



INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

Name of the Student : Rakesh Jana
Roll Number : 156123009
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Name of Thesis Supervisor(s) : Prof. Sukanta Pati
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SHORT ABSTRACT

The study of the properties of graphs via matrices is a widely studied subject that ties together two seemingly unrelated branches of mathematics; graph theory and linear algebra. Graham and Pollak in 1971 proved a remarkable result which tells that the determinant of the distance matrix of a tree only depends on the number of vertices in the tree. This impressive result created a lot of interest among the researchers. Since then many generalizations have been proposed in order to understand the distance matrix better. Yet, the understanding seems to be far from complete. We present one such point of view here showing how many more combinatorial objects are linked together.

For a bipartite graph, we do not require the complete adjacency matrix to display the adjacency information in the graph. Godsil in 1985, used a smaller size matrix to display this information. Later on this matrix was named the bipartite adjacency matrix. The bipartite adjacency matrix has appeared in many places in the literature and it is used widely in the study of the inverse of the bipartite graph with unique perfect matching. In this thesis, similar to the bipartite adjacency matrix, we define the *bipartite distance matrix* of a bipartite graph with a unique perfect matching. We show that the bipartite distance matrix of a nonsingular tree is always invertible and describe its determinant by using the structure of the tree. For a nonsingular tree, we observe that the inverse of its bipartite distance matrix is the sum of a generalization of the usual Laplacian matrix of a tree (which we called the *bipartite Laplacian matrix*) and a rank one matrix. Further, we provide a combinatorial description of all the minors of the bipartite Laplacian matrix of a nonsingular tree. We consider two types of q -analogue versions of the bipartite distance matrix of a nonsingular tree, namely the q -bipartite distance matrix and the exponential bipartite distance matrix. We then completely characterized the determinant of those two types of q -analogue versions of the bipartite distance matrix.