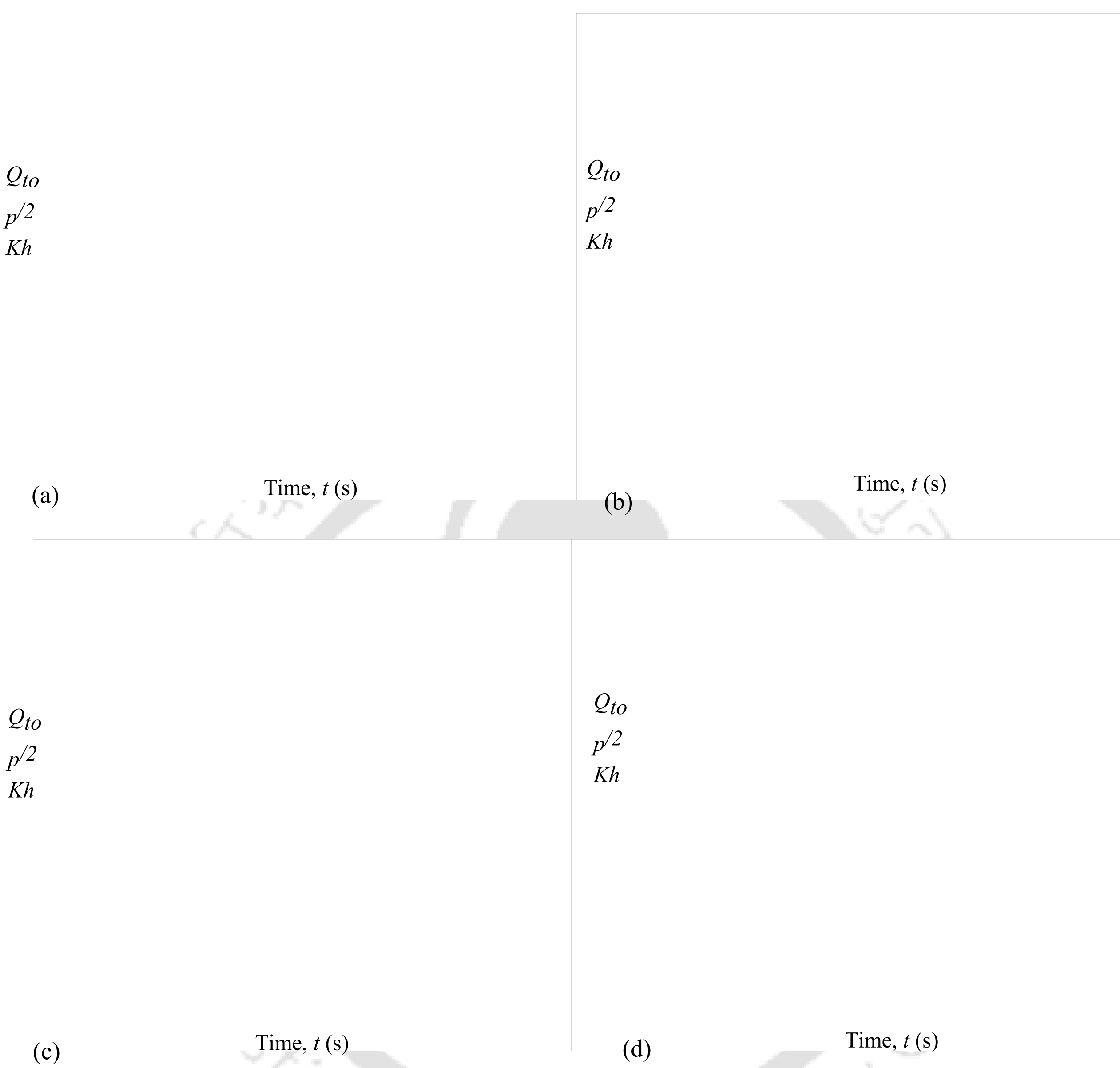


Fig. 4.16. Variation of ratio with time as obtained from the proposed analytical model for different values of h (with i.e., ditches are running empty) when the parameters of Fig. 4.1 are taken as (a) (b) (c) (d)

Fig. 4.17. Variation of ratio with time as obtained from the proposed analytical model for different values of h (with i.e., ditches are running empty) when the parameters of Fig. 4.2 are taken as

(a) (b) (c)

Fig. 4.18 shows the variation of the normalized top discharge function for a drainage situation of Fig. 4.1 for two different times. It is interesting to note from this figure that the discharge distribution at the surface of a ponded soil in a three-dimensional ditch drainage system at a very early time of simulation is relatively much more uniform than that corresponding to a later time. This uniformity, however, breaks down at large times and as may be observed in Fig. 4.18, the percentage of water seeping from different surficial locations of a uniformly ponded drainage scenario may be a pretty uneven one at a large time of simulation of the system, with most of water seeping into drains is being contributed from areas lying close to the drains. The uniformity of surface discharge distribution in a three-dimensional ponded ditch drainage setting of the nature as shown in Fig. 4.19 can also be improved by subjecting the soil to a progressively increasing variable ponding distribution towards halfway distances between the drains. This is being amply demonstrated in Fig. 4.22 where the imposition of a variable ponding field of the

