



INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI  
SHORT ABSTRACT OF THESIS

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Programme of Study : Ph.D.  
Thesis Title : Study of Certain Partition Functions: Arithmetic Properties, Quantitative Estimates, Asymptotic Formulae, and Partition Inequalities  
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Thesis Submitted to the Department/ Center : Mathematics  
Date of completion of Thesis Viva-Voce Exam : 13 June 2025  
Key words for description of Thesis Work : Number Theory, Partition Theory

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In this thesis, along with the ordinary partition function, we investigate certain restricted partition functions such as  $t$ -regular partition functions, mex-related partition functions, and  $\text{PDO}_t(n)$  partition function. We study arithmetic properties of  $t$ -regular partition functions, mex-related partition functions, and  $\text{PDO}_t(n)$  partition function. For these partition functions, we discover infinite families of Ramanujan-type congruences. We also study the distribution of some of these partition functions modulo some positive integers. We use classical  $q$ -series techniques, the theory of modular forms, Radu's algorithm, and the theory of Diophantine equations to prove our results. Analogous to Ono's and Radu's work on Subbarao's conjecture, we prove non-congruences for certain  $t$ -regular partitions, for some values of  $t$ . Motivated by the conjecture of Parkin and Shanks, we obtain some quantitative estimates for the distribution of 6-, 10-, and 14-regular partitions. For mex-related partition functions, we derive Hardy-Ramanujan type asymptotic formulae. We find the generating functions for the number of hooks of particular lengths in some  $t$ -regular partitions. Moving to partition inequalities, we completely uncover the hook length biases for the ordinary partition function. We also prove several hook length biases for  $t$ -regular partition functions. We use generating functions and several combinatorial tools and techniques for proving these partition inequalities.