INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

This thesis provides efficient numerical methods for solving singular perturbation problems (SPPs) of convection-diffusion and reaction-diffusion types with boundary layers. A differential equation is called singularly perturbed when a small parameter is multiplied with the highest-order derivative and this small parameter is called the perturbation parameter. Because of the presence of the parameter in the solution of the differential equation, steep, thin layers occur at the boundaries or/and interior of the domain. Solution of singularly perturbed problems normally has smooth and singular components and its singular component is called the boundary layer function which varies very rapidly in the boundary layer region and behaves smoothly in the outer region. Due to this layer phenomena, it is a very difficult and challenging task to provide parameter-uniform numerical methods for solving SPPs. Parameter-uniform numerical methods means those numerical methods in which the approximate solution converges to the corresponding exact solution of SPPs independently with respect to the perturbation parameter(s).

It is well-known that uniform meshes with classical schemes fail to converge uniformly with respect to the singular perturbation parameter. It is desirable to develop methods which converges uniformly. In this thesis we develop and analyze superconvergence properties of the non-symmetric interior penalty Galerkin (NIPG) method for solving SPPs in one- and two-dimensions on layer-adapted meshes.

We begin the thesis with an introduction followed by a section describing the objectives and the motivation for solving SPPs. Then, we discuss preliminaries which are used throughout the thesis. Next, we move forward to the main work of the thesis. First, we have studied the superconvergence properties of the NIPG for singularly perturbed two-point boundary-value problems (BVPs) of reaction-diffusion and convection-diffusion types. Then, we have considered two-parameter singularly perturbed convection-diffusion-reaction BVPs. Here, in order to discretize the domain, we used the layer-adapted piecewise-uniform Shishkin mesh, the Bakhvalov mesh and the exponentially-graded mesh. Also, here, we have established the superconvergence result of the NIPG method. Further, singularly perturbed coupled system of two-point BVPs of reaction-diffusion type is considered. The solutions of these problems

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exhibit twin overlapping exponential boundary layers. We applied the NIPG method on layer-ad uniform Shishkin mesh. Numerical results are presented to support the theoretical results.

Then we discuss the numerical solution of singularly perturbed 1D parabolic PDEs. For the dis parabolic PDEs, one often uses the space semidiscretization, also called the method of lines. In this approach, discontinuous Galerkin (DG) discretization is applied only to the spatial variable, whereas the temporal variable remains continuous. These method have lower-order of accuracy in time. In order to improve the order of convergence in time also, here, we have used DG discretization for both the variables and showed the superconvergence properties of the DG method. Finally, we dealt with the numerical solution of singularly perturbed 2D elliptic BVPs. Here, we applied the NIPG method on the layer-adapted piecewise-uniform Shishkin mesh and superconvergence results of the NIPG method are established. Numerical results are produced to validate the theoretical error estimates.

At the end of the thesis, conclusion and possible future works are discussed.

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