

Gravitational Wave Signatures of the Genesis of Dark Matter and Baryon Asymmetry

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A thesis submitted for the degree of

Doctor of Philosophy

Supervisor

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A thesis submitted by

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to

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Supervisor

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“In all chaos there is a cosmos, in all disorder a secret order”

-Carl Jung.



This thesis is dedicated to

my

Maa and Baba.





Declaration



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I hereby declare that works presented in the thesis entitled **“Gravitational Wave Signatures of the Genesis of Dark Matter and Baryon Asymmetry”** have been carried out by me under the supervision of **Dr. Debasish Borah** at the Department of Physics, Indian Institute of Technology Guwahati, India. The thesis has not been submitted anywhere else for any degree. Works presented in the thesis are all my own unless referenced to the contrary in the thesis.

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It is certified that the work contained in the thesis entitled “**Gravitational Wave Signatures of the Genesis of Dark Matter and Baryon Asymmetry**” by Mr. Indrajit Saha (Roll No. - 206121015), a Ph.D. student in the Department of Physics, Indian Institute of Technology Guwahati is carried out under my supervision and has not been submitted elsewhere for the award of any other degree.

Date: October 13, 2025


Signature



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-Indrajit Saha

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Abstract

This thesis investigates the longstanding cosmological puzzles of baryon asymmetry and dark matter, with a focus on probing their origins through stochastic gravitational wave (GW) signatures arising from early Universe dynamics. The persistent null results from conventional dark matter searches, direct detection experiments such as XENONnT, LUX-ZEPLIN, PandaX-II as well as Large Hadron Collider (LHC) experiments and the inaccessibility of high-scale baryogenesis mechanisms in these experiments strongly motivate the exploration of gravitational wave astronomy as a complementary probe. In particular, first-order phase transitions (FOPT), associated with spontaneous symmetry breaking in the early Universe, are highlighted as natural sources of stochastic GW while simultaneously playing a crucial role in particle mass generation and asymmetry production. The thesis begins with a review of the observational evidences for dark matter and baryon asymmetry together with popular theoretical frameworks explaining their origins. It then introduces some of the mechanisms of gravitational wave production in the early Universe, with emphasis on FOPT, domain walls, and primordial black holes (PBH) generated signals within the reach of existing and near future detector sensitivity, such as NANOGrav, μ ARES, LISA, DECIGO, CE, LIGO, etc. Based on these foundations, we have investigated some production mechanisms of dark matter and baryon asymmetry connecting them to phase transitions associated with symmetry breaking that play an important role in the dynamics of the early Universe. A central concept discussed is the *mass-gain* mechanism, wherein particles remain massless in the symmetric phase and acquire mass upon crossing the bubble wall during a FOPT, thereby impacting efficiency of baryogenesis via leptogenesis and dark matter relic abundance. The thesis also explores the *filtering effect* during the expansion of true vacuum bubbles in a FOPT, where selective transmission of dark sector particles with non-zero chemical potential across the phase boundary can lead to the formation of PBH. Depending on their mass, such PBH may evaporate to produce dark matter particles or survive themselves as present day dark matter candidates.

After showing the possibility of a doubly-peaked GW spectrum in a scenario with ultra-light PBH formation during a FOPT, we also propose a novel filteredogenesis mechanism which connects PBH dark matter and visible sector asymmetry through selective transmission of dark fermions across expanding bubble walls. Furthermore, the thesis examines gravitational wave signals arising from topological defects, such as domain walls, generated via discrete symmetry breaking in scenarios like minimal Dirac seesaw and Left-Right Symmetric Models (LRSM). The annihilation of such domain wall networks can produce GW in the nano-Hz range, offering a compelling interpretation of recent PTA observations and a novel connection between neutrino mass generation, leptogenesis, and gravitational wave phenomenology.

Overall, this thesis provides a comprehensive and testable framework where the origins of dark matter and baryon asymmetry are encoded in the gravitational wave signatures of symmetry-breaking dynamics in the early Universe. In Chapter 1, the observational evidence for both baryon asymmetry and dark matter is presented, setting the stage for the subsequent discussions. Chapter 2 introduces some of the stochastic gravitational wave sources in the early Universe. The objectives of the thesis are outlined at the end of Chapter 2. In the next three chapters, we explore distinct mechanisms through which the production of dark matter and baryon asymmetry can concurrently occur, with gravitational waves serving as an observational probe.

Permissions and Attributions

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 - (a) **JHEP 11 (2022) 136** in collaboration with Debasish Borah and Arnab Dasgupta.
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Introduction

The desire to understand the Universe drives us to explore its fundamental building blocks and the forces that govern their interactions. In this pursuit, the electromagnetic, weak, and strong interactions have been successfully combined within the theoretical framework of the Standard Model (SM) of particle physics [1–5]. This framework has demonstrated remarkable agreement with a wide range of experimental results. However, despite its success, SM fails to address some of the most profound questions about our existence. One such question is the observed asymmetry between matter and antimatter. Why does the Universe appear to contain more matter than antimatter, even though fundamental theories suggest they should have been created in equal amounts during the Big Bang? Another unresolved mystery is the existence of dark matter, which constitutes about 27% of the Universe’s energy density at present. This form of matter is non-luminous, non-baryonic, and interacts only gravitationally, making it invisible to us. The SM does not provide a viable candidate for dark matter. These gaps strongly suggest the need for physics beyond the Standard Model (BSM) and motivate a broad spectrum of theoretical and experimental efforts in modern particle physics and cosmology.

In this thesis, we investigate the two aforementioned problems: the origin of the baryon asymmetry of the Universe and dark matter, focusing on the possibility of probing their production mechanisms via gravitational wave signatures. We begin by discussing the observational evidence, theoretical foundations, and solutions underlying each of these puzzles.

1.1 Observational evidences of Baryon Asymmetry

The Universe has a surprising imbalance between matter and antimatter, known as baryon asymmetry. There is much more baryonic matter, made up of protons and neutrons, compared to antibaryonic matter. This is unexpected because fundamental theories suggest that matter and antimatter should have formed in equal amounts in the early Universe. Understanding the origin of this asymmetry is one of the biggest puzzles in modern cosmology and particle physics, as it is essential to explain how galaxies, stars, and even life came to be.

There is strong and consistent evidence of baryon asymmetry in the Universe, supported by several independent observations in cosmology. The most direct indication comes from the absence of significant antimatter in the Universe. Astronomical observations, including cosmic ray detections and gamma-ray surveys, reveal no evidence of large-scale antimatter regions, such as anti-galaxies or anti-stars, which would produce characteristic annihilation signatures if they existed in contact with matter-dominated regions [6]. For instance, high-energy gamma rays resulting from matter-antimatter annihilation are not observed in such quantities that would suggest the presence of antimatter regions within our observable Universe. This implies that the Universe is overwhelmingly matter-dominated, at least within the observable horizon.

The most important evidence for baryon asymmetry lies in the cosmic microwave background (CMB). Precision measurements carried by Planck Collaboration ⁱ⁾ provide constraints on the baryon density of the Universe, quantified by the baryon-to-photon ratio, $\eta_B = n_B/n_\gamma$, where n_B is the number density of baryons and n_γ is the number density of photons. The CMB power spectrum, particularly the acoustic peaks, is highly sensitive to the baryon density, as baryons influence the gravitational potential wells and the dynamics of photon-baryon fluid oscillations in the early Universe. The Planck 2018 results yield a baryon-to-photon ratio of $\eta_b \approx 6.1 \times 10^{-10}$, indicating a small but non-zero excess of baryons over antibaryons [7]. This value

ⁱ⁾ [Planck](#) was a space observatory run by the European Space Agency (ESA).

is an improvement of multiple CMB experiments like WMAP[8], reinforcing the reality of the asymmetry. The baryon asymmetry is commonly expressed using the baryon-to-photon ratio, denoted as η_B , which is defined by the equation:

$$\eta_B = \frac{n_B - n_{\bar{B}}}{n_\gamma}, \quad (1.1)$$

where n_γ represents the photon number density, given by $n_\gamma = \frac{2\zeta(3)}{\pi^2}T^3$. The baryon asymmetry can also be represented as the baryonic yield, denoted as Y_B :

$$Y_B = \frac{n_B - n_{\bar{B}}}{s}, \quad (1.2)$$

where s is the entropy density. In the current Universe, the relationship between η_B and Y_B is given by $\eta_B = 7.05Y_B$.

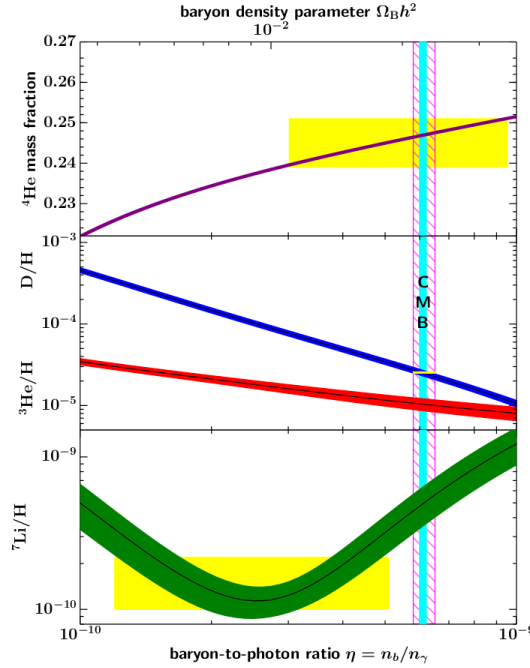


Figure 1.1: Predicted primordial abundances of light elements (^4He , D , ^3He , and ^7Li) from the standard Big Bang nucleosynthesis model. The shaded bands represent the 95% confidence level (CL) prediction ranges, while the yellow boxes denote the observed abundances of the corresponding light elements. The narrow vertical cyan band shows the baryon density inferred from CMB measurements, and the wider magenta meshed band indicates the 95% CL concordance region between deuterium and ^4He within the BBN framework. The figure has been taken from Ref. [9].

Big Bang Nucleosynthesis (BBN) provides a critical probe of baryon asymmetry. BBN describes the production of light elements, such as D, ^3He , ^4He , and ^7Li , in the first few minutes after the Big Bang. The abundances of these elements are highly sensitive to the baryon density at the time of nucleosynthesis. Theoretical predictions of primordial element abundances, when compared with precise measurements from astrophysical environments, such as low metallicity stars and high-redshift gas clouds, yield a baryon-to-photon ratio in excellent agreement with CMB measurements [7]. For example, the observed deuterium to hydrogen ratio, $D/H \approx 2.5 \times 10^{-5}$, is a particularly sensitive indicator of η_B [10]. The close agreement between measurements from BBN and the CMB measurements, as shown in Fig. 1.1, strengthens our confidence in the measured value of the baryon asymmetry and points to its origin in the early stages of the Universe.

1.2 Observational signatures of Dark matter

Dark matter is one of the most enigmatic components of the Universe, inferred to exist through its gravitational influence on visible matter, radiation, and the large-scale structure of the cosmos. Despite constituting approximately 27% of the Universe's total energy density, according to the latest cosmological observations [7], dark matter remains invisible to direct electromagnetic detection, earning its name from its elusive nature. Its presence is critical to explaining various astrophysical and cosmological phenomena, yet its fundamental properties like composition, particle nature, and non-gravitational interactions remain unknown. This thesis explores the nature of dark matter, beginning with the observational evidence that establishes its existence and investigating theoretical frameworks, including a speculative connection to the origin of baryon asymmetry, as a potential avenue for understanding its origin within extensions of the Standard Model of particle physics.

The concept of unseen matter traces back to the late 19th and early 20th centuries, when astronomers like Lord Kelvin and Henri Poincaré speculated about “dark bodies” influencing visible stars through gravity as discussed in a review [11]. A turning point came in 1933 when Fritz Zwicky, analysing the Coma Cluster, found that the

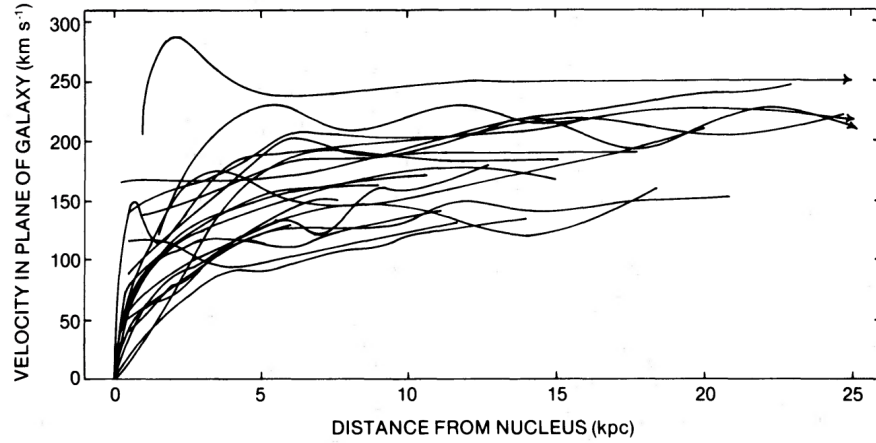


Figure 1.2: Rotational velocity as a function of radial distance from the galactic centre, as observed by Rubin et al. [13] and collected from there.

visible mass of galaxies was insufficient to explain the high velocity dispersion observed, invoking the presence of “dark matter” to reconcile the discrepancy using the virial theorem [12]. However, it wasn’t until the 1970s that the case for dark matter became compelling, largely due to observations of galactic rotation curves. In the 1970s, Vera Rubin and her collaborators conducted measurements of stellar rotational velocities in spiral galaxies, such as the Andromeda Galaxy, as a function of their distance from the galactic centre [13]. According to Newtonian gravity, the rotational velocity should decrease with increasing radius beyond the visible matter distribution. However, observations revealed that velocities remain approximately constant at large radii, as shown in Fig. 1.2, suggesting the presence of a massive, invisible halo extending well beyond the visible galaxy. These “flat” rotation curves imply the presence of a substantial amount of unseen mass, now attributed to dark matter, which provides the additional gravitational potential needed to sustain these velocities.

Gravitational lensing provides another powerful probe of dark matter. According to general relativity, massive objects bend spacetime, causing light from distant sources to follow curved paths. Observations of strong and weak gravitational lensing, such as in galaxy clusters, reveal mass distributions that cannot be explained by visible matter alone. A striking example is the Bullet Cluster (1E0657-558), where the collision of two galaxy clusters results in a separation between the baryonic matter in the pink region (detected via X-ray emissions) and the gravitational mass in the



Figure 1.3: The Bullet Cluster (1E0657-558), captured during the collision of two galaxy clusters. Pink regions show the distribution of hot baryonic gas observed by the Chandra X-ray Observatory, while blue regions mapped through gravitational lensing reveal the presence of dark matter. This figure has been taken from Ref. [14].

blue region (inferred from lensing) as shown in Fig. 1.3, which is attributed to dark matter [14]. The lensing data indicate that dark matter is collisionless or weakly interacting as it passes through the collision without significant scattering, unlike hot gas. These observations provide direct evidence for dark matter's presence and constrain its interaction properties.

The CMB measurements offer precise cosmological evidence for dark matter. The CMB, a snapshot of the Universe at the epoch of recombination, exhibits temperature fluctuations that encode information about the Universe's composition. The power spectrum of these fluctuations, measured with high precision by experiments like the Planck, is sensitive to the density of cold dark matter (CDM). Dark matter influences the growth of density perturbations, affecting the positions and amplitudes of the acoustic peaks in the CMB power spectrum. The Planck 2018 results yield a cold dark matter density parameter of $\Omega_{\text{DM}}h^2 \approx 0.120$, corresponding to roughly 27% of the Universe's energy budget [7]. This measurement is consistent

with other probes, reinforcing the necessity of dark matter in the standard Λ CDM cosmological model.

Dark matter plays a pivotal role in the formation of large-scale structures, such as galaxies and galaxy clusters. In the early Universe, dark matter provides the gravitational scaffolding for structure formation, as its density perturbations grow under gravity, unaffected by radiation pressure, unlike baryonic matter. N-body simulations of structure formation, assuming cold dark matter, accurately reproduce the observed distribution of galaxies and clusters, including the cosmic web of filaments, walls, and voids [15]. These simulations require a significant dark matter component to match observations from surveys like the Sloan Digital Sky Survey (SDSS) [16]. The success of the Λ CDM model in predicting large-scale structure further confirms dark matter's role in cosmology.

1.3 Need for physics beyond the SM

In this section, we discuss the origin of the aforementioned baryon asymmetry and dark matter, along with the most popular theoretical frameworks that aim to explain them.

1.3.1 Origin of Baryon asymmetry

The survival of a small but non-zero baryon excess suggests the presence of physical processes that violate fundamental symmetries and create an imbalance during the early Universe. Under the common assumption that the Universe began with equal amounts of baryons and antibaryons, a dynamical mechanism, known as *baryogenesis*, must have generated this net asymmetry. In 1967, Andrei Sakharov outlined three necessary conditions for generating a baryon asymmetry, now known as the Sakharov conditions [17]. These conditions are: (1) baryon number (B) violation to allow the creation of more baryons than antibaryons; (2) violation of C (charge conjugation) and CP (charge-parity) symmetries, to ensure that processes favour matter over antimatter; and (3) departure from thermal equilibrium, to prevent the washout of any asymmetry through thermal processes. While the Standard

Model contains mechanisms for baryon number violation (e.g., through sphaleron processes [18–20] in the electroweak sector) and CP violation (e.g., through the CKM matrix [21, 22]), these effects are insufficient to produce the observed asymmetry [23]. The electroweak phase transition in the Standard Model is also unlikely to provide the necessary out-of-equilibrium conditions [24, 25], prompting the exploration of physics beyond the Standard Model.

Several theoretical mechanisms have been proposed to explain baryon asymmetry, including grand unified theory (GUT) baryogenesis [26–28], electroweak baryogenesis [19, 29, 30], and Affleck-Dine baryogenesis [31]. GUT baryogenesis proposes that the decay of heavy particles in Grand Unified Theories can produce a baryon asymmetry through CP-violating and out-of-equilibrium processes. However, this mechanism faces significant challenges due to the high energy scales involved, which remain inaccessible to current experiments. A major difficulty arises from the need to suppress proton decay, which typically requires the masses of the mediating scalar or gauge bosons to be greater than 10^{15} GeV in conventional GUT frameworks. At the same time, precise measurements of the Cosmic Microwave Background (CMB) place an upper bound on the scale of inflation [32], which, in turn, limits the reheating temperature to be below 10^{13} GeV. This discrepancy between the GUT scale and the achievable post-inflationary temperatures prevents the efficient thermal production of GUT-scale particles, thus severely limiting the viability of thermal GUT baryogenesis. Electroweak baryogenesis relies on a strongly first-order electroweak phase transition to create out-of-equilibrium conditions together with additional source of CP violation which can be realised in different extensions of SM. A detailed review of electroweak baryogenesis can be found in [33]. Affleck-Dine baryogenesis involves scalar fields in supersymmetric theories where the coherent evolution or oscillation of a scalar field in the early Universe generates baryon asymmetry. Among these, leptogenesis [34] has emerged as an interesting mechanism due to its connection to other unsolved problems in particle physics, such as the origin of neutrino masses [10]. Leptogenesis is a theoretical framework in which the baryon asymmetry is generated indirectly through an initial asymmetry produced in the lepton sector, which is subsequently converted into a baryon asymmetry via sphaleron processes [18, 19]. Given our interest in baryogenesis via leptogenesis, we now proceed to discuss the framework below.