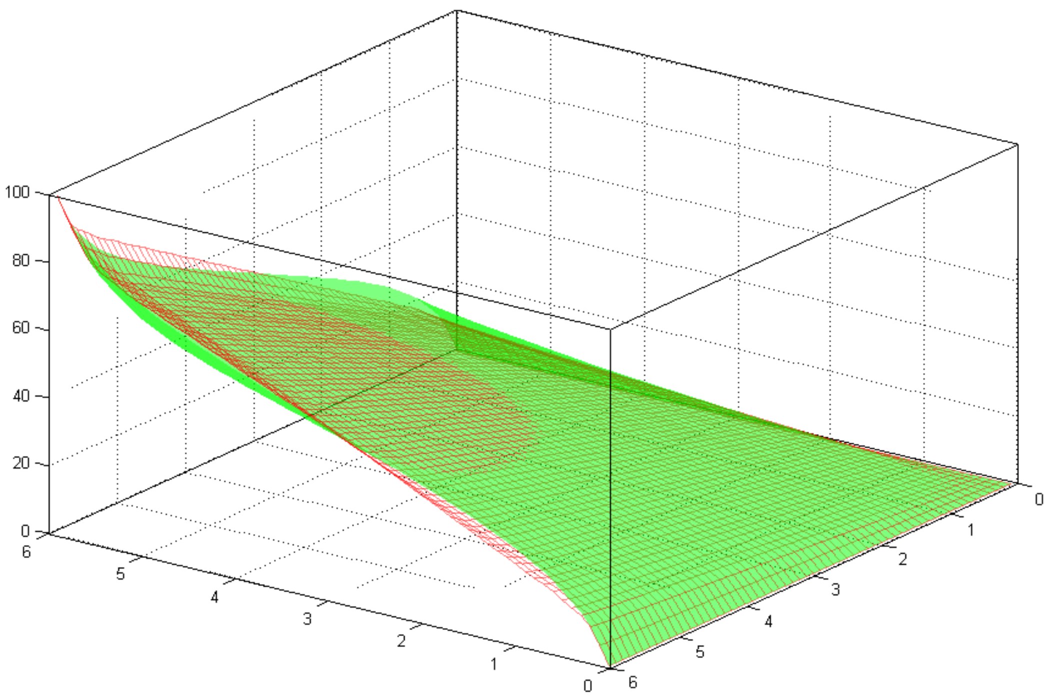
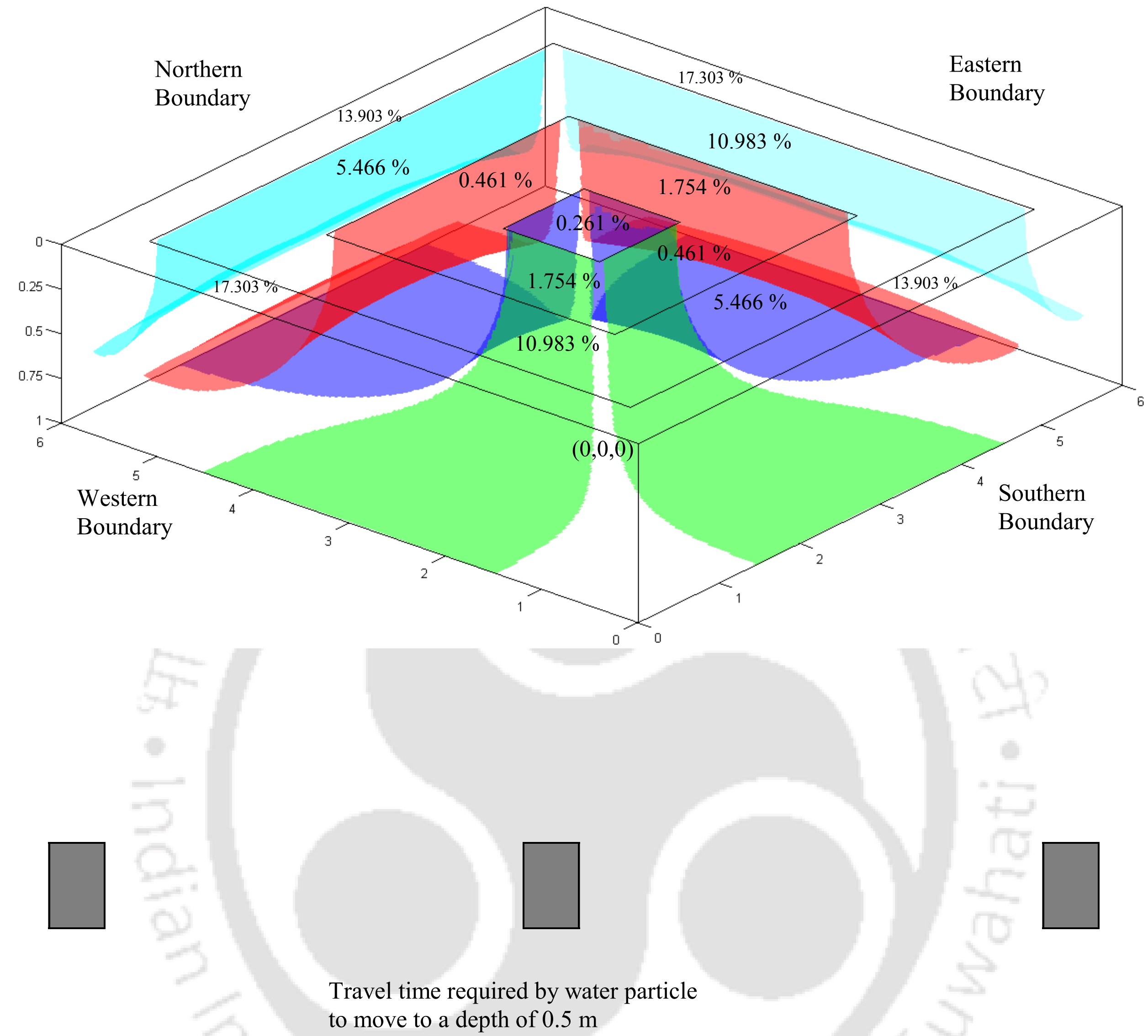


Fig. 4.18. Variations of as obtained from the proposed analytical model of the flow problem of Fig. 4.1 for two different times and when the parameters of Fig. 4.1 are taken as

type as shown on the considered flow geometry has resulted in a relatively more uniform distribution of discharge at the surface of the soil as compared to a situation when the ponded surface of the drainage setting of Fig. 4.19 is being subjected to only a uniform ponding distribution. In Fig. 4.19, the steady state top discharge contributions from various rectangular strips with which the surface of the drainage space of Fig. 4.19 is being divided, is vividly shown for the chosen drainage configuration; it is easy to notice that, for the concerned uniform ponding drainage situation, most of the discharge from the surface of the soil is being confined to areas lying in close proximity to the drains with the inner rectangular core accounting for only 0.2614 % of the total surface discharge. If, however, the flow configuration of Fig. 4.19 is been subjected to a gradually increasing ponding distribution of the type as shown in Fig. 4.22 towards the halfway spacing between the drains, then the uniformity of the top discharge distributions gets noticeably improved with the inner rectangular core now contributing about 2.5833 % of the total surface discharge to the drains. Figs. 4.19 and 4.22 further show that the introduction of the variable ponding on the drainage situation of Fig. 4.1 is also causing the travel times of water particles along the tested streamlines to improve so far as uniformity of travel times are concerned – whereas particles moving in the pathlines originating from the coordinate locations (1.5,2,0), (2.5,2,0), (3.5,2,0), (4.5,2,0), (1.5,4,0), (2.5,4,0), (3.5,4,0) and (4.5,4,0) in the uniform ponding situation of Fig. 4.19 are taking 3.325, 7.222, 7.222, 3.325, 3.325, 7.222 and 3.325 days, respectivley to complete their journey to the recipient ditches, the corresponding figures for the variable ponding drainage situation of Fig. 4.22 are turning out to be only 1.476, 2.483, 2.457, 1.615, 1.476, 2.483, 2.457 and 1.615 days, respectively. Thus, to a reclaim a salt affected soil within a specified time, the proposed solutions can be suitably utilized to work out appropriate ponding distribution, depth and spacing of ditch drains so that adequate quantities of water seep through different locations of the drainage space within the desired time. It can also be observed that by simply changing the vertical conductivity of the flow situation of Fig. 4.19 from 1 m/day to 0.2 m/day in Fig. 4.20, keeping the other parameters same as before, visible improvement on the uniformity of the top discharge distribution as well as on the water particle travel times on the pathlines, can be accomplished (Fig. 4.20). This decrease in the vertical conductivity is also, expectedly, causing the stream surfaces and the pathlines to penetrate relatively shorter distances in the drainage situation of Fig. 4.20 in comparison to the


