

**ESTIMATION OF MULTI-FAULT PARAMETERS IN A
CRACKED-WARPED ROTOR SYSTEM INTEGRATED
WITH AN ACTIVE MAGNETIC BEARING**

*A Thesis Submitted in Partial Fulfilment of the Requirements for
the Degree of*

DOCTOR OF PHILOSOPHY

by

NILAKSHI SARMAH



**DEPARTMENT OF MECHANICAL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
GUWAHATI 781039, INDIA**

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Department of Mechanical Engineering
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Certificate

It is certified that the work contained in this Thesis titled '**Estimation of Multi-Fault Parameters in a Cracked-Warped Rotor System integrated with an Active Magnetic Bearing**' submitted by **Ms. Nilakshi Sarmah** (Roll no. **156103018**) to the Indian Institute of Technology Guwahati for the award of the degree of Doctor of Philosophy has been carried out under my supervision in the Department of Mechanical Engineering, Indian Institute of Technology Guwahati. This work has not been submitted elsewhere for the award of any other degree or diploma.

April 2021
IIT Guwahati

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Acknowledgments

“Gratitude is not only the greatest of virtues, but the parent of all others.”

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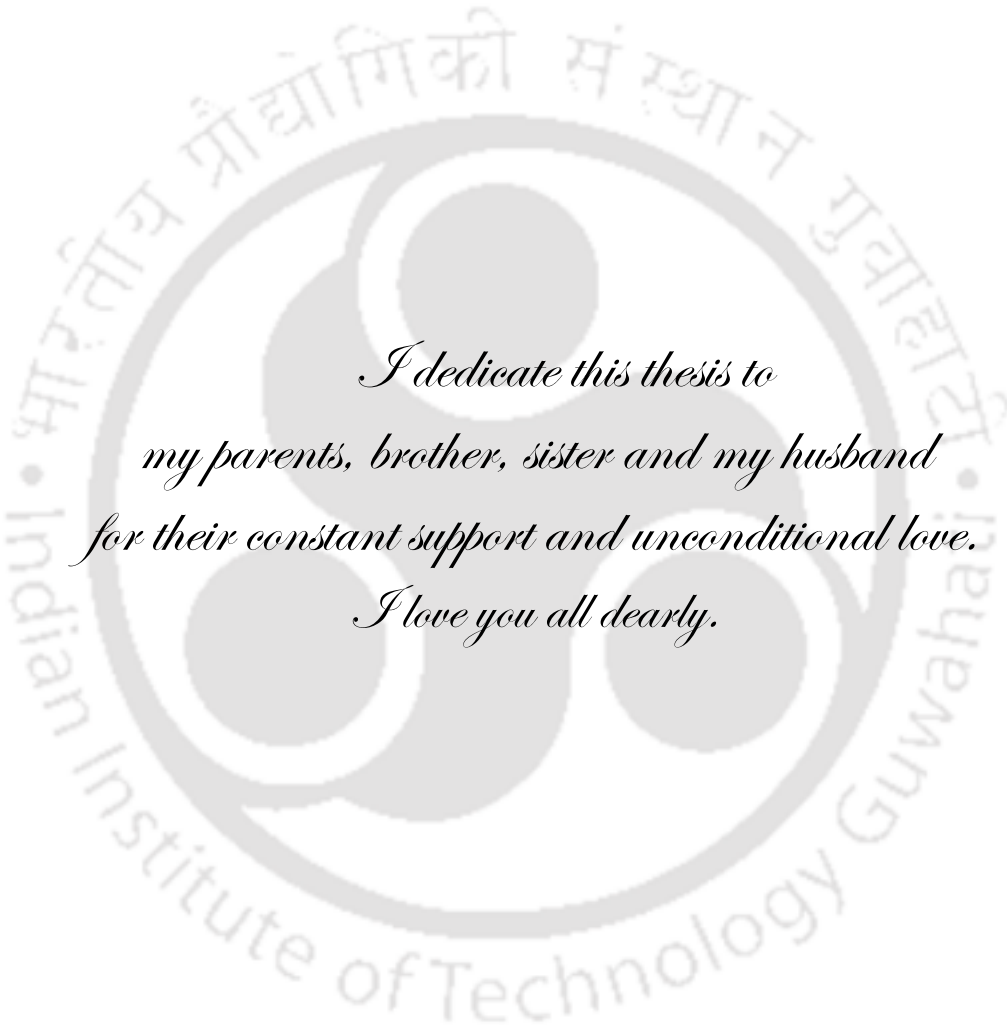
Last but not the least, I want to offer my gratitude and thanks to Almighty for giving me the strength and courage to embark on this journey of my life as well as showering His blessings to enable me to make this minute contribution towards the advancement of research in the enormous field of rotor dynamics.

“Grow flowers of gratitude in the soil of prayer”.

April 2021

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*I dedicate this thesis to
my parents, brother, sister and my husband
for their constant support and unconditional love.
I love you all dearly.*



Abstract

Rotating machinery such as compressors, turbines, pumps, flywheels often suffer severe vibration due to several mechanical faults such as unbalances, residual bow, fatigue cracks, misalignment, rotor-stator rub, internal damping, and temperature variation during operating conditions. The prolonged exposure to the fatigue stress generated due to rotor faults may result in premature failure of the rotating components. Therefore, at the initial stage, the analysis of the dynamic behavior of rotors is highly recommended to acquire the information for fault diagnosis, detection, and isolation of sustained machinery problems. The early detection of vibrational problems that might arise in machinery operation allows the personnel to take necessary corrective measures for the safety and reliability of machines. To identify precisely the unique nature of defects present in a rotating system, the signal processing of traditional FFT is modified to full spectrum response that contains information of both the positive harmonics (forward whirl) and negative harmonics (backward whirl). With the advent of technological advancement, smart condition monitoring (SMC) techniques are employed with the help of active magnetic bearing (AMB) in the system. Also, identifying multiple faults is desirable to avoid [co-occurrence of various](#) malfunctions in the machinery. However, due to the interacting nature of most of the faults, their simultaneous prediction is difficult to achieve.

In the presented work, a model based multiple fault identification technique is illustrated that utilizes the vibrational measurements and control current of active magnetic bearing (AMB) for simultaneous estimation of incipient faults in rotating machinery. The proposed work forwards the mathematical development of the cracked-warped internally

damped rotor-bearing system equipped with an AMB to estimate mechanical faults foregrounded on the full spectrum response of the rotor's complex vibrational and current signals. The mathematical model comprises individual faults such as unbalance, transverse crack, internal and external damping, residual bow faults, and AMB displacement-stiffness and current stiffness parameters. Moreover, the interaction of crack and bow fault, which would be synergistic, is also incorporated into the model to analyze the combined effect of crack and residual bow (warp) in a system. The internal damping that arises owing to the rubbing of the cracked faces during whirling is also considered in the modeling. The threshold speed of instability in a system can be estimated with the help of internal damping parameter by which a personnel can determine the safe operating speed of machinery. In the rotor system, the crack model is based on the assumption of switching crack (sudden opening and closure of the cracked faces), and the combined effect of crack and residual shaft bow yields excitation at multiple harmonics both in forward and backward whirl. Full spectrum responses are obtained through fast Fourier transform (FFT) of numerically generated displacement and AMB current responses. Due to random instants of measurement of the signals in the time domain, phase ambiguity arises in full spectrum harmonics. Therefore, the full spectrum amplitudes and phases after phase compensation with the multi-harmonic reference signal are utilized in the identification algorithm. The estimation of the multi-fault parameters (additive and multiplicative faults) such as unbalance and residual bow with their phases, additive crack stiffness, multiplicative crack and bow, external and internal damping, and the AMB displacement stiffness and current-stiffness parameters is done based on the least square fitting technique. The sensitiveness of the identification algorithm is tested against measurement noise and modeling or bias errors and found to be robust. The

verification of the developed identification methodology is undertaken through the measurement of the displacement and control current responses from the rotordynamic test bench of a fatigue crack and warp (permanent bend) in the shaft equipped with a radial active magnetic bearing (RAMB). The closed loop test is conducted on the developed laboratory test rig in conjunction with the PID (*proportional-integral-derivative*) control strategy in RAMB control action. The acquired time domain responses from the experimental test rig are converted into full spectrum encompassing multi harmonic amplitudes and phases of the forward and backward whirl of the rotor system. These amplitude and phase of vibrational and current responses are input to the identification algorithm to estimate the fault parameters. The experimentally estimated multiple fault parameters are found to be consistent with different rotor spin speed ranges.

Gyroscopic stiffening effect is the most important factor a rotating system experiences, which depends on the spin speeds and the position of the mountings of different rotating masses with respect to the bearing axis of the system. At high speed whirls, the rotating disc slowly gets tilted that creates an angular displacement in the rotor system. Therefore, the gyroscopic effect is included in the second modeling of the cracked warped internally damped rotor-AMB system with an offset disc where the system possesses both translational as well as rotational DOFs. Here, the difficulty in measurement of the angular displacements with accuracy in a practical rotor system needs a way to overcome it. Thus, a modified condensation scheme inclusive of gyroscopic effect i.e. gyroscopic condensation scheme (GDC) is utilized to eliminate the rotational DOFs and then based on the condensed equation of motion of the system identification algorithm is developed. The developed identification algorithm can estimate the fault parameters such as unbalance and residual

bow with their orientations, translational and rotational crack additive stiffness, external and internal damping, AMB displacement-stiffness, and current-stiffness parameters, and the multiplicative crack-bow fault simultaneously. An experimental investigation is also done to test the developed identification algorithm in a laboratory test rig used in the previous system mounted with an offset disc.

The identification methodology with the implementation of the GDC scheme is also extended to the practical rotor model using the finite element method (FEM) approach in the subsequent model based identification technique. The FEM modeling is demonstrated on the multiple discs and AMBs mounted on a cracked warped rotor bearing system. The support (conventional) bearings are considered to be flexible in contrast to the previous models where these are assumed as rigid ones. The development of the identification algorithm is done based on the EOMs in the frequency domain. Numerical test is conducted to verify the applicability of the developed identification algorithm considering two discs cracked warped internally damped rotor-AMB system supported on flexible bearings to estimate fault parameters including the bearing dynamic parameters unlike in the previous models. Eventually, to mimic the real rotor system, different levels of the random noise signal as well as rotor modeling or bias errors are added in the developed identification methodology to check the efficacy and sensitiveness of the algorithm. This model based identification technique can effectively identify simultaneous rotor faults with prior knowledge of some of the characteristics of the rotor system.

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Nomenclature

Symbols	Definition
A_1	Regressor or Regression matrix
b_1	Regressand or known vector
c	Damping
C_E	External damping matrix
C_H	Internal damping matrix
d	Shaft diameter
E	Young's modulus of elasticity
e_u	Disc eccentricity
e_b	Residual shaft bow magnitude
f_0	Initial load
f	Force
g	Acceleration due to gravity
G	Gyroscopic matrix
h	Flexibility of the shaft
I	Area moment of inertia
I	Identity matrix
i_c	Complex current
i	Current
I_i	Current response at i^{th} harmonic
j	$\sqrt{-1}$
k	Shaft stiffness
K	Stiffness matrix
K_C	Circulatory skew-symmetric matrix
K_B	Bearing stiffness
ΔK	Crack additive stiffness matrix

$\Delta k_{c22}, \Delta k_{c44}$	Translational and rotational additive crack stiffness respectively
k_s, k_i	Displacement-stiffness and current-stiffness factor of AMB
K_P, K_I, K_D	Proportional, integral and derivative gains respectively
L	Shaft length
$\mathbf{l}$	Location vector
m	Disc mass
$\mathbf{M}$	Mass matrix
N	Number of coil turns
p_i	Participation factor due to crack only
q_i	Participation factor due to crack and bow effect
$\mathbf{q}$	Dynamic displacement vector
s	Complex displacement
S_i	Displacement response at i^{th} harmonic
s_g	Air gap between stator and rotor
$\mathbf{T}$	Transformation matrix
u, v	Translation displacements
$\mathbf{x}_1$	Unknown vector of fault parameters
z	Bearing axis

Subscripts

AMB	Denoting AMB
bow	Denoting residual bow
b	Bias
cr	Denoting crack
cx, cy	Control current in vertical and horizontal direction respectively
CMA	Series combination of controller, sensor and magnetic bearing
E	External damping
H	Internal damping
Im	Imaginary

m	Master degrees of freedom
0	Due to healthy or uncracked shaft
s	Slave degrees of freedom
st	Static
sn	Due to sensor gain
tf	Transfer function
x, y	Vertical and horizontal displacements in inertial frame of reference respectively
$1, 2$	Denoting disc 1 and disc 2

Superscripts

cb	Due to support bearing
gd	Gyroscopically condensed
id	Due to internal damping
L	Denoting left
R	Right
T	Transpose

Greek symbols

α	Half angle of pole pair at the center of the actuator
δ	Distribution of residual bow throughout the shaft
Δ	Additive or reduction in cracked shaft
ξ, η	Displacements in rotating frame of reference
φ	Transverse rotational displacement of rotor
ϕ_u	Unbalance phase
ϕ_b	Shaft residual bow phase
σ	Periodic, time-varying steering function of crack
μ_0	Magnetic permeability
θ	Disc rotation with time instance
ω	Spin speed

Abbreviations

AMB	Active Magnetic Bearing
ACM	Active Condition Monitoring
AI	Artificial Intelligence
CWT	Continuous Wavelet Transform
CNC	Computerized Numerical Control
DOF	Degrees of freedom
DSP	Digital signal processing
EOM	Equation of Motion
FFT	Fast Fourier Transform
FEM	Finite Element Method
GDC	Gyroscopic condensation
PWM	Pulse width modulation
PID	Proportional–Integral–Derivative
SCEF	Switching crack excitation function
SMC	Smart Condition Monitoring
VFD	Variable frequency drive

Chapter 1

Introduction and Literature Review

To improve the overall performance of industrial machinery, nowadays various control strategies are exploited involving both sophisticated design techniques of digital systems and complex hardware, which comprises input-output sensors, actuators, components, and processing units. This increases the probability of failure of such complex systems and as a result, there is an utmost requirement of supervision of the closed-loop operation for detection and malfunction isolation as early as possible. The parameterized or model based identification or analytical redundancy method consists of using an accurate prototype of the system to mimic the real behavior of the process and helps in detection and diagnosing the faults in a system through the faulty and healthy signals. This procedure is more economical, reduce productivity loss and safety related issues, which does not require additional components but having some potential disadvantages of the need of modeling accuracy of the real system suitable for robust diagnosis to a great extent (Isermann, 1997; Isermann and Balle, 1997; Frank et al., 2000; Chen and Patton, 2012).

In practice, there are unmanageable mismatches between the theoretical model of the rotor-AMB system with the experimental data due to several factors, such as tolerances in fabrication, alignment of the shaft, simplified assumptions in modeling, and potentially undefined dynamics. To predict the faults precisely, there are requirements of improved or corrected mathematical models and hence, system identification techniques are to be implemented for obtaining an accurate mathematical model. Once the mathematical

modeling is done adequately, by varying the input system parameters to the model, theoretically it is possible to detect the faults. The *identification* comes after fault *detection*, where the task is to find the nature of faults present in equipment after they were detected. The identification techniques with the help of regression analysis of the input-output (or output-only) data based on the developed mathematical model of the system are utilized (Draper and Smith, 1998; Natke, 2014). This approach is also directed as a *parameterized identification* method, which is employed for the identification of faults in dynamic systems.

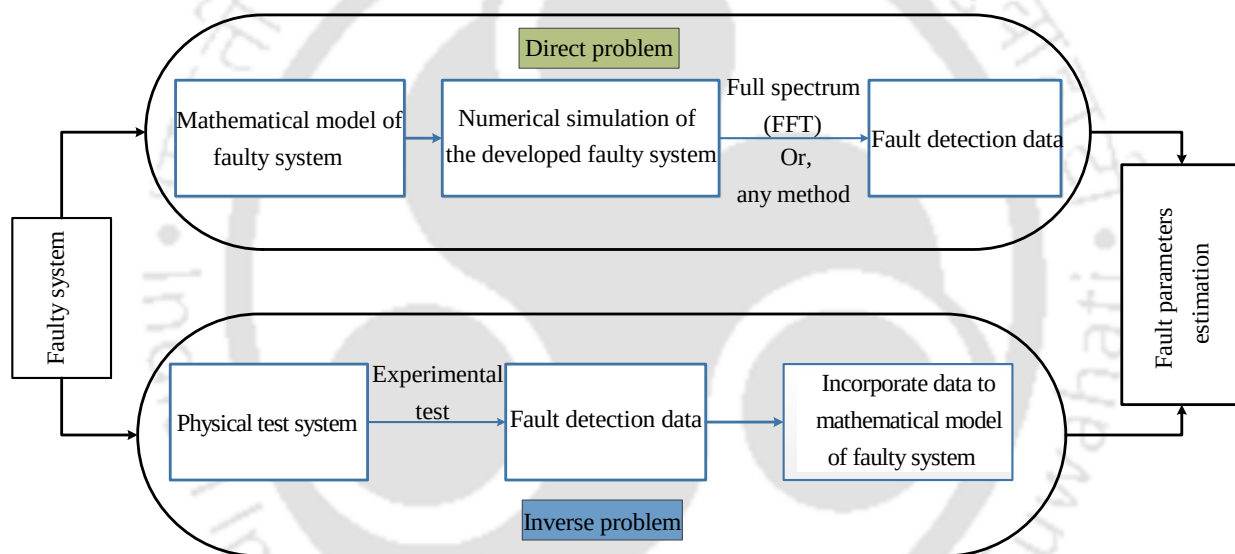


Figure 1.1 Procedure of direct versus inverse method of fault detection and parameter estimation

Over the last few decades, *inverse methods* gained popularity in fault detection and location, using measured vibrational data of the rotating machinery. This technique requires the use of a mathematical model, and the identification and estimation of the physical properties affected by faults (e.g., reduction of stiffness due to the presence of cracks or looseness in the parts) which are unknown from this model in contrast to the

direct straight forward method. In direct method, estimation is done of the system's modal properties from the measured response to determine the changes in dynamics caused by the faults. The direct methods have some limitations though it is straightforward, e.g. estimation of high-frequency modal properties. Figure 1.1 shows the direct and inverse method technique for fault detection in rotating machines (Edwards et al., 2000; Carden and Fanning, 2004; Dos Santos et al., 2008; Friswell, 2008).

Nowadays, the use of active magnetic bearing as an auxiliary support in the rotor-bearing system is a part of the active condition monitoring (ACM) technique, which comes under *smart* condition monitoring (SMC). In the SMC, the concept of monitoring as well as controlling action can be achieved together in machinery so as to identify significant changes in not only the system parameters but also of controlling parameters, which are indicative of developing faults in a system. This technique can provide real-time spectral behavior of a system as well as quantification of faults (Larsonneur and Richard, 2008; Burrows et al., 2009; Schweitzer and Maslen, 2009; Tiwari and Chougale, 2014).

Fault identification in the ACM technique starts with mathematical modeling of the physical model consisting of rotor, actuator, and bearings. The external excitations due to rotor faults are introduced into the EOMs of the system, viz. i.e. the unbalance, residual bow, crack force, misalignment, internal damping, magnetic force excitations, etc. in the numerical experiment. After that, experiments are carried out on the physical rotordynamic test system in a laboratory to acquire the test data and analyze the dynamic response, i.e. the suppressed vibrational displacement and control current along with their phases at predetermined locations on the shaft. The experiments are repeated at various speed ranges and the displacement and current signals for each speed range are fed into the *regression*

equation to estimate the unknown system parameters. The regression equation is a combination of segregated EOMs of the system at various speed ranges in the matrix form, which consists of a regressor, i.e. coefficients of an unknown system parameter vector and a known parameters matrix, i.e. the mass, stiffness, and gyroscopic parameters. The accuracy of the estimation depends on the precision of the numerical model and the experimental test readings. The more accurate is the numerical modeling and experimental test data, the more accuracy in estimating parameters can be expected. The correctness of the estimated fault parameters can be checked through numerical experiments of the regression equations by comparing the estimated values with assumed ones. The experimental data is used in the regression equation for real-time or online estimation of fault parameters in the rotor-AMB system (Bachschmid et al., 2002a; Tiwari and Chakravarthy, 2009).

The current research work focuses on the multiple fault identification in the rotor-AMB system and the relevant methods and literature review are presented in the following sections.

1.1 Fault Detection Approaches

Several forces act on rotating machinery when it is in operation and induces different forces due to mechanical faults as well as electrical faults. These forces originate due to imperfect rotor (like a slight bent in the shaft or a mass eccentricity), design faults, and faults in bearings, cracks, misalignment, rotor-stator rub, internal damping, etc. These forces may grow to an unacceptable level when the rotor is operational near its critical speeds. A characteristic of this frequency is that little energy is required to give rise to enormous amplitudes. Hence, dynamics of the rotor systems are enormously studied, and also the fault detection and diagnosis techniques find a lot of applications. The most effective way

of supervising the mechanical faults (or to get the inside and outside internal condition of a machine while in operation) is the vibration based monitoring technique in the time, frequency, or modal domains. All healthy machines have certain vibration signatures and the development of some faults in them changes the vibration characteristics in a way that would be related to that faults. This leads to an increase in the amplitude of vibration in a system. Vibration data are acquired by the sensors mounted on machines and this data is widely used in fault diagnosis of rotors, bearings, gearboxes, and electric driven machinery (Gosiewski and Muszyńska, 1992; Muszynska, 2005; Tiwari, 2017).

The fault detection approaches for rotating machines are shown in Figure 1.2. The approaches are broadly classified into three categories viz. model based, signal processing based and data driven approaches. The model based fault detection method is a traditional method and it can give deeper insight into the functioning of the system, which can best describe the physics involved in the development of mathematical model of the system. To develop a mathematical model a lot of expertise in the relevant system to be modeled is required. Thus, to ensure that while taking assumptions in modeling, the model is neither too simplified nor highly complex to identify the system. In the model based fault detection technique, besides output signal, input system parameters and the mathematical model to be developed (parameter identification) or assumed (observer based) are used. The performances of different model based fault diagnosis methods are shown in Table 1.1 (Isermann, 1997; Markert et al., 2001). The signal based fault detection method utilizes the features (e.g., spectrum) extracted from the vibrational data and compares that feature with the desired healthy signal features.

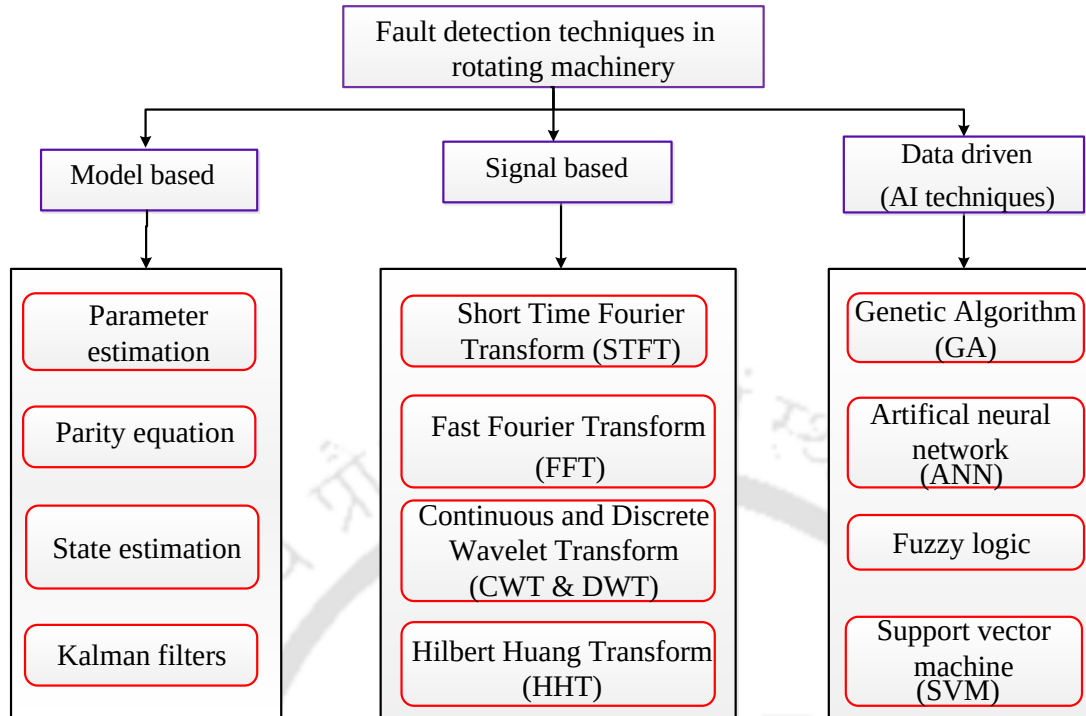


Figure 1.2 Different fault detection approaches in rotating machinery

Many endeavours have been reported by researchers about the implementation of artificial intelligence (AI) in fault detection and diagnosis due to their robustness and adaptation capabilities along with no requirement of a prior physical knowledge of the system. There are several AI techniques including classifiers and statistical learning methods for monitoring of the machine, such as the Artificial Neural Network (ANN), Fuzzy Logic (FL), Support Vector Machines (SVM) and Deep learning (Korbicz et al., 2012; Liu et al., 2018). These techniques use heuristic and model based knowledge with measured data to develop an expert system for the fault diagnosis. The review articles on the application of fault detection and diagnosis methods, such as the model based, data driven and signal based methods are illustrated in Refs. (Ma and Jiang, 2011; Soeffker et al., 2016).

Table 1.1 Model based method performances in fault diagnosis

Model based methods	Performance
Parity equations	Useful for additive fault ¹ diagnosis
State estimation	Useful for additive faults in deterministic situation
Parameter estimation	Useful for multiplicative fault ² diagnosis
Kalman filter	Useful for additive faults in a system with stochastic disturbances

For the health monitoring of a machine, the signal based monitoring systems are usually widely used as a tool, and this technique is generic and not machine specific. Moreover, the information contained in the vibrational response of the machine to be monitored is not fully utilized, and interpretation of the vibrational data is based on qualitative information and not quantitative. On the other hand, the model based identification of fault detection technique, the recorded vibrational response along the physical model of the specific rotor system are utilized for the identification of rotor fault parameters. Besides this, the intact or healthy system vibrational data is also utilized in the identification methodology. With this methodology, the fault type, its severity, and location can be estimated by including a priori information of some of the known system parameters in the identification process. Thus, the estimation can be expected with more accuracy, reliability and in most cases during operation without intervening in the operation of the machine (Platz et al., 2000; Markert et al., 2001). Schematic of the model based and signal based technique in fault detection is depicted in Figure 1.3.

¹ An additive fault such as sensor bias and process leak leads to residuals that are independent of values of an observed system variable.

² A multiplicative fault such as surface contamination reflects changes in plant parameters, leading to residuals that are dependent on system variables.

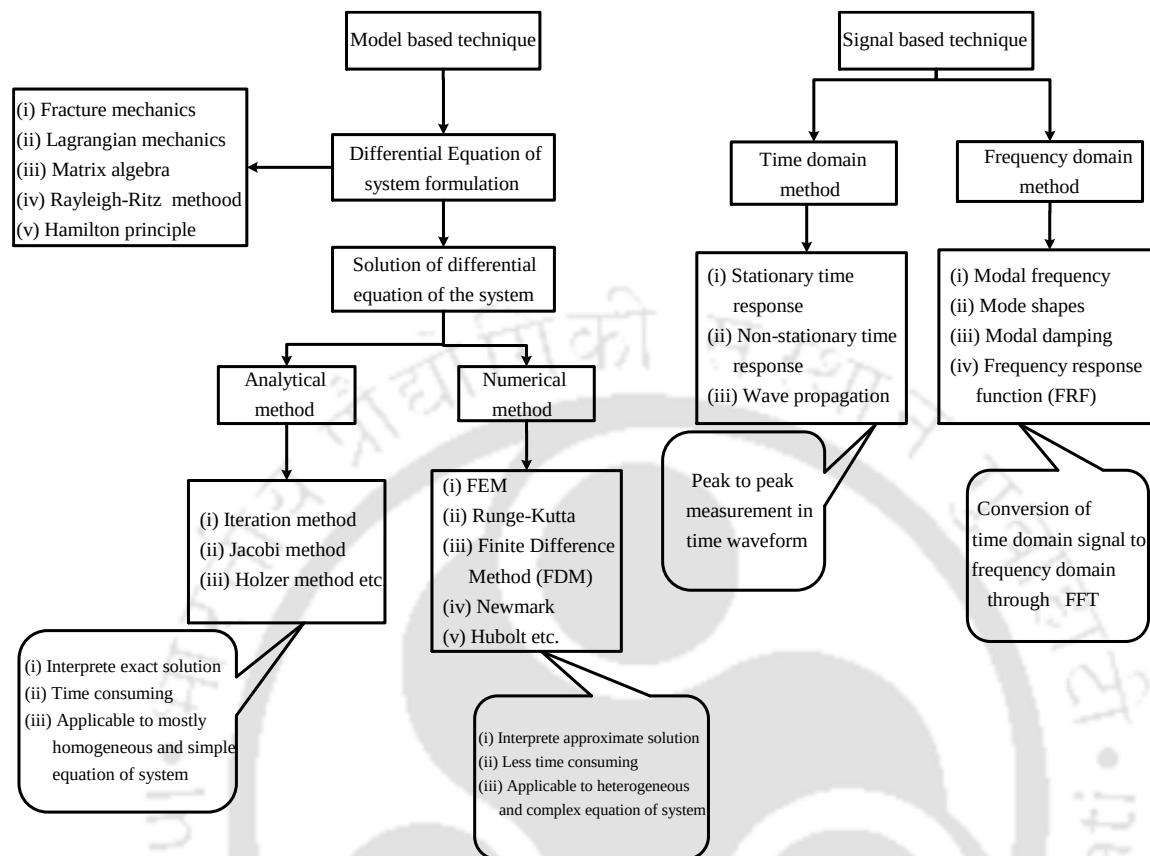


Figure 1.3 Comparison of model based and signal based methods in fault detection

Multiple rotordynamic faults in dynamics systems can be detected by the proper design of model based methods (Clark, 1978; Watanabe and Himmelblau, 1982). If the faults are not included in the modeling stage, they cannot be detected at all, and further, the robustness against measurement noise and modeling errors, and uncertainties need to be considered in the whole identification procedure as real test signals have some inconsistencies in the data due to inevitable measurement and other errors (Chow and Willsky, 1984; Lou et al., 1986; Patton and Chen, 1997; Ma et al., 2013).

1.2 Active Magnetic Bearing System

In industrial application, active suspension of the shaft in a magnetic field with the advancement of electronic control strategy was sufficiently developed in the early '50s and receiving attention in research in recent years. The main dominating research in high-speed machinery is found to achieve digital controlling and vibration elimination (Gosiewski and Muszyńska, 1992; Gryboś, 1994). AMBs help in achieving optimized efficiency, high reliability, and sustainability in rotating machinery. It plays a vital role by allowing a contact-free hovering of the rotor with no lubrication and thus offering many interesting features. On the other hand, an AMB is prone to electromagnetic (e.g., eddy current, Ohmic) losses that are proportional to the total flux at the bearings (Baloh et al., 2000; Ghosh et al., 2000; Knospe, 2000). Thus, for the efficient design of AMBs, minimization of flux is very much essential. AMBs allow very high operating speeds (around 1, 00,000 rpm) in smaller rotors and moderately high speeds in rotors weighing several tons. Also it can handle variable loads without any friction and wear, and thereby, increasing the life of the machine. It does not require high maintenance, which increases the use of AMBs in high performance applications in the rotating machinery domain as in Refs (Schweitzer, 2011; Yoon et al., 2012; Uzhegov et al., 2017; Li et al., 2018).

1.1.1 Working Principle and Components of AMBs

The components of AMB comprise a stator with rectangular poles, coil windings, power amplifiers, a controller with a suitable control strategy, and the rotor core. AMBs can provide very low radial stiffness, allowing the rotor to operate in its center for automatic balancing. In the active magnetic bearing system, the rotor to be levitated is in the desired

position relative to the stator by the electromagnetic actuator generating a significant amount of controlling forces. The inductive sensors continuously monitor the position of the rotor, and the amplified current passes through the windings of the electromagnets of the actuator and produces magnetic forces around the poles to keep the rotor in its equilibrium position for stable operation. The graphical representation of the complete AMB system is depicted in Figure 1.4.

A magnetic rotor is suspended by an electromagnetic stator. To achieve active control of the rotor, its position is measured by a position sensor. The position signal is then sent to the controller, which gives a current set point. This corrected signal is then amplified by the power amplifier, to get the necessary actuator current and sends it to electromagnets of the AMB stator. This control current in a closed loop induces a magnetic field around the electromagnets, which attracts the displaced rotor back to its equilibrium position for stable operation. In real-time, the whole process takes place thus the levitation of rotor and active vibration control can be achieved during machinery operation. The basic components of AMB are shown in Figure 1.4.

Magnetic Actuator and Rotor Core

The actuator of an AMB is made of ferromagnetic material for conducting the magnetic field to the air gap between the stator and rotor. The magnetic permeability, as well as magnetic saturation of the material, has to be high. To minimize eddy current losses, the magnetic actuator (stator) and the rotor core usually consist of insulated laminated sheets. The rotor core is to be mounted on the non-magnetic shaft with tight manufacturing tolerances in order to avoid unbalance. Electromagnets are arranged in pairs with coils for each degree-of-freedom (DOF) in a rotor system. To generate magnetic forces around the

electromagnets in two perpendicular directions, at least three electromagnetic poles wound with coils are necessary. Various design aspects of the magnetic actuator based on the static and dynamic load-carrying capacity of the rotor-bearing system are available in Refs. (Chen, 1989; Larssonneur, 1990; Bornstein, 1991; Maslen and Schweitzer, 2009; Pilat, 2010).

Controller

For real-time processing, Digital Signal Processors (DSPs) are popular. The analog position signal of the rotor measured through the position sensor is converted to a digital signal with the use of an A/D converter and then it is sent to the plant controller. That digitized signal is converted back to an analog voltage signal through a D/A converter for sending that to power amplifiers. The performance and flexibility of digital controllers used in AMBs are investigated in Ref. (Williams et al., 1990). Controller exhibits phase lead or lag characteristics.

Power Amplifiers

The power amplifiers are the electronic circuit for amplification of the low output signal of the controller to a high power input signal to the magnetic actuator. There are basically three categories of power amplifiers, such as linear analog amplifiers, switching mode amplifiers (Liu et al., 2004), and hybrid amplifiers. In the industrial application of magnetic bearing drives, switching power amplifiers are gaining importance. The linear analog amplifier consists of the push-pull transistor at the output having a current rating of less than 10 A and has low efficiency and serious heating effect in heat sinks. On the other hand, switching amplifiers are compact in size, highly efficient and offers a good dynamic characteristic due to which it is widely used. Hybrid amplifiers are acted as a combination

of the above two types based on the current flow, i.e. the push-pull transistors operate as a linear amplifier at low current whereas at high current they operate in switching mode.

The *pulse-width-modulation* (PWM) amplifiers are integrated with switch logic circuits, driving circuits, and PWM circuits inside of the power drive. In PWM switching mode amplifiers operate on the principle of switch ON/OFF employing virtually there is no voltage drop across the switch and power is being transferred to the load. While at ON condition, practically no current flow through the switch during OFF condition for the directed signal from the controller. The main advantage of PWM switching amplifiers is that power loss is very low. Detailed information is available in Refs. (Changsheng and Zhiwei, 2005; Ren and Fang, 2011).

Sensors

Sensors are an integral part of the AMB and their performance depends on the characteristics of the displacement sensors used. Eddy current sensors are the most widely used gap sensors in AMB applications. These sensors work in the principle of magnetic induction. When AC flows through the sensor coil of the eddy current probe, the magnetic field is generated and thus induces the eddy current around the target object. The eddy current circulates opposite of the coils thus, reducing the magnetic field and thereby its inductance in the active electromagnet coil. As the movement of the target or rotor changes, there is an impedance change at the coil terminals. An eddy current sensor has a resolution of up to nanometre size displacements and measures 10 microns to 1 mm of total accuracy. The typical sensitivity of these probes is 787.44 V/mm. Other types of non- contactable displacement sensors are inductive, capacitive, magnetic, optical, combined displacement-velocity, photo-electric, and laser sensors (Schweitzer and Maslen, 2009; Tiwari, 2017).

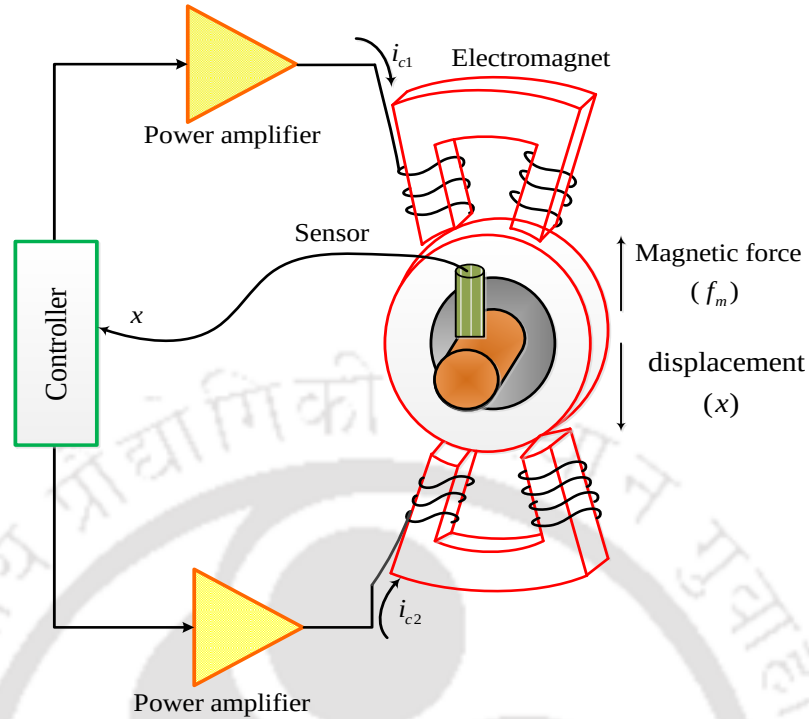


Figure 1.4 Working principle of AMB assembly

1.1.2 Construction of AMBs and their Characteristics

Active magnetic bearings are designed to support radial as well as thrust (axial) loads. The radial one possesses a cylindrical air gap design, a flat air gap for the axial one, and AMBs that support both radial and axial one have a conical air gap between the stator and the rotor. The construction and shape design of the active magnetic bearing (AMB) stator is a crucial and most important aspect of the magnetic field generated around the target object to control and levitate.

The linearized force for active controlling is the superposition of two reluctance forces in opposite directions. These forces are generated between the air gaps of the rotor and the stator of the magnetic actuator. The bias current (i_b) flowing through the coils provides bias flux around the air gaps (s_g) and produces destabilizing force if the rotor is

not at the equilibrium position. That force tries to shift the rotor position further away from the center position. The controller sends the control current (i_c) to generate corrective flux sending through the actuator coils. That corrective flux is added where the air gap is more and subtracted from the lower gap of rotor and actuator stator. The force for a pair of electromagnets is the difference in forces between the positive and the negative directions. When magnetic force attracts the rotor, the distance between the rotor and the stator becomes $(s_g + x)$ and in the other direction $(s_g - x)$. For this case, the actuator currents are $(i_b - i_c)$ and $(i_b + i_c)$ in the positive and negative directions, respectively, to make the rotor bring back to the equilibrium position. The magnetic force is expressed as in Ref. (Schweitzer and Maslen, 2009)

$$f_{mx} = f_+ - f_- = k \left\{ \frac{i_b + i_c^2}{s_g - x^2} - \frac{i_b - i_c^2}{s_g + x^2} \right\} \cos \alpha \quad (1.1)$$

where $k = \frac{1}{4} \mu_0 N^2 A_a$ with number of turns of coil as N , the cross-sectional area of flux in the air that is equal to the area of the iron core as A_a , half angle of pole pair at the center of the actuator as α , and the magnetic permeability of actuator material as μ_0 .

On linearizing the magnetic force in Equation (1.1), considering $x \ll s_g$ and $i_c \ll i_b$, and using Taylor's series expansion, is represented as

$$f_{mx} = \frac{4ki_b^2}{s_g^2} (\cos \alpha) i_c + \frac{4ki_b^2}{s_g^3} (\cos \alpha) x = k_i i_c + k_s x \quad (1.2)$$

where, $k_s = \frac{4ki_b^2}{s_g^3} (\cos \alpha)$ and $k_i = \frac{4ki_b^2}{s_g^2} (\cos \alpha)$

Therefore,
$$k_s = k_i \left(\frac{i_b}{s_g} \right) \quad (1.3)$$

The force-displacement coefficient k_s is the open loop stiffness of the bearing, which is negative and the force-current coefficient k_i is the actuator gain. On the basis of the magnetic polarities around the stator with respect to the rotor, homopolar and heteropolar actuator in two different structural configurations are found for radial AMBs.

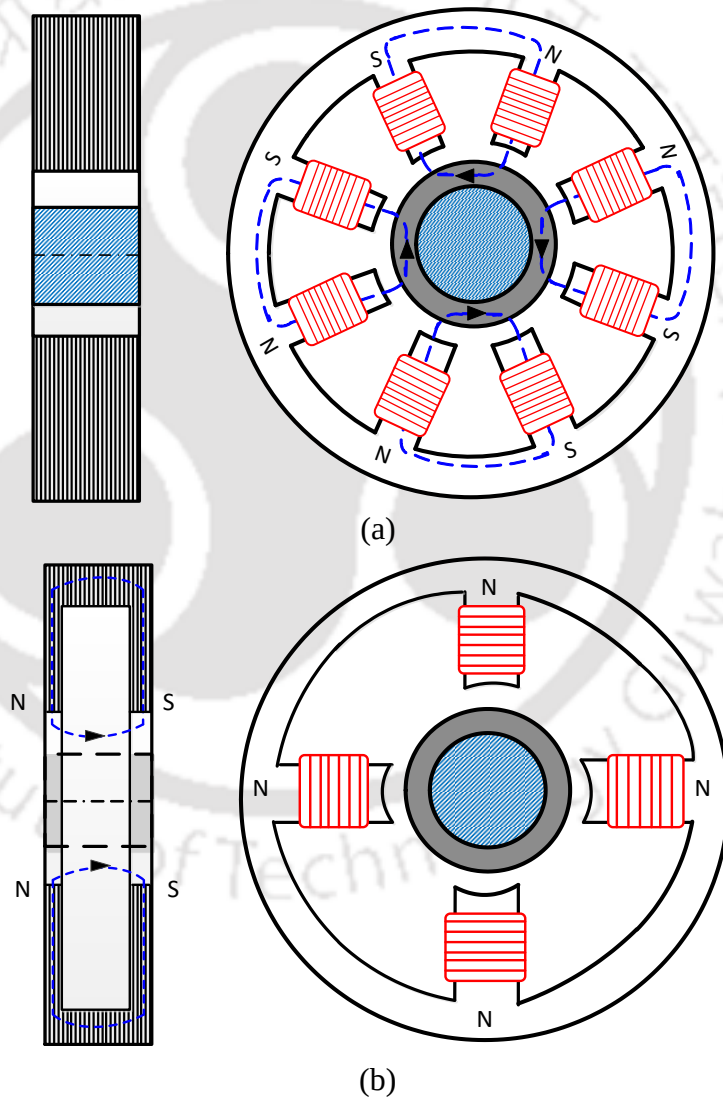


Figure 1.5 (a) Heteropolar AMB with alternating poles (b) Homopolar AMB with same poles

If the stator poles in a given rotor transverse plane have the same magnetic polarities, then it is termed as homopolar whereas if polarities of stator poles in a given rotor transverse plane vary, it is termed as heteropolar actuator. The magnetic flux path in the two such configurations is shown in Figure 1.5. More details on the design and construction of AMB can be found in Refs. (Schweitzer and Maslen, 2009; Pilat, 2010; Mushi et al., 2012).

1.1.3 Control Strategy

AMBs can actively damp out the vibrations of the rotor by letting the rotor motion about its inertial axis and eliminate the dynamic forces caused by unbalance and other mechanical faults. A magnetic actuator enables to control individually along each transverse axis in a rotating system. The implementation of digital controllers permits the magnetic bearing to use a wide range of control algorithms for stabilization of rotor as well as vibration attenuation. The control strategies encountered in the magnetic bearing suspension control system are closed loop feedback, which impacts the stability of the rotor, and open loop feed-forward control, which utilizes a superimposed external control force with the feedback control force for vibration reduction in the rotor system. Many control architectures are found in literature, which can be categorized for active magnetic bearings, such as the decentralized SISO (single input single output) control and the centralized MIMO (multiple input multiple output) control. In the decentralized SISO, each actuator works independently from the other and exploits sensor information, and uses a dedicated control law. On the other hand, in the centralized MIMO control strategy, the action of actuators takes place together with the sensor information. A single control action is used to feed the power amplifiers (Schweitzer and Maslen, 2009).

Several control strategies have been implemented on rotors equipped with active magnetic bearing, such as the gain scheduled control, adaptive control, H_∞ control, μ -synthesis, robust sliding mode control, genetic algorithm control, fuzzy logic control, etc. (Ortega and Tang, 1989; Matsumura et al., 1996; Hong and Langari, 2000; Fittro and Knospe, 2002).

1.1.4 Advantages of AMBs

AMBs have numerous advantages over conventional bearings, such as the fluid-film bearings and rolling element bearings, which physically interface with the rotor and require some sort of lubrication thus, leading to chances of friction due to the contact stresses between two mating surfaces, whereas, in magnetic bearing as the target rotor is suspended in the magnetic field resulting in a contactless bearing system with extremely low friction. Some of the advantages are listed here Ref. (Schweitzer and Maslen, 2009)

- No contact between rotating and stationary components resulting in minimal friction thus reducing routine maintenance and cost-effective from an economical point of view.
- Low power losses allow the machines to achieve high operating speed (range from cryogenics up to 450°C) encompassing high efficiencies and increasing the machine life, unlike the conventional mechanical bearing.
- AMBs can operate in adverse environmental conditions such as extremely low and high temperatures and corrosive environments.
- Sophisticated electronic control to keep the rotor position with high accuracy as the rotor is displaced inside the air gap, which provides an accurate tool positioning in machine tool applications.

- The magnetic bearing has built-in controller firmware, which can easily measure and if required can change the dynamic properties, like the stiffness and damping of magnetic bearing. On the contrary, if the dynamic properties of conventional bearings change, it requires remodeling, remanufacture, and reinstallation of the bearings, which is more cumbersome, much more expensive, and time consuming.
- Health monitoring, dynamic testing, etc., are inbuilt in the control electronics of magnetic bearing, which is not found in conventional bearings and requires external supply and implementation.

1.1. 4 Challenges of AMB

Despite several advantages cited above, AMBs have some limitations also that arise from two reasons: the design technology and material, and the basic physical relations as in Ref. (Schweitzer, 2002).

- Excessive heat generation due to iron losses caused by hysteresis and eddy currents, and air drag losses in the stator and rotor core laminations. Secondly, the wire resistance in the copper wound coils increases temperature resulting in copper loss. The heat may break down the coil insulation and cause electrical short circuits thereby eventually destroying the magnetic bearing with limiting the maximal achievable carrying force.
- Fault-tolerant as the placement of backup bearings, which are mostly the rolling element bearings and pressurized bearings may change the rotordynamic behavior of the system and threats to the life of the backup bearings.

- Inadequate direct stiffness of magnetic bearing requires large surface area in the stator core to carry the loads over it. Magnetic bearings have low stiffness compared to hydrodynamic and externally pressurized bearings.
- To get high precision of the position of the rotor equilibrium point (in the range of μm) requires high resolution sensors and adequate signal processing to separate disturbance signals from the desired ones.

1.1. 5 Applications of AMB

The application of AMBs is continuously developing and finding ideal applications in high-speed machinery, such as turbochargers, compressors, spacecraft, high-speed drive shafts, centrifuges, vacuum pumps, etc. with low vibration that can help in overcoming challenges in industries. There are two trends in AMB application: firstly, operating the machinery in the field of vacuum technology, machining at high speeds gaining more importance. Secondly, numerous use of digital control over analog one, which offers better performance including system monitoring capabilities, easier system integration with reduced design time and offers much more flexibility to adapt the advantage of the possibilities provided by AMBs (Schweitzer, 2002).

AMBs find wide application in the gas turbine industry by improving the turbine engines with lightweight, enabling its operation in high altitude, no oil, lube, and cooling system, mounting bearing in the hot sector of the engine, control of blade tip clearance, fault tolerance, controlling of shaft unbalance and dynamic parameters of bearing. Furthermore, the magnetic bearing replaces the gearbox driven starter and alternator in gas turbines with integral starter generator (ISG) technology (Storace et al., 1995; Clark et al., 2004). High speed machinery, such as the high efficiency blowers, organic Rankine cycle

systems, maglev trains, compressors for natural gas facilities, chillers in large urban buildings, etc., employ AMB for high reliability and efficiency.

The increasing demand of the cost effective space mission has led towards the development of compact spacecraft to attitude control with compact and low mass magnetic bearing actuators in the reaction wheel (MBRWs), thus, withstanding the micro-vibration disturbances generated by the conventional ball bearing reaction wheel (BBRW) (Heimel, 2011; Le, 2017). The contactless operation of active magnetic bearing also can accommodate various anomalies in the distribution of mass automatically to allow the shaft to spin around its center of mass with minimal vibration and attenuates effectively the nutation and rotor precession. The more application of AMB and its technology is accessible in Refs. (Kasarda, 2000; Schweitzer, 2011).

1.2.1.1 Active Vibration Suppression and Control Approaches in performance of AMBs

Active balancing and active vibration control on rotors can be achieved in real-time with the use of AMBs. In the active vibration technique, the lateral control force generated by an actuator of magnetic bearing is applied to the rotor. That lateral force can suppress the synchronous vibration, transient vibration, and non-synchronous vibration of rotating machinery. Furthermore, with the help of suitable control algorithms for the rotor system, the magnetic bearing can precisely control the rotor position within microns and attenuate vibrations. The survey of active balancing and active vibration control is detailed in Ref. (Zhou and Shi, 2001). Robust control in rotating machines is discussed elaborately in Ref. (Lantto, 1999). A state-of-the-art review on vibration control in the flexible rotor with AMB application is overviewed by Srinivas et al. (2018).

Shiau et al. (1997) developed a hybrid model of a flexible rotor–AMB system and H_∞ control strategy to apply in it to suppress the spillover effect, and thus the vibration. The hybrid model includes the finite element as well as generalized polynomial expansion method and is employed to model the flexible rotor system. AMBs are often configured to operate with a proportional–integral–derivative (PID) feedback control scheme. Chen and Knospe (2005) employed a feedback linearization with the robust control technique for single-axis control of a rotor-AMB test rig in current control mode. Polajžer et al. (2006) implemented a cascade decentralized PD and PI control technique in cascade connection form to the rotor-AMB system and was able to achieve high damping and stiffness of the overall system. The design of the control system is based on a linearized model for a single axis using the root-locus method.

Rao and Tiwari (2009) proposed a methodology of an optimal design of double-acting hybrid thrust active magnetic bearings. Multi-objective Genetic Algorithm (GA) for two different bearing geometry and different operating parameters are utilized while considering the objective functions as the minimum power loss, reducing weight, control input, and maximization of the load capacity with geometrical, electrical, and control constraints. Ding et al. (2010) applied the PD controller, direct fuzzy controller, and fuzzy PD controller schemes through the dSPACE platform in a rigid rotor-AMB system. They measured the step response, tracking a sinusoidal reference signal and simulation rotating debugging to analyze the performance of each controller.

Psonis et al. (2015) proposed a PID controller strategy on the stability performance of linearized magnetic bearing and the boundary conditions for the existence of the stability of the system are found based on the system's characteristics equation and the Routh-

Hurwitz criterion. Huang et al. (2015) developed an optimal controller with a proper compensation in speed variation to suppress the vibration occurrence due to chatter in the milling process. Later, an adaptive control algorithm based on the Fourier series is accomplished to achieve a high removal rate without wear in the machining operation. Most of the researches, considers the design of PID controller considering constant stiffness of radial AMB (RAMB) irrespective of speeds as in Refs. (Schmied et al., 1999; Fang et al., 2012; Mushi et al., 2012). With the increase of frequency, the stiffness of RAMB gets reduced leading towards instability. At high speeds, the amplitude and phase lag of stiffness may occur due to eddy current affecting the dynamic performance and system stability of the overall AMB system (Knospe and Zhu, 2010; Walter et al., 2013). Sun et al. (2018) proposed a PID parameter tuning strategy for radial AMB system based on dynamic stiffness to improve the efficiency and reduce debugging risk under high speed operation. The extensive use of PID controllers is found in advanced motion control, manufacturing industries, power electronics, hydraulics and in modern applications, such as the self-driving cars, unmanned aircraft vehicle, and autonomous robots as in Ref. (Díaz-Rodríguez et al., 2019). Zheng et al. (2019) illustrated a controlling technique of synchronous vibration due to imbalance force in rotor-AMB system. They proposed a model-free control (MFC) method based on the Newton-type ILC algorithm. The development of classical PID to the integration of intelligent control used for tuning of PID and its applications in various domains is reviewed by Borase et al. (2020).

Several control laws have been developed, implemented, and compared in pursuit of better vibration control in rotor systems with AMBs. It is observed that time-tested SISO controllers still find popularity for low speed applications, while MIMO control laws have

found application in high-speed rotating systems. A comprehensive discussion of AMB control technology is done in Huang et al. (2019). In the next section, various faults in a rotor and their detection and diagnosis in rotating machinery are presented. The fault detection with AMB is also enunciated in the next section.

1.3 Diagnosis of Faults in Rotating Machinery

Fault diagnosis in rotating machinery is a crucial aspect and often increase in vibrations leads to instability due to self-excited vibrations caused by various factors. Moreover, rotors often suffer vibration because of the several external forces and moments on the rotors due to residual unbalance, cracked and warped shaft, misalignments, rotor-stator rub, etc. The vibration due to common faults, such as the unbalances and misalignments affects critical components of the rotating system, such as the gears, motor, seals, couplings, bearings, etc. Review articles on fault diagnosis of multiple rotordynamic faults in rotating machinery are presented by (Edwards et al., 1998; Bachschmid et al., 2002a; Lees et al., 2009).

1.3.1 Crack Fault Detection and Diagnosis

Fatigue cracks are prone to cause catastrophic failures in rotating machinery, if undetected and undiagnosed in time. The occurrence of cracks in real machines, such as the steam turbines or generators of European power plants has been documented in the early 1960-70s and discovered the propagation of transverse crack detection before it traverses the critical path leading towards catastrophic failure (Schmerling and Hammon, 1976; Jack and Paterson, 1977; Greco et al., 1978; Nilsson, 1982). It is expected to check the occurrence of a rotor crack using ultrasonic testing method or dye testing method in

periodical inspections during general system maintenance. However, these methods are costly and find difficulty in early crack detection and are not very popular among maintenance engineers as in Ref. (Inagaki et al., 1982). Different crack modeling approaches and their detection is surveyed in Refs. (Sabnavis et al., 2004; Kumar and Rastogi, 2009; Soeffker et al., 2016).

1.3.1.1 Various Types of Cracks and its Causes

Continuous operation of rotordynamic equipments are subjected to heavy loads and suffers from low and high cyclic loading or stress concentration in the sharp geometric areas, thus, expecting to produce fatigue failure in the machine. The sequential order of a typical machine failure due to fatigue crack is best illustrated in (Bloch and Geitner, 1997; Stephens et al., 2000). Crack initiation, propagation, and failure are the three stages of failure due to cracks in machinery. The crack initiates like a tiny discontinuity in a rotating part due to asymmetry in geometry, stress concentration in keyways, shrink fit, dent, or, some metallurgical factors such as the inclusion of debris, porosity, etc. After the initiation of the crack, its propagation in the induced component starts due to continuous cyclic loading, which has detrimental effects on the reliability of the machine. When the crack growth accelerates under different operating conditions and reaches its critical size due to loading, failure occurs in the rotating equipment. Furthermore, depending on the geometries the crack can be classified into different categories, such as the transverse crack, longitudinal crack, slant crack, and circumferential crack. The development of cracks in the generator and blades of the steam turbine is picturized in Figure 1.6.

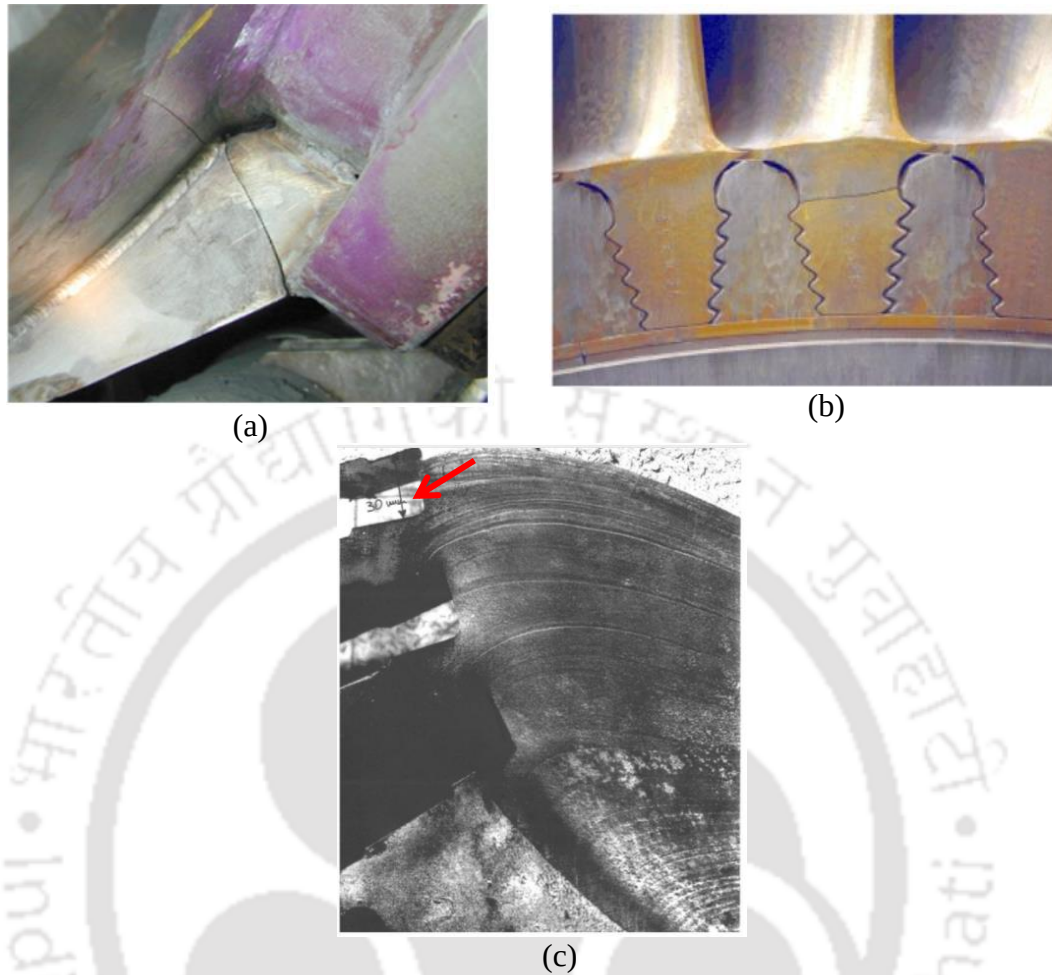


Figure 1.6 Development of crack (a) stiffening plate of a generator (b) blade root of a steam turbine (c) cracked surface of a generator Ref. (Bachschmid et al., 2010)

The most common and serious crack geometry occurs perpendicular to the axis of the shaft by sinking the cross-sectional area near the cracked portion causing severe damage to the system known as the *transverse crack*. The result of this type of crack is due to fluctuating (cyclic) bending stress that has a great influence on the dynamic characteristics of the rotor system (Gasch, 1993; Dirr et al., 1994; Pennacchi et al., 2006). If the rotor system is subjected to cyclic torsion, then the *longitudinal crack* appears, which is parallel to the shaft axis and this kind of crack has comparatively lesser influence on the dynamics of the

system (Wauer, 1990b; Darpe et al., 2004; Abdi et al., 2014). If the geometry of the crack appears at an inclination or an angle with the shaft axis, is commonly designated as the *slant crack*. These cracks are analogous to the helicoidal crack and are primarily generated due to torsional loading. The rotor having slant crack is much stiffer in lateral and longitudinal directions, however more flexible in torsion as compared to that with the transverse crack (Sekhar and Prasad, 1997; Darpe, 2007; Lin and Chu, 2010). *Circumferential cracks* appear and grow perpendicular to the rotor axis due to cyclic torsion in the shaft.

1.3.1.2 Different Crack Modeling Approaches and their Detection

The model based identification primarily requires an accurate mathematical model that closely narrates the dynamics of the system. For the crack identification, modeling of crack fault, which affects the dynamics of the rotor-bearing system, needs to be given significant attention based on the opening and closure of the cracked face periodically during the revolution of the shaft. This phenomenon represents the breathing mechanism of the crack in the rotating system, unlike in the stationary structure this mechanism is a function of the applied load on it. In this section, various modeling approaches adopted by the researchers have been categorized in detail.

Isolating the vibration signature of the damaged rotor system requires finding efficient tools for the different types of cracks modeling, which may help in studying the actual dynamic behavior of the cracked system. Once the dynamic behavior is identified via correct theoretical model, real life vibrational signatures can be verified through experimental data. Modeling of crack mainly depends on the stiffness, damping, and non-linearity in the shaft system. Crack modeling based on strain energy release rate (SERR) is

reviewed by Papadopoulos (2008). Few papers on modeling of a crack in a rotating structure due to thermal effects (Bachschmid et al., 2003) and damping (Wauer, 1990a) are available.

In the initial stage, literature dealing with the crack modeling in rotors, mainly studied the bending vibration conceptualizing the fracture mechanics approach to evaluate the asymmetry in the shaft due to the crack. The main idea of most of these papers was to model the crack with a 6×6 (or less) flexibility matrix, which had non-diagonal terms indicating coupling between the longitudinal and lateral vibrations (Papadopoulos and Dimarogonas, 1987; Wauer, 1990a; Papadopoulos and Dimarogonas, 1992). Mayes and Davies (1980) recommended a sinusoidal stiffness variation (of steering function

$$f(t) = \frac{1 + \cos \Omega t}{2} \text{ with } \Omega \text{ is the spin speed) in order to model the breathing behavior of}$$

the crack, since a rotor crack is expected to open and close gradually under the action of gravity. Grabowski (1984) proposed the crack model where switching of the stiffness magnitude taken place from an intact rotor (closed face of the cracked rotor) to those of a cracked rotor (fully open) at a particular rotor angular position. The stiffness variation of a cracked portion of the rotor is represented by way of a truncated series using known stiffness matrices corresponding to a fully open, a partial-open-partial-closed, and a fully closed crack by Papadopoulos and Dimarogonas (1987, 1988, 1990). Nelson and Nataraj (1986) developed a finite element model of crack considering the rotational stiffness variation, which depends on the bending curvature of the rotor at the location of the crack. The switching crack excitation function is expanded by Fourier series and the analysis is done by perturbation method with iterative technique. Li et al. (1989) introduced a generalized hinge model of crack for both horizontal and vertical rotors with variable

stiffness. The crack was modeled using the finite element model (FEM) and introduced at the node point of the element. The stiffness variation depends on the direction of the bending moment at the crack cross-section. Ostachowicz and Krawczuk (1992) developed a FEM model of an open crack based on the fracture mechanics technique. They used a modified stiffness matrix to account for the torsional coupling along with bending vibration. Sekhar and Prabhu (1994) carried out a finite element analysis of a simply supported shaft containing a transverse crack on it. The local flexibility due to crack is represented by 6×6 matrix with six DOFs shaft element. They investigated the presence of backward whirl due to the crack and addressed the severity of bending stress fluctuation in shafts due to the backward whirl effect.

Gasch (1993; 2008) modified the approach of (Grabowski, 1984) and suggested that switching of stiffness happened not at a particular angular position, but only when there is a change in the sign of rotor deflection perpendicular to the crack edge direction. Here, a hinge model is introduced to show the behavior of crack during rotation. The hinge modeling of crack comprises a discrete spring (hinge model) which can be divided into two parts that are pinned at the crack location and the crack is evaluated by the rotational and torsional springs. The hinge opens in the tension zone and closes in the compression zone of the shaft. He derived a linear equation of motion of a Laval rotor considering the assumption of weight dominance, which was valid even in the unstable ranges of rotation. A steering function of the crack excitation force is developed as a square wave in Fourier series showing opening and closure of cracked faces for smaller cracks and for deeper cracks Mayes steering function is mentioned earlier as shown in Figure 1.7.

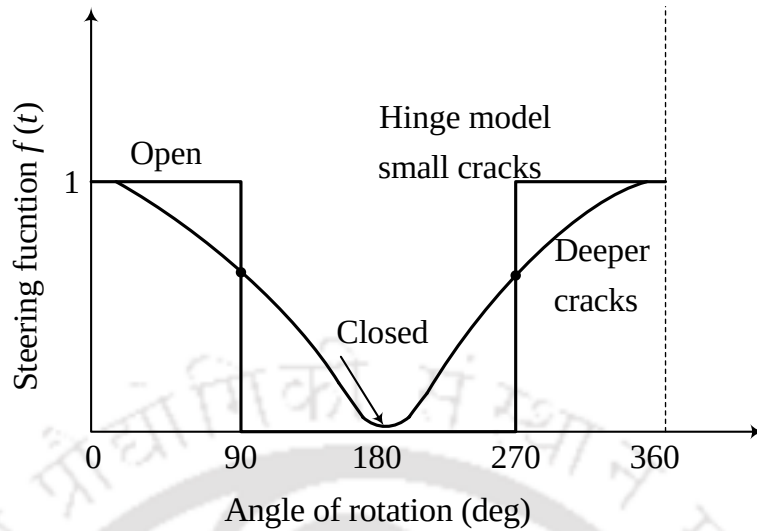


Figure 1.7 Crack opening and closure behavior for the hinge model (weight dominance assumption) and Mayes cosine function for deeper cracks

The crack response (with no unbalance) is observed, which is proportional to the static deflection of the shaft and the stiffness loss due to the crack, i.e. the additive stiffness. The hinge model is not limited to only weight dominant rotors, it can be extended to other applications of rotors as well. The stability analysis is performed with Floquet's theory. Friswell and Penny (2002) studied the performance of different crack models including the breathing crack effect where the stiffness is bilinear in nature, for structural health monitoring purposes. They concluded that crack model with beam elements is more adequate in low frequency vibration monitoring. Keiner and Gadala (2002) discussed the transient analysis of a 3D beam finite element model of crack by avoiding the assumptions of the shaft and crack geometry, local crack compliance, crack breathing mechanism, and dominant static deflection.

Chasalevris and Papadopoulos (2008) investigated the finite element modeling considering Euler-Bernoulli beam elements of multi cracks on a stationary shaft. The

modelling of crack is done considering a spring which is function of the rotational angle and the crack depth with a 2×2 compliance matrix comprising two non-diagonal terms. The experimental observation of the intense coupled bending response phenomenon with deeper cracks and double peaks due to the coupled effect are discussed. They also mentioned that the phases of the two cracks if it is in the same phase providing strong coupling vibration whereas out of phase condition gives weaker that.

Patel and Darpe (2008) investigated the switching/hinge crack model, in which the stiffness of the rotor switches from one corresponding to the closed crack to the stiffness corresponding to the fully open crack state, i.e. sudden opening and closure of crack. Al-Shudeifat (2013) addressed a finite element modeling of an open transverse crack of a rotor system as in heavy duty machines and found after analysis that advanced propagation of breathing crack may be expected to be dominated by open cracks. He considered the open behavior of crack in a transverse edge crack and formulated the periodic finite element stiffness matrix. The time-varying area moments of inertia of the cracked section are employed and the Harmonic balance method is utilized for solving the system equation. It was observed that the presence of open crack and unbalance response of the system is responsible to excite the rotor at $1/2$ and $1/3$ of the backward critical whirling speed unlike in the breathing crack models, where the unique whirl orbits are found while passing through the subcritical forward whirling speeds. Yang et al. (2021) reviewed various analytical crack models for the rotating blade and limitations are discussed. They illustrated modified the crack models such as stress-based breathing crack in Fracture mechanics approach, bilinear and cosine breathing crack models. The analysis of the dynamics of the

system with these modified crack models shows much physical interpretability through vibration of system.

1.3.1.3 Identification of Cracks in Rotating Machinery

Vibration based monitoring techniques are global non-destructive testing methods, which are based on the fact that any deviations introduced in a system, including faults, change its physical properties, and this, in turn, changes the vibrational response. Shaft crack detection is investigated by many researchers using signal-based approaches and documented (Bently and Muszynska, 1986; Gasch and Liao, 1996). A generalized online crack detection based on vibration signature analysis to detect the incipient transverse crack in rotors is developed by Imam et al. (1989). The crack detection procedure can be broadly classified into two groups as depicted in Figure 1.8. The pattern recognition approach is available in many literatures (Sohn et al., 2001; Manson et al., 2003; Trendafilova and Heylen, 2003). In model updating technique, a crack model is adequate for the task, which utilizes time as well as frequency domain to identify and locate the crack (Doebeling et al., 1998; Bachschmid et al., 2000; Hu et al., 2001; Markert et al., 2001).

Dong et al. (2004) did a vibration analysis of an Euler-Bernoulli beam model of an open crack considering the local flexibility and developed an identification algorithm to predict the location with crack depth in a static structure. The identification uses the deflections of the first mode shape at two symmetric points and the contour diagram of crack location versus crack depth for the first two normalized Eigen frequencies. Signal-based methods are easy to implement for damage detection purposes, but this technique lacks a sound physics of the system with damages. The feature-based choices for one system may not work for others and give a qualitative understanding of measured data rather than the

quantification of faults in a system. Model based approaches that help in this regard are discussed below. The comparison of the model based and signal based crack detection is broadly discussed by Soeffker et al. (2016).

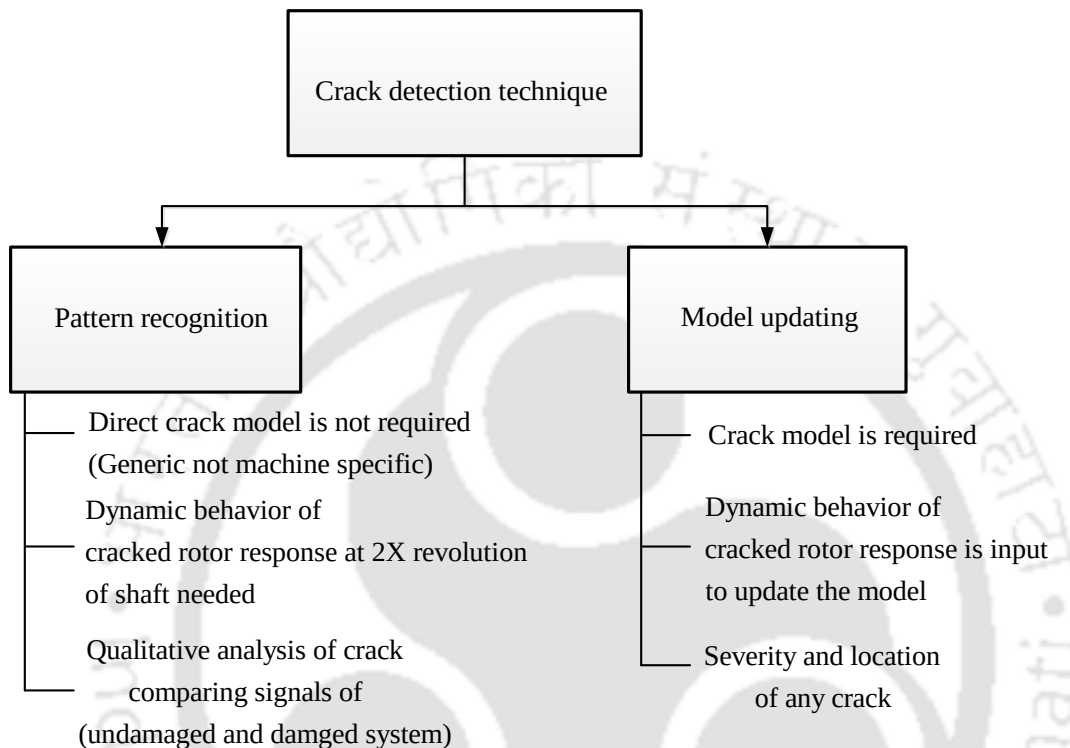


Figure 1.8 Two different categories of the crack detection procedure

In recent years, model based method finds the application in online identification of rotor malfunctions. The models use equivalent loads due to the faults, such as rubbing and unbalance, as virtual forces and moments acting on the linear undamaged system model to generate a dynamic behavior that is identical to the measured one of the damaged system. The identification then carried out by least-squares fitting in the time domain is proposed by (Bach and Markert, 1997; Platz et al., 2000; Markert et al., 2001). Jun et al. (1992) illustrated an identification method for the crack depth in a system considering switching crack behavior of cracked face opening and closure. They observed that the

varying amplitude of harmonics and harmonic dependence of the unbalance orientations would have been a great identification of crack. Bach and Markert (1997) presented a model-based technique for the identification of crack identification, in which, with the prior knowledge of a few measured quantities the full system state is reconstructed by an observer during an iterative procedure and estimated crack parameters through modal expansion.

Sekhar (2003) developed a model based identification of crack in a system using finite element crack modeling. The identification is done based on the equivalent loads concept and the depth and location of the crack are identified. Further, the symptoms of the crack fault are observed through continuous wavelet transform (CWT). Sekhar (2004) again studied the same methodology as above for the identification of crack and its symptom is further observed using fast Fourier transform (FFT). Dharmaraju et al. (2004) investigated the identification of crack flexibility coefficients and the crack depth based on the force–response function. Euler-Bernoulli beam is considered for finite element modeling of the system with an open crack on it. The problem of the excessive number of measurements required for the estimation was overcome by the static (Guyan) reduction scheme to eliminate the need for measurements that are not feasible. An external force of known amplitude and frequency is applied to dynamically excite the beam. The theoretical error function and the flexibility coefficients are utilized for the crack depth identification with the help of the least-squares technique in conjunction with the bisection root–searching method. The identification algorithm was found robust against a moderate level of measurement noise. Pennacchi et al. (2006) proposed a model based crack identification in a suitable industrial machine. They considered three different cracks

having depths of small crack (14% of the diameter) and the third a deep crack (47% of the diameter) and an open crack with a slot of 34% depth. Identification is attained with the observation of 1X, 2X, and 3X revolution components, and the location of the crack is precisely identified by 2X and 3X harmonic components. Xiang et al. (2007) developed a finite element model of *B*-spline wavelet on the interval (FEM BSWI) considering crack as variation in local stiffness to estimate crack location and size in shafts. Identification equations are solved using the surface-fitting technique and contour-plotting method, and illustrated the experimental validation of the algorithm with test data. Sekhar (2008) identified multiple cracks in a structure, such as the beams, rotors, pipes, etc. in different positions and analyzed the complex dynamics of it. The state-of-the-art of modeling aspects of crack, its dynamics, and identification are also discussed.

Shravankumar and Tiwari (2013) identified crack parameter in a Laval rotor with the use of switching crack excitation function. The identification algorithm is solved by the least-squares fitting technique. The crack symptom is analyzed through full spectrum response, which shows forward and backward whirling due to crack and full spectrum amplitude and phases are utilized in the identification algorithm for estimation purposes. Cavalini Jr et al. (2016) proposed an open and breathing transverse crack identification based on a nonlinear approach. The finite element modeling of crack is done using the linear fracture mechanics theory. They use the combinational resonances together with the Differential Evolution (DE) to characterize the signatures of the crack in the spectral responses of the flexible rotor. A preliminary experiment test is done on a horizontal flexible shaft containing a transverse crack, two rigid discs, and two self-aligning ball

bearings to compare with the numerical results. Guo et al. (2017) investigated a fatigue crack in a Jeffcott rotor using empirical mode decomposition (EMD) experimentally.

1.3.2 Residual Bow in Rotors

Residual bow in the shaft rotating machinery may be induced due to various reasons, such as gravity sag, thermal distortions, shrink fits between the rotor and stator, mechanical bow due to prior unbalance, etc. during operation and or a manufacturing deviation. A warp or bend in a system gives rise to synchronous vibration as unbalance creates in a system, but both of them possess different characteristics. A rotating machinery experiences both unbalance and bend or residual bow simultaneously and it requires a clear understanding of the two effects on the system.

Nicholas et al. (1976a, 1976b) published one of the first works about shaft bow and illustrated the effect of bow and unbalance for a single-disc Laval-Jeffcott rotor system through theoretical dynamic analysis. In the second part, they established the balancing of a rotor with a residual bow both theoretically and experimentally. The self-balancing of a machine with both unbalance and bow occurs when the bow and unbalance are out of phase, i.e. 180° apart. The differences in whirl are observed in a lightly damped Jeffcott rotor subjected to the bow, runout, and unbalance and are described by Flack et al. (1982). The Bode and Nyquist plots showed the differences in the bow and runout effects on the rotor. The low speed compensation (the response of the rotor at balance speed is subtracted from that of the low speed case) can be able to balance the system response with runout. On the contrary, the amplitude responses of a rotor with unbalance and bow cannot correct the response to an ideal case, i.e. no bow and no runout case. Shiau and Lee (1989) showed the changes in the dynamic behavior of an unbalanced bowed shaft with disc skewness.

Edwards et al. (2000) used excitation due to bent rotor and unbalance in the model to identify excitation as well as field support parameters of the rotor bearing foundation system, experimentally. Bent in the system is also identified and using the measured test data the balancing of the test rotor system is successfully done by reducing the vibrational amplitude up to one-tenth of the prior one.

Rao (2001) pointed out the importance of phase measurements for the analysis of warped rotor considering different cases of amplitude and phase variations of the rotor bow relative to the unbalance. Important conclusions were drawn of phase behavior for various cases of a warped rotor that are different from that of the purely unbalanced rotor case. Nelson (2002) presented the balancing procedure of the multiple shafts, multiple DOFs system associated with residual unbalance and residual bow effect based on the work of (Nicholas et al., 1976a, 1976b) using linear finite element model. A coordinate reduction strategy using static condensation was implemented to retain only the translational DOFs so that the resulting unbalance correction vector only includes forces and no unbalance moments.