1.3.1.1 Leptogenesis

In its simplest form, leptogenesis involves the out-of-equilibrium decays of heavy right-handed neutrinos, which are introduced in extensions of the Standard Model, such as the seesaw mechanism that generates sub-eV masses of light neutrinos. These decays violate lepton number and CP symmetry, producing net lepton asymmetry in the early Universe. Sphaleron processes, which are active at high temperatures in the electroweak symmetric phase, then partially convert this lepton asymmetry into a baryon asymmetry. The attraction of leptogenesis is its simplicity and efficiency. It uses particles that are already needed to explain neutrino masses, which have been confirmed by experiments studying neutrino oscillations [35, 36]. This mechanism can naturally be realised within the Type-I seesaw framework [37–40], where the Standard Model is extended by introducing three right-handed neutrinos (RHN), denoted as N . The relevant gauge-invariant Lagrangian is given by

$$\mathcal{L}_\nu = -\mathcal{Y}_\nu \bar{l} \tilde{\Phi} N - \frac{1}{2} \bar{N}^c M_R N + \text{h.c.}, \quad (1.3)$$

where l and Φ represent the left-handed SM lepton doublet and Higgs doublet, respectively, and $\tilde{\Phi} = i\sigma_2 \Phi^*$, with σ_2 being the second Pauli matrix. The second term introduces a Majorana mass for the RHN and is responsible for lepton number violation. Following electroweak symmetry breaking, the Dirac neutrino mass matrix takes the form $m_D = \mathcal{Y}_\nu v/\sqrt{2}$, where $v \simeq 246$ GeV is the Higgs vacuum expectation value. In the limit $m_D \ll M_R$, the effective light neutrino mass matrix generated via the seesaw mechanism is approximately $m_\nu \simeq -m_D M_R^{-1} m_D^T$. To explain the observed sub-eV masses of SM neutrinos, this framework typically requires a heavy Majorana mass scale of $M_R \sim \mathcal{O}(10^{14})$ GeV, assuming Yukawa couplings $\mathcal{Y}_\nu \sim \mathcal{O}(1)$. At sufficiently high temperatures, the RHN can remain in thermal equilibrium with the plasma. However, as the Universe cools down to temperatures below their mass scale $T \lesssim M_i$, RHN can undergo lepton number violating out of equilibrium decays. These include both $N_i \rightarrow \ell_\alpha + \Phi$ and $N_i \rightarrow \bar{\ell}_\alpha + \Phi^\dagger$, where N_i denotes the heavy neutrino mass eigenstates with mass M_i (for $i = 1, 2, 3$) and $\alpha = e, \mu, \tau$ labels the lepton flavours. The CP asymmetry parameter in a given lepton flavour α for the

decay of the heavy Majorana neutrino N_i is defined as

$$\epsilon_{i\alpha} = \frac{\Gamma(N_i \rightarrow \ell_\alpha + \Phi) - \Gamma(N_i \rightarrow \bar{\ell}_\alpha + \Phi^\dagger)}{\Gamma(N_i \rightarrow \ell_\alpha + \Phi) + \Gamma(N_i \rightarrow \bar{\ell}_\alpha + \Phi^\dagger)}, \quad (1.4)$$

where Γ denotes decay rate. At tree level, the decay rates of N_i into leptons and antileptons are equal, leading to zero asymmetry, i.e., $\epsilon_{i\alpha} = 0$. However, a non-zero asymmetry arises at the one-loop level due to the interference between the tree-level diagram and loop corrections, which include both self-energy and vertex diagrams. This loop induced CP violation is a crucial ingredient for successful leptogenesis.

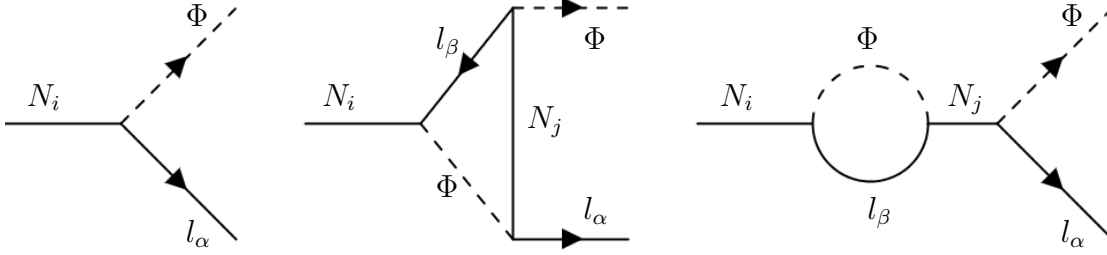


Figure 1.4: Tree-level and one-loop (vertex and self-energy) diagrams contributing to the CP violating decay of heavy Majorana neutrino N_i .

A non-vanishing $\epsilon_{i\alpha}$ ensures that the decays of heavy neutrinos produce a net lepton number asymmetry in the early Universe, which can then be partially converted into a baryon asymmetry through sphaleron processes. The relevant Feynman diagrams that contribute to the CP-violating decay of heavy Majorana neutrinos include both the one-loop vertex and self-energy corrections, as shown in Fig. 1.4. These loop diagrams interfere with the tree-level diagram to produce a non-zero CP asymmetry parameter $\epsilon_{i\alpha}$, given by [34, 41–43]

$$\epsilon_{i\alpha} = \frac{1}{8\pi(\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ii}} \sum_{j \neq i} \left[F \left(\frac{M_j^2}{M_i^2} \right) \text{Im} [(\mathcal{Y}_\nu^*)_{\alpha i} (\mathcal{Y}_\nu)_{\alpha j} (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ij}] + G \left(\frac{M_j^2}{M_i^2} \right) \text{Im} [(\mathcal{Y}_\nu^*)_{\alpha i} (\mathcal{Y}_\nu)_{\alpha j} (\mathcal{Y}_\nu^\dagger \mathcal{Y}_\nu)_{ij}^*] \right], \quad (1.5)$$

where the loop functions corresponding to the vertex and self-energy diagrams respectively, are given by

$$F(a) = \sqrt{a} \left[\frac{1}{1-a} + 1 - (1+a) \ln \left(\frac{1+a}{a} \right) \right], \quad (1.6)$$

$$G(a) = \frac{1}{1-a}. \quad (1.7)$$

The total CP asymmetry ϵ_i is obtained by summing over all flavours α :

$$\begin{aligned} \epsilon_i &= \sum_{\alpha} \epsilon_{i\alpha} \\ &= \frac{1}{8\pi(\mathcal{Y}_{\nu}^{\dagger}\mathcal{Y}_{\nu})_{ii}} \sum_{j \neq i} \text{Im} [(\mathcal{Y}_{\nu}^{\dagger}\mathcal{Y}_{\nu})_{ij}^2] F\left(\frac{M_j^2}{M_i^2}\right). \end{aligned} \quad (1.8)$$

In this setup, the initial temperature of the Universe T_i is assumed to be higher than the mass of the lightest right-handed neutrino, M_1 . Furthermore, we neglect the decay contributions of the heavier right-handed neutrinos N_2 and N_3 , under the assumption that either they do not generate any significant $B - L$ asymmetry, or any asymmetry they produce is efficiently washed out and does not affect the final outcome. Within this minimal framework, the Boltzmann equations governing the evolution of the comoving number density of N_1 and the $B - L$ asymmetry can be written as follows [44]:

$$\frac{dN_{N_1}}{dz} = -(\mathcal{D} + \mathcal{S}) (N_{N_1} - N_{N_1}^{\text{eq}}), \quad (1.9)$$

$$\frac{dN_{B-L}}{dz} = -\epsilon_1 \mathcal{D} (N_{N_1} - N_{N_1}^{\text{eq}}) - \mathcal{W} N_{B-L}, \quad (1.10)$$

where $z = M_1/T$ is a dimensionless temperature parameter, and all number densities are normalised with respect to the photon number density in a comoving volume. The terms appearing in Eqs. (1.9) and (1.10) receive contributions from four classes of processes: Decays: $N_1 \rightarrow \ell + \Phi$ and their CP-conjugates, Inverse decays: $\ell + \Phi \rightarrow N_1$, $\Delta L = 1$ scattering processes (e.g., involving quarks, leptons, Higgs, and gauge bosons), $\Delta L = 2$ processes mediated by virtual heavy neutrinos. The first three types of processes contribute to the evolution of the N_1 abundance, driving it toward equilibrium. The term $\mathcal{D} = \Gamma_D/(Hz)$ accounts for both decays and inverse decays, while $\mathcal{S} = \Gamma_S/(Hz)$ denotes the contribution from $\Delta L = 1$ scatterings. The decays

of N_1 also act as the source of the $B - L$ asymmetry, represented by the first term in Eq. (1.10). All other lepton number violating processes (inverse decays, $\Delta L = 1$ and $\Delta L = 2$ scatterings) contribute to the total washout rate, $\mathcal{W} = \frac{\Gamma_W}{Hz}$, which tends to erase the generated asymmetry. The Hubble expansion rate H in the radiation-dominated era is given by:

$$H(z) \simeq \sqrt{\frac{8\pi^3 g_*}{90}} \frac{M_1^2}{M_{\text{Pl}} z^2} \simeq 1.66 \frac{\sqrt{g_*} M_1^2}{M_{\text{Pl}} z^2}, \quad (1.11)$$

where $g_* = 106.75$ corresponds to the total effective number of relativistic species contributing to the energy density in the Standard Model, and $M_{\text{Pl}} = 1.22 \times 10^{19}$ GeV denotes the Planck mass. The inverse decay rate can be written in terms of the decay rate as

$$\Gamma_{\text{ID}}(z) = \Gamma_D \frac{N_N^{\text{eq}}(z)}{N_l^{\text{eq}}(z)}, \quad (1.12)$$

where Γ_D is the decay rate of the heavy neutrino, and $N_N^{\text{eq}}(z)$, $N_l^{\text{eq}}(z)$ are the equilibrium number densities of the heavy neutrino and the lepton, respectively. The washout term due to inverse decay is then given by

$$W_{\text{ID}}(z) = \frac{1}{4} K z^3 K_1(z), \quad (1.13)$$

where K is the decay parameter and $K_1(z)$ is the modified Bessel function of the second kind of order 1. The decay parameter K is defined as

$$K = \frac{\Gamma_D}{H(z=1)}, \quad (1.14)$$

where $H(z=1)$ is the Hubble expansion rate evaluated at $z=1$, i.e., when $T = M_1$. Scattering washout processes such as $\ell\Phi \leftrightarrow q\bar{t}$ for $\Delta L = 1$ and $\Delta L = 2$ interactions like $\ell\Phi \leftrightarrow \bar{\ell}\bar{\Phi}$ mediated by heavy Majorana neutrinos, can in principle wash out the generated asymmetry [44]. However, in the weak washout regime with small Yukawa couplings, their rates remain subdominant compared to inverse decays. Hence, choosing their impact to be minimal, we neglect them in this analysis for simplicity. We numerically solve the Boltzmann equations (1.9) and (1.10) for different values of the decay parameter K , and present the resulting evolution of N_1 and N_{B-L} in Fig. 1.5 and Fig. 1.6, respectively. In the left panel of Fig. 1.5, we assume a vanishing initial abundance for N_1 , while in the right panel we consider

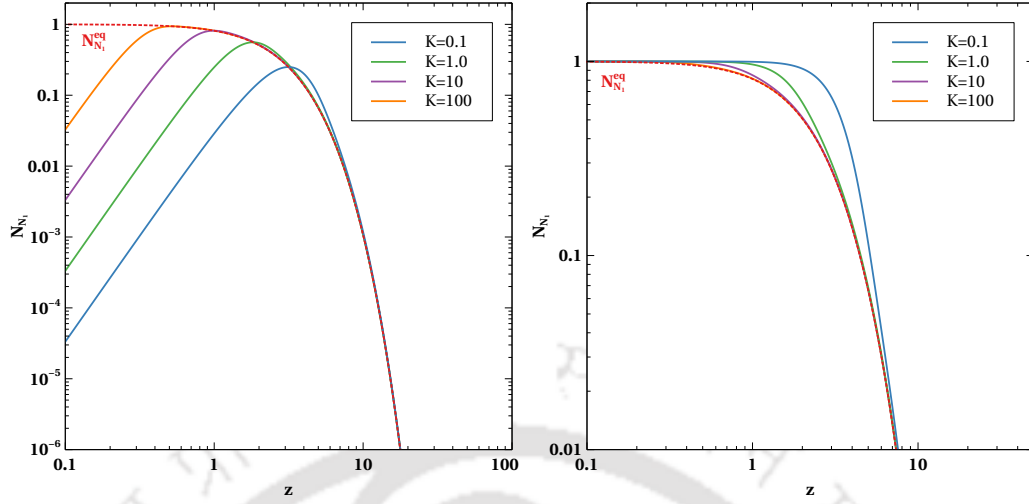


Figure 1.5: Left panel: Evolution of RHN N_1 for different values of K with initial N_1 number density zero. Right panel: Evolution of RHN N_1 for different values of K with initial number density of N_1 , $N_{N_1} = N_{N_1}^{eq}$.

an initial abundance equal to its equilibrium value, $N_{N_1}^{eq}$. Figure 1.6 shows the evolution of the $B - L$ asymmetry corresponding to the two cases of N_1 evolution shown in Fig. 1.5. A larger value of K corresponds to a larger decay width, which drives N_1 towards equilibrium more efficiently, as seen in Fig. 1.5. However, a larger decay width also enhances the inverse decay processes, which tend to wash out the generated asymmetry. As shown in Fig. 1.6, a large K leads to strong washout effects, whereas a small K results in weaker washout. The “kink” in the left panel marks the point where the particle N_1 reaches thermal equilibrium, causing the $B - L$ asymmetry to undergo a sign flip. This highlights the dual role of inverse decays, which not only contribute to the generation of N_1 from an initially vanishing abundance but also to the washout of the asymmetry.

The baryon asymmetry ultimately observed in the Universe originates from the initial $B - L$ asymmetry through the action of sphaleron processes, which conserve $B - L$ but violate $B + L$. These processes redistribute the asymmetry between baryons and leptons with a conversion factor C_{sph} , given by [45]:

$$N_B = \frac{8n_f + 4n_\Phi}{22n_f + 13n_\Phi} N_{B-L} \equiv C_{sph} N_{B-L}, \quad (1.15)$$

where n_f is the number of fermion generations and n_Φ is the number of Higgs doublets. For the Standard Model with $n_f = 3$ and $n_H = 1$, this yields $C_{sph} = \frac{28}{79}$.

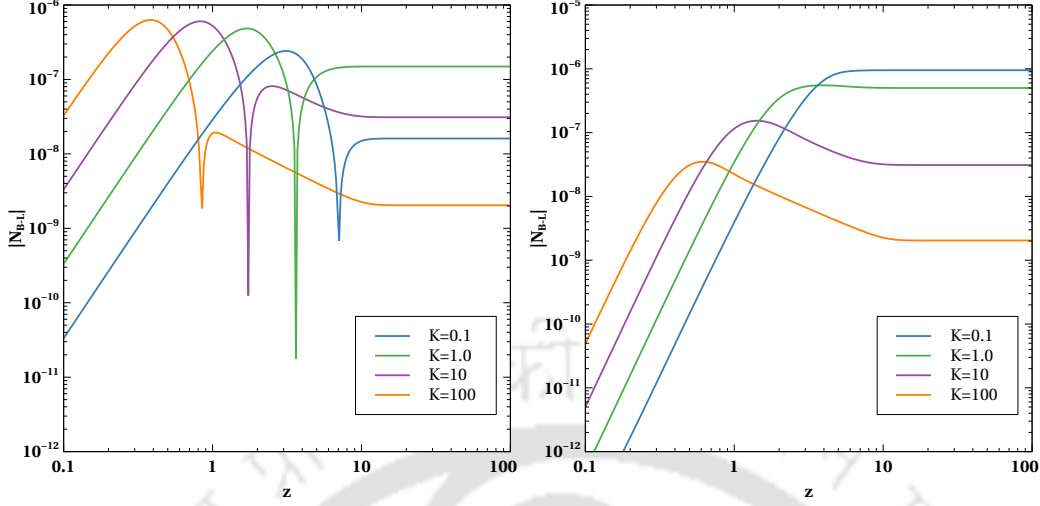


Figure 1.6: Left panel: Evolution of B-L asymmetry for different values of K with initial N_1 number density zero. Right panel: Evolution of B-L asymmetry for different values of K with initial number density of N_1 , $N_{N_1} = N_{N_1}^{eq}$. Here, $\epsilon = 10^{-6}$.

The final baryon-to-photon ratio η_B can be approximately expressed as [44]:

$$\eta_B = \frac{C_{\text{sph}}}{f} \epsilon_1 \kappa, \quad (1.16)$$

where ϵ_1 denotes the CP asymmetry generated from the decay of the lightest heavy neutrino, and κ is the efficiency factor that accounts for washout effects. The factor $f \approx \frac{106.75}{3.91} \simeq 27.3$ accounts for the change in the number of relativistic degrees of freedom from the leptogenesis scale to the epoch of recombination.

The CP asymmetry plays an important role in leptogenesis and is sensitive to both the masses of right-handed neutrinos and the light neutrino mass spectrum. In the framework of thermal leptogenesis, achieving the observed baryon-to-photon ratio requires a CP asymmetry $\epsilon_1 \gtrsim 10^{-6}$. For such a level of asymmetry, and given the experimentally measured light neutrino masses, the mass of the lightest right-handed neutrino must typically satisfy $M_1 \gtrsim 4 \times 10^9 \text{ GeV}$, a constraint known as the Davidson-Ibarra bound [46]. While this bound is derived within the context of the type-I seesaw mechanism, analogous lower bounds also apply to leptogenesis scenarios in type-II [47–49] and type-III [50] seesaw models. However, these stringent bounds can be significantly relaxed in the case of resonant leptogenesis [51], where the CP asymmetry is enhanced due to nearly degenerate right-handed neutrinos. In

such scenarios, successful leptogenesis and the correct light neutrino masses can be achieved even with right-handed neutrino masses at the TeV scale. Also, leptogenesis provides testable predictions, such as potential signatures in neutrino mixing parameters or rare processes like neutrinoless double-beta decay [52], which could be probed in future experiments.

The baryon asymmetry of the Universe is a cornerstone of modern cosmology, bridging particle physics and cosmology. Its explanation requires physics beyond the Standard Model, and leptogenesis stands out as a compelling and elegant solution as discussed above. By exploring the interplay between observational evidence and theoretical mechanisms, this thesis aims to illuminate the origins of this asymmetry and its implications for our understanding of the early Universe.

1.3.2 Dark Matter Paradigms

While multiple observations confirm the existence of dark matter across various cosmic scales, its fundamental nature remains unknown. The properties inferred from observations, such as being electrically neutral, massive, and stable over cosmological timescales, exclude all Standard Model particles except neutrinos. However, neutrinos are too light and relativistic in the early Universe, which prevents them from seeding the formation of cosmic structures. Additionally, their abundance is insufficient to account for the total dark matter density. These shortcomings indicate that dark matter cannot be explained within the Standard Model framework. Instead, we require new physics beyond the Standard Model that introduces a viable dark matter candidate consistent with observational constraints. Furthermore, we outline the different types of dark matter candidates that are investigated in the main chapters of this thesis.