Northern Boundary
Eastern Boundary
Western Boundary
Southern Boundary
13.903 %
17.303 %
5.466 %
10.983 %
0.461 %
1.754 %
0.261 %
0.461 %
17.303 %
1.754 %
13.903 %
5.466 %
10.983 %
(0,0,0)
Travel time required by water particle
to move to a depth of 0.5 m

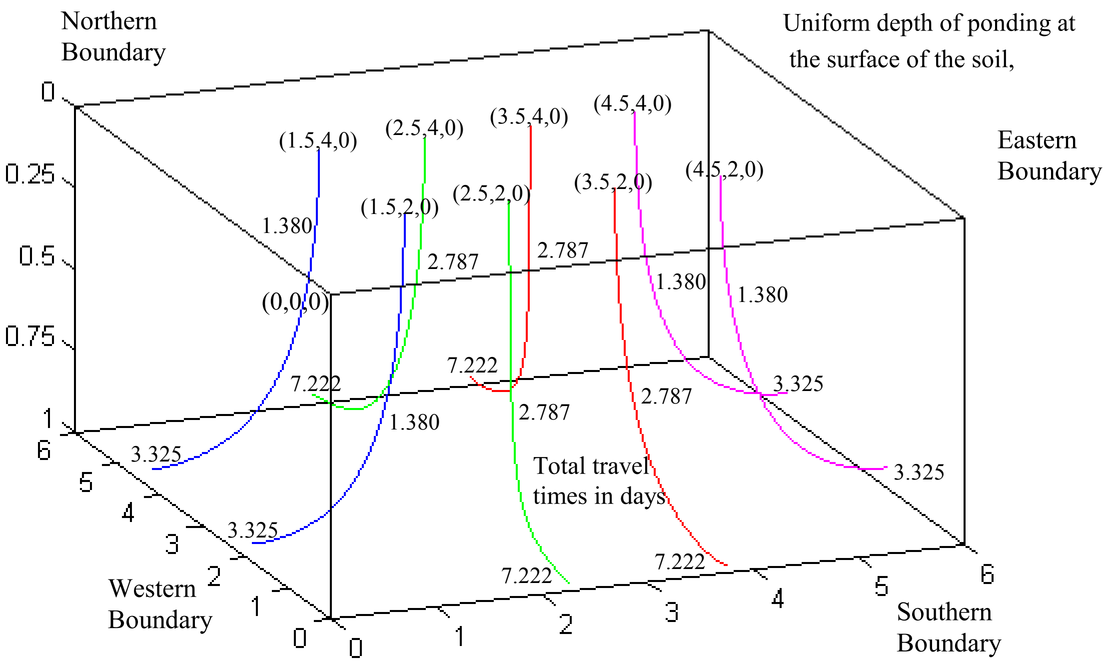
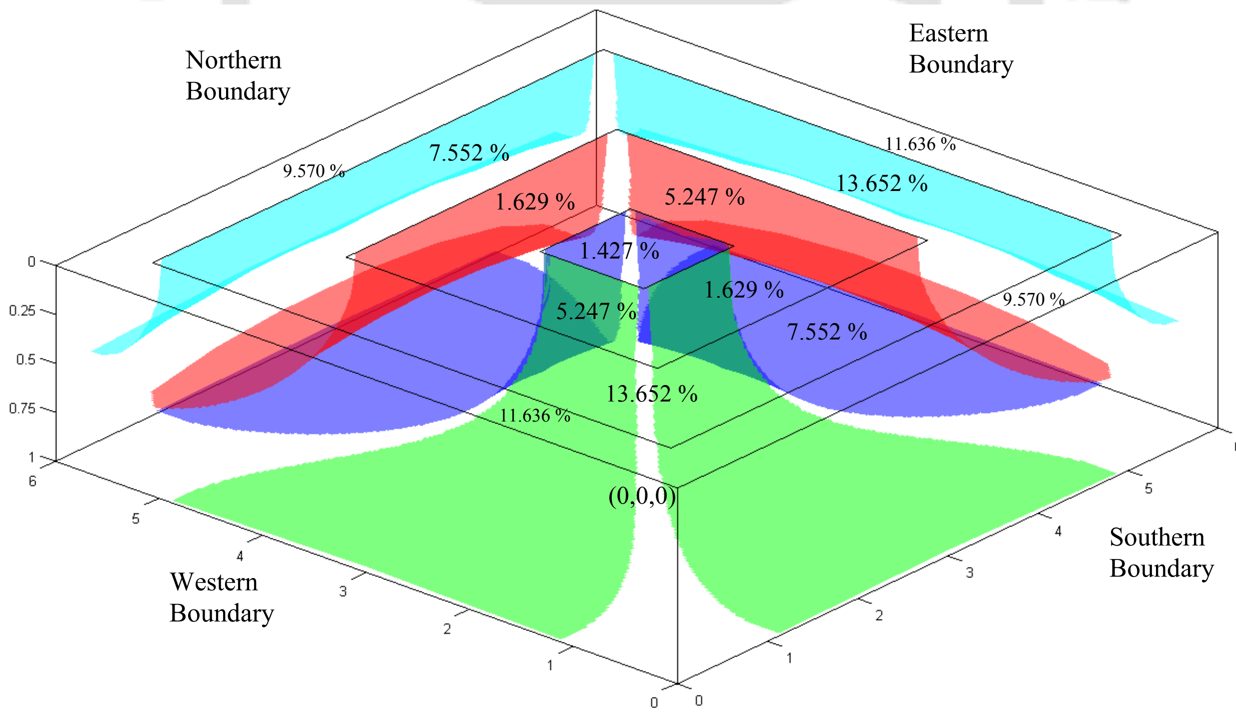


Fig. 4.19. A few stream surfaces and travel times (in days) of water particle on a few pathlines starting from the surface of the soil when the parameters of Fig. 4.1 are taken as



