INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

Name of the Student	:	**Subhra Sankar Kalita**
Roll Number	:	**126103006**
Programme of Study	:	**Ph.D.**

Thesis Title: **Convergence Acceleration of Fluid-Flow Computation with Special Emphasis on Multigrid Technique**

Name of Thesis Supervisor(s)	:	**Prof. Anoop K. Dass**
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SHORT ABSTRACT

This thesis is concerned with the study and analysis of various convergence-acceleration strategies that can help CFD practitioners to obtain their simulation results quickly. Convergence-acceleration can be achieved either by choosing a fast algorithm or by parallelizing the solution strategy. The multigrid method lies in the former category and is considered as one of the most powerful convergence-acceleration strategies. CUDA, which is a relatively new method for parallelizing CFD codes falls under the second category and as a part of the convergence-acceleration efforts embodied in this thesis, CUDA is used to parallelize LBM-based computations.

Multigrid (MG) methods, which are based on many levels of grids, have the ability to overcome the slow convergence characteristic of the single-grid methods for large-scale problems. This method has been established as a powerful tool for accelerating numerical convergence and, thus reducing the computational time. At the heart of computation of nonlinear problems, many times there lies a linearized problem. Therefore, this thesis work starts with numerical experiments for the 2D heat conduction problem using multigrid. The result shows that though successive overrelaxation (SOR) with the optimum relaxation parameter substantially accelerates Gauss-Seidel iterations, linear multigrid does it even better and its performance relative to SOR improves with grid refinement.

Apart from adopting the right strategies in the context of multigrid itself, we observe that the final computational time depends greatly on the selection of the original flow solution schemes and algorithms themselves. The streamfunction-velocity formulation is a relatively new method to solve the Navier-Stokes equations. The formulation allows Dirichlet boundary conditions to be used for both streamfunction and velocity, which are iteration-independent. This makes the method particularly amenable for multigrid acceleration as unchanging boundary information is quickly passed to the interior. The accuracy of the results is established through a careful comparison exercise and faster convergence is indicated by time records of single and multigrid computations. Overall the combination of multigrid and streamfunction-velocity formulation affords a highly efficient solution procedure. To demonstrate the power of the solution procedure, a number of fluid-flow and heat-transfer problems are solved. The staggered cavity flow problem has been solved by the present computational procedure for the first time. Here, the laminar investigations are seen to be carried out at the highest value of Re=3200 and for this Re, the present solution procedure obtains results in only 41 seconds with a speed-up of about 28. For the first time multigrid accelerated ψ-V formulation has been used to compute multiple steady solutions in two- and four-sided cavity flows, demonstrating that multigrid has the ability to compute flows in those classes of problems that exhibit nonunique solutions. Even though the convergence is slower near the critical Re at which the flow bifurcates, the present method achieves fast convergence. In all the situations, solutions are obtained in a few seconds on a grid that provides accurate solutions, giving substantial convergence acceleration over a single-grid.

A part of the thesis is concerned with the lattice-Boltzmann simulation of various fluid flows and heat transfer problems. Self-written C/C++ codes have been developed for both the single-relaxation-time (SRT) and multiple-relaxation-time (MRT) lattice Boltzmann method (LBM). The comparative study between MRT-and SRT-based LBM indicates that in most cases, both models provide comparable solutions while dealing with laminar flow regimes. Nevertheless, MRT-LBM has the edge over SRT-LBM due to its better stability in solving complex flow problems. The LBM algorithm is highly amenable to acceleration through parallelization. We have used graphics processing unit (GPU) to parallelize the LBM code. A number of two- and three-dimensional fluid-flow and heat-transfer problems have been parallelized. Once two-dimensional LBM has been successfully parallelized using CUDA (Compute Unified Device Architecture), extending it to three-dimensional is found to be relatively straightforward. The acceleration of two-dimensional steady mixed convection heat transfer in a differentially heated square cavity for both aiding and opposing mechanisms has been successfully carried out using CUDA. It has been found that parallelized LB code computes about 8 times faster than the serial one and it signifies that LB code is very efficient in GPU environment. Hence, CUDA-based LBM computations are very efficient at a high value of Re that requires a finer grid. Finally, to gain an understanding of how the multigrid-assisted LBM method stands with respect to other multigrid-assisted computations, a two-dimensional heat diffusion problem was solved using MG-LBM and it was found that with a four-level multigrid-LBM we can achieve a speed-up of around 269.

Overall, in the present work, a wide variety of fluid flow and heat transfer problems of varying complexities have been studied using multigrid accelerated techniques and the parallel lattice Boltzmann method. Both SRT and MRT versions of LBM are used to compute flow problems involving natural- and mixed convection. Furthermore, the scope of using the multigrid assisted ψ-V algorithm has been extended to flow configuration having nonunique solutions. Sufficient care has been taken to obtain highly accurate solutions. All presented results are grid-independent. All the results in this thesis are produced using computer codes written in C/C++ (for parallel codes, CUDA was used) that have been carefully developed by the author from scratch and no help from any third-party tool or library has been taken. The results are validated through an extensive comparison with the published results. The ability of multigrid to efficiently compute the wide variety of flows included in this thesis demonstrates not only the utility of the algorithm but also its robustness. Some of the applications deal with problems in which the multigrid method was not used earlier.