1.3.2.1 WIMP

Despite compelling observational evidence, the particle nature of dark matter remains a mystery. The Standard Model of particle physics does not contain a suitable candidate for dark matter, as none of its particles possess the necessary properties: stability, weak interaction with visible matter, and sufficient relic density to match

cosmological observations. Leading candidates include weakly interacting massive particles (WIMP) [53, 54], axions, and sterile neutrinos, each motivated by extensions to the Standard Model. WIMP, with masses in the GeV to TeV range [55], are particularly appealing due to the “WIMP miracle”, where their thermal relic density naturally aligns with the observed dark matter density. A comprehensive review can be found in Ref. [56]. In this framework, the dark matter (DM) particle possesses interaction strengths with the SM sector sufficient to maintain thermal equilibrium in the early Universe. As the Universe expands and cools, the DM interaction rate eventually falls below the Hubble expansion rate. At this point, the DM particles decouple from the thermal bath, and their comoving number density becomes effectively constant, and the process is known as the *freeze-out* mechanism. To determine the evolution of the DM number density and compute its freeze-out abundance, one must solve the Boltzmann equation for the DM number density n_{DM} . However, direct detection experiments, such as XENONnT [57], LUX-ZEPLIN (LZ) [58] and PandaX-II [59], and indirect searches via gamma-ray observatories, like Fermi-LAT [60], have yet to confirm WIMP signals, prompting exploration of alternative scenarios. Considering a dark matter annihilation process of the form $\text{DM DM} \rightarrow \text{SM SM}$

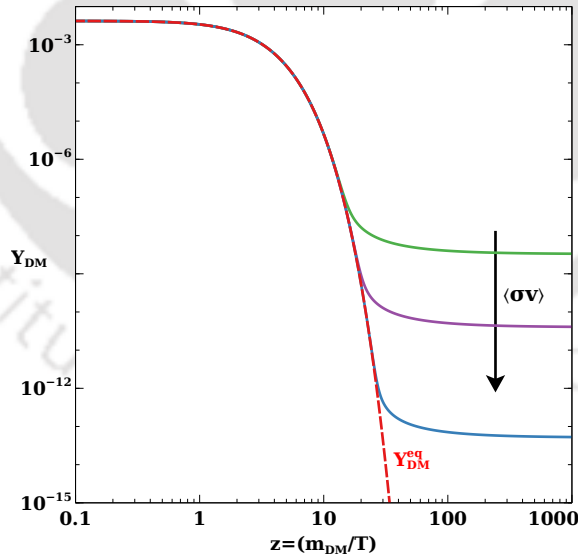


Figure 1.7: The evolution of comoving number density of WIMP DM.

with a thermally averaged annihilation cross-section $\langle\sigma v\rangle$, the evolution of the dark matter comoving number density can be described by the Boltzmann equation [53]:

$$\frac{dY_{\text{DM}}}{dz} = -\frac{s\langle\sigma v\rangle}{Hz} (Y_{\text{DM}}^2 - (Y_{\text{DM}}^{\text{eq}})^2). \quad (1.17)$$

where $z = m_{\text{DM}}/T$ and s is entropy density. The above Eq. (1.17) is governed by its thermal averaged annihilation cross-section $\langle\sigma v\rangle$ and it is defined as [61]

$$\langle\sigma v\rangle = \frac{g_{\text{DM}}^2}{8m_{\text{DM}}^4 T K_2^2\left(\frac{m_{\text{DM}}}{T}\right)} \int_{s_0}^{\infty} \sigma(s) \sqrt{s(s - 4m_{\text{DM}}^2)} K_1\left(\frac{\sqrt{s}}{T}\right) ds. \quad (1.18)$$

The typical thermal history of a WIMP dark matter candidate is depicted in Fig. 1.7. As seen in the figure, increasing the annihilation cross-section causes dark matter to remain in thermal equilibrium longer and freeze out at a lower temperature, resulting in a smaller final abundance. After the freeze-out of dark matter, the present-day relic abundance of dark matter can be readily calculated as:

$$\Omega_{\text{DM}} h^2 = 2.755 \times 10^8 \left(\frac{m_{\text{DM}}}{\text{GeV}}\right) Y_{\text{DM}}(z = \infty). \quad (1.19)$$

1.3.2.2 FIMP

For the interaction strength between dark matter and the Standard Model particles is extremely small couplings less than $\mathcal{O}(10^{-7})$ [62], DM particles fail to thermalise with the SM plasma in the early Universe. The conventional thermal freeze-out scenario is no longer viable in such a regime. Instead, DM can be produced through a non-thermal mechanism, where it is gradually generated via the decay or scattering of SM particles present in the thermal bath. This production process continues as long as the number density of the bath particles remains significant. Once the temperature drops and the SM particle densities become Boltzmann-suppressed, DM production effectively stops. The resulting comoving number density stabilises, leading to a fixed DM abundance, and it is referred as *freeze-in*. DM candidates produced in this manner are known as Feebly Interacting Massive Particles (FIMP)[62, 63]. However, it remains out of the reach of any direct experiments. A detailed review of FIMP is provided in Ref. [64]. In the freeze-in scenario, dark matter is produced through the decay or scattering of particles that are in thermal equilibrium with the SM bath. A minimal example of such a mechanism involves the decay of a heavier particle X into a pair of dark matter particles, via the process $X \rightarrow \text{DM DM}$. The dark matter never reaches thermal equilibrium with the SM plasma. Instead, its population

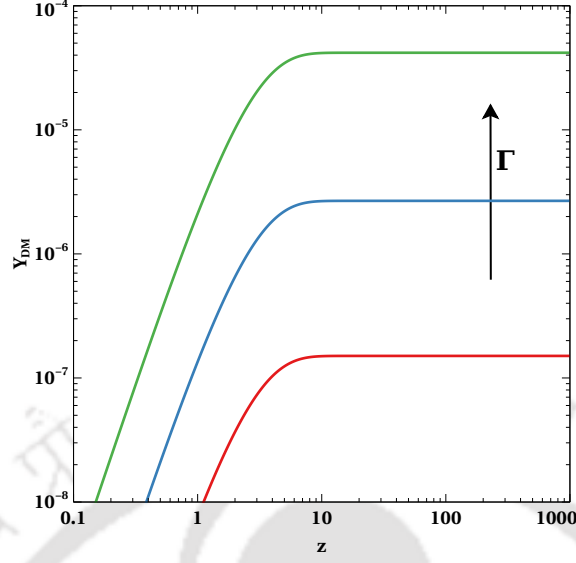


Figure 1.8: The evolution of comoving number density of FIMP DM.

builds up gradually over time as X decays. To study this process quantitatively, it is convenient to express the Boltzmann equation in terms of the comoving number density $Y_{\text{DM}} = n_{\text{DM}}/s(T)$, where $s(T)$ is the entropy density of the Universe, and the dimensionless variable $z = m_{\text{DM}}/T$. Assuming that the parent particle X remains in equilibrium and has decay width Γ_X , the Boltzmann equation governing the freeze-in production of DM takes the form [62]

$$\frac{dY_{\text{DM}}}{dz} = 2 \frac{K_1(z)}{K_2(z)} \frac{\Gamma_X}{Hz} Y_X^{\text{eq}}(z), \quad (1.20)$$

where Y_X^{eq} is the equilibrium comoving number density of X . The factor of 2 accounts for the fact that each decay produces two dark matter particles. A numerical solution of this equation, as shown in Fig. 1.8, illustrates a key feature of the freeze-in mechanism: the final abundance of dark matter increases with the strength of the interaction, i.e., with larger decay width Γ_X . This is in direct contrast to the WIMP scenario, where increasing the interaction strength leads to late freeze-out and a smaller relic abundance due to more efficient annihilation. Under the assumption that particles in the thermal bath follow a Maxwell-Boltzmann distribution, an approximate analytical expression for the final relic density of DM can be derived.

The result is given by

$$\Omega_{\text{DM}} h^2 \approx 4.48 \times 10^8 \left(\frac{g_X}{g_{*s} g_*^{1/2}} \right) \left(\frac{\Gamma_X M_{\text{Pl}}}{m_X^2} \right) \left(\frac{m_{\text{DM}}}{\text{GeV}} \right), \quad (1.21)$$

where m_{DM} and m_X are the masses of the dark matter and the decaying particle respectively, g_X is the internal degrees of freedom of X , g_* is the effective number of relativistic degrees of freedom, and M_{Pl} is the Planck mass. This expression explicitly shows that the final relic abundance is proportional to the decay width Γ_X , reaffirming that stronger interactions yield more dark matter production in this scenario. However, the interaction strength cannot be arbitrarily large. If the coupling between DM and the thermal bath becomes too strong, the dark matter particle may thermalise, thereby transitioning from a *freeze-in* to a *freeze-out* regime. In that case, the underlying assumptions of the FIMP mechanism would no longer hold, and the nature of dark matter would effectively resemble that of a WIMP. Therefore, for the freeze-in mechanism to remain valid, the coupling must be sufficiently small to avoid thermal equilibrium, yet large enough to ensure that the correct relic abundance is generated by the time the Universe cools.

1.3.2.3 PBH

Primordial Black Holes (PBH) are a class of non-particle dark matter candidates [65, 66] that could have formed in the early Universe due to the gravitational collapse of large density fluctuations [67], phase transitions [68], or the collapse of Fermi balls [69], etc. Unlike astrophysical black holes, PBH do not originate from stellar evolution and can span a wide range of masses, from sub-planetary scale to several solar masses, depending on the formation mechanism. If sufficiently long-lived, PBH can contribute to the present-day abundance of dark matter. Their viability as dark matter candidates is subject to a variety of observational constraints, including evaporation of extragalactic γ rays [70], gravitational lensing [71], cosmic microwave background distortions [72], gravitational wave events [73, 74], and dynamical effects on astrophysical systems [75, 76]. Despite these bounds, certain mass windows, particularly in the asteroid-mass and solar-mass ranges, remain unconstrained or partially allowed, making PBH an actively studied dark matter candidate [77]. Different observational constraints on the fraction of PBH as dark matter

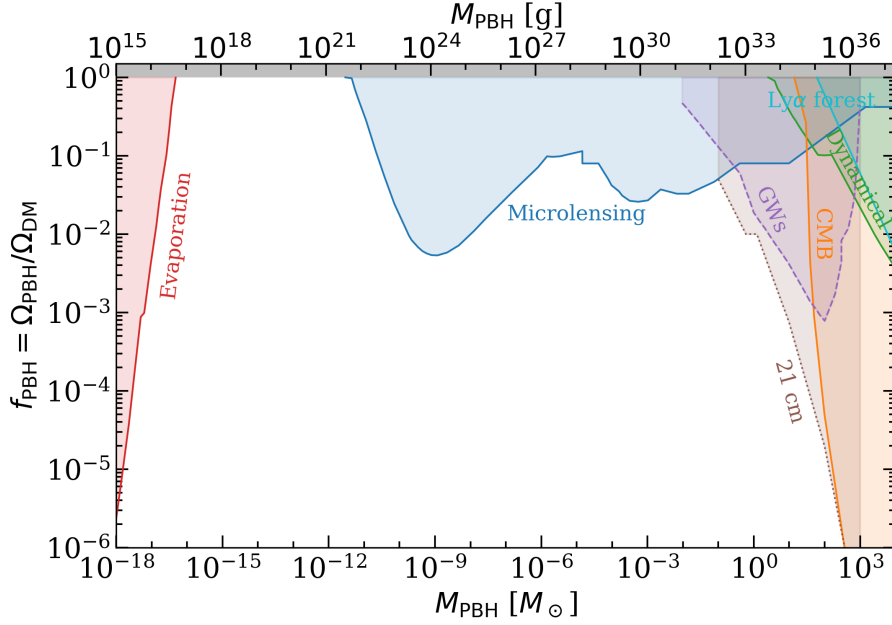


Figure 1.9: Constraints on the PBH fraction relative to dark matter as a function of PBH mass, assuming a monochromatic mass function. This figure is collected from Ref. [77].

are shown in Fig. 1.9, with certain mass ranges still allowing PBH to constitute a viable component of dark matter.

1.4 Conclusion

The thesis begins by reviewing the observational evidence for baryon asymmetry and dark matter, tracing their discovery and characterisation from the 20th century. These evidences are not limited to cosmological observations alone but also include crucial astrophysical signatures. To address these longstanding puzzles, various theoretical frameworks have been developed, some of which are extensively employed in this thesis. In the context of baryogenesis, leptogenesis emerges as one of the most compelling and well-motivated mechanisms and is discussed in detail. For dark matter, we explore a range of candidates, including particle based options such as WIMP and FIMP, as well as non-particle candidates like primordial black holes, which can leave both cosmological and astrophysical imprints. These foundational ideas are further explored and developed in the main chapters of the thesis.

Gravitational wave sources in the early Universe

The groundbreaking discovery of gravitational waves (GW) from a binary black hole merger by the LIGO collaboration [78] marked the beginning of gravitational wave astronomy as a powerful tool for exploring the Universe. Beyond detecting astrophysical sources, GW offer a unique observational window into the early Universe, far beyond the reach of electromagnetic radiation. As GW interact extremely weakly with matter, they can propagate almost unimpeded from their source to the present day, carrying imprints of high energy events in the early cosmos that are otherwise inaccessible. Recently, pulsar timing array (PTA) experiments, including NANOGrav [79], European Pulsar Timing Array (EPTA) together with Indian Pulsar Timing Array (InPTA) [80], Parkes Pulsar Timing Array (PPTA) [81] and Chinese Pulsar Timing Array (CPTA) [82] have reported evidence for a low frequency stochastic gravitational wave background. While the origin of this signal is yet to be definitively determined, a leading astrophysical interpretation suggests it arises from a stochastic gravitational wave background generated by a population of inspiraling supermassive black hole binaries (SMBHB) [83, 84], which are expected to emerge from hierarchical galaxy mergers and emit in the nanohertz frequency range. Alternatively, the signal may have a cosmological origin, possibly emerging from early Universe phenomena such as a strong first-order phase transition (FOPT) in the early Universe, the annihilation of domain walls, or other exotic

mechanisms [85]. In parallel, upcoming space based GW detectors like LISA [86], DECIGO [87], and BBO [88] are expected to significantly enhance sensitivity to such stochastic signals across a wide frequency range, offering a promising route to explore high scale physics far beyond the reach of present or even future particle colliders.

As the Universe expanded and cooled, it may have undergone a sequence of symmetry-breaking phase transitions, analogous to the freezing of water into ice, where the vacuum configuration of fundamental fields changed. These transitions, including the electroweak and possibly GUT-scale transitions, played a role in shaping the structure and composition of the Universe. In many extensions of the Standard Model, such phase transitions can be first-order, proceeding via the nucleation and expansion of bubbles of true vacuum within a metastable false vacuum. The dynamics of such transitions can produce a stochastic background of GW, providing a direct probe of symmetry breaking at high energies.

Moreover, certain symmetry-breaking, particularly those involving discrete symmetries can lead to the formation of topological defects such as domain walls. While stable domain walls are cosmologically problematic, their eventual annihilation can release a significant amount of energy in the form of gravitational radiation. In many well motivated particle physics scenarios, including those aimed at explaining the origin of dark matter or the baryon asymmetry of the Universe, such domain wall configurations arise naturally and may produce observable GW signatures upon decay.

Primordial black holes formed in the early Universe can have an impact on the evolution of the Universe. Depending on their formation mechanisms and mass spectrum, PBH can emit gravitational waves through mergers, scalar-induced production, or evaporation. Importantly, PBH are also compelling dark matter candidates and may participate in baryogenesis in certain scenarios, further motivating their inclusion as a cosmological GW source.

In this chapter, we investigate in detail three distinct mechanisms for gravitational wave production in the early Universe, each linked to fundamental questions about the origin of visible matter and dark matter, which will be discussed further in this thesis. In Section 2.1, we discuss gravitational waves from first-order phase

transitions, beginning with spontaneous symmetry breaking, the effective potential, the thermodynamic conditions for transition and production of GW. We explore domain wall formation from discrete symmetry breaking, their evolution and annihilation, and the resulting gravitational wave signals in Section 2.2. Next, in Section 2.3, we analyse one possible formation mechanism of PBH and the corresponding gravitational wave signatures, as well as their potential roles in dark matter and baryogenesis. Finally, we summarise the objective of this thesis in Section 2.5.

2.1 Gravitational Waves from First-Order Phase Transitions

To understand gravitational waves originating from first-order phase transitions in the early Universe, it is essential to first examine the underlying mechanism of spontaneous symmetry breaking. As the Universe cools, symmetry-breaking transitions can alter the vacuum structure of fields, potentially leading to FOPT. These transitions involve bubble nucleation and out-of-equilibrium dynamics, key ingredients for gravitational wave production. In this section, we outline the theoretical foundation starting from symmetry breaking, the effective potential, and the nature of thermal phase transitions, leading to the generation of GW from FOPT.

2.1.1 Spontaneous Symmetry Breaking

Symmetry principles are the core of modern theoretical physics and underlie the classical and quantum field theory formulation. The Lagrangian describing a physical system is invariant under certain continuous or discrete transformations, corresponding to conserved quantities stated by Noether's theorem [89]. However, the ground state (or vacuum) of a theory does not need to respect the same symmetry. When the system's dynamics select a vacuum that breaks the symmetry of the underlying laws, we encounter the phenomenon known as spontaneous symmetry breaking (SSB). Ferromagnetic materials, such as iron, are well-known examples of systems that exhibit spontaneous symmetry breaking. The action remains unchanged under spatial rotations in these systems, and the magnetisation vector characterises

their magnetic properties. This vector indicates the orientation of the atomic spins within the material. Above a critical temperature, the ground state of the system has zero magnetisation and retains full rotational symmetry. However, below this critical temperature, the system develops a non-zero magnetisation in its ground state, with spins aligning in a specific direction to form magnetic domains. This alignment selects a preferred spatial direction, spontaneously breaking the rotational $SO(3)$ symmetry. As a result, the ground state degenerates, with different magnetisation orientations related to spatial rotations.

Spontaneous symmetry breaking in field theory is inherently associated with the degeneracy of the vacuum. To distinguish between symmetric and asymmetric vacuum states, one introduces an operator that is not invariant under the corresponding symmetry transformations. This operator acquires a non-zero expectation value in any of the degenerate vacuum states, indicating that the symmetry has been spontaneously broken. Such an operator is called the *order parameter operator*. In quantum field theory, this operator is typically considered a scalar field operator, denoted by $\hat{\phi}$. To illustrate the idea, consider a real scalar field ϕ governed by the Lagrangian:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi), \quad (2.1)$$

where the potential is given by

$$V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4, \quad \lambda > 0. \quad (2.2)$$

The sign of the mass parameter μ^2 plays a crucial role in determining the nature of the vacuum depicted in Fig. 2.1. When $\mu^2 > 0$, the potential has a single minimum at $\phi = 0$, which preserves the $\mathbb{Z}_2$ symmetry $\phi \rightarrow -\phi$. However, when $\mu^2 < 0$, the potential takes the form:

$$V(\phi) = -\frac{1}{2}|\mu^2|\phi^2 + \frac{1}{4}\lambda\phi^4, \quad (2.3)$$

resulting in a double-well shape. The minimum of this potential occurs at

$$\langle \phi \rangle = \pm v = \pm \sqrt{\frac{\mu^2}{\lambda}}. \quad (2.4)$$

While the Lagrangian remains symmetric under the transformation $\phi \rightarrow -\phi$, the

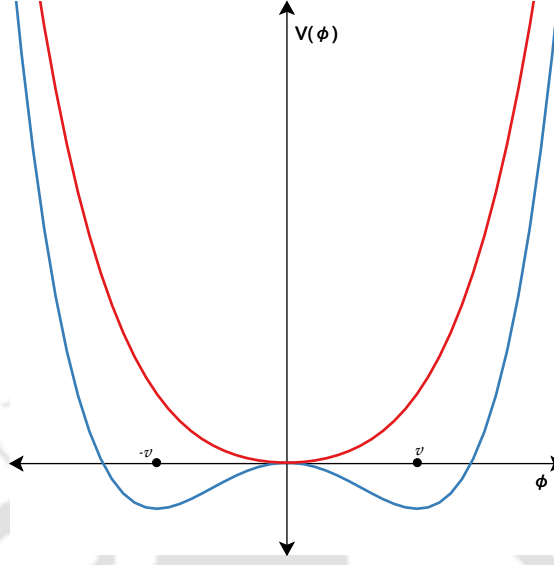


Figure 2.1: The potential profile $V(\phi)$ as a function of ϕ is shown for two cases: the red curve corresponds to $\mu^2 > 0$, representing a single minimum at the origin, while the blue curve corresponds to $\mu^2 < 0$, leading to spontaneous symmetry breaking with a double-well structure.

vacuum spontaneously selects one of these minima, thus breaking the symmetry. This phenomenon is called spontaneous symmetry breaking: the equations remain invariant, but the solution in the vacuum does not. To study the excitations around this vacuum, one can expand the field about one of the minima, say $\phi = +v$:

$$\phi(x) = v + \eta(x), \quad (2.5)$$

where $\eta(x)$ represents fluctuations about the vacuum. Substituting this into the Lagrangian, we find that η is a real scalar field with mass

$$m_\eta^2 = \left. \frac{d^2 V}{d\phi^2} \right|_{\phi=v} = 2\mu^2 = 2\lambda v^2. \quad (2.6)$$

Thus, the spontaneous breaking of symmetry leads to a massive scalar excitation. Till now, we have examined spontaneous symmetry breaking in a theory with a real scalar field and a discrete symmetry. We now generalise this idea to a continuous symmetry, focusing on an invariant complex scalar field theory under a global $U(1)$ transformation. This framework introduces new phenomena, most notably the emergence of a massless scalar excitation known as the Goldstone boson [90]. Consider

a complex scalar field ϕ with the Lagrangian:

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - V(\phi), \quad (2.7)$$

where the potential takes the form:

$$V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2, \quad \lambda > 0. \quad (2.8)$$

This Lagrangian is invariant under global $U(1)$ phase rotations: $\phi \rightarrow e^{i\alpha} \phi$, where α is a real constant. The symmetry corresponds to the conservation of a global charge. The behaviour of the vacuum depends critically on the sign of μ^2 : if $\mu^2 > 0$, the potential has a single minimum at $\phi = 0$, preserving the $U(1)$ symmetry and for $\mu^2 < 0$, the potential develops a continuous ring of degenerate minima: $|\phi| = v = \sqrt{\frac{-\mu^2}{\lambda}}$ as shown in Fig. 2.2.