Darpe et al. (2006) did a steady state and transient analysis of the rotor containing both transverse crack and shaft residual bow. Qualitative and/ or quantitative changes in the dynamics of the rotor due to the effect of shaft residual bow is assessed and observed that bow mainly the sensitivity of orbital response of cracked rotor to unbalance phase at half the critical speed. Thus, the influence of the unbalance phase in a cracked rotor at this speed cannot be used for fault detection purposes. Pennacchi and Vania (2008) discussed a model based identification of faults in a gas turbine (GT) plant. They considered a crack in a shaft and due to the propagation of a crack in the stub shaft of the coupling, a bow

occurs which influences the vibrations in the GT system. The magnitude and phase are estimated using suitable bending moments due to crack propagation and shaft bow. Kang et al. (2011) investigated the dynamic characteristics of a geared rotor supported on a viscoelastic shaft. They considered the gear mesh transmission error, gear eccentricity, and shaft residual bow effect and observed that the residual bow magnitude and phase greatly affecting the first critical speed of the system mounted on stiff viscoelastic support. Song et al. (2013) identified the faults in a rotor with a shaft residual bow. The multiple fault signals are measured for the experimental test rig and features are extracted using the wavelet and nonlinear manifold learning methods. Then, the residual bow fault is identified with the help of the principal component analysis (PCA) technique. Sanches and Pederiva (2016) identified the occurrence of two simultaneous faults, like the unbalance and residual bow using an approach based on correlation analysis in the time domain. Only the measured DOFs are used in deriving the identification equation implementing Lyapunov matrix and the least-squares fitting approach is utilized for fault identification both theoretically and experimental measurements. Chen (2019) examined the dynamic behavior of the double-stage geared rotor-bearing system due to the shaft deformation (residual shaft bow). Using Lagrange's method, the equations of motion are developed considering the contact ratio and pressure angle of the gear pair as time-dependent variables. The results obtained showed that the residual shaft bow increases the amplitudes of lateral displacement, contact ratio, and phase angle of gear pairs in the system.

Residual shaft bow can pose a serious problem in large machinery due to the deformation of the shaft in a definite length of the service time (permanent deformation) which can be

suppressed through design improvements based on the understanding of system characteristics.

1.3.3 Internal Damping in Rotors

The demand for high speed machinery introduces rotor instability. The instabilities in rotors are a result of bearing effects, dry friction, and internal damping in rotors as illustrated in (Kimball Jr, 1924; Newkirk and Taylor, 1925; Ehrich, 1964). The *Internal* or *rotating* damping is a well-known source of potential dynamic instability of shafts operating at supercritical speeds. This kind of destabilizing damping may be present due to energy dissipation in the shaft's material or rubbing between rotating components. The identification of the physical form of destabilizing phenomenon to avoid damage is a crucial subject of research and still requires extensive research. The influence of internal damping on the dynamic behavior of rotating systems considering nonlinearities in the elastic restoring forces due to alternatively compressed and stretched shaft fibres during whirling are studied in Refs. (Dimentberg, 2005; Vatta and Vigliani, 2008). The system undergoes this situation all the time when the angular speed of the rotor with respect to its deflected axis line differs from the whirl speed. The tension and compression occurring between the stationary and rotating parts, such as the disc and rotating shaft due to frictional force is commonly known as rubbing phenomenon. In rubbing, one part gets elongated and the other tries to oppose the friction between them resulting in damping force, which is directly proportional to the rate of shaft deformation. This damping force provides the hysteretic damping effect on the rotor system. Researchers investigated two types of internal damping, viz. the viscous internal damping and the hysteretic damping, and thus to analyze the instability effect in the system, appropriate modeling of material damping is

essential. In literature the stability analysis of the system experiencing these two types of damping is studied with great emphasis.

Internal damping in rotors influences isotropic circulatory and dissipative forces. When the circulatory force exceeds the dissipative one, orbital instability comes into existence. The damping in rotating systems triggers destabilization in supercritical speed regions. The statement that internal viscous damping has stabilizing effect in the subcritical speed region and destabilizing effect in the supercritical speed region while hysteretic damping is always destabilizing at any speed is found in many books and journals vide in Ref. (Dimentberg, 1961; Lund, 1974). This concept is analyzed by Genta (2004) and concludes that the previous statement on hysteretic damping in instability is misleading and states with an example that the internal hysteretic damping in the form of dissipation energy at very low speed is not enough to withstand the vibration in a system. Hence, hysteretic damping is destabilizing at all speed ranges is almost irrelevant.

Zorzi and Nelson (1977) performed an evaluation of damped rotor stability in a finite element based rotor model consisting of both internal viscous and hysteretic damping. The study of forward and backward whirl speed of an improved C^0 three noded element of Timoshenko beam finite element model of a rotor bearing system with the effect of internal damping was done by (Chen and Ku, 1991; Melanson and Zu, 1998). They considered combined effects of both the internal damping as considered in Ref. (Zorzi and Nelson, 1977) such as the viscous damping coefficient (η_v) and the hysteretic loss factor (η_H) for the speed of Ω , which are represented as

$$\eta_a = \frac{1 + \eta_H}{\sqrt{1 + \eta_H^2}} \text{ and } \eta_b = \frac{\eta_H}{\sqrt{1 + \eta_H^2}} + \Omega \eta_v \quad (1.4)$$

Equation (1.4) is used in the strain and dissipation energy formulation neglecting the transverse shear deformation. This work demonstrates that the transverse shear deformation effect and the centrifugal force can decrease the first forward and backward whirl speeds and thus, the instability in the system. The analysis of external and internal damping effects in an asymmetric shaft mounted on anisotropic flexible bearings is done and stability has been studied with Floquet theory by Wettergren and Olsson (1996). They showed that instabilities in the rotor system can often be avoided with the right combination of damping and asymmetric shaft stiffness and bearing damping, and its anisotropic stiffnesses.

Samantaray et al. (2006) mathematically modeled a nonlinear internal and external damping two DOFs system and analyze the stability. The work is based on the principle of work balance over an orbit with the use of the effect of nonlinear cross-coupling terms of damping and circulatory force. Montagnier and Hochard (2007) investigated the stability of a rotating shaft with internal damping mounted either on elastic dissipative bearings or infinitely rigid bearings with viscoelastic suspensions using the finite element model approach. The efficiency can be increased by incorporating bearings with viscoelastic suspension as viscoelastic supports to increase the stability of long shafts, unlike in conventional bearings. Sino et al. (2007) did a parametric study of the dynamic instabilities in a rotating system with the internal damping effect based on the finite element beam approach considering realistic properties of the rheological model. Chouksey et al. (2010) studied the dynamic behavior of the rotor experiencing rotating or internal damping force due to material damping in the shaft and investigated the influence of internal damping force on the mode shape and the directional frequency response of the rotor system. Roy

and Tiwari (2020b) developed a model based identification algorithm to estimate the internal damping and crack parameters in a rotor-bearing system mounted having a rigid disc at the shaft mid-span. Numerical testing of the developed algorithm was performed using full spectrum forward and backward whirling due to multi-harmonic crack excitation force induced in the system. Lastly, the robustness of the algorithm was checked against measurement noise. The experimental test was done on the laboratory test rig of a cracked rotor bearing system using the same methodology discussed in Roy and Tiwari (2020a).

1.3.4 AMB Dynamic Parameters Identification

The electromagnetic force of AMB is mainly influenced by the dynamic force coefficients of AMB and their identification is crucial. These parameters play a very important role, such as the stability, modes of vibration, and bending critical speeds of the rotors and are directly related to the feedback controller parameters.

Williams et al. (1990) developed an algorithm to estimate equivalent stiffness and damping coefficients based on a theoretically derived PD (proportional derivative) feedback controller transfer function. They showed that the controller sampling rate and the changes in second derivative feedback affect the bearing properties. Yue and Lie (2001) developed a rigid rotor-AMB model to identify the AMB support parameters. The identification methodology is based on the multiple frequency excitation approach, in combination with the least-squares method. This experimental identification approach did not consider the rotor flexibility into account resulting in inconsistent results at different rotating speeds. Lim and Cheng (2007) proposed an online estimation of stiffness and damping parameters of a one-axis electromagnetic suspension system based on the frequency domain. Thus, the cross-coupling of control forces can be avoided. The

experimental test rotor is excited by the multi-frequency Schroeder Phased Harmonic Sequences (SPHS) test and a digital PID control is established. Wavelet transform algorithm to identify the magnetic damping and magnetic stiffness coefficients of a drive rod equipped with 4 pole AMBs is illustrated by Tsai et al. (2011). Due to AMB force magnetization, the dynamics of the rod is non-linear in nature and it was found that axial direction of the rod-AMB is unstable under specific displacement and speed condition of the rod.

Tiwari and Chougale (2014) developed an identification algorithm to estimate the AMB dynamic parameters and residual unbalance of a flexible rotor fully levitated on AMBs. The identification methodology is based on the least-squares fitting approach in the frequency domain. The numerically estimated parameters are validated with the experimental data collected from the five-disc flexible rotor test rig levitated on AMBs. Tiwari and Talatam (2015) proposed an identification algorithm for estimation of speed dependent displacement stiffness and current stiffness parameters of AMBs and residual unbalance of a rigid rotor levitated on AMBs. Rotor run-up data of displacement and current generated by SIMULINK™ based on four-DOF rigid rotor-AMB system are used in the identification procedure. The equivalent stiffness and equivalent damping of a multi-DOF AMB-rotor system with multi-frequency excitation are estimated by Jiang et al. (2015). The peak value of the multi-frequency excitation should not be large, otherwise, it can lead to large vibration and may cause AMB force saturation. On the other hand, the amplitude of excitation is decreasing for every component, this will lead to weak perturbations in the frequency domain and thus increases the error in the identification procedure. Therefore, achieving the lowest peak value signal in multi-frequency excitation

with judicious selection of relative phasing of each component is needed to minimize the identification error.

Zhou et al. (2016) proposed an identification method of closed-loop AMB stiffness and damping coefficients of a flexible rotor-AMB system using unbalance response. Identification is done in two steps as initially, the experimental unbalance response at a given rotor speed is utilized in the derived rigid rotor system to find the nominal values of closed loop AMB stiffness and damping parameters. Secondly, the FEM model is utilized to compare the error response surface constructed for each of the AMB parameters in the estimation of identification error due to rotor flexibility. The AMB stiffness and damping coefficients identified through experimental unbalance response are combined with the finite element model of the rotor-AMB system from, which model unbalance responses are generated through simulation and compared with the experimentally measured data at various speeds. Xu et al. (2017) identified the dynamic parameters of AMB (k_s and k_t) using the Timoshenko beam finite element model. Experimental data of the unbalanced rotor response at the speed range of 1200 rpm to 30,000 rpm is utilized to identify these parameters. The identified results of stiffness and damping coefficients of AMB vary along with the rotating speed in transverse directions, indicating that the magnetic force is also affected by the operating speed. Prasad and Tiwari (2019) proposed an identification algorithm to estimate the speed-dependent dynamic parameters of AMB and residual unbalances in an AMB levitated rotor system. PID controllers are implemented to control the vibration in the system. A gyroscopic condensation scheme is used and based on those condensed EOMs in frequency domain, the identification algorithm is developed.

1.3.5 Multiple Fault Identification

In real machines, the presence of multiple faults is quite common apart from only unbalance. In the previous sections, various types of faults in rotating machinery are discussed. Here, the identification of multiple rotodynamic faults in a system based on the model is addressed.

Bachschnid et al. (2002b) studied the identification of multiple faults, such as the unbalance, bow, and rigid coupling misalignment, transverse crack, and axial asymmetry in a rotor system using model based technique. They have considered these faults in a 320 MW turbo generator. Least-squares fitting approach in the frequency domain for multidimensional residual minimization is used for the identification of multiple faults. Experimental tests have been done on a test rig to validate the results of numerical experiments. Sekhar (2005) developed a model based on online identification of the unbalance and shaft cracks together in a rotor system using the finite element method. He considered equivalent load in the mathematical model. Equivalent loads are nothing but the virtual forces and moments induced in a linear healthy system to generate the dynamic behavior similar to the measured one for a faulty system.

Jing and Meng (2009) proposed a blind source (BSS) technique for the diagnosis of multiple faults, such as the crack, rub-impact, and oil-whirl in a rotor system. Faults are separated from the vibrational features produced due to multiple faults. An analysis is done by employing the statistic variable-based algorithm and decorrelation method. Jalan and Mohanty (2009) identified the unbalance and misalignment fault together in a system using residual generation model based technique as in Ref. (Markert et al., 2001). Tiwari and Chakravarthy (2009) developed two algorithms for the simultaneous estimation of residual

unbalance and bearing dynamic parameters in a rigid rotor-bearing system. The first algorithm uses the two independent impulse responses of the shaft measured at the bearing housing in both vertical and horizontal directions and the second method uses the synchronous unbalance response at different orientations in the clockwise and counter-clockwise directions. Sanches and Pederiva (2011) illustrated the identification of unbalance and shaft bow theoretically for a two-piece disc rotor using Genetic Algorithm (GA). Lal and Tiwari (2012) developed model based identification algorithm to estimate simultaneous fault parameters (i.e., unbalance, coupling misalignment, and bearing dynamic parameters) based on forced response information. A flexible joint model is considered to generate the misalignment effect between the shafts. The dynamic analysis and estimation of crack, unbalance and AMB dynamic parameters of a rotor-AMB system having a transverse crack using the least square regression technique are discussed in Ref. (Singh and Tiwari, 2016). Garoli et al. (2021) investigated the unbalance and bearing wear faults in a rotating machine supported by lubricated bearings based on both deterministic and stochastic approaches. They identified the unbalance moment, phase and the axial location at the shaft, as well as two wear parameters: depth and angular position in the bearing using the Markov chain Monte Carlo method and the Bayesian inference with the polynomial chaos expansion technique.

1.4 Fault Detection using AMBs

Active condition monitoring techniques (ACM), as explained in Section 1.1, find application in fault detection of various faults, such as the crack, misalignment, unbalance etc. present in a rotor system. These kinds of mechanical fault detection in rotor systems

using AMBs are investigated in many researches as in Refs. (Humphris, 1992; Iwatsubo et al., 1992; Marshall et al., 2001; Mani et al., 2006).

Aenis et al. (2002) studied the model based fault diagnosis in a magnetically suspended centrifugal pump. They investigated different force measurement techniques those are the *i-s* method, a reluctance network method, and flux based method with the implementation of Hall sensors to get the accuracy in the frequency response function for diagnosis of faults. Zhu et al. (2003) studied the dynamics of a cracked rotor system integrated with an AMB. They observed that optimal controller parameters must be chosen to avoid instability in the cracked rotor-AMB system at the rotational speed region. To diagnose the crack, super-harmonic components of 2X and 3X rotational frequency in the sub-critical speed region were monitored unlike in traditional methods, where crack detection is done based on the amplitude of vibrational components 2X and 3X components of super harmonics frequency in supercritical speed regions. Bash (2005) presented in his thesis the implementation of AMB as a force actuator to study the health of the rotor supported on conventional bearings. Multiple forms of the force input signal are applied such as the Chirp, the Gaussian input to the rotating structure to analyze faults, like the shaft rub and EDM (electric discharge machine) cut notch in the system. At different progressive depths of 10, 25, and 40% of shaft diameter notches are made in the shaft mid-span to analyze the behavior of crack analogous to the breathing crack.

Quinn et al. (2005) illustrated the use of combination resonance among critical shaft frequencies, shaft rotational speed, and external frequency of AMB actuator excitation of a test rig of a cracked shaft supported on rolling element bearings. Multiple scale analysis is used to estimate the combination resonance to identify the magnitude of the time-

dependent stiffness arising due to the breathing behavior of the shaft crack. Sawicki (2009) described an online crack detection technique using forces of AMB actuator both theoretically and experimentally. He also implemented the combination resonance method, where if the cracked rotor is excited by a force at combination resonance, a re-excitation of the first natural frequency occurs with high amplitude in vibration response as compared to a healthy shaft system. It is described that with the increase in the crack depth the response amplitude increases. On the other hand, if a healthy (non-cracked) rotor is excited by combination resonance, at first natural frequency there is no increase in the amplitude of the vibrational response. Friswell et al. (2010) also used the combination resonance for identification of cracks in a rotor-AMB system and showed through recurrence plot as in Ref. (Litak et al., 2009).

Bouaziz et al. (2011) did a dynamic analysis of misaligned rotors mounted on two AMBs. They considered three different models of AMB, viz. 4, 6 and 8-pole AMB. The angular misalignment considered has a predominant amplitude of vibration in the second and third components of the revolution speed and its amplitude varies with different AMB models. Morais et al. (2012) demonstrated the control of breathing mechanism of the crack in a rotating machine by using a mid-span mounting electromagnetic actuator (EMA) numerically. They used four actuators that are arranged along two perpendicular directions to apply attraction forces for producing bending moments to maintain the crack closed, i.e. the *self-healing* of the cracked face during operation. The minimization of the crack opening, however, increases the vibration in the system leading to instability, and to overcome this problem a heuristic method known as Particle Swarm Optimization (PSO) was utilized. The experimental investigation was performed on early detection of crack at

a depth of around 5 percent of shaft radius in a rotor-bearing system mounted on fluid-film bearings using an AMB (Pantelakis et al., 2015). Real-time crack detection is done with the coupling behavior due to local asymmetry that a crack or a notch or cut introduces into the rotating system and the concept of localization of coupling in time domain was combined with the AMB excitation force. Hilbert transforms, Fast Fourier transform (FFT) and Continuous wavelet transform (CWT) signal processing techniques are used on the response measured. Singh and Tiwari (2016) illustrated the dynamic behavior of the breathing mechanism of the crack in a rotor-bearing system supported on AMB. The radial AMB acts as a controller, which is in contrast with the traditional method of using AMB as an exciter and suppresses the vibration due to unbalance and crack excitation force. The attenuated vibrational displacement and control current of AMB is utilized in the fault parameter identification technique.

Apart from the crack, coupling misalignment fault in a rotor is also identified using vibration and current signal of AMB by Srinivas et al. (2019). Kumar and Tiwari (2020) developed a trial misalignment approach to identify the misalignment in two AMBs mounted on a rigid rotor containing two offset discs. They developed an algorithm to estimate the unbalance eccentricities and phases and force-displacement and force-current stiffnesses of AMBs. The misalignment amount is evaluated using the estimated AMB parameters for both residual and additional trial misalignment cases.

In the end, some new research prospects are pointed out and it is believed that the review of literature has synthesized the topic of fault identification in rotating machinery.

1.5 Gaps from the Literature Review

In literature, numerous techniques employed for performing fault diagnosis in rotating machinery have been reviewed.

- A lot of researches have been going on analyzing the dynamics of the rotor with permanent bend since most industrial turbine machines operate with shaft warp, but hardly any work has been reported on its quantification. The residual shaft bow in a cracked rotor may influence multiple harmonics of shaft revolution, thus estimation of shaft bow along with its location eliminates the removing of bow effect from 1x harmonic in signal processing in the slow run condition during signal processing as done in previous works.
- Scope exists, in analyzing the effect of shaft residual bow on a misaligned rotor with coupled-bending vibration in the model-based identification technique for fault detection and diagnosis purposes.
- The effect of rubbing of crack faces during rotation of the machinery, which intensifies the internal damping already present in the system. The rotating internal damping initiates self-induced instabilities in the rotor system in the supercritical speed region, which needs to be quantified to avoid the instability threshold.
- Numerous modeling of crack based on fracture mechanics is widely studied to obtain the stress intensity factors. Likewise, the dissimilar moments of inertia theory used for crack modeling has some pros and cons as multiple rotor faults exhibit similar behavior.

- Various crack models of open crack, slant crack, fatigue crack have been studied earlier. The nonlinearity in breathing mechanism and coupled vibration analysis has more scopes of research.
- In literature, for experimental study of the cracks in rotor mainly it involves wire EDM cut or welded shafts, and very few works have been found on transverse fatigue cracks. Thus, more investigation on the artificial generation of fatigue crack with residual shaft bow and its study on the dynamics of the system becomes necessary.
- Earlier works have been devoted to crack identification based on frequency responses functions. The traditional frequency spectrum (half spectrum (FFT)) and the unique directional featured full spectrum response are discussed mostly in crack and other fault identification through numerical simulation. Thus, full spectrum technique can be exploited with model based identification for simultaneous occurrence of multiple faults using experimental measurements.
- In most of the literature, AMB is used as a replacement for the mechanical external exciter. Multi-fault detection using AMB in the rotor from a smart condition monitoring perspective has the scope and still requires comprehensive research.

1.6 Aims and Objectives of the Current Research

After an extensive literature survey, it can be inferred that apart from levitation of rotors, AMBs are used as an exciter as well as controller and ensures versatile fault detection in rotating machinery. In literature, AMBs application in fault identification of rotors is rarely found. In view of the above fact, the attenuated vibrational displacement of the rotor and the current signal of AMB may be a useful technique for fault identification in rotating

machinery. Multiple fault detection and quantification of the fault parameters, such as the crack, residual bow, internal damping, and misalignment in a rotor integrated with an AMB as well as active vibration control of the rotor would be an extensive area of research. As illustrated above, with the limited amount of literature available on the identification of shaft bow of a cracked internally damped rotor-AMB system, a comprehensive analytical and experimental procedure is necessary for the complete understanding of the dynamics and quantitative estimation of a system. The main objective of the present study is to analyze the dynamics of a cracked-warped-rotor-AMB system as well as to develop a methodology to estimate the fault parameters associated with the system. The utilization of vibrational displacement and control current of AMB for identification of the crack, residual shaft bow, internal and external damping, and AMB parameters is the significant purpose of the work. From the aforementioned discussions following objectives are derived for the current research:

(i) **Development of Mathematical Models:** To develop equations of motion (EOMs) for cracked rotor-bearing system integrated with an AMB considering (a) without and (b) with gyroscopic effect, respectively, based on the existing literature. A switching crack model is to be linearized and implemented in the present work considering the ‘*weight dominance effect*’. The mathematical model of crack force is to be developed comprising additive crack stiffness in both translational and rotational directions. The internal damping due to the rubbing of cracked surfaces during the operation of the system is also included in the model. As the presence of residual bow affects the dynamics of the cracked rotor system, a new mathematical model of a force consisting of additive crack stiffness and residual bow to analyze the interdependency of faults is

to be developed. To comply with a more realistic system, finite element modeling (FEM) is to be done for cracked-bowed flexible rotor with multiple discs mounted on flexible bearings and integrated with AMBs. The end support condition of the rotor with flexible bearings is to be modeled as a spring-mass-damper system considering anisotropic behavior.

(ii) **Development of Identification Algorithm:** To develop an identification algorithm for the simultaneous estimation of the additive crack stiffness, residual unbalances, residual bow, external and internal damping, AMB stiffness parameters, along with multiplicative crack-bow fault for above developed rotor models. The full spectrum amplitude and phase of displacement and current signals are to be utilized in the developed algorithm to estimate the fault parameters.

(iii) **Numerical Simulation:** To numerically simulate the developed rotor-AMB system through SIMULINK™ block. To analyze the displacement and current responses with and without the presence of residual bow based on full spectrum amplitude and phase. To check the sensitiveness and robustness of the developed algorithm against measurement noise and modelling errors, different levels of measurement noise in responses and modeling errors in some of the physical parameters of the rotor system to be added.

(iv) **Development of the Test Rig:** To develop a cracked-warped rotor test bench incorporated with a radial 8-pole magnetic actuator with a digital dSPACE controller and power amplifiers. To fabricate the cracked shaft with artificial fatigue crack generation in a three-point bending test.

(v) **Experimental Investigation:** To conduct a closed-loop feedback control a PID control strategy will be implemented through differential driving mode in the developed rotor-AMB test bench. A comparative study of the dynamics of the healthy and the cracked-warped shaft is to be carried out. To experimentally validate the developed identification algorithms for the above mentioned models in the rotor test bench. The shaft displacement and control current of AMB signals acquired from the test rig will be used in the algorithm for the estimation of multiple fault parameters associated with the test rig. The parameters so estimated are to be validated for their feasible ranges.

1.7 Outline of the Thesis

Fault diagnosis implementing model based technique in a dynamical system using the analytical model. The present study illustrates the various faults in a machine by mathematical models of it and its required fidelity along with the identification procedure for quantification of fault diagnosis. The main aim of the work is to develop a mathematical model for model based fault diagnosis of a process and to develop the identification algorithm based on the system model using full spectrum responses to estimate multiple fault parameters.

A linear model of the system under investigation will be developed and using this model an identification procedure for estimation of multiple fault parameters associated with the system will be done from data affected by noises and acquired from simulated and/or test rig. The thesis consists of seven chapters and the main contributions are presented in Chapters 2 through 6. The conclusion and scope for future work are

summarized in Chapter 7. Chapters are devoted to the particular problem of multiple fault identification of rotating system and these are organized as follows.

Chapter 1 gives the introduction and reviews the state of the art of smart condition monitoring using active magnetic bearing and model based fault diagnosis techniques of various rotor faults.

Chapter 2 demonstrates the development of the identification algorithm in order to estimate the multiple faults of cracked rotor system integrated with AMB. The identification technique presented requires a linear mathematical model of the system under investigation and the assumptions made in the modeling of the system are a rigid disc mounted on a massless shaft with no gyroscopic effect. The handiness of this procedure is that it requires prior knowledge of some of the system parameters consisting of faults as the linear model of the system and exploits the fault parameters in the equation relating to the dynamic characteristics of the system.

Chapter 3 presents an experimental investigation of multi fault identification of cracked warped internally damped rotor with AMB. For that purpose, in the present work a laboratory test rig is developed. The fabrication of radial eight-pole AMB actuator and artificial crack generation on the shaft are described here. The implementation of PID control strategy on the rotor-AMB system is elaborated with conducting experiments on it. The experimental data acquired from the test rig for both healthy and faulty rotors are compared and the dynamic responses are analyzed. Utilization of the test data in the developed identification algorithm in Chapter 2 for estimation of the multiple fault parameters associated with the developed test rig is presented. Lastly, estimated parameters are used in the numerical simulation to generate the responses from the cracked-warped

internally damped rotor-AMB model, and the same is compared with test rig responses for validating the estimated parameters. Furthermore, the balancing of the rotor system is investigated by the use of the estimated unbalance parameter.

Chapter 4 illustrates multi-fault identification of cracked warped rotor system integrated with AMB with gyroscopic effect. Development of the mathematical model, its numerical testing, and development of identification algorithm is extended to cracked-warped internally damped rotor-AMB with an offset disc. Here, the complexity arises due to the appearance of the rotational DOFs apart from translational ones as the physical measurement of angular displacements is not very accurate. Thus, a *condensation* scheme with a gyroscopic effect implemented to overcome this difficulty for developing identification equations is explained in this chapter.

Chapter 5 describes the identification of multiple faults using the developed identification algorithm in Chapter 4 in the laboratory test rig of a cracked-warped internally damped shaft with an offset disc mounting and equipped with an AMB. Here, also, the dynamics of a healthy and a cracked warped shaft with offset disc dynamics are compared. The test data of the rotor-AMB system in the time domain are converted into full spectrum responses. The amplitude and the compensated phases with the multi-harmonic reference signal of the full spectrum responses are used in the developed algorithm for estimation is presented in this chapter.

Chapter 6 aims to define a comprehensive study of the fault identification methodology relating to a real rotor system and with this, a finite element model (FEM) is developed of the cracked warped internally damped flexible rotor system equipped with AMB. Identification algorithm is developed based on the EOM in the frequency domain,

which includes fault parameters, such as the external and internal damping, unbalance and its orientation relative to the cracked face, residual shaft bow and its phase, translational and rotational additive crack stiffness parameter, bearing stiffness and damping parameters and AMB force-displacement and force-current factors. The results of the simulation present the estimated fault parameters without noise as well as the data affected by random noise and modeling or bias errors.

Chapter 7 summarizes the contributions of the research work and provides some recommendations for potential in further research topics as an extension of this work.



Chapter2

Analysis and Identification of Faults in a Rotor-AMB System without Gyroscopic Effect

This chapter presents the mathematical formulations of a cracked-warped-internally damped rotor-bearing system integrated with an active magnetic bearing (AMB). The work focusses on the dynamic analysis of unique nature of fault as well as interdependency of two or more faults together in the rotor-AMB system and the estimation of multiple fault parameters simultaneously. The novel identification algorithms based on the developed equations of motion (EOMs) for estimation of multiple fault parameters both additive and multiplicative ones are described. The numerical testing of the developed algorithm in presence of measurement noise and modeling or bias errors for the viability of the proposed algorithms are also discussed. These studies conclusively highlight the robustness of the proposed model based identification methodology for quantitative estimation of multiple faults even in the presence of noises.

2.1 Introduction

A fatigue crack in a system provides asymmetric stiffness in general and that leads to excitation forces and subsequently responses in both forward and backward whirls. The cracked faces during whirling encompass rubbing between them, which aggravates the internal (rotating) damping leading towards instability in a system. Moreover, propagation of crack in a heavy machinery creates a permanent bend or warped in a system that affects

the dynamics of the system. To have characterization of simultaneous occurrence of such fault models, it is essential that estimation procedure should incorporate full spectrum to have physically meaningful multi fault parameters.

The simple rotor-bearing system containing a transverse crack equipped with AMB is mathematically developed. Here, the disc is considered to be mounted at the middle of the shaft to avoid the gyroscopic effect. The internal damping due to the rubbing effect of the cracked faces as a material hysteresis is also included into the model. Two mathematical models are developed to show the comparison of the without shaft warp (residual bow) and with shaft warp condition. The shaft with a crack close to the disc position at the mid-span is considered. The modeling of crack is derived based on crack local flexibility with assumption of '*weight dominance*' for switching crack model in the rotating frame of reference. The vibrational displacements due to different rotor faults have unique characteristics at various speeds and the control currents of AMB for attenuating the rotor vibration are simulated with the developed EOMs. The excitation forces due to the combination of unbalance, crack, internal damping, and AMB together provide excitation at multiple harmonics of the rotor spin frequency. Besides this, the shaft residual bow along with crack fault in a rotor incorporates multiple harmonic component of the shaft revolution, unlike in only residual bow fault (excites at 1X harmonic revolution component), and gives rise to multiplicative fault (crack and bow) behavior in addition to the additive fault behavior. The simultaneous excitation forces due to unbalance, shaft residual bow, crack, internal damping and magnetic bearing forces, the rotor whirls in the same direction of spin (i.e. the forward whirl) and in reverse direction of spin (i.e., the backward whirl) at multiple harmonics.

Subsequently, an identification algorithm has been developed based on the obtained EOMs. The full spectrum responses of the multiple harmonics are fed to the identification algorithm to estimate the additive and multiplicative fault parameters of the system simultaneously. A numerical simulation is performed onto the rotor system configuration data using MATLAB® SIMULINK™ environment to obtain the time domain responses and then to test the identification algorithm. To check the robustness of the developed identification algorithm, various percentages of random noises and modeling (or bias) errors have been implemented to time domain signals and some of the physical system parameters, respectively.

2.2 Mathematical Modeling of Rotor Bearing System

In this section, assumptions considered in the modeling of the system are stated. Also a detailed description of the warped (bent) rotor having a transverse crack and internal damping incorporated with AMB is presented. In the present work, a simple rotor model configuration is considered with a disc at mid-span of shaft in presence of transverse crack as shown in Figure 2.1 (a, b). The rotor supported on two conventional bearing are assumed to be rigid in the modeling.

2.2.1 Key Assumptions and Description of Rotor Model

A simple rotor is considered for mathematical modeling and analysis purposes as depicted in Figure 2.1(a, b) consists of a massless flexible shaft of length ' L ', having a permanent deformation (residual bow) and a transverse crack, a rigid disc of mass ' m ' at its mid-span, supported on two rigid bearings and equipped with an AMB. For the simplicity, initially the gyroscopic effect and shaft elastic coupling is not considered in the modeling. The

crack is considered in the middle of the shaft due to fatigue loading and a switching crack model is chosen based on the hinge model (Gasch, 2008).

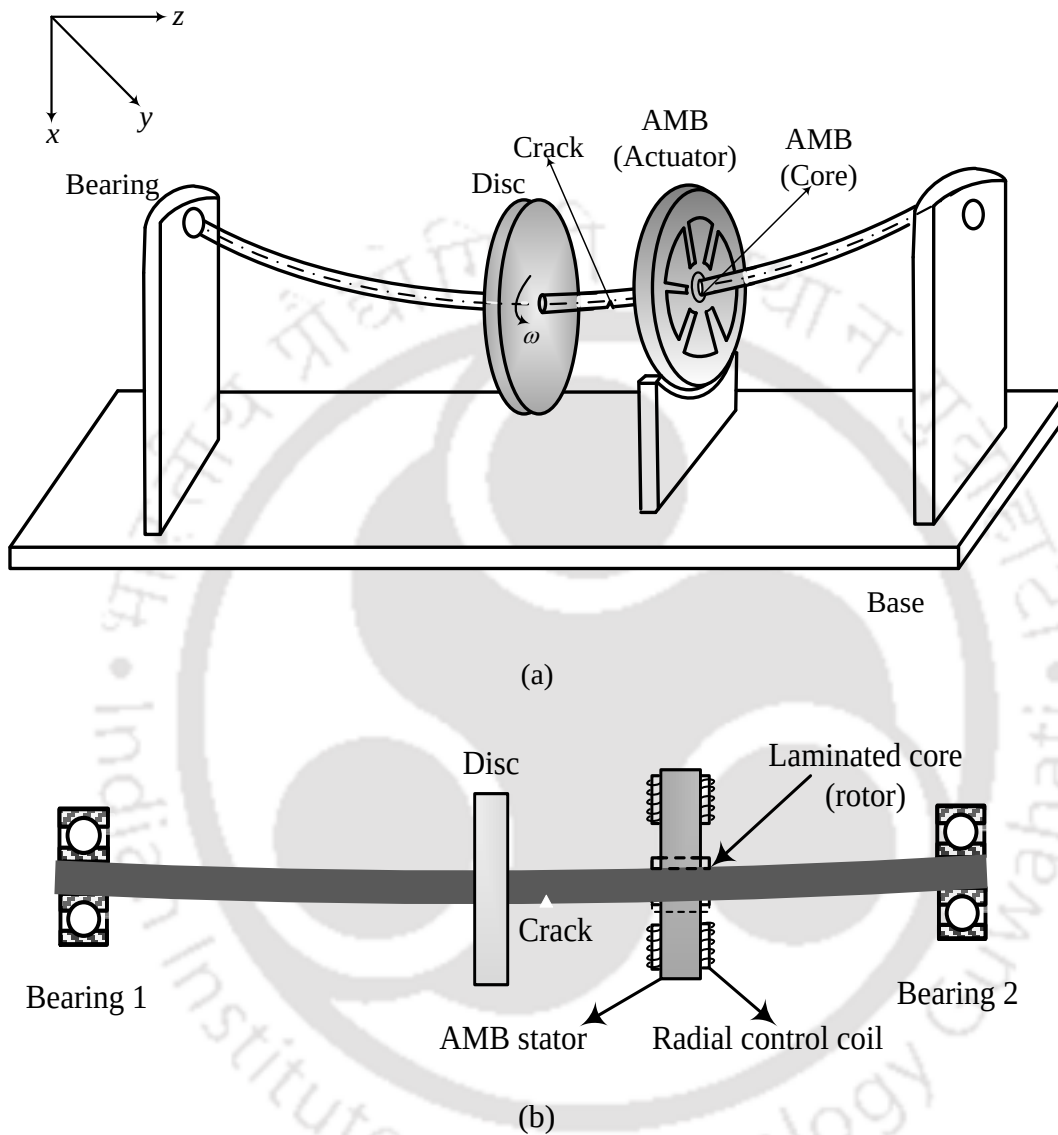
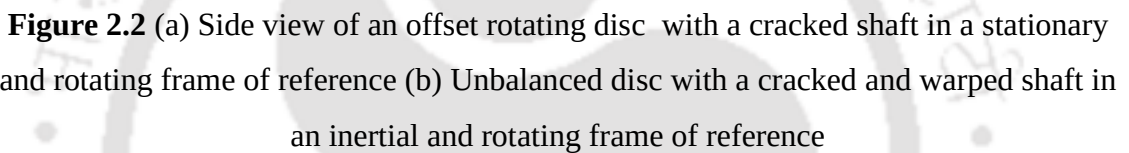


Figure 2.1 Typical rotor bearing system configuration containing a cracked warped shaft with a rigid disc at its mid-span integrated with AMB (a) overall 3D view (b) 2D plane view



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dead-weight is denoted by u_{x_0} as defined in Figure 2.2(a). Figure 2.2(b) shows the cracked shaft containing a residual bow of magnitude ' e_b ' and ' ϕ_b ' is the orientation angle of it. The initial position of the warped shaft at time $t = 0$ be $(u_{x0} + e_b \cos \phi_b, e_b \sin \phi_b)$. For any other time, t , the disc center position is $(u_{x0} + u_x(t) + u_{x_b}(t), u_y(t) + u_{y_b}(t))$ as depicted in Figure 2.2. The dynamic displacements due to the residual shaft bow effect on the rotor are defined as in u_{x_b} , and u_{y_b} the vertical and horizontal directions, respectively.

2.2.2 Fault Models of Rotor System

In the model based identification, mathematical modeling of the faults associated with a rotor model by relating the fault parameters with the force is done. The detailed fault modeling considered for the analysis, such as the unbalance, crack, internal damping, and residual shaft bow are explained in the following section.

2.2.2.1 Internal damping fault model

Internal damping often imposes a destabilizing effect when the operating speed of the rotor is in the supercritical speed region. The internal damping force, appearing as a result of the rubbing of the crack faces of the shaft during the whirling of the rotor, is included as one of the fault parameters in the mathematical model. The internal damping force would act on the cracked shaft along with the ξ and η rotating frame of reference as shown in Figure 2.1. The components of internal damping force acting on the rotor along the stationary coordinates, such as x and y axes, are modified as illustrated in Ref. (Tiwari, 2017)

$$f_{dx} = c_H \dot{\xi} \cos \theta - c_H \dot{\eta} \sin \theta \quad (2.1)$$

$$\text{and,} \quad f_{dy} = c_H \dot{\xi} \sin \theta + c_H \dot{\eta} \cos \theta \quad (2.2)$$

where f_{dx} and f_{dy} are the internal damping force exerted onto the rotor in the vertical and horizontal directions, respectively. In complex form on combining Equations (2.1) and (2.2), we get,

$$f_d = f_{dx} + jf_{dy} = (c_H \dot{\xi} \cos \theta - c_H \dot{\eta} \sin \theta) + j(c_H \dot{\xi} \sin \theta + c_H \dot{\eta} \cos \theta) = c_H (\dot{\xi} + j \dot{\eta}) e^{j\theta} \left. \vphantom{f_d} \right\} \quad (2.3)$$

$$= c_H \dot{\zeta} e^{j\theta}$$

where the complex displacement of the rotor in the rotating as well as stationary frame of reference, be $\zeta = \xi + j\eta$ and $s = (u_x + u_{x0}) + ju_y$, respectively, with $j = \sqrt{-1}$. The internal (rotating) damping force after transforming from the rotating to inertial coordinate systems, using $\zeta = se^{j\theta}$ in Equation(2.3), it takes the following matrix form

$$\mathbf{f}_d = \begin{bmatrix} c_H & 0 \\ 0 & c_H \end{bmatrix} \begin{Bmatrix} \dot{u}_x \\ \dot{u}_y \end{Bmatrix} + \begin{bmatrix} 0 & \omega c_H \\ -\omega c_H & 0 \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\omega c_H u_{x0} \end{Bmatrix} \quad (2.4)$$

Equations of motion of the uncracked rotor with the external and internal damping, static force ($\mathbf{f}_{st}$), and unbalance ($\mathbf{f}_{unb}$) excitation force becomes,

$$\mathbf{M}\ddot{\mathbf{s}} + (\mathbf{C}_E + \mathbf{C}_H)\dot{\mathbf{s}} + (\mathbf{K} + \mathbf{K}_C)\mathbf{s} = \mathbf{f}_{st} + \mathbf{f}_{unb} \quad (2.5)$$

$$\text{where, } \mathbf{M} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}; \mathbf{C}_E + \mathbf{C}_H = \begin{bmatrix} (c_V + c_H) & 0 \\ 0 & (c_V + c_H) \end{bmatrix}; \mathbf{K} + \mathbf{K}_C = \begin{bmatrix} k_0 & \omega c_H \\ -\omega c_H & k_0 \end{bmatrix}$$

$$\mathbf{f}_{st} = \begin{Bmatrix} mg \\ 0 \end{Bmatrix}; \mathbf{f}_{unb} = \begin{Bmatrix} me_u \cos(\theta + \phi_u) \\ me_u \sin(\theta + \phi_u) \end{Bmatrix}; \mathbf{s} = \begin{Bmatrix} u_x \\ u_y \end{Bmatrix}$$

where, $\mathbf{M}$, $\mathbf{C}_E$, $\mathbf{C}_H$, $\mathbf{K}$ and $\mathbf{K}_C$ is the mass, external and internal damping, intact shaft stiffness, and circulatory stiffness matrices of the rotor system; $\mathbf{f}_{st}$ is the static force vector due to self-weight of the rotor, $\mathbf{f}_{unb}$ is the unbalance excitation force vector due to disc

unbalance and $\mathbf{s}$ is the dynamic displacement vector of the rotor at the disc location. The intact shaft stiffness is defined as k_0 , and an acceleration due to gravity by g .

2.2.2.2 Bow Fault Model

The presence of a residual shaft bow is also known as the warped or bowed rotor. The residual bow in the shaft creates an asymmetrical axial distribution of strain on the cross-section of the shaft resulting in the permanent deflected shaft. The excitation amplitude, due to residual shaft bow, is a constant quantity. The shaft bow produces a force, which is mainly prominent in the slow roll condition of the rotor system. The unbalance and shaft bow orientations are in the same direction, the rotor experiences severe vibration whereas at 180° out of phase condition the bow effect is nullified, thus amplitude of vibration decreases. The disc eccentricity and residual shaft bow encountered in real practice are in a different phase relative to the perpendicular axis (ζ axis) of the cracked face in the shaft considered in the modeling. The shaft residual bow force in the rotating frame of reference is represented as (Rao, 2001; Tiwari, 2017)

$$\mathbf{f}_{bow} = \mathbf{K}_{0,\xi\eta} \mathbf{e}_{b,\xi\eta} = \mathbf{K}_{0,\xi\eta} \begin{Bmatrix} e_b \cos \phi_b \\ e_b \sin \phi_b \end{Bmatrix} \quad (2.6)$$

Equation (2.6) is transformed from the rotating to the stationary frame of reference using standard transformation matrix $\mathbf{T}(t)$ as

$$\mathbf{f}_{bow} = \mathbf{T} \mathbf{K}_{0,\xi\eta} \mathbf{e}_{b,\xi\eta} \text{ with } \mathbf{T}(t) = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad (2.7)$$

On expanding Equation (2.7) can be expressed as

$$\mathbf{f}_{bow} = k_0 e_b \begin{Bmatrix} \cos(\theta + \phi_b) \\ \sin(\theta + \phi_b) \end{Bmatrix} \quad (2.8)$$

2.2.2.3 Crack Fault Model

The presence of crack in a rotor, it opens and closes during once per revolution of the shaft under the self-weight of the rotor. The opening and closure behavior of the crack face during rotation of the shaft depends on the orientation of the cracked front. The asymmetry in the shaft due to crack on it changes the flexibility of the system and encompasses coupling of lateral, axial and torsional vibration. For the sake of simplicity, in the present analysis only lateral vibration effects are taken into consideration.

The stiffness matrix of a cracked rotor generally nonlinear and time variant in nature comprising of diagonal stiffness matrix ($\mathbf{K}_{0\xi\eta}$) of uncracked rotor and the reduction in stiffness (additive stiffness, $\Delta\mathbf{K}_{\xi\eta}(u,t)$), which is dependent on both on the vibrational displacement ($u(t)$) of rotor and the angle of rotation of the rotor ($\theta = \omega t$). The angle θ is the crack orientation with respect to a fixed axis as shown in Figure 2.2 and determines the extent of the crack face opening. The rotor system contains time varying stiffness in the rotating frame of reference and can be represented as,

$$\mathbf{K}_{\xi\eta}(t) = \mathbf{K}_{0\xi\eta} + \Delta\mathbf{K}_{\xi\eta}(u,t) \quad (2.9)$$

According to assumption of the weight dominance chosen in the present analysis, where the vibrational displacement will remain small as compared to the static deflection of the rotor under its own weight, i.e. ($u_x(t) \ll u_{x_0}$), on application of this, Equation (2.9) is linearized as

$$\mathbf{K}_{\xi\eta}(t) = \mathbf{K}_{0\xi\eta} + \Delta\mathbf{K}_{\xi\eta}(t) \quad (2.10)$$

The deflection due to the crack flexibility matrix for the 'hinge' model in rotating frame of reference is written as (Gasch, 2008),

$$\begin{Bmatrix} u_\xi \\ u_\eta \end{Bmatrix} = \begin{bmatrix} h_0 & 0 \\ 0 & h_0 \end{bmatrix} + \sigma(t) \begin{bmatrix} \Delta h_\xi & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} f_\xi \\ f_\eta \end{Bmatrix} \quad (2.11)$$

where h_0 is the flexibility of the uncracked shaft, Δh_ξ is the additional flexibility due to the presence of the crack on the shaft, and f_ξ , f_η are excitation forces along the ξ and η rotating coordinate axes, respectively. During rotation of the shaft, if the cracked face is in compressive zone, then the additive flexibility, Δh_ξ becomes zero and the shaft behaves as a healthy shaft. The opening and closure mechanism of crack is best represented by the steering excitation function, $\sigma(t)$. A small crack, at its early propagation is illustrated by sudden opening and closure of the crack faces while whirling of shaft as mentioned in Ref. (Gasch, 2008). The steering excitation function ($\sigma(t)$) shows the crack profile opening and closure by

$$\sigma(t) = \begin{cases} 1, & \text{for } u_\xi = 0 \text{ (open)} \\ 0, & \text{for } u_\xi > 0 \text{ (close)} \end{cases} \quad (2.12)$$

A realistic breathing model of the crack has been proposed for the attainment of more accurate estimation of stiffness. The generalized proposed breathing function of crack, encountering forward (positive components) and backward (negative components) whirl frequencies, is represented as,

$$\sigma(t) = \sum_{i=-n}^n \frac{1}{2} (1 + \cos(i\theta)) \quad (2.13)$$

Inverting the flexibility matrix of Equation (2.11) into stiffness matrix, it is defined as

$$\begin{Bmatrix} f_\xi \\ f_\eta \end{Bmatrix} = \begin{bmatrix} h_0 & 0 \\ 0 & h_0 \end{bmatrix} - \sigma(t) \begin{bmatrix} \Delta h_\xi & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} u_\xi \\ u_\eta \end{Bmatrix} \quad (2.14)$$

Equation (2.14) can be written as

$$\mathbf{f}_{rot} = \mathbf{K}_{0\xi\eta} + \sigma(t) \Delta \mathbf{K}_{\xi\eta} \mathbf{s}_{rot} \quad (2.15)$$

where, $\frac{1}{h_0} = k_0$, $\frac{1}{h_0 + \Delta h_\xi} = k_0 - \Delta k_{c22}$

Equation (2.15) is transformed from the rotating to fixed coordinate axes using a transformation matrix $\mathbf{T}$, assuming the uncracked rotor is axi-symmetric as

$$\mathbf{T}^T \mathbf{f}_{rot} = \mathbf{T}^T (\mathbf{K}_{0\xi\eta} + \mathbf{T}^T (\Delta \mathbf{K}_{\xi\eta}(t)) \mathbf{T}) \mathbf{s}_{rot} \quad (2.16)$$

where, transformation matrix $\mathbf{T} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$

Evaluated Equation (2.16) is written as

$$\begin{Bmatrix} f_x \\ f_y \end{Bmatrix} = \begin{bmatrix} k_0 & 0 \\ 0 & k_0 \end{bmatrix} - \sigma(t) \Delta k_{c22} \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} \quad (2.17)$$

or,

$$\begin{Bmatrix} f_x \\ f_y \end{Bmatrix} = \begin{bmatrix} k_0 & 0 \\ 0 & k_0 \end{bmatrix} - \frac{1}{2} \sigma(t) \Delta k_{c22} \begin{bmatrix} 1 + \cos 2\theta & \sin 2\theta \\ \sin 2\theta & 1 - \cos 2\theta \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} \quad (2.18)$$

The crack force is represented from Equation (2.18) as

$$\mathbf{f}_{cr} = \sigma(t) \Delta \mathbf{K} \mathbf{s}_0 = \frac{1}{2} \sigma(t) \Delta k_{c22} \begin{Bmatrix} (1 + \cos 2\theta) u_{x_0} \\ \sin 2\theta u_{x_0} \end{Bmatrix} \quad (2.19)$$

The breathing behaviour in the actual crack is nonlinear in nature and its Fourier expansion brings the participation factors into the picture. The switching crack excitation function is considered for the present analysis. The crack face in switching function corresponds to the sudden opening and closure of the crack face during rotation. The rectangular square wave of switching crack function is represented as (Gasch, 1993)

$$\sigma(t) = \frac{1}{2} + \frac{2}{\pi} \cos \omega t - \frac{2}{3\pi} \cos 3\omega t + \frac{2}{5\pi} \cos 5\omega t - \frac{2}{7\pi} \cos 7\omega t + \frac{2}{9\pi} \cos 9\omega t - \dots \quad (2.20)$$

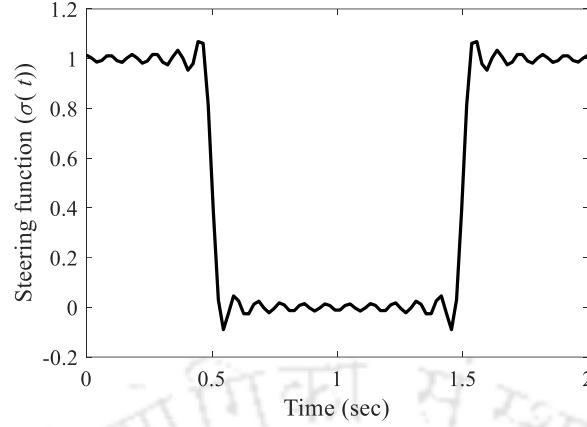


Figure 2.3 Switching crack steering function with 50% duty cycle

The switching crack excitation function is a square wave with 50% duty cycle, which describes the opening and closure behavior of cracked faces as shown in Figure 2.3.

2.2.2.4 Magnetic Force Model

The magnetic force depends on the overall displacement and the control current circulating in the coil of the magnetic actuator. The magnetic control force, in the proposed system at a linear operating point after neglecting the influence of magnetic saturation and leakage, can be expressed as (Schweitzer and Maslen, 2009),

$$\mathbf{f}_{\text{AMB}} = \begin{Bmatrix} -k_s u_x + k_i i_{cx} \\ -k_s u_y + k_i i_{cy} \end{Bmatrix} \quad (2.21)$$

where i_{cx} and i_{cy} are controlling currents in the x and y directions. The k_s and k_i are the force-displacement and force-current stiffness coefficients of the AMB, respectively. In the proposed system an eight-pole AMB is considered to be isotropic for which the control currents i_{cx} and i_{cy} are to be same in both the directions for same rotor displacement in respective directions. In the proposed model for the stable operation of the rotor system, a

close loop feedback control system of AMB is used to facilitate the control current with the use of rotor responses. A PID (proportional-integral-derivative) controller is deployed to achieve the control current. The control current from the PID controller as an output in the coils of the actuator poles in orthogonal directions are represented as

$$i_{cx} = K_p u_x + K_I \int u_x dt + K_D \frac{du_x}{dt} ; \quad i_{cy} = K_p u_y + K_I \int u_y dt + K_D \frac{du_y}{dt} \quad (2.22)$$

where K_p, K_I and K_D are the proportional, integral and derivative controller parameters.

The static deflection of the rotor when the rotor is at stationary condition i.e. at $t = 0$ becomes

$$\begin{bmatrix} k_0 & 0 \\ 0 & k_0 \end{bmatrix} \begin{Bmatrix} u_{x0} \\ u_{y0} \end{Bmatrix} = \begin{Bmatrix} mg \\ 0 \end{Bmatrix} \quad (2.23)$$

From Equation(2.23), the static displacements of the disc are found as,

$$u_{x0} = \left(\frac{mg}{k_0} \right) \quad (2.24)$$

2.2.2.5 Overall Equations of Motion of the Rotor System

The EOM of the rotor model without residual shaft bow effect is written as

$$\mathbf{M}\ddot{\mathbf{s}}_v + (\mathbf{C} - \omega \mathbf{G})\dot{\mathbf{s}}_v + \mathbf{K}\mathbf{s}_v = \mathbf{f}_{cr} + \mathbf{f}_{unb} - \mathbf{f}_{AMB} - \omega \mathbf{C}_H \mathbf{s}_0 \quad (2.25)$$

And EOM with the residual shaft bow effect onto the rotor is written as

$$\mathbf{M}\ddot{\mathbf{s}}_v + (\mathbf{C} - \omega \mathbf{G})\dot{\mathbf{s}}_v + \mathbf{K}\mathbf{s}_v = \mathbf{f}_{cr} + \mathbf{f}_{unb} + \mathbf{f}_{bow} - \mathbf{f}_{AMB} - \omega \mathbf{C}_H \mathbf{s}_0 \quad (2.26)$$

Expanding Equation (2.25) for the vertical direction motion, we get

$$\left. \begin{aligned} m\ddot{u}_x + (c_v + c_H)\dot{u}_x + k_0 u_x + \omega c_H u_y &= m\omega^2 \cos(\theta + \phi_u) - \omega c_H u_{x0} \\ &+ \frac{1}{2} \sigma(t) \Delta k_{c22} \left\{ (1 + \cos 2\theta) u_{x0} + \sin 2\theta u_{y0} \right\} - (-k_s u_x + k_i i_{cx}) \end{aligned} \right\} \quad (2.27)$$

For the horizontal direction motion, we have

$$\left. \begin{aligned} m\ddot{u}_y + (c_v + c_H)\dot{u}_y + k_0 u_y - \omega c_H u_x &= me\omega^2 \sin(\theta + \phi_u) \\ &+ \frac{1}{2} \sigma(t) \Delta k_{c22} \{ \sin 2\theta u_{x0} + (1 - \cos 2\theta) u_{x0} \} - (-k_s u_x + k_i i_{cy}) \end{aligned} \right\} \quad (2.28)$$

Similarly, expanding Equation (2.26) for the vertical direction motion, we have

$$\left. \begin{aligned} m\ddot{u}_x + (c_v + c_H)\dot{u}_x + k_0 u_x + \omega c_H u_y &= me\omega^2 \cos(\theta + \phi_u) + k_0 e_b \cos(\theta + \phi_u) \\ &+ \frac{1}{2} \sigma(t) \Delta k_{c22} \{ (1 + \cos 2\theta)(u_{x0} + u_{xb}) + \sin 2\theta u_{yb} \} - (-k_s u_x + k_i i_{cx}) - \omega c_H u_{x0} \end{aligned} \right\} \quad (2.29)$$

For the horizontal direction motion, we have

$$\left. \begin{aligned} m\ddot{u}_y + (c_v + c_H)\dot{u}_y + k_0 u_y - \omega c_H u_x &= me\omega^2 \sin(\theta + \phi_u) + k_0 e_b \sin(\theta + \phi_u) \\ &+ \frac{1}{2} \sigma(t) \Delta k_{c22} \{ \sin 2\theta(u_{x0} + u_{xb}) + (1 - \cos 2\theta) u_{yb} \} - (-k_s u_y + k_i i_{cy}) \end{aligned} \right\} \quad (2.30)$$

Defining complex variables for the vibrational displacements and control current of AMB is written as

$$s = (u_x + ju_y) \quad \text{and} \quad i_c = i_{cx} + ji_{cy} \quad \text{with} \quad j = \sqrt{-1} \quad (2.31)$$

Multiplying j with Equation (2.28) and adding with Equation (2.27) for the cracked rotor-AMB system, without bent on it, is written as

$$\left. \begin{aligned} m\ddot{s} + (c_v + c_H)\dot{s} + k_0 s - j\omega c_H s &= \underbrace{me\omega^2 e^{j(\theta + \phi_u)}}_{\text{unbalance force}} + \underbrace{\frac{1}{2} \sigma(t) \Delta k_{c22} u_{x0} (1 + e^{2j\theta})}_{\text{crack force}} - \underbrace{(-k_s s + k_i i_c)}_{\text{magnetic force}} \\ &\quad - j\omega c_H u_{x0} \end{aligned} \right\} \quad (2.32)$$

Equation (2.32) is rearranged as,

$$m\ddot{s} + (c_V + c_H)\dot{s} + k_0s - j\omega c_H s = me_u \omega^2 e^{j(\theta+\phi_u)} + \Delta k_{c22} u_{x0} \sum_{i=-n}^n p_i e^{ji\theta} - k_s s - k_i i_c - j\omega c_H u_{x0} \quad (2.33)$$

On multiplying ‘j’ with Equation (2.30) and adding with Equation (2.29) for the warped-cracked rotor-AMB system, we get

$$m\ddot{s} + (c_V + c_H)\dot{s} + k_0s - j\omega c_H s = \underbrace{me_u \omega^2 e^{j(\theta+\phi_u)}}_{\text{unbalance force}} + \underbrace{\frac{1}{2}\sigma(t)\Delta k_{c22} u_{x0} (1 + e^{2j\theta})}_{\text{crack force}} + \underbrace{k_0 e_b e^{j(\theta+\phi_b)}}_{\text{residual shaft bow force}} + \underbrace{\frac{1}{2}\sigma(t)\Delta k_{c22} e_b \cos \phi_b e^{j\theta}}_{\text{crack additive stiffness and shaft bow (multiplicative form)}} - \underbrace{(-k_s s + k_i i_c)}_{\text{magnetic force}} - j\omega c_H u_{x0} \quad (2.34)$$

Equation (2.34) is rearranged as,

$$m\ddot{s} + (c_V + c_H)\dot{s} + k_0s - j\omega c_H s = me_u \omega^2 e^{j(\theta+\phi_u)} + \Delta k_{c22} u_{x0} \sum_{i=-n}^n p_i e^{ji\theta} + k_0 e_b e^{j(\theta+\phi_b)} + \Delta k_{c22} e_b \cos \phi_b \sum_{i=-n}^n q_i e^{ji\theta} - k_s s - k_i i_c - j\omega c_H u_{x0} \quad (2.35)$$

Here, the p_i ’s and q_i ’s are the participation factors of the steering function of the crack faces opening and closure behavior for the i^{th} harmonic of the crack excitation force and bow force, respectively, are represented as,

$$p_i = \frac{1}{2}\sigma(t)(1 + e^{j2\theta}) \quad , \text{and} \quad q_i = \sigma(t)e^{j\theta} \quad (2.36)$$

The magnitude of the participation factors of the switching behavior of the crack excitation function in the form of multiple harmonic response is presented in Table 2.1.

Table 2.1 The participation factor magnitudes due to crack excitation forces and crack and bow force

Multiple harmonics	Participation factors	
	p_i	q_i
0	0.25	0.159
1X	0.319	0.25
2X	0.25	0.159
3X	0.106	0
4X	0	-0.053
5X	-0.021	0
6X	0	0.032
7X	0.009	0
-1X	0.106	0
-2X	0	-0.053
-3X	-0.021	0
-4X	0	0.032
-5X	-0.009	0
-6X	0	-0.026

2.2.3 Development of Identification Algorithm

An identification algorithm is proposed to estimate fault parameters based on rotor system modeling presented in the previous section. To solve developed EOM, solution assumed for multiple harmonic of the crack excitation force is

$$\mathbf{s}(t) = \sum_{i=-n}^n \mathbf{S}_i(t) e^{ji\theta} \quad (2.37)$$

The real time output of PID control current also contains multiple harmonics of same frequency components as displacement harmonics and the assumed solution of the complex current signal at any time instant is represented as

$$\mathbf{i}_c(t) = \sum_{i=-n}^n \mathbf{I}_i(t) e^{ji\theta} \quad (2.38)$$

Putting Equations (2.37) and (2.38) in Equations (2.33) and (2.35) to convert into frequency domain as,

Without shaft residual bow effect

For $i \neq 1$,

$$\left\{ -i^2 \omega^2 m + j i \omega c_v + j(i-1) \omega c_H + k_0 - k_s \right\} S_i = \Delta k_{c22} u_{x0} \sum_{i=-n}^n p_i - k_i I_{ci} - j \omega c_H u_{x0} \quad (2.39)$$

and, for $i=1$,

$$\left\{ -i^2 \omega^2 m + j i \omega c_v + j(i-1) \omega c_H + k_0 - k_s \right\} S_i = m e_u \omega^2 e^{j \phi_u} + \Delta k_{c22} u_{x0} \sum_{i=-n}^n p_i - k_i I_{c1} - j \omega c_H u_{x0} \quad (2.40)$$

With shaft residual bow effect,

For $i \neq 1$,

$$\left\{ -i^2 \omega^2 m + j i \omega c_v + j(i-1) \omega c_H + k_0 - k_s \right\} S_i = \Delta k_{c22} u_{x0} \sum_{i=-n}^n p_i + \Delta k_{c22} e_b \cos \phi_b \sum_{i=-n}^n q_i - k_i I_{ci} - j \omega c_H u_{x0} \quad (2.41)$$

For $i=1$,

$$\left\{ -i^2 \omega^2 m + j i \omega c_v + j(i-1) \omega c_H + k_0 - k_s \right\} S_i = m e_u \omega^2 e^{j \phi_u} + \Delta k_{c22} u_{x0} \sum_{i=-n}^n p_i + \Delta k_{c22} e_b \cos \phi_b \sum_{i=-n}^n q_i + k_0 e_b e^{j \phi_b} - k_i I_{c1} - j \omega c_H u_{x0} \quad (2.42)$$

Equations (2.39) to (2.42) be used for development of the identification equations in frequency domain for both without and with residual bow case to estimate fault parameters.

The frequency domain equations of motion Equations (2.39) and (2.42) separately are rearranged to have all the identifiable parameters on the LHS and the known quantities on the RHS.

Without residual bow case,

$$\begin{aligned}
& -j\omega c_v S_i + j\omega((i-1)S_i + u_{x0})c_H - \Delta k_{c22} u_{x0} \sum_{i=-n}^n p_i - m e_u \omega^2 e^{j\phi_u} - k_s S_i + k_i I_{ci} \\
& = (-i^2 \omega^2 m + k_0) S_i \quad (2.43)
\end{aligned}$$

With residual bow case,

$$\left. \begin{aligned}
& -j\omega c_v S_i + j\omega((i-1)S_i + u_{x0})c_H - \Delta k_{c22} u_{x0} \sum_{i=-n}^n p_i - m e_u \omega^2 e^{j\phi_u} - k_0 e_b e^{j\phi_b} - \Delta k_{c22} e_b \cos \phi_b \sum_{i=-n}^n q_i \\
& - k_s S_i + k_i I_{ci} = (-i^2 \omega^2 m + k_0) S_i
\end{aligned} \right\} \quad (2.44)$$

For the estimation of unknown fault parameters of the system, a linear regression method has been adopted, as

$$\mathbf{A}_1 \mathbf{x}_1 = \mathbf{B}_1 \quad (2.45)$$

where $\mathbf{A}_1$ is the regression matrix or the regressor, $\mathbf{B}_1$ is the vector of known quantities and $\mathbf{x}_1$ is the vector containing the unknowns (both additive faults for without residual bow case and both additive as well as multiplicative fault parameters for with residual bow case) to be determined in estimation.

Equations (2.43) and (2.44) in matrix form by splitting the complex displacement and current harmonics into real and imaginary terms as $S_i = S_{i,Re} + jS_{i,Im}$ and $I_i = I_{i,Re} + jI_{i,Im}$, we get

$$\begin{bmatrix}
-\omega S_{1,lm} & 0 & -u_{x0} p_1 & -m\omega^2 & 0 & -k_0 & 0 & -q_0 & -S_{1,Re} & I_{1,Re} \\
0 & \omega S_{0,lm} & -u_{x0} p_0 & 0 & 0 & 0 & 0 & -q_1 & -S_{0,Re} & I_{0,Re} \\
-2\omega S_{2,lm} & -\omega S_{2,lm} & -u_{x0} p_2 & 0 & 0 & 0 & 0 & -q_2 & -S_{2,Re} & I_{2,Re} \\
-3\omega S_{3,lm} & -2\omega S_{3,lm} & -u_{x0} p_3 & 0 & 0 & 0 & 0 & 0 & -S_{3,Re} & I_{3,Re} \\
-4\omega S_{4,lm} & -3\omega S_{4,lm} & 0 & 0 & 0 & 0 & 0 & -q_4 & -S_{4,Re} & I_{4,Re} \\
-5\omega S_{5,lm} & -4\omega S_{5,lm} & -u_{x0} p_5 & 0 & 0 & 0 & 0 & 0 & -S_{5,Re} & I_{5,Re} \\
-6\omega S_{6,lm} & -5\omega S_{6,lm} & 0 & 0 & 0 & 0 & 0 & -q_6 & -S_{6,Re} & I_{6,Re} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\omega S_{-1,lm} & 2\omega S_{-1,lm} & -u_{x0} p_{-1} & 0 & 0 & 0 & 0 & -q_{-1} & -S_{-1,Re} & I_{-1,Re} \\
2\omega S_{-2,lm} & 3\omega S_{-2,lm} & 0 & 0 & 0 & 0 & 0 & -q_{-2} & -S_{-2,Re} & I_{-2,Re} \\
3\omega S_{-3,lm} & 4\omega S_{-3,lm} & -u_{x0} p_{-3} & 0 & 0 & 0 & 0 & 0 & -S_{-3,Re} & I_{-3,Re} \\
4\omega S_{-4,lm} & 5\omega S_{-4,lm} & 0 & 0 & 0 & 0 & 0 & -q_{-4} & -S_{-4,Re} & I_{-4,Re} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\omega S_{1,Re} & 0 & 0 & 0 & -m\omega^2 & 0 & -k_0 & 0 & -S_{1,lm} & I_{1,lm} \\
0 & \omega(S_{0,Re} + u_{x0}) & 0 & 0 & 0 & 0 & 0 & 0 & -S_{0,lm} & I_{0,lm} \\
2\omega S_{2,Re} & \omega S_{2,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{2,lm} & I_{2,lm} \\
3\omega S_{3,Re} & 2\omega S_{3,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{3,lm} & I_{3,lm} \\
4\omega S_{4,Re} & 3\omega S_{4,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{4,lm} & I_{4,lm} \\
5\omega S_{5,Re} & 4\omega S_{5,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{5,lm} & I_{5,lm} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-\omega S_{-1,Re} & -2\omega S_{-1,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{-1,lm} & I_{-1,lm} \\
-2\omega S_{-2,Re} & -3\omega S_{-2,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{-2,lm} & I_{-2,lm} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
\end{bmatrix}
\begin{Bmatrix}
c_V \\
c_H \\
\Delta k_c \\
e_{u,Re} \\
e_{u,lm} \\
e_{b,Re} \\
e_{b,lm} \\
\Delta k_{c22} e_{b,Re} \\
k_s \\
k_i
\end{Bmatrix}
=
\begin{Bmatrix}
(\omega^2 m - k_0) S_{1,Re} \\
-k_0 S_{0,Re} \\
(4\omega^2 m - k_0) S_{2,Re} \\
(9\omega^2 m - k_0) S_{3,Re} \\
(16\omega^2 m - k_0) S_{4,Re} \\
(25\omega^2 m - k_0) S_{5,Re} \\
(36\omega^2 m - k_0) S_{6,Re} \\
\vdots \\
(\omega^2 m - k_0) S_{-1,Re} \\
(4\omega^2 m - k_0) S_{-2,Re} \\
(9\omega^2 m - k_0) S_{-3,Re} \\
(16\omega^2 m - k_0) S_{-4,Re} \\
\vdots \\
(\omega^2 m - k_0) S_{1,lm} \\
-k_0 S_{0,lm} \\
(4\omega^2 m - k_0) S_{2,lm} \\
(9\omega^2 m - k_0) S_{3,lm} \\
(16\omega^2 m - k_0) S_{4,lm} \\
(25\omega^2 m - k_0) S_{5,lm} \\
\vdots \\
(\omega^2 m - k_0) S_{-1,lm} \\
(4\omega^2 m - k_0) S_{-2,lm} \\
\vdots
\end{Bmatrix}
\quad (2.46)$$

Equation (2.46) can be expressed in a typical matrix form and estimates the identifiable parameters by applying the pseudo-inverse technique, as

$$\mathbf{x}_1 = \left(\mathbf{A}_1(\omega)^T \mathbf{A}_1(\omega) \right)^{-1} \mathbf{A}_1(\omega)^T \mathbf{B}_1(\omega) \quad (2.47)$$

with $\mathbf{x}_1 = [c_V \quad c_H \quad \Delta k_{c22} \quad \Delta k_{c22} e_{b,Re} \quad e_{Re} \quad e_{lm} \quad k_s \quad k_i]^T$ without bow case

and, with $\mathbf{x}_1 = [c_V \quad c_H \quad \Delta k_{c22} \quad \Delta k_{c22} e_{b,Re} \quad e_{Re} \quad e_{lm} \quad e_{b,Re} \quad e_{b,lm} \quad k_s \quad k_i]^T$ bow case.

The range of spin speed of rotor plays a vital role in the performance of the estimation procedure as the multiple spin speeds contain larger data sets, which give better

$$\begin{bmatrix} \mathbf{A}_1(\omega_1) \\ \mathbf{A}_1(\omega_2) \\ \vdots \\ \mathbf{A}_1(\omega_r) \end{bmatrix} \mathbf{x}_1 = \begin{Bmatrix} \mathbf{B}_1(\omega_1) \\ \mathbf{B}_1(\omega_2) \\ \vdots \\ \mathbf{B}_1(\omega_r) \end{Bmatrix} \quad (2.48)$$

2.3 Numerical Experiment

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The excitation forces associated with the rotor model is shown in Figure 2.4. The simulation is done with a sampling rate of 2^{14} in the embedded fourth-order Runge- Kutta method (i.e., using Matlab function ode45).

Table 2.2 Physical parameters of the rotor-bearing system

Rotor Properties	Assumed values
Disc mass, m	2 kg
Shaft length, l	0.50 m
Shaft diameter, d	0.016 m
Young's modulus, E	$2.06 \times 10^{11} \text{ Nm}^{-2}$
External damping, c_v	140 Nsm^{-1}
Internal damping, c_H	75 Nsm^{-1}
Intact shaft stiffness, k_0	$3 \times 10^5 \text{ Nm}^{-1}$
Additive crack stiffness, Δk_{c22}	$7.5 \times 10^3 \text{ Nm}^{-1}$
Disc eccentricity, e	$2 \times 10^{-4} \text{ m}$
Disc eccentricity phase, ϕ_u	60°
Shaft translational bow, e_b	$8 \times 10^{-4} \text{ m}$
Translational bow phase, ϕ_b	30°

Table 2.3 Magnetic bearing actuator properties

Actuator parameters	Assumed Values
Force -displacement factor, k_s	217683 Nm^{-1}
Force-current factor, k_i	43.537 NA^{-1}

Table 2.4 Controller gains

PID controller gains	Assumed Values
Proportional gain, K_p	$8,000 \text{ Am}^{-1}$
Integral gain, K_i	$3,000 \text{ A(m s)}^{-1}$
Derivative gain, K_D	3 Asm^{-1}

2.3.1 Stability Analysis

The stability of the controller is the absolute requirement for the system model with respect to the changing dynamic character of the rotor over time. The PID controller parameters of AMB, which are inherently unstable, implemented in the present analysis, are based on Routh-Hurwitz criterion. For the assessment of the stability, the transfer function of the system is commonly required. The stability criterion using the Nyquist plot allows predicting the stability and performance of a closed-loop (CL) system by observing its open-loop behaviour in the frequency domain. The schematic view of the closed loop feedback system of the proposed rotor model is shown in Figure 2.5.

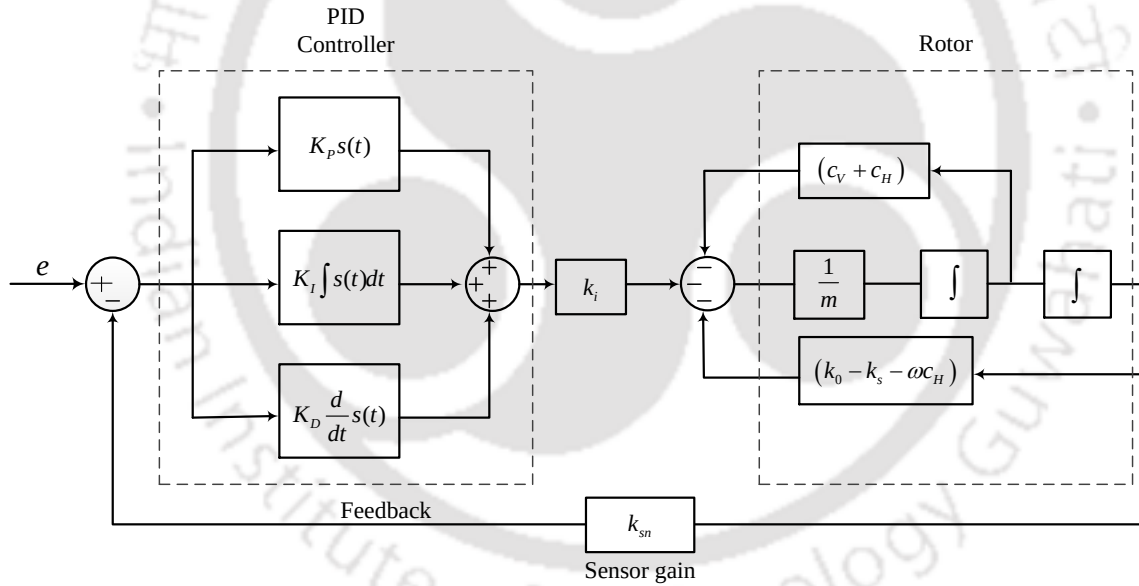


Figure 2.5 Feedback system closed loop rotor system

The transfer function of the system containing the controller, sensor and magnetic bearing in series be represented as G_{CMA} , the overall sensor correction gain as k_{sn} , the new vertical displacement (u_x), which gives the displacement of the rotor from the static equilibrium position presented in Appendix A. Here for this case, the error (e) is the displacement of

the rotor from the centre position as sensed by the position sensors and the output would be the controlling current to produce controlling forces at the magnetic bearings. Essentially, Equation (2.25) employs the Laplace transform and the transfer function (G_{tf}) of the above closed loop feedback system with the external force as input and the control current as output in modified form as in Ref. (Tiwari, 2017) is expressed as

$$G_{tf} = \frac{G_{CMA}}{1 + G_{CMA}G_{sn}} \quad (2.49)$$

$$G_{tf} = \frac{(K_p s^2 + K_I s + K_D s^3) k_i k_s}{(ms^3 + (c_v + c_H + K_D k_i k_s k_{sn})s^2 + (k_0 - j\omega c_H - k_s + K_p k_i k_x k_{sn})s + K_I k_i k_s k_{sn})} \quad (2.50)$$

The condition of the Nyquist stability criterion is given as

$$N = Z - P \quad (2.51)$$

where, P defines the number of poles and N is the number of encirclements $((-1+j.0)$ of the point) of the origin of $(1+G_{CMA}G_{sn})$ domain in the clockwise direction of the complex plane. According to the Nyquist criterion, the zeros (Z) of the characteristic equation of Equation (2.50), i.e. the closed loop system poles, are not present in the right half of the s -plane to make the system stable.

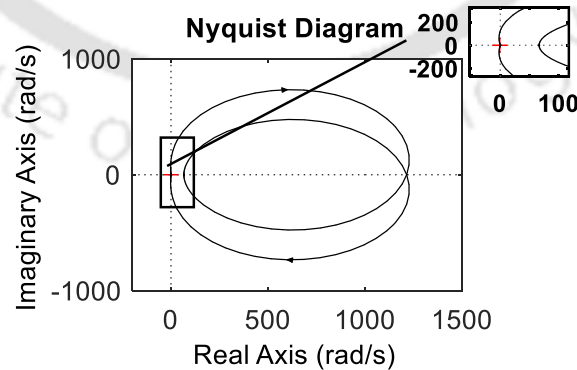


Figure 2.6 Nyquist stability plot of the uncracked rotor system with internal damping integrating AMB operating at the speed range of 33 Hz to 60 Hz

In the present analysis, the Nyquist plot of AMB integrated uncracked internally damped rotor operating for a spin speed range of 33 Hz to 60 Hz with an enlarged view is shown in Figure 2.6. The following conclusions have been drawn from Figure 2.6 as per the stability criterion

- The characteristic equation (Equation (2.50)) of the closed-loop transfer function has no zeros in the right half (RHP) of the s -plane.
- The graph does not cross the critical point in the frequency domain plane, i.e. $(-1+j\ 0)$ point is not encircled indicating that poles do not lie in the right half of s -plane in the frequency domain.

From the above two points according to the Nyquist stability criterion, the system is stable. Therefore, we can infer that the controller parameters considered in the analysis can operate the proposed rotor system in a stable condition.

2.3.2 Forced Response Analysis

The rotor is excited by various rotor faults, such as the unbalance, crack, internal damping, shaft residual bow, and magnetic bearing force. The dynamic behavior of the rotor is affected by different faults and is analyzed by generating its time domain signal. The ramp up response of the proposed rotor system without and with AMB is simulated for 20 s with a constant angular acceleration of 40 rads^{-2} as shown in Figure 2.7(a, b) respectively, to choose the speed range for performing the estimation procedure. The rotor with the AMB gets stiffer as the PID controller used in AMB provides a stiffening effect due to the proportional gain and thus, the critical speed of the rotor shifts as we can observe in Figure 2.7(b). Due to this reason, the rotating machinery safely crosses the critical speed without any intervention in damage to the critical rotor components. The critical speed of the rotor

without AMB is found to be 389.6 rad/s and with the incorporation of AMB the same is found to be 481.8 rad/s. With the inclusion of AMB as auxiliary bearing in rotor system can control the vibration demonstrating its advantage as an active controlling purpose.

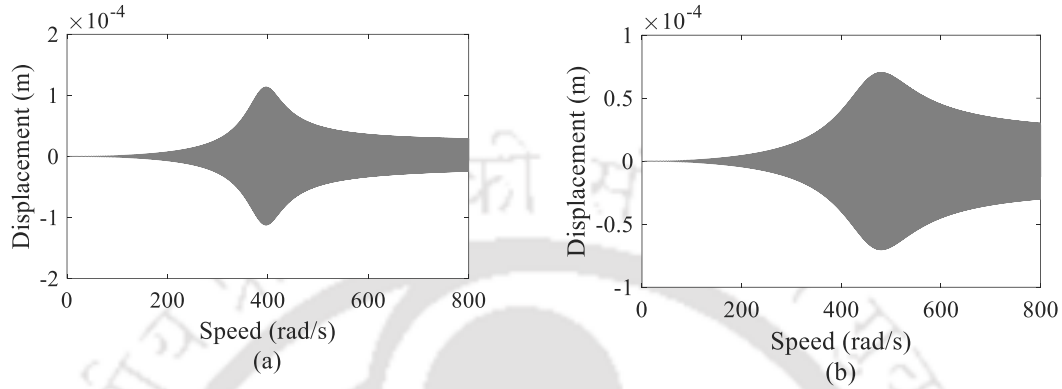


Figure 2.7 Ramp up response of the rotor (a) without AMB (b) with AMB

The time domain signal of the rotor without shaft bow fault is shown in Figure 2.8(a, b) with the amplitude of vibration of 1.15×10^{-5} m and 1.85×10^{-5} m the vertical and horizontal displacements, respectively. Similarly, the control current of AMB required to suppress the vibration emerging due to multiple faults is 0.108 A in the vertical direction and 0.106 A in the horizontal direction, as depicted in Figure 2.8(c, d). The orbit of both displacement and current signal shows a loopy structure due to the presence of crack in the rotor as can be observed in Figure 2.8(e, f).