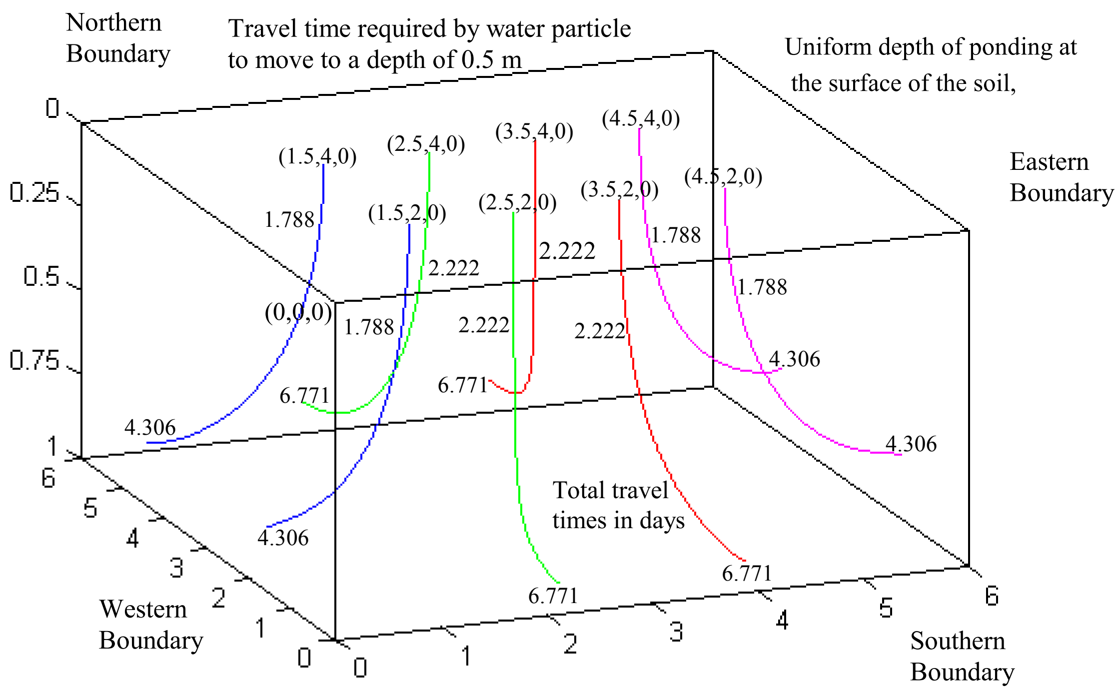
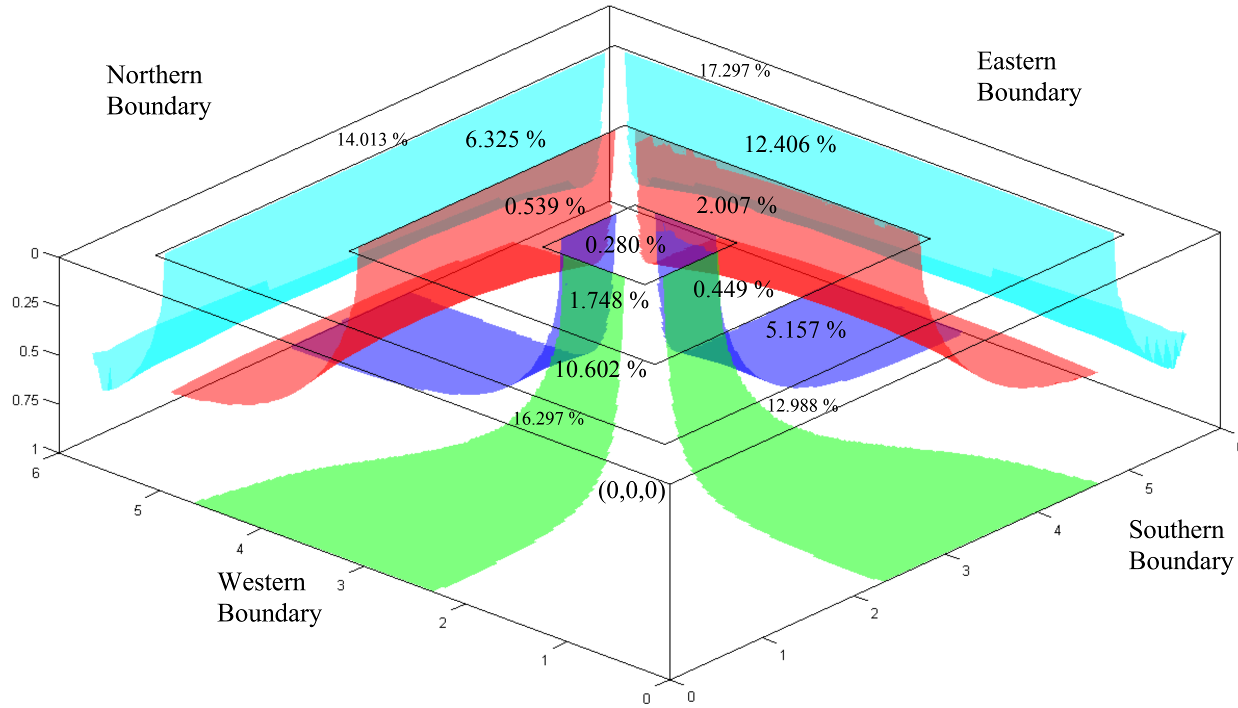
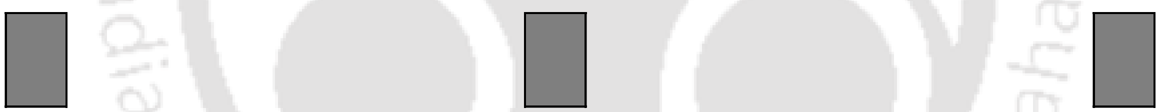


Fig. 4.20. A few stream surfaces and travel times (in days) of water particle on a few pathlines starting from the surface of the soil when the parameters of Fig. 4.1 are taken as


Northern
Boundary
Eastern
Boundary
Western
Boundary
Southern
Boundary
14.013 %
17.297 %
6.325 %
12.406 %
0.539 %
2.007 %
0.280 %
1.748 %
0.449 %
5.157 %
10.602 %
16.297 %
12.988 %
(0,0,0)
0
0.25
0.5
0.75
1
6
5
4
3
2
1
0
0
1
2
3
4
5
6