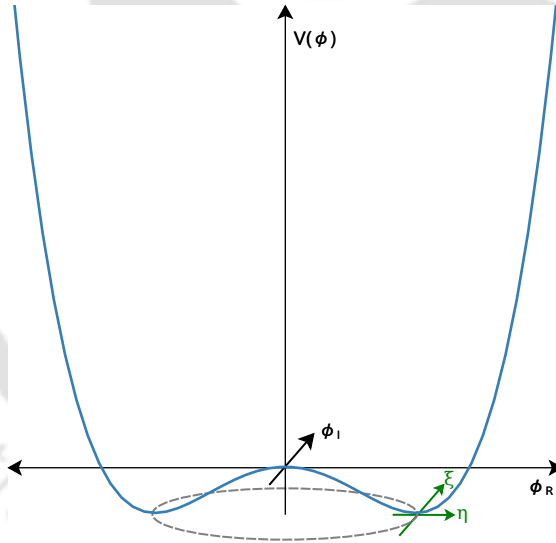


Figure 2.2: The potential profile $V(\phi)$ for a complex scalar field for the case $\mu^2 < 0$ and $\lambda > 0$.

In the latter case, the field acquires a non-zero vacuum expectation value (VEV), and the $U(1)$ symmetry is spontaneously broken. The vacuum must choose a specific phase, breaking the global symmetry. To analyse the physical excitations around the vacuum, it is convenient to parametrise the field as:

$$\phi(x) = \frac{1}{\sqrt{2}} [v + \eta(x)] e^{i\xi(x)/v}, \quad (2.9)$$

where $\eta(x)$ and $\xi(x)$ are real scalar fields describing fluctuations in the radial and angular directions, respectively. Substituting this into the Lagrangian and expanding, we find that: the field $\eta(x)$ represents a massive scalar excitation with mass $m_\eta^2 = 2\lambda v^2$, and the field $\xi(x)$ remains massless. This is the Goldstone boson, which arises due to the spontaneous breaking of the continuous global symmetry. This result is a manifestation of *Goldstone's theorem*, which states that for every spontaneously broken continuous global symmetry, there exists a corresponding massless scalar boson. Although massless Goldstone bosons are not observed in nature for all symmetries, they arise in various effective theories, such as pions in low-energy QCD correspond to pseudo-Goldstone bosons resulting from chiral symmetry breaking. In contrast, when the symmetry is gauged (i.e., local rather than global), the would-be Goldstone boson is absorbed by the gauge field, generating the latter's mass via the Higgs mechanism: a scenario that we will explore in the next section.

2.1.2 The Higgs Mechanism

In the case of global symmetry, spontaneous symmetry breaking leads to the appearance of a massless scalar excitation known as the Goldstone boson. However, when the symmetry is local (gauged), the consequences are different. The Goldstone boson is no longer physical; instead, it is absorbed by the gauge field, which acquires a mass or equivalently, a longitudinal component. This elegant mechanism, known as the *Higgs mechanism* [2, 3, 5], allows gauge bosons to become massive while preserving the gauge invariance of the theory. Consider a complex scalar field $\phi(x)$ interacting with an Abelian gauge field $A_\mu(x)$ through a local $U(1)$ symmetry. The Lagrangian is given by:

$$\mathcal{L} = (D_\mu \phi)^* (D^\mu \phi) - V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad (2.10)$$

where

$$D_\mu = \partial_\mu + ieA_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (2.11)$$

and the scalar potential is the same as before:

$$V(\phi) = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2. \quad (2.12)$$

When $\mu^2 < 0$, the scalar field acquires a non-zero VEV, spontaneously breaking the local $U(1)$ symmetry: $\langle \phi \rangle = \frac{v}{\sqrt{2}}$, where $v = \sqrt{\frac{-\mu^2}{\lambda}}$. After symmetry breaking, we expand the field around the vacuum using the parametrisation: $\phi(x) = \frac{1}{\sqrt{2}} [v + \eta(x)] e^{i\xi(x)/v}$. Here, $\eta(x)$ describes the physical scalar excitation, and $\xi(x)$ is the would-be Goldstone boson. Under a local gauge transformation, we can remove the $\xi(x)$ field entirely by choosing a gauge where $\phi(x)$ is real. This is called the *unitary gauge*, where:

$$\phi(x) \rightarrow \frac{1}{\sqrt{2}} [v + \eta(x)], \quad A_\mu \rightarrow A'_\mu = A_\mu - \frac{1}{ev} \partial_\mu \xi. \quad (2.13)$$

In this gauge, the Lagrangian becomes:

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \eta) (\partial^\mu \eta) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} e^2 v^2 A_\mu A^\mu + \dots, \quad (2.14)$$

revealing two important results:

- The scalar field $\eta(x)$ is a massive particle with mass $m_\eta^2 = 2\lambda v^2$.
- The gauge field A_μ has acquired a mass: $m_A = ev$.

Thus, the Higgs mechanism provides a natural way for gauge bosons to gain mass without explicitly breaking gauge invariance. The Goldstone boson has been absorbed into the longitudinal polarisation of the now massive vector boson [90], preserving the consistency and renormalizability of the theory.

2.1.3 The Standard Model

The Standard Model of particle physics is a quantum field theoretical framework based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, describing the strong, weak, and electromagnetic interactions among all known elementary fermions and gauge bosons and Higgs boson. The electroweak theory developed by Glashow, Weinberg, and Salam unifies the weak and electromagnetic interactions within a single gauge framework based on the group $SU(2)_L \times U(1)_Y$ [1, 4, 91]. This framework, known as the electroweak model, is a gauge theory that describes leptons and their interactions with gauge bosons via both charged and neutral currents. In the Standard

Model, each gauge group is associated with n massless spin-1 gauge bosons, where n refers to the dimension of adjoint representation of the gauge group. The strong interaction group $SU(3)_C$ gives rise to eight gluons g_μ^a , which transform as $(8, 1, 0)$ under the full gauge group. The weak interaction group $SU(2)_L$ is associated with three gauge bosons W_μ^i , transforming as $(1, 3, 0)$. Finally, the hypercharge group $U(1)_Y$ contributes one gauge boson B_μ , transforming as $(1, 1, 0)$.

In addition to the gauge bosons associated with the Standard Model gauge symmetries, the theory includes several other fundamental particles. These consist of five types of spin- $\frac{1}{2}$ fermions and one spin-0 scalar field, classified by their transformation properties under the SM gauge group. The scalar sector features a single complex Higgs doublet $\Phi \sim (1, 2, \frac{1}{2})$, which is responsible for electroweak symmetry breaking. The fermions are divided into quarks and leptons depending on whether they transform under $SU(3)_C$. The quark sector includes left-handed doublets $Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3, 2, 1/6)$, along with right-handed up-type quarks $u_R \sim (3, 1, 2/3)$ and right-handed down-type quarks $d_R \sim (3, 1, -1/3)$. The lepton sector includes left-handed doublets $L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1, 2, -1/2)$ and right-handed charged leptons $e_R \sim (1, 1, -1)$. These fields are replicated across three generations, reflecting the observed structure of matter. Fermion masses in the Standard Model arise through gauge-invariant Yukawa interactions between the fermion fields and the Higgs doublet. These couplings are expressed as:

$$\mathcal{L}_{\text{Yukawa}} = -y_u^{ij} \bar{Q}_L^i \tilde{\Phi} u_R^j - y_d^{ij} \bar{Q}_L^i \Phi d_R^j - y_e^{ij} \bar{L}_L^i \Phi e_R^j + \text{h.c.}, \quad (2.15)$$

where $\tilde{\Phi} = i\sigma^2 \Phi^*$ denotes the conjugate Higgs doublet. Combining all sectors, the full Lagrangian of the SM is:

$$\begin{aligned} \mathcal{L}_{\text{SM}} = & -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & + (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) + \mathcal{L}_{\text{Yukawa}}, \end{aligned} \quad (2.16)$$

where $G_{\mu\nu}^a$, $W_{\mu\nu}^a$, and $B_{\mu\nu}$ are the $SU(3)$, $SU(2)$, and $U(1)$ field strength tensors, respectively. To incorporate mass for the weak gauge bosons while maintaining

gauge invariance, the model employs the Higgs mechanism. A complex scalar doublet acquires a vacuum expectation value, spontaneously breaking the electroweak symmetry $SU(2)_L \times U(1)_Y$ down to the electromagnetic subgroup $U(1)_{\text{EM}}$. This spontaneous symmetry breaking generates masses for the $W^\pm$ and Z bosons while leaving the photon massless, thereby preserving the unbroken electromagnetic gauge invariance. The scalar Higgs field Φ transforms as a complex doublet under $SU(2)_L$ and carries hypercharge under $U(1)_Y$:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \text{with } Y = +\frac{1}{2}. \quad (2.17)$$

The Lagrangian for the scalar field is:

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi), \quad (2.18)$$

where the covariant derivative is defined as:

$$D_\mu = \partial_\mu - ig \frac{\tau^a}{2} W_\mu^a - ig' \frac{Y}{2} B_\mu, \quad (2.19)$$

with τ^a being the Pauli matrices. The corresponding field strengths are:

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc} W_\mu^b W_\nu^c, \quad (2.20)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (2.21)$$

The scalar potential is given by:

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2. \quad (2.22)$$

When $\mu^2 < 0$, the potential develops a non-trivial minimum, and the Higgs field acquires a VEV:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \text{with } v = \sqrt{\frac{-\mu^2}{\lambda}}. \quad (2.23)$$

This choice breaks the symmetry: $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{EM}}$. We can write the gauge boson mass terms that arise from the kinetic term of the Higgs field:

$$(D_\mu \Phi)^\dagger (D^\mu \Phi) \supset \frac{1}{2} v^2 \left[\frac{g^2}{2} W_\mu^+ W^{\mu-} + \frac{1}{2} \begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} \right], \quad (2.24)$$

where we have defined the charged gauge bosons: $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2)$. Diagonalising the neutral gauge boson mass matrix yields the physical fields:

$$Z_\mu = \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu, \quad (2.25)$$

$$A_\mu = \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu, \quad (2.26)$$

where A_μ is the massless photon, and Z_μ is the massive neutral weak boson. The Weinberg angle θ_W is defined by $\tan \theta_W = \frac{g'}{g}$. The masses of the physical gauge bosons are:

$$m_W = \frac{1}{2} g v, \quad (2.27)$$

$$m_Z = \frac{1}{2} \sqrt{g^2 + g'^2} v, \quad (2.28)$$

$$m_\gamma = 0. \quad (2.29)$$

Thus, the Higgs mechanism in the SM naturally explains the masses of the weak gauge bosons while preserving the masslessness of the photon. The ratio of the W and Z boson masses is: $\frac{m_W}{m_Z} = \cos \theta_W$, a relation that is experimentally verified to high precision. After electroweak symmetry breaking, the remaining degree of freedom in the Higgs doublet manifests as a real physical scalar particle known as the Higgs boson, with mass: $m_H^2 = 2\lambda v^2$. In SM, the charged fermions acquire masses through Yukawa interaction as $m_f = \frac{y_f v}{\sqrt{2}}$, after electroweak symmetry breaking.

2.1.4 The Effective Potential in the Standard Model

The concept of effective potential arises from the need to understand how quantum fluctuations modify the vacuum structure of a quantum field theory. In classical field theory, the vacuum is defined by the minimum of the scalar potential. However, in quantum theory, this picture is altered due to virtual processes and loop corrections

that shift and reshape the total scalar potential. The effective potential captures these quantum effects and is important in studying spontaneous symmetry breaking, vacuum stability, and the nature of phase transitions. In the SM, the effective potential for the Higgs field governs both the vacuum structure and the nature of the electroweak phase transition (EWPT).

2.1.4.1 Zero-Temperature Corrections

The effective potential includes all scalar self-interactions without derivatives and is constructed by summing all one-particle-irreducible (1PI) diagrams with scalar external legs evaluated at zero external momentum [92, 93]. This momentum condition ensures that kinetic terms are excluded, focusing solely on potential-like contributions. The quantum-corrected vacua of the theory can be found by minimising this potential, making it crucial for studying symmetry breaking and vacuum structure beyond tree level. The action correspond to scalar field ϕ can be written as

$$S[\phi] = \int d^4x \mathcal{L}(\phi(x)). \quad (2.30)$$

To formalise this, we begin with the generating functional in the presence of an external source $J(x)$:

$$Z[J] = \int \mathcal{D}\phi \exp \left(i \int d^4x [\mathcal{L} + J(x)\phi(x)] \right) = \exp[iW[J]], \quad (2.31)$$

where $W[J]$ is the generating functional for connected Green's functions.

The expectation value of the field in the presence of the source defines the classical field:

$$\frac{\delta W[J]}{\delta J(x)} = \langle \Omega | \phi(x) | \Omega \rangle_J \equiv \phi(x). \quad (2.32)$$

The *effective action* $\Gamma[\phi]$ is obtained via the Legendre transform of the generating functional of connected diagrams $W[J]$:

$$\Gamma[\phi] = W[J] - \int d^4x J(x)\phi(x). \quad (2.33)$$

It satisfies the following stationary condition in the absence of sources:

$$\left. \frac{\delta \Gamma[\phi]}{\delta \phi(x)} \right|_{J=0} = 0. \quad (2.34)$$

In a translationally invariant vacuum where $\phi(x) = \phi_{\text{cl}}$ is constant, the effective action simplifies to:

$$\Gamma[\phi_{\text{cl}}] = - \int d^4x V_{\text{eff}}(\phi_{\text{cl}}), \quad (2.35)$$

thereby defining the *effective potential* V_{eff} .

By Fourier transforming the effective action and evaluating the resulting expression at zero external momenta, we obtain:

$$V_{\text{eff}} = - \sum_{n=0}^{\infty} \frac{\phi_{\text{cl}}^n}{n!} \Gamma^{(n)}(p_i = 0), \quad (2.36)$$

$$= \Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)} + \Gamma^{(3)} + \dots$$

where $\Gamma^{(n)}$ are the n -point 1PI Green's functions. This construction ensures that the vacuum expectation value of the scalar field corresponds to the minimum of the effective potential:

$$\frac{\partial V_{\text{eff}}}{\partial \phi_{\text{cl}}} = 0. \quad (2.37)$$

Thus, the effective potential provides a natural generalisation of the tree-level potential to include quantum corrections and can be minimised to obtain the true vacuum structure of the theory. To compute the quantum corrections to the scalar potential, one of the standard techniques is to use the one-loop effective potential in the $\overline{MS}$ scheme. This method provides an efficient way to calculate loop corrections, and it is particularly useful for analysing vacuum structure and phase transitions. The one-loop effective potential was originally derived in Refs. [92], and is widely applied using the Landau gauge ($\xi = 0$ in R_ξ gauges) and dimensional regularisation.

The tree-level (zeroth loop order contribution) potential for a real scalar is expressed as

$$V_0(\phi) = \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4, \quad (2.38)$$

$$= \text{---} + \text{X}$$

where m^2 is the mass parameter, and λ is the quartic self-coupling.

In *Coleman-Weinberg*(CW) potential, all one-loop 1PI vacuum diagrams are included with zero external momentum, where the loop can be scalars, fermions, and vector bosons.

$$V_{\text{cw}} = \text{---} + \text{---} + \text{---} + \dots$$

All diagrams consist of a scalar loop with an arbitrary even number of external ϕ lines connected to a ring of quartic interactions. The contribution from each diagram

with n vertices (and $2n$ external legs of the classical field ϕ) is:

$$V_{\text{cw}}(\phi) = i \sum_{n=1}^{\infty} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2n} \left[\frac{\lambda \phi^2}{2(k^2 - m^2 + i\epsilon)} \right]^n, \quad (2.39)$$

where $m^2(\phi) = \mu^2 + \lambda \phi^2/2$ is the squared field-dependent mass. Summing over n leads to a geometric series, which re-sums into a logarithm:

$$V_{\text{cw}}(\phi) = -\frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \log \left(1 - \frac{\lambda \phi^2}{2(k^2 - m^2 + i\epsilon)} \right). \quad (2.40)$$

We now perform a Wick rotation to Euclidean space ($k^0 \rightarrow ik_E^0$, $k^2 \rightarrow +k_E^2$), yielding:

$$V_{\text{cw}}(\phi) = \frac{1}{2} \int \frac{d^4 k_E}{(2\pi)^4} \log \left(1 + \frac{\lambda \phi^2}{2(k_E^2 + m^2)} \right). \quad (2.41)$$

After neglecting field independent terms and using shifted mass, the above expression can be written in terms of the field-dependent mass as:

$$V_{\text{cw}}(\phi) = \frac{1}{2} \int \frac{d^4 k_E}{(2\pi)^4} \log (k_E^2 - m^2). \quad (2.42)$$

The final expression can be written as

$$V_{\text{cw}}(\phi) = \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \log (k^2 + m^2(\phi)). \quad (2.43)$$

To handle ultraviolet divergences, we perform dimensional regularisation in $d = 4 - \epsilon$ dimensions. Introducing a renormalisation scale μ to keep the potential dimensionally correct, the expression becomes:

$$V_{\text{cw}}(\phi) = \frac{1}{2} \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} \log (k^2 + m_s^2(\phi)). \quad (2.44)$$

The divergent part can be isolated and subtracted using the $\overline{\text{MS}}$ scheme. The finite one-loop correction to the effective potential is:

$$V_{\text{cw}}(\phi) = \frac{1}{64\pi^2} m^4(\phi) \left[\log \left(\frac{m^2(\phi)}{\mu^2} \right) - \frac{3}{2} \right], \quad (2.45)$$

where $m_s^2(\phi) = m^2 + \frac{\lambda}{2}\phi^2$ is the field-dependent mass squared.