The effect of shaft residual bow on the dynamics of the cracked rotor is shown in Figure 2.9, where the amplitude of vibration increases as compared to without bow case as seen in Figure 2.8. The shaft bow dominates the crack force effect onto the rotor, which can be noticed in the time domain response of Figure 2.9(a-f). The vibrational displacements in the vertical and horizontal directions are 8.65×10^{-5} m and 1.03×10^{-4} m, respectively, and the control current of 0.728 A and 0.848 A in Figure 2.9(a-d). The orbit plot is shown in

Figure 2.9(e, f). It is clearly observed that the residual shaft bow effect dominant over the crack dynamics thus, overshadows the crack effect as no loopy orbits seen due to the increase in vibration amplitude. Similarly, the AMB control current required to suppress the vibrational amplitude of the cracked-warped rotor is high, thus increase in energy consumption by the AMB.

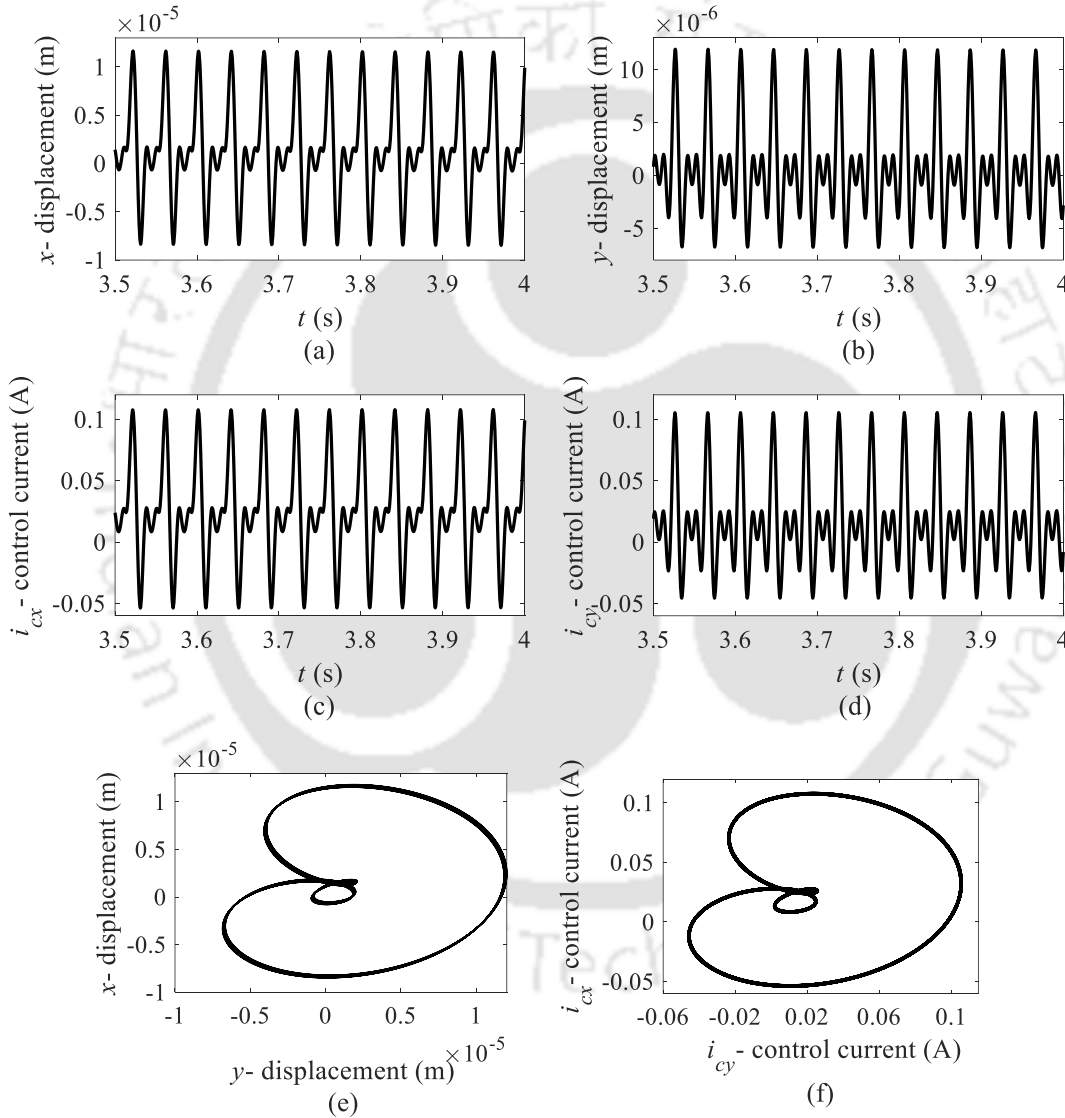


Figure 2.8 Time waveform of rotor without shaft residual bow at 25 Hz spin speed (a) displacement in vertical direction (b) displacement in horizontal direction (c) control current of AMB in vertical direction (d) control current of AMB in horizontal direction (e) orbit plot of displacement signal (f) orbit plot of current signal

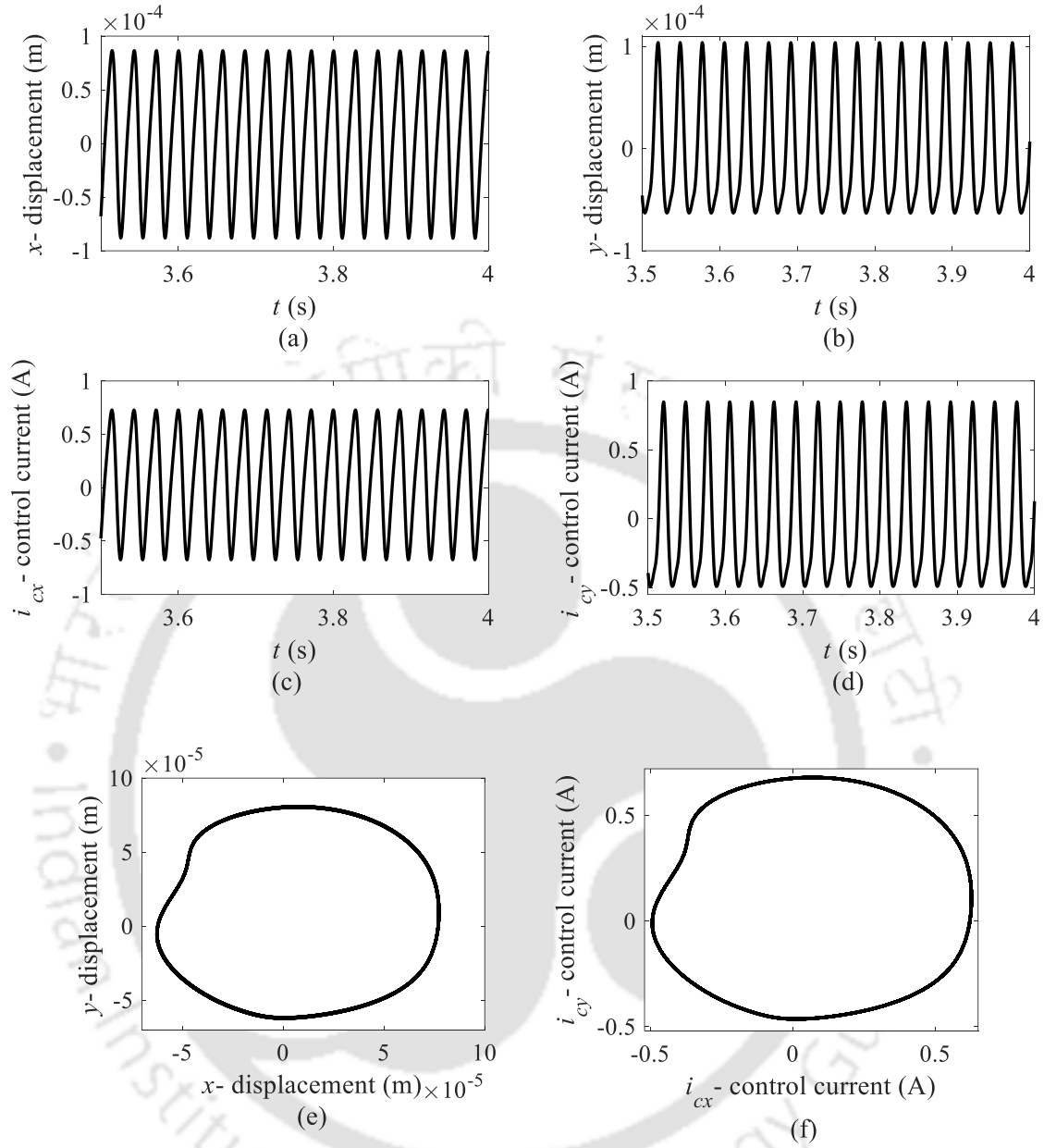


Figure 2.9 Time waveform of rotor with residual bow at 25 Hz spin speed (a) displacement in vertical direction (b) displacement in horizontal direction (c) control current of AMB in vertical direction (d) control current of AMB in horizontal direction (e) orbit plot of displacement signal (f) orbit plot of current signal

The orbit plots of displacement and control current are the superposition of all the harmonics in the forward and backward sense of rotation. It presents orbits describing the path of the cracked and warped shaft with the disc unbalance.

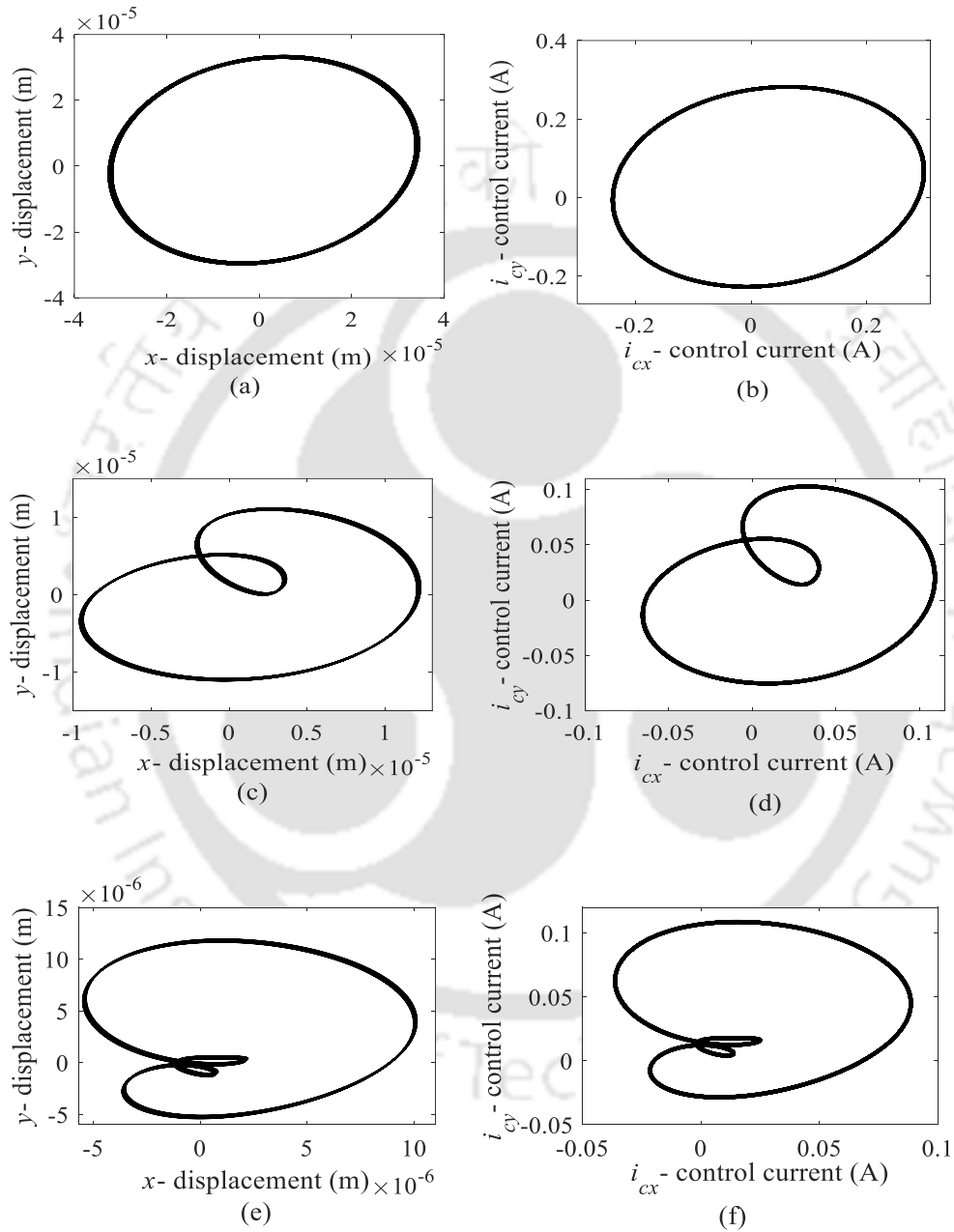


Figure 2.10 Rotor without residual shaft bow orbit plot of displacement and control current (a, b) one per revolution (c, d) twice per revolution (e, f) thrice per revolution of the shaft with crack excitation force and unbalance

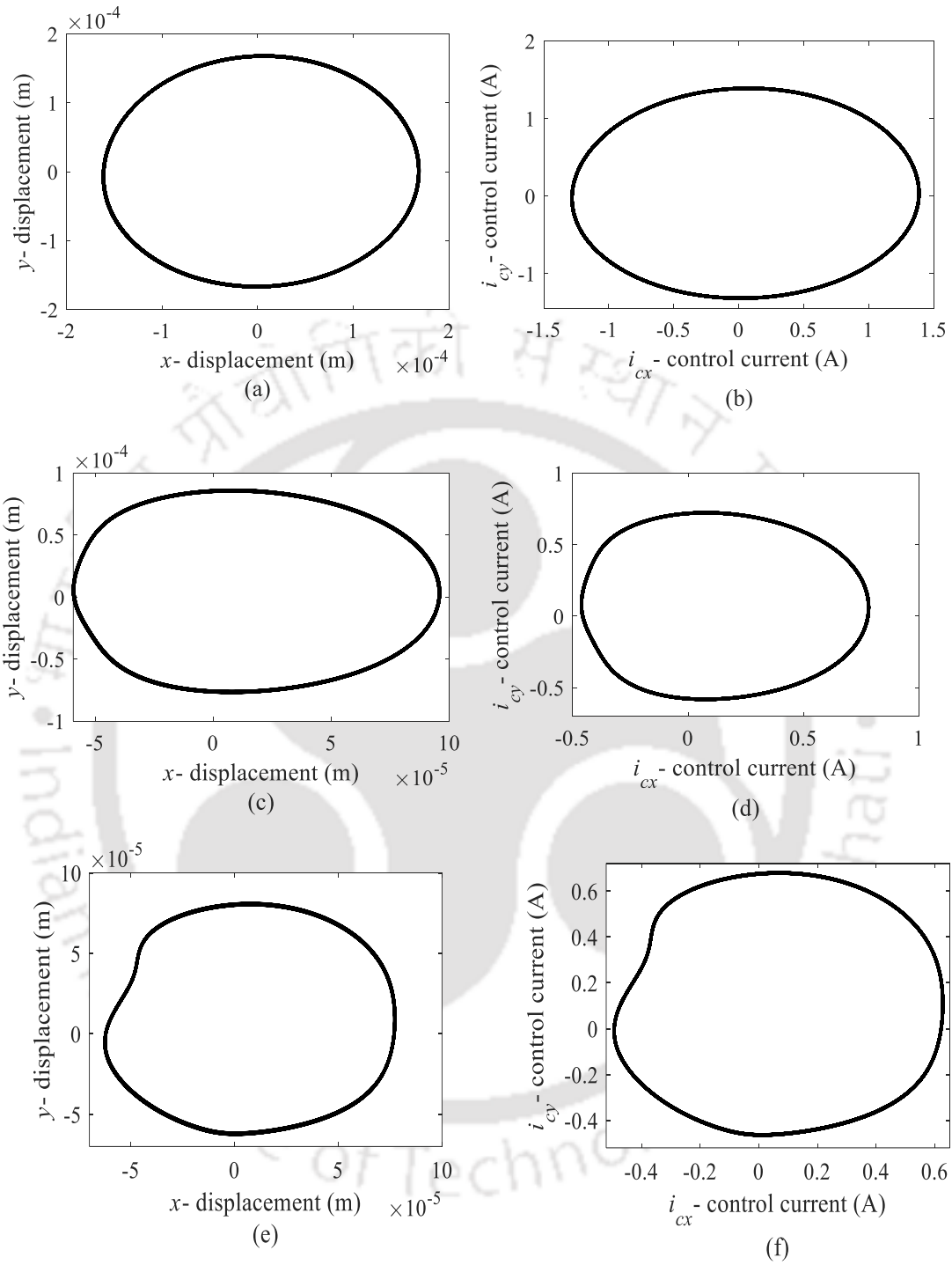


Figure 2.11 Rotor with residual shaft bow orbit plot of displacement and control current (a, b) one per revolution (c, d) twice per revolution (e, f) thrice per revolution of the shaft with crack excitation force and unbalance and shaft residual bow

Orbits describing the path of the cracked and warped shaft with the disc unbalance as shown in Figure 2.10 and Figure 2.11, respectively, depict the orbit plot of a cracked rotor without and with shaft residual bow for ones, twice, thrice, and fourth per revolution of the main excitation force. It is observed from Figure 2.10(a, b) that the orbit is elliptical with a single loop in shape due to resonance at one revolution of the excitation forces. Whereas double loops and triple loops in orbital plots as shown, respectively, in Figure 2.10(c, d) and Figure 2.10(e, f) corresponding to twice and thrice per revolution of the excitation force frequencies. Similarly, orbit plots of displacement and current of a cracked rotor with shaft residual bow as shown in Figure 2.11(a-f) do not show loops in the orbit due to the presence of bow in the cracked rotor. Therefore, it is very much essential to remove the residual shaft bow from the vibrational displacement at slow roll condition and/or, estimate the magnitude of shaft bow and its orientation to precisely detect the crack in a rotor system.

2.3.2.1 Full Spectrum Analysis

Full spectrum tool applied in vibration based analysis gives an insight to recognize and diagnose unique defects in a rotating system pioneered by (Goldman and Muszynska, 1999; Muszynska, 2005). The full spectrum magnitude of the complex rotor vibration signal, the positive (negative) frequencies interpret the forward (backward) components and the negative frequencies spectrum is not the mirror image of the positive ones. The whirling motion of the rotor if it is in the same sense of rotor spin then it is defined as the forward whirl and in opposite sense by the backward whirl. To distinctly evaluate these two senses of rotation, i.e. the forward and backward components, the Fourier transform of time domain complex signal in exponential form is plotted, where the real part represents the

forward precession, and synonymously, the negative part the backward precessional mode. In the present analysis, the crack excitation force in a rotor generates both the forward and the backward whirl in multiple harmonics. Thus, the time domain signal is converted into frequency domain amplitude and phase in full spectrum response showing the directional nature of the rotor. Generally, the experimental measurement of time domain signal of vibration and current in test rigs is done at arbitrary instants of time. This often results in non-integer number of cycles, i.e. incomplete cycle in the acquired signal and when that signal is converted to full spectrum using FFT, a leakage error may be expected in the frequency domain. Thus, phase compensation algorithm for correction of phase is developed to avoid that leakage error from the signal by subtracting the phase of the acquired signal of interest with that of the reference signal of vibration as well as current signal of test setup (Tiwari, 2017). In the present analysis, estimation of parameters is done using the full spectrum with the phase compensation methodology.

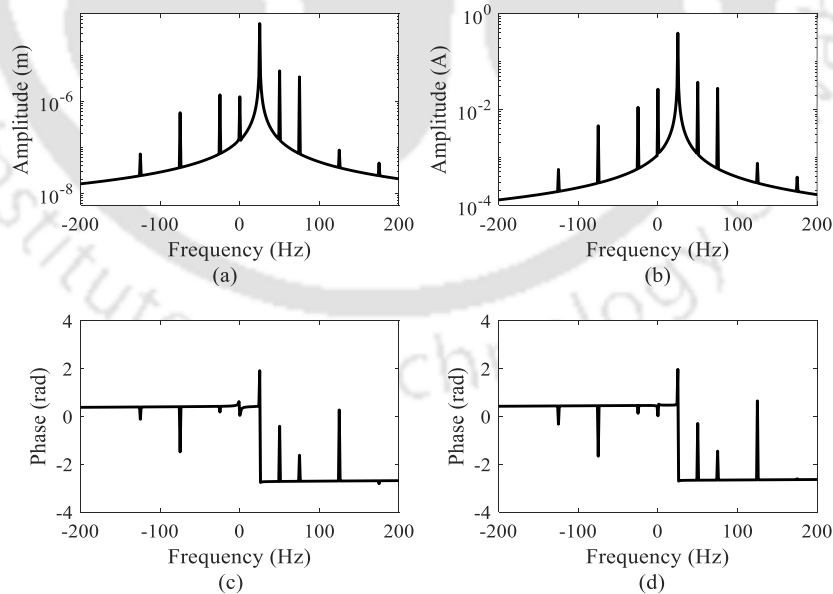


Figure 2.12 Full spectrum response of rotor without residual bow at 25 Hz spin speed (a) amplitude (c) phase of displacement (b) amplitude (d) phase of control current of AMB

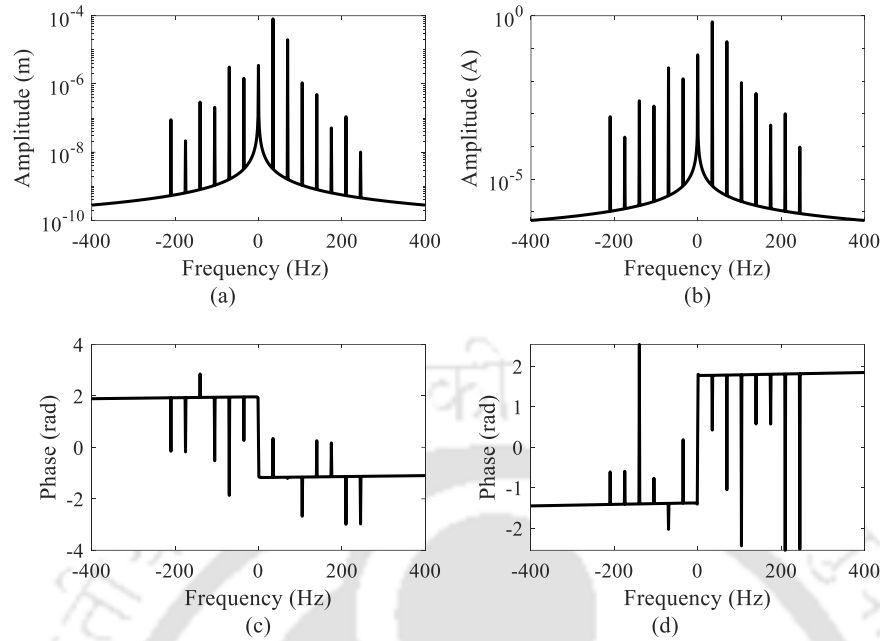


Figure 2.13 Full spectrum response of rotor with residual bow at 25 Hz spin speed (a) amplitude (c) phase of displacement (b) amplitude (d) phase of control current of AMB

Figure 2.12 and Figure 2.13 show the full spectrum responses consisting only of odd harmonics including the second forward harmonic for rotor without residual shaft bow, and all odd and even harmonics in forward and backward whirling for rotor with shaft residual bow, respectively. The full spectrum amplitude of displacement signal S_i and control current signal I_i and its phases for multiple harmonic signals are obtained from the FFT analysis and are summarized in Table 2.5. It is observed from that table, full spectrum amplitudes are obtained for odd harmonics including the second one with an exception of the rotor without bow case whereas rotor system with shaft bow contains all the harmonics irrespective of odd or even in full spectrum response. One more thing observed herein is that the shaft residual bow increases the magnitudes of displacement and current in full spectrum harmonics. These magnitudes and phases will be used in the developed identification algorithm for estimation of multiple fault parameters.

Table 2.5 Full spectrum amplitude and phase of a cracked rotor with and without warped shaft

No. of harmonic of signal (i)	Rotor without residual shaft bow				Rotor with residual shaft bow			
	Displacement signal (S_i)		Control signal (I_i)		Displacement signal (S_i)		Control signal (I_i)	
	Amplitude	Phase	Amplitude	Phase	Amplitude	Phase	Amplitude	Phase
	(m)	(rad)	(A)	(rad)	(m)	(rad)	(A)	(rad)
0	1.26×10^{-6}	0.043	0.026	0.027	2.83×10^{-6}	0.01	0.062	0.019
1	4.94×10^{-5}	1.914	0.395	1.97	8.52×10^{-5}	0.98	0.681	1.04
2	4.61×10^{-6}	-0.409	0.037	-0.291	1.10×10^{-5}	-0.39	0.089	-0.279
3	3.41×10^{-6}	-1.623	0.028	-1.45	3.41×10^{-6}	-1.62	0.027	-1.45
4	-	-	-	-	1.32×10^{-6}	0.54	0.011	0.778
5	8.73×10^{-8}	0.272	7.4×10^{-4}	0.651	8.719×10^{-8}	0.27	7.47×10^{-4}	0.652
6	-	-	-	-	2.75×10^{-7}	-2.88	0.002	-2.578
7	4.57×10^{-8}	-2.82	3.8×10^{-4}	-2.61	4.58×10^{-8}	-2.81	3.84×10^{-4}	-2.607
-1	1.39×10^{-6}	0.183	0.011	0.132	1.38×10^{-6}	0.18	0.011	0.129
-2	-	-	-	-	2.02×10^{-6}	-2.67	0.016	-2.791
-3	5.67×10^{-7}	-1.48	0.005	-1.66	5.66×10^{-7}	-1.48	0.005	-1.663
-4	-	-	-	-	7.90×10^{-7}	2.52	0.007	2.291
-5	7.19×10^{-8}	-0.118	5.5×10^{-4}	-0.324	7.19×10^{-8}	-0.11	5.5×10^{-4}	-0.325
-6	-	-	-	-	2.16×10^{-7}	-0.19	0.002	-0.509

The full spectrum harmonics generated will be utilized as an input to the developed identification algorithm to estimate the parameters considered in the developed mathematical modeling. The concept of phase compensation comes into the picture due to the measurement of the phase with respect to the reference phase of the rotor system. The

phase compensation technique is used as a difference of the phase of the system with that of the reference phase.

2.3.3 Simultaneous Estimation of Multiple Fault Parameters using Full Spectrum

This section elaborates the numerical testing of the identification algorithm developed in Section 2.2.3 and it is performed using Equations (2.44) and (2.45) for different cases of rotor spin speed range and comparison of the estimated fault parameters with the assumed ones are presented. The identification algorithm development flow chart for multiple fault estimation is depicted in Figure 2.14.

In the testing of the identification algorithm all the fault parameters are treated as unknown to be estimated using full spectrum amplitude and phase. To start the testing, different ranges of speed are chosen for which the displacement and control current responses are considered. The resonance of once per revolution of the crack excitation force of an unbalanced bowed rotor occurs at 390.93 rads^{-1} (62.22 Hz) with corresponding peak displacement $6.687 \times 10^{-4} \text{ m}$ as depicted in Figure 2.15(a). Similarly, the resonance of twice and thrice per revolution of the excitation force of crack is observed at 195.46 rads^{-1} (31.1 Hz) and 130.31 rads^{-1} (20.74 Hz), respectively, as shown in Figure 2.15(b, c) with corresponding maximum displacements of $1.067 \times 10^{-4} \text{ m}$ and $6.873 \times 10^{-5} \text{ m}$.

The critical speed of the rotor system and the corresponding peak displacement at this point need to be studied to avoid the measurement error near these speeds relative to the presence of resonance both above and below the spin speed commonly known as half-power points ($S=0.707S_{\text{resonance}}$) on either side of the resonant amplitude.

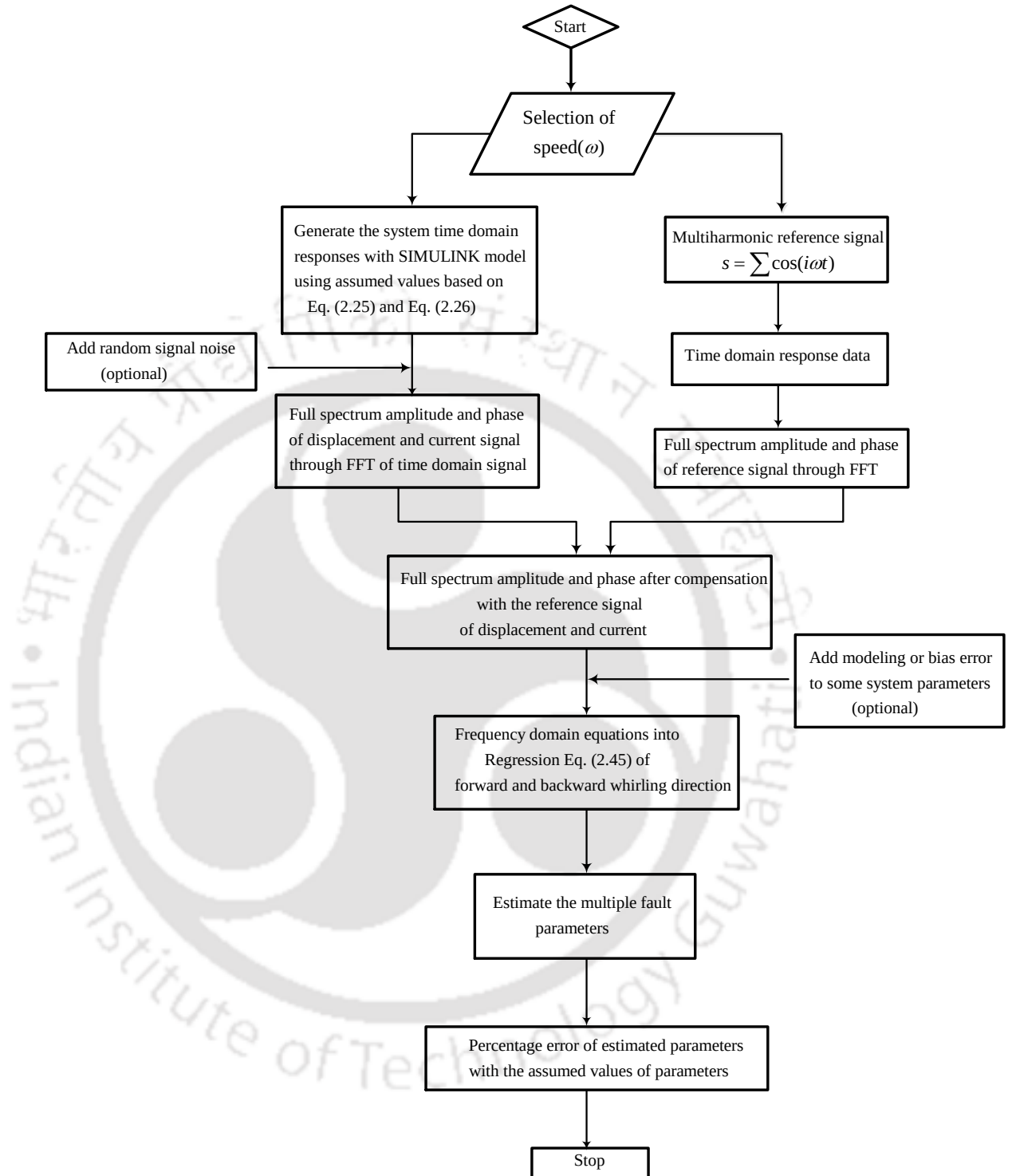


Figure 2.14 Flow chart of the identification algorithm

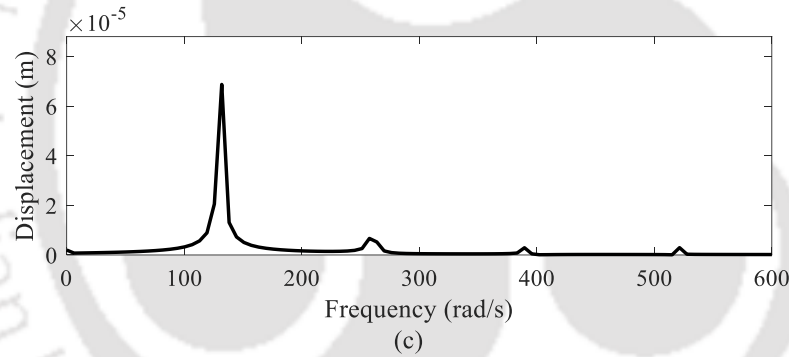
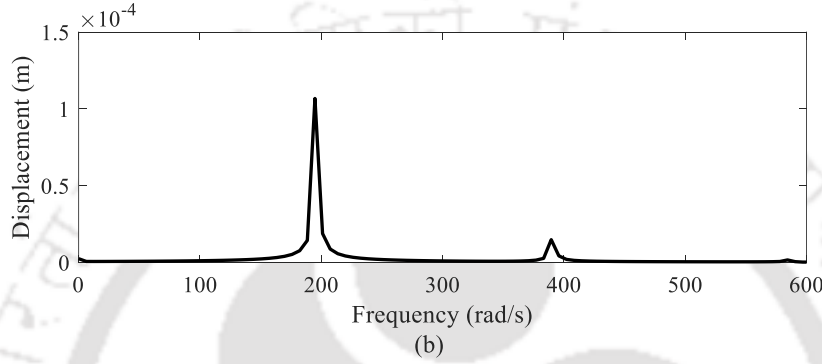
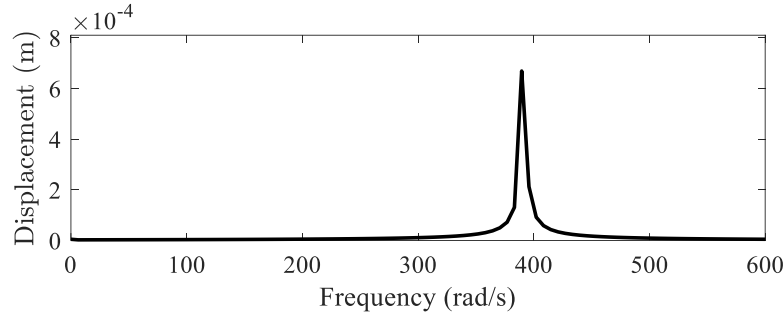


Figure 2.15 Resonance plot at (a) one (b) two (c) three revolution of excitation forces

It is observed that the half-power points correspond to the displacement limit for 1X, 2X, and 3X resonance are 4.7277×10^{-4} m, 7.544×10^{-5} m, and 4.859×10^{-5} m, respectively. Now, the spin speed of the rotor corresponding to half-power points are $388.93 \text{ rad/s}^{-1}$ (61.9 Hz) and $392.93 \text{ rad/s}^{-1}$ (62.54 Hz) corresponding to resonance at once per revolution of the excitation are observed from Figure 2.15(a). Likewise, the half-power point frequencies relative to resonance at twice per revolution of excitation as noted from Figure 2.15(b) are $189.16 \text{ rad/s}^{-1}$ (30.11 Hz) and $201.76 \text{ rad/s}^{-1}$ (32.11 Hz). Lastly, for thrice per revolution of

excitation that speeds are 124.1 rads^{-1} (19.75 Hz) and 136.50 rads^{-1} (21.72Hz) and shown in Figure 2.15(c).

Though the crack excitation force possesses multiple harmonics, the magnitude of higher harmonics such as fifth, seventh, etc., compared to first, second and third are very less, that is why these frequency ranges are not considered for estimation purpose. The frequency ranges are selected for the 1X, 2X, and 3X frequency components considering the above facts of signal processing. The speed cases are chosen for the numerical illustration, such as Case I from 10 Hz to 18 Hz (excluding the lower limit of half-powerpoint of 3X resonance), Case II from 23 Hz to 31 Hz (excluding the upper limit of half-power point of 3X resonance to that of the lower limit of 2X resonance), Case III from 33 Hz to 60 Hz (excluding the upper limit of half-power point of 2X resonance to that of the lower limit of 1X resonance) from Figure 2.15(a-c). The responses are generated at multiple spin speeds with an interval of 1 Hz used further for estimation.

In the present work, two cases of cracked rotor-AMB system is proposed where one of the rotor does not experiences shaft residual bow effect and the other is having the shaft residual bow. Firstly, the simultaneous estimation of parameters, such as the external damping and internal damping, translational additive crack stiffness, unbalance, and AMB force-displacement and force-current constants for the rotor without residual bow is performed at multiple spin speeds using Equation (2.48). The inherent and external noises are always present in the measured rotor system responses and it is very difficult to remove all the noises from the system responses completely. Therefore, it is important to add random noises to the numerically simulated responses to confirm the adequacy of the developed identification algorithm in the estimation procedure. Different levels of random noise of 3%,

5%, and 10% have been introduced to the uncontaminated time domain response and system parameters to check the sensitiveness of the algorithm as shown in Table 2.6 for the speed range of 23 Hz to 31 Hz.

Table 2.6 Estimates of multiple fault parameters with no shaft residual bow for the measurement speed range of 23 Hz to 31 Hz with various percentage of random noise in response

Fault parameters	Assumed parameters	Speed range of 23 to 31 Hz)			
		Random noise			
		0%	3%	5%	10%
c_v	140 (Nsm ⁻¹)	144.479	140.245	140.222	140.193
c_H	75 (Nsm ⁻¹)	74.571	74.595	74.587	74.575
Δk_{c22}	7.5×10^3 (Nm ⁻¹)	74578.331	74597.841	74585.546	74568.952
k_s	217683 (Nm ⁻¹)	218113.362	218080.313	218084.592	218083.839
k_i	43.537 (NA ⁻¹)	43.251	43.261	43.255	43.245
e_u	2×10^{-5} (m)	1.980×10^{-5}	1.977×10^{-5}	1.976×10^{-5}	1.975×10^{-5}
ϕ_u	60° (deg)	59.987	59.957	59.958	59.955

The percentage deviation of the estimated parameters from the assumed values for 3%, 5% and 10% random noises for the combined speed case is shown in Figure 2.16. It is observed from Figure 2.16 that the highest deviating parameter is the external damping parameter of 3.02% from the assumed values with uncontaminated signal. The rest parameters are not showing error with high severity by the noise addition into time domain signal.

Secondly, multiple fault parameters estimation for the cracked rotor-AMB experiencing residual bow is presented. Using Equation (2.48) for the multiple speed cases, simultaneous

estimation of external and internal damping, translational additive crack stiffness, residual shaft bow, multiplicative fault of crack and bow, unbalance, and AMB force-displacement and force-current constants are performed. It is observed from Table 2.7 and Figure 2.17(a) that at 10% noise addition, the force-stiffness factor (k_s) of AMB is having the highest percentage deviation of -1.60% and the least deviating parameter is the additive stiffness (Δk_{c22}) of -0.257% from the assumed ones. Rest all parameters are well agreement with the assumed values. Similarly, Table 2.8 presented the estimated fault parameters for Case II, where the highest percentage deviation is found to be -0.192% in Δk_{c22} parameter and the least affected one is k_s with -0.054% as shown in Figure 2.17(b). For Case III, estimated parameters are shown in Table 2.9 and Figure 2.17(c) shows the force-stiffness factor (k_s) is the highly deviated parameter with -1.65% from the assumed values.

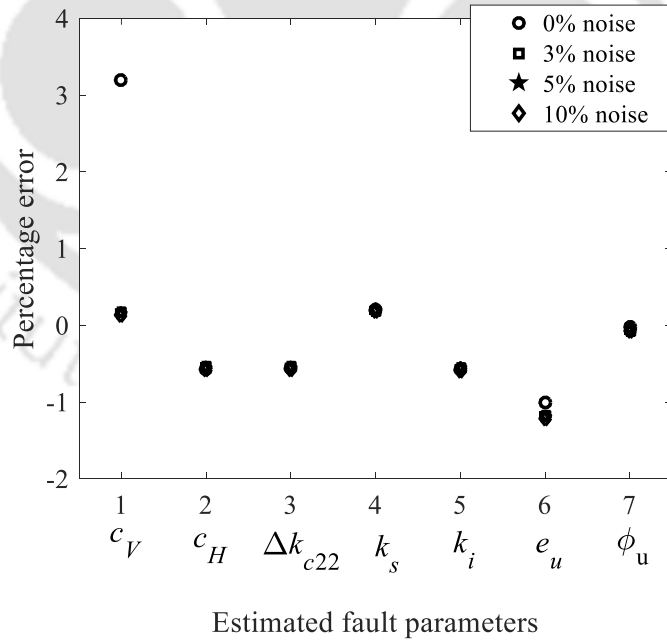


Figure 2.16 Error percentage of estimated fault parameters without residual bow for various random noise levels for the speed range of 23 Hz to 31 Hz

Table 2.7 Estimates of multiple fault parameters with residual bow for the measurement speed range of Case I with various percentage of random noise in response

Fault parameters	Assumed parameters	Case I (speed range of 10 to 19 Hz)			
		Random noise			
		0%	3%	5%	10%
c_v	140 (Nsm ⁻¹)	139.829	139.886	139.921	139.997
c_H	75 (Nsm ⁻¹)	74.941	74.837	74.7631	74.559
Δk_{c22}	7.5×10^3 (Nm ⁻¹)	74956.991	74837.340	74754.316	74535.532
k_s	217683 (Nm ⁻¹)	217708.391	217601.729	217508.562	217198.930
k_i	43.537 (NA ⁻¹)	43.461	43.398	43.352	43.223
e_u	2×10^{-5} (m)	1.999×10^{-5}	1.998×10^{-5}	1.998×10^{-5}	1.999×10^{-5}
ϕ_u	60° (deg)	59.99	59.85	59.76	59.52
e_b	8×10^{-5} (m)	7.999×10^{-5}	7.987×10^{-5}	7.981×10^{-5}	7.969×10^{-5}
ϕ_b	30° (deg)	29.99	29.99	29.99	29.99
$\Delta k_{c22} e_{b,Re}$	5.99 (N)	5.191	5.185	5.182	5.171

Table 2.8 Estimates of multiple fault parameters with residual bow for the measurement speed range of Case II with various percentage of random noise in response

Fault parameters	Assumed parameters	Case II (speed range of 23 to 31 Hz)			
		Random noise			
		0%	3%	5%	10%
c_v	140 (Nsm ⁻¹)	139.862	139.862	139.862	139.862
c_H	75 (Nsm ⁻¹)	74.958	74.958	74.958	74.958
Δk_{c22}	7.5×10^3 (Nm ⁻¹)	74967.518	74967.518	74967.518	74967.518
k_s	217683 (Nm ⁻¹)	217694.999	217694.999	217694.999	217694.999
k_i	43.537 (NA ⁻¹)	43.469	43.469	43.469	43.469
e_u	2×10^{-5} (m)	1.998×10^{-5}	1.998×10^{-5}	1.998×10^{-5}	1.998×10^{-5}
ϕ_u	60° (deg)	60.01	60.01	60.01	60.01
e_b	8×10^{-5} (m)	7.995×10^{-5}	7.995×10^{-5}	7.995×10^{-5}	7.995×10^{-5}
ϕ_b	30° (deg)	29.99	29.99	29.99	29.99
$\Delta k_{c22} e_{b,Re}$	5.99 (N)	5.192	5.192	5.192	5.192

Table 2.9 Estimates of multiple fault parameters for the measurement speed range of Case III with various percentage of random noise in response

Fault parameters	Assumed parameters	Case III (speed range of 33 to 60 Hz)			
		Random noise			
		0%	3%	5%	10%
c_V	140 (Nsm ⁻¹)	139.956	140.089	140.197	140.535
c_H	75 (Nsm ⁻¹)	74.948	74.861	74.777	74.476
Δk_{c22}	7.5×10 ³ (Nm ⁻¹)	74955.52	74950.476	74928.276	74807.054
k_s	217683 (Nm ⁻¹)	217636.853	217108.654	216515.649	214197.492
k_i	43.537 (NA ⁻¹)	43.461	43.400	43.330	43.051
e_u	2×10 ⁻⁵ (m)	1.999×10 ⁻⁵	1.999×10 ⁻⁵	1.998×10 ⁻⁵	1.992×10 ⁻⁵
ϕ_u	60° (deg)	60.02	60.05	60.06	60.07
e_b	8×10 ⁻⁵ (m)	7.995×10 ⁻⁵	7.996×10 ⁻⁵	7.998×10 ⁻⁵	8.001×10 ⁻⁵
ϕ_b	30° (deg)	30.00	29.99	29.99	29.96
$\Delta k_{c22}e_{b,Re}$	5.99 (N)	5.192	5.188	5.184	5.171

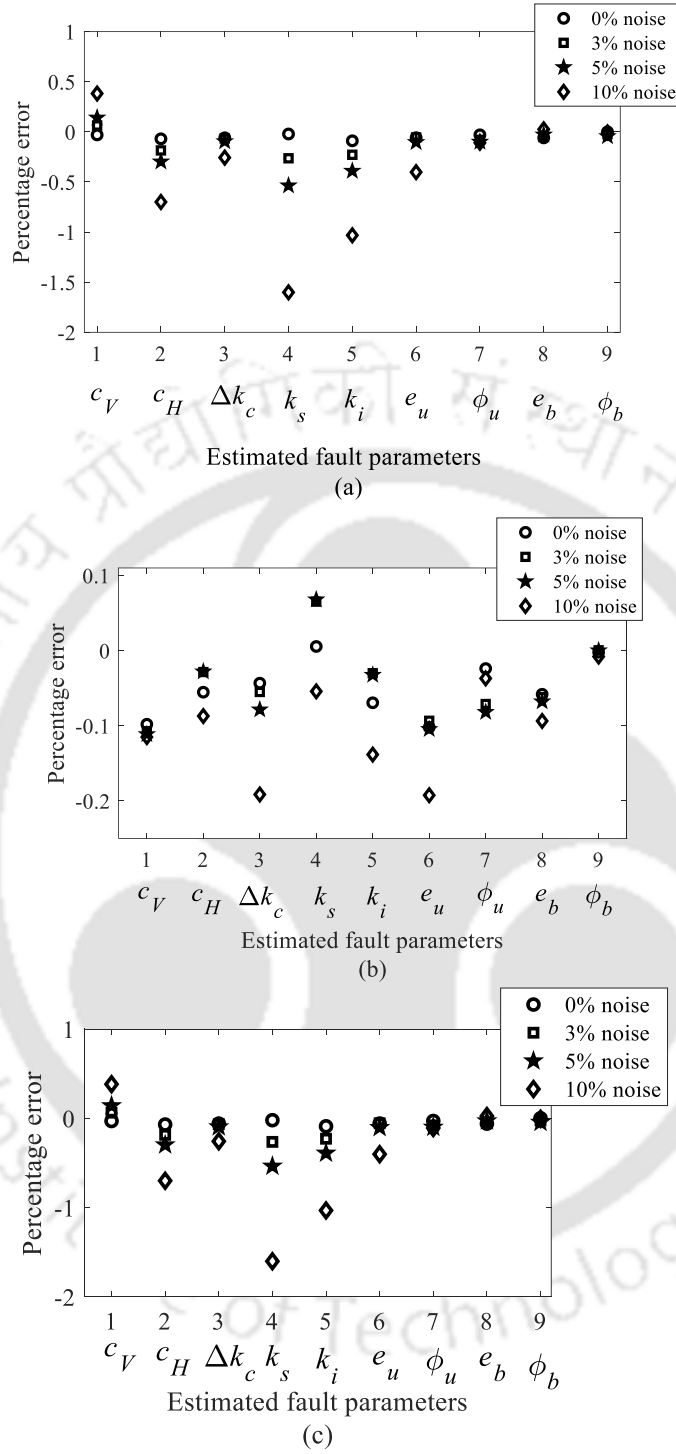


Figure 2.17 Error percentage of estimated fault parameters for various random noise levels for the speed range of (a) Case I (10 Hz to 19 Hz) (b) Case II (23 Hz to 31 Hz) (c) Case III (33 Hz to 60 Hz)

2.3.3.1 Introduction of Modeling or Bias Errors into the Proposed Model

In view of the practical difficulty in accurately measuring the physical parameters of a rotor system, like mass, intact shaft stiffness due to inadequate measuring instruments, human error, etc. pertain some errors which will be present in the estimation equations. Instead of this, the assumptions considered, such as the lumped parameter model, rigid end support in the mathematical modeling also encompass some permanent set of errors in the estimation procedure. Therefore, modeling or bias error is introduced in the rotor properties, such as density, length, and Young's modulus of the system to analyze the sensitivity of the developed identification algorithm. The response is generated using Equations (2.44) and (2.45) and modeling (or bias) error of different levels (0%, $\pm 3\%$ and $\pm 5\%$) is introduced in the model parameters of Equations (2.34) and (2.36) and estimates the fault parameters presented in Table 2.10 and Tables (2.11-2.13) for without and with residual bow effect onto the rotor respectively.

Table 2.10 Estimates of multiple fault parameters with no residual shaft bow fault for the measurement speed range of 23 Hz to 31 Hz with various percentage of modeling or bias error

Fault parameters	Assumed parameters	Speed range of 23 to 31 Hz				
		Modeling or bias error				
		0%	3%	5%	-3%	-5%
c_v	140 (Nsm ⁻¹)	144.479	144.451	147.263	136.043	133.238
c_H	75 (Nsm ⁻¹)	74.571	76.835	78.326	72.359	70.867
Δk_{c22}	7.5×10^3 (Nm ⁻¹)	74578.331	76838.814	78330.830	72362.767	70870.751
k_s	217683 (Nm ⁻¹)	218113.362	224620.921	228982.492	211536.207	207174.636
k_i	43.537 (NA ⁻¹)	43.251	44.561	45.426	41.965	41.099
e_u	2×10^{-5} (m)	1.980×10^{-5}	1.977×10^{-5}	1.977×10^{-5}	1.977×10^{-5}	1.977×10^{-5}
ϕ_u	60° (deg)	59.987	59.957	59.958	59.958	59.958

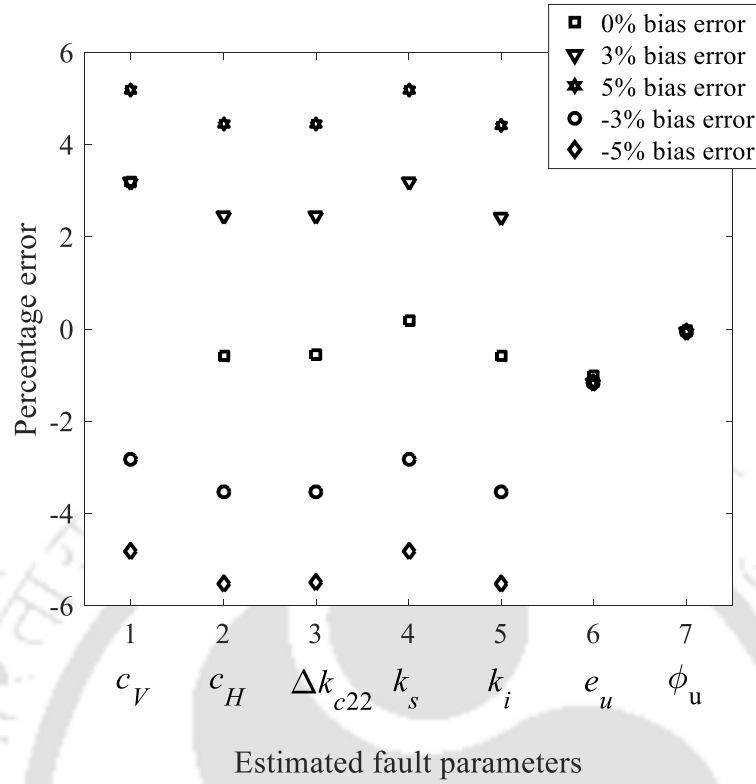


Figure 2.18 Error percentage of estimated fault parameters for various levels of modeling or bias error for the speed range of 23 Hz to 31 Hz

It is observed from Figure 2.18 of the without bow case, the that the much affected parameters, after introducing 5% modeling or bias error into the model parameters, the displacement stiffness (k_s) is 5.65% and the external damping (c_V) of 5.25 % with respect to the assumed values are taken for response generation.

The percentage error plots for cases I through III of rotor experiencing residual bow effect are shown in Figure 2.19(a-c). It is observed from the error plots from Table 2.11 and Figure 2.19(a) that for Case I at 5% bias error the maximum deviation in the force-stiffness factor (k_s) with 5.02% and at -5% bias error the maximum percentage error is found to be in internal damping (c_H) parameter of -5.12% with the absolute values.

Table 2.11 Estimates of multiple fault parameters for the measurement speed range of Case I with various percentage of modeling or bias error in system parameters

Fault parameters	Assumed parameters	Case I (speed range of 10 to 19 Hz)				
		Modeling or bias error				
		0%	3%	5%	-3%	-5%
c_V	140 (Nsm ⁻¹)	139.83	144.02	146.82	135.63	132.84
c_H	75 (Nsm ⁻¹)	74.94	77.19	78.69	72.69	71.19
Δk_{c22}	7.5×10 ³ (Nm ⁻¹)	74956.99	77205.70	78704.84	72708.28	71209.14
k_s	217683 (Nm ⁻¹)	217708.39	224239.64	228593.81	211177.14	206822.97
k_i	43.537 (NA ⁻¹)	43.46	44.77	45.63	42.16	41.29
e_u	2×10 ⁻⁵ (m)	1.999×10 ⁻⁵	1.997×10 ⁻⁵	1.997×10 ⁻⁵	1.997×10 ⁻⁵	1.997×10 ⁻⁵
ϕ_u	60° (deg)	59.99	59.97	59.97	59.97	59.97
e_b	8×10 ⁻⁵ (m)	7.999×10 ⁻⁵	7.994×10 ⁻⁵	7.994×10 ⁻⁵	7.994×10 ⁻⁵	7.994×10 ⁻⁵
ϕ_b	30° (deg)	29.99	29.99	29.99	29.99	29.99
$\Delta k_{c22}e_{b,Re}$	5.99 (N)	5.19	5.35	5.45	5.04	4.93

Table 2.12 Estimates of multiple fault parameters for the measurement speed range of Case II with various percentage of modeling or bias error in system parameters

Fault parameters	Assumed parameters	Case II (speed range of 23 to 31 Hz)				
		Modeling or bias error				
		0%	3%	5%	-3%	-5%
c_V	140 (Nsm ⁻¹)	139.86	144.06	146.86	135.68	144.06
c_H	75 (Nsm ⁻¹)	74.96	77.22	78.71	72.71	77.20
Δk_{c22}	7.5×10 ³ (Nm ⁻¹)	74967.52	77216.54	78715.89	72718.49	77216.54
k_s	217683 (Nm ⁻¹)	217694.99	224225.85	228579.75	211164.14	224225.85
k_i	43.537 (NA ⁻¹)	43.47	44.77	45.64	42.17	44.77
e_u	2×10 ⁻⁵ (m)	1.998×10 ⁻⁵	1.998×10 ⁻⁵	1.998×10 ⁻⁵	1.998×10 ⁻⁵	1.998×10 ⁻⁵
ϕ_u	60° (deg)	60.01	60.01	60.01	60.01	60.01
e_b	8×10 ⁻⁵ (m)	7.995×10 ⁻⁵	7.995×10 ⁻⁵	7.995×10 ⁻⁵	7.995×10 ⁻⁵	7.995×10 ⁻⁵
ϕ_b	30° (deg)	29.99	29.99	29.99	29.99	29.99
$\Delta k_{c22}e_{b,Re}$	5.99 (N)	5.19	5.35	5.45	5.04	5.35

Table 2.13 Estimates of multiple fault parameters for the measurement speed range of Case III with various percentage of modeling or bias error in system parameters

Fault parameters	Assumed parameters	Case III (speed range of 33 to 60 Hz)				
		Modeling or bias error				
		0%	3%	5%	-3%	-5%
c_v	140 (Nsm ⁻¹)	139.956	144.146	146.945	135.749	132.869
c_H	75 (Nsm ⁻¹)	74.948	77.197	78.696	72.700	71.211
Δk_{c22}	7.5×10 ³ (Nm ⁻¹)	74955.52	77205.020	78704.147	72707.641	71219.143
k_s	217683 (Nm ⁻¹)	217636.85	224171.58	228524.43	211113.0	206810.25
k_i	43.537 (NA ⁻¹)	43.46	44.77	45.64	42.16	41.29
e_u	2×10 ⁻⁵ (m)	1.999×10 ⁻⁵	1.999×10 ⁻⁵	1.998×10 ⁻⁵	1.999×10 ⁻⁵	1.998×10 ⁻⁵
ϕ_u	60° (deg)	60.02	60.02	60.02	60.02	60.01
e_b	8×10 ⁻⁵ (m)	7.995×10 ⁻⁵	7.995×10 ⁻⁵	7.995×10 ⁻⁵	7.995×10 ⁻⁵	7.995×10 ⁻⁵
ϕ_b	30° (deg)	30.00	29.99	29.99	29.99	29.99
$\Delta k_{c22} e_{b,Re}$	5.99 (N)	5.19	5.35	5.45	5.04	4.93

Similarly, for Case II, the highly affected parameter is found to be k_s with 5.01% at 5% bias error and external damping (c_v) with -5.09% at -5% bias error introduction into the model parameters shown in Table 2.12 and Figure 2.19(b). For Case III, the maximum percentage error is found to be 5.01% in the force-stiffness parameter at 5% error and for -5% bias error, the highest deviation is found to be in external damping of -5.09% with the assumed values as shown in Table 2.13 and Figure 2.19(c). It is clear from the above discussions that the force-stiffness factor of AMB, which is very much important parameter in characterizing the magnetic bearing performance much deviated from the reference value assumed with the introduction of highest percentage of bias error. However, at the lowest percentage of bias error in the rotor model, estimates the parameters from where external and internal damping are highly affected ones. Trends are same in the all the three speed

range cases. The values of the remaining estimated parameters are well confirmed with the absolute ones.

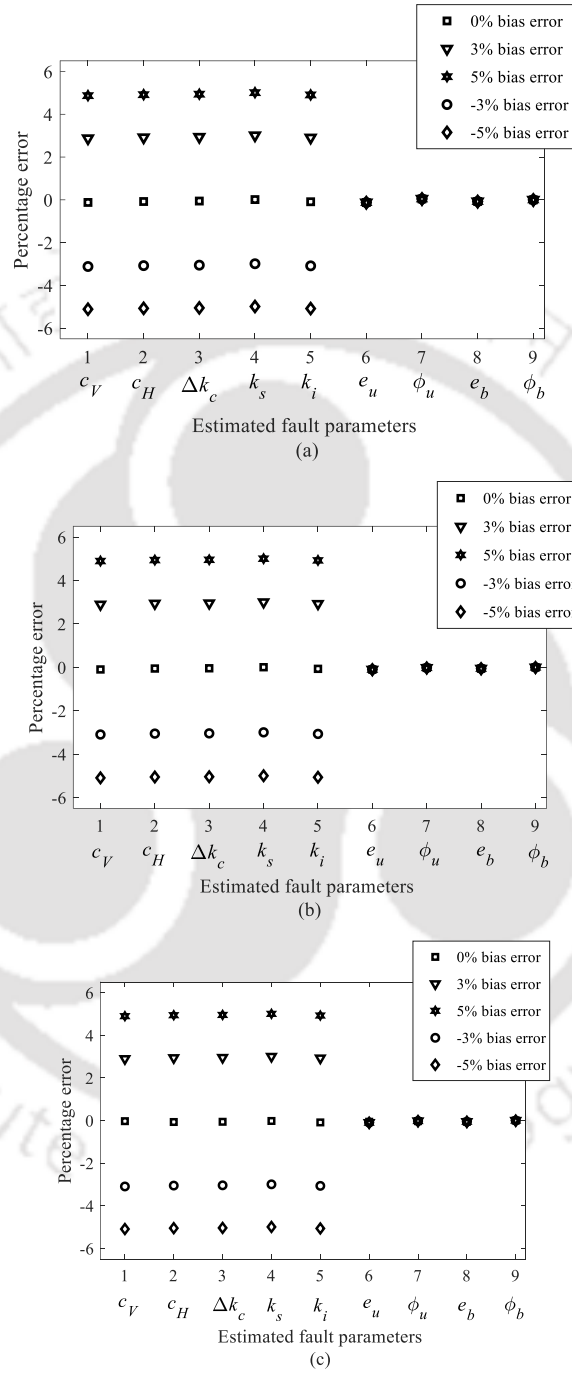


Figure 2.19 Error percentage of estimated fault parameters for various levels of modeling or bias error for the speed range of (a) Case I (10 Hz to 19 Hz) (b) Case II (23 Hz to 31 Hz) (c) Case III (33 Hz to 60 Hz)

Thus, it is summarized that as long as the speed ranges are chosen in the present analysis with a fairly accurate model of rotor bearing system with the disc displacement and control current measurement, the multiple fault parameters may be estimated through the developed identification algorithm with a high degree of accuracy, even in the presence of noise and modeling or bias errors.

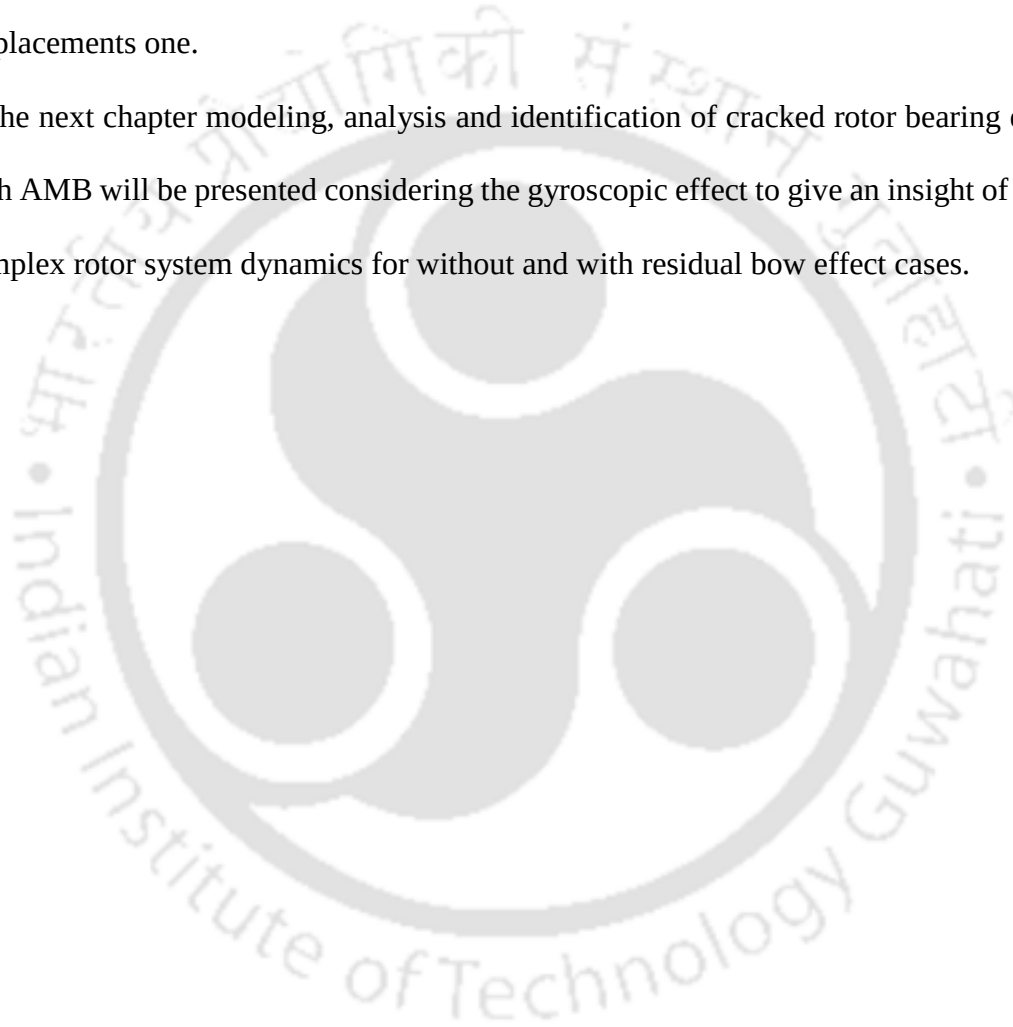
2.4 Concluding Remarks

In the present chapter two cases of cracked rotor bearing system integrated with AMB are considered (i) cracked rotor not experiencing residual bow effect (ii) cracked rotor experiences residual bow effect. EOMs are developed for the above mentioned two cases where some assumptions are taken viz. (i) massless shaft and rigid disc (ii) weight dominance effect (iii) disc is considered at the mid-span of the shaft and the crack position is near to the disc. Multi harmonic forward and backward whirl precessional mode components using full spectrum are analyzed and found the effect of bow on the cracked rotor produces all the even and odd harmonics unlike the cracked rotor without residual bow case. This introduces a new fault parameter in multiplicative form of bow and crack which influences the dynamics of the system greatly. The active controlling of the proposed system is achieved by the magnetic bearing to maintain its stability, proper functioning, and monitoring.

Identification algorithms are developed based on the two EOMs in frequency domain and the full spectrum response of amplitudes and phases (after compensation with the multi harmonic reference signal) are utilized here to estimate multiple additive and multiplicative fault parameters simultaneously. The developed algorithm is found to be robust enough against different levels of measurement noises and modeling or bias errors.

To comply with a more realistic model, the positioning of disc mounted on the rotor not exactly at the mid span is ruled out and for offset location with respect to the conventional bearing position, the rotor experiences tilting motion introducing gyroscopic effect. The inclusion of gyroscopic effect into the mathematical modeling for the multiple fault estimation procedure needs angular deflection measurements along with the transverse displacements one.

In the next chapter modeling, analysis and identification of cracked rotor bearing equipped with AMB will be presented considering the gyroscopic effect to give an insight of the more complex rotor system dynamics for without and with residual bow effect cases.





Chapter 3

Experimental Estimation of Multiple Fault Parameters of a Cracked-Warped Rotor-AMB System with a Central Disc

Experimental investigation is carried out in a laboratory rotordynamic test bench to implement the identification methodology developed in Chapter 2 for multiple fault parameters estimation. Detailed steps of the development of the test bench, PID control strategy for conducting close loop test, instrumentation, data acquisition, and signal processing are illustrated in the present chapter. The static and dynamic analyses of both healthy and cracked shaft system are analyzed. The experimentally estimated fault parameters are utilized in the mathematically developed EOMs to simulate responses from the updated rotor model and the same is compared with the responses of the real test data to verify the estimated parameters. Furthermore, estimated parameters of unbalance is implemented as balance mass for balancing the system without and with AMB control.

3.1 Introduction

The model based identification of multiple fault parameters with the real test data from the rotordynamic test bench installed in Advanced Dynamics & Vibration Laboratory of IIT Guwahati is illustrated now. The test bench consists of a cracked shaft mounted with a disc at its middle integrated with a radial AMB as auxiliary support. The crack in the shaft is artificially generated in the in-house facility of the Strength of Materials laboratory. Due

to the application of cyclic loading on the shaft for fatigue crack initiation, the shaft experienced a permanent bend. The fabrication of the magnetic bearing actuator and its electrical connections with the power amplifiers are also described. The instrumentation of the dSPACE controller with the assembled test bench is enunciated. The amplitude and phase information of the measured displacement and AMB current signals have been extracted from full spectrum analysis with associated phase corrections. To verify the developed identification algorithm, the full spectrum amplitudes and phases after phase compensation of the measured data from the test bench are utilized for simultaneous estimation of unbalance and residual bow along with their orientations, additive stiffness of crack, multiplicative crack-bow fault, external and internal damping and AMB displacement-stiffness and current-stiffness parameters.

3.2 Description of the Rotordynamic Test Bench

A shaft containing a fatigue crack is supported by two conventional bearings (deep groove ball bearing) and an auxiliary support of AMB to apply transverse magnetic force on it. The rotor-bearing test assembly is mounted on a cast iron bed. The rotor system is driven by a CNC spindle motor through a flexible coupling. The radial AMB is an eight-pole heteropolar type and the current supplied by two servo (pulse width modulation) PWM power amplifiers in current control mode to suppress the vibration in the transverse direction. The AMB is controlled using a standard PID scheme through dSPACE DS1202 platform. The power amplifier used in current control mode is a voltage-to-current amplifier. The design layout of the test bench is shown in Figure 3.1. Description of the components in the test bench assembly are now provided below.

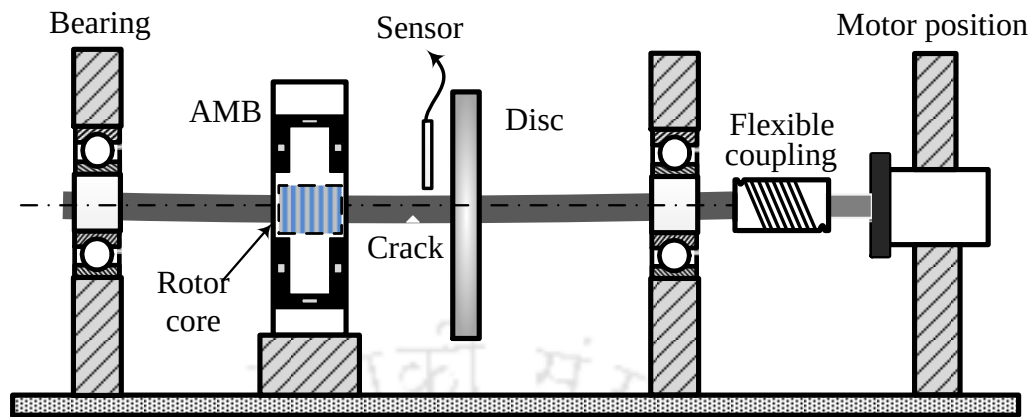


Figure 3.1 Layout of the rotordynamic test bench in the laboratory

3.2.1 Rotor Shaft and Disc

The shaft containing an artificial fatigue crack is made of mild steel. The length of the shaft from bearing to bearing is 47 mm and the diameter is 16 mm. The disc made of stainless steel material is of 150 mm diameter. The weight of the disc is 1.7 kg that is mounted on the mid-span shaft containing a crack near to it.

3.2.2 Flexible Coupling

The flexible coupling consisting of aluminium alloy with the rotor end fitting is of 15 mm diameter and the motor spindle end fitting of 14 mm diameter. The coupling length is 50 mm. It comes with a helical cut in the middle along its length that makes it flexible so they can be fit to two shafts even if they are not perfectly co-linear and will help reduce misalignment effects. Due to the helical cut, it makes the coupling behave like a spring only in the transverse direction and not in the rotational direction for which it shows less backlash as shown in Figure 3.2. The specification of the flexible coupling is presented in Table 3.1.



Figure 3.2 Flexible coupling with helical cut

Table 3.1 Specifications of flexible coupling

Outer diameter (OD)	20 mm
Inner diameter (ID1)	14 mm
Inner diameter (ID2)	15 mm
Set screws	2×M6
Length (L)	50 mm

3.2.3 Conventional Bearing

Single-row deep-groove ball bearings are versatile and can accommodate the radial and axial loads in both the directions. The bearing used here is open type SKF standard bearing as shown in Figure 3.3 and its specification are listed in Table 3.2.



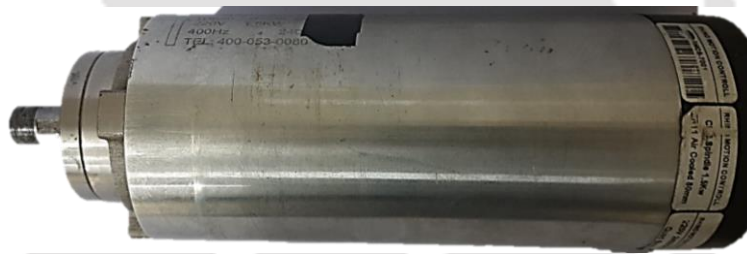
Figure 3.3 Deep groove ball bearing

Table 3.2 Specifications of bearing

Model no.	61802
Outer diameter	24 mm
Inner diameter	15 mm
Width	5 mm
Dynamic load rating	1.9 kN
Static load rating	1.1 kN
Fatigue load limit	0.048 kN
Limiting speed	38,000 rev./min

3.2.4 Electric motor (Drive)

An ER11 Air-cooled CNC-spindle motor is a sensor-less Vector-Micro AC drive used in the laboratory test rig with specification tabulated in Table 3.3. Figure 3.4 shows the CNC-spindle motor used during experimentation.

**Figure 3.4** CNC spindle motor**Table 3.3** Specification of the spindle motor

Material	Stainless Steel
Power	1.5 kW
Voltage	220 V AC
Current	6 A
Frequency	0-400 Hz
Speed	10-24000 rpm
Cooling	Air-cooled
Runout off	Less than 0.01 mm
Precision tapered bore	Less than 0.005 mm
Bearing Lubrication	Grease
Bearings	4 Ceramic bearing
Net Weight	4.1 kg

3.2.5 Variable Frequency Drive

The variable frequency drive (VFD) of Delta-M series AC motor drive is used as shown in Figure 3.5. The AC motor drive consists of digital keypad that provide parameter display along with the operation status and the programming and control interface. The VFD- M AC motor has 157 AC drive parameters for easy setting before start-up the motor operation.



Figure 3.5 Delta VFD speed controller

Table 3.4 General specification of VFD

Motor output (kW)	1.5 kW (2 HP)
Rated output current (A)	7 A
Maximum output voltage (V)	3-Phase proportional to input voltage
Output frequency (Hz)	0.1~ 400 Hz
Rated voltage ,frequency	200-240 V, 50/60Hz
Cooling method	Fan cooled
Control system	Sinusoidal pulse width modulation (SPWM) control (V/F or sensor less control)
Frequency setting resolution	0.1 Hz
Ambient temperature	-10 ⁰ to 40 ⁰ C

3.2.6 dSPACE Control Desk System

The basic features of dSPACE platform are described briefly. The MicroLabBox is used to control, test or measurement applications to develop and use of new control concepts into reality. It provides computation power combined with low I/O real-time performance. A programmable FPGA used in running fast control loops, as required in applications, such as the electric motor control or active noise and vibration cancellation. dSPACE control desk is a user interface. The DS2102 board of 6.3 version is considered a platform on which a simulation is run. From this environment, one can able to download applications to the DS2102, configure virtual instrumentation that can be used to control, monitor and automate experiments, and develop controllers. In PID control, the proportional gain is a measure of system stiffness and does immediate action, with the increase of this gain increases the speed of the response in the controller. The integral gain eliminates the deviation, the derivative gain restrains the vibration. The PID control method determines the control with no deviation, high accuracy, and stable operation. The controller produces a combined response or controlled output, which is applied to test system according to the PID algorithm.

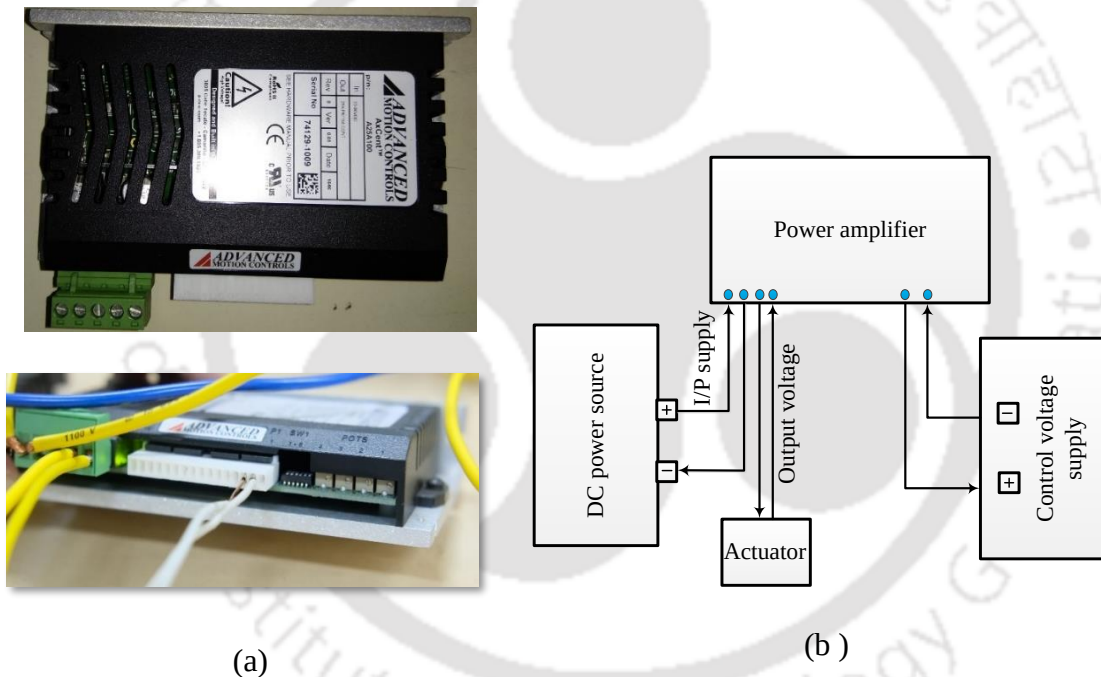
3.2.7 Power Amplifier

The A25A100 PWM servo drive is used to amplify the control current passes through the electromagnetic actuator of magnetic bearing at a high switching frequency. The specification of the A25A100 servo amplifier used in the experimentation is presented in Table 3.5. The drive can interface with digital controllers or be used stand-alone, and requires only a single unregulated DC power supply. Loop gain, current limit, input gain and offset could be adjusted using the potentiometers as shown in Figure 3.6(a) and the

power amplifier connections with the magnetic bearing actuator is shown in line diagrams of Figure 3.6(b). The detailed description of the connections and tuning of these power amplifiers are illustrated in Chapter 5.

Table 3.5 Specification of the PWM servo drive

Peak Current	15 A
Continuous Current	7.5 A
Supply Voltage	20 - 80 VDC
Switching frequency	20 kHz



3.2.8 Fabrication of Electromagnetic Actuator of AMB

The magnetic bearing actuator is designed based on the physical system data of the developed rotor-bearing test rig. The design criterion of the magnetic bearing actuator are discussed in Appendix B. The complex geometry of the actuator during fabrication need

to be machined and laminated sheets of the CRGO (cold rolled grain oriented) steel (of thickness of each sheet 0.36 mm) to be stacked together. Electrical steel are silicon based and highly ferromagnetic in nature finding application in electrical transformer manufacturing industry. The laminated sheets are cut in a SLP-400 ND-Yag laser machine available at the central workshop of IIT Guwahati. The actuator configuration and picture are presented in Table 3.6 and Figure 3.7, respectively.