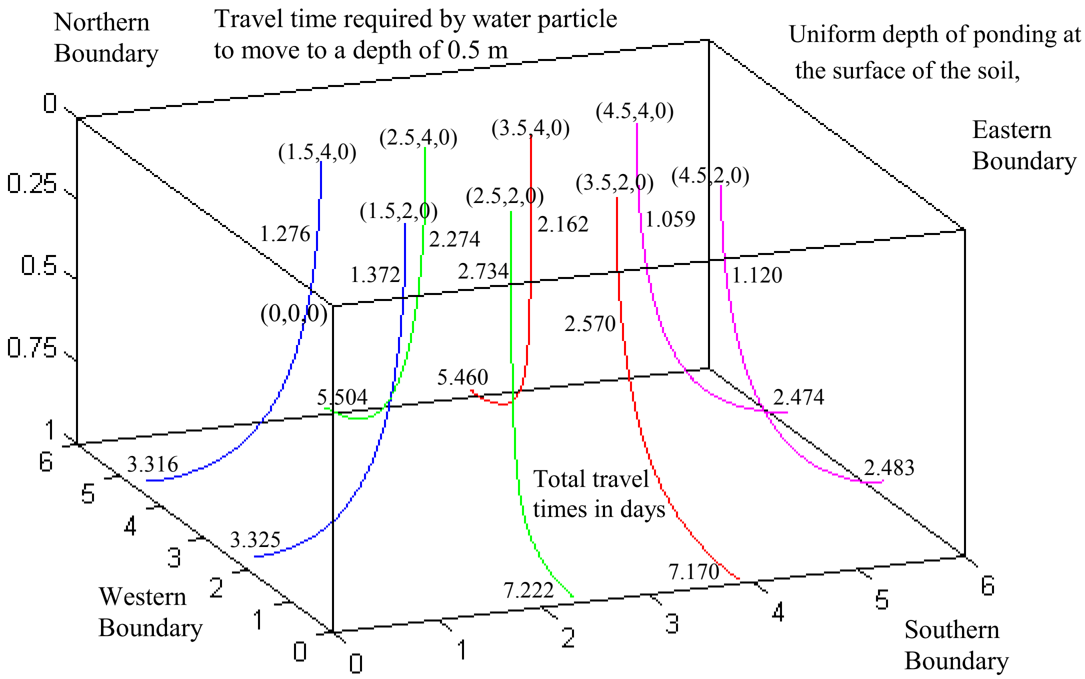
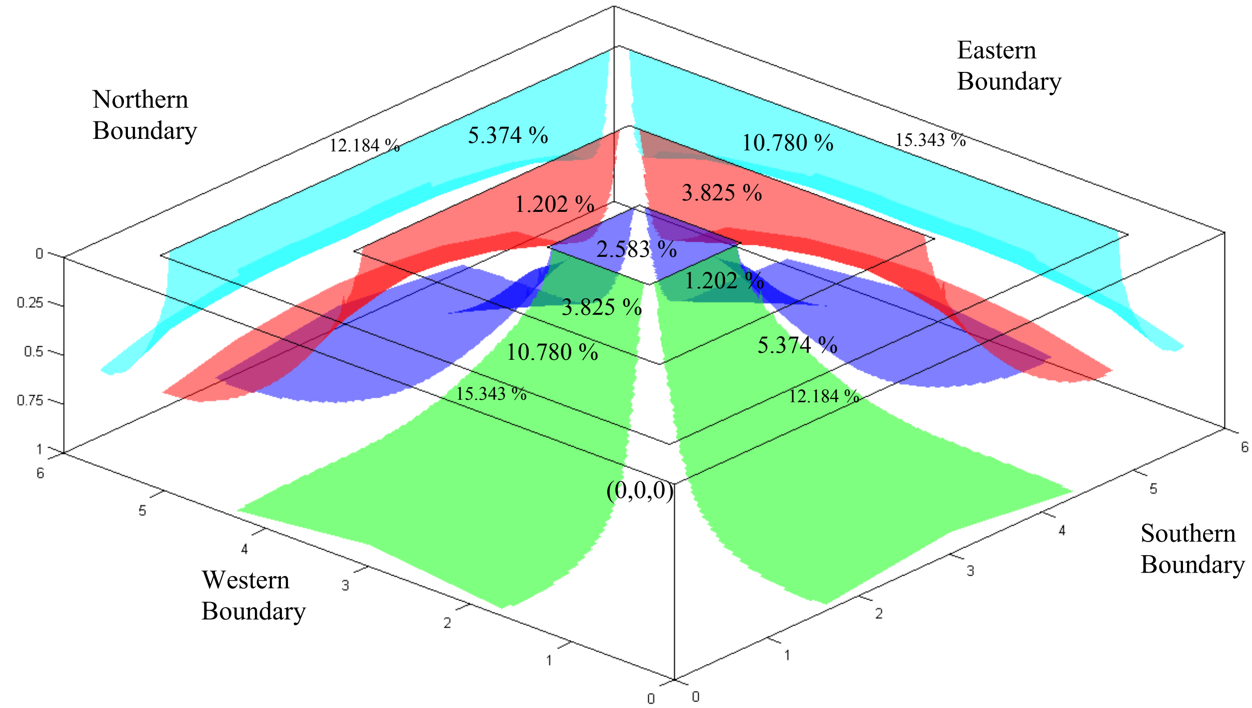


Fig. 4.21. A few stream surfaces and travel times of water particle on a few pathlines starting from the surface of the soil when the parameters of Fig. 4.1 are taken as


Northern Boundary
Eastern Boundary
Southern Boundary
Western Boundary
12.184 %
5.374 %
10.780 %
15.343 %
1.202 %
3.825 %
2.583 %
1.202 %
3.825 %
10.780 %
5.374 %
15.343 %
12.184 %
(0,0,0)
0
0.25
0.5
0.75
1
6
5
4
3
2
1
0
0
1
2
3
4
5
6

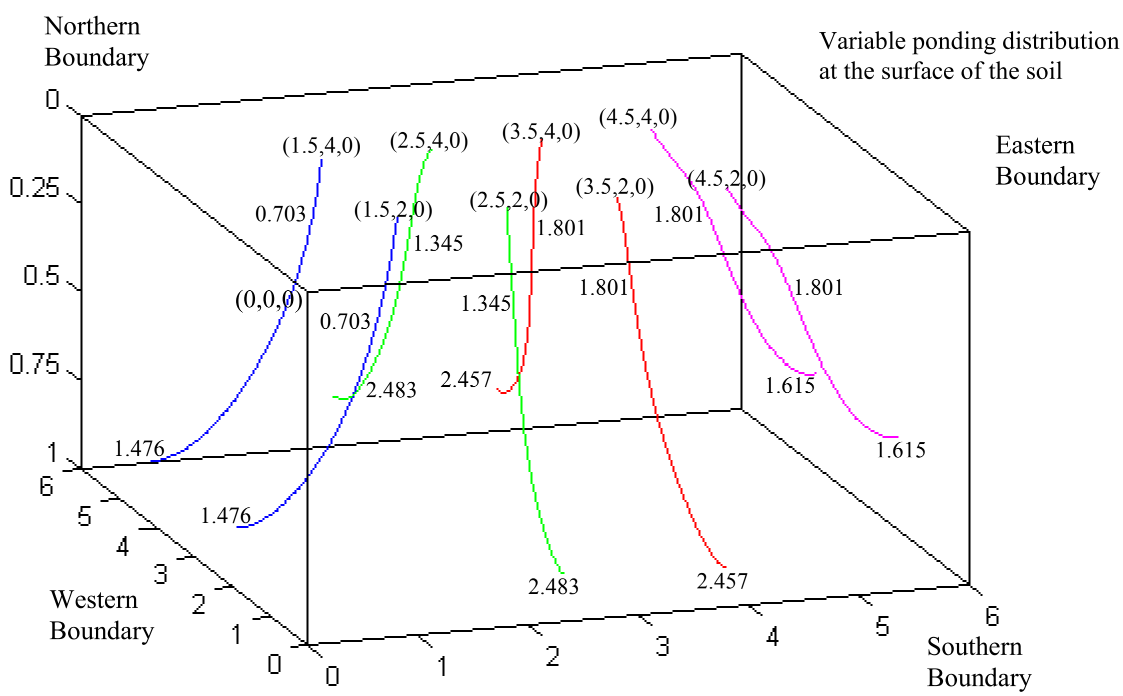


Fig. 4.22. A few stream surfaces and travel times (in days) of water particle on a few pathlines starting from the surface of the soil when the parameters of Fig. 4.1 are taken as