The result above can be generalised to multiple scalar fields, fermions, and gauge bosons. For a set of real scalar fields $\{\phi_i\}$ with field-dependent squared masses $m_s^2(\phi_i)$, the scalar contribution is:

$$V_{\text{cw,scalar}} = \frac{1}{64\pi^2} \sum_s m_s^4(\phi) \left[\log \left(\frac{m_s^2(\phi)}{\mu^2} \right) - \frac{3}{2} \right]. \quad (2.46)$$

The fermionic and gauge boson contributions, using Weyl fermions and massive vectors, respectively, are:

$$V_{\text{cw,fermion}} = -\frac{1}{64\pi^2} \sum_f m_f^4(\phi) \left[\log \left(\frac{m_f^2(\phi)}{\mu^2} \right) - \frac{3}{2} \right], \quad (2.47)$$

$$V_{\text{cw,gauge}} = \frac{1}{64\pi^2} \sum_v m_v^4(\phi) \left[\log \left(\frac{m_v^2(\phi)}{\mu^2} \right) - \frac{5}{6} \right]. \quad (2.48)$$

The renormalisation conditions also ensure that the effective potential satisfies:

$$\left. \frac{\partial(V_{\text{CW}} + V_{\text{CT}})}{\partial\phi_i} \right|_{\phi_i=v_i^{\text{tree}}} = 0, \quad (2.49)$$

$$\left. \frac{\partial^2(V_{\text{CW}} + V_{\text{CT}})}{\partial\phi_i\partial\phi_j} \right|_{\phi_i=v_i^{\text{tree}}} = 0, \quad (2.50)$$

where V_{CW} is the Coleman-Weinberg potential and V_{CT} is the counterterm potential. These conditions ensure that both the position of the minimum and the masses remain at their tree-level values, enabling a physical interpretation of parameters. In the Standard Model, the Higgs field can be represented by all of its components as follows:

$$\Phi = \begin{pmatrix} \chi_1 + i\chi_2 \\ \frac{\phi+h+i\chi_3}{\sqrt{2}} \end{pmatrix}. \quad (2.51)$$

Here, ϕ denotes the constant real background field, h represents the physical Higgs excitation, and χ^a (with $a = 1, 2, 3$) are the three Goldstone bosons. The tree-level (zeroth loop order contribution) potential for Higgs scalar can be expressed in terms

of the background field as

$$V_0(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{\lambda}{4}\phi^4, \quad (2.52)$$

with positive λ and m^2 , the tree level minimum is $\pm\sqrt{m^2/\lambda}$. The field dependent masses of Higgs and Goldstone bosons are

$$\begin{aligned} m_h^2(\phi) &= 3\lambda\phi^2 - m^2, \\ m_\chi^2(\phi) &= \lambda\phi^2 - m^2. \end{aligned} \quad (2.53)$$

The gauge bosons contributing to the effective potential are $W^\pm$ and Z , the field dependent masses are

$$\begin{aligned} m_W^2(\phi) &= \frac{g^2}{4}\phi^2, \\ m_Z^2(\phi) &= \frac{g^2 + g'^2}{4}\phi^2. \end{aligned} \quad (2.54)$$

The top quark is the only fermion which has significant contribution and the field dependent mass is

$$m_t^2(\phi) = \frac{y_t^2}{2}\phi^2. \quad (2.55)$$

2.1.4.2 Finite-Temperature Corrections

Thermal effects are essential when studying phase transitions in the early Universe. At high temperatures, the shape of the scalar potential can be significantly modified. For instance, a potential that exhibits spontaneous symmetry breaking at zero temperature (such as the Higgs potential with a non-zero vacuum expectation value) can become symmetric again at high temperature, with the global minimum returning to the origin [94–97]. We use the framework of finite-temperature quantum field theory (QFT). While ordinary QFT is designed to describe processes in a cold, vacuum background, finite temperature QFT extends to systems in thermal equilibrium at temperature T . It combines quantum field theory with principles from statistical mechanics, allowing us to study particle interactions in a thermal bath.

In statistical mechanics, the thermodynamic properties of a system are encoded in the partition function:

$$Z = \text{Tr} \left(e^{-\beta \mathbb{H}} \right), \quad \text{with } \beta = \frac{1}{T}, \quad (2.56)$$

where $\mathbb{H}$ is the Hamiltonian of the system and T is the temperature (we use natural units with $k_B = 1$). In quantum field theory, the partition function is formulated as a path integral over all possible field configurations. The temperature-dependent effective action $\Gamma_T[\phi]$ is related to the generating functional $W[J]$ (as in zero-temperature QFT) by:

$$\begin{aligned} Z &= e^{iW[J]}, \\ \phi(x) &= \frac{\delta W[J]}{\delta J(x)}, \\ \Gamma_T[\phi] &= W[J] - \int d^4x \phi(x) J(x), \\ \left. \frac{\delta \Gamma_T[\phi]}{\delta \phi(x)} \right|_{J=0} &= 0. \end{aligned} \quad (2.57)$$

In thermal equilibrium and for a homogeneous field configuration, $\phi(x) = \phi_{\text{cl}}$ (a spacetime-independent classical field), the effective action simplifies to:

$$\Gamma_T[\phi_{\text{cl}}] = - \int d^4x V_{\text{eff}}^T(\phi_{\text{cl}}), \quad (2.58)$$

defining the finite-temperature effective potential as V_{eff}^T . This potential plays the role of the free energy density, and its minimisation determines the thermal vacuum configuration.

To evaluate thermal effects, we employ the imaginary-time (or Matsubara) formalism [98], which replaces the real time coordinate with imaginary time $\tau = it$, and changes the time dimension to a circle of circumference $\beta = 1/T$. This converts spacetime from Minkowski to Euclidean and leads to the following substitutions:

$$p^\mu = (i\omega_n, \vec{k}), \quad (2.59)$$

$$\omega_n = \begin{cases} 2\pi n T, & \text{for bosons,} \\ (2n+1)\pi T, & \text{for fermions,} \end{cases} \quad (2.60)$$

where $n \in \mathbb{Z}$. The integral over energy becomes a discrete sum over Matsubara frequencies:

$$\int \frac{dp_0}{2\pi} \rightarrow \frac{1}{\beta} \sum_n. \quad (2.61)$$

As a representative example, consider the one-loop thermal correction from a real scalar field with field-dependent mass $m_s(\phi)$. The relevant expression is:

$$V_T^S(\phi, T) = \frac{1}{2\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \log \left(\omega_n^2 + \vec{k}^2 + m_s^2(\phi) \right). \quad (2.62)$$

This sum is divergent, but the divergent part is field-independent and can be discarded when focusing on ϕ -dependent physics. Using standard techniques used in ref. [97], the sum over Matsubara frequencies can be evaluated:

$$V_T^S(\phi, T) = \int \frac{d^3k}{(2\pi)^3} \left[\frac{\omega_k}{2} + \frac{1}{\beta} \log \left(1 - e^{-\beta\omega_k} \right) \right], \quad (2.63)$$

where $\omega_k = \sqrt{\vec{k}^2 + m_s^2(\phi)}$. The first term reproduces the zero-temperature one-loop correction, while the second term captures the thermal contribution and can be expressed in terms of the thermal integral function J_B :

$$V_T^S(\phi, T) = \frac{T^4}{2\pi^2} J_B \left(\frac{m_s^2(\phi)}{T^2} \right), \quad (2.64)$$

$$J_B(y^2) = \int_0^\infty dx x^2 \log \left(1 - e^{-\sqrt{x^2+y^2}} \right). \quad (2.65)$$

In the high-temperature limit ($y^2 \ll 1$):

$$J_B^{\text{HT}}(y^2) = -\frac{\pi^4}{45} + \frac{\pi^2}{12} y^2 - \frac{\pi}{6} y^3 - \frac{1}{32} y^4 \log \left(\frac{y^2}{a_B} \right) + \dots, \quad (2.66)$$

where $a_B = 16\pi^2 e^{3/2-2\gamma_E}$ and γ_E is the Euler-Mascheroni constant. In the Low-temperature limit ($y^2 \gg 1$) [99]:

$$J_B^{\text{LT}}(y^2) \approx - \sum_{n=1}^m \frac{1}{n^2 y^2} K_2(ny), \quad (2.67)$$

where K_2 is the modified Bessel function of the second kind and $m = 2$ or 3 typically suffices for accurate approximation.

For Weyl or Dirac fermions, the procedure is similar but the Matsubara frequencies differ. The one-loop correction is:

$$V_T^F(\phi, T) = -n_F \int \frac{d^3k}{(2\pi)^3} \left[\frac{\omega_k}{2} + \frac{1}{\beta} \log(1 + e^{-\beta\omega_k}) \right], \quad (2.68)$$

where n_F is the number of fermionic degrees of freedom and $\omega_k = \sqrt{\vec{k}^2 + m_f^2(\phi)}$. The thermal part becomes:

$$V_T^F(\phi, T) = -\frac{n_F T^4}{2\pi^2} J_F\left(\frac{m_f^2(\phi)}{T^2}\right), \quad (2.69)$$

$$J_F(y^2) = \int_0^\infty dx x^2 \log(1 + e^{-\sqrt{x^2 + y^2}}). \quad (2.70)$$

Summing the scalar, fermionic, and vector boson contributions, the full one-loop thermal correction to the effective potential (in the Landau gauge) is:

$$V_T = \frac{T^4}{2\pi^2} \left[\sum_i n_\phi J_B\left(\frac{m_{\phi_i}^2}{T^2}\right) + \sum_j n_V J_B\left(\frac{m_{V_j}^2}{T^2}\right) - \sum_k n_F J_F\left(\frac{m_{f_k}^2}{T^2}\right) \right], \quad (2.71)$$

where $n_\phi = 1$ for scalars, $n_V = 3$ for massive vector bosons (or 2 for massless), and $n_F = 2$ for Weyl fermions (or 4 for Dirac fermions). This expression is widely used in studies of finite-temperature phase transitions, especially for determining the effective potential's structure at different temperatures.

In perturbative calculations of the finite-temperature effective potential, a significant challenge arises due to the breakdown of perturbation theory at high temperatures [95, 96]. This breakdown comes from infrared (IR) divergences and the poor convergence of loop expansions in the thermal background. To address this, one must reorganise the perturbative series through a technique known as *resummation*. The necessity for resummation can be seen most clearly in the high-temperature expansion of the effective potential.

The most widely used resummation scheme in finite-temperature field theory is the *ring resummation*, also known as *daisy resummation* [97, 100]. It involves summing

over a class of diagrams that dominate in the IR limit specifically, the so-called “ring” or “daisy” diagrams, which consist of a one-loop self-energy insertion repeated an arbitrary number of times. These diagrams effectively modify the propagators by including thermal mass corrections. That is, instead of using the bare field-dependent masses $m_i^2(\phi)$, one uses thermally corrected masses:

$$\tilde{m}_i^2(\phi, T) = m_i^2(\phi) + \Pi_i(T), \quad (2.72)$$

where $\Pi_i(T)$ is the finite-temperature self-energy (thermal mass) of the i -th particle. This correction is particularly important for bosonic fields, whose zero modes are responsible for the IR sensitivity. Two common methods used for implementing this resummation are the Parwani method [101] and the Arnold–Espinosa method [102]. In the Parwani approach, the thermally corrected masses are substituted everywhere in the one-loop effective potential, both in the zero-temperature Coleman–Weinberg part and the thermal part. In contrast, the Arnold–Espinosa prescription modifies only the bosonic cubic thermal contributions, which are the source of the infrared divergence, by replacing the bare masses with thermal masses. To implement this resummation, one modifies the one-loop thermal potential by subtracting the temperature-dependent mass in the cubic term of the high-temperature expansion. The temperature-dependent cubic term is extracted as follows:

$$V_T^{\text{daisy}}(\phi, T) = -\frac{T}{12\pi} \sum_i n_i [\tilde{m}_i^3(\phi, T) - m_i^3(\phi)]. \quad (2.73)$$

Here, n_i denotes the number of degrees of freedom of each bosonic mode. This form subtracts the unresummed term (which is already included in V_T) and adds the properly resummed contribution, ensuring no overcounting.

The thermal self-energies $\Pi_i(T)$, which determine the thermal masses, are computed from the leading order (typically one-loop) diagrams at high temperatures. For scalars and longitudinal gauge bosons, they are proportional to T^2 . For example, for a scalar ϕ with quartic self-coupling λ , the thermal mass correction is: $\Pi_\phi(T) \sim \lambda T^2$. In the case of gauge bosons, only the longitudinal components acquire thermal masses (Debye masses). The transverse components remain massless at leading order and are responsible for remaining IR divergences that cannot be cured within

standard perturbation theory. Hence, the effective potential of a scalar field is

$$V_{\text{eff}}(\phi, T) = V_0(\phi) + V_{\text{cw}}(\phi) + V_T(\phi, T) + V_T^{\text{daisy}}(\phi, T). \quad (2.74)$$

2.1.5 Cosmological Phase Transitions

Phase transitions in the early Universe are crucial in connecting high-energy particle physics with cosmology. As the Universe cooled from its hot, dense origins, it may have undergone a series of symmetry-breaking transitions that shaped the current structure and content of the cosmos. These transitions are closely tied to the vacuum structure of the underlying field theory and can leave observable imprints such as relics, baryon asymmetry, and gravitational waves.

2.1.5.1 Thermal History and Symmetry Restoration

In the early Universe, temperatures were high enough that spontaneous symmetry breaking, such as that responsible for the Higgs mechanism, did not occur [96]. All symmetries of the Lagrangian were effectively restored due to thermal fluctuations. As the Universe expanded and cooled, certain symmetries were spontaneously broken through phase transitions, leading to a hierarchy of physical scales and the emergence of distinct particle species. Let us denote the temperature of the Universe as T and the Hubble rate as $H(T)$. The thermal behaviour of a scalar field ϕ is governed by the finite-temperature effective potential $V_{\text{eff}}(\phi, T)$, which determines the preferred vacuum at a given temperature. The shape of this potential changes with T , and the transition from a symmetric phase ($\langle\phi\rangle = 0$) to a broken phase ($\langle\phi\rangle \neq 0$) constitutes a cosmological phase transition.

2.1.5.2 Classification of Phase Transitions

Phase transitions are classified according to the order of discontinuity in the derivatives of the effective potential:

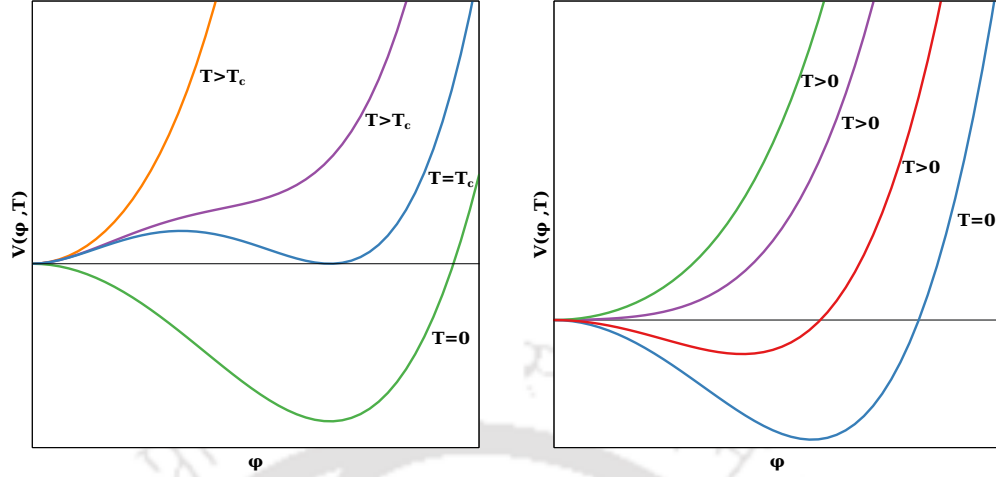


Figure 2.3: Left: Potential profile with scalar field for first-order phase transition, Right: Potential profile with scalar field for second order phase transition

- **First-Order Phase Transition:** The potential develops a second, local minimum separated by a barrier from the symmetric phase. At the critical temperature T_c , both minima are degenerate. The system undergoes a discontinuous transition by nucleating bubbles of the broken phase. The order parameter (VEV) changes discontinuously as shown in the left panel of Fig. 2.3.
- **Second-Order Phase Transition:** The minimum at the origin gradually shifts toward a non-zero value as temperature decreases, as depicted in the right panel of Fig. 2.3. The order parameter (VEV) changes continuously.

2.1.5.3 Nature of the Phase Transition in Standard Model

Within the Standard Model, the electroweak phase transition governed by the Higgs potential is not a first-order transition but rather a second-order transition or a smooth crossover, depending on the Higgs mass. For the physical Higgs mass of 125 GeV, lattice simulations confirm that the transition is a crossover [25, 103–105], meaning the Higgs vacuum expectation value changes continuously with temperature. This behaviour is illustrated in Fig. 2.4, where the VEV smoothly decreases as the Universe cools. Interestingly, for a much lighter Higgs boson, around 35 GeV, the electroweak transition would have been strongly first-order, with a discontinuous

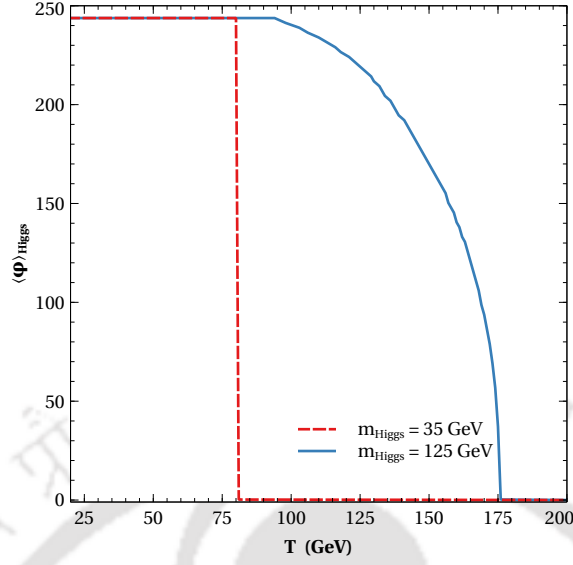


Figure 2.4: The variation of the vacuum expectation value of Higgs in the Standard Model with temperature for two different values of Higgs mass.

jump in the VEV [25]. However, such a scenario is excluded by experimental results from the LHC [106], which confirmed the Higgs mass to be approximately 125 GeV, ruling out the possibility of a first-order phase transition in the Standard Model. The order of the electroweak phase transition depends on the relative strength of the effective cubic term in thermal correction. For a strong first-order phase transition, a barrier must persist between the vacua at the critical temperature T_c , where both vacua are degenerate, which is ensured by the cubic term. But in the Standard Model, the coefficient of the cubic term is not sufficient to form a barrier. The condition for a strong first-order EWPT, relevant for baryogenesis, is $\frac{\phi_c}{T_c} \gtrsim 1$, where ϕ_c is the Higgs field value at the broken minimum at T_c . This criterion ensures that sphaleron processes, which violate baryon number, are suppressed inside the broken phase. Hence, the Standard Model is insufficient for successful electroweak baryogenesis.

2.1.5.4 Beyond the Standard Model Needs

To overcome these limitations, several Beyond Standard Model extensions are proposed to have first-order phase transition:

- Additional scalar fields (e.g., scalar singlets [107–109], doublets [110–112], or triplets[113, 114]) that enhance the cubic term ϕ^3 .
- Modified Higgs potentials via higher-dimensional operators (e.g., ϕ^6 terms)[115–117].
- Strong coupling or non-perturbative effects (e.g., in composite Higgs or extra-dimensional models) [118, 119].

These extensions can make the EWPT strongly first-order and fulfil the condition for successful electroweak baryogenesis.