Table 3.6 Actuator configuration

Number of poles	8
Stator outer diameter	90 mm
Stator inner diameter	32 mm
Core outer diameter	32.4 mm
Core inner diameter	16 mm
Pole leg width	8 mm
Pole leg length	35 mm
Coil diameter	0.813mm (21 SWG Copper wire)
Number of coil turns in one pole	100



(a)



(b)

Figure 3.7 (a) Eight pole actuator with rotor core (b) Close view of pole length of eight-pole actuator without core

The machine can cut up to 2 mm of thickness one at a time and 70 sheets are made ready for stamping. The rotor core is also made of same material of 16 mm inner diameter and 32 mm outer diameter. The stator of the actuator has 90 mm outer diameter and 32.8 mm inner one and air gap between stator and rotor is 0.8 mm. There are 4-pole pairs of electromagnets, which can generate magnetic force around the electromagnets of the actuator and the laminated rotor core.

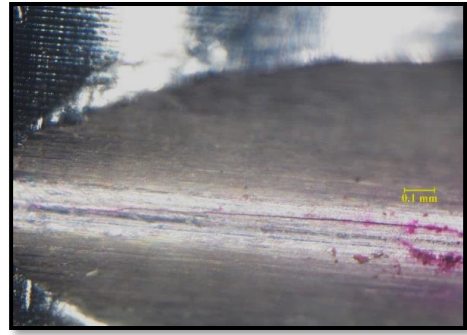
3.2.9 Generation of Fatigue Crack on Shaft

The fatigue crack was made on the shaft by 3-point bending test on the Instron test machine with cyclic loading in the Strength of Materials laboratory in the Department of Mechanical Engineering of IIT Guwahati as shown in Figure 3.8. For the generating a fatigue crack, a sharp V-notch having a depth of 2 mm through 90° milling cutter is generated at the middle of the shaft in Central Workshop of the institute. Then the shaft with the notch is subjected to cyclic loading in the INSTRON machine at Strength of Materials Laboratory of the institute. In the three-point bending test, the load onto the shaft was 6 kN to 1.8 kN and applied cyclically.

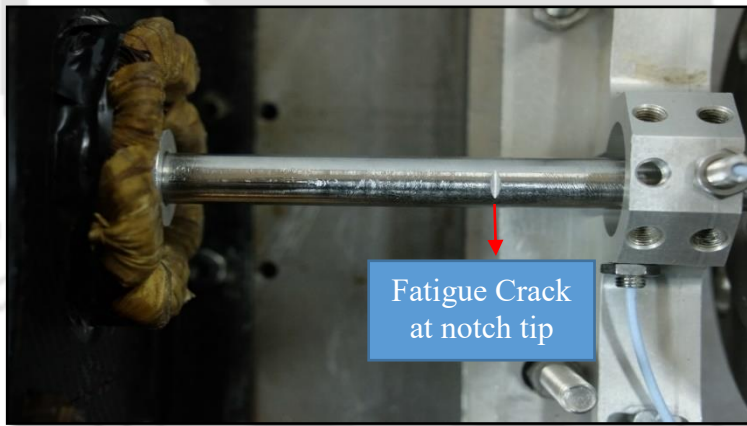
The positional amplitude at the initial condition at the notch portion of the shaft was 0.246 mm after 2500 cycles. After application of cyclic loading, the positional amplitude increased to 0.28 mm, which is 12.15% of the initial one. This increase of the positional amplitude in the notch portion determines the initiation of crack in the shaft. Figure 3.9 (a, b) shows the total number of cycles of the load repetition versus the positional amplitude of the shaft at the notch tip. A permanent bend occurs in the shaft due to cyclic loading applied on the shaft opposite to the notched area.



(a)

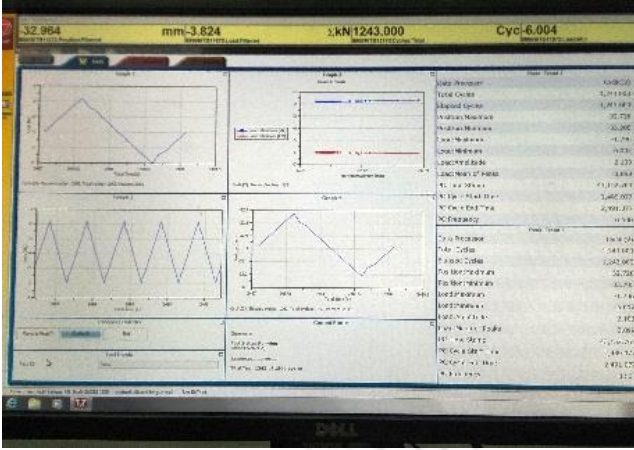


(b)

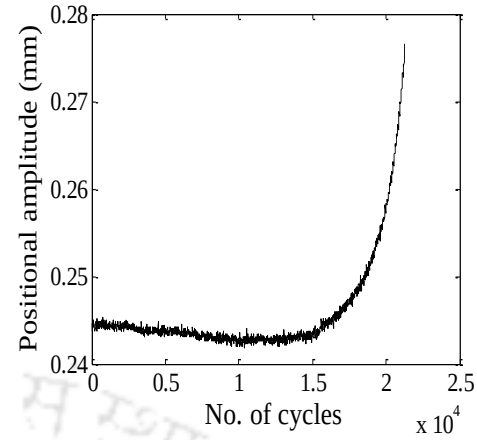


(c)

Figure 3.8 (a) Cyclic loading applied on the shaft in the INSTRON machine using three-point bending fixture (b) Close view of the crack in the V-notch tip of the shaft (c) Fatigue crack on the shaft with notch



(a)



(b)

Figure 3.9 (a) Computer screen during fatigue crack generation in INSTRON machine
(b) Positional amplitude versus number of cycles of loading in three-point bend test

3.2.10 PID Control Scheme Implementation

A proportional plus integral plus differential (PID) control scheme is implemented in differential driving mode as shown in Figure 3.10. In the driving mode, one pair of magnetic poles are driven by the sum of the bias (i_b) and control currents (i_c), i.e. ($i_b + i_c$), and the other by the difference ($i_b - i_c$) allows the rotor hovering in its original position (Bleuler, Cole et al. 2009). In the current experimentation, these currents in the upper and the lower actuator poles generate the magnetic force around the electromagnets and the ferromagnetic rotor core to suppress the rotor vibration. In this data acquisition process through dSPACE, real-time tuning of controller gains are suitably adjusted that stabilizes the rotor-AMB system.

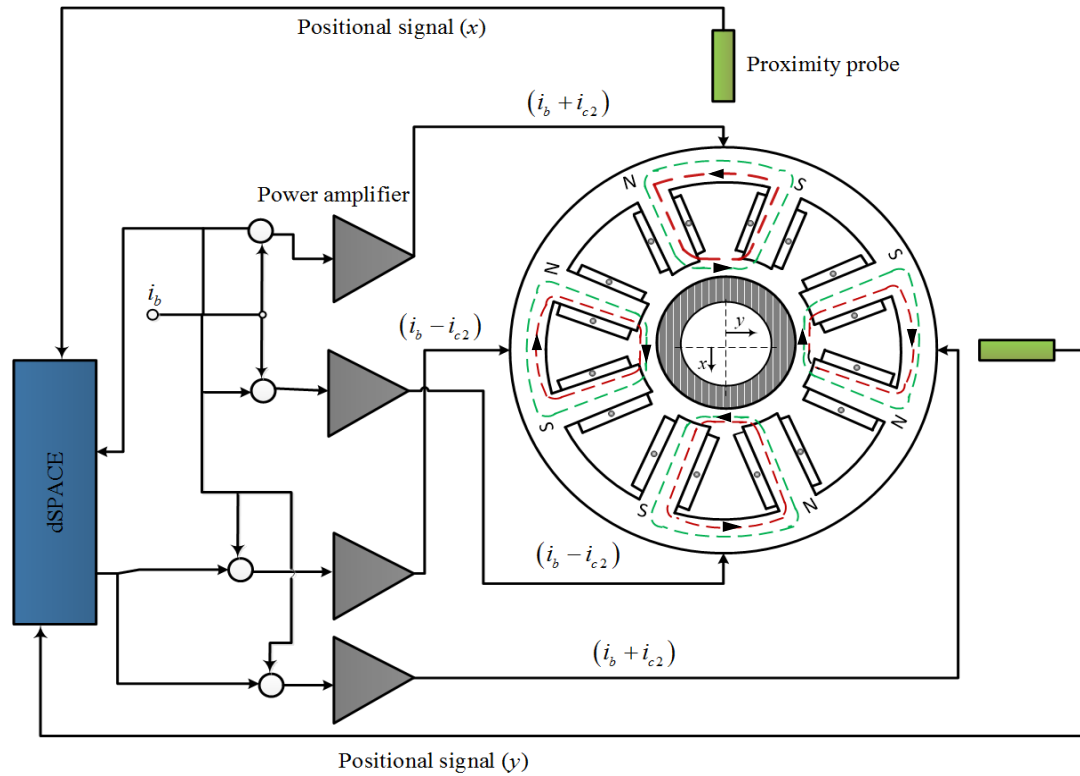


Figure 3.10 A block diagram of electrical circuit for working of differential type actuator

3.2.11 Assembly of the Test Rig and Data Measurement Procedure

The rotordynamic test bench in an assembled position is installed in the laboratory as depicted in Figure 3.11. From the test rig, measurement is done for the rotor vibrational signal and real-time control is achieved through PID control scheme by tuning the controller gain parameters. The procedure of data measurement and data-acquisition in dSPACE control desk is summarized now.

A 5-channel Bently Nevada eddy-current probe power-cum-amplifier system was used to measure the rotor motion in the transverse direction as well as of the reference signal. The signal was acquired from the eddy current sensor by adjusting the gap of the sensor observing the shaft in the voltage range of -5.75 ± 0.5 Vdc or 0.8 mm. The sensitivity

of the non-contact sensor was 1.27 V/mm. The collected vibration displacements of the rotor were sent to the ADC/DAC input-output panel. The digital signal goes to the dSPACE

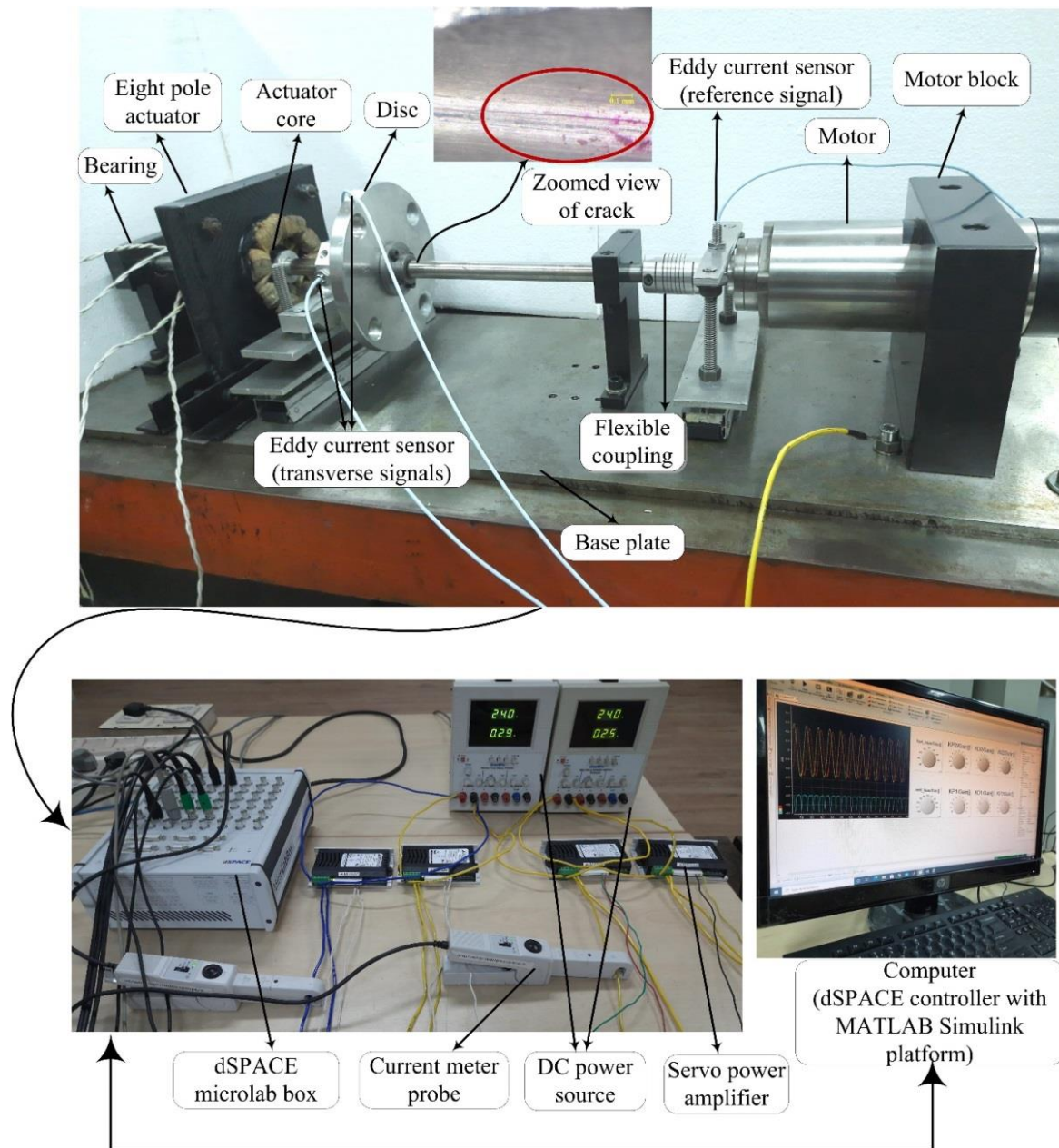


Figure 3.11 Pictorial representation of a cracked-warped shaft integrated with AMB in laboratory

controller (processing is done at a sampling rate of 10,000 samples per second) from which the digital signal of voltage input is fed into the power amplifiers, which amplifies the analog voltage signal in the ratio of 1 V/A as gain. Then, it is fed to the magnetic bearing actuator for producing electromagnetic force around the poles. Two 1146B KEYSIGHT AC/DC current probes (AC current accuracy of 100 mV/A, and 10 mV/A with the sensitivity of 0.2 to 0.5 V/div) are used for measuring control current flowing through the coils to reduce the vibration in the transverse direction. In the following sections, the measured signals are shown in meter by conversion of voltage (V) to meter (m) using the sensor sensitivity.

3.3 Study of Rotor Bearing System Responses

The measured vibration and current responses from the test rig are analyzed in the time and frequency domain approach both in the non-rotating and rotating conditions as explained in the following sub-sections.

3.3.1 Study of Non-Rotating Cracked and Healthy Rotor System Free Vibration Responses

In the experimental analysis, first, the rotor is excited with an impulsive force with an impact hammer in the standstill condition to evaluate the natural frequency of both healthy shaft and cracked shaft configuration of the rotor described above without and with AMB condition. The natural frequency of the healthy shaft system is found to be 62.86 Hz for without AMB case; and for with AMB case, it is 66.83 Hz as shown in Figure 3.12 (a, d).

On the other hand, the natural frequency of the cracked shaft is evaluated based on the different directions of cracked faces. For this, the impact is given on the shaft in the vertical direction at three conditions as mentioned below

- when the crack on the shaft is in the downward direction (stiffness is minimum due to opening of the cracked face due to dead weight of the rotor),
- when the cracked face is in the horizontal direction (stiffness is maximum as the crack is in the fully closed condition with notch effect also minimum and act as an intact (healthy) shaft) and,
- lastly, when the crack is in the upward direction (not fully closed condition due to tension in the shaft).

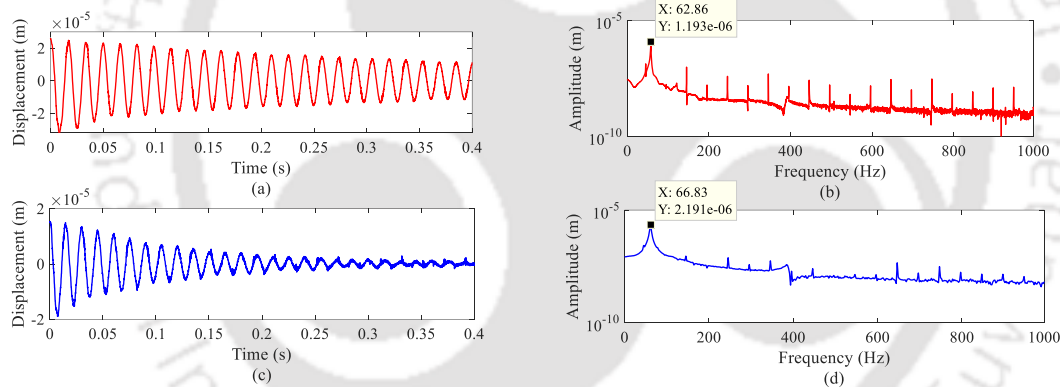


Figure 3.12 Natural frequency of a healthy shaft without (— solid line) and with (— solid line) AMB (a) time domain ,and (b) frequency domain response without AMB (c) time domain, and (d) frequency domain response with AMB

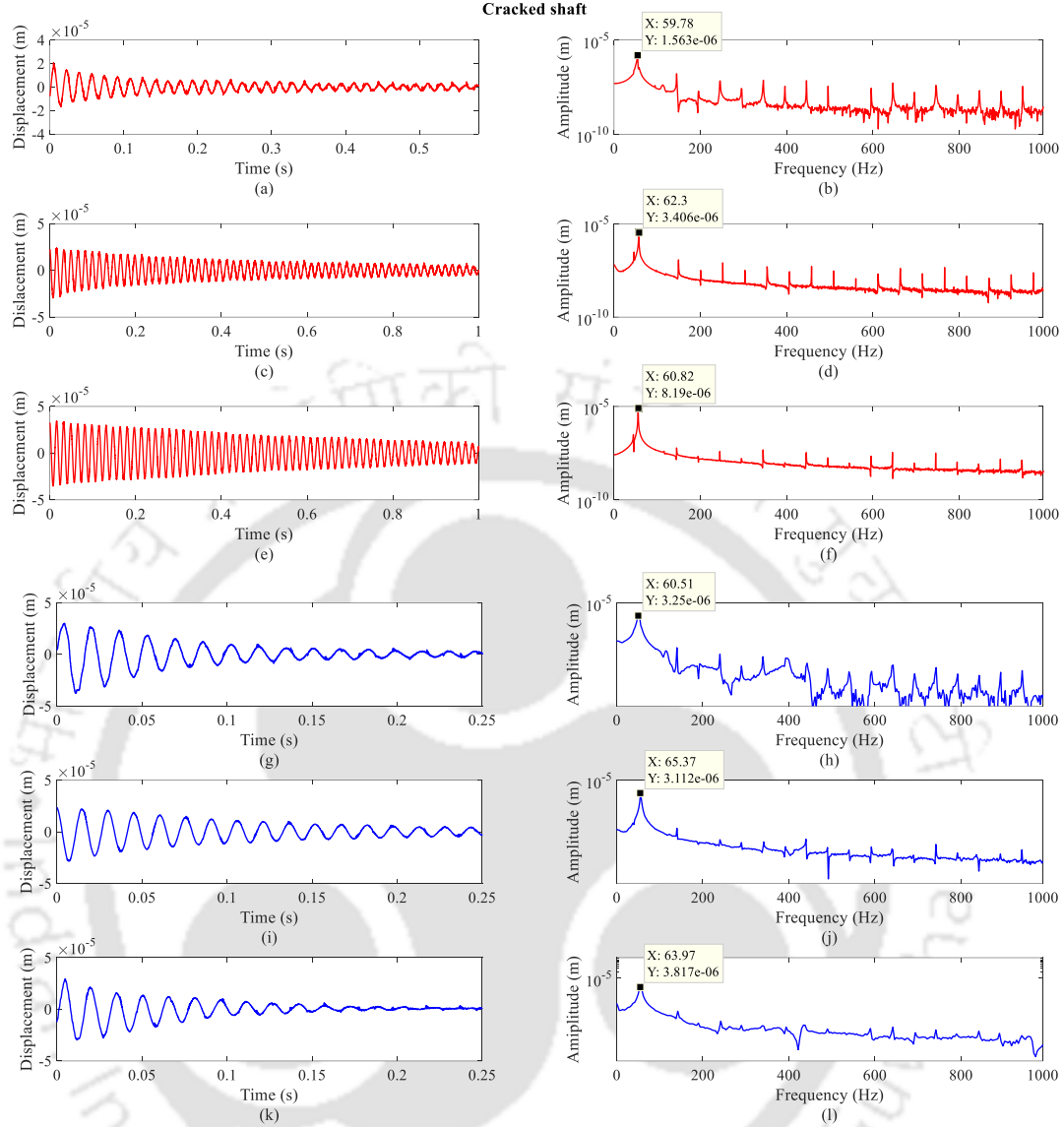


Figure 3.13 Natural frequency of cracked shaft with different crack orientation (time and frequency domain responses) without (— solid line) and with (— solid line) AMB
 (a, b, g, h) crack face is in the downward direction (c, d, i, j) crack face is in the horizontal direction (e, f, k, l) crack face is in the upward direction

The time and frequency domain responses of the system without and with AMB case for the above mentioned conditions are shown in Figure 3.13 (a, l). The natural frequencies of the cracked rotor for different cracked face orientations without AMB case when the cracked face is in the downward direction are 59.78 Hz and 62.30 Hz (maximum due to

the least notch effect as compared to when the crack face is at the upward direction) as shown in Figure 3.13 (b, d). The cracked face if is upward direction, the frequency found is 60.51 Hz (as the cracked shaft experiences lesser tension as compared to the crack is in the downward direction) as depicted in Figure 3.13(f). Similarly, Figure 3.13(g, l) shows the natural frequencies of the cracked rotor system with AMB case are found to be 60.51 Hz, 65.37 Hz and 63.97 Hz for downward, horizontal (or sideways) and upward direction of the cracked face, respectively. The natural frequencies of the healthy shaft and the cracked shaft (when the crack is fully closed condition) are found to have some difference due to the notch effect.

3.3.2 Study of Vibration Responses of Rotating Cracked and Healthy Rotor System

In the rotating of shaft case, operation of the test bench is first performed the static and dynamic analysis on a shaft without any crack, i.e. using the healthy shaft. Then, the cracked shaft was incorporated into the test rig and the experiment was started with the operation of the system at slow roll condition. The residual shaft bow (permanent bent) of the shaft is much prominent in slow roll (slow speed of 5 Hz) conditions. The cracked shaft possesses a permanent bend at the central position of the shaft due to cyclic loading during artificial fatigue crack initiation, which affects the dynamics of the system. In the previous works (Tiwari 2017, Roy and Tiwari 2019, Shravankumar and Tiwari 2019) the effect of shaft bow was removed by elimination of the 1X harmonic from the measured signals in frequency domain. But it has been found that the bow affects the multiple harmonics in the presence of the crack excitation forces as discussed in Chapter 2. In lieu of the above fact, the residual shaft bow is not removed from the measured signals, rather analyzed its effect

on the cracked shaft and estimated its magnitude and orientation. Nevertheless, how the shaft residual bow affects the dynamics of the system is compared through response measured from the test rig with a bow and without bow case.

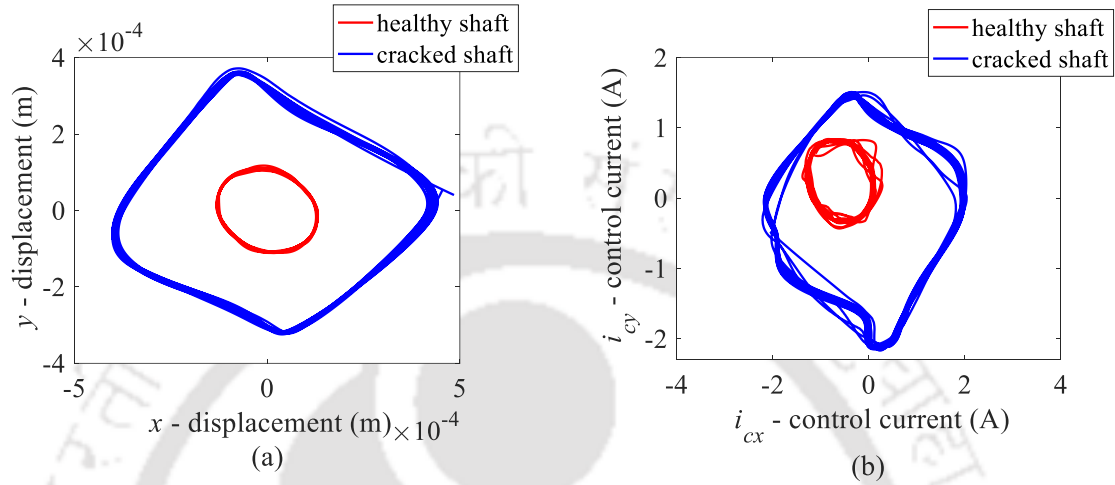


Figure 3.14 Comparison of orbit plots of healthy (— solid line) and cracked shaft (— solid line) (a) Displacement response and (b) Control current response

The comparison of the orbit plots of shaft displacement and AMB control current signal of the healthy shaft and the cracked shaft is represented in Figure 3.14(a, b). The amplitude of vibration in the case of the healthy shaft due to the presence of only unbalance excitation force is less (1.335×10^{-4} m) as compared to the response of the cracked shaft system due to reduction in stiffness is 4.255×10^{-4} m as shown in Figure 3.14(a). Similarly, as the vibration is less in a healthy shaft system, the control current of AMB required to suppress that vibrational amplitude is less as observed in Figure 3.14(b).

The residual shaft bow affects the dynamics of the system by increasing the amplitude of vibration and it overshadows the effect of crack present in a system. Here, Figure 3.15(a, b) clearly shows reduction of 82.1% in the amplitude of a response signal after removing the residual bow from the 1X harmonic of the measured signal. The orbit of the shaft

centerline is shown without and with bow case is shown in Figure 3.15(c). It can be observed that the shaft bow influences the dynamics much and after removal of it, a clear change in orbitals shape of the cracked rotor is observed. Hence, the shaft residual bow effect on the rotors needs to be explored in the present research and its quantification is also very much essential.

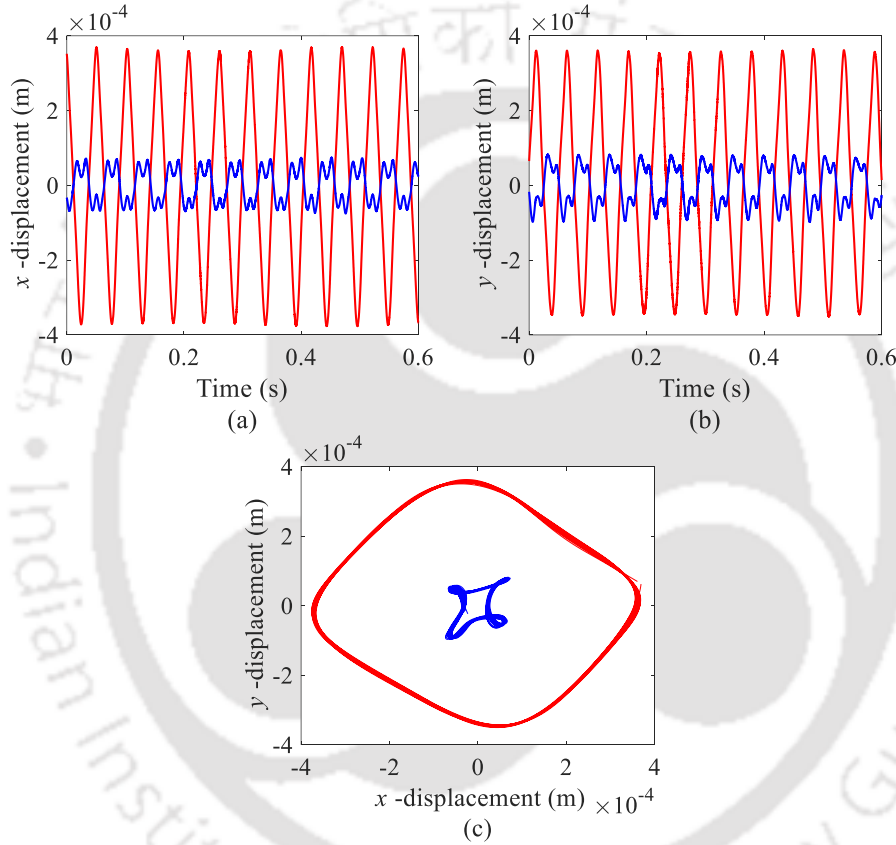


Figure 3.15 Comparison of without (— solid line) and with (— solid line) bow response of a cracked shaft (a) Vertical displacement vs time response (b) Horizontal displacement vs time response and (c) orbit plot of displacement response

The comparison of the full spectrum response of displacement and current of the healthy and cracked shaft is depicted in Figure 3.16. It is seen from the full spectrum amplitude response that there is 1X and -1X harmonic due to unbalance and friction at rolling

bearing, respectively, in healthy shaft response whereas the cracked shaft constitutes the multiple harmonic responses such as $\dots, -3X, -2X, -1X, 0, 1X, 2X, 3X, \dots$ which represents the behavior of the breathing of crack.

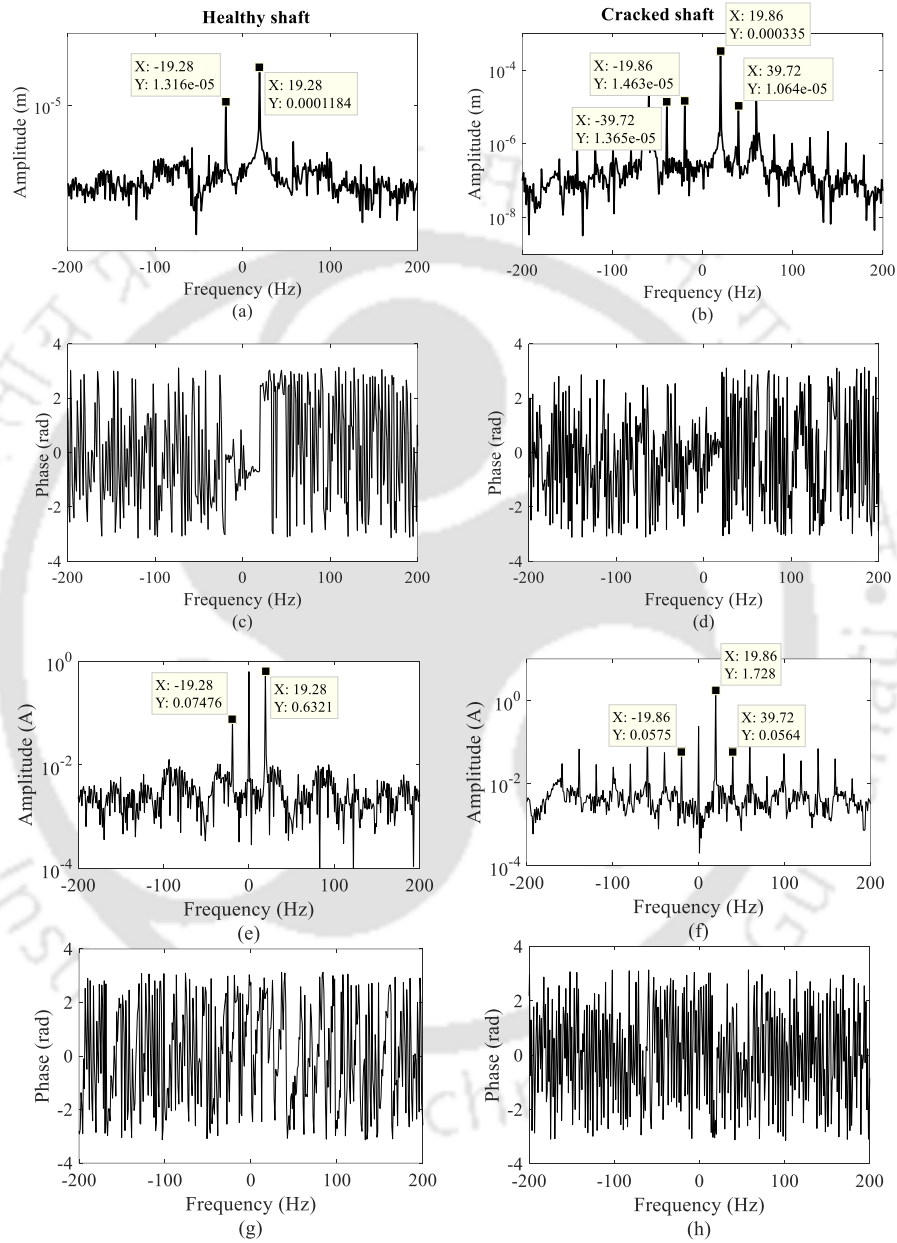


Figure 3.16 Comparison of full spectrum response of healthy and cracked shaft operating at approximately 20 Hz speed (a, e) Amplitude of displacement and control current signal of healthy shaft (c, g) phases of healthy shaft (b, f) Amplitude of displacement and control current signal of cracked shaft (d, h) phases of cracked shaft

The reference signal was also measured in each spin speed of the rotor to compensate for the phase lag or lead of the acquired vibration signals in the orthogonal direction. An eddy current sensor was mounted over the motor spindle from which the reference signal was triggered once per revolution of the slot in the motor spindle, like key phasor. The spindle of the motor contains two slots over it and the reference signal is collected from one of the peaks for the present analysis as shown in Figure 3.17(a, b). In the whole analysis, the peak with higher amplitude of the reference signal has been chosen for phase measurement of the transverse displacement and control current signals measured for different operating speeds.

Time domain signals measured from the test rig operating at approximately 5 Hz and 26 Hz speeds consisting of the vertical and horizontal displacements, control currents in both the directions, and corresponding the orbit plots of displacements and control currents are depicted in Figures 3.18 and 3.19, respectively.

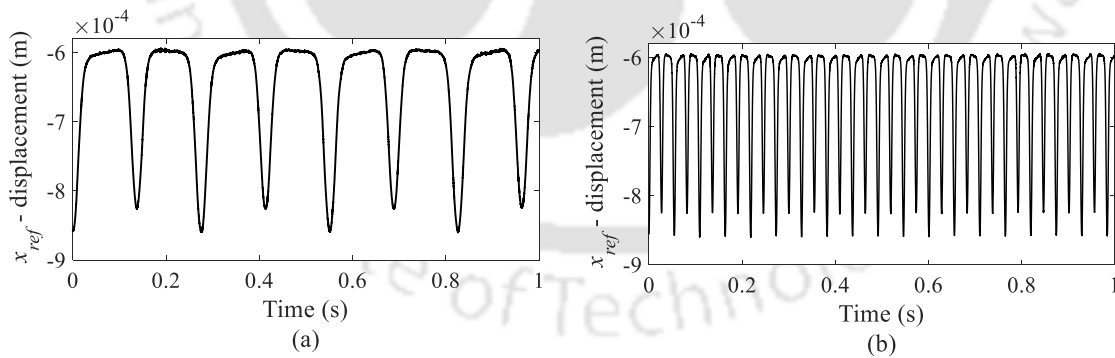


Figure 3.17 Reference signals of the system operating at approximately (a) spin speed 5 Hz (b) spin speed 25 Hz

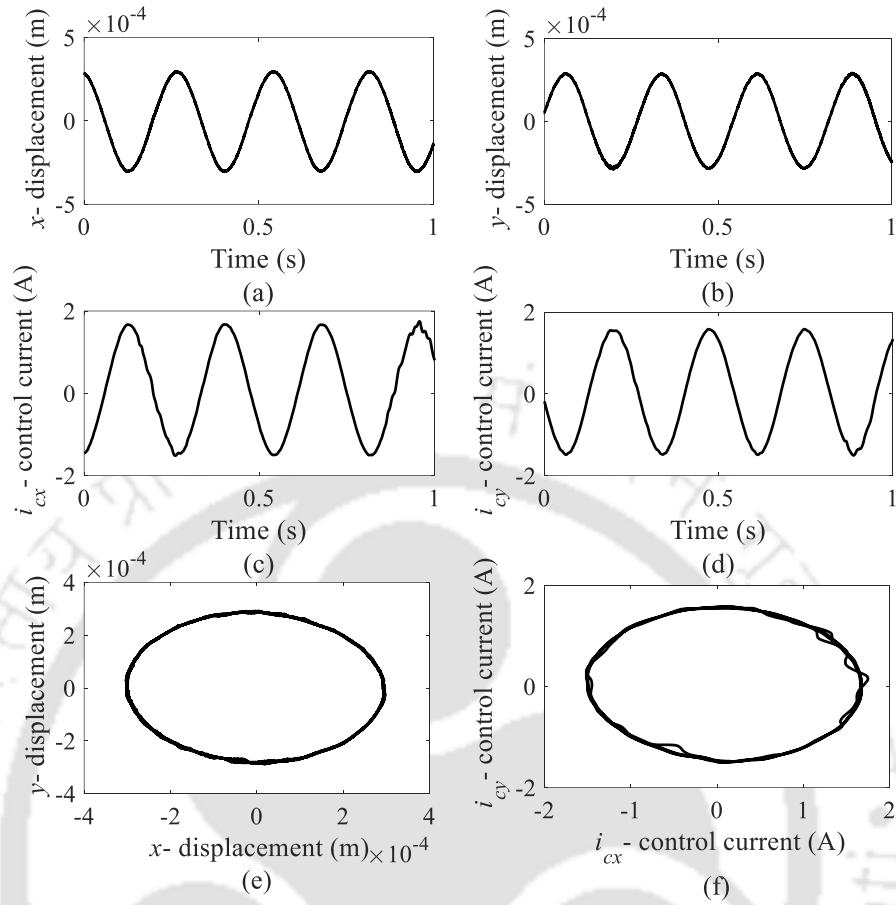


Figure 3.18 Time domain responses of the system operating at approximately 5 Hz (a) vertical displacement (b) horizontal displacement (c) control current in vertical direction (d) control current horizontal direction (e) orbit plot of displacement signal (f) orbit plot of control current signal

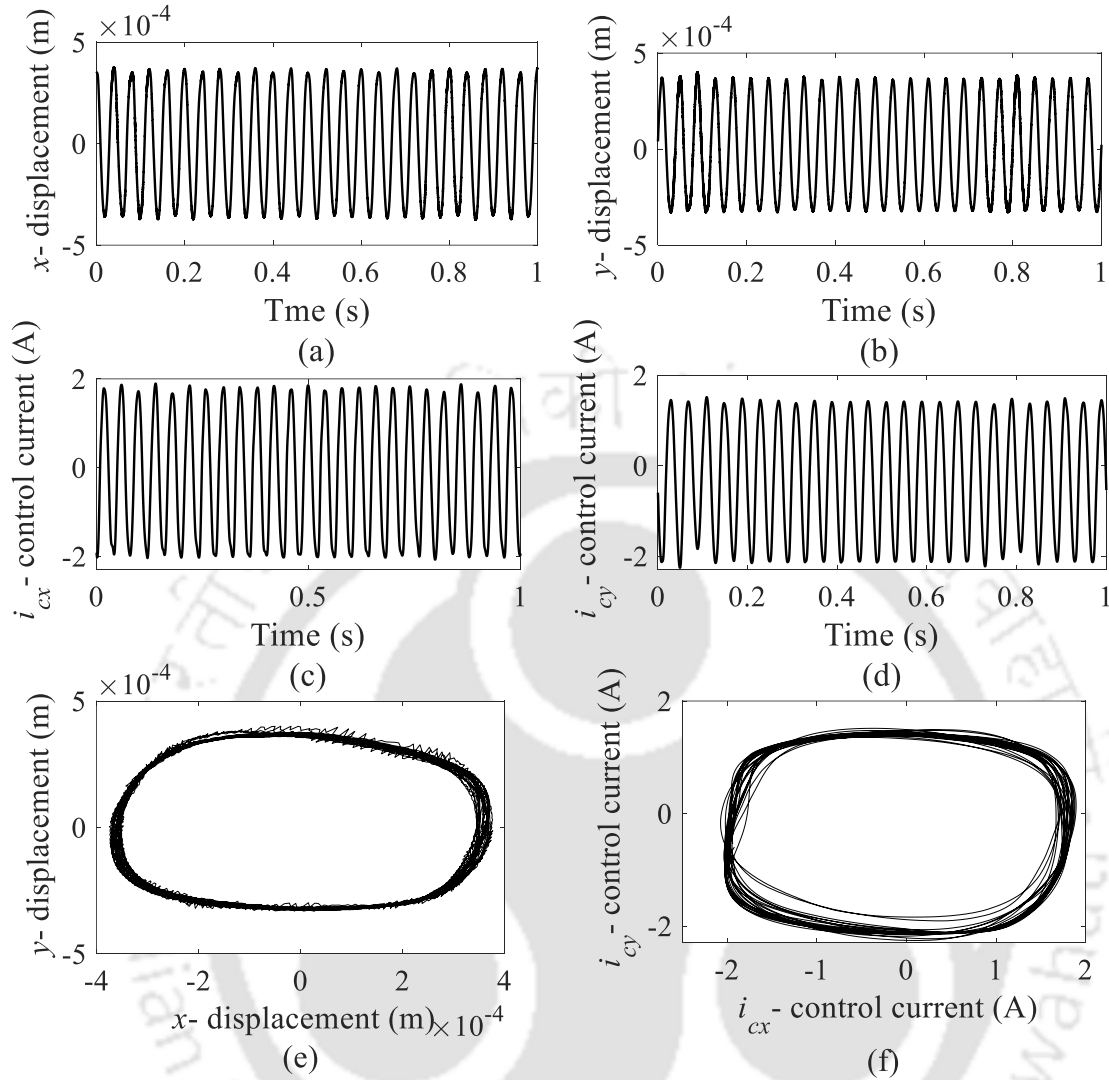


Figure 3.19 Time domain responses of the system operating at approximately 5 Hz (a) vertical displacement (b) horizontal displacement (c) control current in vertical direction (d) control current horizontal direction (e) orbit plot of displacement signal (f) orbit plot of control current signal

Using the Fast Fourier Transform (FFT), time domain signals are converted into frequency domain commonly known as full spectrum response, which contains the forward and backward whirls, i.e. the directional features of the system. In the present analysis, phase compensation (subtraction of the phase of the measured signal with that of the reference

signal for each operating condition) technique is implemented as discussed in Chapter 2. Full spectrum plots of the reference, displacement and control current signals for two operating conditions of 5 Hz and 26 Hz speeds are shown in Figures 3.20 and 3.21, respectively.

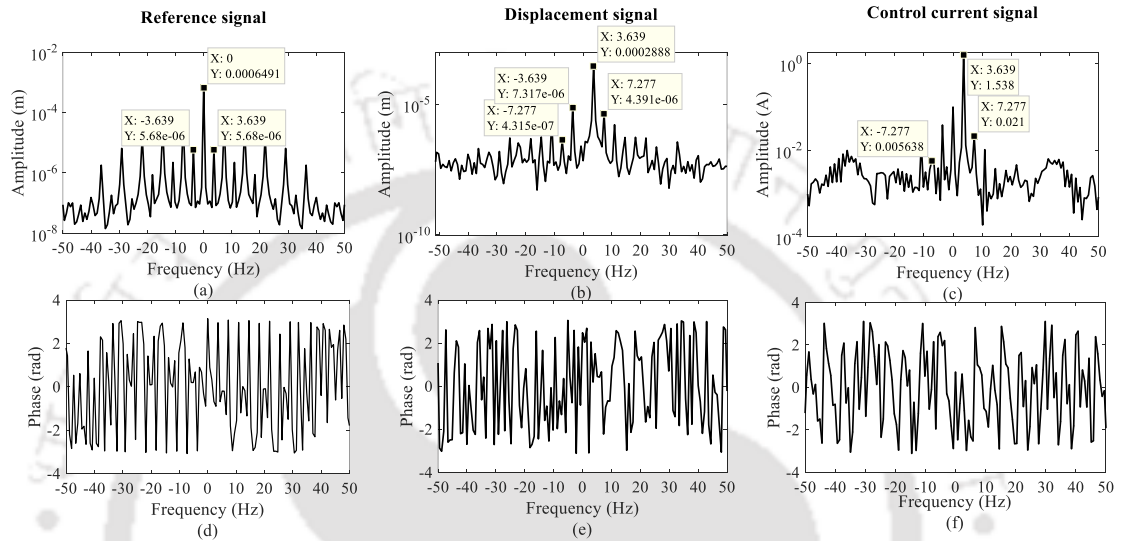


Figure 3.20 Full spectrum amplitude and phase of rotor signal measured at approximately 4 Hz (a, d) reference (b, e) vertical direction (c, f) horizontal direction

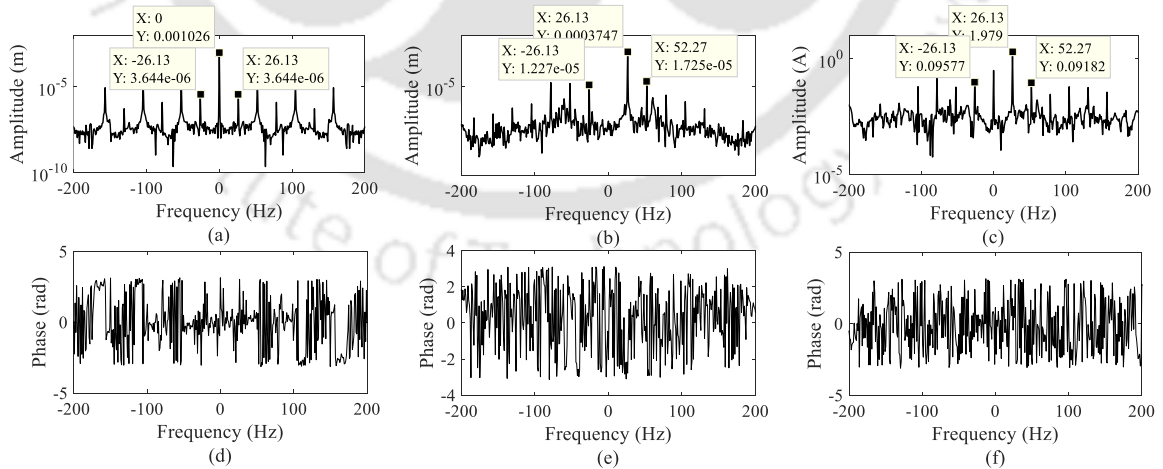


Figure 3.21 Full spectrum amplitude and phase of rotor signal measured at approximately 26 Hz (a, d) reference (b, e) vertical direction (c, f) horizontal direction

Table 3.7 Full spectrum amplitude and phases of reference, displacement, and control current signal of the test rig

<i>i</i>	Reference signal				Displacement signal				Control current signal			
	Amplitude (m)		Phase compensation (rad)		Amplitude (m)		Phase compensation (rad)		Amplitude (A)		Phase compensation (rad)	
	A*	B*	A*	B*	A*	B*	A*	B*	A*	B*	A*	B*
0	6.49×10^{-4}	0.001	0.00	0.00	6.826×10^{-8}	2.85×10^{-7}	2.47	-2.03	0.09	0.315	0.72	1.14
1X	5.68×10^{-6}	3.64×10^{-6}	2.95	3.03	2.89×10^{-4}	3.75×10^{-4}	0.19	-2.78	1.54	1.98	-2.97	0.45
2X	4.38×10^{-5}	3.06×10^{-5}	3.078	2.71	4.39×10^{-6}	1.73×10^{-5}	-2.22	1.55	0.02	0.09	1.06	-1.44
3X	2.21×10^{-6}	1.25×10^{-6}	2.97	-2.96	1.65×10^{-6}	4.22×10^{-6}	1.33	1.19	0.01	0.03	-1.87	-1.45
4X	2.94×10^{-5}	1.91×10^{-5}	-1.88	-2.70	5.35×10^{-7}	2.16×10^{-6}	-0.09	2.05	3.75×10^{-3}	0.06	2.88	-1.81
5X	8.63×10^{-7}	5.32×10^{-7}	-2.33	-2.65	5.43×10^{-7}	1.03×10^{-6}	2.56	-2.28	3.38×10^{-3}	0.05	-1.50	3.018
6X	1.55×10^{-6}	9.01×10^{-6}	-3.02	-2.47	5.10×10^{-7}	1.53×10^{-7}	-1.67	-1.79	4.5×10^{-3}	0.04	1.64	0.74
-1X	5.68×10^{-6}	3.64×10^{-4}	-2.95	-3.03	7.32×10^{-6}	1.23×10^{-5}	-0.77	2.37	0.04	0.09	2.21	-0.60
-2X	4.38×10^{-5}	3.06×10^{-5}	-3.078	2.95	4.32×10^{-7}	1.40×10^{-5}	-1.32	-1.67	5.64×10^{-3}	0.06	2.15	-1.60
-3X	2.21×10^{-6}	1.25×10^{-6}	-2.97	2.96	2.19×10^{-6}	1.56×10^{-5}	-0.08	-1.16	0.02	0.14	3.01	1.95
-4X	2.94×10^{-5}	1.91×10^{-5}	1.88	2.70	5.58×10^{-7}	1.70×10^{-6}	-2.45	-0.79	2.61×10^{-3}	0.06	-3.11	-3.04
-5X	8.63×10^{-7}	5.32×10^{-7}	2.33	2.65	4.79×10^{-7}	1.11×10^{-6}	-2.85	-0.49	1.64×10^{-3}	0.01	-2.95	-2.17
-6X	1.55×10^{-6}	9.01×10^{-6}	-3.02	2.47	2.26×10^{-7}	5.99×10^{-7}	-0.89	-2.05	2.25×10^{-3}	0.01	1.29	-0.15

i: No. of harmonics; A*: slow speed (4 Hz) ; B*: high speed (26 Hz)

3.4 Identification of Multiple Faults from Experiments

The identification of the rotor faults from the measured signals of the laboratory test rig is explained for various speed range cases in the present section. The measured displacement responses are utilized as per the same methodology as described in Chapter 2 of the numerical experiment for estimation of multiple faults associated with the cracked-warped rotor-bearing integrated with AMB installed in the laboratory.

3.4.1 Estimation of Multiple Faults

The full spectrum amplitude and phase compensation represented in Table 3.7 are utilized in the developed identification algorithm as in Equation (2.46) for different speed range cases to estimate additive as well as multiplicative fault parameters associated with the test rig. Equation (2.47) is segregated as known and unknown parameters based on the developed EOM of Equation (2.45) in frequency domain for estimation process is written as

$$\begin{bmatrix}
 -\omega S_{1,lm} & 0 & -u_{x0} p_1 & -m\omega^2 & 0 & -k_0 & 0 & -q_0 & -S_{1,Re} & I_{1,Re} \\
 0 & \omega S_{0,lm} & -u_{x0} p_0 & 0 & 0 & 0 & 0 & -q_1 & -S_{0,Re} & I_{0,Re} \\
 -2\omega S_{2,lm} & -\omega S_{2,lm} & -u_{x0} p_2 & 0 & 0 & 0 & 0 & -q_2 & -S_{2,Re} & I_{2,Re} \\
 -3\omega S_{3,lm} & -2\omega S_{3,lm} & -u_{x0} p_3 & 0 & 0 & 0 & 0 & 0 & -S_{3,Re} & I_{3,Re} \\
 -4\omega S_{4,lm} & -3\omega S_{4,lm} & 0 & 0 & 0 & 0 & 0 & -q_4 & -S_{4,Re} & I_{4,Re} \\
 -5\omega S_{5,lm} & -4\omega S_{5,lm} & -u_{x0} p_5 & 0 & 0 & 0 & 0 & 0 & -S_{5,Re} & I_{5,Re} \\
 -6\omega S_{6,lm} & -5\omega S_{6,lm} & 0 & 0 & 0 & 0 & 0 & -q_6 & -S_{6,Re} & I_{6,Re} \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 \omega S_{-1,lm} & 2\omega S_{-1,lm} & -u_{x0} p_{-1} & 0 & 0 & 0 & 0 & -q_{-1} & -S_{-1,Re} & I_{-1,Re} \\
 2\omega S_{-2,lm} & 3\omega S_{-2,lm} & 0 & 0 & 0 & 0 & 0 & -q_{-2} & -S_{-2,Re} & I_{-2,Re} \\
 3\omega S_{-3,lm} & 4\omega S_{-3,lm} & -u_{x0} p_{-3} & 0 & 0 & 0 & 0 & 0 & -S_{-3,Re} & I_{-3,Re} \\
 4\omega S_{-4,lm} & 5\omega S_{-4,lm} & 0 & 0 & 0 & 0 & 0 & -q_{-4} & -S_{-4,Re} & I_{-4,Re} \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 \omega S_{1,Re} & 0 & 0 & 0 & -m\omega^2 & 0 & -k_0 & 0 & -S_{1,lm} & I_{1,lm} \\
 0 & \omega(S_{0,Re} + u_{x0}) & 0 & 0 & 0 & 0 & 0 & 0 & -S_{0,lm} & I_{0,lm} \\
 2\omega S_{2,Re} & \omega S_{2,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{2,lm} & I_{2,lm} \\
 3\omega S_{3,Re} & 2\omega S_{3,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{3,lm} & I_{3,lm} \\
 4\omega S_{4,Re} & 3\omega S_{4,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{4,lm} & I_{4,lm} \\
 5\omega S_{5,Re} & 4\omega S_{5,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{5,lm} & I_{5,lm} \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
 -\omega S_{-1,Re} & -2\omega S_{-1,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{-1,lm} & I_{-1,lm} \\
 -2\omega S_{-2,Re} & -3\omega S_{-2,Re} & 0 & 0 & 0 & 0 & 0 & 0 & -S_{-2,lm} & I_{-2,lm} \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
 \end{bmatrix}
 \begin{Bmatrix}
 c_v \\
 c_H \\
 \Delta k_c \\
 e_{u,Re} \\
 e_{u,lm} \\
 e_{b,Re} \\
 e_{b,lm} \\
 \Delta k_c e_{b,Re} \\
 k_s \\
 k_i
 \end{Bmatrix}
 =
 \begin{Bmatrix}
 (\omega^2 m - k_0) S_{1,Re} \\
 -k_0 S_{0,Re} \\
 (4\omega^2 m - k_0) S_{2,Re} \\
 (9\omega^2 m - k_0) S_{3,Re} \\
 (16\omega^2 m - k_0) S_{4,Re} \\
 (25\omega^2 m - k_0) S_{5,Re} \\
 (36\omega^2 m - k_0) S_{6,Re} \\
 \vdots \\
 (\omega^2 m - k_0) S_{-1,Re} \\
 (4\omega^2 m - k_0) S_{-2,Re} \\
 (9\omega^2 m - k_0) S_{-3,Re} \\
 (16\omega^2 m - k_0) S_{-4,Re} \\
 \vdots \\
 (\omega^2 m - k_0) S_{1,lm} \\
 -k_0 S_{0,lm} \\
 (4\omega^2 m - k_0) S_{2,lm} \\
 (9\omega^2 m - k_0) S_{3,lm} \\
 (16\omega^2 m - k_0) S_{4,lm} \\
 (25\omega^2 m - k_0) S_{5,lm} \\
 \vdots \\
 (\omega^2 m - k_0) S_{-1,lm} \\
 (4\omega^2 m - k_0) S_{-2,lm} \\
 \vdots
 \end{Bmatrix}
 \quad (2.52)$$

The identification equation can be applied for a set of rotating speed is organized in a vector for n number of rows and r number of speeds as

$$\begin{bmatrix} \mathbf{A}_1(\omega_1) \\ \mathbf{A}_1(\omega_2) \\ \vdots \\ \mathbf{A}_1(\omega_r) \end{bmatrix}_{(2n+1)r \times 10} \mathbf{x}_{1(10 \times 1)} = \begin{Bmatrix} \mathbf{B}_1(\omega_1) \\ \mathbf{B}_1(\omega_2) \\ \vdots \\ \mathbf{B}_1(\omega_r) \end{Bmatrix}_{(2n+1)r \times 1} \quad (2.53)$$

Applying least square fitting technique for best fit data to Equation (2.53) as

$$\mathbf{x}_1 = \left(\mathbf{A}_1(\omega)^T \mathbf{A}_1(\omega) \right)^{-1} \mathbf{A}_1(\omega)^T \mathbf{B}_1(\omega) \quad (2.54)$$

With $\mathbf{x}_1 = \left[c_V \quad c_H \quad \Delta k_{c22} \quad \Delta k_{c22} e_{b,Re} \quad e_{Re} \quad e_{Im} \quad e_{b,Re} \quad e_{b,Im} \quad k_s \quad k_i \right]^T$ are the parameters to be estimated.

In the present identification procedure, fourteen numbers of harmonics of the vibration response and current signal are chosen for the analysis. To get more data sets and maintaining consistency with a high degree of accuracy, identification of parameters is done for combined speed cases such as from 10 Hz to 20 Hz, 15 Hz to 26 Hz, and 10 Hz to 26 Hz of the three different spin speed ranges of the rotor system as summarized in Table 3.8. The above mentioned three speed cases measured data are fed to the regression matrix $\mathbf{A}_1(\omega_r)_{(2n+1)r \times 10}$ and the known matrix $\mathbf{B}_1(\omega_r)_{(2n+1)r \times 10}$ of Equation (2.52) for the quantitative estimation of multiple faults.

It is observed from the estimated parameters from Table 3.8 that the external damping of the rolling element bearing (deep groove ball bearing) parameter in all the three speed cases is in a range of 330 Nsm^{-1} to 550 Nsm^{-1} based on SKF standard bearings with slight lubricant in bearing. The internal damping (c_H) parameter values are in the same range to that of the estimated parameter in Roy and Tiwari (2020). The additive crack stiffness is a good indicator of crack in a system. The comparison of the estimated additive crack stiffness ($1.0229 \times 10^5 \text{ N}$) to that of the intact or healthy shaft

(7.044×10^5 N) can be done and the ratio of intact to cracked shaft stiffness is related to the ratio of diameter of intact shaft to the equivalent diameter of the cracked shaft. The additive stiffness due to fatigue crack is found to be around 58% of the intact shaft stiffness whereas stiffness variation for the notched shaft is 47.70% (since stiffness is proportional to the fourth power of diameter i.e. $k_{\text{intact}} / k_{\text{cracked}} = d_{\text{intact}}^4 / d_{\text{cracked}}^4$, so it will be equivalent to 2.67% reduction in overall shaft diameter due to crack) due to crack growth. The severity of the fatigue crack can be assessed from the magnitude of the additive crack stiffness parameter and preventive measure can be ensured for the machinery.

Table 3.8 Experimentally estimated multiple fault parameters for different speed range cases

Estimated parameters	10 Hz to 20 Hz	15 Hz to 26 Hz	10 Hz to 26 Hz
c_v (Nsm ⁻¹)	416.679	309.551	288.997
c_H (Nsm ⁻¹)	342.310	276.028	259.074
Δk_{c22} (Nm ⁻¹)	1.0229×10^5	1.8614×10^5	1.6389×10^5
e_u (m)	3.341×10^{-4}	2.014×10^{-4}	1.776×10^{-4}
ϕ_u (deg)	64.884	89.84	85.809
e_b (m)	1.622×10^{-4}	2.002×10^{-4}	1.904×10^{-4}
ϕ_b (deg)	28.29	28.65	28.66
k_s (Nm ⁻¹)	2.2192×10^5	1.6274×10^5	1.7132×10^5
k_i (NA ⁻¹)	8.348	9.727	8.806
$\Delta k_{c22} e_{b,Re}$ (N)	8.597	3.847	7.622

The shaft residual bow is much noticeable in the slow speed condition and from that, the permanent bend of the shaft can be identified easily by analyzing the amplitude of 1X harmonic as it is much prominent in 1X only irrespective of other harmonics of the response of the full spectrum response. Here, the residual shaft bow estimated through the identification algorithm is in the similar range of the amplitude of slow speed, i.e.

2×10^{-4} m in the vibrational signal of the test rig. The force-displacement factor and force-current factor of AMB is also in the range of the analytically calculated ones provided in the numerical simulation section, which was obtained from the magnetic bearing design considerations. The estimated parameters are also validated indirectly using the numerical simulation in the next section.

3.4.2 Analysis of the Behavior of the Crack and Residual Bow Forces

The crack force and the bow force behaviour is analyzed separately from the experimentally identified parameters. Moreover, the effect of the residual bow on the crack force is also studied in this section.

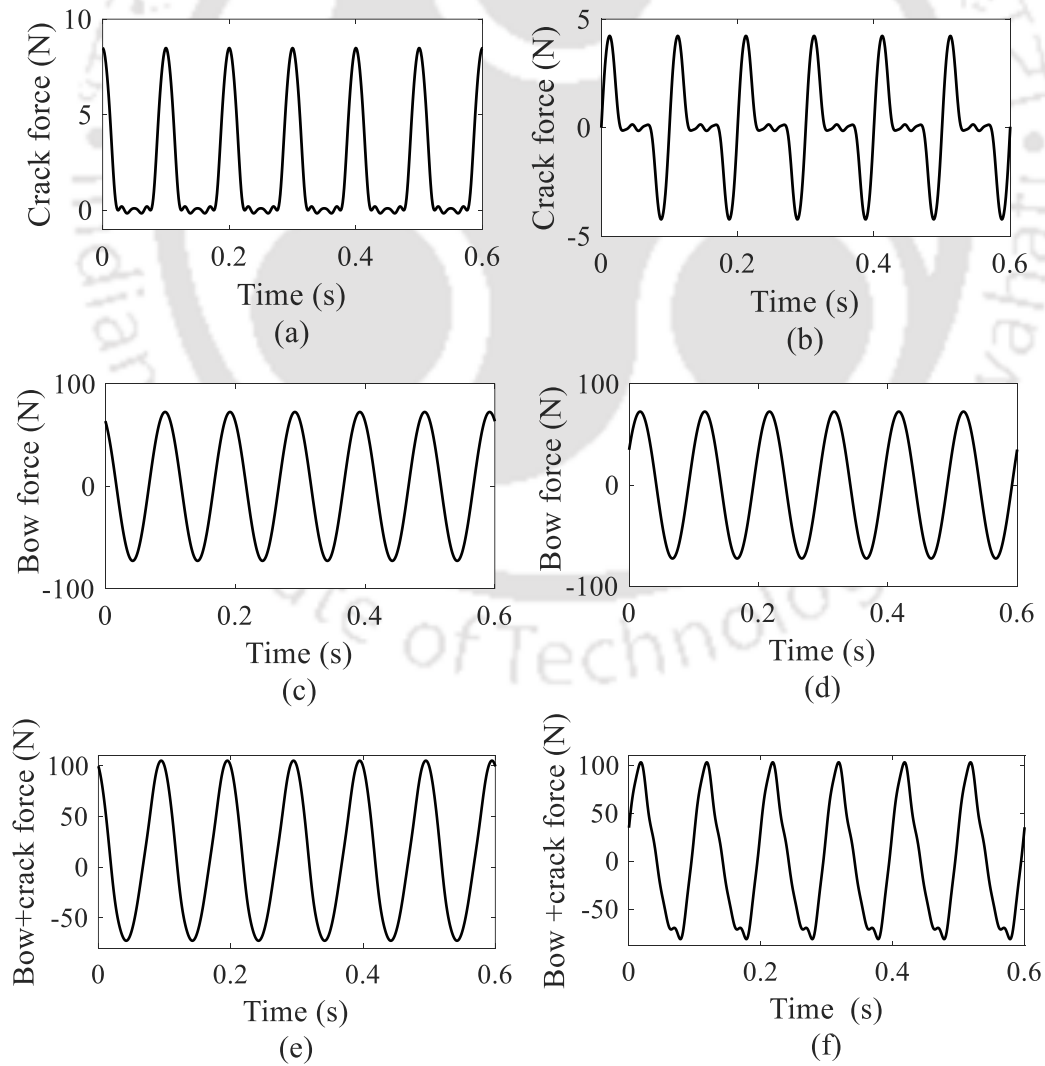


Figure 3.22 Crack force without residual bow (a) vertical direction (b) horizontal direction Residual bow force acting on the rotor system (a) vertical, and (b) horizontal direction. Overall crack force, multiplicative fault effect, and residual bow force (c) vertical direction (d) horizontal direction

It is observed from Figure 3.22(a, b) that the crack force behaves like a sudden opening and closure as considered in the modeling as switching excitation model. The residual bow force greatly affects the dynamics of the system, which can be observed from Figure 3.22(c, d). The rotor system experiences around 80 N residual bow force as seen from the plots. The most important analysis is the effect of the residual bow on the cracked rotor where the rotor experiences multiplicative fault due to crack and bow as discussed in Equation (2.45) in Chapter 2. Figure 3.22(e, f) depicts the effect of residual bow on the overall force experienced by the rotor system due to residual bow, crack and the multiplicative fault. The plot shows that the residual bow force overshadowed the effect of crack in a system by increasing the amplitude of vibration as well as the force of 100 N in the system.

The estimated disc unbalance and its phase angle in the system are used in the balancing procedure in the following section to determine its precision. The estimated parameters are also validated indirectly using the numerical simulation in the next subsection.

3.4.3 Validation of the Numerical Model with Experimentally Estimated Parameters

The experimental results need to be verified through the numerical model once. Due to several uncertainties involved in the rotordynamic test bench and the data acquisition system, which may risk falsifying the identification of the estimation of multiple faults.

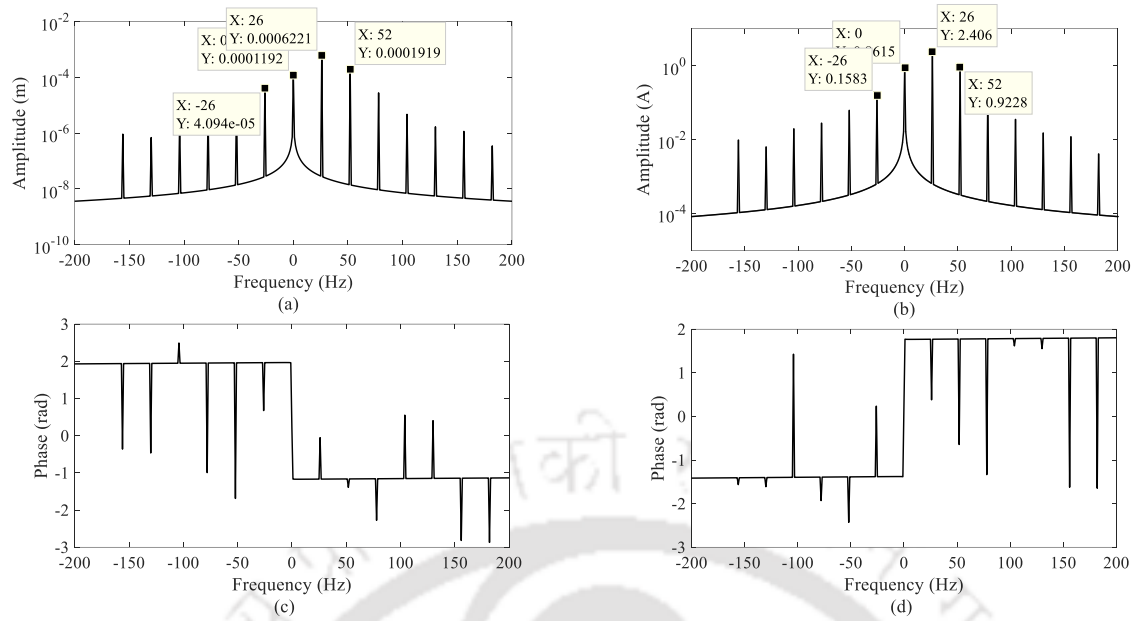


Figure 3.23 Simulated full spectrum response utilizing experimentally estimated parameters at spin speed of 26 Hz (a, c) amplitude and phase of displacement signal (b, d) amplitude and phase of control current signal

For verification purposes, an example of the full spectrum response measured from the test set-up and of the simulated model, operating at the speed of 26 Hz is shown in Figure 3.21 and Figure 3.23, respectively. The amplitude of multiple harmonics in experimental response and numerically simulated response are shown in Table 3.9. There is a very close agreement between the measured responses with the simulated ones and the pattern of the responses in the two cases are very similar, which verifies the identification methodology well.

Table 3.9 Comparison of experimental and simulated full spectrum response amplitudes of displacement and control current signal operating around 26 Hz of spin speed

No. of harmonics	Experimental full spectrum response		Simulated full spectrum response	
	Displacement amplitude (m)	Control current amplitude (A)	Displacement amplitude (m)	Control current amplitude (A)
0		0.29	1.192×10^{-4}	0.861
1X	3.75×10^{-4}	1.93	6.22×10^{-4}	2.41
2X	1.73×10^{-5}	0.08	4.09×10^{-5}	0.31
3X	4.22×10^{-6}	0.03	2.93×10^{-6}	0.01
4X	2.16×10^{-6}	0.05	4.49×10^{-6}	0.02
5X	1.03×10^{-6}	0.05	1.64×10^{-7}	8.67×10^{-4}
-1X	1.53×10^{-7}	0.08	3.95×10^{-5}	0.05
-2X	1.23×10^{-5}	0.05	1.21×10^{-5}	0.05
-3X	1.40×10^{-5}	0.13	4.55×10^{-6}	0.02
-4X	1.56×10^{-5}	0.05	2.21×10^{-6}	0.01

Table 3.10 Balancing of rotor system with estimated parameters of unbalance from the identification algorithm

Rotor spin speed	Balance mass (g)	Vibrational amplitude (m)		% reduction
		Without AMB	With AMB	
Approximately 18 Hz	0	1.175×10^{-4}	7.431×10^{-5}	36.60
	7	1.072×10^{-4}	6.623×10^{-5}	40.17
	11	1.034×10^{-4}	5.781×10^{-5}	50.81
Approximately 20 Hz	0	1.272×10^{-4}	4.931×10^{-5}	61.18
	7	1.129×10^{-4}	3.232×10^{-5}	74.57
	11	1.093×10^{-4}	2.801×10^{-5}	77.95

Moreover, the unbalances experience by the cracked-warped rotor system estimated using the developed identification algorithm is utilized for balancing the rotor system. The estimated unbalance is used as a balance mass at an orientation opposite to

the corresponding phases estimated in the holes of the disc in the system. The percentage reductions of the vibrational response of the system operating at approximately 18 Hz and 20 Hz speed for before and after balancing as well as with and without AMB control with two different balance masses are summarized in Table 3.10.

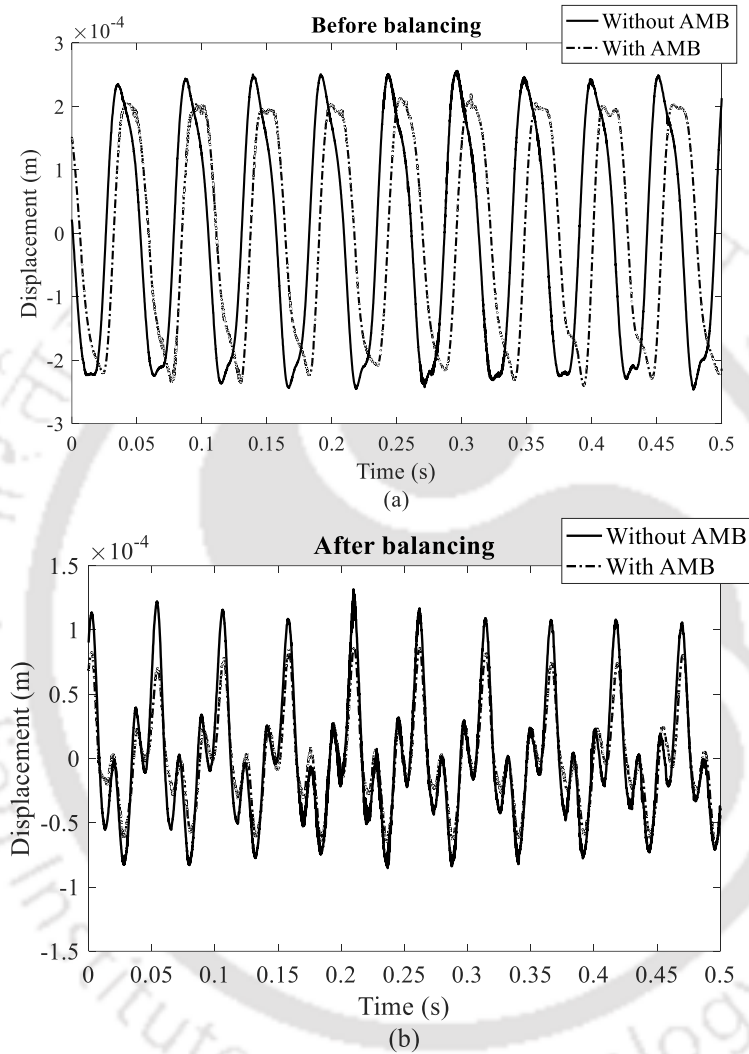


Figure 3.24 Time domain response of with and without balancing (a) without AMB control at 20 Hz speed (b) with AMB control at 20 Hz speed

The responses with the reduction in amplitude for the above cases for the spin speed of approximately 20 Hz system is shown in Figure 3.24. From Table 3.10, it is observed that the maximum percentage reduction of the vibrational amplitude operating at the spin speed of 20 Hz after balancing with a balance mass of 11 g and with AMB control as

77.95% and so as reduces the amount control current to attenuate the vibration of the system. From the present analysis, it can be concluded that the developed identification algorithm is accurate enough to describe the system dynamics and for the quantification of multiple faults in a rotor bearing system.

3.5 Concluding Remarks

In the current chapter, the experimental identification of additive and multipliative faults associated with a cracked-warped rotor-bearing system mounted a centrally located disc on the shaft integrated with a radial eight-pole AMB. The rotor test data acquired from the laboratory test rig development is described. Fabricaton of the radial magnetic bearing actuator and generation of fatigue crack in three-point bending test is also elaborated in this chapter. The present chapter also gives an insight in the successful implementation of PID control startegy for generating control current to work active magnetic bearing actuator as a controller. The data acquisition of the test data through dSPACE MicroLabbox are utilizing in converting multiple harmonic full spectrum to use in the developed identification algorithm for estimation of fault parameters, those are present in the test bench, is broadly described here.

The current experimental investigation on the laboratory test rig are summarizes below:

- PID control strategy is successfully implemented for controlling action in the developed laboratory test rig.
- As the shaft residual bow affects the amplitude of vibration immensely and its effect along with crack on the shaft shows multilple harmonics by inducing mutiplicative fault effect, the bow effect is not removed from the 1X harmonic during signal processing as done in previous researcchs. In fact the residual bow

magnitude and its orientation along with the multiplicative fault of bow and crack is estimated quite accurately along with other rotor fault parameters.

- The estimated fault parameters are utilized in the numerical simulation for generating full spectrum responses and its viability is assessed with the experimental full spectrum plots. Moreover, the unbalance estimated is utilizing for balancing the rotor system experimentally and a good amount of reduction of vibrational amplitude of the test rig is achieved.
- Crack forces experiences by the test rig through additive stiffness estimated is studied.

To analyze a more realistic rotor system, gyroscopic effect is added into the mathematical modelling of a cracked rotor-AMB system for both without and with residual shaft bow effect. The development of identification algorithm for both the cases for multiple fault estimation will be described in next chapter. The sensitivity and the robustness of the developed algorithm against measurement noise and modelling (or bias) errors is also illustrated through numerical simulation.

Chapter 4

Identification of Multiple Faults in a Rotor-AMB System having an Offset Disc

Gyroscopic effects arise in the rotor due to the presence of rotating masses of mountings, such as the gears, pulley, wheels, etc. and their orientations or positions with respect to the bearings axis. Gyroscopic effect is proportional to the rotational speed and the moment generated by this effect tries to bring the rotor into its self-centering position by increasing the stiffening of the rotor. The foundation of the analysis of the cracked rotor-AMB system with gyroscopic effect in presence of residual bow and without bow is laid here. The complexity due to the presence of gyroscopic effect in the rotor for measuring the rotational DOFs precisely during measurement is envisioned. The development of the identification algorithm for multiple rotor faults estimation based on the EOMs of the rotor-AMB system containing a disc axially offset with the center of the shaft. To check the robustness of the algorithm numerical experiments are carried out in presence of random noise and modeling or bias errors.

4.1 Introduction

Governing equations of motion are derived for a cracked-warped rotor equipped with AMB having a rigid disc mounted offset from the middle of the shaft for both without and with residual bow effect. The inclusion of the offset disc offers a tilting motion due to which

rotor experiences gyroscopic effect producing skew-symmetry in the gyroscopic matrix. Therefore, there is a need for analyzing the rotor based on 4-DOFs, where translational and rotational motion of rotor to be considered. The present chapter explores the method of identification of multiple faults discussed in Chapter 2 for the rotor system with an offset disc. The complexity arises in the development of the identification algorithm for the present case, as it requires the rotational DOFs. Nevertheless, measuring angular displacements in practical situation may not be so accurate or difficult to measure, therefore, to overcome this difficulty, *gyroscopic condensation scheme (GDC)* is implemented to the developed EOMs for eliminating rotational DOFs. The condensed EOMs are then utilized to develop the identification algorithm to estimate fault parameters. Full spectrum amplitude and phase compensation are utilized to identify the additive as well as multiplicative fault parameters in the rotor system. Numerical experimentation is performed against various levels of random noise and modeling errors to check the robustness of the algorithm.

4.2 Background of the Proposed Rotor Model

The assumptions made in the modeling of the rotor system are now summarized. The mathematical model considers a bowed rotor containing a transverse crack integrated with an AMB as depicted in Figure 4.1. The unbalanced disc is placed axially offset on to the flexible massless shaft. The gyroscopic effect on the disc depends on the disc dimension and disc position on the rotor. The supports are considered as simply supported. The gyroscopic effect due to the offset disc is considered. The residual shaft bow, which is the permanent deformation of the shaft, is also taken into consideration. The *weight dominance* effect is considered in crack modeling, where the vibration response due to the static

deflection of the system is much higher than that of the dynamic response (Gasch, 1993).

AMB considers the PID controller.

4.1.1 Depiction of the Cracked-Warped Rotor-AMB Model

The mechanical model of the proposed cracked rotor system to be analyzed is depicted in Figure 4.1. To develop the proposed mathematical model two sets of co-ordinate systems are used. The $(x-y)$ fixed coordinate system is coincident with the shaft center, while the $(\zeta-\eta)$ rotating co-ordinate system has the same origin but it rotates with the shaft (Refer Figure 2.2 in Chapter 2). The transverse angular displacements, as shown in Figure 4.2, are φ_x and φ_y in the $(y-z)$ and $(x-z)$ planes, respectively. The mass of the disc and the stiffness of the intact shaft are denoted by m and k , respectively. The cross-section of crack is considered perpendicular to the ζ -direction and the effect of crack in the η -direction is ignored. The internal viscous damping is of the rotating nature and exerts forces in the ζ and η directions as depicted in Figure 2.2, where c_H is the viscous internal damping parameter.