corresponding penetrations as being observed for the drainage setting of Fig. 4.19. It is also interesting to observe from Figs. 4.19 and 4.21 that the flow to the drains can be strongly influenced by playing with the water level heights in the drains. Thus, if the water level height in the East and North drains for the drainage configuration of Fig. 4.19 is made to increase from 0.5 m to 0.75 m (as measured from the surface of the soil), the discharge contributions from the surface of the soil can then also be seen (Fig. 4.21) to increase noticeably from that of the flow situation of Fig. 4.19. This is an important observation because it tells us that that considerable alteration of the flow behavior inside a three-dimensional ponded drainage space can be bought about by simply playing with the water level heights of the drains – a drain with a less depth of water in it tends to capture more flows than that of a drain with a high water level depth in it. The solutions provided here can also be utilized to design ditch drains for determining the upper limit of fall of water level of a waterlogged soil. Consider, for example, the flow situation as mentioned in Fig. 4.8; for this drainage setting, the volume of water seeping from the surface of the soil in the first 1 hour has turned out to be 0.1378 m^3 and in the first 5 hours to be 0.6924 m^3 . Thus, the upper limit of fall of water for this drainage setting will be 5.738 mm at the end of the first hour and 28.840 mm at the end of the 5 hours. These are upper limits of fall because while determining them, we have intrinsically assumed the depth of ponded water of 0.1 m over the surface of the soil to remain constant throughout the simulation times. In reality, however, the depth of ponding and hence the water head over the surface of the soil, will not remain constant but will decrease with the increase of time resulting in less volume of water seeping into the drains in a fixed time in comparison to the volume of water seeping into the drains as obtained using the constant ponding depth assumption. Nevertheless, an upper limit of fall of water level of a waterlogged soil within a stipulated time vide a ditch drainage system is an important information as it provides insight about the efficacy of the chosen drainage system in controlling waterlogging of the tested soil profile within the concerned time interval.

4.6 Conclusions

Analytical solutions have been developed for predicting three-dimensional transient seepage of water into a network of ditch drains surrounding a horizontal, saturated, homogeneous and anisotropic soil medium being subjected to a variable ponding distribution at the surface of the soil. During the development of the models, the soil column has been assumed to be underlain by a horizontal impervious barrier and the ditches have also been assumed to be dug all the way up to this barrier. Solutions have been obtained for two general situations, namely, when the soil column is being surrounded on all four of its vertical sides by ditch drains having different levels of water in them and when soil column is surrounded on three of its vertical sides by ditch drains with different water levels in them but the fourth vertical side is a no-flow zone. The separation of variable method along with a judicious combination of the double and triple Fourier runs have been made use of to obtain solution to these problems – the double Fourier runs have been utilized to tackle the boundary conditions and the triple Fourier runs have been used to take care of the initial conditions of the problems. The transient expressions for the hydraulic head, top and side discharges pertinent to these problems can be easily reduced to that of the steady state situation by simply allowing the time variable in them to go to infinity; this will make the transient terms in them to disappear living behind only the steady state terms. For clarity of presentations, simple procedures for tracing pathlines and stream surfaces utilizing the proposed solutions, have also been presented. The validity of the proposed solutions have been checked for a few simplified situations by comparing predictions as obtained from the proposed solutions with the corresponding predictions obtained from the analytical and experimental works of others. Numerical checks on both the proposed solutions have also been carried out using the PMWIN platform.

From the solutions proposed here, it is clearly seen that flow to a network of ditch drains from a ponded field is mostly three-dimensional in nature, particularly in areas lying close to the drains. Even for a ponded drainage situation where the separation between two its vertical drainage boundaries is given a large value, three-dimensional nature of the pathlines is still seen to prevail in locations adjacent to the drains. However, on a vertical plane lying further away from both the largely separated boundaries, two-dimensional flow situation can roughly be assumed without introducing any appreciable error in the hydraulics associated with this plane. The study also shows that the nature of a side boundary – i.e., whether a ditch or a no-flow boundary – in a three-dimensional ponded ditch drainage system plays a significant role in deciding the hydraulics associated with such a system, again mainly in drainage areas lying in close proximity to this boundary. We also observe from the study that the time taken by a three-dimensional ponded ditch drainage system to go to steady state may be considerable for a soil with low directional conductivities and high specific storage, particularly for situations where the ditches are installed relatively deeper into the ground. Further, the hydraulics associated with such a system has also been observed to be sensitive to the spacing, depth and water level heights of the ditches as well as on the nature and magnitude of the ponding field being imposed at the surface of the soil. It is seen that by just playing with the level of water in a ditch, noticeable changes to the distribution of pathlines in locations close to the ditch as well as to the overall discharge to the ditch, can be brought about. This is an important observation as it lets us know that the flow in a drainage space can also be visibly altered by just changing the water level heights of the drains. From the study, it is also seen that by suitably playing with the ponding field at the surface, considerable improvement in the distribution of the flow lines as well as on the water particle travel times along these flow lines, can be achieved. Thus, by suitably adjusting a ponding field over a salt affected soil being considered for reclamation vide a subsurface ponded ditch drainage system, considerable improvement on the uniformity of cleaning of the profile can be accomplished. This is an important point note since reclaiming a salt affected soil by a ponded ditch drainage system by subjecting it to only a uniform ponding depth at the surface of the soil most often leads to situations where the regions close to the drains get over-washed and the regions away from the drains under-washed. Thus, the analytical models proposed here, owing to their abilities to accommodate three-dimensional flows, variable ponding distributions and unequal water level heights of the drains, are expected to come up with better ditch drainage designs for cleaning salt affected soils as compared to designs based on drainage solutions developed utilizing more restrictive assumptions. The solutions proposed here can also be used to design ditch drains for draining a waterlogged field by a desired amount within a stipulated time and are, thus, important from the point of view of reclamation of a flooded and waterlogged field as well.