2.1.6 False Vacuum Decay

For multiple vacua of a scalar field, the Universe may undergo a transition from a higher energy (false) vacuum to a lower energy (true) vacuum. This process is known as *false vacuum decay*. It is crucial to understand first-order cosmological phase transitions and their associated non-equilibrium dynamics. The decay of the false vacuum can proceed in two distinct ways, as shown in Fig. 2.5. In one scenario, the field transitions over the potential barrier, typically driven by thermal fluctuations; however, this process becomes suppressed as the temperature decreases. On the other scenario, the decay occurs through quantum or thermal tunnelling across the barrier, where the field configuration interpolates between the false and true vacua via non-perturbative solutions known as *bounces*. These solutions to the Euclidean field equations govern the rate of vacuum decay and the nucleation of bubbles of true vacuum in a false vacuum background. The evolution of a scalar field in the presence of an effective potential during a phase transition is governed by the Euclidean action in d -dimensional space. This action captures the dynamics of tunnelling between vacua and is given by [120, 121]:

$$S_d[\phi] \propto \int d\rho \rho^{d-1} \left[\frac{1}{2} \left(\frac{d\phi}{d\rho} \right)^2 + V_{\text{eff}}(\phi) \right],$$

where $d = 4$ corresponds to zero-temperature quantum tunnelling (vacuum decay), and $d = 3$ applies to finite-temperature tunnelling. The radial coordinate ρ arises

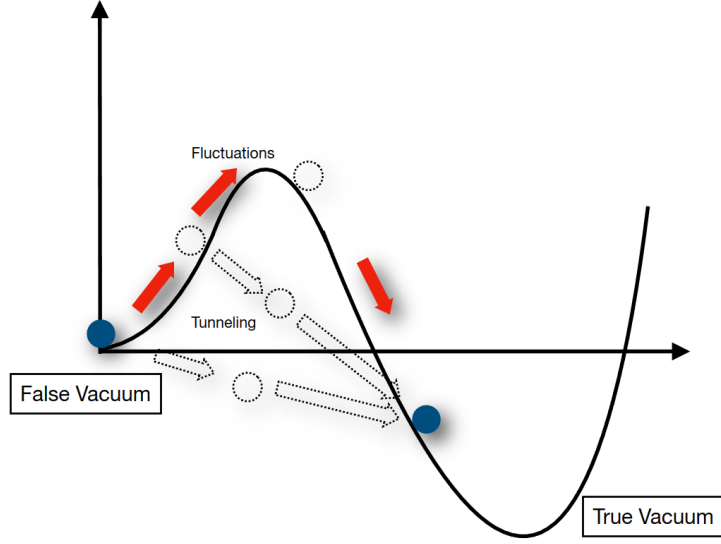


Figure 2.5: The transition of the Universe from false vacuum to true vacuum.

due to $O(d)$ symmetric nature of the system, which allows the field configuration describing a single nucleated bubble to depend only on the radial distance from the centre. To find the field configuration that minimises the action, we apply the principle of stationary action, leading to the so-called bounce equation:

$$\frac{d^2\phi}{d\rho^2} + \frac{d-1}{\rho} \frac{d\phi}{d\rho} = \frac{dV_{\text{eff}}}{d\phi} \Big|_{\phi=\langle\phi\rangle}, \quad (2.75)$$

where we denote the background field by ϕ . The bounce solution $\phi(\rho)$ describes the field profile interpolating between the false vacuum (outside the bubble) and the true vacuum (inside the bubble). It satisfies the boundary conditions:

$$\phi(\rho \rightarrow \infty) = 0, \quad \frac{d\phi}{d\rho} \Big|_{\rho=0} = 0,$$

which ensures that the field is localised and smooth. This configuration represents the critical bubble at the point of nucleation, the solution that dominates the tunnelling process. Depending on the effective potential, multiple bounce solutions can exist. Among them, the one associated with the smallest Euclidean action, either S_4 (in four dimensions) or S_3/T (in three dimensions for finite temperature), corresponds to the most probable tunnelling path. Consequently, it determines the rate of nucleation when the Universe transitions to true vacuum. The tunnelling rate

correspond to O(4) symmetric Euclidean action can be written as [122]

$$\Gamma(T) \sim T^4 \left(\frac{S_4(\phi)}{2\pi} \right)^2 e^{-S_4}. \quad (2.76)$$

Also for O(3) symmetric bounce solution, the tunnelling rate can be expressed as [122]

$$\Gamma(T) \sim T^4 \left(\frac{S_3}{2\pi T} \right)^{3/2} e^{-S_3/T}. \quad (2.77)$$

2.1.6.1 Thin-Wall Approximation

In the limit where the potential barrier is high and the vacuum energy difference is small (i.e., $\epsilon \ll V$), the resulting bubble wall is much thinner than its radius. This is known as the thin-wall regime. Let us consider a scalar field ϕ with a potential of the form:

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2 + \epsilon\phi, \quad (2.78)$$

where ϵ introduces a small explicit symmetry-breaking term that generates a slight energy difference between the two minima located near $\phi \approx \pm v$, as illustrated in Fig. 2.6. The potential thus describes a metastable false vacuum and a true (global) vacuum. The system initially resides in the false vacuum ϕ_{fv} , but can transition to the true vacuum ϕ_{tv} via tunnelling. This decay occurs through the nucleation of true vacuum bubbles which expand if the free energy of the true vacuum is lower than that of the false vacuum.

Under the *thin-wall approximation*, the bubble radius is much larger than the thickness of the bubble wall, one can neglect the second term in the bounce equation (Eq. (2.75)), reducing it to:

$$\frac{d^2\phi}{d\rho^2} = V'(\phi). \quad (2.79)$$

In the limit $\epsilon \rightarrow 0$, the solution of the above equation satisfies:

$$\rho = \int_{\phi_{fv}}^{\phi_{tv}} \frac{d\phi}{\sqrt{V(\phi, T)}}. \quad (2.80)$$

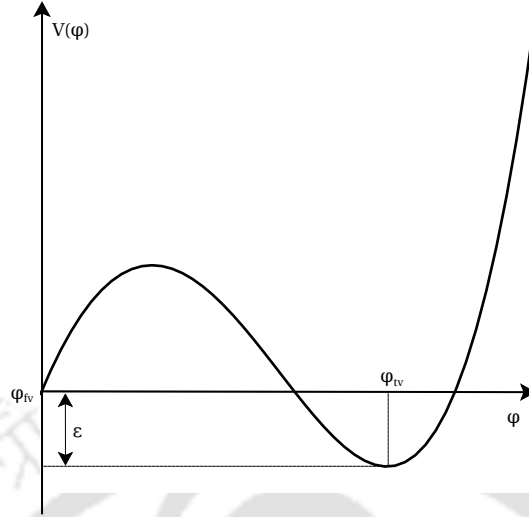


Figure 2.6: Scalar potential illustrating the thin-wall approximation with a small energy difference between the vacua.

The Euclidean action for an $O(4)$ -symmetric bounce configuration is:

$$\begin{aligned} S_4 &= 2\pi^2 \int_0^\infty \rho^3 d\rho \left[\frac{1}{2} \left(\frac{d\phi}{d\rho} \right)^2 + V(\phi) \right] \\ &= -\frac{1}{2} \epsilon \pi^2 \rho^4 + 2\pi^2 \rho^3 S_1, \end{aligned} \quad (2.81)$$

where the first term corresponds to the volume (energy difference) contribution and the second term to the surface energy of the bubble wall. The surface tension (or one-dimensional action) S_1 is given by:

$$S_1 = \int_0^\infty d\rho \left[\frac{1}{2} \left(\frac{d\phi}{d\rho} \right)^2 + V(\phi) \right] = \int_{\phi_{fv}}^{\phi_{tv}} d\phi \sqrt{2V(\phi)}. \quad (2.82)$$

Minimising the total action S_4 with respect to the bubble radius yields the critical action:

$$S_4 = \frac{27\pi^2 S_1^4}{2\epsilon^3}. \quad (2.83)$$

For the $O(3)$ -symmetric case relevant at finite temperature, the Euclidean action takes the form:

$$\begin{aligned} S_3 &= 4\pi \int_0^\infty \rho^2 d\rho \left[\frac{1}{2} \left(\frac{d\phi}{d\rho} \right)^2 + V(\phi) \right] \\ &= -\frac{4}{3}\pi\rho^3\epsilon + 4\pi\rho^2 S_1. \end{aligned} \quad (2.84)$$

Again, minimising the action with respect to ρ gives:

$$S_3 = \frac{16\pi S_1^3}{3\epsilon^2}. \quad (2.85)$$

The $O(3)$ symmetric bounce configuration at finite temperature leads to the minimum action. The bounce action $S_3(T)$ controls the tunnelling probability. The *nucleation temperature* T_n is defined as the temperature at which bubble nucleation becomes efficient enough for the phase transition to proceed. This typically occurs when the nucleation rate per Hubble volume per Hubble time becomes order one:

$$\frac{\Gamma(T_n)}{H^4(T_n)} \sim 1, \quad (2.86)$$

ensuring that at least one bubble nucleates per Hubble volume per Hubble time. Using Eq. (2.86), for electroweak phase transition around 100 GeV, this condition gives:

$$\frac{S_3(T_n)}{T_n} \approx 140. \quad (2.87)$$

Another important temperature is the *percolation temperature* T_p , at which a significant fraction, i.e. 34 % of space, has transitioned to the true vacuum. This occurs slightly after T_n as bubbles expand and overlap.

2.1.6.2 Phase Transition Parameters

A first-order phase transition can be quantified by several key quantities, and they are defined as follows.

- **Critical Temperature T_c :** The temperature at which the two vacua of the potential become degenerate, i.e., $V_{\text{eff}}(\phi_{\text{false}}, T_c) = V_{\text{eff}}(\phi_{\text{true}}, T_c)$.
- **Nucleation Temperature T_n :** The temperature at which the probability for bubble nucleation becomes equal to the Hubble expansion rate as defined in Eq. 2.86.
- **Latent Heat (Strength Parameter) α :**

$$\alpha = \frac{\Delta\rho_{\text{vac}}}{\rho_{\text{rad}}} = \frac{1}{\rho_{\text{rad}}} \left[\Delta V_{\text{eff}} - \frac{T}{4} \frac{\partial \Delta V_{\text{eff}}}{\partial T} \right], \quad (2.88)$$

where ΔV is the difference in free energy between the false and true vacua, and $\rho_{\text{rad}} = \frac{\pi^2}{30} g_* T^4$ is the radiation energy density. Since α quantifies the amount of vacuum energy released during the phase transition relative to the radiation energy density, a larger value of α corresponds to more released energy, which in turn leads to a stronger gravitational wave signal.

- **Inverse Duration β :**

$$\beta = - \left. \frac{d(S_3/T)}{dt} \right|_{t=t_n} \simeq H_n T_n \left. \frac{d(S_3/T)}{dT} \right|_{T_n}, \quad (2.89)$$

which characterises how fast the transition proceeds. A smaller β implies a stronger and longer-lasting GW signal.

2.1.7 Gravitational Waves

Gravitational waves are perturbations of the spacetime metric that propagate at the speed of light, as predicted by Einstein's general theory of relativity. They carry energy and information from regions where spacetime is strongly curved, and they are emitted by systems with time-varying quadrupole moments. While astrophysical sources such as binary black hole mergers have already been detected by the LIGO-VIRGO-KAGRA (LVK) collaboration, cosmological processes in the early Universe, like first-order phase transitions, also provide a promising source of GW which, unlike the ones detected by LVK, are of stochastic nature.

During a first-order phase transition, the nucleation and expansion of bubbles of true vacuum have dramatic physical consequences. Once nucleated, these bubbles expand due to the pressure difference between the true and false vacuum phases, eventually colliding and completing the transition as the entire Universe converts to the broken phase. The expansion and collision of these bubbles within the thermal plasma generate anisotropic stresses in the energy-momentum tensor. These stresses act as a source of gravitational waves, potentially leaving behind a stochastic GW background observable today as a relic imprint of early-Universe dynamics. In a Minkowski spacetime background with metric $g_{\mu\nu} = \eta_{\mu\nu}$, we consider linear perturbations to the spatial part of the metric introduced by a transverse, traceless tensor E_{ij}^T , satisfying $\partial^j E_{ij}^T = 0$ and $E^T{}^i{}_i = 0$. The perturbed line element then reads:

$$ds^2 = -dt^2 + (\delta_{ij} + 2E_{ij}^T)dx^i dx^j.$$

The Einstein field equations reduce to a wave equation for the tensor perturbation $E_{ij}^T(\mathbf{x}, t)$, sourced by the transverse-traceless part of the anisotropic stress tensor $\Pi_{ij}^T(\mathbf{x}, t)$. In Fourier space, this becomes:

$$\ddot{E}_{ij}^T(\mathbf{k}, t) + k^2 E_{ij}^T(\mathbf{k}, t) = 16\pi G \Pi_{ij}^T(\mathbf{k}, t),$$

where G is Newton's gravitational constant. Assuming the source is active within a finite interval $t \in [t_i, t_f]$, the solution can be expressed using Green's function methods. The general solution is:

$$E_{ij}^T(\mathbf{k}, t) = A_{ij}(\mathbf{k}) \sin[k(t - t_f)] + B_{ij}(\mathbf{k}) \cos[k(t - t_f)],$$

with the source-dependent coefficients given by:

$$A_{ij}(\mathbf{k}) = \frac{8\pi G}{k} \int_{t_i}^{t_f} dt \cos[k(t - t_f)] \Pi_{ij}^T(\mathbf{k}, t),$$

$$B_{ij}(\mathbf{k}) = \frac{8\pi G}{k} \int_{t_i}^{t_f} dt \sin[k(t - t_f)] \Pi_{ij}^T(\mathbf{k}, t).$$

The physically relevant observable is the GW energy density, defined as:

$$\rho_{\text{GW}}(t) = \frac{\langle \dot{E}_{ij}^T \dot{E}_{ij}^T \rangle}{8\pi G} = \int_0^\infty |\dot{E}^T(k, t)|^2 d \ln k.$$

This leads to the dimensionless GW abundance per logarithmic frequency interval, expressed as:

$$\Omega_{\text{GW}}(k, t) = \frac{1}{\rho_{\text{tot}}} \frac{d\rho_{\text{GW}}}{d \ln k},$$

where ρ_{tot} is the total energy density of the Universe.

To understand the generation of anisotropic stress, consider the simplest case where two bubbles collide and generate gravitational radiation through a time-varying quadrupole moment [123]. The energy-momentum tensor of a spherically symmetric scalar field configuration describing a bubble is:

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \mathcal{L}(\phi).$$

Since only the first term contributes to anisotropic stress, the spatial components simplify to:

$$T_{ij} = \partial_i \phi \partial_j \phi.$$

In Fourier space, this becomes:

$$T_{ij}(\mathbf{k}, \omega) = \frac{1}{2\pi} \int dt e^{i\omega t} \int d^3x \partial_i \phi \partial_j \phi e^{-i\mathbf{k} \cdot \mathbf{x}}.$$

From the quadrupole moment of the colliding configuration, one can derive the spectral energy distribution of gravitational waves emitted in the collision [123]:

$$\frac{dE}{d\omega} = \frac{32\pi}{15} G \omega^2 |\Delta(\omega)|^2,$$

where the quadrupole source is given by:

$$\Delta(\omega) = T_{zz} - \frac{1}{2}(T_{xx} + T_{yy}).$$

In summary, the nucleation, expansion, and collision of true vacuum bubbles during a first-order phase transition lead to spacetime-dependent variations in the scalar field. These variations source gravitational radiation via the energy-momentum

tensor's anisotropic stress. Furthermore, a significant portion of the energy released during the transition may be transferred to the surrounding plasma, giving rise to turbulent motion or long-lasting sound waves, both of which can also act as sources of a stochastic gravitational wave background. Hence, the three main mechanisms contribute to GW production during a FOPT:

- **Bubble wall collisions**, where expanding scalar field configurations overlap and radiate [123–129].
- **Sound waves** generated in the plasma due to the bubble expansion [130–134].
- **Plasma turbulence**, which stirs the plasma and sustains motions that source GW over extended times [135–139].

While precise predictions require detailed modelling of field and fluid dynamics, valuable insight into the expected GW amplitude can be obtained using dimensional analysis. The key assumption is that the GW energy must be proportional to Newton's constant G , as gravitational radiation arises from the quadrupole moment of the energy-momentum tensor. In addition, the strength of the source is governed by the vacuum energy released during the transition, denoted by ρ_V , and the fraction κ of that energy converted into anisotropic stresses that can radiate GW. The expansion and collisions of bubbles are controlled by the bubble wall velocity v_w , while the overall timescale of the transition is characterised by the inverse of the parameter β , which represents the typical rate of bubble nucleation. Assuming no other relevant mass scales, the total energy density in GW must scale as [140, 141]:

$$E_{\text{GW}} \sim G \kappa^2 v_w^3 \rho_V^2 \beta^{-5}, \quad (2.90)$$

which reflects that GW production is enhanced by efficient energy transfer (through κ), and favoured by slow transitions (small β) that allow coherent GW generation over large volumes. For comparison, the total vacuum energy released during the transition scales as:

$$E_V \sim \rho_V v_w^3 \beta^{-3}. \quad (2.91)$$

The ratio between these gives the GW efficiency:

$$r \equiv \frac{E_{\text{GW}}}{E_V} \sim G \kappa^2 \rho_V \beta^{-2}. \quad (2.92)$$

We now relate this to latent heat parameters by introducing the strength of the transition, $\alpha = \frac{\rho_V}{\rho_R}$, where ρ_R is the radiation energy density at the transition temperature. Using the Friedmann equation $H^2 \sim G\rho_{\text{tot}}$, and expressing $\rho_V/\rho_{\text{tot}} = \alpha/(1+\alpha)$, the above ratio becomes:

$$r \sim \kappa^2 \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{H}{\beta} \right)^2. \quad (2.93)$$

This estimate implies that the efficiency of GW generation is enhanced for stronger phase transitions (large α), more efficient energy conversion (large κ), and slower transitions (small β/H). These three parameters α , κ , and β/H serve as key benchmarks for evaluating the detectability of cosmological GW signals. Finally, the GW energy density amplitude today, expressed per logarithmic frequency interval, can be estimated as:

$$\Omega_{\text{GW}}(f) \sim \frac{1}{\rho_{\text{tot}}} \frac{d\rho_{\text{GW}}}{d \ln f} \sim \kappa^2 \left(\frac{\alpha}{1+\alpha} \right)^2 \left(\frac{H}{\beta} \right)^2, \quad (2.94)$$

This framework provides an intuitive understanding of the main parametric dependencies of the GW signal and is widely used to guide experimental forecasts and model-building efforts.

2.1.7.1 Bubble Collisions

One of the primary sources of anisotropic stress during a first-order phase transition arises from the collision of expanding bubbles, which breaks the spherical symmetry of the initially isolated bubble walls. This disruption generates anisotropies in the energy-momentum tensor, making bubble collisions a significant contributor to gravitational wave production. Analytically, this source is often modelled using the envelope approximation, which assumes that only the uncollided portions of the bubble walls contribute to the gravitational wave signal. This is typically combined with the thin-wall approximation, wherein the bubble walls are treated as infinitesimally thin surfaces. Although originally formulated to describe vacuum transitions, where no thermal plasma is present to slow down the expanding bubble walls, the envelope approximation has been shown to yield reasonably accurate predictions even in the presence of a thermal medium [125]. For a more refined treatment that

accounts for thick-walled bubble profiles, see Ref. [142]. The theoretical understanding of gravitational waves from bubble collisions with an analytic derivation of the GW spectrum is presented in Ref.[123, 124, 126, 128]. Apart from these, most estimates relied on numerical simulations, such as those in Ref.[125, 143], which were widely referenced in earlier literature [144]. The analytic results rely on both the envelope and thin-wall approximations, offering an efficient and physically intuitive framework for estimating the gravitational wave signal from such early Universe processes. Following Ref. [145], GW spectrum for bubble collision can be written as

$$\Omega_\phi h^2 = 1.67 \times 10^{-5} \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{H_*}{\beta} \right)^2 \left(\frac{\kappa_\phi \alpha_*}{1 + \alpha_*} \right)^2 \frac{0.11 v_w^3}{0.42 + v_w^2} \frac{3.8 (f/f_{\text{peak}}^{\text{PT},\phi})^{2.8}}{1 + 2.8 (f/f_{\text{peak}}^{\text{PT},\phi})^{3.8}}, \quad (2.95)$$

where the peak frequency [145] is

$$f_{\text{peak}}^{\text{PT},\phi} = 1.65 \times 10^{-5} \text{Hz} \left(\frac{g_*}{100} \right)^{1/6} \left(\frac{T_n}{100 \text{ GeV}} \right) \frac{0.62}{1.8 - 0.1 v_w + v_w^2} \left(\frac{\beta}{H_*} \right). \quad (2.96)$$

The Hubble expansion parameter at nucleation temperature is denoted as $H_* = H(T_n)$ and $\alpha_* = \alpha|_{T=T_n}$. The efficiency factor κ_ϕ for bubble collision can be expressed as [146]

$$\kappa_\phi = \frac{1}{1 + 0.715 \alpha_*} \left(0.715 \alpha_* + \frac{4}{27} \sqrt{3 \alpha_*/2} \right). \quad (2.97)$$

2.1.7.2 Sound Waves

Relativistic sound waves, corresponding to the coherent bulk motion of the plasma, can carry significant energy and act as an important source of gravitational waves. This energy contributes directly to the energy-momentum tensor and, hence, to spacetime curvature, leading to GW generation. In the case of first-order phase transitions, the energy and pressure contained in the expanding bubble walls are transmitted into the surrounding plasma. After nucleation and during bubble collisions, this energy is converted into bulk fluid motion that propagates as sound waves. The possibility that sound waves produced during FOPT could source gravitational radiation was first proposed nearly four decades ago [146–148], though only in the past decade has their role been investigated in detail [130, 131, 134, 139, 149, 150].