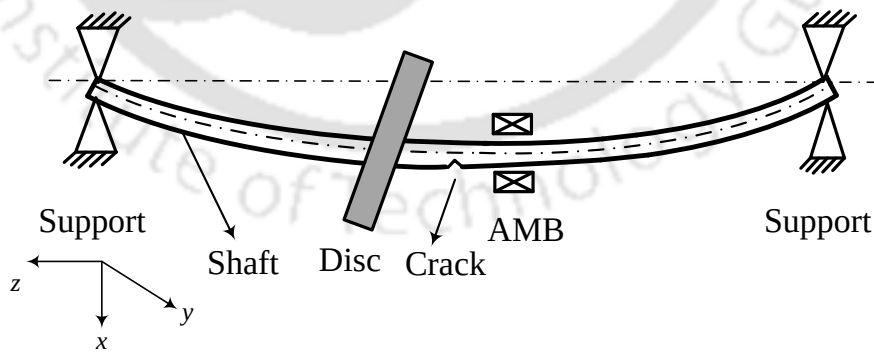


Figure 4.1 Schematic view of a cracked-warped shaft having an offset disc integrated with an AMB disc must be perpendicular to elastic line

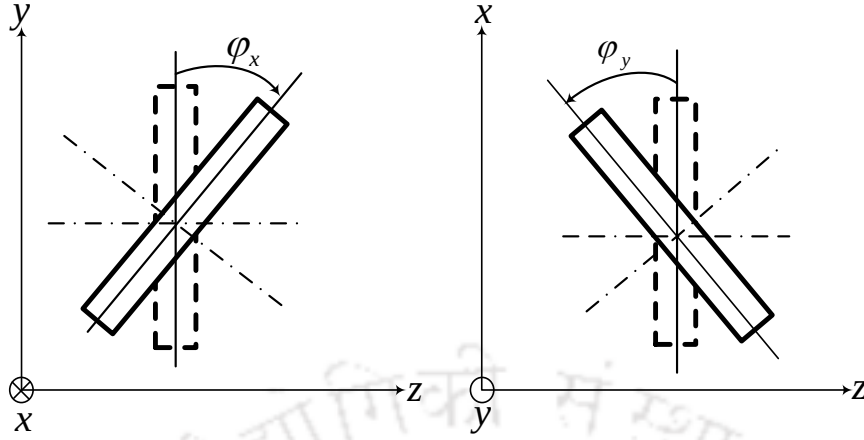


Figure 4.2 Schematic view of the offset disc orientation (a) y-z plane (b) x-z plane

4.1.2 Mathematical Formulation of the Proposed Model

In the present rotor modeling, three different kinds of faults have been considered. The generalized equations of motion, in the presence of a gyroscopic effect, are four degrees of freedom system. Initially, EOMs in the inertial frame of reference, with the static and the unbalance forces without crack, internal damping residual bow and, magnetic forces can be expressed in matrix form as in Refs. (Genta, 2007; Friswell and Penny, 2006)

$$\mathbf{M}\ddot{\mathbf{q}}(t) + (\mathbf{C} - \omega \mathbf{G})\dot{\mathbf{q}}(t) + \mathbf{K}_0\mathbf{q}(t) = \mathbf{f}_{st} + \mathbf{f}_{unb}(t) \quad (4.1)$$

The $\mathbf{M}$, $\mathbf{C}_e$, $\mathbf{G}$ and $\mathbf{K}_0$ are the mass, external damping, gyroscopic and intact shaft stiffness matrices, respectively. The external damping, c_{22} is equivalent viscous damping at bearings. In the stiffness subscript 2 represents to force/translational displacement and 4 represent moment/ rotational displacement. The uniform spin speed of the shaft is denoted by, ω . The static force vector, $\mathbf{f}_{st}$, and the unbalance force vector, $\mathbf{f}_{unb}$, are affecting the dynamic behavior of the rotor system. The vector $\mathbf{q}(t)$ is the total displacement of the disc

and it is a combination of the vibration response vector, $\mathbf{q}_v(t)$ of the disc and the static deflection vector, $\mathbf{q}_0$ of the disc due to its self-weight.

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_d & 0 \\ 0 & 0 & 0 & I_d \end{bmatrix}; \mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p \\ 0 & 0 & -I_p & 0 \end{bmatrix}; \mathbf{C} = \begin{bmatrix} c_{22} & 0 & 0 & 0 \\ 0 & c_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

with $\mathbf{K}_0 = \begin{bmatrix} k_{22} & 0 & k_{24} & 0 \\ 0 & k_{22} & 0 & k_{24} \\ k_{24} & 0 & k_{44} & 0 \\ 0 & k_{24} & 0 & k_{44} \end{bmatrix}; \mathbf{f}_{\text{unb}} = \begin{Bmatrix} me\omega^2 \cos(\theta + \phi_u) \\ me\omega^2 \sin(\theta + \phi_u) \\ 0 \\ 0 \end{Bmatrix}; \mathbf{f}_{\text{st}} = \begin{Bmatrix} k_{22}u_{x_0} \\ k_{22}u_{y_0} \\ k_{44}\phi_{y_0} \\ k_{44}\phi_{x_0} \end{Bmatrix}$ (4.2)

$$\mathbf{q}(t) = \mathbf{q}_0 + \mathbf{q}_v(t) = \begin{Bmatrix} u_{x_0} \\ u_{y_0} \\ \phi_{y_0} \\ \phi_{x_0} \end{Bmatrix} + \begin{Bmatrix} u_x(t) \\ u_y(t) \\ \phi_y(t) \\ \phi_x(t) \end{Bmatrix}$$

Internal and External Damping Force Model

The internal (rotating) damping occurs due to several reasons including the rubbing of the crack faces while the rotor is rotating. The internal damping would affect in the cracked shaft along the ξ and η rotating coordinate system as shown in Figure 2.2. In the matrix form of 4-DOF system, the internal damping force after transforming from rotating coordinates to inertial frame of reference coordinates is represented as

$$\mathbf{f}_{id} = \begin{bmatrix} (c_V + c_H) & 0 & 0 & 0 \\ 0 & (c_V + c_H) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u}_x \\ \dot{u}_y \\ \dot{\phi}_y \\ \dot{\phi}_x \end{Bmatrix} + \begin{bmatrix} 0 & \omega c_H & 0 & 0 \\ -\omega c_H & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ \phi_y \\ \phi_x \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\omega c_H u_{x_0} \\ 0 \\ 0 \end{Bmatrix} \quad (4.3)$$

The internal damping force represented in Equation (4.3) will be used in the present analysis.

Crack Force Model

The crack forces can be modeled considering the behavior of opening and closure of the cracked faces during rotation of the shaft. In the present case, the switching crack model is considered for modeling of crack forces as discussed in Chapter 2 elaborately. The opening and closure behaviour of the crack face during rotation of the shaft depends on the orientation of the cracked front. The crack parameter $\Delta \mathbf{K}(t)$ can be expanded as a Fourier series with the assumption of the choice of the switching crack excitation function. With the same concept of crack modelling in 2-DOF system, the time-varying stiffness matrix of the cracked shaft arises due to the breathing crack is given as

$$\mathbf{K}_{\xi\eta}(t) = \mathbf{K}_{0\xi\eta} + \Delta \mathbf{K}_{\xi\eta}(t) \quad (4.4)$$

$$\mathbf{K}_{0\xi\eta} = \mathbf{K}_0, \text{ and } \Delta \mathbf{K}_{\xi\eta}(t) = \begin{bmatrix} \Delta k_{c22} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \Delta k_{c44} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.5)$$

Where, $\mathbf{K}_0$ is the diagonal matrix of the intact shaft stiffness and $\Delta \mathbf{K}_{\xi\eta}(t)$ is the additive stiffness of crack in rotating coordinate system, which can be expressed in Fourier series considering a breathing crack excitation function. The translational and rotational additive stiffness of crack denoted by Δk_{c22} and Δk_{c44} , respectively are considered in the analysis. Subscripts 2 corresponds to shearing force, subscripts 4 correspond to the bending moment about two lateral directions on the cracked face of the shaft and c defines the crack. In literature, hardly any attempt is made to include the rotational additive stiffness (Δk_{c44}) in the analysis and the identification. This is may be due to difficulty in measuring rotational DOFs but it is an important component for the transverse crack in shafts. In the present

case, the difficulty associated with this has been overcome by the gyroscopic dynamic condensation scheme (Prasad and Tiwari, 2019). Transformation of the intact and cracked shaft components in Equation (4.4) from rotating to an inertial frame of reference is detailed in Ref. (Gasch, 2008) based on the weight dominance assumption, as

$$\mathbf{K}(t) = \mathbf{T}^T \mathbf{K}_{0\xi\eta} \mathbf{T} + \mathbf{T}^T (\Delta \mathbf{K}_{\xi\eta}(t)) \mathbf{T} \quad (4.6)$$

with transformation matrix as,

$$\mathbf{T} = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 \\ -\sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & \cos \theta & \sin \theta \\ 0 & 0 & -\sin \theta & \cos \theta \end{bmatrix} \quad (4.7)$$

The overall stiffness of the shaft becomes,

$$\mathbf{K}(t) = \begin{bmatrix} k_{22} & 0 & k_{24} & 0 \\ 0 & k_{22} & 0 & k_{24} \\ k_{42} & 0 & k_{44} & 0 \\ 0 & k_{42} & 0 & k_{44} \end{bmatrix} - \frac{1}{2} \begin{bmatrix} \Delta k_{c22}(1 + \cos 2\theta) & \Delta k_{c22} \sin 2\theta & 0 & 0 \\ \Delta k_{c22} \sin 2\theta & \Delta k_{c22}(1 - \cos 2\theta) & 0 & 0 \\ 0 & 0 & \Delta k_{c44}(1 + \cos 2\theta) & \Delta k_{c44} \sin 2\theta \\ 0 & 0 & \Delta k_{c44} \sin 2\theta & \Delta k_{c44}(1 - \cos 2\theta) \end{bmatrix} \quad (4.8)$$

Introducing the steering function of crack $\sigma(t)$, which shows periodic changes of sudden opening and closure behavior of crack faces (without partial opening or closure), the additive stiffness of crack becomes

$$\Delta \mathbf{K}(t) = \sigma(t) \Delta \bar{\mathbf{K}}(t) \quad (4.9)$$

The crack force vector in a generalized case of the static deflection due to the self-weight of the rotor system without the residual shaft bow effect is represented as,

$$\mathbf{f}_{cr} = \Delta \mathbf{K}(t) \mathbf{q}_0 = \mathbf{f}_{crack} = \frac{1}{2} \sigma(t) \begin{Bmatrix} \Delta k_{c22} (1 + \cos 2\theta) u_{x_0} + \Delta k_{c22} \sin(2\theta) u_{y_0} \\ \Delta k_{c22} \sin(2\theta) u_{x_0} + \Delta k_{c22} (1 - \cos 2\theta) u_{y_0} \\ \Delta k_{c44} (1 + \cos 2\theta) \varphi_{y_0} + \Delta k_{c44} \sin(2\theta) \varphi_{x_0} \\ \Delta k_{c44} \sin(2\theta) \varphi_{y_0} + \Delta k_{c44} (1 - \cos 2\theta) \varphi_{x_0} \end{Bmatrix} \quad (4.10)$$

With weight-dominance assumption, in Equation (4.10) the crack forces and moments are represented as the product of translational and rotational additive stiffness parameter, an appropriate steering function, and the static deflection in both transverse and rotational directions, respectively.

Residual Bow Force Model

The shaft bow (which is rotating) force is independent of speed. It originates different amplitude and phase relationship in contrast with the mass unbalance, which is a function of the square of the speed (Shinou and Lees, 2000; Tiwari, 2017). As mentioned in Chapter 2, the effect of the residual shaft bow on the stiffness variation of the shaft due to the presence of transverse fatigue crack imposes a multiplicative form of fault parameters

The residual shaft bow force in rotating co-ordinate system is represented as

$$\mathbf{f}_{bow} = \mathbf{K}_{0,\xi\eta} \mathbf{e}_{b,\xi\eta} = \mathbf{K}_{0,\xi\eta} \begin{Bmatrix} e_b \cos \phi_b \\ e_b \sin \phi_b \\ 0 \\ 0 \end{Bmatrix} \quad (4.11)$$

Equation (4.11) after transformation from rotating to inertial co-ordinate system becomes

$$\mathbf{f}_{bow} = \mathbf{T}(\mathbf{K}_{0,\xi\eta} \mathbf{e}_{b,\xi\eta}) \quad (4.12)$$

Expanding Equation (4.12) residual bow force becomes

$$\mathbf{f}_{bow} = \begin{bmatrix} k_{22} & 0 & k_{24} & 0 \\ 0 & k_{22} & 0 & k_{24} \\ k_{24} & 0 & k_{44} & 0 \\ 0 & k_{24} & 0 & k_{44} \end{bmatrix} \begin{Bmatrix} e_b \cos(\theta + \phi_b) \\ e_b \sin(\theta + \phi_b) \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} e_b k_{22} \cos(\theta + \phi_b) \\ e_b k_{22} \sin(\theta + \phi_b) \\ e_b k_{24} \cos(\theta + \phi_b) \\ e_b k_{24} \sin(\theta + \phi_b) \end{Bmatrix} \quad (4.13)$$

The residual bow force in Equation (4.13) will be used in the subsequent analysis.

Magnetic Force Model

The magnetic force is dependent on the air gap between the stator and the rotor, and the coil current circulation in the actuator. The magnetic force near the operating point for isotropic AMB, as discussed in Chapter 2, for 4 DOF rotor-bearing system, it is written as

$$\mathbf{f}_{amb} = \begin{Bmatrix} k_s u_x + k_l i_{cx} \\ k_s u_y + k_l i_{cy} \\ 0 \\ 0 \end{Bmatrix} \quad (4.14)$$

The magnetic force is acting only in the vertical and horizontal direction of the rotor to suppress the transverse vibration. The control current i_{cx} and i_{cy} in Equation (4.14) are obtained from the PID control strategy application.

Static and Dynamic Displacements of the Warped Rotor

In the static condition of the rotor system (when the rotating and stationary axis systems are coincident as shown in Figure 4.3(a)), the self-weight of the system can be represented as

$$\begin{bmatrix} k_{22} & 0 & k_{24} & 0 \\ 0 & k_{22} & 0 & k_{24} \\ k_{24} & 0 & k_{44} & 0 \\ 0 & k_{24} & 0 & k_{44} \end{bmatrix} \begin{Bmatrix} u_{x_0} \\ u_{y_0} \\ \varphi_{y_0} \\ \varphi_{x_0} \end{Bmatrix} = \begin{Bmatrix} mg \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (4.15)$$

$$\text{Therefore, } u_{x_0} = \frac{mgk_{44}}{(k_{22}k_{44} - k_{24}^2)}, u_{y_0} = \varphi_{y_0} = \frac{mgk_{24}}{(k_{24}^2 - k_{22}k_{44})}, \varphi_{x_0} = 0 \quad (4.16)$$

The dynamic response due to the presence of only residual shaft bow is represented as (refer Figure 4.3(b))

$$\begin{bmatrix} k_{22} & 0 & k_{24} & 0 \\ 0 & k_{22} & 0 & k_{24} \\ k_{24} & 0 & k_{44} & 0 \\ 0 & k_{24} & 0 & k_{44} \end{bmatrix} \begin{Bmatrix} u_{x_b}(t) \\ u_{y_b}(t) \\ \varphi_{y_b}(t) \\ \varphi_{x_b}(t) \end{Bmatrix} = \begin{bmatrix} k_{22} & 0 & k_{24} & 0 \\ 0 & k_{22} & 0 & k_{24} \\ k_{24} & 0 & k_{44} & 0 \\ 0 & k_{24} & 0 & k_{44} \end{bmatrix} \begin{Bmatrix} e_b \cos(\theta + \phi_b) \\ e_b \sin(\theta + \phi_b) \\ 0 \\ 0 \end{Bmatrix} \quad (4.17)$$

$$\text{Therefore, } u_{x_b} = e_b \cos(\theta + \phi_b), u_{y_b} = e_b \sin(\theta + \phi_b), \varphi_{y_b} = 0, \varphi_{x_b} = 0 \quad (4.18)$$

The complete generalized EOMs for four-DOFs rotor system containing the external and internal damping as well as the crack, residual unbalance and residual bow and magnetic force are described as,

$$\mathbf{M}\ddot{\mathbf{q}}_v + (\mathbf{C} - \omega \mathbf{G})\dot{\mathbf{q}}_v + \mathbf{K}\mathbf{q}_v = \mathbf{f}_{cr} + \mathbf{f}_{unb} + \mathbf{f}_{bow} - \mathbf{f}_{AMB} - \omega \mathbf{C}_H \mathbf{q}_0 \quad (4.19)$$

As in the present work, AMB is incorporated in the rotor as a controller, not as an exciter, the magnetic bearing forcing is put with as negative sign in the right side of Equation (4.19) to achieve attenuation of vibration of the system experiencing from the crack, unbalance, bow and internal damping faults.

$$\mathbf{K} = \mathbf{K}_0 + \mathbf{K}_c = \begin{bmatrix} k_{22} & 0 & k_{24} & 0 \\ 0 & k_{22} & 0 & k_{24} \\ k_{24} & 0 & k_{44} & 0 \\ 0 & k_{24} & 0 & k_{44} \end{bmatrix} + \begin{bmatrix} 0 & \omega c_H & 0 & 0 \\ -\omega c_H & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.20)$$

All other matrices in Equation (4.19) are defined in the previous sections.

The crack force after introduction of the shaft residual bow into the mathematical model is represented as

$$\mathbf{f}_{crack} = \frac{1}{2} \sigma(t) \begin{Bmatrix} \Delta k_{22} (1 + \cos 2\theta) (u_{x_0} + u_{x_b}) + \Delta k_{22} \sin(2\theta) (u_{y_0} + u_{y_b}) \\ \Delta k_{22} \sin(2\theta) (u_{x_0} + u_{x_b}) + \Delta k_{22} (1 - \cos 2\theta) (u_{y_0} + u_{y_b}) \\ \Delta k_{44} (1 + \cos 2\theta) (\varphi_{y_0} + \varphi_{y_b}) + \Delta k_{44} \sin(2\theta) (\varphi_{x_0} + \varphi_{x_b}) \\ \Delta k_{44} \sin(2\theta) (\varphi_{y_0} + \varphi_{y_b}) + \Delta k_{44} (1 - \cos 2\theta) (\varphi_{x_0} + \varphi_{x_b}) \end{Bmatrix}; \quad \mathbf{q}_v = \begin{Bmatrix} u_x(t) \\ u_y(t) \\ \varphi_y(t) \\ \varphi_x(t) \end{Bmatrix}; \quad \mathbf{q}_b = \begin{Bmatrix} u_{x_b}(t) \\ u_{y_b}(t) \\ \varphi_{y_b}(t) \\ \varphi_{x_b}(t) \end{Bmatrix}$$

The stiffness matrix $\mathbf{K}$ considered is the combination of the shaft stiffness matrix and the circulatory matrix ($\mathbf{K}_c$) due to the presence of internal (c_H) damping. The vibrational displacement is represented by vector $\mathbf{q}_v$ and $\mathbf{q}_b$ dynamic response vector due to shaft bow. The crack, unbalance, bow force, and magnetic force excitations applied to the rotor system are represented by $\mathbf{f}_{cr}$, $\mathbf{f}_{bow}$, $\mathbf{f}_{unb}$, and $\mathbf{f}_{AMB}$, respectively. Now defining the complex form of variables as, $s = u_x + ju_y$; $\phi = \phi_y + j\phi_x$ and $i_c = i_{cx} + ji_{cy}$. On putting into Equation (4.19), we get

$$\bar{\mathbf{M}}\ddot{\mathbf{s}} + (\bar{\mathbf{C}} - \omega\bar{\mathbf{G}})\dot{\mathbf{s}} + \bar{\mathbf{K}}\mathbf{s} = \frac{1}{2}\sigma(t) \left\{ \begin{array}{c} \Delta k_{c22}(1 + e^{2j\theta})u_{x_0} + \Delta k_{c22}e_b \cos\phi_b e^{j\theta} \\ \Delta k_{c44}(1 + e^{2j\theta})\phi_{y_0} \\ + \left\{ \begin{array}{c} me\omega^2 e^{j(\theta+\phi_u)} \\ 0 \end{array} \right\} + \left\{ \begin{array}{c} e_b k_{22} \\ e_b k_{24} \end{array} \right\} e^{j(\theta+\phi_b)} + \omega c_H \left\{ \begin{array}{c} u_{x_0} + ju_{y_0} \\ 0 \end{array} \right\} - k_i \left\{ \begin{array}{c} i_c \\ 0 \end{array} \right\} \end{array} \right\} \quad (4.21)$$

where

$$\begin{aligned} \tilde{\mathbf{M}} &= \begin{bmatrix} m & 0 \\ 0 & I_d \end{bmatrix}; \tilde{\mathbf{C}} = \begin{bmatrix} c_{22} & c_{24} \\ c_{42} & c_{44} \end{bmatrix} + \begin{bmatrix} c_H & 0 \\ 0 & 0 \end{bmatrix}; \tilde{\mathbf{G}} = \begin{bmatrix} 0 & 0 \\ 0 & -I_p \end{bmatrix}; \\ \tilde{\mathbf{K}} &= \begin{bmatrix} (k_{22} - k_s) & k_{24} \\ k_{42} & k_{44} \end{bmatrix} + j\omega \begin{bmatrix} c_H & 0 \\ 0 & 0 \end{bmatrix} \end{aligned} \quad (4.22)$$

The complex form of forcing terms in Equation (4.21) is expressed in a combination of harmonic (i^{th}) terms as

$$\begin{aligned} \mathbf{f}_{cr} &= \frac{1}{2}\sigma(t) \left\{ \begin{array}{c} \Delta k_{c22}(1 + e^{2j\theta})u_{x_0} + \Delta k_{c22}e_b \cos\phi_b e^{j\theta} \\ \Delta k_{c44}(1 + e^{2j\theta})\phi_{y_0} \end{array} \right\} = \left\{ \begin{array}{c} \overbrace{\Delta k_{c22}u_{x_0} \sum_{i=-n}^n p_i + \Delta k_{c22}e_b \cos\phi_b \sum_{i=-n}^n q_i}^{\text{multiplicative crack-bow fault}} \\ \Delta k_{c44}\phi_{y_0} \sum_{i=-n}^n p_i \end{array} \right\} e^{ji\theta}; \\ \mathbf{f}_{unb} &= \left\{ \begin{array}{c} me\omega^2 e^{j(\theta+\phi_u)} \\ 0 \end{array} \right\}; \mathbf{f}_{bow} = \left\{ \begin{array}{c} e_b k_{22} \\ e_b k_{24} \end{array} \right\} e^{j(\theta+\phi_b)} = \left\{ \begin{array}{c} e_b k_{22} \\ e_b k_{24} \end{array} \right\} e^{j\phi_b} e^{j\theta}; \mathbf{f}_{amb} = k_i \left\{ \begin{array}{c} i_c \\ 0 \end{array} \right\} e^{ji\omega t} \end{aligned} \quad (4.23)$$

The p_i 's and q_i 's are the participation factors of the steering function of the crack faces opening and closure behaviour for the i^{th} harmonic of the crack excitation force and bow force, respectively are defined in Chapter 2. The response vector of Equation (4.21) is assumed containing the same harmonics as forcing terms as

$$\mathbf{s}(t) = \sum_{i=-\infty}^{\infty} \tilde{\mathbf{s}} e^{ji\theta} \quad (4.24)$$

where $\tilde{\mathbf{s}}$ are the response amplitude vector and their elements are complex quantities. On substituting Equation (4.24) into Equation (4.21), the governing equation will be converted into the frequency domain, as

$$\left((i\omega)^2 \bar{\mathbf{M}} + (j\omega) \bar{\mathbf{C}} + (i\omega^2) \omega \bar{\mathbf{G}} + \bar{\mathbf{K}} \right) \tilde{\mathbf{s}} = \left\{ \begin{array}{c} \Delta k_{c22} u_{x_0} \sum_{i=-n}^n p_i + \Delta k_{c22} e_b \cos \phi_b \sum_{i=-n}^n q_i \\ \Delta k_{44} \varphi_{y_0} \sum_{i=-n}^n p_i \\ + \left\{ \begin{array}{c} m e \omega^2 e^{j\phi_a} \\ 0 \end{array} \right\} + \left\{ \begin{array}{c} e_b k_{22} \\ e_b k_{24} \end{array} \right\} e^{j\phi_b} k_i \left\{ \begin{array}{c} i_c \\ 0 \end{array} \right\} \end{array} \right\} \quad (4.25)$$

The frequency domain equation (4.25) will be used to generate responses for the excitation forces experienced by the rotor to study the influence of additive as well as multiplicative fault parameters. The full spectrum amplitudes and phases of displacement and control current for different harmonics from Equation (4.25) will be used to develop the identification algorithm for estimation of various fault parameters, which will be discussed in the next section.

4.3 Development of Identification Algorithm

In the general case, the direct problem approach analyzes the presence of crack influence on the natural frequency and mode shape by changing the characteristics equation of the

rotor model. In contrast to the direct problem, in the inverse problem the measured vibration data from the system is used in a model to identify the unknown system parameters (Tiwari, 2017). In the present methodology, some of system parameters such as mass of the disc and stiffness of the shaft are considered to be known and the other additive crack stiffness, unbalance, and bow are to be identified. The frequency domain Equation (4.25) is used for the development of the regression equation to estimate fault parameters. The proposed identification algorithms use the full spectrum amplitudes and phases of displacements and currents generated from the cracked rotor equipped with an AMB system to capture the forward and backward whirl information to have more realistic identification. In the four-DOF rotor system, two translational and two rotational displacements are there as generalized coordinates. To overcome the limitation of practical measurement of rotational displacements, reducing the number of DOFs in the analytical model from the limited measured DOFs in comparison to the experimental data with the numerical one during limited information the dynamic condensation scheme is preferred (Chen and Pan, 1988; Paz, 1984; Prasad and Tiwari, 2019), which will be discussed in the next subsection section.

4.3.1 Implementation of Gyroscopic Dynamic Condensation (GDC) Scheme

The condensation scheme is used in mathematical modeling of the rotor itself, where the rotational DOFs are represented in terms of translational ones to avoid the measurement difficulty, and the concept of condensation scheme comes into the picture to help this out. Moreover, the tilting motion of the disc during rotation creates gyroscopic effects and it is very important to bring that effect into condensation scheme. In the present study, the dynamic condensation scheme has been used with the inclusion of the gyroscopic effect to eliminate the

rotational degrees of freedom (Prasad and Tiwari, 2019). This condensation methodology has been used to condense the rotational (degrees of freedom) DOFs, whereas retaining the rotational additive stiffness while developing identification algorithms. The degrees of freedom eliminated in this process are called slaves and those retained for the analysis are called masters. For this, the frequency domain Equation (4.25) can be segregated as the master and slave degrees of freedom first, as

$$\left\{ -(i\omega)^2 \begin{bmatrix} \mathbf{M}_{mm} & 0 \\ 0 & \mathbf{M}_{ss} \end{bmatrix} + j(i\omega) \left(\begin{bmatrix} \mathbf{C}_{mm} & \mathbf{C}_{ms} \\ \mathbf{C}_{sm} & \mathbf{C}_{ss} \end{bmatrix} - \omega \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ 0 & \mathbf{G}_{ss} \end{bmatrix} \right) + \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ \mathbf{K}_{sm} & \mathbf{K}_{ss} \end{bmatrix} \right\} \begin{Bmatrix} \mathbf{p}_m \\ \mathbf{p}_s \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_m \\ \mathbf{f}_s \end{Bmatrix} \quad (4.26)$$

$$\left. \begin{aligned} \mathbf{M}_{mm} &= \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}; \mathbf{M}_{ss} = \begin{bmatrix} I_d & 0 \\ 0 & I_d \end{bmatrix}; \mathbf{C}_{mm} = \begin{bmatrix} (c_{22} + c_H) & 0 \\ 0 & (c_{22} + c_H) \end{bmatrix}; \mathbf{C}_{ss} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; \\ \mathbf{C}_{ms} = \mathbf{C}_{sm} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}; \mathbf{G}_{ss} = \begin{bmatrix} 0 & I_p \\ -I_p & 0 \end{bmatrix}; \mathbf{K}_{mm} = \begin{bmatrix} (k_{22} - k_s) & 0 \\ 0 & (k_{22} - k_s) \end{bmatrix}; \\ \mathbf{K}_{ms} = \mathbf{K}_{sm} &= \begin{bmatrix} k_{24} & 0 \\ 0 & k_{42} \end{bmatrix}; \mathbf{K}_{ss} = \begin{bmatrix} k_{44} & 0 \\ 0 & k_{44} \end{bmatrix}; \mathbf{p}_m = \begin{Bmatrix} S_{i,\text{Re}} \\ S_{i,\text{Im}} \end{Bmatrix}; \mathbf{p}_s = \begin{Bmatrix} \Phi_{i,\text{Re}} \\ \Phi_{i,\text{Im}} \end{Bmatrix} \end{aligned} \right\} \quad (4.27)$$

If the slave DOFs are not experiencing any forces and moments, (initial tilt of the disc is also considered for which appreciable moments occurs), only ignoring the damping and external force terms in Equation (4.26), we get

$$\mathbf{K}_{sm} \mathbf{p}_m + \mathbf{K}_{ss} \mathbf{p}_s + j\omega^2 \mathbf{G}_{ss} \mathbf{p}_s - (i\omega)^2 \mathbf{M}_{ss} \mathbf{p}_s = \mathbf{0} \quad (4.28)$$

$$\text{or, } \mathbf{K}_{sm} \mathbf{p}_m = -\left\{ \mathbf{K}_{ss} + j\omega^2 \mathbf{G}_{ss} - (i\omega)^2 \mathbf{M}_{ss} \right\} \mathbf{p}_s \quad (4.29)$$

$$\text{and} \quad \mathbf{p}_m = \mathbf{I} \mathbf{p}_m \quad (4.30)$$

Introducing Equation (4.29) and Equation (4.30) in the matrix form, as

$$\begin{Bmatrix} \mathbf{p}_m \\ \mathbf{p}_s \end{Bmatrix} = \mathbf{T}^{gd} \mathbf{p}_m \quad (4.31)$$

with
$$\mathbf{T}^{gd} = \begin{bmatrix} \mathbf{I} \\ (\mathbf{K}_{ss} + i\omega^2 \mathbf{G}_{ss} - (i\omega)^2 \mathbf{M}_{ss})^{-1} \mathbf{K}_{sm} \end{bmatrix} \quad (4.32)$$

Here, $\mathbf{T}_i^{gd}$ is the transformation matrix on using the dynamic condensation to remove the rotational DOFs. It should be noted that the known gyroscopic matrix effect is also present in it, which is often ignored along with the unknown damping matrix. In the explicit form, the transformation matrix can be written as

$$\mathbf{T}_i^{gd} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ t_i^d & 0 \\ 0 & t_i^d \end{bmatrix} \text{ with } t_i^d = \left(-\frac{k_{24}}{k_{44} + i\omega^2 I_p - (i\omega)^2 I_d} \right) \quad (4.33)$$

On pre-multiplying by the transpose of $\mathbf{T}_i^d$ and post-multiplying by $\mathbf{T}_i^{gd}$ in Equation (4.26), we get

$$\left[-[\mathbf{T}_i^{gd}]^T (i\omega)^2 \begin{bmatrix} \mathbf{M}_{mm} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{ss} \end{bmatrix} [\mathbf{T}_i^{gd}] + j(i\omega) [\mathbf{T}_i^{gd}]^T \left(\begin{bmatrix} \mathbf{C}_{mm} & \mathbf{C}_{ms} \\ \mathbf{C}_{sm} & \mathbf{C}_{ss} \end{bmatrix} - \omega \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{ss} \end{bmatrix} \right) [\mathbf{T}_i^{gd}] + [\mathbf{T}_i^{gd}]^T \begin{bmatrix} \mathbf{K}_{mm} & \mathbf{K}_{ms} \\ \mathbf{K}_{sm} & \mathbf{K}_{ss} \end{bmatrix} [\mathbf{T}_i^{gd}] \right] \begin{Bmatrix} \mathbf{p}_m \\ \mathbf{p}_s \end{Bmatrix} \right] = [\mathbf{T}_i^{gd}]^T \begin{Bmatrix} \mathbf{f}_m \\ \mathbf{f}_s \end{Bmatrix} \quad (4.34)$$

The reduced matrices of the mass $\mathbf{M}^{gd}$, gyroscope $\mathbf{G}^{gd}$, damping $\mathbf{C}^{gd}$, and stiffness $\mathbf{K}^{gd}$ and total external force vector $\mathbf{f}^{\text{total}}$ after elimination of slave degrees of freedom become,

$$\mathbf{M}^d = [\mathbf{T}_i^{gd}]^T \begin{bmatrix} \mathbf{M}_{mm} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{ss} \end{bmatrix} [\mathbf{T}_i^{gd}] = \begin{bmatrix} m + (t_i^d)^2 I_d & 0 \\ 0 & m + (t_i^d)^2 I_d \end{bmatrix};$$

$$\mathbf{G}^d = [\mathbf{T}_i^{gd}]^T \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{ss} \end{bmatrix} [\mathbf{T}_i^{gd}] = \begin{bmatrix} 0 & I_p (t_i^d)^2 \\ -I_p (t_i^d)^2 & 0 \end{bmatrix};$$

$$\mathbf{C}^d = [\mathbf{T}_i^{gd}]^T \begin{bmatrix} \mathbf{c}_{mm} & \mathbf{c}_{ms} \\ \mathbf{c}_{sm} & \mathbf{c}_{ss} \end{bmatrix} [\mathbf{T}_i^{gd}] = \begin{bmatrix} c_{22} + c_H & 0 \\ 0 & c_{22} + c_H \end{bmatrix};$$

$$\mathbf{K}^d = [\mathbf{T}_i^{gd}]^T \begin{bmatrix} \mathbf{k}_{mm} & \mathbf{k}_{ms} \\ \mathbf{k}_{sm} & \mathbf{k}_{ss} \end{bmatrix} [\mathbf{T}_i^{gd}] = \begin{bmatrix} (k_{22} - k_s) + 2t_i^d k_{24} + (t_i^d)^2 k_{44} & \omega c_H \\ -\omega c_H & (k_{22} - k_s) + 2t_i^d k_{24} + (t_i^d)^2 k_{44} \end{bmatrix}$$

$$\mathbf{f}^{\text{total}} = [\mathbf{T}_i^{gd}]^T \begin{Bmatrix} \mathbf{f}_m \\ \mathbf{f}_s \end{Bmatrix} = \begin{Bmatrix} me\omega^2 \cos \phi_u \\ me\omega^2 \sin \phi_u \end{Bmatrix} + \omega \begin{Bmatrix} 0 \\ c_H q_{w_1} \end{Bmatrix} + \begin{Bmatrix} e_b k_{22} \cos(\theta + \phi_b) + t_i^d e_b k_{24} \cos(\theta + \phi_b) \\ e_b k_{22} \sin(\theta + \phi_b) + t_i^d e_b k_{24} \sin(\theta + \phi_b) \end{Bmatrix}$$

$$+ \begin{Bmatrix} -k_i i_{cx} \\ -k_i i_{cy} \end{Bmatrix} + \frac{1}{2} \sigma(t) \begin{Bmatrix} \Delta k_{c22} q_{w_1} (1 + \cos 2\theta) + \Delta k_{c22} e_b \sin \phi_b \sin 2\theta + t_i^d \Delta k_{c44} q_{w_2} (1 + \cos 2\theta) \\ \Delta k_{c22} q_{w_1} \sin 2\theta + \Delta k_{c22} e_b \sin \phi_b (1 - \cos 2\theta) + t_i^d \Delta k_{c44} q_{w_2} \sin 2\theta \end{Bmatrix}$$

with $q_{w_1} = \frac{mgk_{44}}{(k_{24}^2 - k_{22}k_{44})}$ and, $q_{w_2} = \frac{mgk_{24}}{(k_{24}^2 - k_{22}k_{44})}$

The frequency domain equation obtained, after the dynamic condensation scheme, has been used to develop the identification algorithm to estimate the fault parameters. The complex form of the frequency domain condensed EOM of Equation (4.34) is segregated to have unknown parameters and the known terms to be brought towards left of equality sign and the right side, respectively. The unknown terms to be estimated are the external and internal damping, both translational and rotational additive stiffness, unbalance and phase, residual shaft bow and its phase, AMB parameters, and other known terms, such as the mass, stiffness and gyroscopic one. The segregated EOM is represented as

$$\left. \begin{aligned} & j(i\omega)c_{22}S_i + j\omega\{(1-i)S_i + q_{w_1}\}c_H - me\omega^2 e^{j\phi_u} - \Delta k_{c22} p_i q_{w_1} - t_i^d \Delta k_{c44} p_i q_{w_2} \\ & - \Delta k_{c22} e_b q_i \cos \phi_b - (k_{22} + t_i^d k_{24}) e_b e^{j\phi_b} - k_s S_i + k_i i_{c_i} \\ & = \left[(i\omega)^2 \left\{ m + (t_i^d)^2 I_d \right\} + i\omega^2 I_p (t_i^d)^2 - \left\{ k_{22} + 2t_i^d k_{24} + (t_i^d)^2 k_{44} \right\} \right] S_i \end{aligned} \right\} \quad (4.35)$$

Here, S_i is the complex frequency displacement and i_{c_i} is the complex AMB control current for multiharmonic (i^{th}) signal of the proposed rotor model.

For $i = 0$; complex terms of Equation (4.35) can be represented as,

$$\left. \begin{aligned} & j\omega c_H S_0 + j\omega c_H S_0 + j\omega c_H q_{w_1} - \Delta k_{c22} p_0 q_{w_1} - t_0^d \Delta k_{c44} p_0 q_{w_2} - \Delta k_{c22} e_b q_0 \cos \phi_b \\ & - k_s S_0 + k_i i_{c_0} = \left[- \left\{ k_{22} + 2t_0^d k_{24} + (t_0^d)^2 k_{44} \right\} S_0 \right] \end{aligned} \right\} \quad (4.36)$$

For $i=1$; complex terms of Equation (4.35) can be represented as,

$$\left. \begin{aligned} & j\omega c_{22} S_1 - m e \omega^2 e^{j\phi_u} - \Delta k_{c22} p_1 q_{w_1} - t_1^d \Delta k_{c44} p_1 q_{w_2} - \Delta k_{c22} e_b q_1 \cos \phi_b - (k_{22} + t_1^d k_{24}) e_b e^{j\phi_b} \\ & - k_s S_1 + k_i i_{c_1} = \left[\omega^2 \left\{ m + (t_1^d)^2 I_d \right\} + \omega^2 I_p (t_1^d)^2 - \left\{ k_{22} + 2t_1^d k_{24} + (t_1^d)^2 k_{44} \right\} S_1 \right] \end{aligned} \right\} \quad (4.37)$$

The linear least-squares regression is a primary tool for finding out the best fit data between the data and the system model. The regression form of estimation equation for the general multi-harmonic ($i = -n, \dots, -1, 0, 1, \dots, n$) signal consisting of forward and backward whirl components, and zeroth component that represents the DC point due to the static weight of the system is expressed as

$$\mathbf{A}_{1(2n+1) \times r} \mathbf{x}_{1(r \times 1)} = \mathbf{b}_{1(1 \times (2n+1) \times r)} \quad (4.38)$$

The identifiables of the least-squares estimator are expressed, in the matrix form, as

$$\text{Without residual bow case, } \mathbf{x}_1 = [c_{22} \quad c_H \quad \Delta k_{c22} \quad \Delta k_{c44} \quad e_{Re} \quad e_{Im} \quad k_s \quad k_i]^T \quad (4.39)$$

With residual bow case,

$$\mathbf{x}_1 = [c_{22} \quad c_H \quad \Delta k_{c22} \quad \Delta k_{c44} \quad \Delta k_{c22} e_{b,Re} \quad e_{Re} \quad e_{Im} \quad e_{b,Re} \quad e_{b,Im} \quad k_s \quad k_i]^T \quad (4.40)$$

The $\mathbf{A}_1$ matrix, known as the design matrix, is a rectangular matrix, which often suffers from ill conditioning during its pseudo-inverse process. Often, it is convenient to use a linear least-squares prediction technique to estimate the parameters. The estimation of the unknown parameters obtained from linear least-squares regression is the optimal estimates from a wide-range of possible parameter estimates under the usual assumptions used for the system modeling. In the present case, the $\mathbf{A}_1$ matrix has the number of equations larger

than the number of unknowns to be estimated $\{(2n+1) > r\}$, where n is the number of harmonics to be considered and r is the fault parameters to be estimated, thus the system is overdetermined and in general have no exact solution. Hence, the least-squares approach aims to minimize the residual vector to find out the estimation. The regression matrix and the regressor matrix obtained from Equation (4.38) are given in Appendix C.

Applying pseudo inverse technique to Equation (4.38) has the solution as

$$\mathbf{x}_1 = (\mathbf{A}_1^T \mathbf{A}_1)^{-1} \mathbf{A}_1^T \mathbf{b}_1 \quad (4.41)$$

For the consistent estimation of system parameters as much information as possible should be available about the rotor system. It is possible to get responses at multiple speeds from the present rotor model for the robust estimation of fault parameters as discussed elaborately in Chapter 2 (Section 2.2.3) are implemented in the aforesaid estimation process.

4.4 Numerical Illustration

The responses generated based on the EOM of Equation (4.19) of the proposed rotor model are displacement and current. The current is used to drive the coils consisting of constant current added to the control (varying) current with respect to the rotation of the system. The simulation of the rotor model has been done on Matlab SIMULINK™ environment. The numerical model of the system is described by SIMULINK™ block as shown in Figure 4.3 that provides the rotor displacement and the AMB control current at two orthogonal directions as well as the reference signal for the phase measurement. The numerical simulation is done using the rotor-AMB model parameters as summarized in Table 4.1.

Though the switching function has an infinite number of terms theoretically, for simulation purpose in subsequent analysis very few predominant harmonics will be retained.

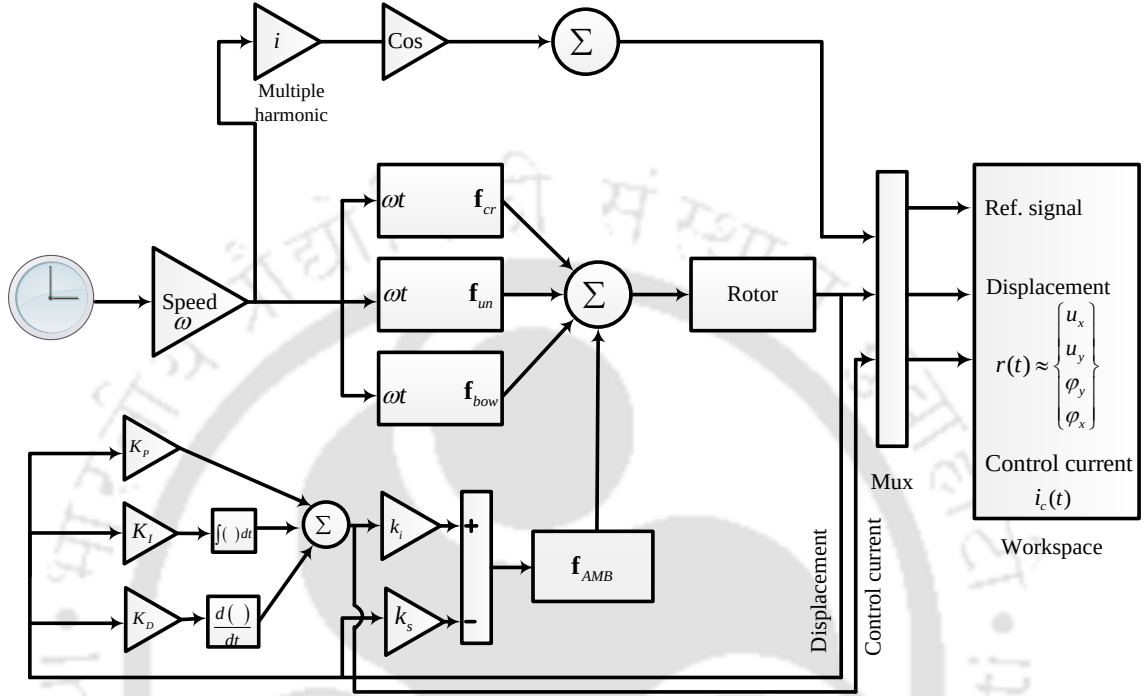


Figure 4.3 Proposed rotor model SIMULINK™ block

The presence of gyroscopic effect due to the tilting of the disc during rotation splits the natural frequency in both the forward and backward whirling directions of the rotor system. An example of the forward and backward whirling of the present rotor system (without and with AMB incorporation into the rotor model) is reported in the Campbell diagram as shown in Figure 4.4(a, b). The first natural frequency is split into two flexural natural frequencies and gives two critical speeds in the rotor system, close to 510 rad/s (81.17 Hz) and 447 rad/s (71.14 Hz) for the forward and backward whirling, respectively, for the rotor with no AMB case. Similarly, rotor with AMB case, the flexural natural frequencies are

564 rad/s (89.76 Hz) and 518 rad/s (82.44 Hz) for forward and backward whirling, respectively.

Table 4.1 Physical properties of the proposed rotor system with actuator values and PID controller gains

Rotor Properties	Assumed values
Disc mass, m	2 kg
Shaft length, l	0.50 m
Young's modulus, E	$2.06 \times 10^{11} \text{ Nm}^{-2}$
Diametral mass moment of inertia, I_d	0.0048 kgm^2
Polar mass moment of inertia, I_P	0.0024 kgm^2
External damping, c_{22}	140 Nsm^{-1}
Internal damping, c_H	125 Nsm^{-1}
Offset location of disc on the shaft, a	0.375 m
and, b	0.125 m
Shaft stiffness using influence co-efficient, k_{22}	$8.3127 \times 10^5 \text{ Nm}^{-1}$
k_{24}	$8.9064 \times 10^4 \text{ N(m-rad)}^{-1}$
k_{44}	$16.7 \times 10^4 \text{ N rad}^{-1}$
Translational additive crack stiffness, Δk_{c22}	$2.05 \times 10^5 \text{ Nm}^{-1}$
Translational additive crack stiffness, Δk_{c44}	$1670 \text{ N(m-rad)}^{-1}$
Disc eccentricity, e	$2 \times 10^{-4} \text{ m}$
Disc eccentricity phase, ϕ_u	30°
Shaft translational bow, e_b	$5 \times 10^{-4} \text{ m}$
Translational bow phase, ϕ_b	60°
Force -displacement factor, k_s	217683 Nm^{-1}
Force-current factor, k_i	43.537 NA^{-1}
Proportional gain, K_p	$8,000 \text{ Am}^{-1}$
Integral gain, K_I	$3,000 \text{ A(m s)}^{-1}$
Derivative gain, K_D	3 Asm^{-1}

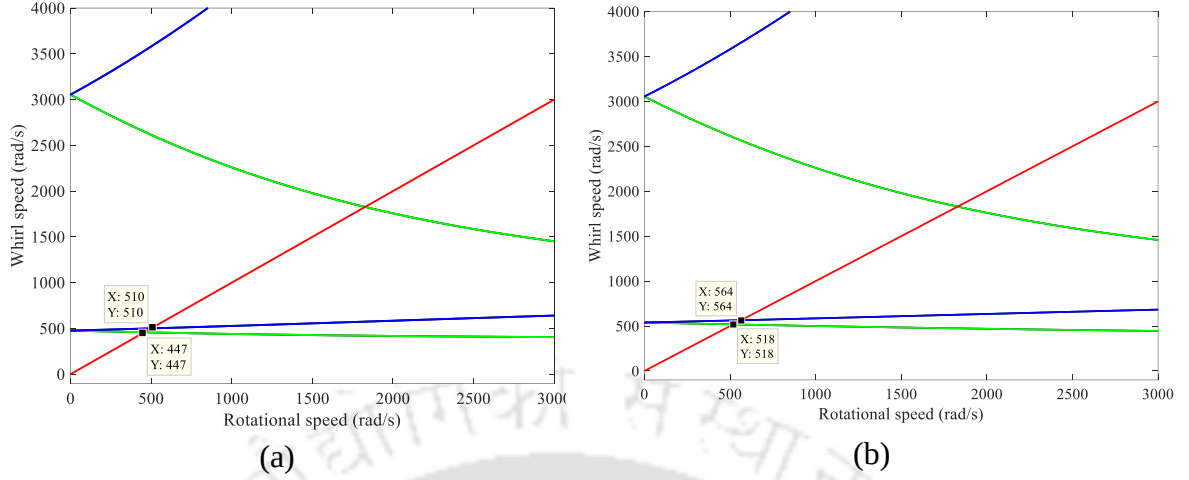


Figure 4.4 Campbell diagram of the proposed rotor system (a) without AMB (b) with AMB

4.4.1 Stability Analysis Rotor System without Crack

The use of AMB often requires arduous control problem to overcome its inherent instability, the stability analysis with the implementation of Nyquist stability criterion is discussed to confirm the stable operation of the rotor-AMB system with the assumed values of the PID controller gains. The closed loop of the magnetic bearing control with rotor system is illustrated in Figure 2.5 in Chapter 2 (Section 2.3.1) schematically.

The modified form of the closed loop (CL) transfer function corresponding to the negative feedback of the rotor-bearing system with gyroscopic effect is represented as (Tiwari, 2017)

$$G_{tf} = \frac{s^2 \left(K_p + \frac{K_I}{s} + K_D s \right) k_i k_s}{(ms^3 + (c_V + c_H - j\omega I_P + K_D k_i k_s k_{sn})s^2 + (k_{22} - j\omega c_H + k_s + K_p k_i k_x k_{sn})s + K_I k_i k_s k_{sn})} \quad (4.42)$$

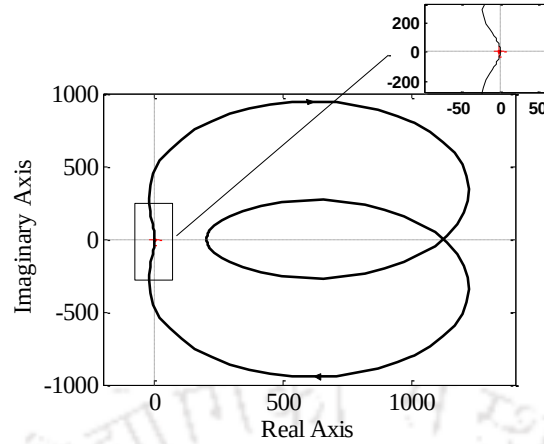


Figure 4.5 Nyquist plot of the rotor system with AMB for the speed range of 25 to 35 Hz

According to the condition of Nyquist stability criterion as discussed in Chapter 2 (Section 2.3.1), The number of zeros of the characteristic equation of Equation (4.42), i.e. the closed loop transfer function poles, are absent in the right-half of the s -plane as depicted in Figure 4.5. Since here, P is zero (for open loop stable system described in Appendix A), the roots of the characteristic equation have negative real parts, using the rotor, actuator and controller data assumed in Table 4.1 and hence do not lie in the right-half of s - plane (RHP). Hence, according to the Nyquist stability criterion the rotor system rotating at the range of spin speed of 25 Hz to 35 Hz for the tabulated PID controller parameters ensures the stability of the proposed rotor system during its operation.

4.4.2 Generation of Time integrated and Full Spectrum Forced Vibration Responses

The simulation is run for 4 s using Runge-Kutta method and last 0.5 s data has been considered for analyzing the response to avoid initial numerical transients. The steady-state responses have been obtained by using Equation (4.19) in the complex form.

The time waveform of the cracked rotor without experiencing residual bow is shown in Figure 4.6. The amplitude of the translational vertical and horizontal displacements are found to be 4.25×10^{-5} m and 4.46×10^{-5} m, respectively, as shown in Figure 4.6(a, b). Similarly, the amplitude of angular displacements is close to the range of 2×10^{-4} rad (0.0115 deg) and 1.2×10^{-4} rad (0.006 deg) as shown in Figure 4.6(c, d), which is very small and from the practical point of view it is difficult to measure. Figure 4.6(e, f) shows the control currents required to attenuate vibration response in the vertical and horizontal directions and are observed to be 0.42 A and 0.4 A, respectively. Cracked rotor displacement and current orbit plots are depicted in Figure 4.6(g, h).

Cracked rotor response beyond instability threshold is analyzed now. A rotor behaves in an unstable manner when the shaft orbit increases with time, without apparent limit. Instability activates when the rotor spin speed exceeds a so-called *threshold* speed. The critical speed of the system without AMB is 476.3 rad/s (75.81 Hz) and with AMB is 539 rad/s (85.78 Hz).

According to the standard formula, we can calculate the threshold speed of the proposed system

$$\omega_{th} \geq \left(1 + \frac{C_H}{C_{22}} \right) \omega_{cr} \quad (4.43)$$

The threshold speed of instability for the proposed rotor system without AMB is 143.49 Hz whereas with AMB is 162.38 Hz. The rotor without AMB operated beyond the threshold speed of instability, such as 148 Hz, is shown in Figure 4.7 (a, e). The amplitude of vibration goes on increasing and the magnitude becomes 0.2 m that is very high amplitude of vibration leading towards instability in the rotor system.

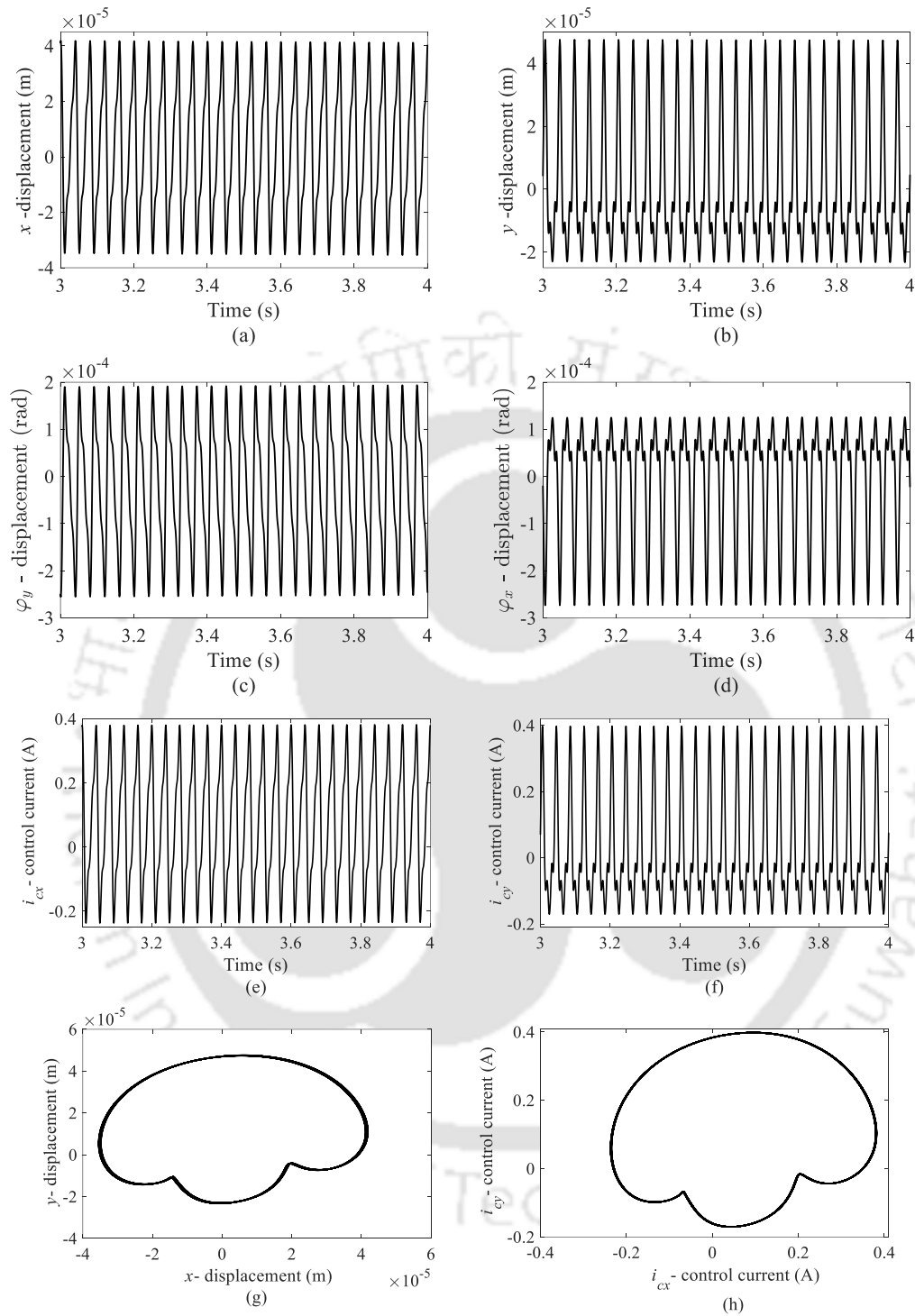


Figure 4.6 Time waveform of cracked rotor without residual bow at the spin speed of 25 Hz (a) Vertical displacement (b) Horizontal displacement (c) Angular displacement about vertical axis (d) Angular displacement about horizontal axis (e) Control current in vertical direction (f) Control current in horizontal direction

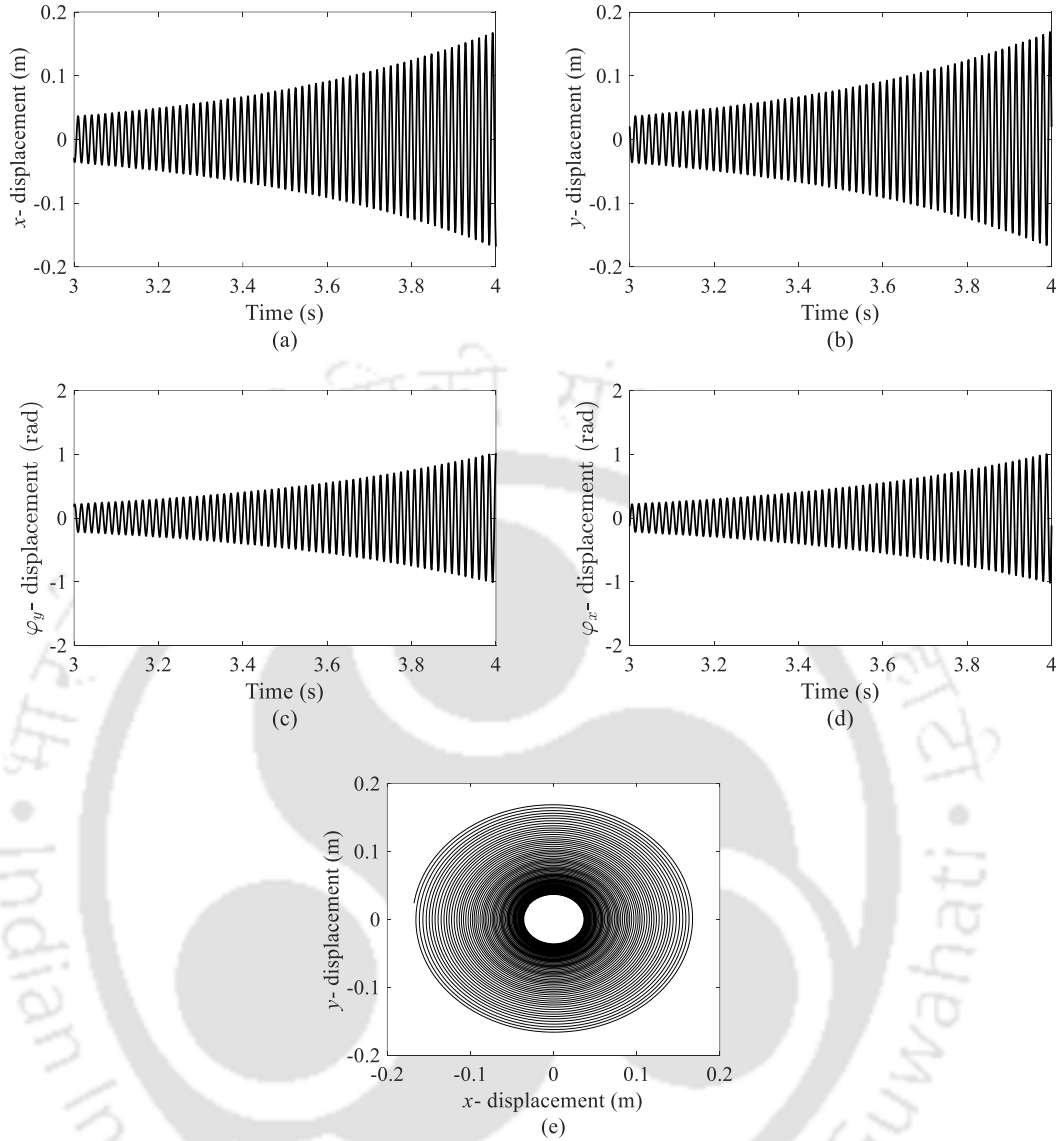


Figure 4.7 Time waveform of cracked rotor without AMB at the spin speed of 148 Hz (a) Vertical displacement (b) Horizontal displacement (c) Angular displacement in (x-z) plane (d) Angular displacement in (y-z) plane (e) Orbit plot of translational displacements

With the incorporation of AMB, the present rotor system operating at the same spin speed mentioned above is found to be rotating in stable condition with reduced amplitude of vibration (1.5×10^{-4} m) response of the rotor with 1.8 A of AMB control current as shown

in Figure 4.8. Figure 4.8(a, h) shows the stable response of the rotor with AMB operating beyond the threshold speed.

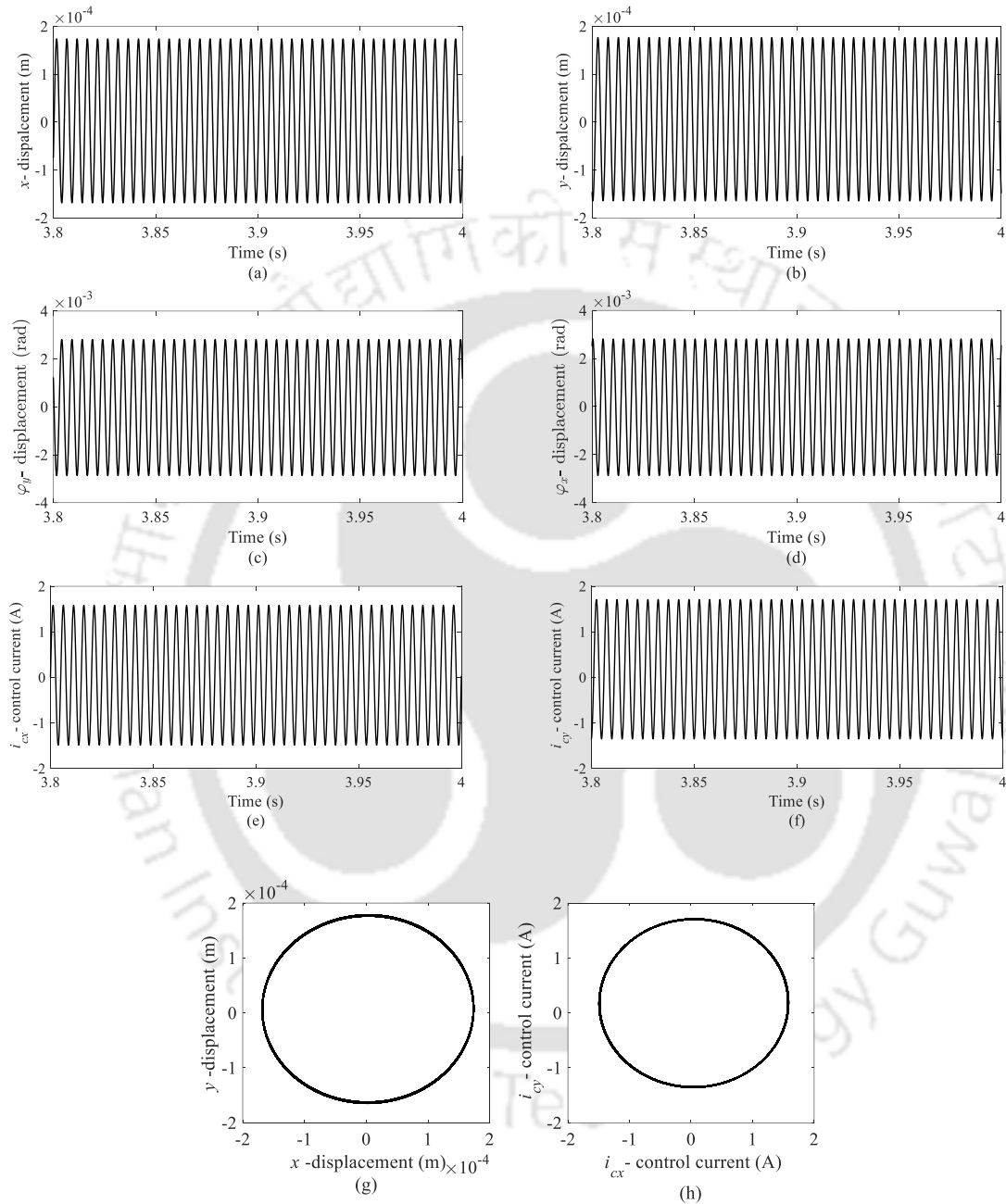


Figure 4.8 Time waveform of cracked rotor with AMB at the spin speed of 148 Hz (a) Vertical displacement (b) Horizontal displacement (c) Angular displacement about (x-z) plane (d) Angular displacement about (y-z) plane (e) Current in vertical direction (f) Current in horizontal direction (g) Orbit plot of translational displacements (h) Orbit plot of control current

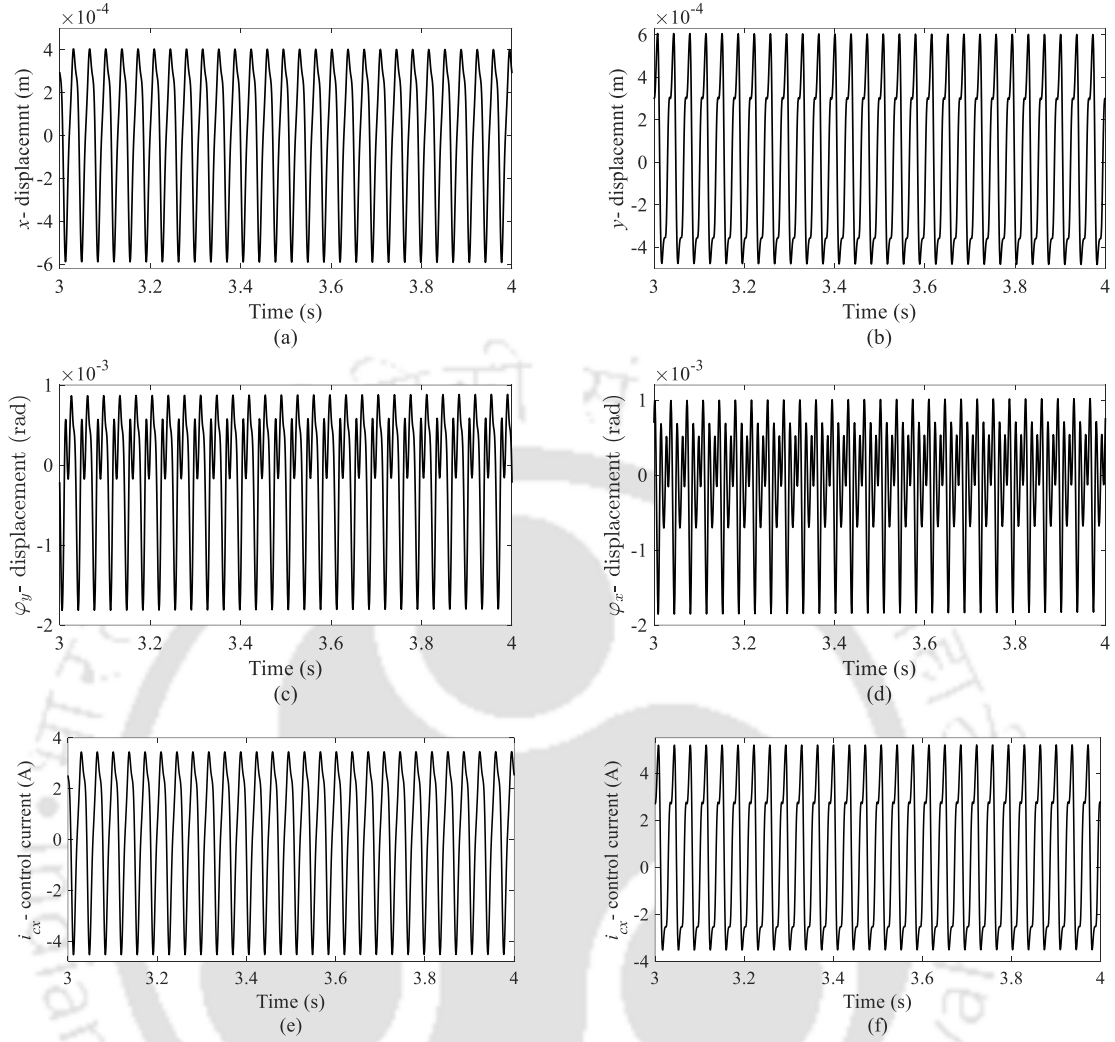


Figure 4.9 Time waveform of cracked rotor experiencing residual bow at the spin speed of 25 Hz (a) Vertical displacement (b) Horizontal displacement (c) Angular displacement about vertical direction (d) Angular displacement about horizontal direction (e) Control current in vertical direction (f) Control current in horizontal direction

Eventually, for observing the dynamic behavior of the rotor system in the presence of a crack, as well as residual shaft bow, the time waveform of the system operating at the spin speed of 25 Hz in the transverse directions are depicted in Figure 4.9. The cracked-warped rotor experiences vibrational amplitude of 4×10^{-4} m and 5.8×10^{-4} m in the vertical and horizontal directions, respectively, as observed in Figure 4.9(a, b) and the angular

displacements correspondingly in x-z and y-z plane are 0.8×10^{-3} rad (0.052 deg) and 1×10^{-3} rad (0.057 deg) as in Figure 4.9(c, d). AMB requires control currents for vibration suppression are 2.8 A and 3.8 A in the transverse translational directions as shown in Figure 4.9(e, f).

The characteristics of orbits (of displacement and control current) for both additive (unbalance, crack, and bow) and multiplicative (effect of bow on crack i.e. crack and bow fault together) are analyzed in the next stage. In the orbit plot of displacement and control current responses, where each of the orbit of rotor experiencing different excitation forces, changes from a double loop to an inside loop (a small loop inside the bigger one) that indicate both the amplitude and the phase changes of the harmonics due to the existence of crack (which imparts internal damping also) and residual bow in the system are shown in Figure 4.10. From the orbit plot, it is observed that the magnitude of vibration is lower in the presence of an additive faults, like the unbalance and only crack as shown in Figure 4.10(a, b) and Figure 4.10(c, d), respectively. On the other hand, the system having only the residual shaft bow and the multiplicative fault parameter effect (crack and residual shaft bow together), magnitudes of the responses is high and at the same time the necessity of control current in AMB is also high to suppress that signal as depicted in Figure 4.10(e, f) and Figure 4.10(g, h), respectively.

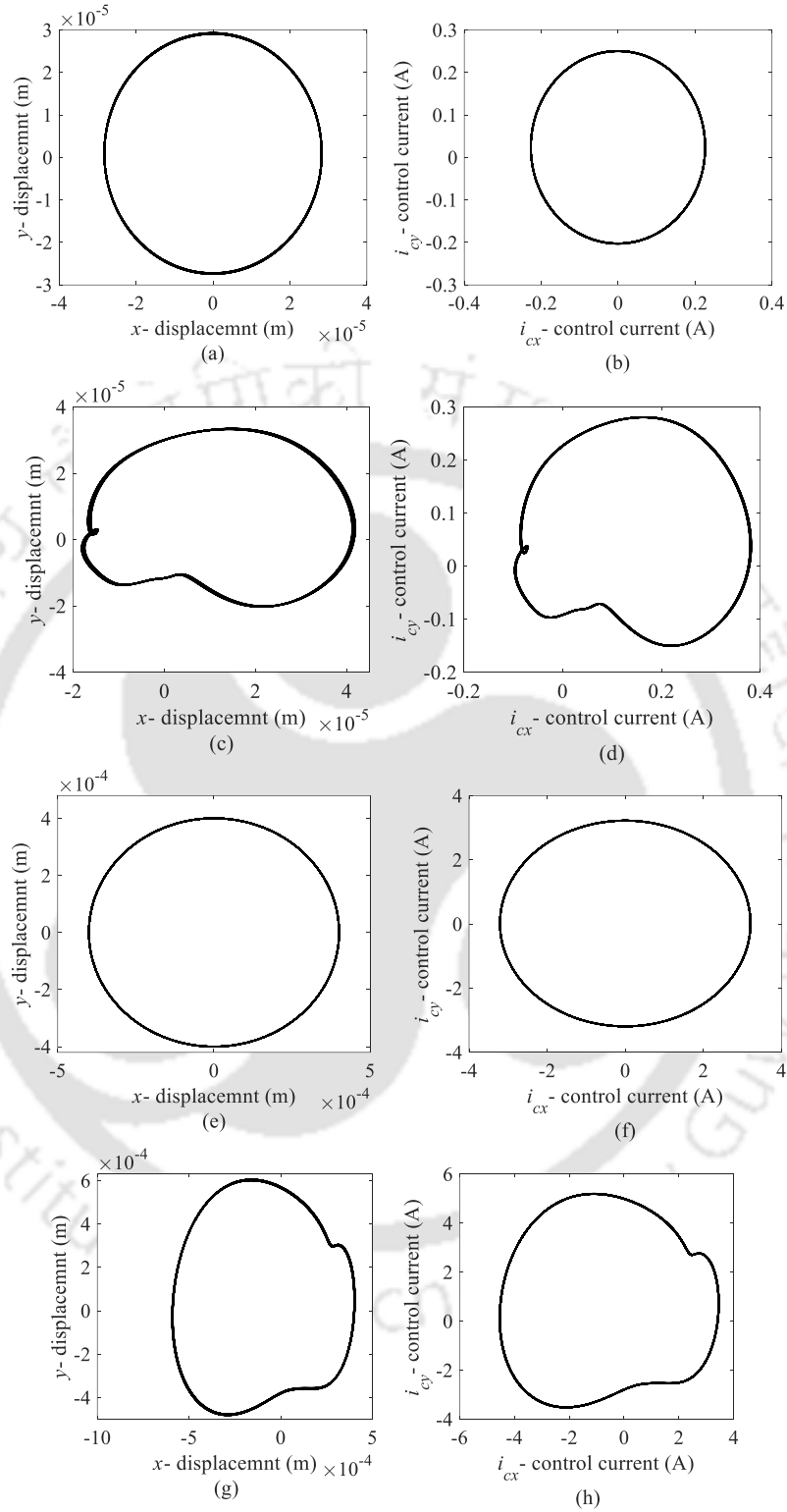


Figure 4.10 (a, b) Orbit plot only with unbalance force (c, d) Orbit plot only with crack force (e, f) Orbit plot only with bow force (g, h) Orbit plot with both unbalance, crack and bow force

4.2.1.1 Adoption of Full Spectrum for Parameter Identification

The full spectrum is a very handy signal processing tool as it contains the information of complex magnitude as well as the phase spectrum between two orthogonal signals. The full spectrum displays the directional frequency components with reference to the shaft spin (Goldman, 1999; Muszynska, 2005; Shravankumar, 2013) as elaborated in Chapter 2. Time domain responses generated from Equation 4.15 are converted into the frequency domain to acquire multi-harmonic amplitudes and corresponding phases. The comparison of the cracked rotor and cracked-bowed rotor are shown in Figures 4.11 and 4.12, respectively. The full spectrum shows some interesting features without the bow. The rotor having only crack fault as mentioned in the previous Chapters 2-3, the full spectrum of 4-DOF cracked rotor without shaft bow shows all odd harmonics except the second harmonic such as $1\times, 2\times, 3\times, 5\times, 7\times, -1\times, -3\times, -5\times, \dots$ of the spin speed with a lower amplitude of vibration as observed in Figure 4.11(a-d). Due to this, the AMB current required is also small of the order of 0.233 A. On the contrary, the rotor having residual bow and crack fault together generates comparatively higher amplitude of vibration of 4.87×10^{-4} m, and hence, 3.90 A current is required by the AMB to suppress that amplitude of vibration. As observed in the full spectrum of 2-DOF rotor system, the responses of displacement and current signals consist of all the harmonics irrespective of odd and even, i.e. $1\times, 2\times, 3\times, 4\times, 5\times, 6\times, 7\times, -1\times, -2\times, -3\times, -4\times, -5\times, -6\times, \dots$ of the rotor spin speed due to the multiplicative effect of parameters of bow-crack faults, unlike only cracked shaft without bow case. In the present work of 4 DOF system also possesses the similar pattern of full spectrum as shown in Figure 4.12(a-d).

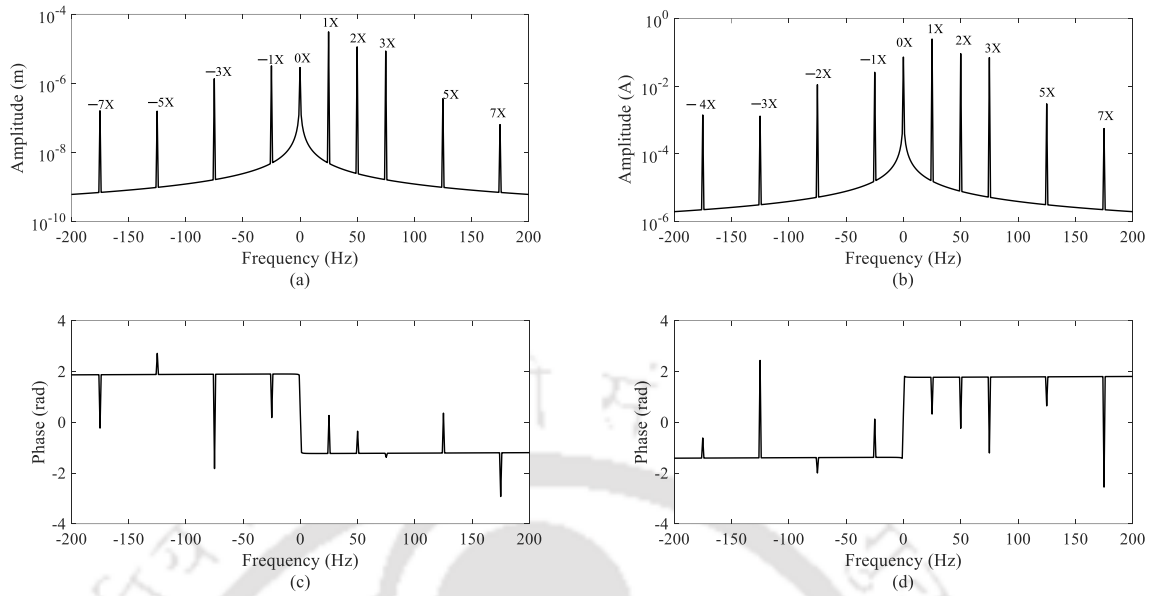


Figure 4.11 Full spectrum response generated without shaft bow of the cracked rotor at the speed of 25Hz (a, c) Displacement amplitude and phase (b, d) Control current amplitude and phase

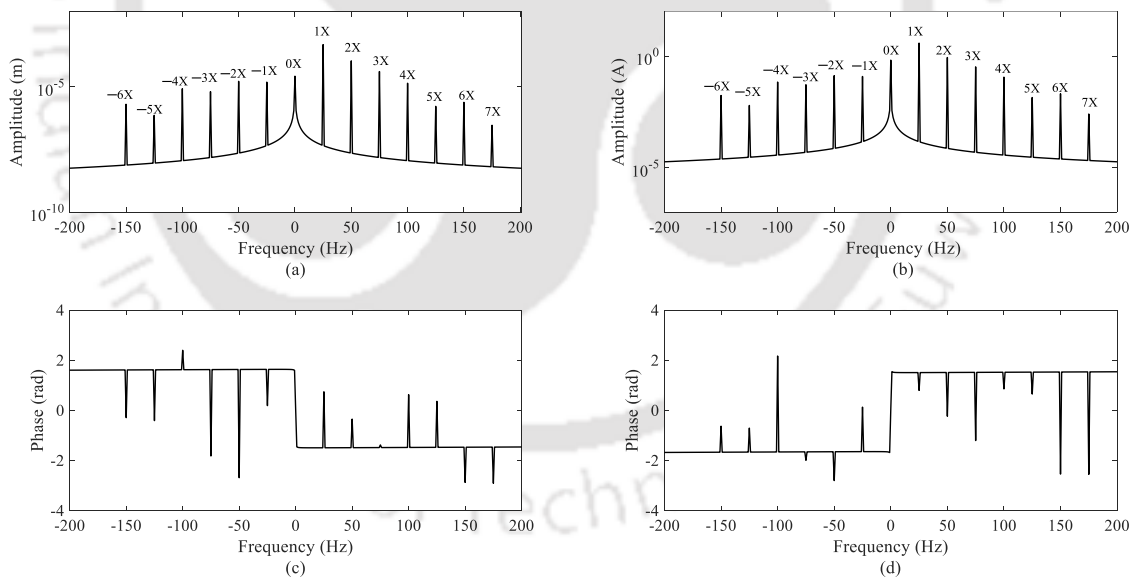


Figure 4.12 Full spectrum response generated with shaft bow of the cracked rotor at the speed of 25 Hz (a, c) Displacement amplitude and phase (b, d) Control current amplitude and phase

The comparison of the amplitude and their phases of control current for the cracked rotor without and with shaft residual bow are summarized in Table 4.4. These full spectrum harmonics generated will be utilized as an input to the developed identification algorithm to estimate the parameters considered in the developed mathematical modeling. The phase compensation technique is used as a difference of the phase of the system with that of the reference phase discussed in Chapter 2.