4.7 List of Notations

The following notation are used in this chapter

= constants with =

1,2,3,...., = *1,2,3,....,* = 1,2,3,...., = *1,2,3,....,* = 1,2,3,...., = *1,2,3,....,*

= 1,2,3,...., = 1,2,3,...., = 1,2,3,...., = 1,2,3,...., = 1,2,3,....,

= 1,2,3,...., and = *1,2,3,....,* ;

spacing between two adjacent ditches in N-S direction of Figs. 4.1 and 4.2 [L];

spacing between two adjacent ditches in N-S direction in transform domain [L];

depth up to impermeable layer of Figs. 4.1 and 4.2 as measured from the surface of the soil [L];

depth of water in the Northern ditch of Fig. 4.1 as measured from the surface of the soil [L];

depth of water in the Southern ditch of Figs. 4.1 and 4.2 as measured from the surface of the soil [L];

depth of water in the eastern ditch of Figs. 4.1 and 4.2 as measured from the surface of the soil [L];

depth of water in the western ditch of Figs. 4.1 and 4.2 as measured from the surface of the soil [L];

hydraulic conductivity of soil in x- direction [LT^{-1}];

hydraulic conductivity of soil in y- direction [LT^{-1}];

hydraulic conductivity of soil in z- direction [LT^{-1}];

anisotropy ratio of the soil (dimensionless);

anisotropy ratio of the soil (dimensionless);

spacing between two adjacent ditches in E-W direction of Figs. 4.1 and 4.2 [L];

spacing between two adjacent ditches in E-W direction in transform domain [L];

number of terms to be summed in the infinite series solutions, 1, 2, 3, …

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with = 1,2,3,...., ;

with p = 1,2,3,...., ;

with q = 1,2,3,...., ;

with q = 1,2,3,...., ;

with r = 1,2,3,...., ;

number of divisions of the ponding surface at the top of the soil of Figs 4.1 and 4.2;

discharge through the northern face of the ditch of Fig. 4.1

discharge through the southern face of the ditch of Fig. 4.1

discharge through the southern face of the ditch of Fig. 4.2

discharge through the eastern face of the ditch of Fig. 4.1

discharge through the eastern face of the ditch of Fig. 4.2

discharge through the westtern face of the ditch of Fig. 4.1

discharge through the westtern face of the ditch of Fig. 4.2

discharge through the top surface of the soil of Fig. 4.1

discharge through the top surface of the soil of Fig. 4.2

specific storage of soil

time variable of the flow problem of Figs. 4.1 and 4.2 [T];

velocity distribution in the *x*-direction of Fig. 4.1

velocity distribution in the *x*-direction of Fig. 4.2

velocity distribution in the *y*-direction of Fig. 4.1

velocity distribution in the *y*-direction of Fig. 4.2

velocity distribution in the *z*-direction of Fig. 4.1

velocity distribution in the *z*-direction of Fig. 4.2

Volume of water seeping through the northern face of the ditch of Fig. 4.1

Volume of water seeping through the southern face of the ditch of Fig. 4.1

Volume of water seeping through the southern face of the ditch of Fig. 4.2

Volume of water seeping through the eastern face of the ditch of Fig. 4.1

Volume of water seeping through the eastern face of the ditch of Fig. 4.2

Volume of water seeping through the western face of the ditch of Fig. 4.1

Volume of water seeping through the western face of the ditch of Fig. 4.2

horizontal coordinate of Figs. 4.1 and 4.2 [L];

horizontal coordinate in transform domain [L];

horizontal coordinate in the transverse direction of Figs. 4.1 and 4.2 [L];

horizontal coordinate in the transverse direction in transform domain [L];

vertical coordinate of Figs. 4.1 and 4.2 [L];

hydraulic head distribution of Fig. 4.1 [L];

hydraulic head distribution of Fig. 4.2 [L];

porosity of flow domain of Figs. 4.1 and 4.2 (dimensionless);

ponding depth for the strip at the soil surface of Fig. 4.1 and 4.2 [L];

width of the ditch banks in the E-W direction of Figs. 4.1 and 4.2

width of the ditch banks in the N-S direction of Figs. 4.1 and 4.2

;

;

CHAPTER 5

SUMMARY AND CONCLUSIONS

Subsurface drainage is necessary in many irrigated areas of the world for combating salinity buildup and waterlogging. It is also essential for the orderly and timely removal of excess water from agricultural fields so as to maintain optimum soil-air-water ambience in the root zones of plants. Subsurface drainage is now also been frequently used for draining paddy fields as a number of studies have conclusively established that by providing mid-season drainage of paddy fields, considerable reduction in the emission of methane from these fields can be obtained, a green house gas whose global warming potential is next only to carbon dioxide by mass. Subsurface drainage can be by ditch or tile drains but in locations where the conductivity of the soils is low and the topography relatively flat, subsurface drainage vide ditch drains are found to be more appropriate than that vide underground tiles. Thus, it is essential that due efforts should be made to study in detail the hydraulics of flow associated with such a system so that efficient ditch drainage networks specific to a purpose can be designed in the field. The hydraulics of a complex flow problem is generally studied by taking recourse to numerical modeling. Numerical models, however, often require a large set of input variables for them to be used, which may often be difficult and/or expensive to obtain. Further, being approximate solutions, the convergence and stability of these solutions need always be established before they can be applied to actual field situations. Another approach of studying hydraulics of a flow system is by using analytical models. Analytical models are generally been developed for comparatively simpler flow situations as compared to numerical models (even though they are now been increasingly developed for complex problems as well) but being exact solutions, they are mostly stable and require much less input data for their use as compared to numerical models. Also, a very important feature of an analytical model is its ability to provide physical insights into the underlying hydraulics associated with a flow situation. In this research work, an effort has been to derive a few analytical models related to subsurface drainage under ponded conditions. The details of these models as well as their outcomes, have already been presented in the earlier chapters; however, for ready reference, we are now again giving below the salient conclusions of each of our investigations that have been carried out in these chapters.

5.1 Salient Conclusions of the Study

1. Equation of the hydraulic head function and from it the stream function have been derived for predicting steady seepage of water into an array of equally spaced parallel ditch drains partially penetrating a multi-layered soil and receiving water from a variably ponded horizontal field of infinite areal extent, the soil being underlain by an impervious layer at a finite depth from the bottom of the ditches. From the proposed model, the following conclusions are drawn.