Sound waves are now recognised as the dominant source of GW in many scenarios, particularly when the bubble walls interact strongly with the surrounding plasma. In such scenarios, the energy transferred to the plasma is efficiently converted into sound waves, which continue to source GW. Unlike the envelope approximation, which neglects overlapping regions of colliding bubbles, a more realistic picture reveals that these regions develop into freely propagating sound waves, sustaining anisotropic stresses in the fluid for a longer duration [130, 150]. As a result, the GW signal from sound waves gains a significant enhancement. The gravitational wave amplitude from sound waves depends on multiple timescales, including the sound wave lifetime τ_{sw} , and the autocorrelation time τ_c , which characterises the timescale over which the anisotropic stress from fluid motion decorrelates. A general expression for the energy density of GW sourced by sound waves can be written as [134, 150, 151]:

$$\Omega_{\text{sw}} \sim K^2 (H_* \tau_{\text{sw}}) (H_* \tau_c) S_{\text{sw}}(k), \quad (2.98)$$

where K denotes the kinetic energy fraction of the fluid and $S_{\text{sw}}(k)$ provides shape of the spectrum. This quantity can be extracted from hydrodynamic simulations and is commonly expressed as $K = \Gamma U_f^2$, with Γ representing the average adiabatic index of the plasma and U_f the root-mean-square fluid velocity. This formulation makes explicit the dependence of the GW amplitude on the efficiency and persistence of the sound wave source. The GW spectrum produced from the sound wave in the plasma can be written as [132, 145, 151]

$$\Omega_{\text{sw}} h^2 = 2.65 \times 10^{-6} \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{H_*}{\beta} \right) \left(\frac{\kappa_{\text{sw}} \alpha_*}{1 + \alpha_*} \right)^2 v_w (f/f_{\text{peak}}^{\text{PT,sw}})^3 \left(\frac{7}{4 + 3(f/f_{\text{peak}}^{\text{PT,sw}})^2} \right)^{7/2} \Upsilon, \quad (2.99)$$

and the corresponding peak frequency is given by [145]

$$f_{\text{peak}}^{\text{PT,sw}} = 1.65 \times 10^{-5} \text{Hz} \left(\frac{g_*}{100} \right)^{1/6} \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{\beta}{H_*} \right) \frac{2}{\sqrt{3}}. \quad (2.100)$$

The efficiency factor for sound wave can be expressed as [152]

$$\kappa_{\text{sw}} = \frac{\sqrt{\alpha_*}}{0.135 + \sqrt{0.98 + \alpha_*}}. \quad (2.101)$$

The suppression factor $\Upsilon = 1 - \frac{1}{\sqrt{1+2\tau_{sw}H_*}}$ depends on the lifetime of sound wave τ_{sw} [132] given by $\tau_{sw} \sim R_*/\bar{U}_f$ where mean bubble separation is $R_* = (8\pi)^{1/3}v_w\beta^{-1}$ and rms fluid velocity is $\bar{U}_f = \sqrt{3\kappa_{sw}\alpha_*/4}$.

2.1.7.3 Turbulence

Following bubble collisions during a first-order phase transition, turbulence may develop in the plasma as large amounts of energy are injected on characteristic length scales. In the early Universe, small shocks can evolve into turbulent flow [136]. The turbulence is connected to the strength of the transition, with the shock timescale inversely proportional to the transition strength. Turbulence is typically described by Kolmogorov's theory [153], in which energy cascades from large distances (of the order of the bubble radius) down to smaller scales until it dissipates via viscous heating. The duration of stirring is limited to the timescale of the phase transition. Moreover, relativistic effects and causal horizons introduce deviations from the standard classical picture of homogeneous and isotropic turbulence [135, 136]. Although the GW signal from turbulence is typically subdominant compared to that from sound waves, turbulent motions can still influence the GW spectrum by transferring energy across scales. While modern simulations and analytical models continue to improve, many widely used fits in the literature, such as those from Ref. [136], remain valuable for estimating the GW contribution from plasma turbulence. The GW spectrum generated by the turbulence in the plasma is given by [145]

$$\Omega_{\text{turb}}h^2 = 3.35 \times 10^{-4} \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{H_*}{\beta} \right) \left(\frac{\kappa_{\text{turb}}\alpha_*}{1 + \alpha_*} \right)^{3/2} v_w \frac{(f/f_{\text{peak}}^{\text{PT,turb}})^3}{(1 + f/f_{\text{peak}}^{\text{PT,turb}})^{11/3} (1 + 8\pi f/h_*)}, \quad (2.102)$$

with the peak frequency as [145]

$$f_{\text{peak}}^{\text{PT,turb}} = 1.65 \times 10^{-5} \text{Hz} \left(\frac{g_*}{100} \right)^{1/6} \left(\frac{T_n}{100 \text{ GeV}} \right) \frac{3.5}{2} \left(\frac{\beta}{H_*} \right). \quad (2.103)$$

The efficiency factor for turbulence is $\kappa_{\text{turb}} \simeq 0.1\kappa_{\text{sw}}$ and the inverse Hubble time at the epoch of GW emission, redshifted to today is

$$h_* = 1.65 \times 10^{-5} \frac{T_n}{100 \text{ GeV}} \left(\frac{g_*}{100} \right)^{1/6}. \quad (2.104)$$

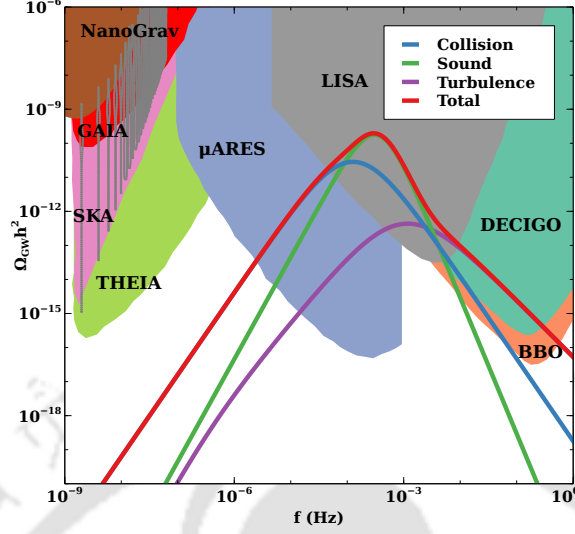


Figure 2.7: Gravitational wave spectrum corresponds to the individual contributions from different sources associated with FOPT, as well as their total contribution. The parameters are fixed at $\alpha_* = 0.5$, $\beta/H_* = 50$, and nucleation temperature $T_n = 60$ GeV.

In Fig. 2.7, we show the gravitational wave spectra for the individual contributions from a FOPT, along with the total spectrum for a specific choice of parameters. The shaded regions correspond to the sensitivities of different experiments namely, NANOGrav [79], SKA [154], THEIA [155], GAIA [155], μ ARES [156], LISA [157], DECIGO [87] and BBO [88] while the grey coloured violin-shaped points correspond to NANOGrav 15-year data [79]. As evident from both the above expressions and Fig. 2.7, the sound wave contribution is the most dominant, and the peak of the total GW spectrum coincides with the peak frequency of the sound wave contribution.

2.2 Gravitational Waves from Topological Defects: Domain Walls

As discussed above in the context of cosmological first-order phase transitions, gravitational waves provide a novel observational window into symmetry-breaking events in the early Universe. Among other possible sources, topological defects formed during such transitions, particularly domain walls [53, 158], can lead to distinct GW signals. These defects typically emerge when a discrete symmetry is spontaneously

broken, resulting in energetically degenerate but physically distinct vacua separated by surfaces of localised energy density. While stable domain walls are cosmologically problematic due to their scaling behaviour and energy domination, their eventual annihilation often triggered by a bias in the potential [159–162] can source a stochastic GW background detectable by current or future GW observatories. In this section, we examine the formation, evolution, and annihilation of domain walls and the associated GW production in the context of physics beyond the Standard Model. We begin by briefly outlining the dynamics of domain walls, followed by a discussion on how they give rise to a stochastic gravitational wave background (SGWB) potentially detectable by current and future gravitational wave observatories.

2.2.1 Discrete Symmetry Breaking and Domain Wall Formation

Domain walls arise naturally in field theories where a discrete symmetry is spontaneously broken. Consider a real scalar field ϕ with a potential of the form:

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2, \quad (2.105)$$

which possesses a $\mathbb{Z}_2$ symmetry: $\phi \rightarrow -\phi$. At high temperatures in the early Universe, the effective potential is minimised at $\phi = 0$. As the Universe cools and undergoes a phase transition, the field acquires a non-zero VEV, choosing one of the two degenerate minima, $\phi = \pm v$. Due to causality, different regions of space make independent choices, and the boundaries between regions with different vacua correspond to domain walls which are two dimensional topological defects with surface energy density. Assuming a static planar domain wall configuration oriented perpendicular to the x -axis in flat Minkowski spacetime, we consider the field profile $\phi(x)$. The classical equation of motion for the scalar field admits an exact analytical solution given by:

$$\phi(x) = v \tanh \left(\sqrt{\frac{\lambda}{2}} v x \right). \quad (2.106)$$

This solution represents a kink centred at $x = 0$, where the field smoothly interpolates from $\phi = -v$ as $x \rightarrow -\infty$ to $\phi = +v$ as $x \rightarrow +\infty$ shown in Fig. 2.8. The surface energy density (or tension) of the domain wall is determined by integrating

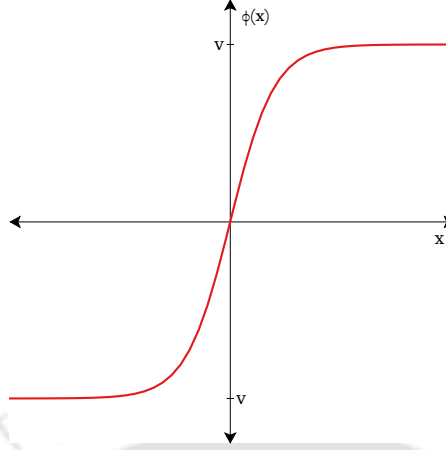


Figure 2.8: Domain wall field solution

the energy density across the wall:

$$\sigma = \int_{-\infty}^{\infty} dx \left[\frac{1}{2} \left(\frac{d\phi}{dx} \right)^2 + V(\phi(x)) \right], \quad (2.107)$$

which evaluates to:

$$\sigma = \frac{2\sqrt{2}}{3} v^3 \sqrt{\lambda}. \quad (2.108)$$

Hence, the domain wall tension is set by the symmetry-breaking scale v and the quartic coupling λ .

2.2.2 Dynamics and Evolution of Domain Walls

Following formation, domain walls evolve in the expanding Universe. Their dynamics are governed by a competition between tension driven collapse and Hubble expansion. Without any explicit symmetry-breaking term, domain walls tend to enter a scaling regime in which their energy density evolves as:

$$\rho_{\text{DW}} \propto \frac{\sigma}{t}, \quad (2.109)$$

which decreases more slowly than radiation ($\propto t^{-2}$) or matter ($\propto t^{-3/2}$). Consequently, if domain walls are stable, they eventually dominate the energy density of the Universe, leading to severe cosmological constraints. Although domain walls may constitute only a negligible fraction of the present-day energy density, their presence

can still induce significant isocurvature fluctuations during the recombination epoch if their surface tension exceeds $\mathcal{O}(\text{MeV}^3)$. This imposes a strong upper bound on the wall tension, commonly referred to as the Zel'dovich-Kobzarev-Okun (ZKO) bound [163], to ensure compatibility with precision cosmological data. So, stable domain walls pose serious cosmological challenges, particularly due to their potential to overclose the Universe or generate large anisotropies in the Cosmic Microwave Background (CMB). One viable resolution is to make the domain wall network unstable by introducing a small energy bias between the degenerate vacua [161]. This allows the walls to eventually decay, thereby avoiding long-term cosmological consequences. It is crucial to emphasise that while a bias is necessary for the decay of domain walls, an excessively large bias [159] from the outset would prevent their formation altogether by removing the vacuum degeneracy before the phase transition. In the presence of an explicit symmetry-breaking term, the energy difference between the two vacua, denoted by V_{bias} , determines this effect. If V_{bias} is too large, the system will favour the lower energy vacuum directly, thereby suppressing the formation of extended false vacuum domains and preventing domain wall formation. Therefore, a hierarchy between the symmetry-breaking scale and the bias term is required to ensure that domain walls form during spontaneous symmetry breaking and subsequently decay as the Universe evolves. To quantify this effect, consider the probabilities p_+ and p_- that the scalar field ends up in the $\phi = +v$ and $\phi = -v$ vacua, respectively, after the phase transition. The ratio of these probabilities can be expressed as [164]

$$\frac{p_-}{p_+} = \exp\left(-\frac{\Delta F}{T}\right), \quad (2.110)$$

where $\Delta F = V_{\text{bias}} \xi^3$ is the difference in free energy between the two vacua, ξ is the correlation length at the time of the transition, and T is the temperature during the phase transition. Following the Ginzburg criterion, the Ginzburg temperature can be estimated as $T \simeq V_0 \xi^3$, where V_0 is the height of the potential barrier separating the minima [159]. Substituting this into the probability ratio gives

$$\frac{p_-}{p_+} \simeq \exp\left(-\frac{V_{\text{bias}}}{V_0}\right). \quad (2.111)$$

This relation shows that any non-zero V_{bias} introduces an asymmetry in the spatial distribution of the two vacua after the phase transition. From percolation theory, the

appearance of an infinite cluster of a given vacuum in a three-dimensional cubic lattice requires that its occupation probability exceeds a critical value $p_c \simeq 0.311$ [165]. For the false vacuum (p_-) to percolate, we require $p_- > p_c$, which together with the normalisation $p_+ + p_- = 1$ implies

$$\frac{V_{\text{bias}}}{V_0} < \ln \left(\frac{1 - p_c}{p_c} \right) \simeq 0.795. \quad (2.112)$$

This result provides a robust, model-independent *upper bound* on the bias term: if V_{bias} exceeds roughly $0.8 V_0$, the false vacuum cannot form a percolating network, and domain walls will not be created during the phase transition. A soft symmetry-breaking term can be introduced to generate a vacuum energy bias. In the case of a softly broken $\mathbb{Z}_2$ symmetry, such a bias term can be written as [164, 166]:

$$\Delta V(\phi) = \epsilon v \phi \left(\frac{1}{3} \phi^2 - v^2 \right), \quad (2.113)$$

where ϵ is a dimensionless parameter characterising the strength of the explicit breaking. Although the potential still possesses minima at $\phi = \pm v$, the degeneracy between them is lifted. The resulting energy difference between the two vacua is given by:

$$V_{\text{bias}} = V(-v) - V(+v) = \frac{4}{3} \epsilon v^4. \quad (2.114)$$

This vacuum energy difference induces a pressure difference across the domain walls, known as the volume pressure, estimated as $p_V \sim V_{\text{bias}}$. This pressure drives the shrinkage of regions in the false vacuum. The domain walls collapse when this pressure exceeds the restoring force due to surface tension, which can be approximated as $p_T \sim \sigma \mathcal{A}/t$, where $\mathcal{A}$ is the area parameter [167]. Equating the two forces provides an estimate for the annihilation time of the domain wall network:

$$t_{\text{ann}} = C_{\text{ann}} \frac{\sigma \mathcal{A}}{V_{\text{bias}}}, \quad (2.115)$$

where C_{ann} is a numerical factor determined by the specific dynamics of the wall evolution. Assuming domain wall annihilation occurs during the radiation-dominated era, the corresponding temperature at the time of decay, t_{ann} , can be estimated by equating the volume pressure from the vacuum energy bias with the tension pressure of the wall. This leads to the following expression for the annihilation

temperature [164]:

$$T_{\text{ann}} = 3.41 \times 10^{-2} \text{ GeV} \times C_{\text{ann}}^{-1/2} \mathcal{A}^{-1/2} \left(\frac{g_*(T_{\text{ann}})}{10} \right)^{-1/4} \left(\frac{\sigma}{\text{TeV}^3} \right)^{-1/2} \left(\frac{V_{\text{bias}}}{\text{MeV}^4} \right)^{1/2}, \quad (2.116)$$

where $g_*(T_{\text{ann}})$ is the effective number of relativistic degrees of freedom at T_{ann} . From Eq. 2.115, it is evident that the lifetime t_{ann} scales inversely with V_{bias} . A sufficiently small energy bias allows domain walls to persist for an extended period. However, constraints on energy injection during the BBN epoch [168, 169] require the lifetime to satisfy $t_{\text{ann}} < 0.01 \text{ sec}$, assuming that a significant fraction of the domain wall energy density is converted into energetic particles. This requirement imposes a lower bound on the magnitude of the energy bias as defined below [164]

$$V_{\text{bias}}^{1/4} > 5.07 \times 10^{-4} \text{ GeV} C_{\text{ann}}^{1/4} \mathcal{A}^{1/4} \left(\frac{\sigma}{\text{TeV}^3} \right)^{1/4}. \quad (2.117)$$

2.2.3 Gravitational Wave Production from Domain Wall Collapse

The annihilation of domain walls is a non-equilibrium process that can generate a stochastic gravitational wave background. The GW spectrum at a given time t is characterised by the energy density per logarithmic frequency interval normalised to the critical energy density:

$$\Omega_{\text{GW}}(t, f) = \frac{1}{\rho_c(t)} \frac{d\rho_{\text{GW}}(t)}{d \ln f}, \quad (2.118)$$

where ρ_{GW} is the GW energy density and f is the frequency. The GW spectrum generated from domain wall annihilation typically exhibits a broken power-law shape. The peak frequency is determined by the Hubble scale at the time of annihilation, while the peak amplitude is governed by the energy stored in the domain wall network and the efficiency of gravitational wave emission. Assuming instantaneous annihilation at $t = t_{\text{ann}}$, the peak amplitude of the GW spectrum at that time is [167, 170]:

$$\Omega_{\text{GW}}(t_{\text{ann}})_{\text{peak}} = \frac{1}{\rho_c(t_{\text{ann}})} \left(\frac{d\rho_{\text{GW}}}{d \ln f} \right)_{\text{peak}} = \frac{8\pi \tilde{\epsilon}_{\text{GW}} G^2 \mathcal{A}^2 \sigma^2}{3H^2(t_{\text{ann}})}, \quad (2.119)$$

where $\tilde{\epsilon}_{\text{GW}}$ is the efficiency parameter of GW emission (estimated to be $\tilde{\epsilon}_{\text{GW}} \simeq 0.7 \pm 0.4$ from simulations), G is Newton's gravitational constant, and $H(t_{\text{ann}})$ is the Hubble rate at the annihilation epoch. Once the domain wall network enters the scaling regime, their typical size becomes comparable to the Hubble radius, i.e., $R \sim t$, and the energy density evolves as $\rho_{\text{DW}} \propto t^{-1}$. Redshifting the GW amplitude to the present time t_0 , the observed peak energy density becomes:

$$\Omega_{\text{GW}} h^2 (t_0)_{\text{peak}} = 7.2 \times 10^{-18} \tilde{\epsilon}_{\text{GW}} \mathcal{A}^2 \left(\frac{\sigma}{\text{TeV}^3} \right)^2 \left(\frac{T_{\text{ann}}}{10^{-2} \text{ GeV}} \right)^{-4}. \quad (2.120)$$

The corresponding redshifted peak frequency today is given by:

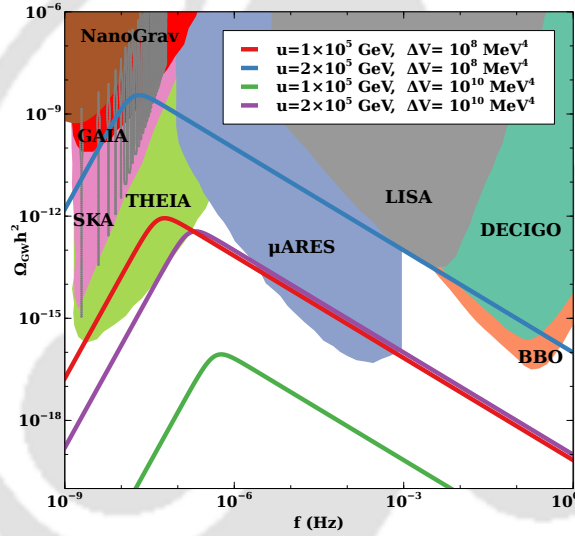


Figure 2.9: Gravitational wave spectrum from domain wall annihilation for some choice of u (VEV) and bias term ΔV .

$$f_{\text{peak}} \simeq \left(\frac{a(t_{\text{ann}})}{a(t_0)} \right) H(t_{\text{ann}}) \simeq 1.1 \times 10^{-9} \text{ Hz} \left(\frac{T_{\text{ann}}}{10^{-2} \text{ GeV}} \right). \quad (2.121)$$

To model the full GW spectrum, one can adopt a phenomenological broken power-law form [151]:

$$\Omega_{\text{GW}} h^2(f) = \Omega_{\text{GW}} h^2_{\text{peak}} \frac{(a+b)^c}{(bx^{-a/c} + ax^{b/c})^c}, \quad (2.122)$$

where x defined as f/f_{peak} , the parameters a , b , and c are all positive real values. Causality enforces $a = 3$, while $b \simeq c \simeq 1$ is supported by numerical results [167]. In Fig. 2.9, we present the gravitational wave spectrum generated from domain wall

annihilation for different values of the bias term and wall tension (expressed in terms of the VEV).

