ABSTRACT

In this thesis, an attempt is made to undertake a systematic analysis of the sensitivity of eigensystems in the natural geometric framework of the spectral portraits of matrices. The ϵ-spectra and the spectral portraits are shown to be efficient graphical tools for sensitivity analysis of eigenvalues and spectral decompositions of matrices. The notion of ϵ-spectra is also shown to be an appropriate logical setting for spectral analysis of matrices which are known only up to a given accuracy.

The geometric separation of eigenvalues of a matrix A which can be read off from the ϵ-spectra of A is shown to be an appropriate measure of sensitivity of eigenvalues and spectral decompositions. For the ℓ^2-norm, a characterisation of the sensitivity of spectral decompositions is provided and in the process a problem raised by Demmel[13] is solved. Sufficient conditions are obtained for the stability of spectral decompositions with respect to operator norms. Several bounds on the magnitude of the perturbations which ensure stability are also derived. Under appropriate assumptions, a conjecture of Demmel[13] on the separation of matrices is also settled.

It is shown that the spectral analysis of the one parameter family $A+tE$, $t \in \mathbb{C}$, the location of its multiple points and the splitting behaviour of its eigenvalues are better analysed in the setting of algebraic geometry. While analysing the effect of the linear perturbations $A + tE$, $t \in \mathbb{C}$, the structure of E is shown to play an important role. Based on the influence of E, the eigenvalues of A are classified into three groups. Characterisations are provided for eigenvalues in each of these groups. Characterisations are also provided for A and $A+tE$ to be isospectral for all $t \in \mathbb{C}$. The notion of ϵ-spectra and the spectral portrait of A with respect to E is introduced. The ϵ-spectra of A with respect to E are shown to provide an efficient graphical tool for analysis of the finer aspects of eigenvalue sensitivity which takes into account the influence of the structure of E.

An attempt is made to carry over these results to the case of operators on Banach spaces. It is shown that many results obtained for matrices do not hold for operators. However, under appropriate assumptions, the analysis of the effect of linear perturbations on the spectra of operators are shown to be carried over on the same lines as that of the matrices. Further, under appropriate hypothesis, characterisations for invariance of the spectra as well as the essential spectra are also provided .