- Flow to the ditch drains from a uniformly ponded field is mostly confined to locations close to the drains and that an increase in the anisotropy ratio of the layers increases and a decrease in the anisotropy of the layers decreases the uniformity of distribution of the streamlines in a ponded drainage flow space.
- Considering all other factors to remain the same, the fraction of the total flow through the bottom of the ditches increases if the anisotropy ratio of the constituent layers is made to decrease and reverse is the case if the anisotropy ratio of the layers is made to increase.
- Ponded drainage with a constant ponding depth at the surface of the soil is mostly limited to areas close to the ditches, unless the sub-soils close to the surface possess very low conductivities like the muddy and plow sole layers of a paddy field.
- Considerable improvement in the uniformity of distribution of the streamlines and in the travel times of water particle in a multi-layered ponded drainage space, can be brought about by imposing a suitable variable ponding field at the surface of the soil.
- A variable ponding field may actually result in a lesser rate of water input to the system as compared to a constant ponding situation. Thus, leaching a contaminated soil utilizing a suitably chosen variable ponding ditch drainage system may lead to a much better, quicker and economical cleaning of the soil profile as compared to a situation when the soil is being leached using only a uniform depth of ponding at the surface of the soil.
- Subsurface drainage in a paddy field is a slow process, particularly in presence of the muddy and plow sole layers, and that water particles may require considerable time to travel to the drains from the surface of paddy fields.
- The presence or absence of the plow sole layer strongly influences the hydraulics of flow associated with a ditch drainage system in a paddy field and thus, should such a layer be present,

it must be accounted for while studying flow dynamics in such a system. As the proposed ditch drainage model for the multi-layered soil is of a pretty general nature in that it can account for not only layeredness of a soil profile and anisotropy of the individual layers but partial penetration of the ditches and a variable ponding field at the surface of the soil as well, it is hoped this solution will lead to better design of drainage networks for reclaiming salt affected soils and in draining paddy fields as compared to designs obtained from more simplistic solutions to the ditch drainage problem.

- The developed models can also be used for estimating the upper limit of water to be pumped for draining a multi-layered waterlogged soil vide a ditch drainage system within a specific time.

2. An analytical solution has also been proposed for predicting transient seepage into an array of equally spaced ditch drains partially penetrating a homogeneous and anisotropic soil underlain by an impervious barrier, the drains being fed by a variable ponding field at the surface of the soil. In the development of the analytical model, the soil has been assumed to be of finite thickness, the surface of the soil to be horizontal and of infinite extent and the drains to be separated from each other by a finite distance. The solution can account for anisotropy of the soil, partial penetration of the drains and finite bottom width of the ditches; thus, this solution is also of a general nature even though it is valid for only a single-layered soil. From the analysis of this solution, the following observations have come out.

- The time taken by a partially penetrating ponded ditch drainage system to go to steady state may be considerable if the directional conductivities of the soil are low and the specific storage and anisotropy ratio of the soil are high. This is all the more true if the thickness of the soil is relatively large and the depth of penetration of the drains is relatively high as scaled with respect to the thickness of the soil.
- The width of the ditch drains receiving water from a ponded field does not seem to have much of an influence on the drain discharge as well as on the discharge taking place through the surface of the ponded field but the level of water in the ditches has been observed to have a strong influence on the surface as well as on the side and bottom discharges of the ditches.
- Transient ditch drainage from a uniformly ponded field is mostly confined to areas close to the drains, particularly at large times of simulation of a system, and that considerable uniformity in the movement of water in the drainage space, both in terms of quantum of flow and in water

particle travel times from the surface of the soil to a recipient ditch or a desired horizontal plane below the surface of the soil (say, up to the root zone of a crop), can be brought about by introducing a proper ponding field over the surface of the soil specific to a drainage situation.

- The exit gradients at the boundaries of a ditch are very sensitive to the time of simulation of a ditch drainage system and also to the position of the water level in the ditches. The exit gradients are found to be higher for early times of simulation of a transient ditch drainage system as compared to later times and are also observed to vary inversely with the level of water in the ditches. Thus, the possibility of breaching of the banks of a stream/ditch due to sudden lowering of water level in the stream/ditch than the surrounding water table and the ensuing subsurface seepage to the stream/ditch because of it, is more likely to occur at an early transient stage of movement of the groundwater to the stream/ditch, particularly at points where the water level in the stream/ditch touches the soil surface.
- The analytical model developed here can also be utilized to design a network of subsurface ditch drains for lowering the level of flood water in an area by a desired amount in a specific time.

3. In order to study the effects of three-dimensional flow in a ponded ditch drainage model under transient conditions, analytical models have also been presented in the thesis for predicting transient seepage of water to a system of ditches in a homogeneous and anisotropic soil of finite thickness and of finite areal extent, the ditches being fed by a variably ponded field at the surface of the soil and the soil being underlain by an impervious barrier. Two situations have been considered for modeling, namely, when the flow domain has been bounded on all the four sides by ditches (i.e., North, South, East and West) and when the Northern boundary of the flow space is a no-flow boundary and all the other side boundaries are ditch boundaries. These models can account for any water level heights in the ditches along with flows taking through the seepage faces of the ditches. From these modeling studies, the following conclusions are arrived at.

- The time required for a three-dimensional ponded ditch drainage system to go to steady state is dependent, to a great extent, on the magnitude of the directional conductivities, anisotropy ratio and specific storage of a soil – a low conductivity and high specific storage soil increases the transient state duration of such a system and converse is the case for a high conductivity and low specific storage soil.

- The distribution of the pathlines and the travel times of water particles in a three-dimensional ponded ditch drainage space, depend not only on the hydro-geological properties of a soil but also on the nature of the imposed hydraulic head distributions at the side and surface boundaries of the flow domain as well.
- Assuming all other factors to remain the same, a mere change in one of the side boundaries from a Dirichlet (specified hydraulic head distribution in a ditch) to that of a Neumann boundary (no flow boundary here) may result in a substantial change in the distribution of the hydraulic heads in the flow profile, particularly in areas close to the concerned boundary.
- The pathlines are observed to exhibit marked three-dimensional flow characteristics in the vicinity of the drains even for situations where the distances between the opposite boundaries of the flow domain are assigned large values. This shows that flow in a ponded ditch drainage system is never truly a two-dimensional one, particularly in areas located close to the drains where appreciable variations from two-dimensional flow may occur. However, the assumption of two-dimensional flow may very well be realized in a vertical plane located in between two drainage boundaries if the separation between these boundaries is given a large value.
- Considering all other factors to remain the same, an increase in the vertical conductivity of soil tends to uncoil the pathlines and push them downward in the flow space and an increase in the horizontal conductivity of soil tends to flatten them and decrease their vertical penetration.
- Also, high conductivity and ponding depths, low water level heights in the ditches and a thin soil column cause the water particle travel times to decrease in a three-dimensional drainage space as compared to situations where the conductivity and ponding depths are low, the level of water in the ditches high and the soil column relatively thick.
- The developed three-dimensional models assist in having a better insight of the hydraulics of flow associated with a ditch drainage system as compared to insights gained through considering only two-dimensional simulation of such a system. Thus, subsurface drainage designs based on these solutions are expected to be more efficient than the ones based on solutions developed with more stringent assumptions. This is particularly true for drainage situations where the flow to the drains cannot be adequately described with the two-dimensional flow assumption.

- Further, these models should also be helpful in checking the validity of complex numerical codes related to subsurface drainage provided these codes can be reduced to the relatively simple drainage settings for which drainage investigations have been carried out in the current study.

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