Table 4.2 Comparison of full spectrum amplitude and phase without and with shaft bow of the proposed cracked rotor system

	With bow				Without bow			
	Amplitude (m)	Phase (rad)	Amplitude (A)	Phase (rad)	Amplitude (m)	Phase (rad)	Amplitude (A)	Phase (rad)
0X	2.68×10^{-5}	0.026	0.672	0.050	2.58×10^{-9}	0.047	0.078	0.277
1X	4.87×10^{-4}	0.736	3.902	0.792	2.31×10^{-5}	0.221	0.233	0.330
2X	1.09×10^{-4}	-0.353	0.882	-0.237	7.73×10^{-6}	-0.409	0.079	-0.171
3X	4.11×10^{-5}	-1.383	0.335	-1.209	4.47×10^{-6}	-1.013	0.048	-0.659
4X	1.40×10^{-5}	0.621	0.116	0.852	—	—	—	—
5X	1.68×10^{-6}	0.356	0.014	0.647	4.00×10^{-7}	0.697	4.77×10^{-3}	1.248
6X	2.43×10^{-6}	-2.881	0.021	-2.546	—	—	—	—
7X	2.95×10^{-7}	-2.911	0.003	-2.556	7.24×10^{-8}	-2.761	9.78×10^{-4}	-2.056
-1X	1.55×10^{-5}	0.185	0.124	0.124	2.55×10^{-6}	0.202	0.026	0.092
-2X	1.67×10^{-5}	-2.692	0.135	-2.806	—	—	—	—
-3X	6.46×10^{-6}	-1.825	0.053	-1.997	7.74×10^{-7}	-2.12	0.008	-2.476
-4X	8.45×10^{-6}	2.392	0.069	2.164	—	—	—	—
-5X	7.34×10^{-7}	-0.414	0.006	-0.713	1.689×10^{-7}	2.366	2.013×10^{-3}	1.814
-6X	2.03×10^{-6}	-0.296	0.017	-0.637	—	—	—	—

4.3 Identification of Fault Parameters

The developed identification algorithm in Equation (4.38) for estimating the various system parameters practiced using Equation (4.35) in order to estimate the multiple fault parameters. The present analysis estimates the internal and external damping, crack

additive stiffness (both translational and rotational), and AMB parameters for without rotor case, as well as estimation of shaft residual bow and its orientation and multiplicative fault of crack-bow for with rotor bow case. In the numerical simulation, a proper range of combined speed of the rotor and its step size through which the vibration signals are generated plays a significant role in the estimation of more accurate parameters. The best fit of data using Equation (4.38) (considering up to 14 harmonics of the full spectrum amplitude and phase of the rotor) is done corresponding to combined rotating speeds of 25 Hz to 35 Hz in the step of 1 Hz for without bow case. With residual bow case, a particular parameter appears as a single parameter as well as in multiplicative form with another parameter (for example, Δk_{c22} , e_b as a single parameter and $(\Delta k_{c22} e_b)$ parameter). Due to this, the number of unknowns increases in the with bow case, thus combined speeds of range 25 Hz to 65 Hz avoiding the critical speed is chosen for simultaneous estimation of fault parameters for better accuracy.

The inherent and external noises are always present in the measured rotor system responses and it is very difficult to remove all the noises from the system responses completely. Therefore, it is important to add random noises to the numerically simulated responses to confirm the adequacy of the developed identification algorithm in the estimation procedure. Different levels of random noises of 1%, 2%, 3%, and 5% have been introduced to the uncontaminated time domain response to check the sensitiveness of the algorithm as shown in Tables 4.5 and 4.6 for without and with residual bow cases, respectively.

(i) *Without Rotor Bow case*

It is observed from Table 4.3 and Figure 4.13 that at 5% random noise, the rotational additive stiffness parameter is the highest deviating parameter by 3.87% from the assumed one. The reason for such deviation of the estimated rotational additive stiffness with the introduction of random noise may be due to an approximation during condensation of rotational DOFs. On the other hand, the AMB displacement-stiffness and current-stiffness factors have less altered, for example, 0.03% and –0.06%, respectively, in the presence of noises. It can be inferred that with the present methodology the reasonable estimates of fault parameters is expected even in the noisy environment.

(ii) *With rotor bow case*

The cracked rotor experience residual bow effect, both additive and multiplicative fault parameters are associated to excite the system. The multiple fault parameters of the cracked-bowed rotor are presented in Table 4.4 for various percentages of random noises.

Table 4.3 Estimation of parameters with various random noises for the spin speed of 25 Hz to 35 Hz

Parameters	Assumed Values	Random noises			
		0%	1%	3%	5%
$c_{22} \text{ Nsm}^{-1}$	140	139.970	139.999	139.055	139.112
$c_H \text{ Nsm}^{-1}$	125	125.988	125.988	125.989	125.990
$\Delta k_{c22} \text{ Nm}^{-1}$	2.05×10^5	2.062×10^5	2.056×10^5	2.045×10^5	2.033×10^5
$\Delta k_{c44} \text{ N(m rad)}^{-1}$	1670	1662.024	1679.942	1695.808	1718.714
$e_u \text{ m}$	2×10^{-4}	2.994×10^{-4}	2.995×10^{-4}	2.995×10^{-4}	2.996×10^{-4}
$\phi_u \text{ deg}$	30°	29.999	30.00	30.00	30.001
$k_s \text{ Nm}^{-1}$	2.17683×10^5	2.17883×10^5	2.17858×10^5	2.17808×10^5	2.17757×10^5
$k_i \text{ NA}^{-1}$	43.537	43.509	43.509	43.511	43.512

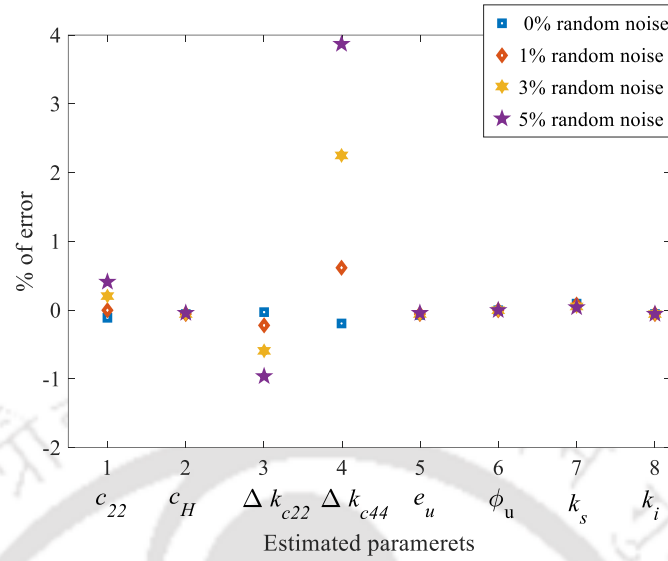


Figure 4.13 Percentage errors of estimated parameters with reference to the assumed values at 0%, 1%, 3% and 5% random noises

Table 4.4 Estimation of additive and multiplicative fault parameters with various random noises for the speed range of 25 Hz to 65 Hz

Parameters	Assumed Values	Random noises			
		0%	1%	3%	5%
Additive fault parameters					
$c_{22} \text{ Nsm}^{-1}$	140	139.86	139.73	139.51	139.32
$c_H \text{ Nsm}^{-1}$	125	124.48	124.65	124.92	125.10
$\Delta k_{c22} \text{ Nm}^{-1}$	2.05×10^5	2.07×10^5	2.134×10^5	2.184×10^5	2.233×10^5
$\Delta k_{c44} \text{ N(m rad)}^{-1}$	1670	1686.44	1631.45	1459.02	1286.73
$e \text{ m}$	2×10^{-4}	1.99×10^{-4}	1.99×10^{-4}	1.99×10^{-4}	1.99×10^{-4}
$\phi_u \text{ deg}$	30°	29.97	29.93	29.84	29.74
$e_b \text{ m}$	5×10^{-4}	4.99×10^{-4}	4.99×10^{-4}	4.99×10^{-4}	4.99×10^{-4}
$\phi_b \text{ deg}$	60°	59.99	59.99	59.99	59.99
$k_s \text{ Nm}^{-1}$	217683	2.17626×10^5	2.17608×10^5	2.17517×10^5	2.17349×10^5
$k_i \text{ NA}^{-1}$	43.537	43.42	43.42	43.41	43.40
Multiplicative fault parameter					
$\Delta k_{c22} e_{b,\text{Re}} \text{ N}$	51.95	51.55	51.54	54.91	56.30

It can be observed from Table 4.4 that all the parameters are getting affected with the addition of different percentage of random noises into the uncontaminated signal. The rotational additive stiffness parameter is highly affected by noise addition. In Figure 4.14, the percentage errors of all the estimated parameters are depicted with respect to its assumed values. The rotational additive stiffness (Δk_{c44}) parameter deviates by -24.81% and translational additive stiffness by 7.06% in the presence of 5% noise into the signal. Whereas the parameters of AMB (k_s) and the external damping (c_{22}) are least affected having an error of -0.026% and -0.097% , respectively.

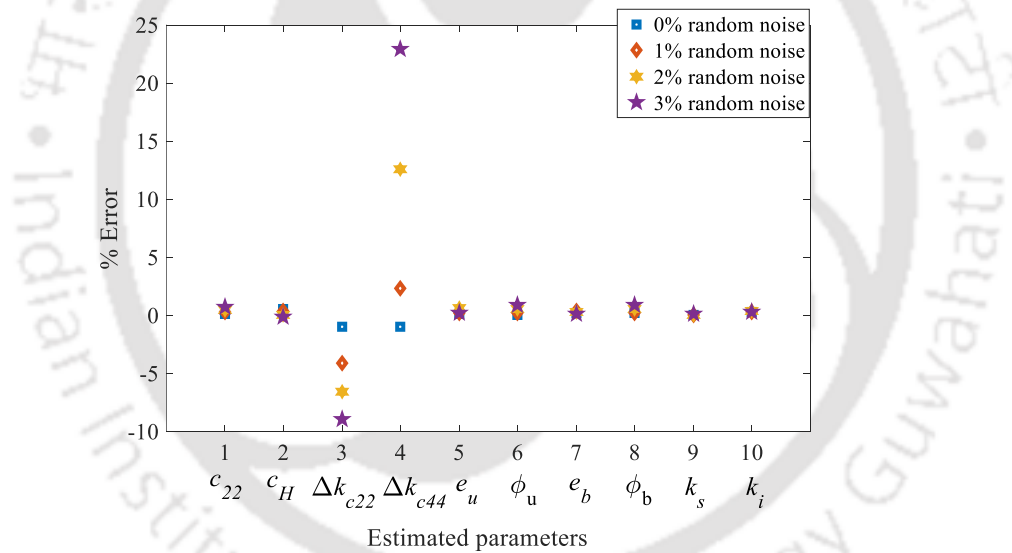


Figure 4.14 Percentage errors of estimated parameters with reference to the assumed values at 0%, 1%, 3% and 5% random noises

The deviations of the estimated parameters are found to be very small in good accordance with the assumed values of the parameters except the rotational additive stiffness parameter. Thus, the developed algorithm of the rotor with residual bow case also shows

its robustness towards the estimation of parameters in the existence of instrumental and external noise.

4.3.1 Introduction of Modeling or Bias Errors into the Proposed Model

The mathematical modeling of the system for the purpose of analysis may have differences in measurement of the system properties, such as the mass and stiffness as in real system. Therefore, one needs to take into account such errors for the analysis. Hence, the modeling (or bias) error plays a very important role in the estimation purposes to check the sensitivity of the developed algorithm. The bias errors have been introduced in the density, length, and Young's modulus of the proposed system for 1%, 3%, and 5% cases as presented in Table 4.5 and Table 4.6 for both without and with rotor bow case, respectively, and tested for the viability of the identification algorithm.

Table 4.5 Estimation of parameters with no residual bow against various modeling errors for the spin speed of 25 Hz to 35 Hz

Parameters	Assumed Values	0% noise	1% noise	3% noise	5% noise
$c_{22} \text{ Nsm}^{-1}$	140	139.981	140.022	140.105	140.189
$c_H \text{ Nsm}^{-1}$	125	124.987	125.019	125.081	125.145
$\Delta k_{c22} \text{ Nm}^{-1}$	2.05×10^5	2.061×10^5	2.059×10^5	2.054×10^5	2.048×10^5
$\Delta k_{c44} \text{ N(m rad)}^{-1}$	1670	1592.212	1689.506	1720.801	1954.518
$e_u \text{ m}$	2×10^{-4}	2.994×10^{-5}	2.992×10^{-5}	2.986×10^{-5}	2.982×10^{-5}
$\phi_u \text{ deg}$	30°	30.00	29.999	29.998	29.998
$k_s \text{ Nm}^{-1}$	2.17683×10^5	2.17883×10^5	2.22641×10^5	2.32155×10^5	2.41667×10^5
$k_i \text{ NA}^{-1}$	43.537	43.508	43.577	43.714	43.851

It is observed from Figure 4.15 that the much affected parameters, after introducing 5% modeling error into the model parameters, the displacement stiffness (k_s) and the rotational additive stiffness (Δk_{c44}) parameters are deviating, respectively, by 11% and 6% with respect to the absolute values. The least affected parameters are additive stiffness (Δk_{c22}) and internal damping (c_{22}) with deviations of -0.46% and 0.70% , respectively. Results show that the identification algorithm gives excellent estimates of the additive stiffness of crack and other fault parameters, such as the internal and external damping, unbalance and AMB parameters.

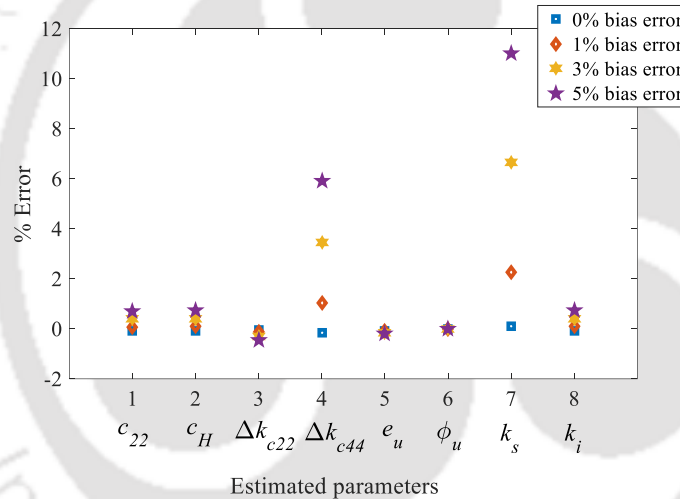


Figure 4.15 Percentage errors of estimated parameters with reference to the assumed values at 0%, 1%, 3% and 5% modeling error

For the cracked rotor experiencing shaft residual bow, the additive and multiplicative fault parameters estimated are presented in Table 4.6. Percentage errors of all the estimated parameters with the introduction of modeling errors are depicted with respect to its assumed values in Figure 4.16. It is observed that the highly affected parameter at 5% bias error is the rotational additive stiffness, which deviates 24.64% with the assumed one. The least affected parameters are found to be the force-displacement

factor (k_s) and the external damping (c_{22}), which deviate by -0.026% and -0.096% , respectively, at 5% bias errors with the assumed parameter values. As the deviations of the parameters are comparatively very small with the assumed values of the system parameters, and hence, confirms the robustness of the developed identification algorithm for rotor with bow case.

Table 4.6 Estimation of parameters with residual bow against various modeling errors for the spin speed of 25 Hz to 65 Hz

Parameters Values	Assumed	Modeling or bias error			
		0%	1%	3%	5%
Additive fault parameters					
c_{22} Nsm ⁻¹	140	139.86	141.54	142.51	143.71
c_H Nsm ⁻¹	125	124.48	122.17	121.00	119.23
Δk_{c22} Nm ⁻¹	2.05×10 ⁵	2.07×10 ⁵	2.053×10 ⁵	1.992×10 ⁵	1.969×10 ⁵
Δk_{c44} N(m rad) ⁻¹	1670	1686.44	1766.95	1971.50	2081.43
e_u m	2×10 ⁻⁴	1.99×10 ⁻⁴	2.00×10 ⁻⁴	2.01×10 ⁻⁴	1.94×10 ⁻⁴
ϕ_u deg	30°	29.97	30.70	32.24	33.36
e_b m	5×10 ⁻⁴	4.99×10 ⁻⁴	4.98×10 ⁻⁴	4.94×10 ⁻⁴	4.89×10 ⁻⁴
ϕ_b deg	60°	59.99	60.02	60.05	60.05
k_s Nm ⁻¹	217683	2.17626×10 ⁵	2.18135×10 ⁵	2.20336×10 ⁵	2.25429×10 ⁵
k_i NA ⁻¹	43.537	43.42	43.43	43.44	43.47
Multiplicative fault parameter					
$\Delta k_{c22} e_{b,Re}$ N	51.95	51.55	51.68	51.71	51.86

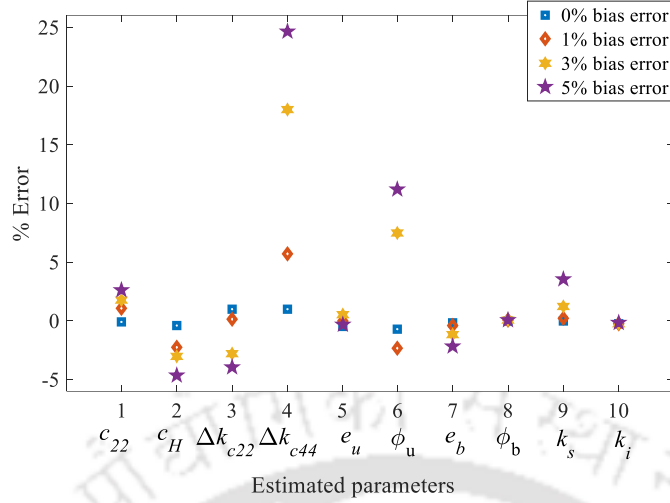


Figure 4.16 Percentage errors of estimated parameters with reference to the assumed values at 0%, 1%, 3% and 5% modeling error

As a whole in this identification process, the rotational additive stiffness is much affected by the noise addition. This is due to its small value compared to the translational additive stiffness parameter. Therefore, a small addition of noise changes the rotational additive stiffness parameter of crack drastically. Moreover, the rotational additive stiffness parameter is associated with the angular deflection of the rotor and in the dynamic reduction scheme, it is ignored initially to remove the rotational DOF. However, the same parameter is again incorporated in the identification algorithm for the estimation. Likewise, the shaft bow is directly related to the permanent deformation of the shaft, for which the force-displacement factor of AMB is much affected by noise.

4.3.2 Effect of Variations Fault Parameters in Response Analysis and Identification

Four different cases of two faults, i.e. the disc eccentricity (e) and the translational shaft bow parameter (e_b), have been analyzed on this model. The variation of faults chosen is

the unbalance force (a function of rotor spin speed) and residual shaft bow (permanent deformation, which is not a function of rotor spin speed). These are the most common faults in real machines, and they affect the dynamics of the system. The influence of e and e_b parameters in the dynamics of the rotor system is interpreted in orbit plots, for different cases as shown in Figure 4.17. The four different cases considered in the present analysis are as follows:

Case I: The disc eccentricity ($e = 2 \times 10^{-4}$ m) is in the same range as of the translational shaft bow amplitude ($e_b = 5 \times 10^{-4}$ m), the orbit plot shows the amplitude of vibration is within 5×10^{-4} m and the AMB control current within 4 A in Figure 4.17 (a-b).

Case II: The magnitude of the disc eccentricity ($e = 5 \times 10^{-4}$ m) is the same as that of the translational shaft bow ($e_b = 5 \times 10^{-4}$ m), corresponding the orbit plot is shown in Figure 4.17 (c-d). The amplitude of vibration is within 5×10^{-4} m and the AMB control current is 4 A.

Case III: It is observed that the higher value the translational shaft bow ($e_b = 8 \times 10^{-4}$ m) than the eccentricity ($e = 5 \times 10^{-5}$ m) of the disc increases the amplitude of vibration to 5×10^{-4} m as well as the control current to 5 A, as depicted in Figure 4.17 (e-f) orbit plots.

Case IV: If the disc eccentricity ($e = 2 \times 10^{-4}$ m) is much higher than the magnitude of the translational shaft bow ($e_b = 5 \times 10^{-5}$ m), then the orbit plot as in Figure 4.17 (g-h) shows the amplitude of vibration of within 5×10^{-5} m and the AMB current required to control that vibration of below 1A.

For above mentioned four cases, the estimation of the fault parameters through the identification algorithm are summarized in Table 4.7. It is observed from the tables that the variation of the disc eccentricity, e , and translational shaft bow, e_b , values mostly affect the additive crack stiffness (Δk_{c22} , Δk_{c44} , $\Delta k_{c22}e_{b,Re}$) parameters. The multiple fault parameters

are identified with a good accuracy in presence of the noisy data, i.e. at 5% random noise, and with the variation of the disc eccentricity e_u , and the translational shaft bow, e_b fault parameters in Table 4.7 except the rotational additive stiffness parameter. Moreover, one more analysis is done to investigate the effect of variation of the phase angles of unbalance and shaft residual bow on the estimated parameters.

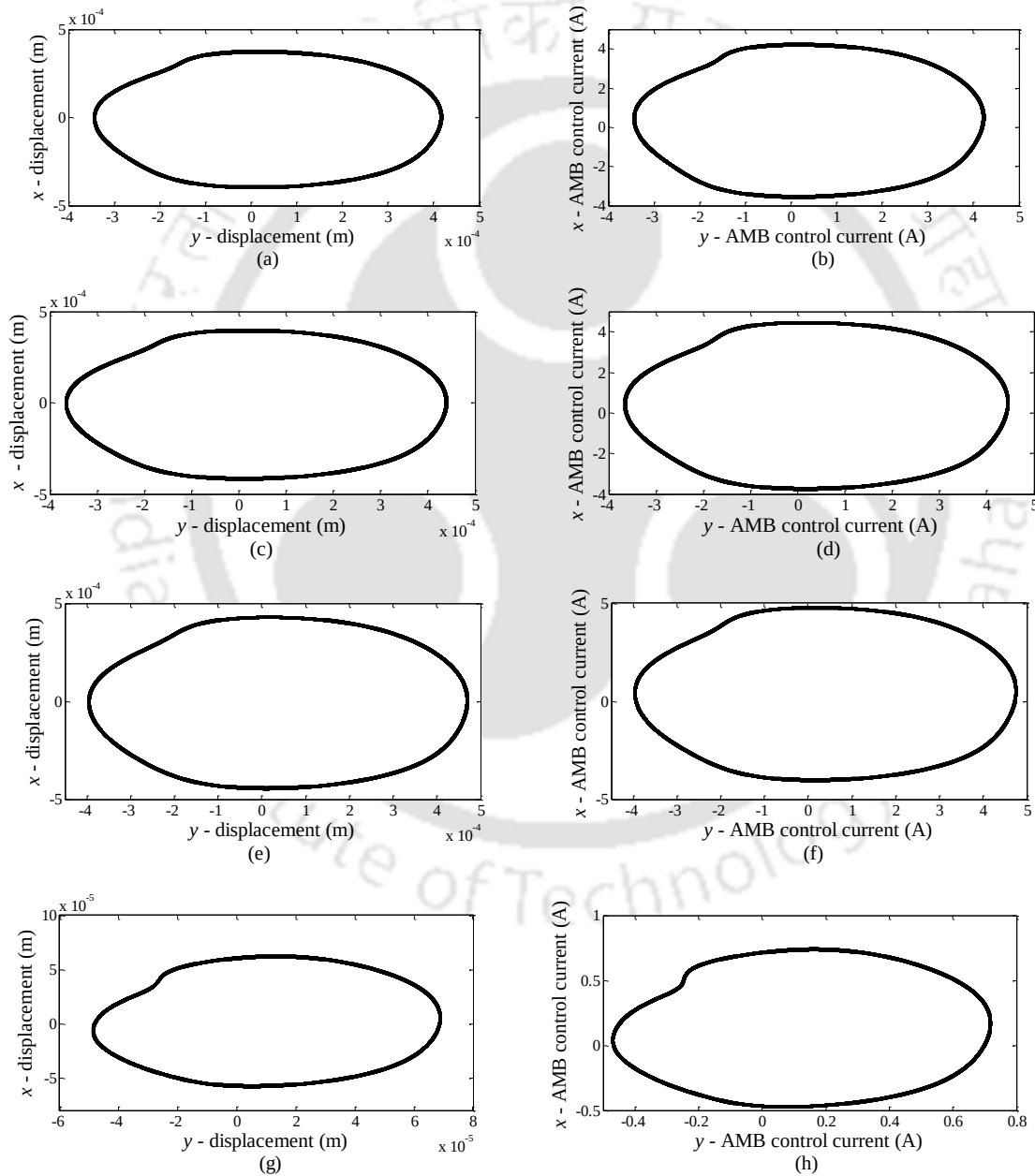


Figure 4.17 Orbit plots of different values of disc eccentricity (e_u) and shaft bow (e_b) for
(a) Case I (b) Case II (c) Case III (d) Case IV

In this regard, Table 4.8 shows a typical result for Case I to IV at 5% random noise as mentioned above to illustrate the variation of estimates due to different phase combinations. Table 4.9 presents the multiple fault parameters estimation with variation of phase angles at 5% random noises.

Table 4.7 Comparison of estimated parameters at 5% random noise levels for different values of disc eccentricity (e_u) and residual shaft bow (e_b)

Parameters	Assumed Values	Estimated parameters at 5% random noise			
		Case I	Case II	Case III	Case IV
		$e = 2 \times 10^{-4}$ m $\phi_u = 30^\circ$, $e_b = 5 \times 10^{-4}$ m, $\phi_b = 60^\circ$ (assumed)	$e = 5 \times 10^{-4}$ m, $\phi_u = 30^\circ$, $e_b = 5 \times 10^{-4}$ m, $\phi_b = 60^\circ$ (assumed)	$e = 5 \times 10^{-5}$ m, $\phi_u = 30^\circ$, $e_b = 8 \times 10^{-4}$ m, $\phi_b = 60^\circ$ (assumed)	$e = 2 \times 10^{-4}$ m, $\phi_u = 30^\circ$, $e_b = 5 \times 10^{-5}$ m, $\phi_b = 60^\circ$ (assumed)
Additive fault parameter					
c_{22} Nsm $^{-1}$	140	139.32	139.95	139.81	140.56
c_H Nsm $^{-1}$	125	125.10	124.37	124.55	122.27
Δk_{c22} Nm $^{-1}$	2.05×10^5	2.233×10^5	2.075×10^5	2.078×10^5	2.057×10^5
Δk_{c44} N(m rad) $^{-1}$	1670	1286.73	1683.37	1686.70	1686.14
e_u m	–	1.99×10^{-4}	4.989×10^{-4}	4.966×10^{-5}	1.99×10^{-4}
ϕ_u deg	–	29.742	29.75	29.80	30.01
e_b m	–	4.99×10^{-4}	4.990×10^{-4}	7.984×10^{-4}	4.991×10^{-5}
ϕ_b deg	–	59.99	60.00	59.99	59.993
k_s Nm $^{-1}$	217683	2.17349×10^5	2.17626×10^5	2.17645×10^5	2.17041×10^5
k_i NA $^{-1}$	43.537	43.40	43.42	43.42	43.36
Multiplicative fault parameter					
$\Delta k_{c22} e_{b,Re}$ N	51.25	56.30	56.69	165.88	10.40

Table 4.8 Comparison of estimated parameters at 5% random noise levels for different values of phase angles of unbalance (ϕ_u) and shaft residual bow (ϕ_b) keeping disc eccentricity (e_u) and translational shaft bow (e_b) same

Estimated parameters	Case I with variation in phase angles of unbalance and shaft bow							
	$\phi_u=60^0$, $\phi_b=30^0$	% error	$\phi_u=60^0$, $\phi_b=45^0$	% error	$\phi_u=30^0$, $\phi_b=65^0$	% error	$\phi_u=90^0$, $\phi_b=36^0$	% error
Additive fault parameter								
c_{22} Nsm ⁻¹	139.305	-0.49	139.95	-0.35	139.32	-0.23	139.31	-0.49
c_H Nsm ⁻¹	123.762	-0.99	123.57	-0.93	125.10	-0.43	123.92	-0.86
Δk_{c22} Nm ⁻¹	2.45×10 ⁵	17.72	2.34×10 ⁵	12.37	2.233×10 ⁵	2.67	2.36×10 ⁵	13.33
Δk_{44} N(m rad) ⁻¹	718.087	-57.00	1012.92	-39.34	1286.73	-13.12	984.44	-41.05
e_u m	1.98×10 ⁻⁴	-0.91	1.98×10 ⁻⁴	-0.76	1.99×10 ⁻⁴	-0.22	1.98×10 ⁻⁴	-0.98
ϕ_u deg	59.94	-0.41	59.89	-0.49	29.742	-0.27	89.92	-0.08
e_b m	4.99×10 ⁻⁴	-0.06	4.99×10 ⁻⁴	-0.09	4.99×10 ⁻⁴	-0.07	4.99×10 ⁻⁴	-0.08
ϕ_b deg	29.97	-0.09	44.96	-0.07	59.99	0.03	35.96	-0.09
k_s Nm ⁻¹	216504.95	-0.54	2.16418×10 ⁵	-0.58	2.17349×10 ⁵	-0.28	2.16397×10 ⁵	-0.59
k_t NA ⁻¹	43.29	-0.48	43.28	-0.50	43.40	-0.35	43.28	-0.59
Multiplicative fault parameter								
$\Delta k_{c22} e_{b,Re}$ N	89.20	4.92	72.85	18.73	51.534	8.36	83.34	8.74

It is observed that the estimated parameters presented in Table 4.9, the percentage error for the most of estimated parameters are in the range of -1 to 1% , except for the translational and rotational additive stiffness. This trend is present for all phase combinations, so relative phase of bow and unbalance has no effect on most of the estimates, except translational and rotational additive stiffness parameters of the crack. Similar trend is observed in multiplicative fault parameter. In the use of dynamic condensation scheme for removal of the slave DOFs (i.e. the rotational displacement of the rotor) brings the higher variation of the rotational additive stiffness parameter as discussed earlier.

4.4 Concluding Remarks

In the present chapter the identification of multiple fault parameters of a cracked rotor system with an offset disc mounting and an AMB support for two rotor conditions is obtained, where one condition is when the cracked rotor does not experience any residual bow and other condition when the cracked rotor experiences residual bow excitation. Following conclusions can be drawn from this chapter as follows:

- Gyroscopic effect due to the location of the disc at the offset position of the shaft, which has peculiar effect on rotors unlike structures during operation. Thus, both translational and rotational additive crack stiffness on the dynamics of the rotor system is studied.
- To overcome the physical measurement of angular displacements, a suitable *gyroscopic dynamic condensation* (GDC) is applied to developed EOMs for both the without and with rotor bow conditions, and the identification algorithms are developed based on the master DOFs only.

- The additive and multiplicative fault (due to crack and bow together) are estimated based on the full spectrum multi-harmonic amplitudes and phase corrections of displacement and control current signals, and found the algorithms are robust against measurement noise and modelling errors.
- The rotational additive stiffness of crack is found to be deviating much from the absolute values in noisy condition. This parameter is associated with the angular deflection of the rotor and in the dynamic reduction scheme, it is ignored initially to remove the rotational DOF. However, the same parameter is again incorporated in the identification algorithm for the estimation has drastically change its value with noisy signal.
- The identification procedure for the rotor bow case provides much more information of influence of one fault onto the other irrespective of its estimation individually.

This framework of use of condensation scheme and multiple fault parameters identification based on the offset disc location in this chapter is discussed here. The experimental identification using the same methodology here will be discussed in the next chapter. The next chapter presents the time and frequency response analysis of the acquired signals from the developed test rig with offset mounting of disc from shaft mid-span and estimation of multiple fault parameters.

Chapter 5

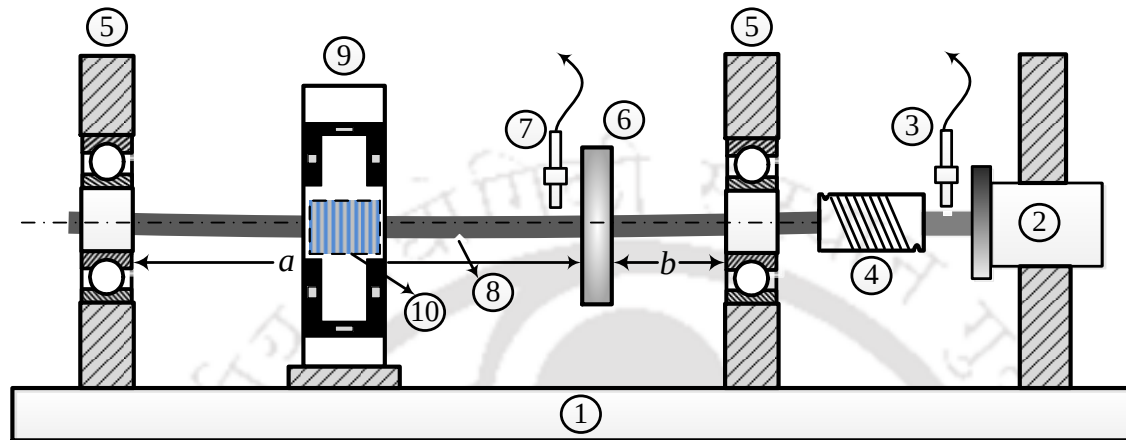
Experimental Estimation of Multiple Fault Parameters in a Cracked-Warped Rotor-AMB System with an Offset Disc

Experimental investigation to estimate the multiple fault parameters through the implementation of the developed identification algorithm in Chapter 4, which uses condensation of rotational DOFs, is addressed. The identification is done by the use of measured transverse translational vibrational displacements and control currents of AMB in the cracked-warped rotor-AMB system with an offset disc mounted over it. The test rig was developed in-house in the Advanced Dynamics & Vibration Laboratory at IIT Guwahati. Experimental validation of the estimated parameters with the numerically simulated responses as presented in Chapter 4 are enunciated here.

5.1 Description of the Test Rig with AMB Connection

The model based identification procedure of faults is achieved by the use of data collected from the developed rotordynamic test rig in the laboratory. Various components of the test rig and their assembly was discussed in Chapter 3. The only difference here is that the rigid disc is mounted offset with the bearings on the cracked shaft. As the disc is at offset location, during operation due to tilting of the disc, the rotor experiences transverse rotatory inertia and gyroscopic effect. These effects are considered in the whole analysis and identification methodology. The schematic diagram of the rotordynamic test bench is

shown in Figure 5.1. It consists of a warped shaft containing an artificially generated transverse fatigue crack, an eight-pole magnetic bearing actuator, two deep-groove ball bearings, flexible coupling, and motor.



① = Cast iron bed; ② = Motor; ③ = Eddy current probe (reference signal); ④ = Flexible coupling; ⑤ = Bearing with block; ⑥ = Disc at offset; ⑦ = Eddy current probe (transverse signal); ⑧ = Fatigue crack on the shaft; ⑨ = Eight-pole magnetic bearing actuator; ⑩ = Magnetic rotor core

Figure 5.1 Schematic of rotordynamic test bench

The picture of the rotor system equipped with AMB with dSPACE control desk with all the circuit connection is shown in Figure 5.1. The integration of AMB in the cracked-warped rotor test rig with the dSPACE controller are illustrated in Chapter 3. The magnetic bearing actuator coil connections with the servo power amplifier (A25A100) are elaborated in the current Chapter. The same connections were used in the experimentation of disc at the middle location of the test rig in Chapter 3.

The analog tuning of the power amplifier is done on switches and potentiometers of the servo drive. Analog drives are operated in four different modes, such as the current control mode, voltage control mode, velocity control mode and IR compensation mode.

We have chosen the amplifiers to work with the drive in the current mode. First, to check the operating status of the drive we have to check the green led (normal operation) is ON or it is red (i.e., OFF). If it is red, then the drive is not in working condition and dipswitches need to be set as given in the Advanced Motion Controls manual. The amplifier is shown in Figure 5.2.

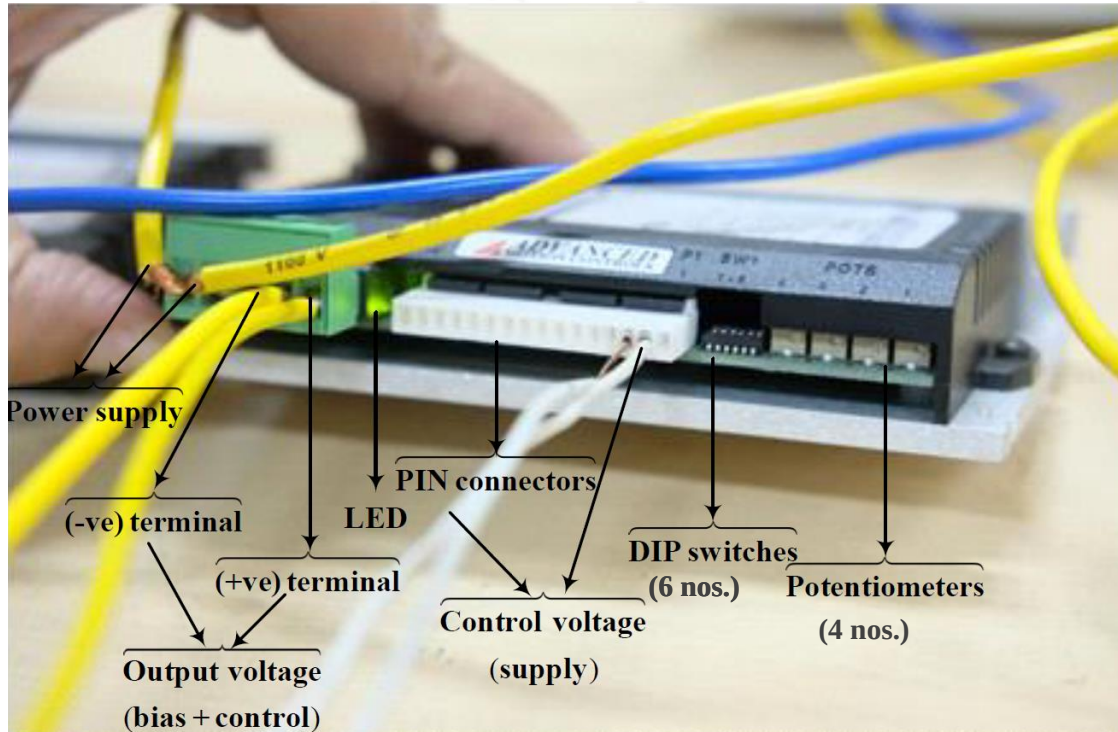


Figure 5.2 Servo power amplifier in ON condition

This drive requires only a single DC power supply with a supply voltage range of 20-80 Vdc. Here, the drive was set at 24 Vdc supply voltage with current range of 0 to 2 A on the power source knob. To set the drive to operate in current control mode, dip switch SW1-1 from left need to be in OFF condition and SW1-3 be in ON. The drive is operating at high switching frequency of 20 kHz. It consists of 16-pin friction lock header P1 connector, where the second (SIGNAL GND) and the fourth pin (+REF IN (± 10 V Operating Range,

± 15 V Maximum Input)) are connected with wire from which the positive and negative terminals are connected to the oscilloscope probe to send the signal to the dSPACE MicroLabBox output channels. Moreover, the drive has P2 connector of 5-pins equally spaced to quick disconnect terminals. Counting from left side, P2 connectors of A stands for negative output, B stands for positive output, C for not connected, POWER GND stands for Ground connection and lastly, HIGH VOLTAGE stands for DC power input.

The potentiometers are to apply the signal to the drive and there are four potentiometers (POTs), viz. loop gain, current limit, reference gain, and offset/test each have 12 active turns and 1 inactive turn. The following are the features of the four potentiometers:

- (i) The loop gain (POT 1) increases the gain in the drive by adjusting the duty cycle in current mode with the help of turning the POT 1 fully in clockwise direction.
- (ii) Current limit (POT 2) or pot is to set the current limit by adjusting the peak and continuous current limit by maintaining their ratio with the help of turning the POT 2 fully in clockwise direction.
- (iii) Reference gain (POT 3) also increases gain by doing the adjustment of the input signal to the output variable signal, such as the voltage, current or velocity by turning POT 3 fully in clockwise direction.
- (iv) Offset/ test (POT 4) need to be adjusted in the negative direction for modifying any imbalance in the input signal or may be in the amplifier.

The current limit is used to protect the servo drive so that it does not get overloaded. After tuning the PWM (pulse width modulation) signal is observed in oscilloscope for two different duty cycles as shown in Figure 5.3. The voltage source is supplied to the analog signal by means of a square wave of on and off pulses where the on-time is the time during

which the DC supply is applied to the load, and the off-time is the period during which the supply is switched off. The current signals are dependent on the on and off condition. The purpose of these amplifiers is to convert low power output signal of the controller to a high power input signal. The amplifier consists of a MOSFET drive with a maximum continuous current rating of 10 A. This drive (power amplifier) is interface with the digital dSPACE DS1202 digital controller as discussed in Chapter 3. The rotor-AMB test rig assembly with an offset disc is depicted in Figure 5.4.

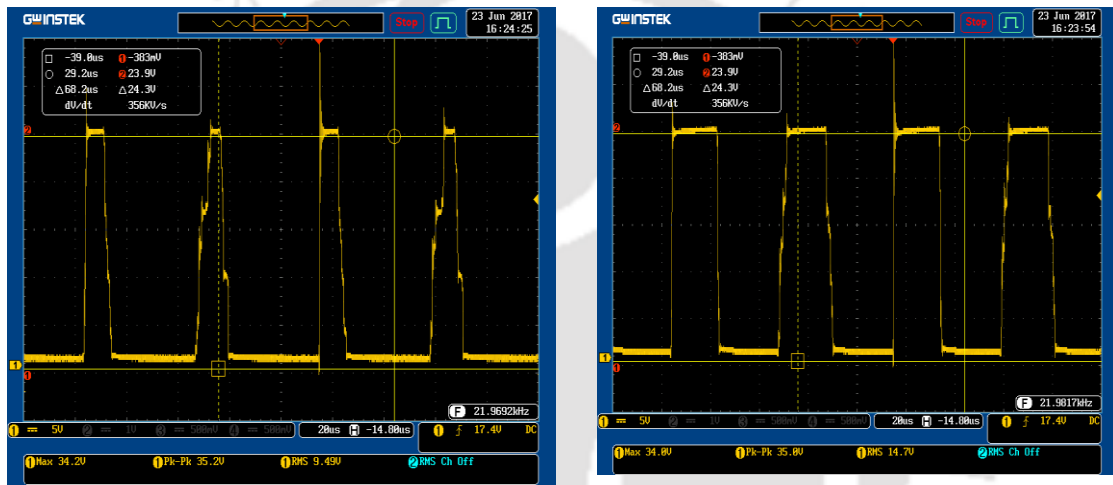


Figure 5.3 Voltage signal of PWM of the servo drive (amplifier) with different duty cycle

5.1.1 Procedure of Acquiring Test Data and Data Acquisition

Two eddy-current proximity sensors are utilized for acquiring vibrational displacements of the cracked-warped rotor-AMB system, which are located in two orthogonal transverse directions near the offset disc as shown in Figure 5.4. One more sensor is placed on the top of the motor spindle ensuring negligible motor transverse displacement to acquire the phase reference signal of the system. The measured test data is fed for data acquisition in dSPACE DS1202 control through I/O panels in the MicroLabBox connected with MATLAB SIMULINK™ interface.

The block diagram in SIMUINK™ platform for the data acquisition and implement the PID control strategy as shown in Figure 5.5.

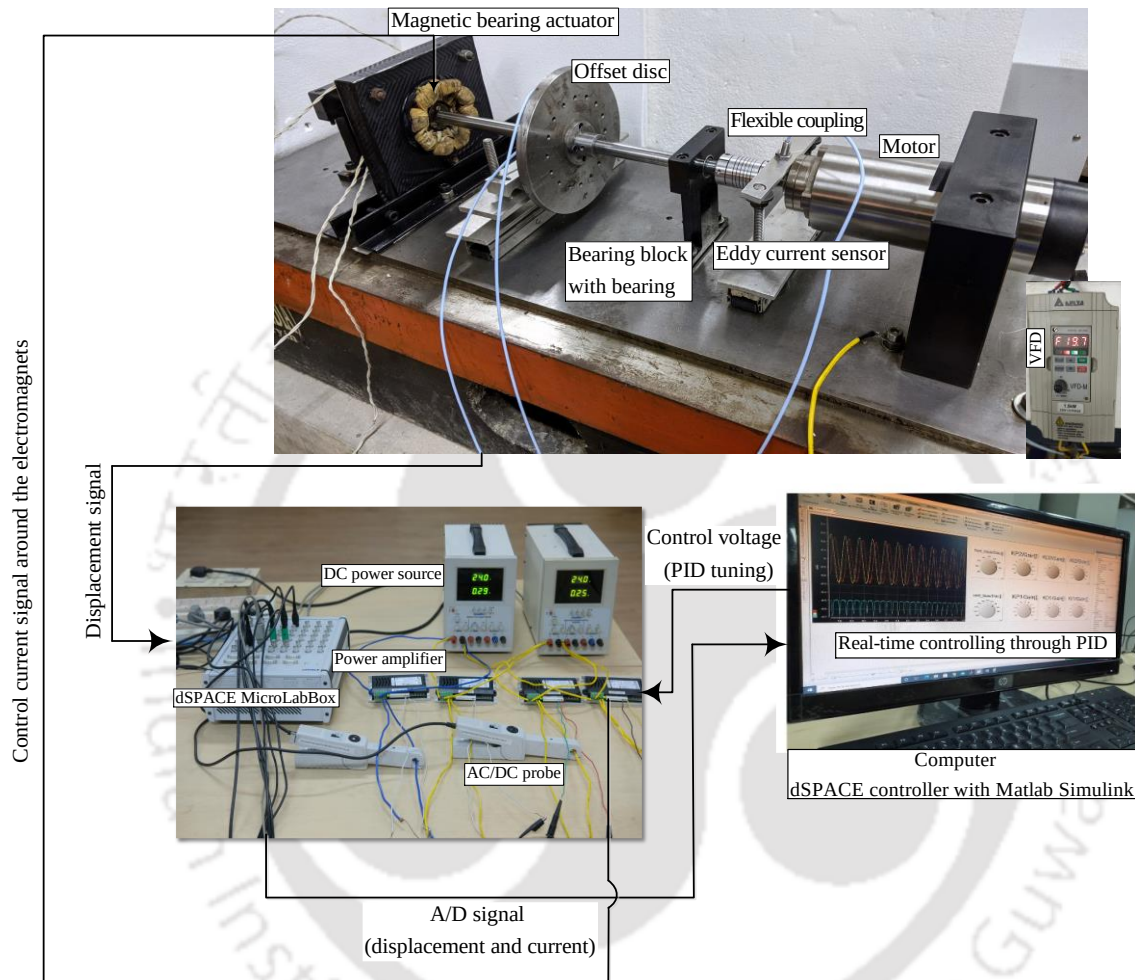


Figure 5.4 Test rig assembly of cracked warped rotor-AMB system with an offset disc with dSPACE controller

The transverse displacement and reference signal measured through the eddy current probes in terms of voltage is fed to ADC block and the signal is shifted to the mean (zero) position. After that the gain block is added to convert the voltage signal to meter form. Then the displacement signal is fed to the PID controller gain blocks in both vertical and horizontal directions. The generation of control current from the PID blocks with respect to displacements

of rotor is transferred to a gain block and multiplied with a gain of 1 V/A and the control voltage signal is added to the bias current of 2 A in both the directions in the bias block. Then the total bias and control voltage is passing through a saturation block to limit the lower and the upper limit of the current passing through the actuator coils to avoid damage to the coils. Lastly, the control voltage is fed to the DAC block and goes to the power amplifiers connected two each in vertical (upper and lower poles) and horizontal (right and left side poles) directions of the eight-pole actuator. That amplified signal of bias and control current together from the amplifiers goes to the actuator electromagnets. The same procedure as illustrated here is applied for the data acquisition of the central disc in the test rig experimentation discussed in Chapter 3. The experimental results of the offset mounting disc on the test bench is discussed in the next section.

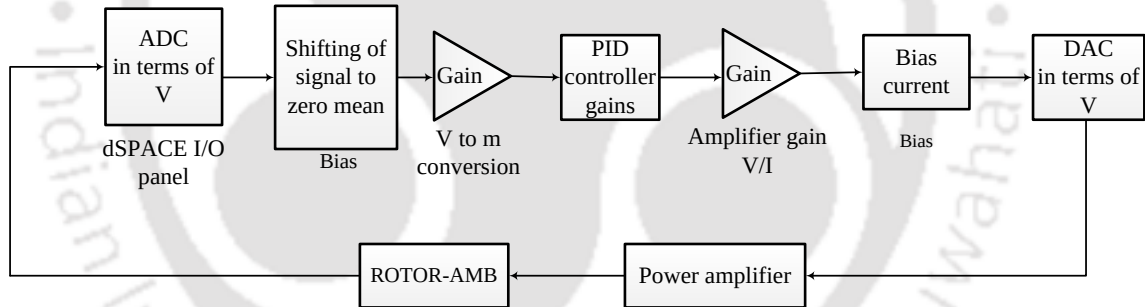


Figure 5.5 SIMULINK block diagram real-time signal acquisition in dSPACE control desk

5.2 Experimental Test Results

The free and forced vibration analyses of the experimentally measured data is discussed in this section.

5.2.1 Free Vibration Analysis

In the free vibration analysis, with the help of an impact hammer, an impulsive force is given on to the rotor in standstill condition. The generation of time domain damped signal is measured and FFT of that time domain signal is used to evaluate the natural frequency of the system. The physical properties of the rotordynamic test bench installed in the laboratory are presented in Table 5.1.

Table 5.1 Physical parameters of rotor test rig

Rotor Properties	Measurement Values
Disc mass, m	2.038 kg
Shaft length, l	0.47 m
Young's modulus, E	$2.06 \times 10^{11} \text{ Nm}^{-2}$
Diametral mass moment of inertia, I_d	0.0198 kgm^2
Polar mass moment of inertia, I_p	0.0397 kgm^2
Offset location of disc on the shaft, a	0.305 m
And, b	0.165 m
Shaft stiffness using influence co-efficient, k_{22}	$6.1099 \times 10^5 \text{ Nm}^{-1}$
k_{24}	$6.272 \times 10^4 \text{ N(m-rad)}^{-1}$
k_{44}	$1.914 \times 10^4 \text{ N rad}^{-1}$

Here, the natural frequency of two configuration of shaft in the test rig are observed.

- (i) healthy shaft (without and with AMB integration) and
- (ii) cracked shaft (without and with AMB integration)

The time domain signal of the healthy shaft without and with AMB case is shown in Figure 5.6(a, c). The frequency domain signals gives the natural frequency of the

healthy shaft system is found to be 66.33 Hz for without AMB case and for with AMB case, it is 69.64 Hz as shown in Figure 5.6(b, d) respectively.

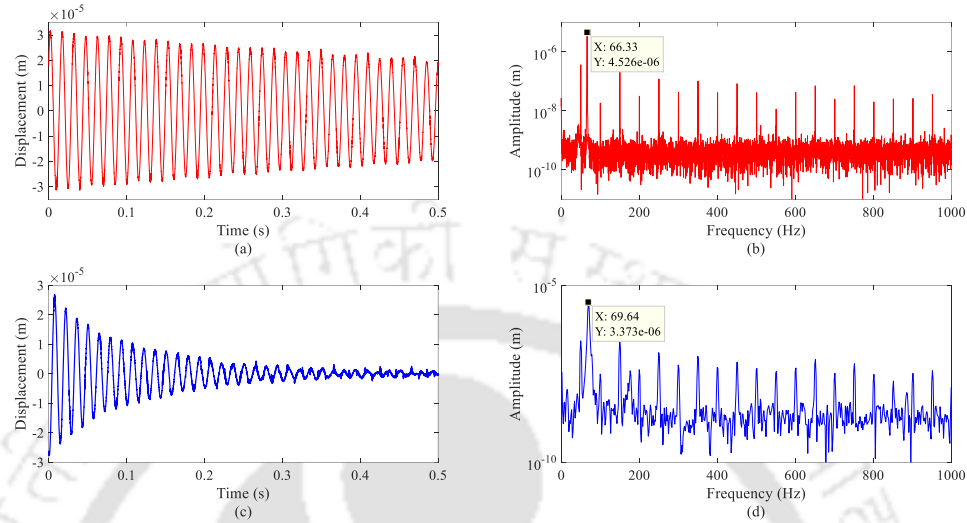


Figure 5.6 Natural frequency of a healthy shaft without (— solid line) and with (— solid line) AMB (a) time domain ,and (b) frequency domain response without AMB (c) time domain, and (d) frequency domain response with AMB

The natural frequency of the cracked shaft without and with AMB integration is evaluated based on the different directions of cracked faces as illustrated in Chapter 3. The cracked shaft is excited with an impulsive force in three conditions such as cracked face is in downward direction, cracked face is in horizontal or side (left and right with respect to the analyst) and when the cracked face is in upward direction. The time domain signals in the three cracked face orientations without AMB integration, respectively, as shown in Figure 5.7 (a, c, e). The natural frequency of the test rotor without AMB integration when the crack in downward direction is the lowest frequency of 61.94 Hz due to the reduction in stiffness while experiencing tension in operation as observed in Figure 5.7(b).

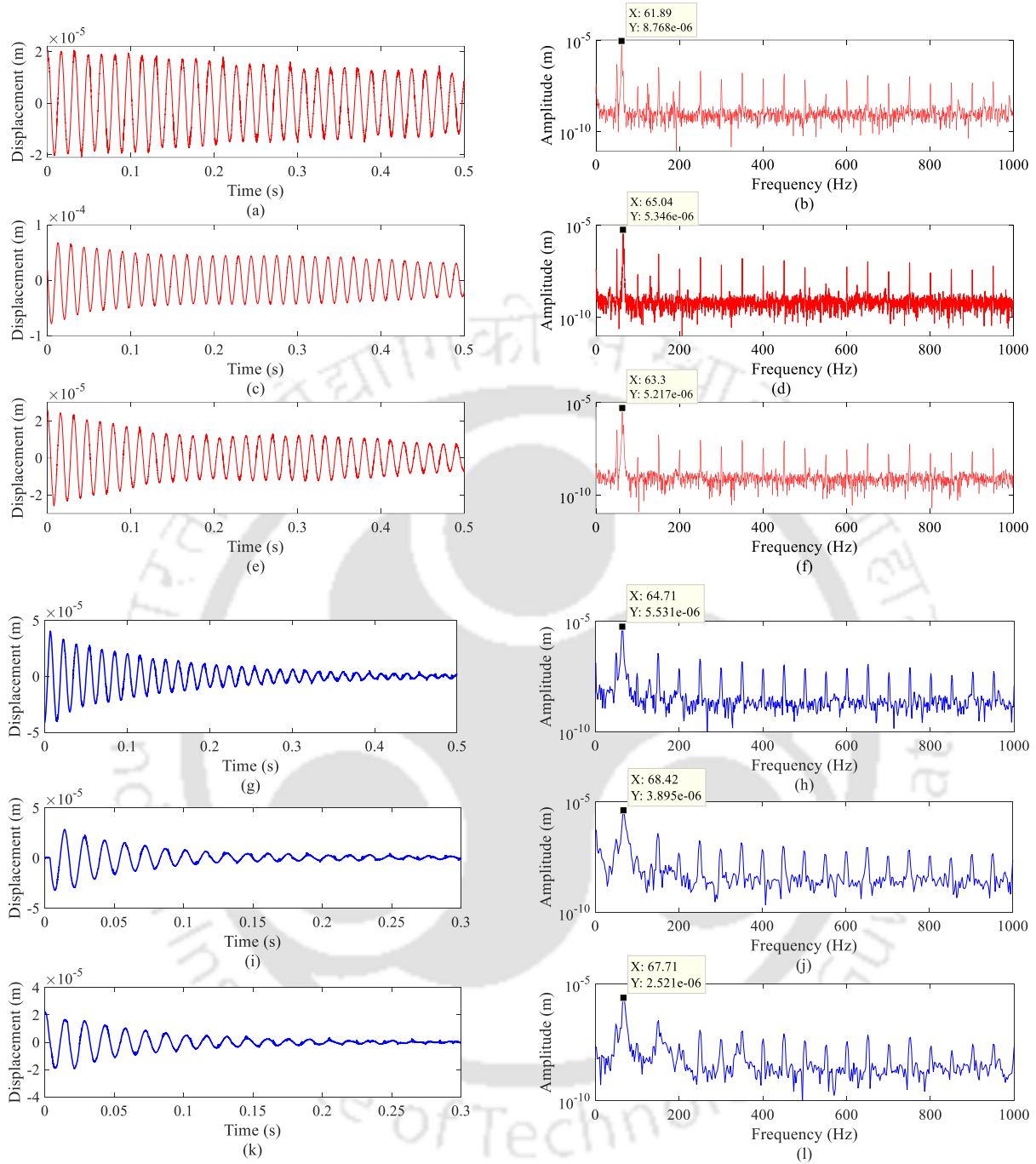


Figure 5.7 Natural frequency of cracked shaft with different crack orientation (time and frequency domain responses) without (— solid line) and with (— solid line) AMB
(a, b, g, h) crack face is in the downward direction (c, d, i, j) crack face is in the horizontal direction (e, f, k, l) crack face is in the upward direction

Similarly, when the cracked face orientation is in the horizontal (side) direction behaves as healthy (uncracked) shaft as that face is in fully closed condition due to compression and the natural frequency becomes 65.36 Hz as shown in Figure 5.7(d). When the cracked face is in upward direction that face is experiencing partly tension and compression having natural frequency of 63.32 Hz. With AMB integration time domain signals of the cracked rotor with three different cracked face orientations at, viz. bottom position, right side at 90° anticlockwise from the top position and top position of the shaft are shown in Figure 5.7(g, i, k). The natural frequencies corresponding to three orientations with AMB case are 64.71 Hz, 68.42 Hz and 67.71 Hz, respectively, depicted in Figure 5.7(h, j, l). AMB as a controller increases the stiffness of the rotor and hence, increases the natural frequency as observed in the above results.

5.2.2 Forced Vibration Analysis

The forced vibration analysis is done on the healthy and cracked shaft mounting on the test rig to show the comparison of the amplitude responses of two of them having same physical configuration. The whirl orbits of the healthy and cracked shaft rotating at various spin speed conditions such as 900 rpm (15 Hz), 1200 rpm (20 Hz), 1800 rpm (30 Hz) and 2040 rpm (34 Hz), respectively, are shown in Figure 5.8(a, d). It is observed from the Figure 5.8 that the amplitude of vibration is more in cracked shaft of magnitude 3.5×10^{-4} m, whereas 1.02×10^{-4} m for healthy shaft condition with the AMB support condition. This necessitates the study of dynamic behaviour of cracked shaft to avoid catastrophic failure of the rotating system.

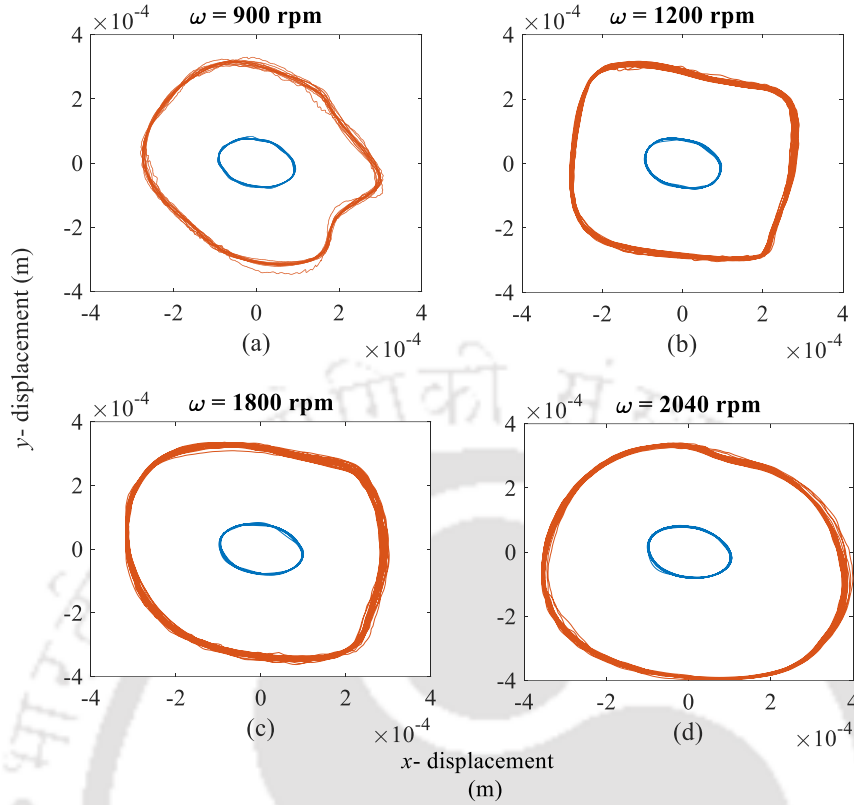


Figure 5.8 Whirl orbits of the cracked and healthy rotor with AMB during the passage through several speeds (the blue solid curve is the orbit of the healthy rotor, and the dotted red curve is the orbit of the cracked rotor)

Further, the residual shaft bow in a warped rotor, much affects the dynamics of the system by increasing the amplitude of vibration. The bow effect is prominent in slow roll condition that will be analysed in the later part. The bow amplitude overshadows the effect of crack present in a system, so the effect of bow is removed from 1X harmonic from the signal to show the cracked response as shown in Figure 5.9. Figure 5.9(a, b) shows the vibrational displacements in transverse directions for the spin speed of 15 Hz having magnitude of 3.45×10^{-4} m and 3.21×10^{-4} m with and without AMB support respectively. After removal of residual bow, the cracked rotor response with AMB control becomes 1.02×10^{-4} m,

where 68% reduction in amplitude from the cracked warped (bowed) rotor without AMB control.

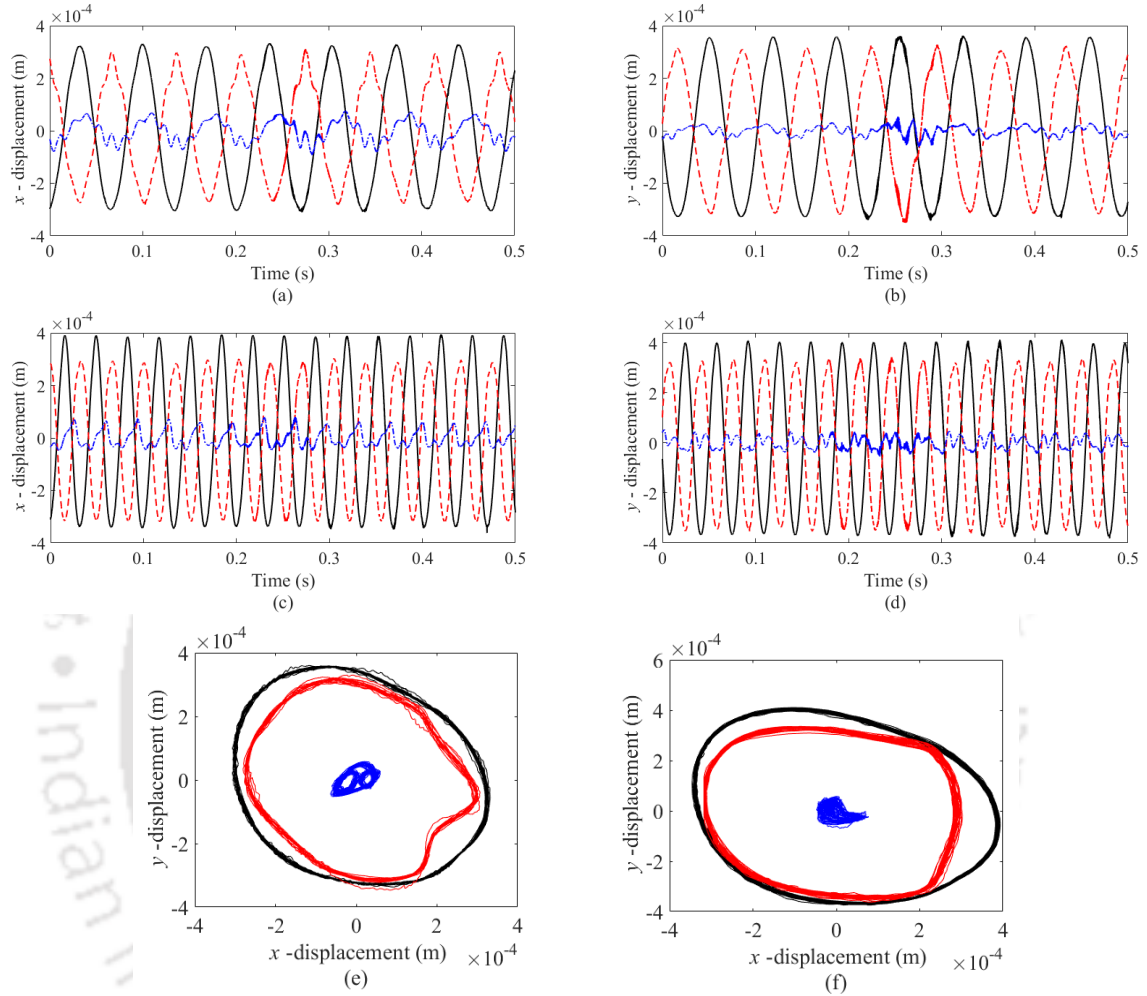


Figure 5.9 Time waveform of the cracked rotor system without and with AMB control

(a) at spin speed of approximately 15 Hz (b) at spin speed of approximately 30 Hz (— line denotes the response with residual bow and without AMB control, - - - dotted line denotes the response with residual bow and with AMB control, and dotted line denotes response with AMB control and without residual bow)

Similarly, the reduction in vibration with AMB control for higher spin speed case of 30 Hz is shown in Figure 5.9(c, d). The orbital shapes without and with bow removal along with the without and with AMB control is observed in Figure 5.9(e, f) for above mentioned two

speed conditions. Thus, estimation of residual bow along with other fault parameters as discussed in Chapter 3 are also identified in the current chapter for the offset disc condition. As previously mentioned that gyroscopic effect depends on the rotating speed and the position of the mountings on the rotor. In the present experimentation, the disc is placed on the shaft at offset position and the shaft itself having a fatigue crack produces flexibility into the rotor system during operation. The rotor experiences the gyroscopic couple effect between the tilted disc and the shaft centreline during whirling.

The dynamic characteristics of the test rig is analyzed by the full spectrum of displacement and current generated from the measured data in the vertical and horizontal directions at various speed ranges. To gain insight into the condition or health of the machine full spectrum analysis in frequency domain is studied and further these full spectrum amplitude and phases after compensation will be used for fault parameters estimation also. The full spectrum response is generated by converting the measured time domain measured data in time domain into frequency domain by Fast Fourier transform (FFT). At first, a comparative analysis of the full spectrum response of the healthy shaft and cracked shaft with AMB control is done. The rotor is operated at the speed of 34 Hz and observed that the healthy shaft full spectrum amplitude and phase contains 1X and -1X harmonics due to unbalance in the system and due to bearing asymmetry (or slight rubbing effect), respectively, as shown in Figure 5.10(a, c). On the contrary, the full spectrum response of cracked shaft contains multiple harmonic in both forward and backward precessional modes due to opening and closure of crack faces due tension and compression in the shaft while whirling, as shown in Figure 5.10(b, d). The amplitude of

vibration of at 1X harmonic of healthy shaft is 9.036×10^{-5} m and 3.622×10^{-4} m for the cracked shaft system are observed from Figure 5.10.

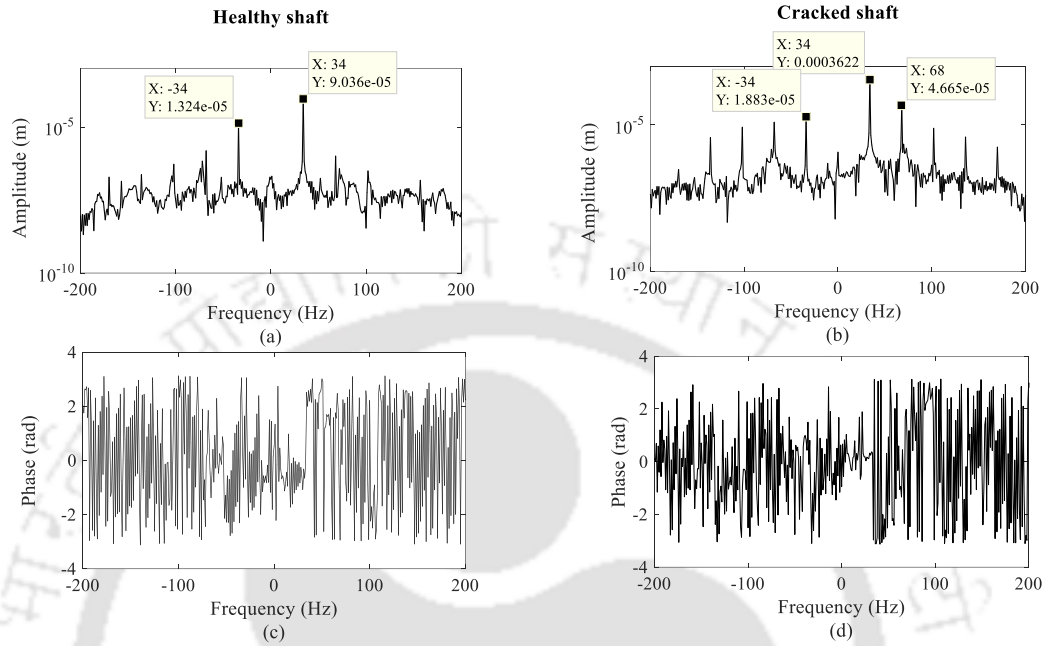


Figure 5.10 Comparison of full spectrum response (a, c) amplitude and phase of healthy shaft (b, d) amplitude and phase of cracked shaft

Further in the identification procedure as mentioned in earlier chapters also, for every single speed case, a reference signal is measured for acquiring displacement signals of shaft and control current signal of AMB for complete multiple shaft rotational cycles, i.e. $\omega t = 2\pi n$ where, n defines the number of complete cycle to avoid leakage error in the signal. Figure 5.11(a, b) shows the measured reference signal of the rotor system for 1 sec. Four complete cycles of the reference signal for 4 Hz (slow run condition) motor speed from the vertical direction sensor near the motor spindle is taken as in Figure 5.11(a). Similarly, rotor operating at approximately at 29 Hz speed, the reference signal for 29 complete cycle is as shown in Figure 5.11(b). For the same time duration, displacement and control current signals are acquired near the disc and AMB location in transverse

directions of the rotor. Figure 5.12(a, b) show the displacements in the vertical and horizontal directions of the rotor for the operating condition of nearly 4 Hz speed. Similarly, the control current required to suppress that rotor vibration is shown in Figure 5.12(c, d) in both the directions. The orbit of shaft displacement and AMB control current is depicted in Figures 5.12(e and f), respectively, for the spin speed of 4 Hz. The time domain response of the rotor operating at 29 Hz is shown in Figure 5.13. The displacement signal and control current signal are shown in Figure 5.13(a, b, c, d). The orbital shape of displacement and control current are depicted in Figure 5.13(e, f). To show the dynamic behaviour of the system, the time domain responses of two speed cases, one at slow speed, and one at high speed are presented. Nevertheless, for the identification process, different ranges of speeds are chosen from 4 Hz to 34 Hz range.

