



INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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Programme of Study : Ph.D.

Thesis Title : A study of class groups of number fields in connection with Greenberg's conjectures.

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Thesis Submitted to the Department/ Center : Mathematics

Date of completion of Thesis Viva-Voce Exam : 19-06-2025

Key words for description of Thesis Work : Class groups, class number, 2-class group, Iwasawa module, Iwasawa invariants, p -rationality, Greenberg's conjectures

SHORT ABSTRACT

For a number field K , whether its ring of integers O_K is a PID or not is a critical question. The importance of class groups was understood as they measure the failure of O_K from being a PID. Identifying the structure of the class group of a number field is a challenging task, making it one of the most interesting objects studied in algebraic number theory. We first discuss the indivisibility of class number by a given prime p in the context of p -rational fields. We find new families of multi-quadratic p -rational fields for infinitely many p and contribute to Greenberg's conjecture on the existence of such fields. We use a variety of techniques including results on densities of infinite sets by Heath-Brown and results on units by Serre.

Next, we deal with problems on class groups in Iwasawa theory, where objects like p -class groups are studied in Z_p -extensions of number fields. Iwasawa's class number formula relates the order of p -class groups at the n -th level in the infinite tower with constants μ , λ , and ν known as Iwasawa invariants. Greenberg conjectured that the invariants μ and λ must be equal to 0 under certain conditions. We study the Z_2 -extension of certain families of real quadratic and biquadratic fields focusing on the growth of rank and order of the 2-class groups of the intermediate number fields. Our results verify Greenberg's conjecture on Iwasawa invariants for the fields considered. We mainly appeal to results from genus theory, class field theory, Galois theory, and group actions to achieve our results.