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ABSTRACT

Keywords. *Semilinear order, C-relation, B-relation, D-relation, Jordan group, homogeneous structure, oligomorphic group, cycle type, uncountable cofinality.*

This thesis is about the interplay between permutation groups and some relational structures. Special emphasis is on a class of permutation groups called Jordan groups in which, in some non-degenerate way, the pointwise stabiliser of a subset is transitive on the complement. The relations considered are semilinear orders, C-relations, B-relations and D-relations. These relations are precisely the ones that appear in the study of Jordan groups. We try to answer the following natural question: Suppose we start with a relational structure whose automorphism group is a Jordan group. Can we expand this structure by imposing extra relations on it so that it still admits a Jordan automorphism group?

First, a C-set Ω and its automorphism group is studied. It is known from the literature that the automorphism group of a C-set is a Jordan group. Two types of automorphisms of (Ω, C) are defined. Roughly, a chain automorphism induces an order-preserving permutation on some chain of the underlying semilinear order Λ fixing setwise that particular chain. And a branch automorphism fixes a node of Λ inducing a permutation on the branches at that fixed node. A relation which restricts the class of branch automorphisms of (Ω, C) is called a branch relation. And a relation which restricts the class of chain automorphisms of (Ω, C) is called a chain relation. It is shown that any element of $\mathrm{Aut}(\Omega, C)$ can be expressed as a product of these two types of automorphisms. A few other relations namely, a binary (2-place) relation $\leq$ and quaternary (4-place) relations V, L and R are also

Abstract-TH-2716_964601

imposed on the C-set. The group $\mathrm{Aut}(\Omega, C, \leq)$ is shown to be 2-homogeneous which is not 2-transitive and $\mathrm{Aut}(\Omega, C, \leq, V)$ is 2-homogeneous but not relatively 3-transitive. Again, $\mathrm{Aut}(\Omega, C, L)$ is 2-transitive but $\mathrm{Aut}(\Omega, C, L, R)$ is not even 2-homogeneous. Further, it is shown that each of these automorphism groups are Jordan groups. For example, the Jordan sets of $\mathrm{Aut}(\Omega, C, \leq)$ are of the forms $\Sigma_{\alpha,P} = \{\alpha q n_1 q_1 n_2 q_2 \ldots n_k q_k \mid q \in P\}$, where α is an element of Λ and P is either a singleton subset or an open interval of $\mathbb{Q}$. The point stabiliser of each of these groups is also determined. A minimal Jordan group G_0 is determined such that the imposition of any extra relation on the underlying structure no longer admits a Jordan automorphism group. It is shown that a branch relation which contains the class of translation branch automorphisms in its automorphism group can be imposed on the C-set and will still admit a Jordan group. And any chain relation imposed on the C-set will still admit a Jordan group.

The structures of (Ω, C) with the extra relations are then studied with reference to homogeneous structures and oligomorphic groups. It is shown that all the structures under consideration are homogeneous and their automorphism groups are oligomorphic. The realisable cycle types of non-identity elements of the automorphism groups are also determined.

Finally, a D-set Ψ and its automorphism group is studied. A few other relations namely, a ternary (3-place) relation K and 5-place relations T and R are imposed on Ψ. It is found that the automorphism groups of each of these structures are Jordan groups. For any element x_0 in Ψ, the relation K induces a linear order on $(\Psi \setminus \{x_0\}, C)$, the relation T induces a 4-place relation L and the 5-place relation R induces the corresponding 4-place relation R as defined in the C-set Ω. The last result obtained is on cofinality. Suppose we have an automorphism group A. If there exists a countable chain $H_0 < H_1 < H_2 < \ldots$ of proper subgroups with union A then we say that A has countable cofinality. Otherwise we say that A has uncountable cofinality. It is shown that the group $\mathrm{Aut}(\Psi, D)$ has uncountable cofinality.

Abstract-TH-2716_964601

Contents