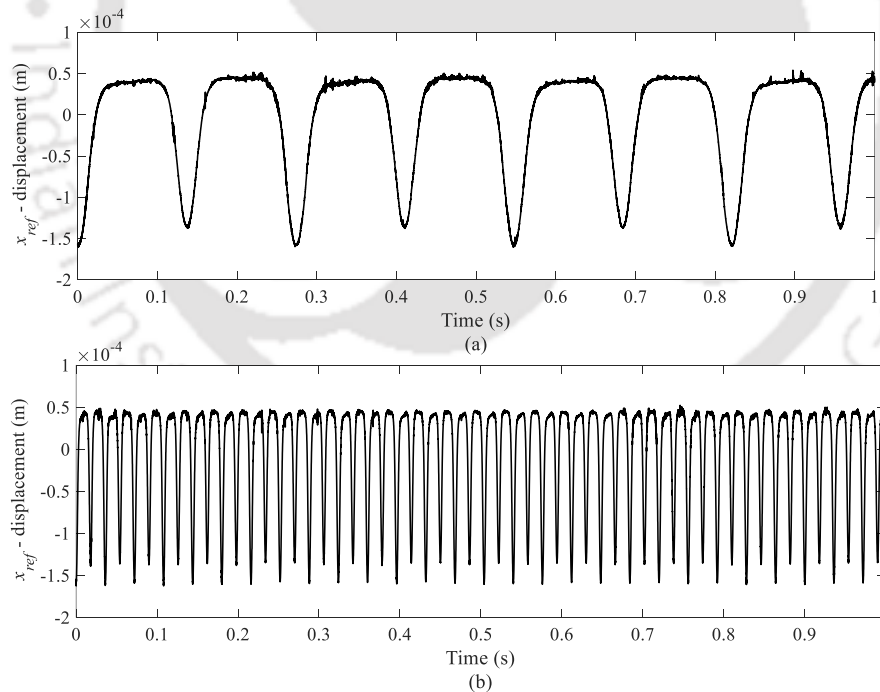


Figure 5.11 Reference signals of the system operating at approximately (a) spin speed 4 Hz (b) spin speed 29 Hz

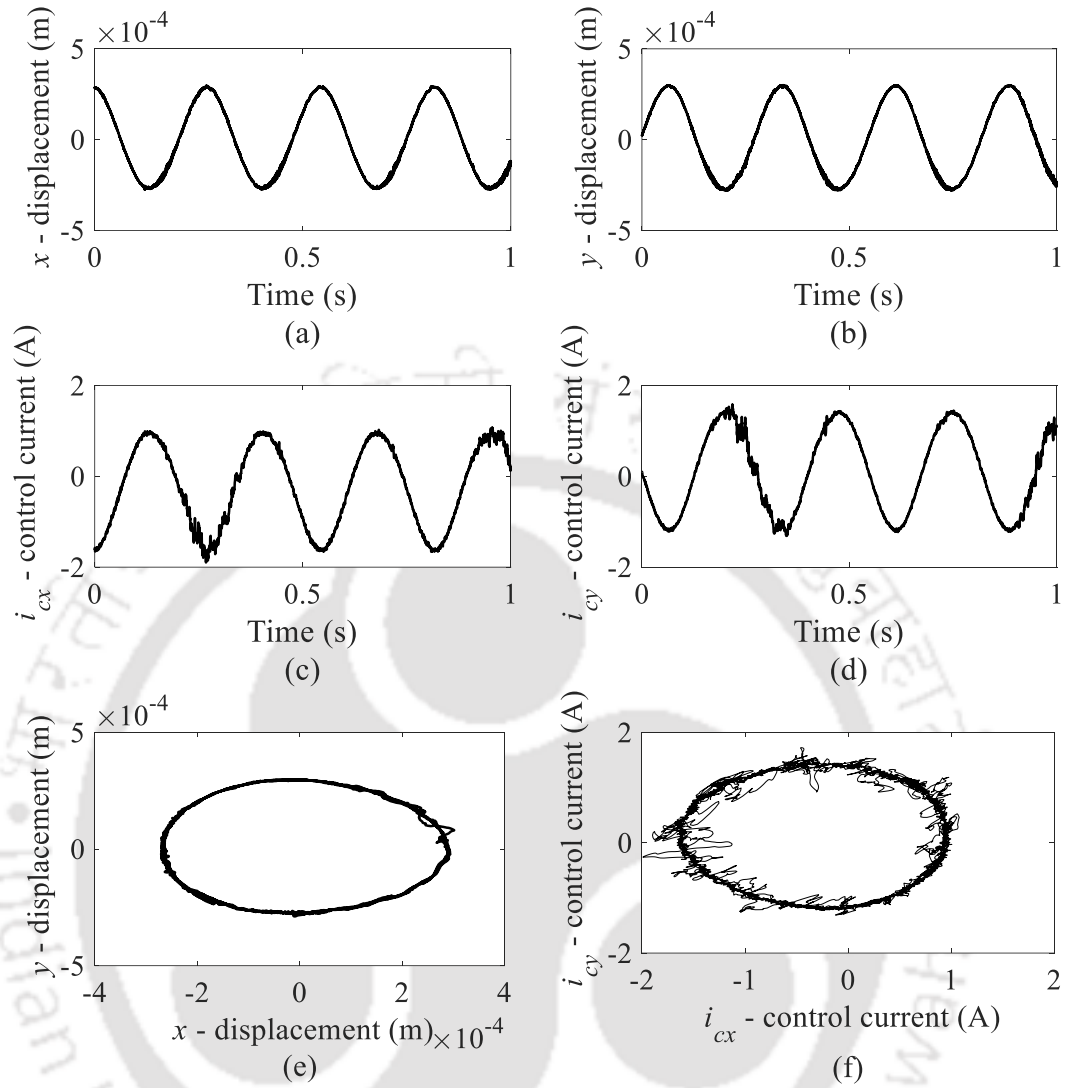


Figure 5.12 Time waveform of rotor at approximately 4 Hz of spin speed (a, b) displacements at vertical and horizontal direction (c, d) control current of AMB in vertical and horizontal direction (e) orbit of displacement signal (f) orbit of control current signal

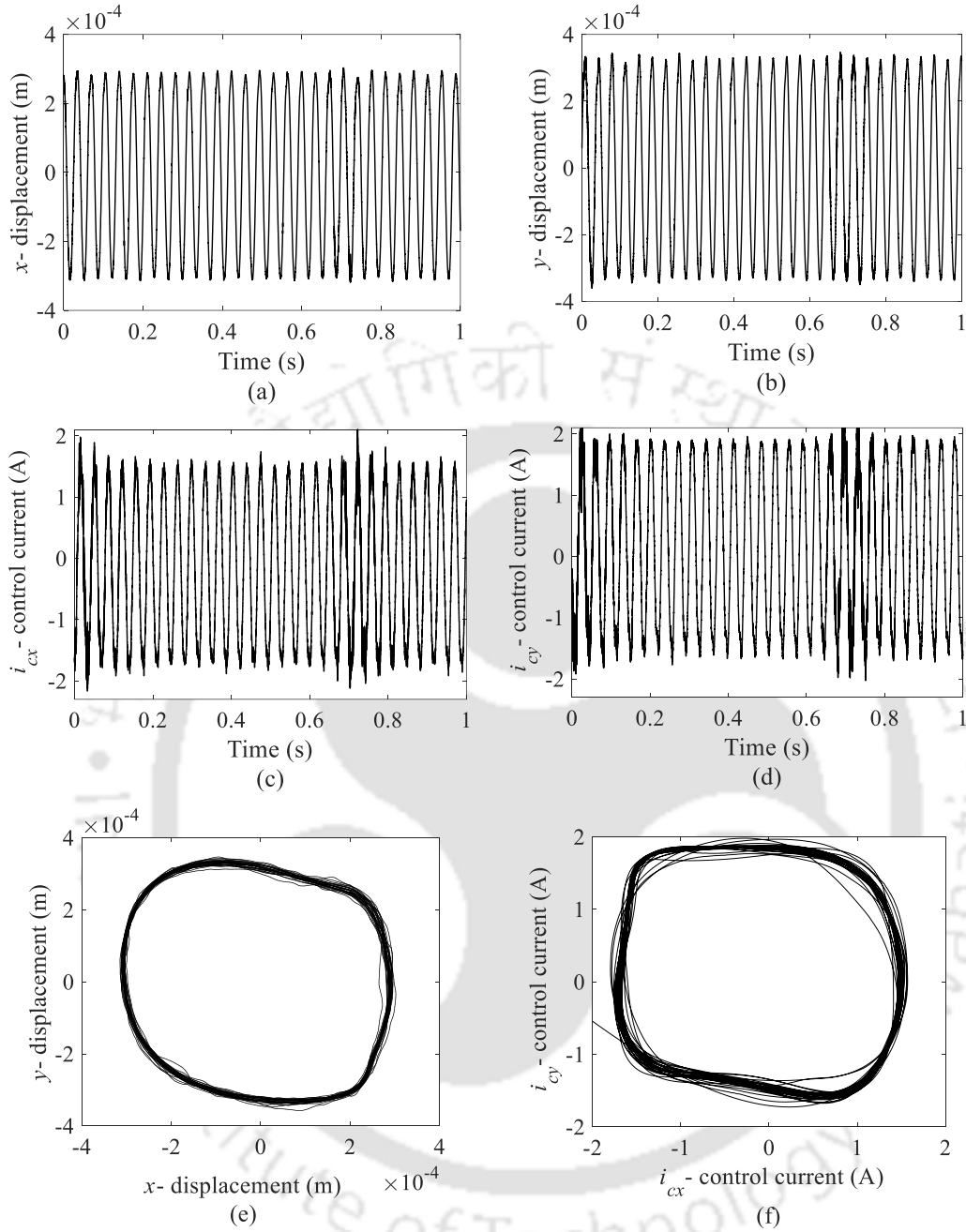


Figure 5.13 Time waveform of rotor at approximately 29 Hz of spin speed (a, b) displacements at vertical and horizontal direction (c, d) control current of AMB in vertical and horizontal direction (e) orbit of displacement signal (f) orbit of control current signal

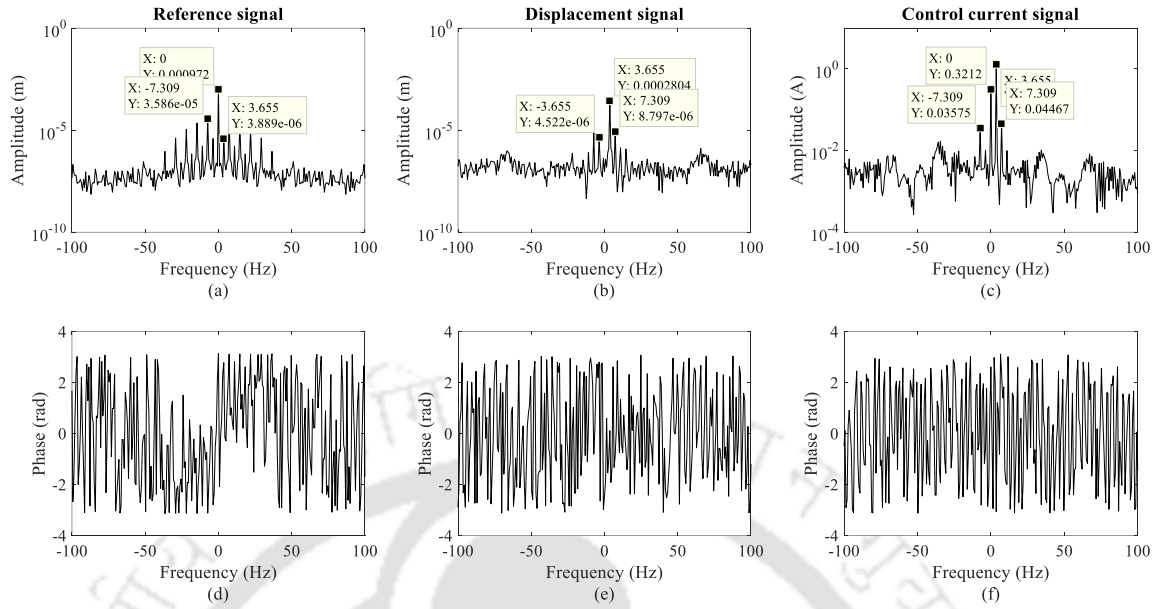


Figure 5.14 Full spectrum amplitude and phase of rotor signal measured at approximately 4 Hz (a, d) reference (b, e) vertical direction (c, f) horizontal direction

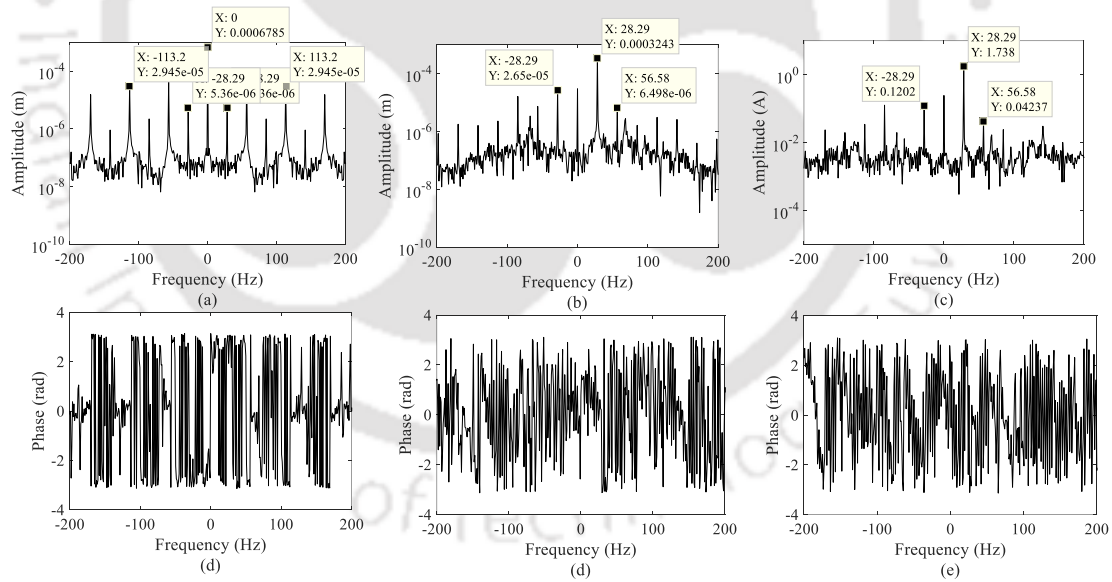


Figure 5.15 Full spectrum amplitude and phase of rotor signal measured at approximately 29 Hz (a, d) reference (b, e) vertical direction (c, f) horizontal direction

Full spectrum responses as discussed in earlier chapters, are analysed for the rotor with offset disc mounting also. The acquired time domain signal from the rotor test data near

the disc location are converted into full spectrum in frequency domain. The multi-harmonic full spectrum response at the slow speed of 4 Hz condition is shown in Figure 5.14. The amplitude and phase of reference signal is depicted in Figure 5.14(a, d). The displacement and control current signal amplitudes and phases after compensated with the phase of reference signal are shown in Figure 5.14(b, c, e, f). Similarly, full spectrum response at the spin speed of 29 Hz is shown in Figure 5.15 (a, b, c, d, e, f). The amplitude and phases of full spectrum responses of reference, displacement, and current signal from the test rig operated at 4 Hz and 29 Hz spin speeds are presented in Table 5.2. These full spectrum amplitudes and phases are further implemented in the identification algorithm for fault estimation.

5.3 Experimental Identification

The amplitudes and phases of full spectrum displacement responses, after application of the phase compensation are utilized in identification of rotor parameters using the regression Equations (4.42) to estimate multiple fault parameters.

$$\mathbf{A}_{1((2n+1) \times r)} \mathbf{x}_{1(r \times 1)} = \mathbf{b}_{1(1 \times (2n+1) \times r)} \quad (5.1)$$

with

$$\mathbf{A}_{1((2n+1) \times r)} = \begin{bmatrix} 0 & \omega S_{0,lm} & -p_0 q_{w_1} & -t_0^d p_0 q_{w_2} & -q_0 & 0 & 0 & 0 & 0 & -S_{0,Re} & I_{0,Re} \\ -\omega S_{0,lm} & 0 & -p_1 q_{w_1} & -t_1^d p_1 q_{w_2} & -q_1 & -m\omega^2 & 0 & -(k_{22} + t_1^d k_{24}) & 0 & -S_{1,Re} & I_{1,Re} \\ -2\omega S_{0,lm} & -\omega S_{1,lm} & -p_1 q_{w_1} & -t_2^d p_2 q_{w_2} & -q_2 & 0 & 0 & 0 & 0 & -S_{2,Re} & I_{1,Re} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \omega S_{-1,lm} & 2\omega S_{-1,lm} & -p_{-1} q_{w_1} & -t_{-1}^d p_{-1} q_{w_2} & -q_0 & 0 & -m\omega^2 & 0 & -(k_{22} + t_1^d k_{24}) & -S_{-1,Re} & I_{-1,Re} \\ 3\omega S_{-2,lm} & 4\omega S_{-2,lm} & -p_{-2} q_{w_1} & -t_{-2}^d p_{-2} q_{w_2} & -q_{-1} & 0 & 0 & 0 & 0 & -S_{-2,Re} & I_{-2,Re} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & (\omega S_{0,Re} + q_{w_1}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_{0,lm} & I_{0,lm} \\ \omega S_{0,Re} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_{1,lm} & I_{1,lm} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\omega S_{-1,Re} & -2\omega S_{-1,Re} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_{-1,lm} & I_{-1,lm} \\ -3\omega S_{-3,Re} & -4\omega S_{-2,Re} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_{-1,lm} & I_{-2,lm} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

And, $\mathbf{x}_1 = [c_{22} \quad c_H \quad \Delta k_{c22} \quad \Delta k_{c44} \quad \Delta k_{c22} e_{b,Re} \quad e_{Re} \quad e_{Im} \quad e_{b,Re} \quad e_{b,Im} \quad k_s \quad k_i]^T$

$$\mathbf{b}_i = \left\{ \begin{array}{c} 0 \\ \left(\omega^2 (m + (t_1^d)^2 I_d) S_{1,Re} + \omega^2 I_p (t_1^d)^2 S_{1,Im} + \left(\omega (t_1^d)^2 + 2\omega t_1^d \right) S_{1,Im} - (k_{22} + (t_1^d)^2 k_{44}) S_{1,Re} \right) \\ \left(4\omega^2 (m + (t_2^d)^2 I_d) S_{2,Re} + 2\omega^2 I_p (t_2^d)^2 S_{2,Im} + \left(2\omega (t_2^d)^2 + 4\omega t_2^d \right) S_{2,Im} - (k_{22} + (t_2^d)^2 k_{44}) S_{2,Re} \right) \\ \left(9\omega^2 (m + (t_3^d)^2 I_d) S_{3,Re} + 3\omega^2 I_p (t_3^d)^2 S_{3,Im} + \left(3\omega (t_3^d)^2 + 9\omega t_3^d \right) S_{3,Im} - (k_{22} + (t_3^d)^2 k_{44}) S_{3,Re} \right) \\ \vdots \\ \left(\omega^2 (m + (t_{-1}^d)^2 I_d) S_{-1,Re} - \omega^2 I_p (t_{-1}^d)^2 S_{-1,Im} - \left(2\omega t_{-1}^d + \omega (t_{-1}^d)^2 \right) S_{-1,Im} - (k_{22} + (t_{-1}^d)^2 k_{44}) S_{-1,Re} \right) \\ \left(9\omega^2 (m + (t_{-3}^d)^2 I_d) S_{-3,Re} - 3\omega^2 I_p (t_{-3}^d)^2 S_{-3,Im} - \left(6\omega t_{-3}^d + 3\omega (t_{-3}^d)^2 \right) S_{-3,Im} - (k_{22} + (t_{-3}^d)^2 k_{44}) S_{-3,Re} \right) \\ \vdots \\ 0 \\ \left(\omega^2 (m + (t_1^d)^2 I_d) S_{1,Im} + \omega^2 I_p (t_1^d)^2 S_{1,Re} - \left(2\omega t_2^d + \omega (t_2^d)^2 \right) S_{1,Re} - (k_{22} + (t_1^d)^2 k_{44}) S_{1,Im} \right) \\ \left(4\omega^2 (m + (t_2^d)^2 I_d) S_{2,Im} + 2\omega^2 I_p (t_2^d)^2 S_{2,Re} - \left(4\omega t_2^d + 2\omega (t_2^d)^2 \right) S_{2,Re} - (k_{22} + (t_2^d)^2 k_{44}) S_{2,Im} \right) \\ \left(9\omega^2 (m + (t_3^d)^2 I_d) S_{3,Im} + 3\omega^2 I_p (t_3^d)^2 S_{3,Re} - \left(6\omega t_3^d + 3\omega (t_3^d)^2 \right) S_{3,Re} - (k_{22} + (t_3^d)^2 k_{44}) S_{3,Im} \right) \\ \vdots \\ \left(\omega^2 (m + (t_{-1}^d)^2 I_d) S_{-1,Im} - \omega^2 I_p (t_{-1}^d)^2 S_{-1,Re} + \left(2\omega t_{-1}^d + \omega (t_{-1}^d)^2 \right) S_{-1,Re} - (k_{22} + (t_{-1}^d)^2 k_{44}) S_{-1,Im} \right) \\ \left(9\omega^2 (m + (t_{-3}^d)^2 I_d) S_{-3,Re} - 3\omega^2 I_p (t_{-3}^d)^2 S_{-3,Im} + \left(6\omega t_{-1}^d + 3\omega (t_{-3}^d)^2 \right) S_{-3,Re} - (k_{22} + (t_{-3}^d)^2 k_{44}) S_{-3,Im} \right) \\ \vdots \end{array} \right\}$$

In identification, the unknown parameter vector $\mathbf{x}_1$, Equation (5.1)

$$\mathbf{x}_1 = (\mathbf{A}_1^T \mathbf{A}_1)^{-1} \mathbf{A}_1^T \mathbf{b}_1 \quad (5.2)$$

The identification of unknown additive and multiplicative fault parameters $\mathbf{x}$ is reliable while using information for the multiple speed ranges of interest. On combining Equation (5.2) for different spin speeds

$$\left\{ \begin{array}{c} \mathbf{A}_1(\omega_1) \\ \mathbf{A}_1(\omega_2) \\ \vdots \\ \mathbf{A}_1(\omega_n) \end{array} \right\} \mathbf{x}_1 = \left\{ \begin{array}{c} \mathbf{b}_1(\omega_1) \\ \mathbf{b}_1(\omega_2) \\ \vdots \\ \mathbf{b}_1(\omega_n) \end{array} \right\} \quad (5.3)$$

Equation (5.3) is utilized for estimation of fault parameters as presented in Table 5.3.

Table 5.2 Full spectrum amplitude and phases of reference, displacement, and current signal of the test rig operating at speed of 4 and 29 Hz

<i>i</i>	Reference signal				Displacement signal				Control current signal			
	Amplitude (m)		Phase compensation (rad)		Amplitude (m)		Phase compensation (rad)		Amplitude (A)		Phase compensation (rad)	
	A*	B*	A*	B*	A*	B*	A*	B*	A*	B*	A*	B*
0	9.07×10^{-4}	6.78×10^{-4}	0.00	0.00	9.26×10^{-8}	2.889×10^{-5}	-0.69	0.78	0.32	0.25	1.93	2.22
1X	3.89×10^{-6}	5.36×10^{-6}	2.78	2.86	2.80×10^{-4}	3.243×10^{-4}	0.12	0.21	1.29	1.74	-3.05	-2.85
2X	3.59×10^{-5}	4.43×10^{-5}	3.12	3.09	8.79×10^{-6}	6.498×10^{-6}	-0.87	-2.96	0.04	0.04	2.38	1.15
3X	1.66×10^{-6}	2.19×10^{-6}	2.83	1.43	1.35×10^{-6}	2.703×10^{-6}	1.36	-0.76	0.003	0.02	-1.99	1.62
4X	2.25×10^{-5}	2.95×10^{-5}	1.42	2.98	1.20×10^{-6}	1.582×10^{-6}	-0.59	0.23	0.004	0.02	1.59	-2.89
5X	6.79×10^{-7}	9.06×10^{-7}	3.12	3.01	2.14×10^{-6}	2.995×10^{-6}	1.47	0.51	0.002	0.01	2.59	-2.05
6X	1.08×10^{-5}	1.53×10^{-5}	2.62	3.07	1.36×10^{-6}	2.531×10^{-6}	0.67	0.15	0.002	0.02	-2.01	0.59
-1X	3.89×10^{-6}	5.36×10^{-6}	-2.78	-2.86	4.52×10^{-6}	2.65×10^{-5}	2.81	-2.43	0.01	0.12	-0.58	0.77
-2X	3.59×10^{-5}	4.43×10^{-5}	-3.12	-3.09	7.27×10^{-6}	7.161×10^{-6}	0.24	1.97	0.04	0.01	2.36	-1.49
-3X	1.66×10^{-6}	2.19×10^{-6}	-2.83	-1.43	8.15×10^{-6}	1.576×10^{-5}	1.62	-2.81	0.007	0.02	-2.31	-3.04
-4X	2.25×10^{-5}	2.95×10^{-5}	-1.42	-2.98	3.412×10^{-7}	2.226×10^{-6}	-2.15	2.17	0.003	0.13	2.04	-1.11
-5X	6.79×10^{-7}	9.06×10^{-7}	-3.12	-3.07	1.59×10^{-7}	1.576×10^{-6}	0.16	2.06	0.005	0.03	-0.08	-1.13
-6X	1.08×10^{-5}	1.53×10^{-5}	-2.62	-2.86	1.31×10^{-7}	1.732×10^{-6}	-0.89	-0.58	0.009	0.02	-2.54	1.07

i: No. of harmonics; A*: slow speed (4 Hz) ; B*: high speed (29 Hz)

Table 5.3 Experimentally estimated multiple faults of the test rig

Estimated parameters	20 Hz to 22 Hz	25 Hz to 28 Hz	29 Hz to 31Hz
	Additive fault parameters		
c_{22} (Nsm ⁻¹)	457.748	420.515	350.782
c_H (Nsm ⁻¹)	333.978	289.153	264.677
Δk_{c22} (Nm ⁻¹)	1.5929×10^5	1.3997×10^5	3.5002×10^5
Δk_{c44} (Nmrad ⁻¹)	1.9799×10^4	3.6897×10^4	4.8168×10^3
e_u (m)	3.8169×10^{-4}	4.538×10^{-4}	2.615×10^{-4}
ϕ_u (deg)	77.812	-32.634	84.322
e_b (m)	1.118×10^{-4}	2.052×10^{-4}	2.108×10^{-4}
ϕ_b (deg)	18.413	31.782	26.215
k_s (Nm ⁻¹)	2.7901×10^5	1.3105×10^5	1.6616×10^5
k_i (NA ⁻¹)	11.345	7.691	16.133
Multiplicative fault parameter			
$\Delta k_{c22} e_{b,Re}$ (N)	20.253	22.088	33.850

Estimation is done for the data acquired from the test rig operating from 20 Hz through 31 Hz spin speeds. Three different speed ranges of 20 Hz through 22 Hz, 25 Hz through 28 Hz and 29 Hz through 31 Hz test data are utilized to obtain the multiple faults associated with the test bench. It is observed from the experimental results that the external damping value is in a same range as found in the experimentation discussed in Chapter 3 (Table 3.7). Moreover, the consistency in estimates is seen in the external damping and the internal damping. The residual bow magnitude is also consistent in all the speed ranges and found in the range of 1X harmonic magnitude at slow roll condition (as bow effect is prominent in slow speed). The translational additive stiffness (Δk_{c22}) is found to be around to be 47% of reduction in the stiffness (k_{22}) as calculated through influence co-efficient provided in Table 5.1. The rotational additive stiffness (Δk_{c44}) gives around 20% reduction in stiffness of the shaft (k_{44}) in first and the second speed range cases. Whereas, the in second speed range (25 Hz through 28 Hz) its estimated value is

not found to be appropriate as the value found to be greater than the shaft stiffness, i.e. value of k_{44} calculated as presented in Table 5.1. The reason would have been due to the implementation of gyroscopic condensation scheme (GDC) implementation to remove the rotational DOFs in the system in the identification procedure as elaborated in Chapter 4. In experimental testing, even the rotational displacements measuring instrument, like laser vibrometer, cannot measure that vibrational displacement accurately due to the presence of crack and residual bow. Due to the residual bow, the center of the shaft with that of the offset disc varies during operation, thus, the tilting of the disc is not able to measure precisely. Further, the effect of bow in the rotational displacements of the rotor is dominant enough than that of the tilting of the disc. Therefore, to overcome the difficulty in accurate measurement of the angular displacements, a condensation scheme is implemented to remove the rotational DOFs by retaining the translational ones in this identification methodology. The AMB force-displacement (k_s) and force-current (k_i) factors are found to be in line with the analytically calculated ones using Equation (2.22).

5.3.1 Justification of the Estimated Results with the Numerical Simulation

In this section, a numerical simulation has been done through MATLAB Simulink™ model based on test rig configuration and estimated fault parameters through experimentally acquired responses and data provided in Tables 5.1 and 5.2. Moreover, balancing of the rotor system is also analysed here. Based on developed EOMs for the offset disc case, the time domain data is generated through SIMULINK™ model for the samples considered as in the experimentations of 10,000 data points. The simulation is run for 5 s for the speed of 20 Hz. The full spectrum response of the numerically simulated and experimentally found signals are compared in Figure 5.16. The amplitude of multiple harmonics in experimental response and numerically simulated response are

shown in Table 5.4. There is a very close agreement between the measured responses with the simulated ones and the pattern of the responses in the two cases are very similar, which indirectly verifies the identification methodology well.

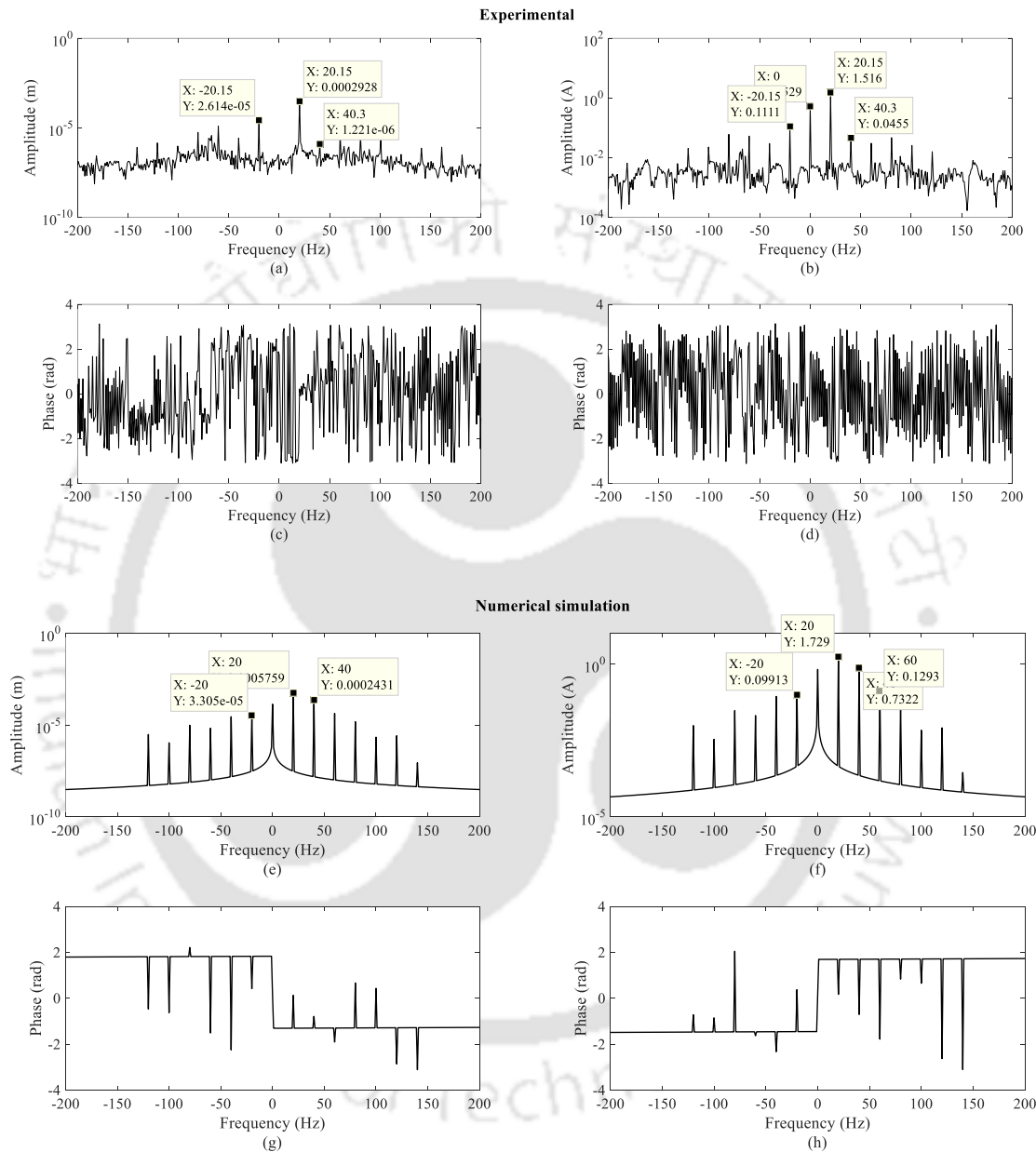


Figure 5.16 Full spectrum amplitude and phase of displacement and control current (a, b, c, d) experimental data (e, f, g, h) numerically simulated

Table 5.4 Full spectrum amplitude and phase of the numerically simulated and experimental test data for validation

No. of harmonics	Experimental full spectrum response		Simulated full spectrum response	
	Displacement amplitude (m)	Control current amplitude (A)	Displacement amplitude (m)	Control current amplitude (A)
0	3.25×10^{-5}	0.29	5.75×10^{-5}	0.861
1X	2.93×10^{-4}	1.52	3.57×10^{-4}	1.73
2X	1.73×10^{-5}	0.08	4.09×10^{-5}	0.13
3X	4.22×10^{-6}	0.03	2.93×10^{-6}	0.01
4X	2.16×10^{-6}	0.05	4.49×10^{-6}	0.02
5X	1.03×10^{-6}	0.05	1.64×10^{-7}	8.67×10^{-4}
-1X	1.53×10^{-7}	0.08	3.95×10^{-5}	0.05
-2X	1.23×10^{-5}	0.05	1.21×10^{-5}	0.05
-3X	1.40×10^{-5}	0.13	4.55×10^{-6}	0.02
-4X	1.56×10^{-5}	0.05	2.21×10^{-6}	0.01

For balancing, the unbalance magnitude estimated are used as balance mass at an orientation opposite to the estimated phase. The balancing is done considering one speed case such as at 20 Hz spin speed condition of the rotor and the time domain displacement response of the system is plotted at the balancing planes as shown in Figure 5.17(a, b). The amplitude of vibration of the test data before balancing and without AMB control is found to be 3.419×10^{-4} m and with AMB case is 3.137×10^{-4} m. After balancing with 15 g balance mass, the system's amplitude of vibration without AMB is reduced to 1.487×10^{-4} m and with AMB control is 1.3646×10^{-4} m. The displacement reduction after balancing and without AMB control is found to be 56.5% after bow removal whereas 8% reduction is found in before balancing and with AMB control system. Further, the amplitude of vibration gets reduced after balancing, bow removal and with AMB control to 60.09%, thus, it can be observed that after balancing, the control current of AMB can suppress the vibration to a fairly good amount. Therefore, it can be inferred that requirement of control current for vibration attenuation will be less if the system is balanced and hence, power consumption of AMB will be low leads to an efficient system (Ranjan and Tiwari, 2020).

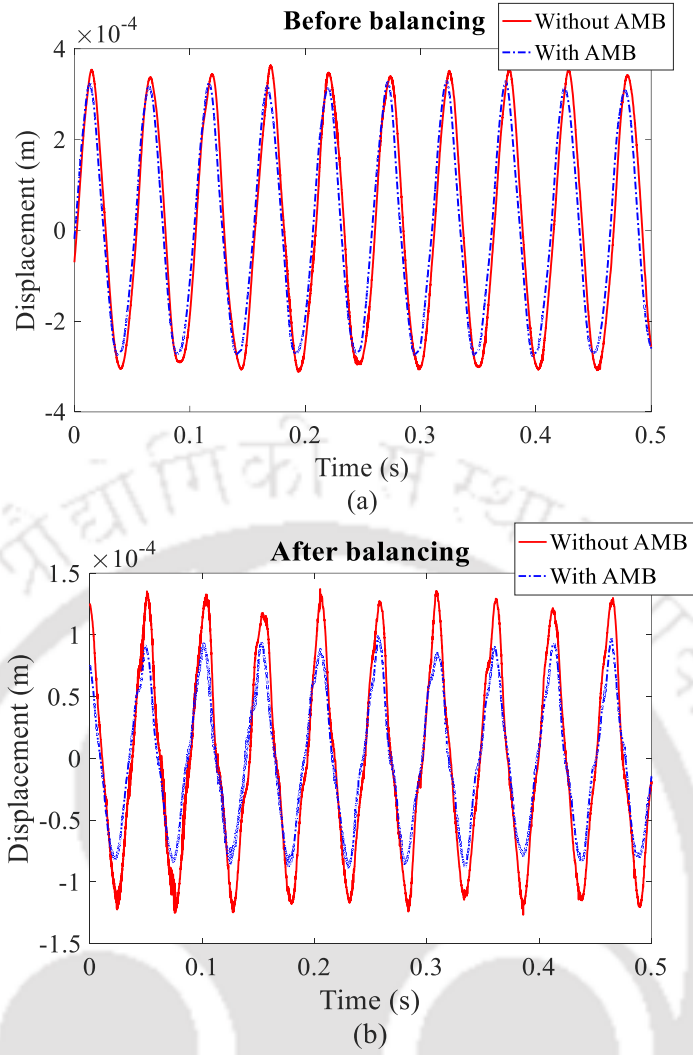


Figure 5.17 Balancing of the rotor at the speed of 20 Hz (a) without AMB control, and (b) with AMB control

5.4 Concluding Remarks

This chapter presents the experimental investigation of identification of fault parameters of an offset disc cracked-warped rotor-AMB system. The estimated fault parameters, such as the external and internal damping, unbalance and residual bow, and translational and rotational additive crack stiffness, AMB control parameters and multiplicative fault parameters of bow and crack of a rotor bearing system integrated with an AMB is studied. The rotor-AMB test data are acquired from the laboratory test rig as discussed in Chapter 3. The measured time domain responses are converted into full spectrum, and the multi-

harmonic amplitudes and phases of the displacement and control current signal are used in the developed identification algorithm method for estimation of fault parameters as has been discussed in Chapter 4 for estimation of different fault parameters. The following are the conclusions drawn from the present Chapter

- (i) The estimated parameters of the faults associated with the test rig are found to be consistent in all the three speed ranges except the rotational crack additive stiffness parameter. This may be due to the condensation scheme applied during identification algorithm development as discussed in Chapter 4. The rotational additive crack parameter may be estimated better through sophisticated instrumentation so that angular displacements can be measured with a great accuracy.
- (ii) The experimentally estimated parameters are utilized in the numerical simulation for validation of the full spectrum responses of the numerically simulated one with that of the experimental. The full spectrum responses shows the similar pattern in both the cases, which determines the accuracy of the experimental procedure of identification.
- (iii) There is a sufficient amount of reduction in vibration after utilizing the estimated unbalance magnitude as balance mass, after balancing, and with AMB control condition.

With the same approach, in the next chapter more practical rotor system is modelled mathematically using finite element approach. The developed EOMs considering multiple discs mounting on the rotor are implemented for the development of the identification algorithm to estimate multiple fault parameters. The sensitivity of the model is also tested through numerical simulation.

Chapter 6

Finite Element Modeling, Analysis and Identification of Multiple Fault Parameters in a Flexible Rotor-AMB System with Multiple Discs

Previous Chapters 2 and 4 emphasize on the modeling and analysis of a faulty rotor-AMB system and methodology has been developed where suppressed vibrational displacement of rotor and the AMB control current information are used for quantification of the fault parameters on simple systems. The current chapter deals with the modeling and analysis of a more realistic, i.e. the practical rotor system of a cracked-warped rotor mounted on flexible support bearings consisting multiple discs integrated with AMBs through finite element approach. An algorithm has been developed with the purpose of identification of additive and multiplicative fault parameters associated with the faulty system. Numerical testing is carried out to verify the identification algorithm for the warped internally damped rotor mounted two discs supported on two flexible bearings and AMB auxiliary support.

6.1 Introduction

The steam turbine-generator (STG) set in a power plants are fast rotating assemblies consisting of a shaft, discs, rim, and blades. There is a tendency to initiate fatigue cracks due to high speeds in the region of high stress concentration. Moreover, the residual shaft bow also highly affects the system dynamics during start-ups. In this study, the focus is in

attempting general formulation of identification of multiple rotor faults in a complex rotor system. For example, the estimation of fault parameter present in a warped internally damped flexible rotor system having a transverse crack integrated with a radial AMB. In the proposed work, dynamic unbalance and residual bow with their orientations are identified as sometimes a system experiences vibrations after balancing of a system due to the presence of residual bow. Thus, quantification of these two faults simultaneously can distinguish the source of vibration in a system. Moreover, the propagation of fatigue crack due to continuous loading experience by a rotor lead to failure of the machine, hence a transverse crack is modelled using a switching crack excitation function behaving the cracked faces with sudden opening and closure in the proposed model. During whirling, the cracked faces get rubbed contributing internal damping inherent material property of the shaft in the system. The identification of external as well as internal damping distinctly is very much needed to analyze the threshold speed of instability as their relative magnitudes contributes the insatiability at various speeds. The internal damping is also included in the model. After mathematical modelling, the identification algorithm is developed based on the condensed EOMs as discussed in Chapter 4 in frequency domain to estimate multiple fault parameters. The estimation of fault parameters (additive and multiplicative) is done through numerical experiment in the presence of measurement noise and modelling or bias error to check its robustness.

6.2 System Description

A steam turbine generator (STG) set supported on anisotropic flexible bearings is shown in Figure 6.1. Here, a typical finite element formulation of flexible warped shaft having a transverse crack equipped with AMB has been performed. The discs are considered to be

rigid with the rotary inertia effect representing turbines blades, flywheels, impellers, disc brakes etc. The schematic view of a general rotor-train is depicted in Figure 6.2. The rotor consists of p number of flexible bearings characterized by stiffness and damping in two orthogonal directions, k number of discs, and n number of AMBs at different locations. The rigid discs are mounted on the flexible shaft containing q number of transverse cracks.

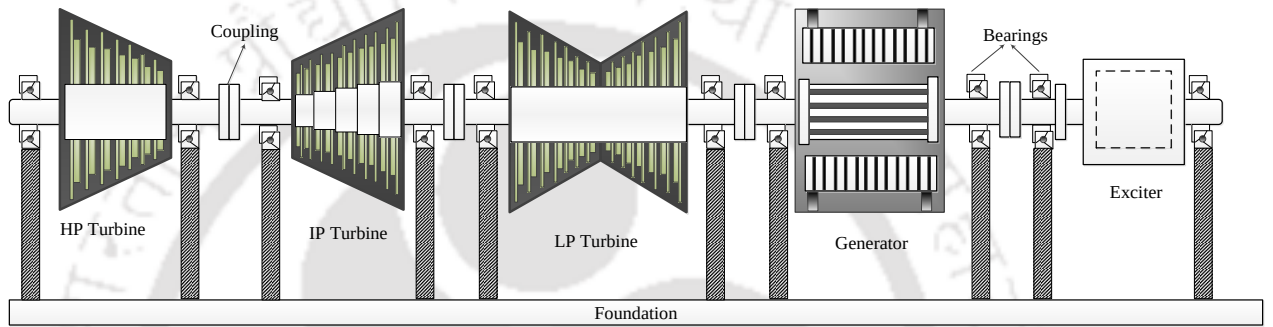


Figure 6.1 A pictorial view of steam turbine generator (STG) system

Unbalances in the rotor are assumed to be concentrated at discrete discs in different magnitudes with different orientations. Moreover, the shaft residual bow due to warped rotor with a permanent bend is also considered. The orientations of the unbalance and bow are quantified with respect to the direction perpendicular to the crack front.

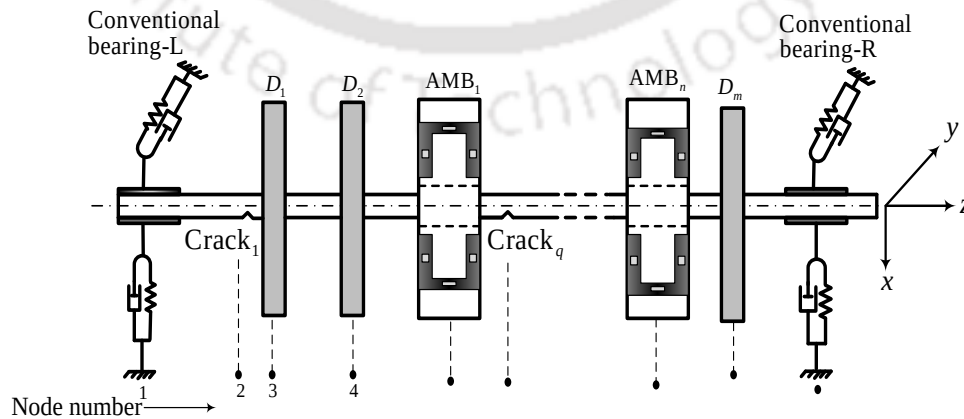


Figure 6.2 Finite element representation of a flexible rotor-AMB system

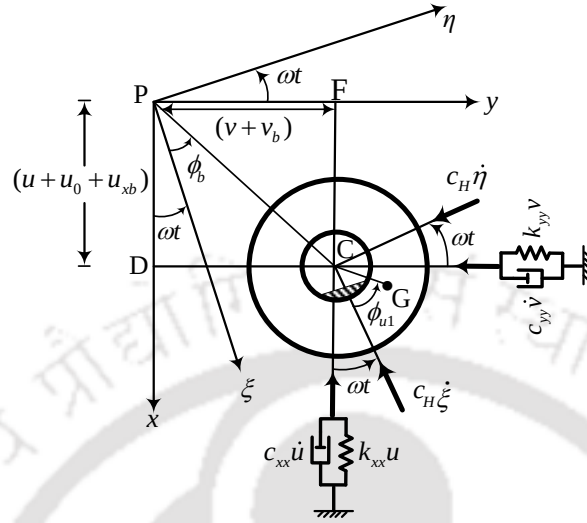


Figure 6.3 Forces acting on the rotor in the stationary and rotating co-ordinate systems

The side view of a single disc and shaft containing crack and forces acting in inertial (x - y) and rotating (ζ - η) frame of reference is shown in Figure 6.3. Here, P is the equilibrium point of the rotor about which it is rotating. The spin speed of the system is defined by ω and traversing in time t as $\omega t = \theta$. The distance CG is the eccentricity of disc; ϕ_{u1} is the unbalance phase, i.e. angle between the axis perpendicular to cross-section of crack and the eccentricity; the shaft bow (e_b) which is rotating in nature, its phase, ϕ_{u1} is considered from rotating co-ordinate system, ζ axis. The static deflection of rotor in translation and rotational direction is denoted by u_{x0} and ϕ_{y0} . The dynamic displacement due to bow is u_b . The vibrational displacements in the vertical and horizontal directions are abbreviated as u and v , respectively. Besides these, the flexible bearing produces damping (c_{xx}, c_{yy}) and restoring (k_{xx}, k_{yy}) forces in the x and y directions, and forces due to internal damping (c_H) in the ζ and η directions as shown in Figure 6.3.

6.2.1 Discussion of Matrices

In the present work, the mass, stiffness and gyroscopic matrices are modeled based on the Euler- Bernoulli beam theory (Tiwari, 2017).

6.2.1.1 Shaft Beam Model

The shaft is discretized into a number of elements where each element has two nodes. Each node of an element corresponds to two translational (u, v) and two rotational (φ_x, φ_y) DOFs as shown in Figure 6.4.

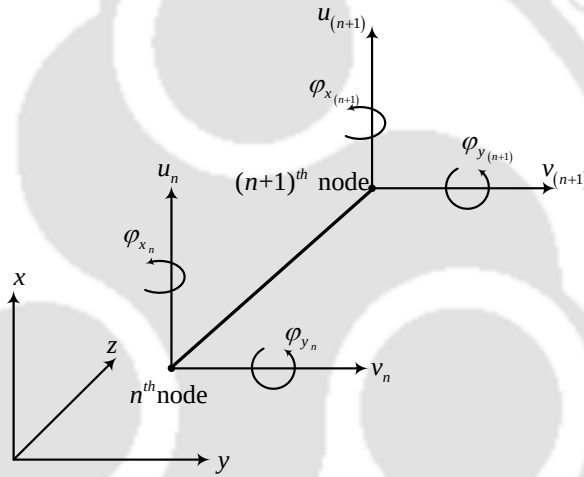


Figure 6.4 Representation of an element with nodal degrees of freedom

The equation of motion for a shaft element spinning at a speed of ω rad/s considering the viscous form of internal damping is written as (Ku, 1998; Das et al., 2008)

$$\mathbf{M}^{(e)} \ddot{\mathbf{q}}^{(ne)} + (\mathbf{K}_B^{(e)} + \omega \mathbf{K}_C^{(e)}) \mathbf{q}^{(ne)} = \mathbf{f}^{(ne)} \quad (6.1)$$

$$\mathbf{q}^{(ne)} = \begin{bmatrix} u_n & \varphi_{y_n} & u_{n+1} & \varphi_{x_{n+1}} & v_n & \varphi_{y_n} & v_{n+1} & \varphi_{x_{n+1}} \end{bmatrix} \quad (6.2)$$

With $\mathbf{q}^{(ne)}$ is the elemental nodal displacement vector and $\mathbf{f}^{(ne)}$ is the elemental force vector.

The elemental mass, bending stiffness, and circulatory matrix (skew-symmetric) due to the shaft internal damping is defined by $\mathbf{M}^{(e)}$, $\mathbf{K}_B^{(e)}$, $\mathbf{K}_C^{(e)}$ respectively.

6.2.1.2 Rigid Disc Model

The disc considered in the mathematical model is rigid, which incorporates the gyroscopic effect into the rotor system. The rigid disc have of the mass and the diametral mass moment of inertia, and are represented by a governing equation, as

$$\mathbf{M}^{(d)}\ddot{\mathbf{q}}^d - \omega\mathbf{G}^{(d)}\dot{\mathbf{q}}^d = \mathbf{f}^d \quad (6.3)$$

where disc displacement vector is defined as $\mathbf{q}^d$ at the location of the disc. The mass and gyroscopic matrices are denoted as $\mathbf{M}^{(d)}$ and $\mathbf{G}^{(d)}$ respectively.

6.2.1.3 Bow Force Model

The residual shaft bow is a permanent deformation due to stiffness variation in the shaft centerline. The distribution of the shaft residual bow (δ) of the rotor system is formulated as a second order polynomial as in Ref. (Chen, 2019)

$$\delta = \left\{ 4 \frac{z_1}{L} \left(1 - \frac{z_1}{L} \right) \quad 4 \frac{z_2}{L} \left(1 - \frac{z_2}{L} \right) \quad \dots \quad 4 \frac{z_n}{L} \left(1 - \frac{z_n}{L} \right) \right\}^T \quad (6.4)$$

where, z represents the distance of nodes from the left bearing along the axis of the shaft, L , the total length of the shaft. In general, the rotor gets permanent deformation commonly known as residual shaft bow due to fatigue, creep, etc. To show the residual bow effect on the rotor system an initial loading (f_o) is considered, which produces deformation of the rotor. The shaft residual bow force considered is represented in a single plane as

$$\mathbf{f}_{bow} = f_o \mathbf{S} \delta e^{i(\theta + \phi_b)} \quad (6.5)$$

with

$$\mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{n \times n} \quad (6.6)$$

and, $\mathbf{S}$ is the location matrix, f_o is the initial load onto the shaft at any arbitrary portion for which the permanent bow distribution occurs over the shaft.

The deflection ($\mathbf{e}_b$) on each element of the shaft due to the initial load can be obtained by multiplying the load with the inverse of the total stiffness (shaft and bearing), $\mathbf{K}$, of the system

$$\mathbf{e}_b = \mathbf{K}^{-1} \mathbf{f}_{bow} \quad (6.7)$$

The dynamic displacement ($\mathbf{q}_b$) at the crack location due to the presence of shaft residual bow (e_b) and its phase (ϕ_b) becomes

$$\mathbf{q}_b = \begin{bmatrix} u_{b_c} & v_{b_c} & \varphi_{yb_c} & \varphi_{xb_c} & u_{b_{c+1}} & v_{b_{c+1}} & \varphi_{yb_{c+1}} & \varphi_{xb_{c+1}} \end{bmatrix}^T \quad (6.8)$$

Subscript 'c' denotes the nodal point at the crack location of the shaft and $u_{b_c} = e_b \cos(\theta + \phi_b)$, $v_{b_c} = e_b \sin(\theta + \phi_b)$ with u_{b_c} and v_{b_c} are the translational dynamic displacements in the vertical and horizontal direction, respectively, as discussed in Chapter 4.

6.2.1.4 Crack Force Model

Large machinery usually offers lower vibrational displacement as compared to the static deflection of the rotor. Assuming a transverse breathing crack is present on the weighted rotor and the shaft crack forces are modeled based on a switching crack assumption, as discussed in Chapter 4, and Equation (4.13) is given as

$$\mathbf{f}_{cr} = \Delta \mathbf{K}(t) \mathbf{q}_0 = \frac{1}{2} \sigma(t) \begin{Bmatrix} \Delta k_{c22} (1 + \cos 2\theta) u_{x_0} + \Delta k_{c22} \sin(2\theta) u_{y_0} \\ \Delta k_{c22} \sin(2\theta) u_{x_0} + \Delta k_{c22} (1 - \cos 2\theta) u_{y_0} \\ \Delta k_{c44} (1 + \cos 2\theta) \varphi_{y_0} + \Delta k_{c44} \sin(2\theta) \varphi_{x_0} \\ \Delta k_{c44} \sin(2\theta) \varphi_{y_0} + \Delta k_{c44} (1 - \cos 2\theta) \varphi_{x_0} \end{Bmatrix} \quad (6.9)$$

The dynamics of the crack force is influenced by the dynamic displacement at the location of crack due to the presence of residual shaft bow, which is represented as

$$\mathbf{f}_{cr} = \frac{1}{2} \sigma(t) \begin{Bmatrix} \Delta k_{c22} (1 + \cos 2\theta) (u_0 + u_b) + \Delta k_{c22} \sin(2\theta) (v_0 + v_b) \\ \Delta k_{c22} \sin(2\theta) (u_0 + u_b) + \Delta k_{c22} (1 - \cos 2\theta) (v_0 + v_b) \\ \Delta k_{c44} (1 + \cos 2\theta) (\varphi_{y_0} + \varphi_{y_b}) + \Delta k_{c44} \sin(2\theta) (\varphi_{x_0} + \varphi_{x_b}) \\ \Delta k_{c44} \sin(2\theta) (\varphi_{y_0} + \varphi_{y_b}) + \Delta k_{c44} (1 - \cos 2\theta) (\varphi_{x_0} + \varphi_{x_b}) \end{Bmatrix} \quad (6.10)$$

In the complex form, Equation (6.10) becomes

$$\begin{aligned} \mathbf{f}_{cr} &= \frac{1}{2} \sigma(t) \begin{Bmatrix} \Delta k_{c22} (1 + e^{2j\theta}) (u_0 + u_{b_0}) + \Delta k_{c22} e_b \cos \phi_b e^{j\theta} \\ \Delta k_{c44} (1 + e^{2j\theta}) \varphi_{y_0} \end{Bmatrix} \\ &= \begin{Bmatrix} \Delta k_{c22} (u_0 + u_{b_0}) \sum_{i=-n}^n p_i + \Delta k_{c22} e_b \cos \phi_b \sum_{i=-n}^n q_i \\ \Delta k_{c44} \varphi_{y_0} \sum_{i=-n}^n p_i \end{Bmatrix} e^{ji\theta} \end{aligned} \quad (6.11)$$

The crack force expression in expanded form by Fourier series helps in the computational work in subsequent steps.

6.2.1.5 Bearing Force Model

Discrete bearing dynamic parameters are considered at corresponding degrees of freedom after assembling different shaft elements. The force induced by the flexible bearing due to damping (c_{xx}, c_{yy}) and stiffness (k_{xx}, k_{yy}) in the transverse direction can be represented as

$$\mathbf{f}_{cb} = \begin{bmatrix} c_{xx} & 0 & 0 & 0 \\ 0 & c_{yy} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{\varphi}_y \\ \dot{\varphi}_x \end{Bmatrix} + \begin{bmatrix} k_{xx} & 0 & 0 & 0 \\ 0 & k_{yy} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ \varphi_y \\ \varphi_x \end{Bmatrix} \quad (6.12)$$

The damping and stiffness of the bearings are taken into account as linear damper-springs on the nodes affecting both translation and rotation. Cross-coupling terms of bearing damping and stiffness are not considered for the sake of simplicity in the modelling.

6.2.1.6 Internal Damping Force Model

The internal force due to the variation of deflection on the shaft during tension and compression leads to the internal damping. The rotating (or internal) damping (c_H) makes the system unstable, while the rotor rotates in the supercritical speed region.

$$\mathbf{f}_{id} = \begin{bmatrix} c_H & 0 & 0 & 0 \\ 0 & c_H & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{\phi}_y \\ \dot{\phi}_x \end{Bmatrix} + \begin{bmatrix} 0 & \omega c_H & 0 & 0 \\ -\omega c_H & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ \phi_y \\ \phi_x \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\omega c_H u_0 \\ 0 \\ 0 \end{Bmatrix} \quad (6.13)$$

The expression of the internal damping force matrix ($\mathbf{f}_{id}$) in Equation (6.13) is consistent with Equation (4.6) in Chapter 4. These internal damping force matrices defined above are used in the finite element formulation of the rotor-bearing system.

6.2.1.7 Magnetic Bearing Force Model

The magnetic actuator and sensor are connected with the feedback loop where the main virtue of the feedback is to attenuate the disturbance error within the bandwidth of the control system. The control current is supplied to the electromagnets through close loop feedback control using a PID controller as discussed in Chapter 2 and Chapter 4. In present model of AMB, the set-point of the closed loop is the center of the shaft and the derivative component need to incorporate a low pass filter scheme to reduce the noise amplification coming from high frequencies. As the AMB is driven in differential mode, the linearized magnetic force for small rotor displacement is represented as

$$\mathbf{f}_{AMB} = -k_s \mathbf{q}_{AMB} + k_i \mathbf{i}_c \quad (6.14)$$

with

$$\mathbf{i}_c = K_p \mathbf{q}_{AMB} + K_I \int \mathbf{q}_{AMB} dt + K_D \frac{d(\mathbf{q}_{AMB})}{dt} \quad (6.15)$$

where k_s and k_i are the displacement-stiffness and current-stiffness factors of AMB, respectively. The controlling current matrix i_c contains the PID controller gains as K_p , K_I and K_D proportional, integral and derivative gain, respectively and, $\mathbf{q}_{AMB}$ is the translational displacement at the AMB location.

Once all the excitation forces are available in terms of the vibration displacement vector defined above are used in the finite element modelling of the rotor-bearing system.

6.2.2 Total Governing Global Equation of Motion

A rotor system consisting a internally damped shaft with a transverse crack mounted with discs and AMB as shown in Figure 6.2, are combined together and boundary conditions applied to obtain the global equations of motion of the rotor is represented as

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{C} - \omega \mathbf{G})\dot{\mathbf{q}} + (\mathbf{K} + \mathbf{K}_B + \omega \mathbf{K}_C)\mathbf{q} = \mathbf{f}_{unb} + \mathbf{f}_{cr} + \mathbf{f}_{bow} - \mathbf{f}_{AMB} - \omega \mathbf{C}_H \mathbf{q}_0 \quad (6.16)$$

where, $\mathbf{M}$, $\mathbf{C}$, $\mathbf{C}_H$, $\mathbf{G}$, $\mathbf{K}_B$ and $\mathbf{K}_C$ are the global mass, the bearing and internal damping, gyroscopic, bearing stiffness and circulatory matrices, respectively. The static deflection due to the self-weight of rotor in the uncracked shaft is represented as

$$\mathbf{q}_0 = (\mathbf{K})^{-1} \{\mathbf{f}_{st}\} \quad (6.17)$$

with

$$\mathbf{f}_{st} = \begin{Bmatrix} m_k g \\ 0 \end{Bmatrix} \text{ for } k \text{ number of discs} \quad (6.18)$$

The generalized unbalance force ($\mathbf{f}_{unb}$) is represented as,

$$\mathbf{f}_{unb} = \sum_{i=1}^k m_i e_{u_i} \omega^2 e^{j\phi_{ui}} \mathbf{l}_{unb_i} \quad (6.19)$$

With $\mathbf{l}_{unb_i} = \{0 \ 0 \ \dots \ 1 \ 0 \ \dots \ 0\}^T$ and, the discs up to k numbers in the rotor system.

The amplitude of the unbalance force vector $\mathbf{f}_{unb}$ is due to the centrifugal force due to unbalance mass m_i located at an eccentricity e_{u_i} from the center of the shaft at an angular position of ϕ_{u_i} for spin speed of ω .

The anisotropic flexible bearing support condition, which introduces force on the rotor system, is

$$\mathbf{f}_{cb} = \sum_{i=1}^h \left(c_{xx_i} \dot{u}_i + j c_{yy_i} \dot{v}_i + k_{xx_i} u_i + j k_{yy_i} v_i \right) \mathbf{l}_{br_i} \quad (6.20)$$

With support bearing locations are given as $\mathbf{l}_{br_i} = \{0 \ 0 \ \dots \ 1 \ 0 \ \dots \ 0\}^T$.

The internal damping force in generalized form is,

$$\mathbf{f}_{id} = (c_H \dot{u} + j c_H \dot{v} + \omega c_H u - j \omega c_H v - j \omega c_H u_0) \mathbf{l}_{cr} \quad (6.21)$$

with, $\mathbf{l}_{cr} = \{0 \ 0 \ \dots \ 1 \ 0 \ \dots \ 0\}^T$ is the location vector of crack present in the rotor system.

The transverse fatigue crack force is given as

$$\mathbf{f}_{cr} = \left\{ \Delta k_{c22} (u_0 + u_b) \sum_{i=-n}^n p_i + \Delta k_{c22} e_{b_i} \cos \phi_b \sum_{i=-n}^n q_i + \Delta k_{c44} \phi_{y_0} \sum_{i=-n}^n p_i \right\} \mathbf{l}_{cr} \quad (6.22)$$

with the location of crack on the shaft $\mathbf{l}_{cr} = \{0 \ 0 \ \dots \ 1 \ 0 \ \dots \ 0\}^T$ and, n represents the number of harmonics.

The generalized magnetic force is given as

$$\mathbf{f}_{AMB} = \sum_{r=1}^b \left(-k_{s_r} q_{AMB_r} + k_{i_r} i_{c_r} \right) \mathbf{l}_{AMB_r} \quad (6.23)$$

with AMB location vector, $\mathbf{l}_{AMB_r} = \{0 \ 0 \ \dots \ 1 \ 0 \ \dots \ 0\}^T$ and, $r=1, 2, \dots, b$ represents the number of AMBs mounted on the rotor system.

6.3 Identification Algorithm Development

In presence of residual shaft bow, the harmonic content of the vibrations induced by unbalance fault in the rotor would show only a 1X rev. component. Besides this, the crack in the rotor may induce multiple harmonics and the presence of bow also encompasses multiple harmonic due to multiplicative fault effect of the crack and bow parameters. Therefore, an identification analysis has been carried out to estimate the unbalances, crack parameters, shaft residual bow, stiffness, and damping of bearings, and internal damping as machine faults as well as the AMB dynamic parameters.

6.3.1 Application of Gyroscopic Dynamic Condensation Scheme

The implementation of dynamic condensation scheme in rotordynamic analysis offers computational efficiency and modeling accuracy. The present work uses the gyroscopic dynamic condensation scheme, which serves the purpose of removal of rotational displacements, as in practical case it is difficult to take measurements of rotational degrees of freedom, as illustrated in Chapter 4. The same transformation is also implemented in FEM modelling and identification as

$$\mathbf{q} = \begin{Bmatrix} \mathbf{q}_m \\ \mathbf{q}_s \end{Bmatrix} = \mathbf{T}^{gd} \mathbf{q}_m \quad \text{with} \quad \mathbf{T}^{gd} = \begin{bmatrix} \mathbf{I} \\ -(\mathbf{K}_{ss} - j(i\omega^2)\mathbf{G} - (i\omega^2)\mathbf{M}_{ss})^{-1} (\mathbf{K}_{sm} - (i\omega^2)\mathbf{M}_{sm}) \end{bmatrix} \quad (6.24)$$

where $\mathbf{T}^{gd}$ is the transformation matrix of the gyroscopic dynamic condensation with m and s representing the master and slave DOFs, respectively. Herein, frequency ω is taken as the geometric mean of frequencies considered in the analysis. The overall equation after using

the gyroscopic dynamic condensation with pre-multiplying and post-multiplying the elemental matrices and force vectors, as described in Chapter 4, takes the following form

$$\mathbf{M}^{gd} \ddot{\mathbf{q}}_m + (\mathbf{C}^{gd} - \omega \mathbf{G}^{gd}) \dot{\mathbf{q}}_m + (\mathbf{K}_S^{gd} + \mathbf{K}_B^{gd} + \omega \mathbf{K}_C^{gd}) \mathbf{q}_m = \mathbf{f}^{gd} \quad (6.25)$$

Where the superscript ‘*gd*’ refers to the gyroscopic condensation of matrices and vectors; $\mathbf{q}_m$ and $\mathbf{f}^{gd}$ are the rotor displacement (master DOFs) and force vectors, respectively; and $\mathbf{M}^{gd}$, $\mathbf{C}^{gd}$, $\mathbf{G}^{gd}$, $\mathbf{K}_B^{gd}$ and $\mathbf{K}_C^{gd}$ are the condensed mass, damping, gyroscopic, stiffness, and circulatory matrices, respectively. These condensed matrices at various speeds are to be conducted in the subsequent analysis. The condensed Equation (6.25) will be utilized in developing the identification algorithm to estimate multiple fault parameters in the subsequent analysis.

The anisotropic bearing support consideration in rotor generates elliptical orbits for all the rotational frequencies. Thus, the solution associated with Equation (6.25) is defined as

$$\mathbf{q}_{m_x} = \mathbf{q}_a \cos(i\theta) + \mathbf{q}_h \sin(i\theta) = \mathbf{c}e^{ji\theta} + \bar{\mathbf{c}}e^{-ji\theta} \quad (6.26)$$

and,
$$\mathbf{q}_{m_y} = \mathbf{q}_b \cos(i\theta) + \mathbf{q}_h \sin(i\theta) = \mathbf{d}e^{ji\theta} + \bar{\mathbf{d}}e^{-ji\theta} \quad (6.27)$$

Similarly, for the control current of AMB is given as

$$\mathbf{i}_{c_x} = \mathbf{i}_a \cos(i\theta) + \mathbf{i}_h \sin(i\theta) = \mathbf{f}e^{ji\theta} + \bar{\mathbf{f}}e^{-ji\theta} \quad (6.28)$$

and,
$$\mathbf{i}_{c_y} = \mathbf{i}_b \cos(i\theta) + \mathbf{i}_h \sin(i\theta) = \mathbf{g}e^{ji\theta} + \bar{\mathbf{g}}e^{-ji\theta} \quad (6.29)$$

where $\mathbf{q}_{m_x} = [u_1 \ \cdots \ u_n]^T$ and $\mathbf{q}_{m_y} = [v_1 \ \cdots \ v_n]^T$ are the translational displacements in the vertical and horizontal directions, respectively, and the vectors $\mathbf{c}$, $\bar{\mathbf{c}}$, $\mathbf{d}$, $\bar{\mathbf{d}}$, $\mathbf{f}$, $\bar{\mathbf{f}}$, $\mathbf{g}$ and $\bar{\mathbf{g}}$ are the periodic functions of time.

In complex form, $\mathbf{q}_m = \mathbf{q}_{m_x} + j\mathbf{q}_{m_y} = (\mathbf{c} + j\mathbf{d})e^{ji\theta} + (\bar{\mathbf{c}} + j\bar{\mathbf{d}})e^{-ji\theta} = \mathbf{p}_i e^{ji\theta} + \mathbf{s}_i e^{-ji\theta}$ and

similarly, the control current $\mathbf{i}_c = (\mathbf{f} + j\mathbf{g})e^{ji\theta} + (\bar{\mathbf{f}} + j\bar{\mathbf{g}})e^{-ji\theta} = \mathbf{r}_i e^{ji\theta} + \mathbf{v}_i e^{-ji\theta}$ with $\mathbf{p}_i = (\mathbf{c} + j\mathbf{d})$

and $\mathbf{q}_i = (\bar{\mathbf{c}} + j\bar{\mathbf{d}})$, $\mathbf{r}_i = (\mathbf{f} + j\mathbf{g})$ and, $\mathbf{v}_i = (\bar{\mathbf{f}} + j\bar{\mathbf{g}})$

For multiple harmonic case the master DOFs for can be expressed as

$$\mathbf{q}_{m_x}(t) = \sum_{i=-n}^n \mathbf{c}_i e^{ji\theta}; \mathbf{q}_{m_y}(t) = \sum_{i=-n}^n \mathbf{d}_i e^{ji\theta}; \mathbf{q}_m(t) = \sum_{i=-n}^n \tilde{\mathbf{q}}_i e^{ji\theta} \text{ and } \mathbf{i}_c(t) = \sum_{i=-n}^n \tilde{\mathbf{i}}_{c_i} e^{ji\theta} \quad (6.30)$$

By introducing Equation (6.30) into the condensed Equation(6.25), the following algebraic equation in the frequency domain is readily obtained

$$\left(-(i\omega)^2 \mathbf{M}^{gd} - j\omega \mathbf{G}^{gd} + \mathbf{K}_B^{gd} \right) \tilde{\mathbf{q}}_i e^{ji\theta} = \mathbf{f}^{gd} \quad (6.31)$$

For $i=0$;

$$\begin{aligned} & \underbrace{-\sum_{r=1}^a \left(\Delta k_{c22_r} p_0 u_{x_{0,r}} + \Delta k_{c22_r} q_0 e_{b_r} \cos \phi_{b_r} + \Delta k_{c44_r} p_0 \varphi_{y_{0,r}} \right) \mathbf{l}_{c_r}}_{\text{Crack force}} - \underbrace{\sum_{r=1}^h \left(j\omega c_{xx_r} c_{b_r} + \omega c_{yy_r} d_{b_r} \right) \mathbf{l}_{b_r}}_{\text{Bearing damping}} \\ & \underbrace{-\sum_{r=1}^a j\omega c_{H_r} (\tilde{q}_0 + u_{x_{0,r}}) \mathbf{l}_{c_r}}_{\text{Internal damping}} - \underbrace{\sum_{r=1}^h \left(k_{xx_r} c_{b_r} - jk_{yy_r} d_{b_r} \right) \mathbf{l}_{b_r}}_{\text{Bearing stiffness}} + \underbrace{\sum_{r=1}^b \left(-k_{s_r} \tilde{q}_{0,AMB_r} + k_{i_r} \tilde{i}_{c_{0,r}} \right) \mathbf{l}_{AMB_r}}_{\text{AMB force}} = -(\mathbf{K}_B^{gd}) \tilde{\mathbf{q}}_0 \end{aligned} \quad (6.32)$$

For $i=1$;

$$\begin{aligned} & \underbrace{-\sum_{r=1}^a \left(\Delta k_{c22_r} p_1 u_{x_{0,r}} + \Delta k_{c22_r} q_1 e_{b_r} \cos \phi_{b_r} + \Delta k_{c44_r} p_1 \varphi_{y_{0,r}} \right) \mathbf{l}_{c_r}}_{\text{Crack force}} - \underbrace{\sum_{r=1}^k m_r e_r \omega^2 e^{j\beta_r} \mathbf{l}_{unb_r}}_{\text{Unbalance force}} \\ & \underbrace{-f_o \mathbf{S} \boldsymbol{\delta}}_{\text{Residual bow force}} - \underbrace{\sum_{r=1}^h \left(j\omega c_{xx_r} c_{b_r} + \omega c_{yy_r} d_{b_r} \right) \mathbf{l}_{b_r}}_{\text{Bearing damping}} - \underbrace{\sum_{r=1}^a \left(j\omega c_{H_r} \tilde{q}_1 + j\omega c_{H_r} u_{x_{0,r}} \right) \mathbf{l}_{c_r}}_{\text{Internal damping}} \\ & \underbrace{-\sum_{r=1}^h \left(k_{xx_r} c_{b_r} - jk_{yy_r} d_{b_r} \right) \mathbf{l}_{b_r}}_{\text{Bearing stiffness}} + \underbrace{\sum_{r=1}^b \left(-k_{s_r} \tilde{q}_{1,AMB_r} + k_{i_r} \tilde{i}_{c_{1,r}} \right) \mathbf{l}_{AMB_r}}_{\text{AMB force}} = \left(\omega^2 \mathbf{M}^{gd} - j\omega \mathbf{G}^{gd} - \mathbf{K}_B^{gd} \right) \tilde{\mathbf{q}}_1 \end{aligned} \quad (6.33)$$

For $i \neq 1$;

$$\begin{aligned}
& - \underbrace{\sum_{r=1}^a \left(\Delta k_{c22r} p_i u_{x_{0,r}} + \Delta k_{c22r} q_i e_{b_r} \cos \phi_{b_r} + \Delta k_{c44r} p_i \phi_{y_{0,r}} \right) \mathbf{I}_{cr_r}}_{\text{Crack force}} - \underbrace{\sum_{r=1}^h \left(j\omega c_{xxr} c_{b_r} + \omega c_{yyr} d_{b_r} \right) \mathbf{I}_{b_r}}_{\text{Bearing damping}} \\
& - \underbrace{\sum_{r=1}^a \left(j\omega c_{Hr} \tilde{q}_i \right) \mathbf{I}_{c_r}}_{\text{Internal damping}} - \underbrace{\sum_{r=1}^h \left(k_{xxr} c_{b_r} - jk_{yyr} d_{b_r} \right) \mathbf{I}_{b_r}}_{\text{Bearing stiffness}} + \underbrace{\sum_{r=1}^b \left(-k_{s_r} \tilde{q}_{1,AMB_r} + k_{l_r} \tilde{i}_{c_{i,r}} \right) \mathbf{I}_{AMB_r}}_{\text{AMB force}} \\
& = \left((i\omega)^2 \mathbf{M}^{gd} - j\omega \mathbf{G}^{gd} - \mathbf{K}_B^{gd} \right) \tilde{\mathbf{q}}_i
\end{aligned} \tag{6.34}$$

In pseudo inverse form, it is given as

$$\mathbf{A}_{1i} \mathbf{x}_1 = \mathbf{B}_{1i} \tag{6.35}$$

The solution of $\mathbf{x}$ is given as

$$\mathbf{x} = \left(\mathbf{A}_i(\omega)^T \mathbf{A}_i(\omega) \right)^{-1} \mathbf{A}_i(\omega)^T \mathbf{B}_i(\omega) \tag{6.36}$$

where $\mathbf{B}_i$ is expressed as

$$\mathbf{B}_i = \left(-(i\omega)^2 \mathbf{M}^{gd} - j\omega \mathbf{G}^{gd} + \mathbf{K}_B^{gd} \right) \tilde{\mathbf{q}}_i \tag{6.37}$$

where, $\mathbf{x}_1$ is the vector of unknowns, i.e. the multiple fault parameters, such as the disc unbalances and its phases, translational and rotational additive crack stiffness, bearing damping and stiffness in vertical and horizontal directions, internal damping, initial load onto the shaft, multiplicative fault of crack and shaft residual bow, and AMB parameters

$$\mathbf{x} = \left[c_{xx}^L \quad c_{yy}^L \quad c_{xx}^R \quad c_{yy}^R \quad k_{xx}^L \quad k_{yy}^L \quad k_{xx}^R \quad k_{yy}^R \quad \Delta k_{c22} \quad \Delta k_{c44} \quad \Delta k_{c22} e_{b,Re} \quad e_{u1} \quad \phi_{u1} \quad e_{u2} \quad \phi_{u2} \quad k_s \quad k_i \quad f_0 \right]^T \tag{6.38}$$

The superscripts ‘L’ and ‘R’ define the left and right flexible bearing considered in the present analysis (this is for illustration purpose only, it can have multiple bearings and AMBs also). To get the best estimated results, the condition number of matrix $\mathbf{A}_1$ shown in Appendix D is improved by acquiring data at multiple speeds as described in previous Chapters.

The proposed identification algorithm is numerically tested with an example and will be discussed in the following sections.

6.4 Numerical Study

As in the present analysis, bearing properties are chosen unsymmetrical, the orbits due to unbalance and crack for synchronous running speed are not circular, rather elliptical in nature. The flexible rotor with a transverse crack and mounted on two discs supported on conventional (flexible bearing) integrated with an AMB is discretized into five elements with six nodes as shown in Figure 6.5 for the numerical simulation. The simulations are conducted for obtaining the vibration data corresponding to twenty and eighteen speeds. It is equally spaced in the range of 15 Hz to 35 Hz (low speed) as well as for 68 to 80 Hz (higher speed) as compared to first critical speed (65.2 Hz as shown in Campbell diagram in Figure 6.6).

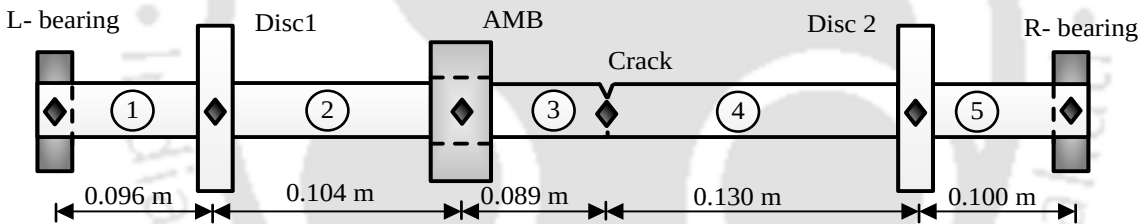


Figure 6.5 Discretized rotor model into the number of elements for the numerical study

For identification procedure, the physical rotordynamic parameters, actuator properties and controller gains as presented in Table 6.1 and Table 6.2, are used in numerical simulation. In the present analysis, mass of the journal or core of the magnetic bearing is assumed very small in comparison to the mass of the discs. Therefore, the mass of the journal of magnetic bearing (i.e. the rings mounted onto the shaft to allow the magnetic flux circuit in the AMB system) is ignored. However, the journal bearing mass can be included in the modeling and may be better estimation can be achieved which is beyond the scope of the current work.

Table 6.1 Physical rotordynamic parameters of the system

Physical parameters		Assumed values
Mass of Disc1 and Disc2	m_1, m_2	1.5 kg, 2 kg
Diametral mass moment of inertia of Disc1 and Disc2	I_{d_1}, I_{d_2}	0.0028 kgm ² , 0.0066 kgm ²
Length of shaft	L	0.5 m
Diameter of shaft	d	0.016 m
Density of shaft	ρ	7850 kg/m ³
Left bearing damping	c_{xx}^L	210 Ns/m
	c_{yy}^L	160 Ns/m
Right bearing damping	c_{xx}^R	180 Ns/m
	c_{yy}^R	200 Ns/m
Left bearing damping	k_{xx}^L	5×10 ⁵ N/m
	k_{yy}^L	4.5×10 ⁵ N/m
Right bearing damping	k_{xx}^R	5.5×10 ⁵ N/m
	k_{yy}^R	4×10 ⁵ N/m
Translational additive stiffness	Δk_{c22}	3×10 ⁵ N/m
Rotational additive stiffness	Δk_{c44}	4000 N/rad
Internal damping	c_H	75 Ns/m
Disc 1 eccentricity and phase	e_{u1}, ϕ_{u1}	9×10 ⁻⁵ m, 60°
Disc 2 eccentricity and phase	e_{u2}, ϕ_{u2}	8×10 ⁻⁵ m, 30°
Initial loading on any section over the shaft	f_0	10 N

Table 6.2 Actuator properties and PID controller parameters

Actuator poles		8 pole
Number of coil turns	N	100 turns
Bias current	i_b	2 A
Air gap	s_g	0.8 mm
Pole angle	α	22.5°
Force-displacement factor	k_s	217368 N/m
Force-current factor	k_i	43.538 N/A
Proportional gain	K_P	8,000 Am ⁻¹
Integral gain	K_I	6,000 A(m s) ⁻¹
Derivative gain	K_D	3Asm ⁻¹

6.4.1 Free Vibration Analysis

The finite element rotor model developed is implemented in MATLAB®-SIMULINK environment and rotates up to a speed of 3000 rad/s (approximately 478 Hz). Figure 6.6 shows the Campbell diagram of the rotor system without and with AMB integrated into the rotor system. The splitting of natural frequencies into the forward and backward whirls can be seen due to the gyroscopic effect.

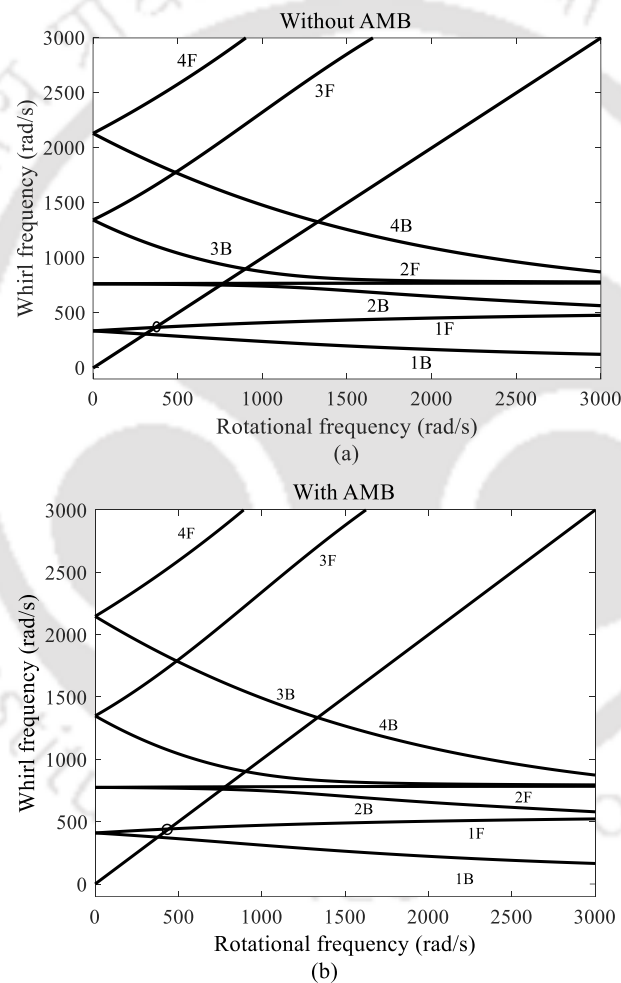


Figure 6.6 Campbell diagram showing the forward and backward whirl of the rotor system (a) without AMB and (b) with AMB

The diagonal line plotted in the Campbell diagram represents the synchronous whirl condition of the rotor. The present rotor system, without AMB, the first forward critical speed is found to be 53.69 Hz (337.2 rad/s) and with AMB is 65.2 Hz (409.5 rad/s).

