INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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Programme of Study	:	**Ph.D.**

Thesis Title: **Stability and error analysis of numerical schemes for 1D and 2D fractional differential equations**

Name of Thesis Supervisor	:	**Prof. Natesan Srinivasan**
Thesis Submitted to the Department/ Center	:	**Mathematics**

Date of completion of Thesis Viva-Voce Exam: **09/08/2024**

Key words for description of Thesis Work	:	**Fractional boundary-value problem, Time-fractional differential equation, Tempered fractional diffusion equation, Caputo fractional derivative, k-fractional derivative, Finite difference method, Spline technique, $L1$-method, $kL2$-1_σ-method, Weak Galerkin finite element method, Dimensional-splitting technique, Locally one-dimensional method, Stability, Error analysis**

SHORT ABSTRACT

This thesis aims to develop and analyze efficient numerical methods for approximating solutions to fractional differential equations (FDEs). These equations are essential in many fields, including fluid dynamics, chemical reactions, electrical networks, and control systems.

FDEs often involve weakly singular kernels, which have milder singularities compared to classical calculus. Because most FDEs lack analytical solutions, numerical methods are crucial. However, standard numerical techniques may not be sufficient, particularly when dealing with weakly singular kernels. Specialized methods and non-uniform mesh generation strategies are therefore needed to handle these singularities and ensure accurate computations.

The research begins by analyzing one-dimensional (1D) steady-state fractional boundary-value problems (FBVPs) with fractional convection terms and variable coefficients. The thesis establishes the existence and uniqueness of solutions and explores numerical methods like the $L1$-method. The discrete comparison principle and error analysis are also conducted using a discrete barrier function. The thesis then moves to a second-order scheme using spline techniques for FBVPs with integral boundary conditions. It proves the existence and uniqueness of solutions and performs error analysis, followed by discussions on discretization methods, supported by numerical experiments.

Next, we address the numerical solution of 1D nonlinear time-tempered k-Caputo fractional diffusion equations. Newton's quasilinearization technique is used to simplify the problem, followed by the $kL2$-1_σ scheme for discretization. Stability analysis is conducted for this scheme. The study also extends to 1D nonlinear time-fractional diffusion equations (TFDEs) with generalized memory kernels, using similar linearization techniques and non-uniform discretization methods to manage singularities. A generalized discrete fractional Grönwall inequality is along with further stability analysis and error estimation in L_2-norm.

Then, we focus on two-dimensional (2D) FDEs. A dimensional-splitting weak Galerkin (WG) approach is introduced for numerically solving 2D TFDEs, aimed at reducing computational complexity and storage needs. The 2D problem is split into individual 1D problems, and stability analysis is provided for each direction. An overall error estimate is established in L_2-norm. Additionally, a locally one-dimensional (LOD) method is proposed for 2D nonlinear space-fractional diffusion equations (SFDEs), which involves linearizing the model and decomposing it into two 1D subproblems to minimize computational cost. The thesis includes stability analysis, convergence analysis in the maximum norm, and empirical validation through numerical examples.

The thesis concludes by summarizing the research findings and suggesting directions for future exploration. It emphasizes the practical significance of the developed numerical methods in real-world applications critical to various scientific knowledge.