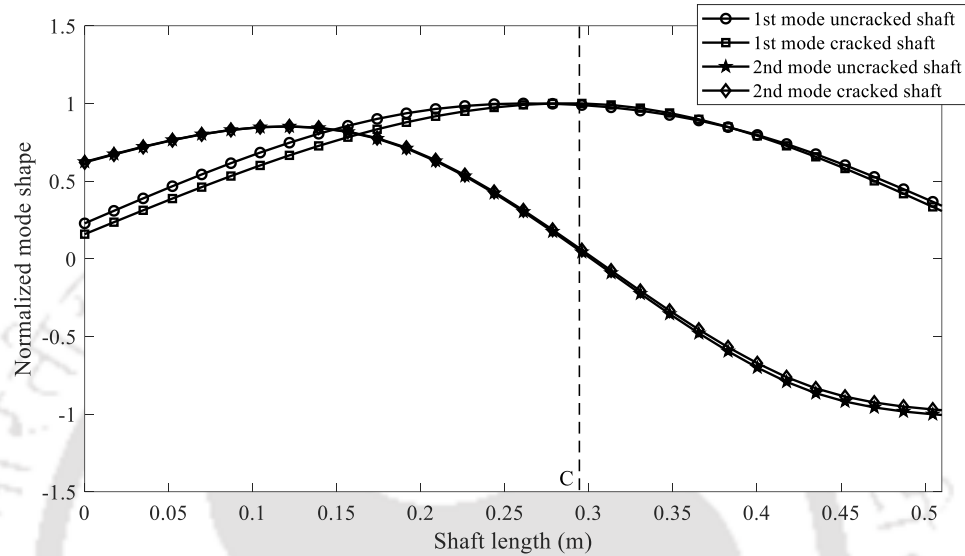


Figure 6.7 Mode shapes of the proposed rotor-AMB system corresponding to the first three natural frequencies with and with crack case

The mode shapes of a system are sensitive enough to local damages and its analysis helps ones to identify the damages from the response. The determination of the normalized mode shapes of the uncracked and cracked shaft with shaft residual bow cases over the shaft length corresponding to the first two natural frequencies of the proposed rotor-AMB configuration are obtained is shown in Figure 6.7. In the cracked case, additive crack stiffness (Δk_{c22}) is added in the respective node location of crack in the rotor-AMB system in the simulation. The cracked shaft shows the variations or shape alteration of the associated modes (1st and 2nd) with the uncracked shaft. The location C denotes the location of the crack over the shaft length from the left bearing. This feature is a good way of identification of crack in a practical rotor system.

Static deflection

The graphical representation of the static deflections and slopes of the shaft due to self-weight of the rotor at each node along the length of the shaft is obtained through Equation (6.17) and is represented in Figure 6.8.

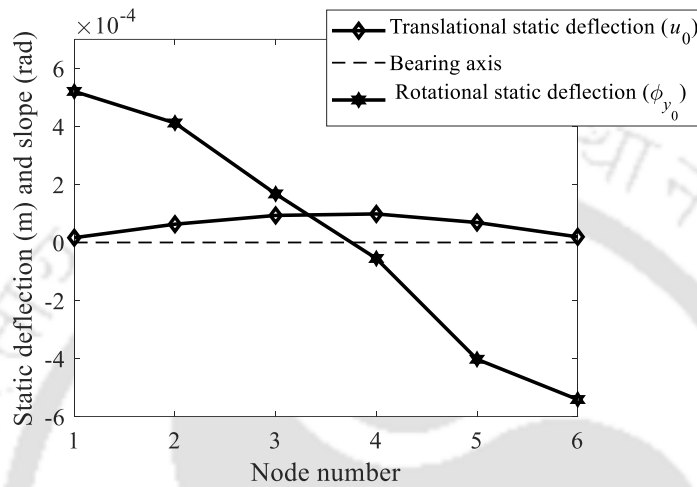


Figure 6.8 Static deflection and slope versus node location on the shaft

6.4.2 Time Domain and Full Spectrum Analysis of Displacement and Current signal

The response of the cracked-warped flexible rotor-bearing system equipped with AMB is generated for the spin speed of 15 Hz. Figure 6.9 shows the time domain response of transverse displacement at bearing, discs, and AMB locations excited by the unbalance, shaft residual bow and crack force. As shown in Figure 6.9(a, b), at the spin speed of 900 rpm (15 Hz) the amplitude of vibration goes up to 4×10^{-5} m at the left bearing location (node 1 of rotor model). Similarly, at the right bearing location (node 5 of the rotor model), the amplitude of vibration observed to be 5.5×10^{-5} m in Figure 6.9(c, d). The disc displacements at node locations 2 and 5 are observed in Figure 6.9(e-h) with the vibration amplitude of 1.2×10^{-4} m and 1.8×10^{-4} m, respectively, for Disc 1 and Disc 2. The active

magnetic bearing suppresses the amplitude of vibration of the rotor bearing system, which is considered at node location 4 of the rotor model. Figure 6.9(i, j) shows the vibration responses at AMB location with amplitude 1.8×10^{-4} m and the corresponding control current required for vibration attenuation is 2 A and 1.8 A, respectively, as shown in Figure 6.9(k, l) in the vertical and horizontal directions.

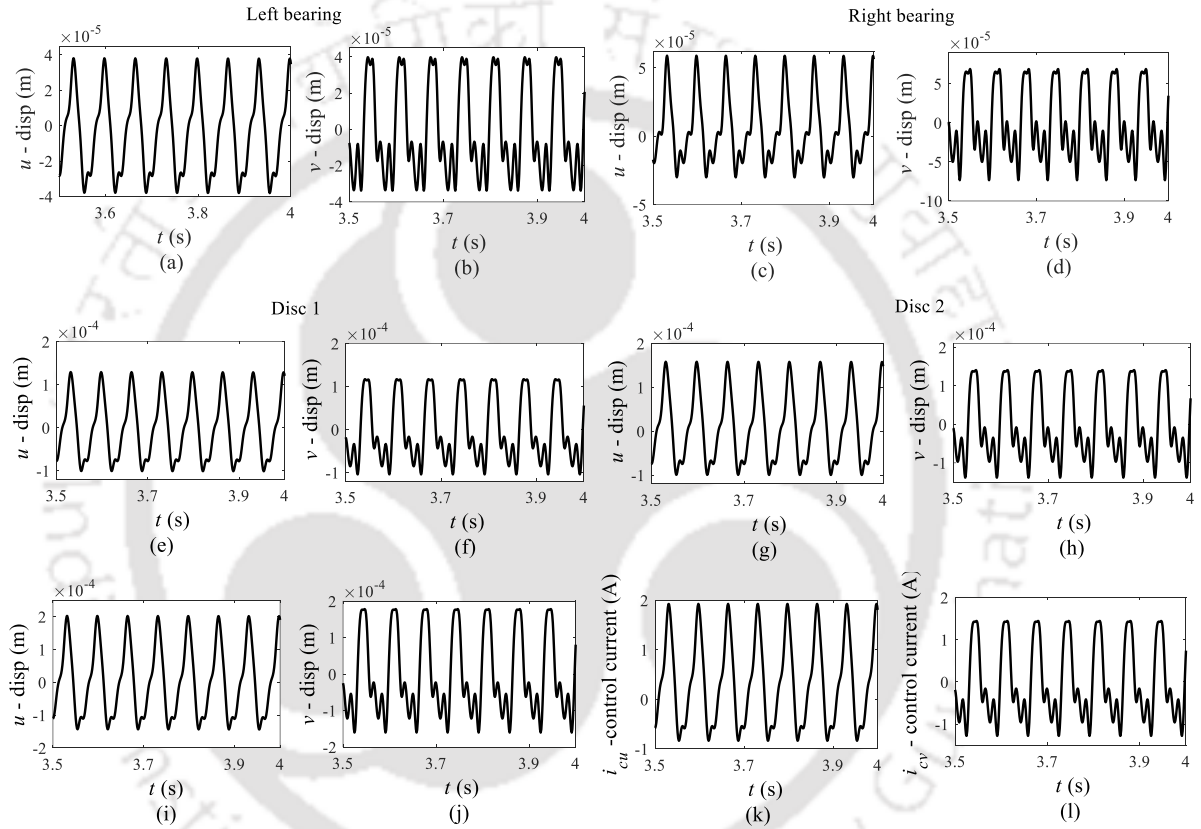


Figure 6.9 Transverse responses at (a, b) left bearing location at node 1 (c, d) right bearing location at node 6 (e, f) Disc 1 location at node 2 (g, h) Disc 2 locations at node 5 (i, j) AMB location at node 3 control current at (k, l) AMB location at node 3 of the proposed rotor model

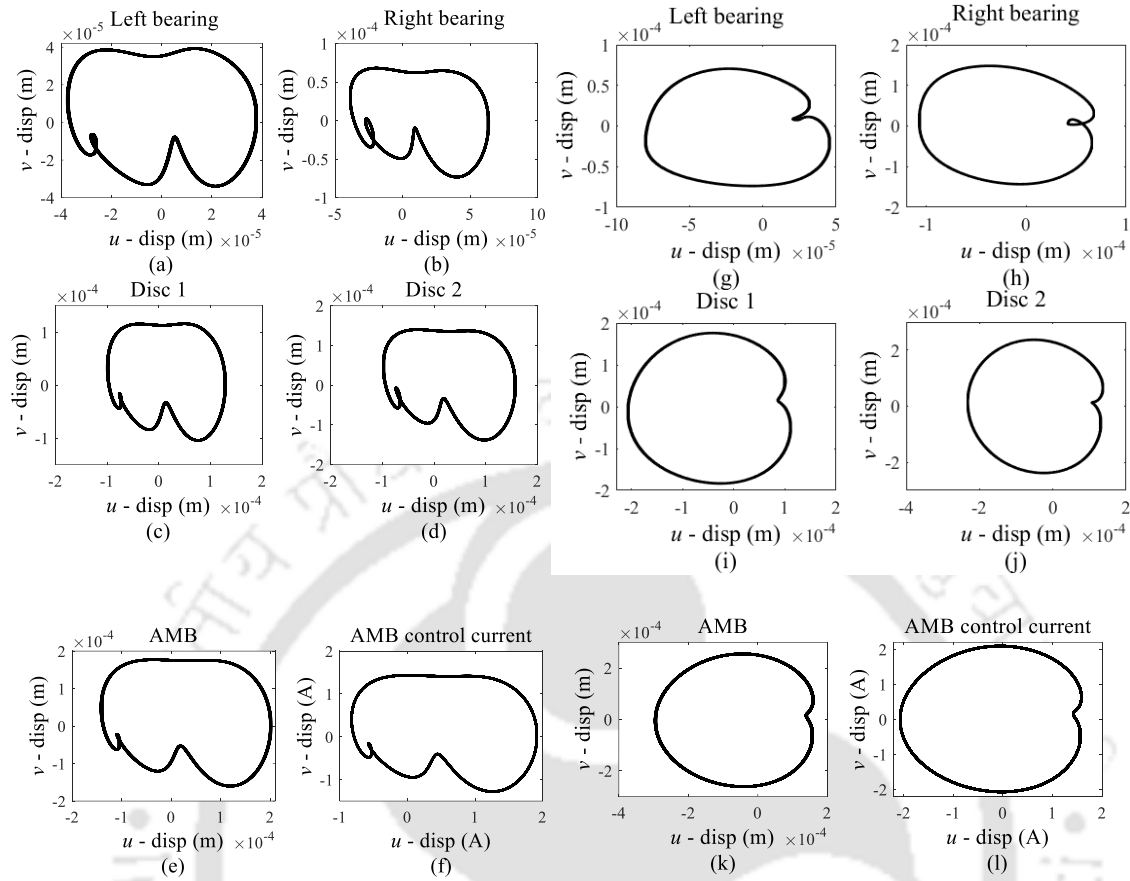


Figure 6.10 Orbit plots at the spin speed of 15 Hz (low speed) and high speed (35 Hz) (a, g) left and right bearing location at node 1 (b, h) left and right bearing location at node 6, (c, i) disc 1 at node 2 (d, j) disc 2 location at node 5 (e, k) AMB location displacement response at node 3 (f, l) AMB location control current at node 3

The dynamic characteristics of the rotor system are affected by one of the key parameters, i.e. the rotational speed. The dynamic response of the rotor is summarized in terms of the horizontal and vertical displacements of the flexible rotor system at bearing locations, discs locations and AMB location of the model by plotting its orbital motion at the low (15 Hz) and high (35 Hz) spin speeds as shown in Figure 6.10(a-l). It is observed from Figure 6.10 that as the speed increases the amplitude of vibration goes on increasing and the amount of control current required to attenuate the vibration is also high.

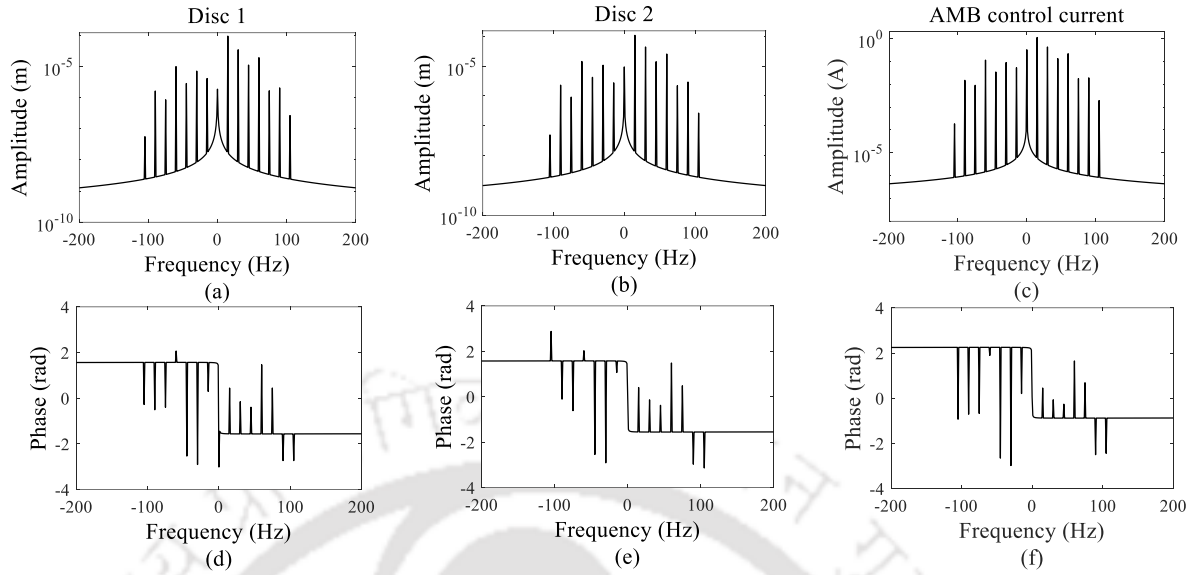


Figure 6.11 Full spectrum of Disc 1, Disc 2 and AMB control current (a) Amplitude of Disc 1 (b) Amplitude of Disc 2 (c) Amplitude of AMB control current (d) Phase of Disc 1 (e) Phase of Disc 2 (f) Phase of control current

The vibrational amplitude and phase of multi-harmonic signal generated from the present rotor system are shown in full spectrum plots as in Figure 6.11(a, d) for the location of disc 1 at node 2 and Figure 6.11(b, e) for disc 2 at node 5 mounted on the shaft. Similarly, the control current required for suppressing vibration at AMB location of node 3 is shown in Figure 6.11(c, f). It can be observed that the interaction of shaft residual bow and crack shows multi-harmonic signals, consisting of all the harmonics, such as $1X$, $2X$, $3X$, $\dots nX$, unlike in the signals containing only crack fault effect as illustrated in Chapters 2 and 4.

6.4.3 Simultaneous Estimation of Multiple Faults

As explained in Section 6.4.2, faults associated with the rotor system are estimated using the numerically generated full spectrum amplitude and phase (displacement and control current), where the rotor is operated in a closed-loop feedback control system. The

estimation of parameters using Equation (6.35) for the successive data of low speed and high speed range cases as mentioned in Section 6.4 are included in the following tables. At par with a more realistic situation, the response of the rotor is contaminated by random noise in the simulation procedure.

Table 6.3 Estimated additive and multiplicative fault parameters of the proposed rotor system for the lower range of speed (15 Hz to 35 Hz with steps of 2 Hz) by introducing random noise

	Additive fault parameters				
	Assumed values	0% noise	1% noise	3% noise	5% noise
c_{xx}^L Ns/m	210	212.59	212.61	212.66	212.71
c_{yy}^L Ns/m	160	163.58	163.63	163.74	163.84
c_{xx}^R Ns/m	180	180.95	181.13	181.48	181.84
c_{yy}^R Ns/m	200	206.49	206.54	206.63	206.73
k_{xx}^L N/m	5×10^5	501142.43	501181.14	501258.52	501335.85
k_{yy}^L N/m	4.5×10^5	451276.28	451299.49	451346.09	451392.93
k_{xx}^R N/m	5.5×10^5	554781.18	554888.68	555103.98	555319.71
k_{yy}^R N/m	4×10^5	402373.45	402472.83	402671.83	402871.11
Δk_{c22} N/m	3×10^5	282126.95	281481.60	280190.51	278898.88
Δk_{c44} N/rad	4000	3596.85	3607.66	3629.44	3651.41
c_H Ns/m	75	74.67	74.74	74.88	75.03
k_s N/m	217368	215042.61	214862.98	214506.09	214152.35
k_i N/A	43.538	43.15	43.13	43.08	43.03
e_1 m	8×10^{-5}	7.982×10^{-5}	7.981×10^{-5}	7.978×10^{-5}	7.976×10^{-5}
ϕ_{u1} deg	60°	60.75	60.81	60.93	61.05
e_2 m	9×10^{-5}	9.043×10^{-5}	9.054×10^{-5}	9.077×10^{-5}	9.100×10^{-5}
ϕ_{u2} deg	30°	30.03	30.03	30.02	30.01
f_0 N	10	10.03	10.03	10.03	10.04
Multiplicative fault parameter					
$\Delta k_{c22} e_{b,Re}$ N	14.45	14.393	14.405	14.428	14.452

Table 6.3 shows the estimated multiple fault parameters of additive and multiplicative faults (effect of shaft residual bow onto the cracked portion of the rotor), respectively, with the addition of random noise to the system at low speed range case.

Table 6.4 Estimated additive and multiplicative parameters of the proposed rotor system for the higher range of speed (68 Hz to 80 Hz with steps of 2 Hz) by introducing random noise

	Additive fault parameters				
	Assumed values	0% noise	1% noise	3% noise	5% noise
c_{xx}^L Ns/m	210	210.84	210.58	210.06	209.53
c_{yy}^L Ns/m	160	158.246	157.914	157.24	156.561
c_{xx}^R Ns/m	180	180.633	180.852	181.585	182.706
c_{yy}^R Ns/m	200	201.206	199.604	196.419	193.258
k_{xx}^L N/m	5×10^5	499542.46	499600.53	499717.62	499835.98
k_{yy}^L N/m	4.5×10^5	450370.11	450230.58	449949.09	449664.36
k_{xx}^R N/m	5.5×10^5	551897.12	552293.11	553075.59	553845.30
k_{yy}^R N/m	4×10^5	402377.90	401139.54	398672.32	396218.02
Δk_{c22} N/m	3×10^5	281941.61	280586.83	277887.09	275200.81
Δk_{c44} N/rad	4000	3449.98	3409.87	3337.58	3276.43
c_H Ns/m	75	74.98	74.64	73.98	73.33
k_s N/m	217368	214095.69	213924.01	213606.97	213325.19
k_i N/A	43.538	43.12	43.11	43.09	43.07
e_1 m	8×10^{-5}	7.996×10^{-5}	8.009×10^{-5}	8.037×10^{-5}	8.064×10^{-5}
ϕ_{u1} deg	60°	60.07	60.06	60.02	59.99
e_2 m	9×10^{-5}	9.003×10^{-5}	9.006×10^{-5}	9.011×10^{-5}	9.016×10^{-5}
ϕ_{u2} deg	30°	29.99	30.01	30.04	30.07
f_0 N	10	10.02	9.96	9.86	9.76
Multiplicative fault parameter					
$\Delta k_{c22} e_{b,Re}$ N	14.45	14.466	14.681	14.114	14.550

Table 6.5 Estimated shaft residual bow and phase with the addition of random noise at both low and high speed range

Parameter		Estimated shaft residual bow (e_b) and its phase angle (ϕ_b) with random noise			
		0% noise	1% noise	3% noise	5% noise
Low speed range (15 Hz to 35)	e_b m	4.906×10^{-5}	4.885×10^{-5}	4.842×10^{-5}	4.800×10^{-5}
	ϕ_b deg	44.14	43.95	43.56	43.19
High speed range (68 Hz to 80)	e_b m	4.913×10^{-5}	4.870×10^{-5}	4.773×10^{-5}	4.747×10^{-5}
	ϕ_b deg	44.32	43.38	42.96	42.71

Similarly, for the higher speed range case, the additive and multiplicative fault parameters are summarized Table 6.4. The identified initial load onto the rotor has been used to obtain the evaluation of the residual shaft bow using Equation (6.7). The residual shaft bow (permanent deformation of the shaft) and its phase are tabulated in Table 6.5.

A comparison of the absolute values of all the identifiable to different levels of noise in responses for two ranges of speed (low and high) cases is shown in Figure 6.12. It is observed from Figure 6.12(a, f) that the deviation of the estimated parameters without noisy signals is less than 3% for 14 parameters out of 17 parameters. The maximum percentage error is found for the translational and rotational additive stiffness ($\Delta k_{c22}, \Delta k_{c44}$) of -5.958% and 10.078% , respectively, and the right bearing damping (c_{yy}^R) with 3.247% . Similarly, for 5% noise addition, the highest deviating parameters are found to be (Δk_{c22}) with 7.033% , (Δk_{c44}) with 8.715% and (c_{yy}^R) with 3.368% , whereas the least affected one is (c_H) with deviation of 0.040% . The remaining identifiable are deviating less than 3% with the absolute values. The percentage error of the identifiable for the high speed range case

is shown in Figure 6.13(a, f) at 0% noise level. Highly affected parameters are (Δk_{c22}) by 6.019%, (Δk_{c44}) by 13.750% and the least affected parameter is (k_{xx}^L) 0.033%. The higher deviations of a few parameters may be due to the consideration of responses from limited locations and that to of only translational DOFs.

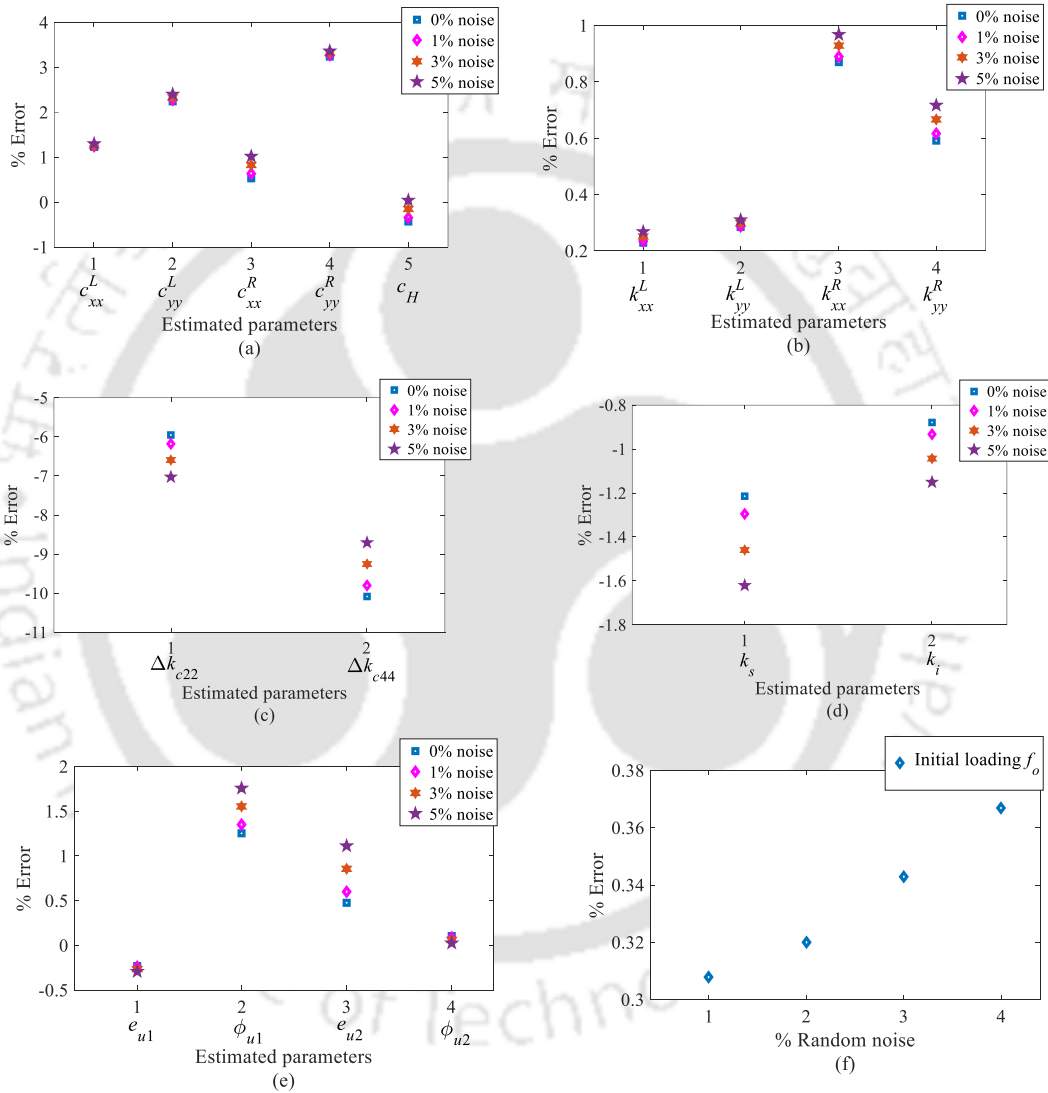


Figure 6.12 Percentage error of fault parameters to the assumed values at low speed range with random noise (a) Bearing damping and internal damping (b) Bearing stiffness (c) Unbalance and phase of Disc 1 and Disc 2 (d) Initial loading on the shaft (e) AMB parameters (f) Translational and rotational additive stiffness of crack

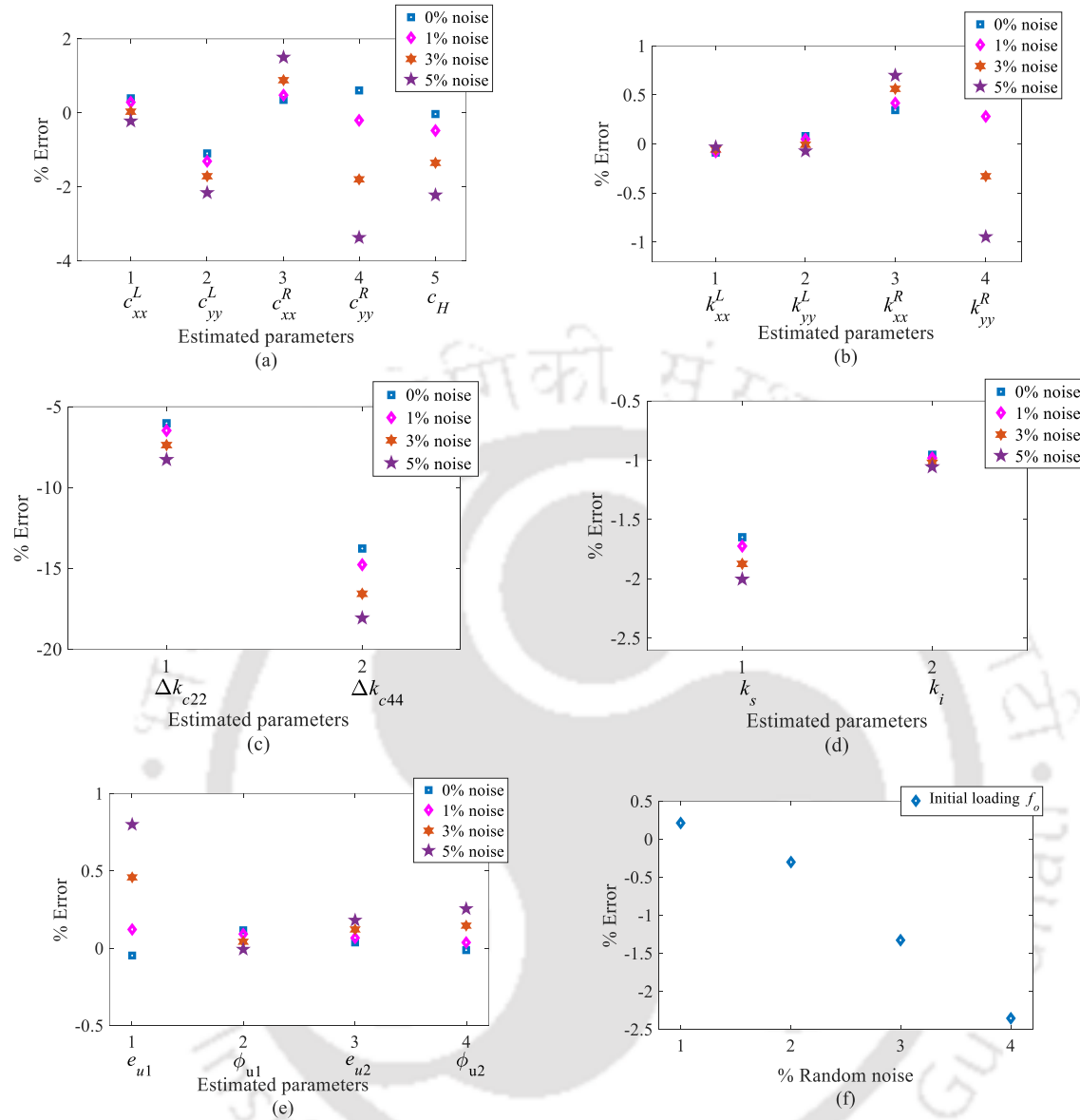


Figure 6.13 Percentage error of fault parameters to the assumed values at high speed range with random noise (a) Bearing damping and internal damping (b) Bearing stiffness (c) Unbalance and phase of Disc 1 and Disc 2 (d) Initial loading on the shaft (e) AMB parameters (f) Translational and rotational additive stiffness of crack

The sensitivity of the developed identification algorithm to bias error in modeling parameters has been studied in the next section.

6.4.4 Introduction of Modeling Error

The measurement of the system parameters of a real rotor with exact values may not be possible due to measuring constraints discussed in previous Chapters. Modeling error is introduced in the critical system parameters as done in previously. In the present analysis, Table 6.6 illustrates the additive and multiplicative fault parameters of the rotor with 1%, 3% and 5% bias error for the low speed range case. For the high speed range case, the estimated parameters of the same are shown in Table 6.7. The shaft residual bow (e_b) and its phase angle (ϕ_b) are found to be 4.818×10^{-5} m and 43.35° , respectively, for no bias error case shown in Table 6.8.

Table 6.6 Estimated additive and multiplicative fault parameters of the present rotor system for the lower range of speed (15 Hz to 35 Hz with steps of 2 Hz) by introducing modeling or bias error

Parameters	Additive fault parameters				
	Assumed values	0% bias error	1% bias error	3% bias error	5% bias error
c_{xx}^L Ns/m	210	212.59	214.30	217.28	220.26
c_{yy}^L Ns/m	160	163.59	166.59	169.02	171.46
c_{xx}^R Ns/m	180	180.96	179.32	178.952	178.59
c_{yy}^R Ns/m	200	206.49	210.41	211.58	212.77
k_{xx}^L N/m	5×10^5	501142.44	507194.17	517524.30	527854.73
k_{yy}^L N/m	4.5×10^5	451276.28	457080.53	466453.44	475826.95
k_{xx}^R N/m	5.5×10^5	554781.18	564146.97	575924.66	587708.04
k_{yy}^R N/m	4×10^5	402373.45	408238.56	416702.60	425168.93
Δk_{c22} N/m	3×10^5	282126.95	286909.28	297296.07	307836.36
Δk_{c44} N/rad	4000	3596.85	3813.25	4118.19	4514.46
c_H Ns/m	75	74.67	75.92	77.75	79.58
k_s N/m	217368	215042.62	216961.60	221450.01	225951.11
k_i N/A	43.538	43.16	43.52	44.41	45.31

e_1 m	8×10^{-5}	7.982×10^{-5}	8.083×10^{-5}	8.246×10^{-5}	8.404×10^{-5}
ϕ_{u1} deg	60°	60.75	59.72	59.21	58.74
e_2 m	9×10^{-5}	9.043×10^{-5}	9.243×10^{-5}	9.504×10^{-5}	9.757×10^{-5}
ϕ_{u2} deg	30°	30.03	29.77	30.07	30.36
f_0 N	10	10.03	10.09	10.27	10.46
Multiplicative fault parameters					
$\Delta k_{c22} e_{b,Re}$ N	14.45	14.08	14.220	14.499	14.778

Table 6.7 Estimated additive and multiplicative fault parameters of the present rotor system for the higher range of speed (68 Hz to 80 Hz with steps of 2 Hz) by introducing modeling error

Parameters	Additive fault parameters				
	Assumed values	0% bias error	1% bias error	3% bias error	5% bias error
c_{xx}^L Ns/m	210	210.85	212.22	215.605	218.97
c_{yy}^L Ns/m	160	158.25	159.11	163.09	166.988
c_{xx}^R Ns/m	180	180.63	179.37	174.91	172.82
c_{yy}^R Ns/m	200	201.21	202.41	209.14	212.43
k_{xx}^L N/m	5×10^5	499542.47	504409.04	513815.86	523239.55
k_{yy}^L N/m	4.5×10^5	450370.11	454616.43	462841.16	471020.65
k_{xx}^R N/m	5.5×10^5	551897.13	558850.81	570276.879	581710.03
k_{yy}^R N/m	4×10^5	402377.90	405466.03	412932.79	418154.58
Δk_{c22} N/m	3×10^5	281941.61	279944.84	292800.53	295790.01
Δk_{c44} N/rad	4000	3449.98	3826.38	4722.86	5573.60
c_H Ns/m	75	74.98	75.63	79.07	81.52
k_s N/m	217368	214095.69	216295.82	222924.40	229414.28
k_i N/A	43.538	43.12	43.58	44.72	45.86
e_1 m	8×10^{-5}	7.996×10^{-5}	7.798×10^{-5}	7.411×10^{-5}	7.040×10^{-5}
ϕ_{u1} deg	60°	60.07	60.51	61.53	62.62
e_2 m	9×10^{-5}	9.003×10^{-5}	8.702×10^{-5}	8.114×10^{-5}	7.554×10^{-5}
ϕ_{u2} deg	30°	29.99	29.27	27.76	26.07
f_0 N	10	10.021	10.179	10.66	11.031
Multiplicative fault parameters					
$\Delta k_{c22} e_{b,Re}$ N	14.45	14.08	14.46	14.23	14.76

Table 6.8 Estimated shaft residual bow and phase with the addition of modeling error at both low and high speed range

Parameter		Estimated shaft residual bow (e_b) and its phase angle (ϕ_b) with bias error			
		0% error	1% error	3% error	5% error
Low speed range (15 Hz to 35 Hz)	e_b m	4.818×10^{-5}	4.857×10^{-5}	4.935×10^{-5}	5.011×10^{-5}
	ϕ_b deg	43.35	43.69	44.39	445.86
High speed range (68 Hz to 80 Hz)	e_b m	4.819×10^{-5}	4.902×10^{-5}	5.055×10^{-5}	5.143×10^{-5}
	ϕ_b deg	43.36	44.10	45.48	4.27

The comparison of the estimated parameters with the absolute values of assumed one for the low and high speed range cases are depicted in Figure 6.14 and Figure 6.15 respectively. It is noticed in the low speed case as shown in Figure 6.14(a, f), the highly deviated identifiables after introducing 5% bias error are found to be 6.098% (c_H), 6.216% (c_{yy}^R), 7.163% (c_{yy}^L), 8.407% (e_2) and 12.861% (Δk_{c44}). Rest of parameters are getting no more deviation by 5%. Similarly, on observing Figure 6.15(a, f) for the high speed range analysis, it is found that the highly affected parameters with 5% bias error are 39.34% (Δk_{c44}), -16.07% (e_2), -11.99% (e_1), 10.315% (f_0) and 6.216% (c_H), whereas least affected parameter deviates by -1.403% (Δk_{c22}) for the high speed range case.

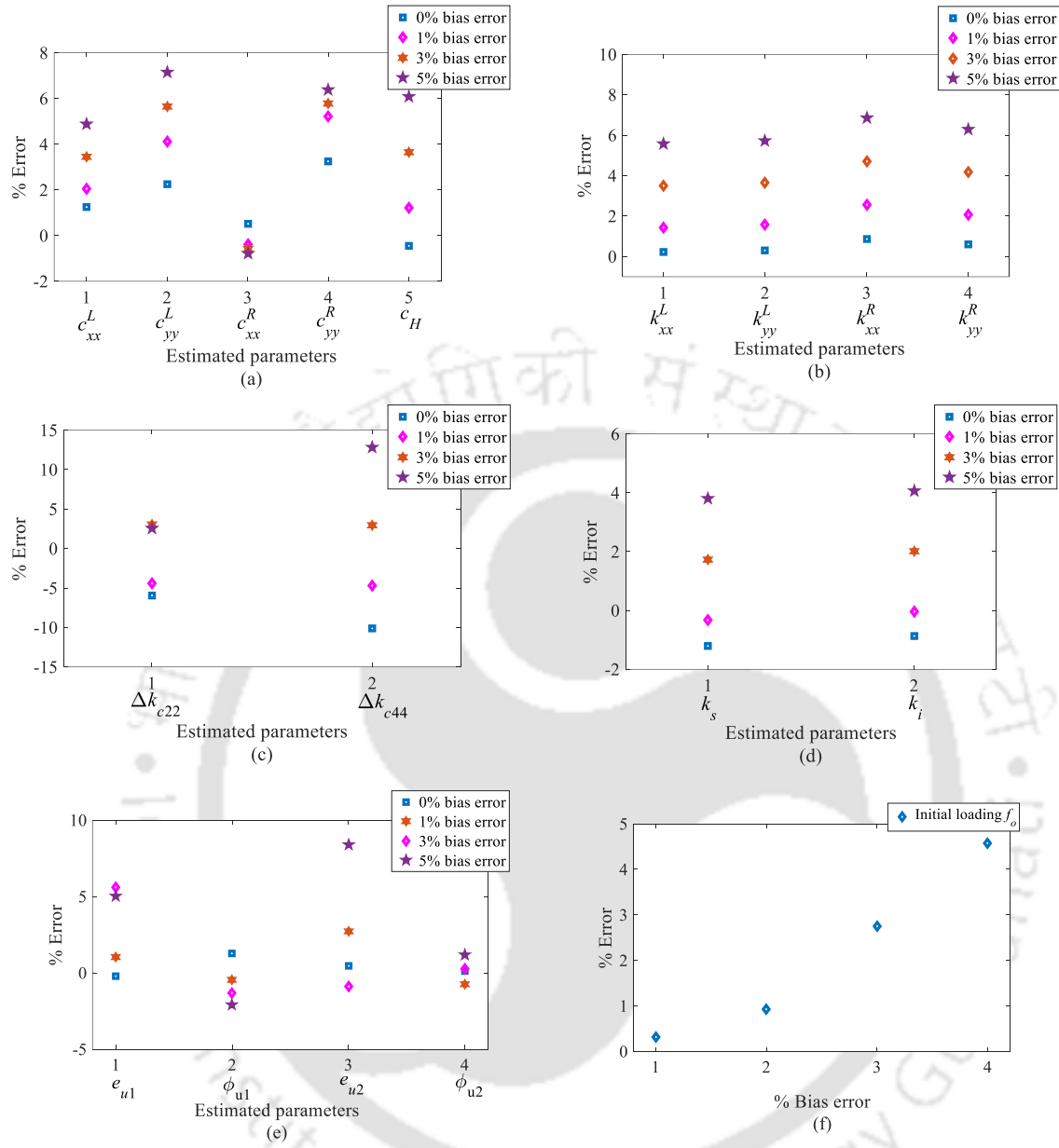


Figure 6.14 Error percentage of fault parameters with respect to the assumed values at low speed range (15 Hz to 35 Hz) after introducing modelling or bias error (a) Bearing damping and internal damping (b) Bearing stiffness (c) Translational and rotational additive stiffness of crack (d) AMB parameters (e) Unbalance and phase of Disc 1 and Disc 2 (f) Initial loading on the shaft

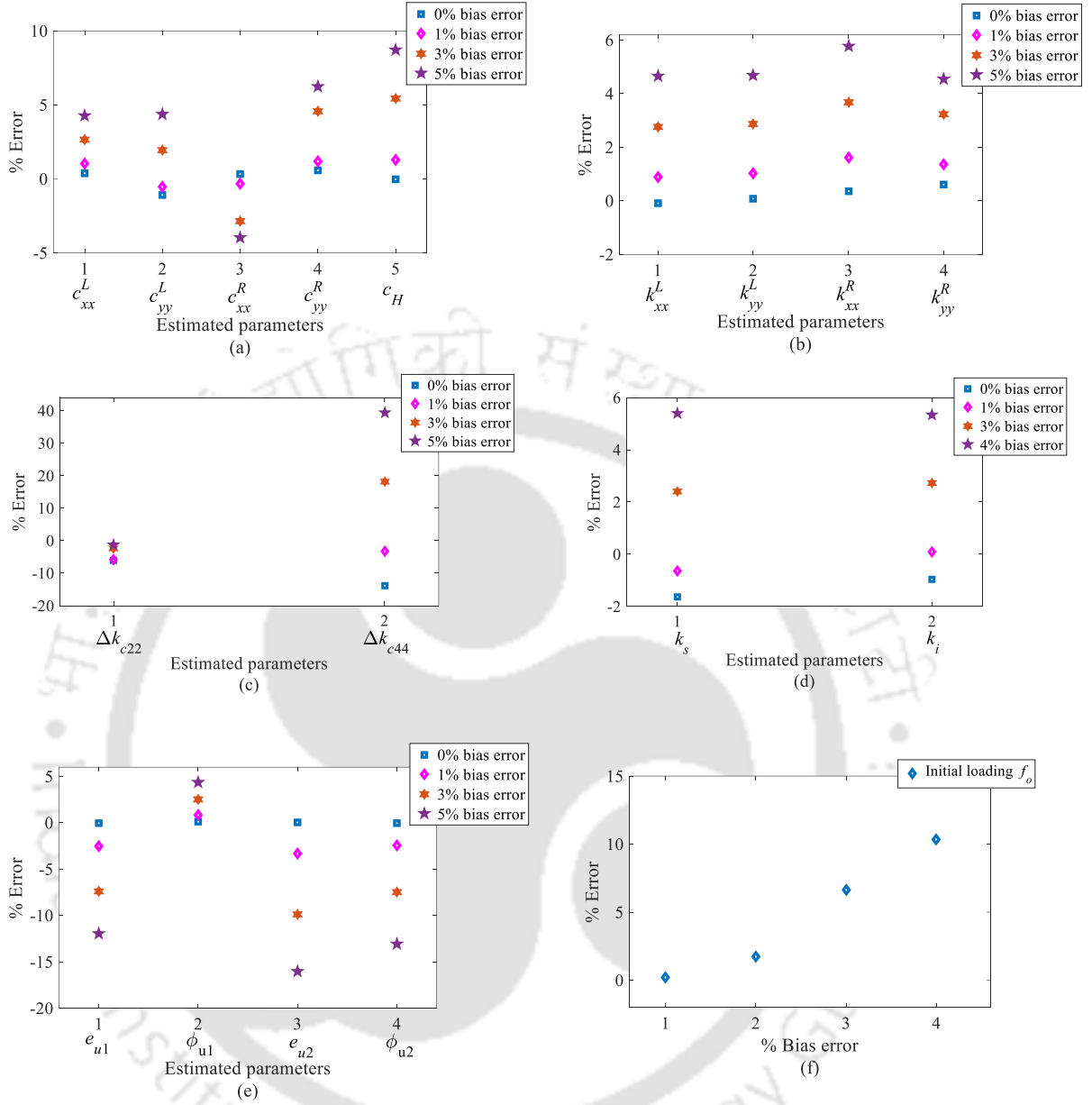


Figure 6.15 Error percentage of fault parameters with respect to the assumed values at high speed range (68 Hz to 80 Hz) after introducing modelling or bias error (a) Bearing damping and internal damping (b) Bearing stiffness (c) Translational and rotational additive stiffness of crack (d) AMB parameters (e) Unbalance and phase of Disc 1 and Disc 2 (f) Initial loading on the shaft

The developed algorithm makes a quite good agreement with the assumed parameters and found to be sensitive to measurement noise as well as modeling or bias errors. The effect of modeling errors in the shaft stiffness and the disc mass, to introduce uncertainty into the shaft and disc parameters, affect the unbalance amplitude and hence deviations are higher in the disc eccentricity. Moreover, one of the main reasons for the higher variation of Δk_{c44} parameter might be owing to the removal of its effect in utilization of the gyroscopic condensation scheme (GDC) and inclusion in the estimation further. In continuation of the present work, it is summarized that the multiple fault parameters may be estimated with a high degree of accuracy, even in the presence of noise and bias errors provided the speed ranges are chosen properly with a fairly accurate model of rotor bearing system with the disc displacement and control current measurement.

6.5 Concluding Remarks

In the present chapter, the cracked-warped rotor-AMB model has been extended to a very generic situation with multiple discs and multiple AMB support as industrial steam turbine generator (STG) machinery and the support dynamics has also been taken into account. The finite element modeling of identification of multiple faults in a flexible rotor has been performed with help of the vibrational displacements and AMB control current signals. The FEM model developed is based on simple Euler- Bernoulli beam theory with gyroscopic effect in discs for generation of simulated responses for use in the identification algorithm. For a demonstration of the procedure the developed identification problem is tested in a numerical model with two. It is found sufficient accuracy in the estimation process even in the presence of measurement noise and modelling errors. The conclusions drawn from the present analysis are as follows

- (i) The signal processing procedure developed in the previous Chapters 2 and 4 for simple rotor models in identification of multiple faults are applicable for complex cracked- warped internally damped rotor–AMB systems.
- (ii) It also establishes an identification procedure for the support dynamic parameters (damping and stiffness of bearing).
- (iii) The sensitivity of the developed identification algorithm is tested against measurement noise and modeling errors. The estimated parameters are found in a good agreement with the absolute values even consideration of flexibility in the support conditions except the rotational additive stiffness parameter of crack like in previous models of estimation.

In the next chapter, a summary of the various cracked-warped internally damped rotor integrated with AMB models of the identification process, complexities addressed and results obtained with those models and assumptions of the rotor–AMB system considered and numerically tested as discussed in Chapter 2, 4 and 6. The experimental investigations of the above discussed fault identification technique considering both with and without gyroscopic effect condition are done on the developed laboratory rotordynamic test bench, as illustrated in Chapters 3 and 5, are also addressed in the next chapter. The major contributions, conclusions, and scopes of future works for the present study will also be discussed in the next chapter.



Chapter 7

Conclusions

This project has centred a concept for a rotor-AMB system topology consisting of a cracked-warped rotor which is integrated with radial AMB for simultaneous identification of multiple faults implementing model based approach. Mathematical modeling including various faults is developed, one without gyroscopic effect, another is with gyroscopic effect in a cracked simple rotor and for flexible rotor mounted on flexible bearings. Then identification algorithms have been developed based on these frequency domains EOMs. Numerical tests are done to estimate the fault parameters through the developed identification algorithm using full spectrum technique. The time domain responses of the rotor are generated through SIMULINK™ model in MATLAB® environment and converted into full spectrum responses using FFT. The full spectrum responses carry the rotor behavior in both forward and backward directions, which are fed to the identification algorithm to estimate multiple faults associated with the rotor-AMB system. To avoid the phase ambiguity in the signal, a multi-frequency reference signal is generated from which the phases of the displacement signal and current signal are subtracted and these compensated phases are then used in the estimation technique. To mimic the real rotor system, random noises are implemented in time domain responses of the numerically generated signal. Moreover, to check the sensitivity, modeling, or bias errors are added to some of the physical rotor parameters. It is observed that the estimated parameters are robust against measurement noise and bias errors. The experimentally acquired rotor

vibrational displacements and AMB control currents from the laboratory test rig operating at various distinct speeds are utilized in the developed algorithm to estimate the multiple fault parameters. The estimated parameters are observed to be in the range of rotor systems, which indicates the efficacy of the proposed technique. Later, to analyze and estimate the parameters of a more realistic model, an identification algorithm is developed using FEM (finite element modeling) based EOMs of a flexible rotor-AMB system mounted on flexible bearing considering all the mentioned faults in the earlier modeling and tested numerically.

7.1 Major Conclusions from the Current Work

- Often estimation of rotor fault parameters has been done, which do not affect each other. In the present study, the interdependency of crack and residual bow fault and their combined effect on the system dynamics is analyzed.
- While considering transverse rotational DOFs in rotor models, the development of an identification algorithm requires the measurement of transverse rotational DOFs and it poses difficulty in physical measurements. Consideration of the gyroscopic dynamic condensation (GDC) helps effectively the elimination of transverse rotational DOFs as compared to the conventional dynamic reduction scheme where the known gyroscopic matrix is neglected.
- With the utilization of the developed algorithm, the simultaneous estimation of multiplicative crack-bow faults along with other rotor faults is now possible for a rotor system and the same is validated with the experimental test rig in the laboratory.
- Bow produces 1X revolution component in the rotor response and its effect can be removed by removing 1X component using signal processing technique from total

response. However, in presence of crack, the bow has been found to contribute at multiple harmonics due to multiplicative nature or interdependency of bow and crack parameters. Therefore, the removal of 1X harmonics will not eliminate effect of bow from the responses, which is often done during experimental estimation of crack parameters through responses.

- Full spectrum response of displacement and current signal shows the directional nature of the vibrations arising due to faults in a rotor system. From full spectrum analysis an observation is made that healthy shaft possesses 1x harmonic due to unbalance and -1x due to slight rubbing (or friction) at rolling bearing, while cracked shaft encompasses multiple harmonics in both forward and backward whirls.

7.2 Major Contributions and Recommendations from the Present Work

The main contributions of the thesis work can thus be summarized as

- Conventionally, in modal testing, the exciters are used to apply known excitation to the dynamic system and output is measured. The input-output information is used to characterize the system parameters, including fault parameters. In the present work, the AMB is used as a controller to suppress rotor vibration. The suppression force (AMB current and suppressed rotor displacement) is used to find multiple fault conditions in the rotor using model-based identification algorithms. The use of the present technique of suppression force instead of excitation force may be used in any dynamic system, which has several advantages.
- From the present modelling of crack and bow together, it has been found that they have multiplicative effect. That means they affect each other. This is very significant

contribution since estimation of multiplicative fault parameters have hardly been attempted in literature. The similar, analysis and identification may be attempted in other combination of fault conditions to investigate interplay among them.

- A multi-fault identification algorithm is developed for the estimation of the residual bow and crack parameters, as well as other rotor fault parameters. Herein, rather than removing the residual bow effect from the signal through signal processing at whole signal is used.
- The experimental implementation of AMB as a controller for estimation of cracked-warped rotor parameters is performed. A radial AMB system is successfully integrated in the test rig with differential driving mode and PID controller. The control current of AMB in real-time from the controller together with the attenuated vibrational displacement of the rotor is used as diagnostic information for the identification of rotor faults.
- The linear static deflection magnitude in real rotors gives rise to an appreciable amount of rotational static deflection and that influences the rotor flexibility which was not considered in the previous works. In the present work, experimental estimation of the rotational additive crack stiffness along with the translational additive crack stiffness is attempted for the first time.
- To comply with the practical rotor system, end support dynamic parameters is also modeled and identified through numerical simulation with other multiple faults such as external and internal damping, unbalance and residual bow, translational and rotational additive crack stiffness, multiplicative bow-crack fault, and AMB dynamic parameters of rotor-AMB system using finite element modeling (FEM) approach.

7.3 Scope for Future Work

Based on the work carried out in this research some of the key challenges and areas, where future research has scopes, are as follows

- In the present work, the FEM based identification algorithm on the flexible rotor-bearing integrated with AMB have been developed and tested numerically. It can be extended to validate the developed technique with a cracked rotor mounted on flexible bearing and integrated with AMB in a laboratory test rig.
- In literature, it has been reported that the crack and misalignment give multiple harmonics in responses and due to this distinct identification of crack and misalignment pose difficulty based on only signal analysis. It will be interesting to explore model-based identification of crack and misalignment together along with other effects considered in the present study, such as the bow, unbalance, internal and external damping, AMB dynamic parameters, etc. With this study definitely, a more in-depth interplay among various faults will be understood with more clarity.
- The location of crack is assumed to be known in the present work. The localization of crack parameters can be included in the mathematical model to identify them. It may be further extended for multiple cracks.
- The present work considers a linearized crack model in the analysis based in the ‘weight dominance’. Herein, the opening and closure of crack faces is determined by the rotation angle of the cracked section. This condition may be relaxed and the nonlinear behavior in crack can be studied along with estimation of non-linear crack force by taking general Fourier series instead of rectangular switching function.

- The modeling of shaft residual bow in translational radial direction is only modeled present work. Still it has scope of modeling the residual bow effect in angular direction and simultaneously the multiplicative fault effect of the rotational crack additive stiffness and the angular bow with its orientation could be quantified.
- In the experimental rig, it was observed that vibration suppression was not significant. It would be interesting to apply different modern adaptive control techniques, such as H_∞ , μ -synthesis, etc. on the developed test rig for effective vibration suppression.
- In the present work, the AMB is used as a controller and not as a supporting bearing. Further research can be carried out on the cracked rotor system levitated on AMBs and similar identification procedure can be applied to quantify faults in the system.

In summary then, the topic of model based identification of multiple faults in a cracked-warped rotor integrated with AMB has been introduced, and its viability with experimental verification conclusively demonstrated. From here, a multitude of possibilities for further research and development on the theme lie waiting to be explored, with numerous interesting and useful discoveries surely to be made.

Appendix

A. Transfer Function of the Proposed Rotor-AMB Model

The closed loop transfer function of the rotor- AMB system, as shown in the block diagram in Figure 2.5 is given as,

$$G_{tf} = \frac{G_{CMA}}{1 + G_{CMA}G_{sn}} \quad (A.1)$$

The open loop transfer function of the rotor bearing system is as follows,

$$1 + G_{CMA}G_{sn} \quad (A.2)$$

where without gyroscopic effect

$$G_{CMA} = G_C G_M G_A = \left(\frac{K_p s + K_I + K_D s^2}{s} \right) \left(\frac{k_i}{\{ms^2 + (c_{22} + c_H)s + (k_{22} - j\omega c_H) + k_s\}} \right) k_a$$

and, with gyroscopic effect

$$G_{CMA} = G_C G_M G_A = \left(\frac{K_p s + K_I + K_D s^2}{s} \right) \left(\frac{k_i}{\{ms^2 + (c_{22} + c_H - jI_p)s + (k_{22} - j\omega c_H) + k_s\}} \right) k_a$$

where, $G_{sn} = k_{sn}$ and G_{CMA} be the transfer function of the system containing a controller, sensor and magnetic bearing in series as shown in Figure 2.5 and, G_{sn} is the overall correction sensor gain. The overall closed loop transfer function of the system becomes,

$$G_{tf} = \frac{s^2 \left(K_p + \frac{K_I}{s} + K_D s \right) k_i k_s}{\left(\{ms^3 + (c_{22} + c_H + K_D k_i k_s k_{sn})s^2 + (k_{22} - j\omega c_H + k_s + K_p k_i k_x k_{sn})s\} + K_I k_i k_s k_{sn} \right)} \quad (A.3)$$

$$\text{and, } G_{tf} = \frac{s^2 \left(K_p + \frac{K_I}{s} + K_D s \right) k_i k_s}{\left(\{ms^3 + (c_{22} + c_H - jI_p + K_D k_i k_s k_{sn})s^2 + (k_{22} - j\omega c_H + k_s + K_p k_i k_x k_{sn})s\} + K_I k_i k_s k_{sn} \right)} \quad (A.4)$$

where k_x is the amplifier gain of the power amplifier. The characteristics equation of motion of the proposed rotor system, the denominator of the Eq. (A.4) is represented as

$$ms^3 + (c_{22} + c_H + K_D k_i k_s k_{sn})s^2 + (k_{22} - j\omega c_H + k_s + K_p k_i k_s k_{sn})s + K_I k_i k_x k_{sn} = 0 \quad (A.5)$$

$$\text{And, } ms^3 + (c_{22} + c_H - jI_p + K_D k_i k_s k_{sn})s^2 + (k_{22} - j\omega c_H + k_s + K_p k_i k_s k_{sn})s + K_I k_i k_x k_{sn} = 0 \quad (A.6)$$

B. Design of Actuator based on Heteropolar Radial Magnetic Bearing

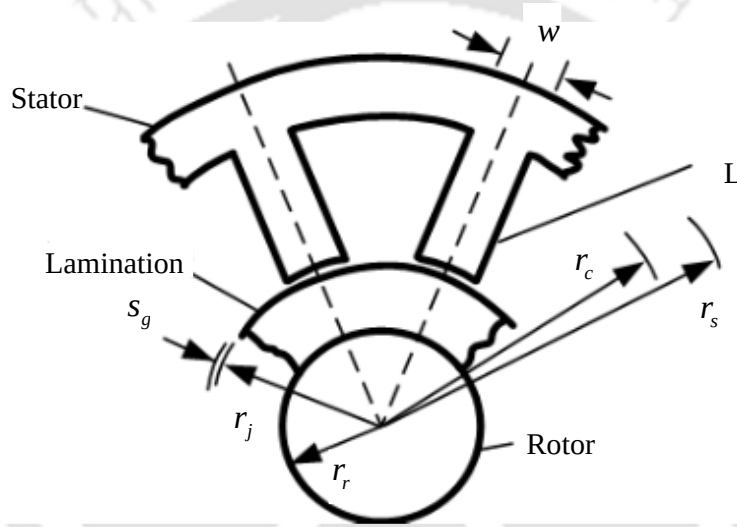


Figure B.1 Cross section of the magnetic bearing actuator

In the cross-section of the actuator, the coil space radius is denoted by r_c , the lamination radius is r_j , the stator radius, r_s the rotor radius, the air-gap length, s_g and w is the pole width as shown in Figure B.1.

Primary specifications of the bearing actuator depends on the peak capacity, $F_{\max}$; the slew rate requirement, dF/dt and the shaft diameter.

The saturation force capacity,

$$F_{\max} = \frac{\beta n B_{\text{sat}}^2 A_g}{2\mu_0} \quad (B.1)$$

where, β is the load capacity factor and value is taken 0.3 for individual coil current for the eight pole actuator configuration as in Ref. (Masleen et al., (2004)).

Saturation current,

$$I_{sat} = \frac{B_{sat} s_g}{\mu_0 N} \quad (B.2)$$

Putting Equation (B.2) in Equation (B.2) becomes,

$$F_{max} = \frac{\beta n A_g \mu_0}{2 s_g^2} N^2 I_{sat}^2 \quad (B.3)$$

The leg width, w , and the journal length, l , are related to the air gap area, A_g as

$$A_g = w l \quad (B.4)$$

To avoid the saturation in the rotor, the laminated radial dimension of the rotor is represented as

$$r_j \geq (r_r + f_s w) \quad (B.5)$$

where, $f_s = 0.5$ for the flux splitting or 1.0 for the without flux splitting. If the iron is not using the split flux condition, the rotating losses becomes low and the current controlling is also simple. Therefore, the journal aspect ratio related with the pole width can be represented as,

$$\gamma = \frac{l}{2r_j} \leq \frac{l}{2(r_r + f_s w)} = \frac{A_g}{2w(r_r + f_s w)} \quad (B.6)$$

Stator Outer Radius:

The stator outer radius can be evaluated as

$$r_s = (r_c + f_s w) \quad (B.7)$$

Stator axial length:

The stator axial length is the iron length plus the coil thickness becomes

$$l_s = l + 2t_c \quad (B.8)$$

Where, the coil thickness, $t_c = r_p \tan\left(\frac{\pi}{n}\right) - \frac{w}{2}$

Pole Tip radius:

The radius of the stator pole becomes

$$r_p = r_j + s_g \quad (\text{B.9})$$

C. Appendix: Regression Equation of Cracked Warped Rotor-AMB System in Chapter 4

The regressor in Equation (4. 38),

$$\mathbf{A}_{1_i} = \begin{bmatrix} 0 & \omega S_{0,lm} & -p_0 q_{w_1} & -t_0^d p_0 q_{w_2} & -q_0 & 0 & 0 & 0 & 0 & -S_{0,Re} & I_{0,Re} \\ -\omega S_{0,lm} & 0 & -p_1 q_{w_1} & -t_1^d p_1 q_{w_2} & -q_1 & -m\omega^2 & 0 & -(k_{22} + t_1^d k_{24}) & 0 & -S_{1,Re} & I_{1,Re} \\ -2\omega S_{0,lm} & -\omega S_{1,lm} & -p_1 q_{w_1} & -t_2^d p_2 q_{w_2} & -q_2 & 0 & 0 & 0 & 0 & -S_{2,Re} & I_{1,Re} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \omega S_{-1,lm} & 2\omega S_{-1,lm} & -p_{-1} q_{w_1} & -t_{-1}^d p_{-1} q_{w_2} & -q_0 & 0 & -m\omega^2 & 0 & -(k_{22} + t_1^d k_{24}) & -S_{-1,Re} & I_{-1,Re} \\ 3\omega S_{-2,lm} & 4\omega S_{-2,lm} & -p_{-2} q_{w_1} & -t_{-2}^d p_{-2} q_{w_2} & -q_{-1} & 0 & 0 & 0 & 0 & -S_{-2,Re} & I_{-2,Re} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & (\omega S_{0,Re} + q_{w_1}) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_{0,lm} & I_{0,lm} \\ \omega S_{0,Re} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_{1,lm} & I_{1,lm} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\omega S_{-1,Re} & -2\omega S_{-1,Re} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_{-1,lm} & I_{-1,lm} \\ -3\omega S_{-3,Re} & -4\omega S_{-2,Re} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_{-1,lm} & I_{-2,lm} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \quad (\text{C.1})$$

and, the known matrix of Equation (4.38) as

$$\mathbf{b}_i = \begin{Bmatrix} 0 \\ \left(\omega^2 (m + (t_1^d)^2 I_d) S_{1,Re} + \omega^2 I_p (t_1^d)^2 S_{1,Im} + \left(\omega (t_1^d)^2 + 2\omega t_1^d \right) S_{1,Im} - (k_{22} + (t_1^d)^2 k_{44}) S_{1,Re} \right) \\ \left(4\omega^2 (m + (t_2^d)^2 I_d) S_{2,Re} + 2\omega^2 I_p (t_2^d)^2 S_{2,Im} + \left(2\omega (t_2^d)^2 + 4\omega c_1^d \right) S_{2,Im} - (k_{22} + (t_2^d)^2 k_{44}) S_{2,Re} \right) \\ \left(9\omega^2 (m + (t_3^d)^2 I_d) S_{3,Re} + 3\omega^2 I_p (t_3^d)^2 S_{3,Im} + \left(3\omega (t_3^d)^2 + 9\omega t_3^d \right) S_{3,Im} - (k_{22} + (t_3^d)^2 k_{44}) S_{3,Re} \right) \\ \vdots \\ \left(\omega^2 (m + (t_{-1}^d)^2 I_d) S_{-1,Re} - \omega^2 I_p (t_{-1}^d)^2 S_{-1,Im} - \left(2\omega c_{-1}^d + \omega (t_{-1}^d)^2 \right) S_{-1,Im} - (k_{22} + (t_{-1}^d)^2 k_{44}) S_{-1,Re} \right) \\ \left(9\omega^2 (m + (t_{-3}^d)^2 I_d) S_{-3,Re} - 3\omega^2 I_p (t_{-3}^d)^2 S_{-3,Im} - \left(6\omega c_{-3}^d + 3\omega (t_{-3}^d)^2 \right) S_{-3,Im} - (k_{22} + (t_{-3}^d)^2 k_{44}) S_{-3,Re} \right) \\ \vdots \\ 0 \\ \left(\omega^2 (m + (t_1^d)^2 I_d) S_{1,Im} + \omega^2 I_p (t_1^d)^2 S_{1,Re} - \left(2\omega t_2^d + \omega (t_2^d)^2 \right) S_{1,Re} - (k_{22} + (t_1^d)^2 k_{44}) S_{1,Im} \right) \\ \left(4\omega^2 (m + (t_2^d)^2 I_d) S_{2,Im} + 2\omega^2 I_p (t_2^d)^2 S_{2,Re} - \left(4\omega c_2^d + 2\omega (t_2^d)^2 \right) S_{2,Re} - (k_{22} + (t_2^d)^2 k_{44}) S_{2,Im} \right) \\ \left(9\omega^2 (m + (t_3^d)^2 I_d) S_{3,Im} + 3\omega^2 I_p (t_3^d)^2 S_{3,Re} - \left(6\omega t_3^d + 3\omega (t_3^d)^2 \right) S_{3,Re} - (k_{22} + (t_3^d)^2 k_{44}) S_{3,Im} \right) \\ \vdots \\ \left(\omega^2 (m + (t_{-1}^d)^2 I_d) S_{-1,Im} - \omega^2 I_p (t_{-1}^d)^2 S_{-1,Re} + \left(2\omega c_{-1}^d + \omega (t_{-1}^d)^2 \right) S_{-1,Re} - (k_{22} + (t_{-1}^d)^2 k_{44}) S_{-1,Im} \right) \\ \left(9\omega^2 (m + (t_{-3}^d)^2 I_d) S_{-3,Re} - 3\omega^2 I_p (t_{-3}^d)^2 S_{-3,Re} + \left(6\omega c_{-1}^d + 3\omega (t_{-3}^d)^2 \right) S_{-3,Re} - (k_{22} + (t_{-3}^d)^2 k_{44}) S_{-3,Im} \right) \\ \vdots \end{Bmatrix} \quad (C.2)$$

D. Appendix: An Euler Bernoulli Beam Model and the Regression Matrix of the Developed Identification Algorithm

D.1 Mass, stiffness and circulatory matrices of the shaft element

Element matrices for the shaft based on Euler-Bernoulli beam finite element are given as Ref. (Tiwari, 2017)

$$\mathbf{M}^e = \frac{\rho A l}{420} \begin{bmatrix} 156 & & & \text{sym} \\ 22l & 4l^2 & & \\ 54 & -13l & 156 & \\ 13l & -3l^2 & 22l & 4l^2 \end{bmatrix} \quad (\text{D.1})$$

$$\mathbf{K}_s^e = \frac{EI}{l^3} \begin{bmatrix} 12 & & & \text{sym} \\ 6l & 4l^2 & & \\ -12 & 6l & 12 & \\ -6l & 2l^2 & 6l & 4l^2 \end{bmatrix} \quad (\text{D.2})$$

$$\mathbf{K}_c^e = \frac{2EI}{l^3} \begin{bmatrix} -6 & & \text{skew} & \text{sym} \\ -3l & 3l & & \\ -6 & -6 & -3l & \\ -3l & 3l & l^2 & -l^2 \end{bmatrix} \quad (\text{D.3})$$

D.2 Rigid Disc Model

$$\text{Mass matrix, } \mathbf{M}^d = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_d & 0 \\ 0 & 0 & 0 & I_d \end{bmatrix} \quad (\text{D.4})$$

$$\text{Gyroscopic matrix, } \mathbf{G}^d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_p \\ 0 & 0 & I_p & 0 \end{bmatrix} \quad (\text{D.5})$$

D.3 Regression Matrix

The regression equation of the matrix containing the co-efficient of the unknown fault parameters is written in the next page.

The known matrix in the right hand side of Equation (6.35) as

$$\mathbf{B}_i = \begin{pmatrix} \vdots \\ (25\omega^2\mathbf{M}^{gd} - 5j\omega\mathbf{G}^{gd} + \mathbf{K}_B^{gd})\tilde{\mathbf{q}}_5 \\ (16\omega^2\mathbf{M}^{gd} - 4j\omega\mathbf{G}^{gd} + \mathbf{K}_B^{gd})\tilde{\mathbf{q}}_4 \\ (9\omega^2\mathbf{M}^{gd} - 3j\omega\mathbf{G}^{gd} + \mathbf{K}_B^{gd})\tilde{\mathbf{q}}_3 \\ (4\omega^2\mathbf{M}^{gd} - 2j\omega\mathbf{G}^{gd} + \mathbf{K}_B^{gd})\tilde{\mathbf{q}}_2 \\ (\omega^2\mathbf{M}^{gd} - j\omega\mathbf{G}^{gd} + \mathbf{K}_B^{gd})\tilde{\mathbf{q}}_1 \\ (-\mathbf{K}_B^{gd})\tilde{\mathbf{q}}_0 \\ (\omega^2\mathbf{M}^{gd} + j\omega\mathbf{G}^{gd} - \mathbf{K}_B^{gd})\tilde{\mathbf{q}}_{-1} \\ (4\omega^2\mathbf{M}^{gd} + 2j\omega\mathbf{G}^{gd} - \mathbf{K}_B^{gd})\tilde{\mathbf{q}}_{-2} \\ (9\omega^2\mathbf{M}^{gd} + 3j\omega\mathbf{G}^{gd} - \mathbf{K}_B^{gd})\tilde{\mathbf{q}}_{-3} \\ \vdots \end{pmatrix} \quad (\text{D.6})$$

Regression matrix $\mathbf{A}_1$ corresponds to various system parameters in the rows of vector $\mathbf{x}$ for multiple harmonic signal segregating in the left hand side of Equation (6.35) is as follows

$$\mathbf{A}_i = \begin{bmatrix} \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots & \dots & 0 & \dots & \vdots & \dots & \vdots & \dots & \vdots & \dots \\ -j5\omega q_{m_5, \text{Re}} \mathbf{l}_{br_i} & \dots & -j\omega q_{m_5, \text{Im}} \mathbf{l}_{br_i} & \dots & q_{m_5, \text{Re}} \mathbf{l}_{br_i} & \dots & -jq_{m_5, \text{Im}} \mathbf{l}_{br_i} & \dots & -j4\omega q_{m_5} & p_5 u_0 \mathbf{l}_{cr} & \dots & p_5 \varphi_{y_0} \mathbf{l}_{cr} & \dots & q_5 \mathbf{l}_{cr} & 0 & \dots & -q_{m_5} \mathbf{l}_{AMB_i} & \dots & i_{c_{5,1}} \mathbf{l}_{AMB_i} & \dots & 0 \\ -j4\omega q_{m_4, \text{Re}} \mathbf{l}_{br_i} & \dots & -j\omega q_{m_4, \text{Im}} \mathbf{l}_{br_i} & \dots & q_{m_4, \text{Re}} \mathbf{l}_{br_i} & \dots & -jq_{m_4, \text{Im}} \mathbf{l}_{br_i} & \dots & -j3\omega q_{m_4} & 0 & \dots & 0 & \dots & q_4 \mathbf{l}_{cr} & 0 & \dots & -q_{m_4} \mathbf{l}_{AMB_i} & \dots & i_{c_{4,1}} \mathbf{l}_{AMB_i} & \dots & 0 \\ -j3\omega q_{m_3, \text{Re}} \mathbf{l}_{br_i} & \dots & -j\omega q_{m_3, \text{Im}} \mathbf{l}_{br_i} & \dots & q_{m_3, \text{Re}} \mathbf{l}_{br_i} & \dots & -jq_{m_3, \text{Im}} \mathbf{l}_{br_i} & \dots & -j2\omega q_{m_3} & p_2 u_0 \mathbf{l}_{cr_i} & \dots & p_2 \varphi_{y_0} \mathbf{l}_{cr_i} & \dots & q_3 \mathbf{l}_{cr} & 0 & \dots & -q_{m_3} \mathbf{l}_{AMB_i} & \dots & i_{c_{3,1}} \mathbf{l}_{AMB_i} & \dots & 0 \\ -j2\omega q_{m_2, \text{Re}} \mathbf{l}_{br_i} & \dots & -j\omega q_{m_2, \text{Im}} \mathbf{l}_{br_i} & \dots & q_{m_2, \text{Re}} \mathbf{l}_{br_i} & \dots & -jq_{m_2, \text{Im}} \mathbf{l}_{br_i} & \dots & -j\omega q_{m_2} & p_1 u_0 \mathbf{l}_{cr_i} & \dots & p_1 \varphi_{y_0} \mathbf{l}_{cr_i} & \dots & q_2 \mathbf{l}_{cr} & m_1 e_1 \omega^2 \mathbf{l}_{unb_1} & \dots & -q_{m_2} \mathbf{l}_{AMB_i} & \dots & i_{c_{2,1}} \mathbf{l}_{AMB_i} & \dots & 0 \\ -j\omega q_{m_1, \text{Re}} \mathbf{l}_{br_i} & \dots & -j\omega q_{m_1, \text{Im}} \mathbf{l}_{br_i} & \dots & q_{m_1, \text{Re}} \mathbf{l}_{br_i} & \dots & -jq_{m_1, \text{Im}} \mathbf{l}_{br_i} & \dots & j\omega(q_{m_0} + u_0 \mathbf{l}_{cr_i}) & p_0 u_0 \mathbf{l}_{cr_i} & \dots & p_0 \varphi_{y_0} \mathbf{l}_{cr_i} & \dots & q_1 \mathbf{l}_{cr} & 0 & \dots & -q_{m_1} \mathbf{l}_{AMB_i} & \dots & i_{c_{1,1}} \mathbf{l}_{AMB_i} & \dots & -S\delta \\ 0 & \dots & -j\omega q_{m_0, \text{Im}} \mathbf{l}_{br_i} & \dots & q_{m_0, \text{Re}} \mathbf{l}_{br_i} & \dots & -jq_{m_0, \text{Im}} \mathbf{l}_{br_i} & \dots & j2\omega q_{m_{-1}} & p_{-1} u_0 \mathbf{l}_{cr_i} & \dots & p_{-1} \varphi_{y_0} \mathbf{l}_{cr_i} & \dots & q_0 \mathbf{l}_{cr} & \dots & \dots & -q_{m_0} \mathbf{l}_{AMB_i} & \dots & i_{c_{0,1}} \mathbf{l}_{AMB_i} & \dots & 0 \\ j\omega q_{m_{-1}, \text{Re}} \mathbf{l}_{br_i} & \dots & -j\omega q_{m_{-1}, \text{Im}} \mathbf{l}_{br_i} & \dots & q_{m_{-1}, \text{Re}} \mathbf{l}_{br_i} & \dots & -jq_{m_{-1}, \text{Im}} \mathbf{l}_{br_i} & \dots & j3\omega q_{m_{-2}} & 0 & \dots & 0 & \dots & q_{-1} \mathbf{l}_{cr} & \dots & \dots & -q_{m_{-1}} \mathbf{l}_{AMB_i} & \dots & i_{c_{-1,1}} \mathbf{l}_{AMB_i} & \dots & 0 \\ j2\omega q_{m_{-2}, \text{Re}} \mathbf{l}_{br_i} & \dots & -j\omega q_{m_{-2}, \text{Im}} \mathbf{l}_{br_i} & \dots & q_{m_{-2}, \text{Re}} \mathbf{l}_{br_i} & \dots & -jq_{m_{-2}, \text{Im}} \mathbf{l}_{br_i} & \dots & j4\omega q_{m_{-3}} & p_{-3} u_0 \mathbf{l}_{cr_i} & \dots & p_{-3} \varphi_{y_0} \mathbf{l}_{cr_i} & \dots & q_{-2} \mathbf{l}_{cr} & \dots & \dots & -q_{m_{-2}} \mathbf{l}_{AMB_i} & \dots & i_{c_{-2,1}} \mathbf{l}_{AMB_i} & \dots & 0 \\ j3\omega q_{m_{-3}, \text{Re}} \mathbf{l}_{br_i} & \dots & -j\omega q_{m_{-3}, \text{Im}} \mathbf{l}_{br_i} & \dots & q_{m_{-3}, \text{Re}} \mathbf{l}_{br_i} & \dots & -jq_{m_{-3}, \text{Im}} \mathbf{l}_{br_i} & \dots & j5\omega q_{m_{-4}} & 0 & \dots & 0 & \dots & q_{-3} \mathbf{l}_{cr} & \dots & \dots & -q_{m_{-3}} \mathbf{l}_{AMB_i} & \dots & i_{c_{-3,1}} \mathbf{l}_{AMB_i} & \dots & 0 \\ \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \dots & \vdots & \dots & \vdots & \dots & \vdots \end{bmatrix} \quad (\text{D.7})$$

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List of Publications from the Present Work

Journals communicated/accepted from the thesis work:

1. **Nilakshi Sarmah** and **Rajiv Tiwari** "Dynamic analysis and identification of multiple fault parameters in a cracked rotor system equipped with active magnetic bearings: a physical model based approach." *Inverse Problems in Science and Engineering*: 1-32 2019.
2. **Nilakshi Sarmah** and **Rajiv Tiwari** "Analysis and identification of the additive and multiplicative fault parameters in a cracked-bowed-unbalanced rotor system integrated with an auxiliary active magnetic bearing." *Mechanism and Machine Theory* 146: 103744, 2020.
3. **Nilakshi Sarmah** and **Rajiv Tiwari** "Application of active magnetic bearings in identification of multiple faults in an internally damped cracked bowed rotor systems through finite element modeling". *Journal of the Brazilian Society of Mechanical Sciences and Engineering* (Revised manuscript submitted)
4. **Nilakshi Sarmah** and **Rajiv Tiwari** "Numerical and experimental study on quantitative assessment of multiple fault parameters in a warped internally damped rotor with a transverse fatigue crack integrated with Active magnetic bearing" *Mechanical Systems and Signal Processing* (Revised manuscript submitted)
5. **Nilakshi Sarmah** and **Rajiv Tiwari** "Development of novel identification algorithm for the fatigue cracked rotor-AMB system undergoing forward and backward whirls" (ready to be submitted)

Conferences:

1. **Nilakshi Sarmah** and **Rajiv Tiwari** “Active Control of Internal damping Instabilities in a Cracked Rotor with Magnetic Bearings”. *13th International Conference on Vibration Problems -ICOVP-2017*, 29th November-2nd December, 2017, Indian Institute of Technology Guwahati, Assam, India.
2. **Nilakshi Sarmah** and **Rajiv Tiwari** “A Stability Analysis Due to Internal Damping of a Cracked Jeffcott Rotor with a Offset Disc Integrated with an Active Magnetic Bearing”. *National Symposium on Rotor Dynamics- NSRD 2017*, December 12th to 13th, 2017, IIT Patna, India
3. **Nilakshi Sarmah** and **Rajiv Tiwari** “Identification of Crack and Internal Damping Parameters using Full Spectrum Responses from a Jeffcott Rotor Integrated with an Active Magnetic Bearing”. *International Federation for the Promotion of Mechanism and Machine IFTOMM-2018*, Sep. 23rd to 27th. Rio de Janeiro, Brazil
4. **Nilakshi Sarmah** and **Rajiv Tiwari** "Experimental Investigation of Active Control of Cracked Rotor-Bearing System Equipped With Magnetic Bearing." *Gas Turbine India Conference. Vol. 83525. American Society of Mechanical Engineers*, 2019, 5th December-6th December, 2019, IIT Madras, Chennai
5. **Nilakshi Sarmah** and **Rajiv Tiwari** "Experimental identification of unbalance and crack parameters in a rotor system integrated with active magnetic bearings". *Proceedings of the XVI Vibration Engineering & Technology of Machinery Conference VETOMAC - 2021, Dec 16-18, 2021*, B.M.S. College of Engineering, Bengaluru. (Full length paper accepted)