2.3 Gravitational Waves from Primordial Black Holes

In this section, we first explore the formation of primordial black holes, followed by a discussion of their gravitational wave signatures. Unlike black holes formed from the collapse of massive stars, PBH are thought to originate in the early Universe through non-stellar mechanisms. The idea was first proposed by Zeldovich, Novikov, and Hawking [171, 172], and it was soon recognised that PBH could potentially account for a fraction or even the entirety of the dark matter content [65, 67, 173]. Beyond their role as dark matter, PBH have also been considered as sources of initial inhomogeneities for cosmic structure formation [66] and as possible seeds for the formation of supermassive black holes [174]. On the other hand, low mass PBH can have interesting cosmological roles due to its evaporation during the evolution of the Universe [175].

2.3.1 PBH formation

PBH may originate from a variety of early Universe phenomena, including inflationary fluctuations [67, 171, 176–178], dynamics of first-order phase transitions [68, 69, 179–189], and the gravitational collapse of topological defects such as domain walls or cosmic strings [190, 191]. In this thesis, we focus specifically on those formation scenarios that are linked to a first-order phase transition. Such transitions can give rise to PBH through several distinct pathways, including:

2.3.1.1 Bubble Collisions and Overdensities

During a first-order cosmological phase transition, quantum tunnelling proceeds stochastically, creating a nonzero probability that an entire Hubble region remains

trapped in the false vacuum longer than its surroundings. As the Universe expands, radiation energy density dilutes rapidly, while the false vacuum energy remains largely constant. Consequently, these delayed-decay patches accumulate an overdensity compared to neighbouring regions. Once the vacuum finally decays, these overdense pockets can undergo gravitational collapse, leading to the formation of primordial black holes [192–195]. Numerical simulations by Liu et al. [68] show that this mechanism leads to a nearly monochromatic PBH mass spectrum and naturally relates the abundance of PBH to the gravitational wave signal expected from the same transition. The collision of bubble walls can trap regions of false vacuum, forming high-energy-density pockets. After bubble percolation, these trapped regions can gravitationally collapse if the local overdensity exceeds a threshold: $\delta\rho/\rho \gtrsim \delta_c \sim 0.3 - 0.5$ [196–198]. Such trapped false vacuum domains can thus collapse into PBH.

2.3.1.2 Collapse of Fermi Balls

Another mechanism for primordial black hole formation during a first-order phase transition involves the creation and subsequent collapse of non-topological solitons known as Fermi-balls [69, 199]. In this scenario, a fermion species acquires a large mass in the true vacuum and becomes trapped in remnants of the false vacuum after bubble nucleation. These trapped fermions form high-density Fermi-balls when the false vacuum regions shrink. A short range attractive Yukawa interaction among the fermions eventually destabilises these objects, causing them to collapse into PBH [69, 184, 200]. This model offers a minimal and natural pathway from FOPT dynamics to PBH driven entirely by heavy fermion trapping and soliton collapse.

Hence, these mechanisms responsible for PBH formation are inherently connected to gravitational wave production.

2.3.2 Gravitational Wave Production from PBH

PBH can generate the stochastic gravitational wave background through multiple physical processes. These include: (a) emission of gravitons via Hawking radiation, which leads to ultra-high frequency gravitational waves [201]; (b) gravitational wave

bursts from PBH binary mergers, particularly relevant for LIGO-VIRGO-KAGRA type observations [202]; (c) second-order gravitational waves sourced by large primordial scalar perturbations that led to PBH formation during inflation [203]; and (d) gravitational waves induced by spatial inhomogeneities in PBH distribution after their formation, due to scalar density fluctuations [204–206]. In this thesis, we focus on the last mechanism, gravitational wave production due to PBH induced scalar density fluctuations in the post-formation epoch. This scenario has several appealing features. Firstly, the resulting gravitational waves fall within the sensitivity bands of upcoming space-based interferometers and currently operational pulsar timing array (PTA) experiments, making them potentially observable in the near future. Secondly, this mechanism is largely independent of the specific details of the PBH formation process, relying instead on the generic outcome of PBH clustering and the resultant matter inhomogeneities. Most importantly, the gravitational wave production from such scalar inhomogeneities is closely linked to an early matter-dominated (EMD) phase induced by PBH domination. This makes the resulting SGWB a unique probe of early matter domination driven by PBH. Once PBH form in the early Universe, they behave as non-relativistic matter. If they become sufficiently abundant, they can come to dominate the total energy density, triggering a phase of matter domination prior to the Big Bang Nucleosynthesis (BBN).

After formation, the PBH are not uniformly distributed across space; their number density follows the imprint of the primordial scalar fluctuations. These fluctuations source anisotropic stress at second order, which in turn generates gravitational waves. The efficiency of this production mechanism is enhanced during the matter-dominated era, since scalar perturbations grow linearly with the scale factor in this phase, in contrast to the frozen super-horizon behaviour during radiation domination. The induced gravitational wave spectrum typically exhibits a peak frequency determined by the PBH mass and abundance, and its amplitude depends on the nature of the fluctuation spectrum. The present-day gravitational wave energy density can be expressed as [204]

$$\Omega_{\text{GW},0}(k) h^2 = 0.39 \left(\frac{g_*(T_c)}{106.75} \right)^{-1/3} \Omega_{r,0} h^2 \Omega_{\text{GW},c}(k), \quad (2.123)$$

where $\Omega_{\text{GW},c}(k)$ is the gravitational wave energy density evaluated at a characteristic

temperature T_c during the radiation-dominated epoch after PBH evaporation, when the GW spectrum becomes constant. Since gravitational waves redshift like radiation, the amplitude scales with the radiation energy density. The present radiation energy density is $\Omega_{r,0}h^2 \simeq 4.18 \times 10^{-5}$ [7]. The dominant contribution to $\Omega_{\text{GW},c}(k)$ arises from modes with $k \gg k_{\text{eq}}$, where k_{eq} denotes the scale that enters the horizon when PBH energy density becomes equal to that of radiation. This is because the GW amplitude is proportional to $k\tau_{\text{evap}} \sim k/k_{\text{evap}}$, enhancing contributions from small scales. The UV cutoff scale k_{UV} is related to the PBH evaporation scale k_{evap} by [207]

$$\frac{k_{\text{UV}}}{k_{\text{evap}}} \simeq 2.3 \times 10^6 \left(\frac{g_{*,H}(T_{\text{BH}})}{108} \right)^{-1/3} \left(\frac{10^4 \text{ g}}{m_{\text{in}}} \right)^{-2/3}, \quad (2.124)$$

where m_{in} is the initial PBH mass and $g_{*,H}(T_{\text{BH}})$ is the effective number of relativistic degrees of freedom at the PBH temperature, T_{BH} . Furthermore, the ratio of the equality scale to the ultraviolet cutoff can be expressed as $\frac{k_{\text{eq}}}{k_{\text{UV}}} \simeq \sqrt{2} \gamma^{1/3} \beta_{\text{PBH}}^{2/3}$, where γ denotes the ratio of the primordial black hole mass to the horizon mass at the time of formation [207]. Even for $m_{\text{in}} \sim 1 \text{ g}$, this enhancement factor can reach $\sim \mathcal{O}(10^3 - 10^4)$. Near the resonance at $k \sim k_{\text{UV}}$, the GW energy density is approximately [207]

$$\Omega_{\text{GW},c}^{\text{res}}(k \sim k_{\text{UV}}) \simeq \frac{1}{24576 \pi 2^{1/3} \sqrt{3}} \left(\frac{k_{\text{UV}}}{k_{\text{evap}}} \right)^{17/3} \left(\frac{k_{\text{eq}}}{k_{\text{UV}}} \right)^8. \quad (2.125)$$

This can be rewritten in terms of the PBH parameters β_{PBH} and m_{in} as [207]

$$\Omega_{\text{GW},c}^{\text{res}}(k \sim k_{\text{UV}}) \simeq 10^{30} \beta_{\text{PBH}}^{16/3} \left(\frac{\gamma}{0.2} \right)^{8/3} \left(\frac{g_{*,H}(T_{\text{BH}})}{108} \right)^{-17/9} \left(\frac{10^4 \text{ g}}{m_{\text{in}}} \right)^{-34/9}, \quad (2.126)$$

where β_{PBH} is the PBH energy fraction at the time of formation. Clearly, the amplitude is significantly enhanced unless β_{PBH} is extremely small. The observed GW spectrum near its peak can be approximated by [204]

$$\Omega_{\text{GW}}(t_0, f) \simeq \Omega_{\text{GW}}^{\text{peak}} \left(\frac{f}{f_{\text{peak}}} \right)^{11/3} \Theta(f_{\text{peak}} - f), \quad (2.127)$$

where the peak amplitude is given by

$$\Omega_{\text{GW}}^{\text{peak}} \simeq 1.19 \times 10^{-2} \left(\frac{\beta_{\text{PBH}}}{10^{-8}} \right)^{16/3} \left(\frac{m_{\text{in}}}{10^8 \text{ g}} \right)^{34/9}, \quad (2.128)$$

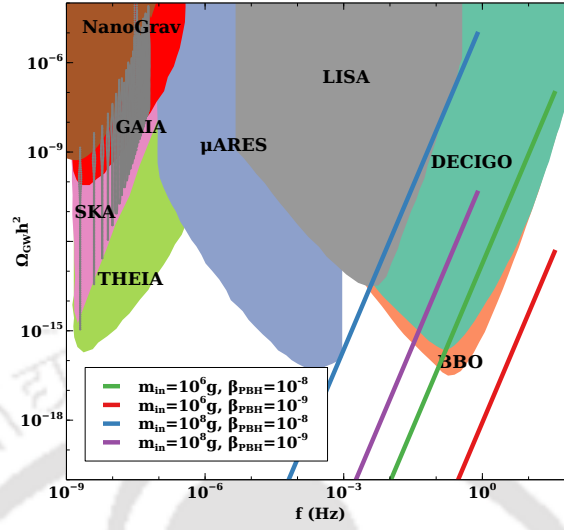


Figure 2.10: Gravitational wave spectrum from density fluctuations during PBH evaporation for different PBH initial masses and initial energy fractions.

and the peak frequency corresponding to k_{UV} is

$$f_{\text{peak}} \simeq 0.79 \text{ Hz} \left(\frac{g_{*,H}(T_{\text{BH}})}{108} \right)^{1/6} \left(\frac{g_*(T_{\text{evap}})}{106.75} \right)^{1/4} \left(\frac{g_{*,s}(T_{\text{evap}})}{106.75} \right)^{-1/3} \left(\frac{10^8 \text{ g}}{m_{\text{in}}} \right)^{5/6}. \quad (2.129)$$

These results highlight that gravitational waves sourced by PBH-induced density perturbations during an early matter-dominated era can lead to a sizeable stochastic background, with characteristic peak frequencies accessible to future detectors. The gravitational wave spectrum generated during PBH evaporation via density fluctuations is shown in Fig. 2.10 for different PBH parameters. Both the amplitude and the spectral shape exhibit strong sensitivity to the PBH parameters, making such signals valuable probes of early-Universe cosmology. It can be observed that the peak frequency depends solely on the initial mass of the PBH, whereas the amplitude is determined by both the initial PBH mass and the energy fraction.

2.3.3 Cosmological Constraints and Observational Signatures

PBH formed from early FOPT are subject to various constraints, depending on their mass:

- **Evaporation (Hawking Radiation):** Light PBH ($M \lesssim 10^{15}$ g) evaporate via Hawking radiation [175] within the age of the Universe. Their emitted products, such as photons [208], neutrinos [70], etc., contribute to cosmic backgrounds which constrain their mass.
- **Dark Matter Candidates:** PBH in the mass range $\sim 10^{17} - 10^{23}$ g are viable dark matter candidates. FOPT that generate the correct abundance can provide an alternative to particle dark matter.
- **Microlensing Surveys:** PBH with masses $\gtrsim 10^{24}$ g are constrained by microlensing data from EROS [209], OGLE [210], and Subaru-HSC [71, 211, 212].
- **Gravitational Waves:** The same FOPT that produces PBH also generates stochastic gravitational waves. On the other hand, massive primordial black holes can generate their own gravitational wave events, such as mergers [73, 74]. A correlated detection of both signals would offer strong evidence of early-universe FOPT.

2.4 Detectability

Detecting GW from above sources requires the signal to be strong enough and fall within the sensitivity band of detectors. Several upcoming and proposed experiments aim to probe the stochastic GW background from the early Universe, including:

- The space-based gravitational wave observatories, such as **LISA**[157] and **μ ARES**[156], are engineered to explore gravitational wave signals within the frequency band of 10^{-6} Hz to 0.4 Hz.
- The planned detectors **DECIGO**[87] and **BBO**[88] are expected to be sensitive to gravitational waves within the 10^{-3} to 10 Hz frequency range.
- **Einstein Telescope** [213] and **Cosmic Explorer** [214] are ground-based detectors optimised for astrophysical sources but potentially sensitive to high-frequency (kHz) cosmological signals.

- **NANOGrav** [79], **SKA** [154], **THEIA** [155], and **GAIA** [155] operate in the nanohertz regime, making them suitable for detecting ultra-low-frequency gravitational waves.
- The detectability of a stochastic gravitational wave background in the frequency range of ground-based interferometers such as **LIGO** [215], **VIRGO** [216], and **KAGRA** [217] depends on both the predicted spectral shape and amplitude in comparison to the detector sensitivity curves. The LVK network operates optimally in the frequency band $\mathcal{O}(10\text{--}10^2)$ Hz. While its ongoing observations are able to constrain new physics scenarios like FOPT [218, 219], PBH [73, 74, 220] and domain wall [221, 222].

2.5 Objective of the thesis

This thesis addresses two longstanding puzzles in cosmology and particle physics: the origin of baryon asymmetry and dark matter in the Universe, through a few frameworks with detectable gravitational waves arising from symmetry-breaking phenomena in the early Universe. The persistent null results from dark matter searches, including direct detection experiments, indirect search experiments, collider-based production attempts, and inability to directly probe high-scale mechanisms related to baryogenesis, strongly motivate the pursuit of complementary, non-conventional detection avenues. By probing energy scales and dynamics inaccessible to traditional particle detectors, gravitational wave astronomy provides a novel and promising observational window into the mechanisms responsible for dark matter genesis and baryogenesis. Particular emphasis is placed on the role of FOPT, where the dynamics of spontaneous symmetry breaking is not only responsible for mass generation but also leave imprints in the form of stochastic GW backgrounds. Since particle number densities are exponentially sensitive to their masses, the mechanism of mass generation is important for baryo(lepto)genesis. Simultaneously, the relic abundance of dark matter is also significantly affected by the thermal and dynamical history governed by such transitions. In scenarios involving a FOPT, the associated bubble nucleation and expansion generate unique GW signatures, with peak frequencies set by the symmetry-breaking scale, ranging from the sensitivity of space-based interferometers to pulsar timing arrays. In the early Universe, the breaking of a discrete

symmetry can result in the formation of domain walls, whose subsequent annihilation generates gravitational wave signatures that may serve as probes of specific models of baryon asymmetry generation like minimal Dirac seesaw or left-right symmetric model. On the other hand, scenarios involving primordial black hole domination, such as an early matter-dominated era can produce gravitational waves during PBH evaporation, while simultaneously accounting for the observed abundance of dark matter, produced from the same PBH evaporation.

In Chapter 3, we explore a unified framework for dark matter and leptogenesis, both originating from a FOPT. The leptogenesis mechanism operates via the *mass-gain* effect, wherein right-handed neutrinos acquire mass upon crossing the bubble wall. Concurrently, dark matter production is analysed in both thermal (WIMP) and non-thermal (FIMP) scenarios within the same setup. In Chapter 4, we investigate the formation of primordial black holes originating from a FOPT. We first consider scenarios where ultra-light PBH evaporate, yielding the observed dark matter relic density. In contrast, PBH in the asteroid-mass range can survive until the present epoch and constitute the entire dark matter. Furthermore, the formation of PBH is inherently linked to the generation of baryon asymmetry, allowing for a unified cogenesis framework that connects PBH dark matter and baryogenesis. Next, in Chapter 5, we examine the implications of recently observed nanoHertz-range stochastic gravitational wave signals for constraining the scale of Dirac leptogenesis. Finally, in Chapter 6, we summarise the thesis and discuss some future directions. Overall, this thesis presents a comprehensive exploration of how first-order phase transitions, and more broadly symmetry-breaking dynamics in the early Universe, can offer a unified and testable origin for both dark matter and the baryon asymmetry through gravitational waves.

Leptogenesis and Dark Matter from mass-gain mechanism

In this chapter, we consider a minimal scenario where both leptogenesis and DM are triggered by a strong first order phase transition with observable GW signatures. Similar to the baryogenesis mechanism adopted in [223] followed by leptogenesis implementation in [224, 225], we consider a scenario where DM as well as right handed neutrino responsible for leptogenesis acquire masses in a FOPT (*mass-gain* mechanism) by crossing the relativistic bubble wallsⁱ⁾. Unlike additional gauge symmetries considered in these works, here we consider a minimal scenario without any gauge extension of the SM. Adopting a classical conformal symmetry required to generate masses of gauge singlet fermions via FOPT as well as to enhance the strength via supercooling, we first consider the conformal version of the minimal scotogenic model [228]. While this model contains DM as well as right handed neutrino (RHN) responsible for leptogenesis, we find that it is not possible to get the desired leptogenesis from RHN decay. This is due to strict constraints on the hierarchy of massive fields like DM and RHNs coupling to the singlet scalar field which is also driving the FOPT. We then adopt a hybrid setup with type I and scotogenic origin of light neutrino masses and discuss the resulting phenomenology of DM, leptogenesis and GW.

ⁱ⁾For further details on other scenarios connecting FOPT to baryogenesis, see the recent works in Refs. [226, 227].

With only five additional BSM fields, the model remains successful and predictive at experiments operational at different frontiers.

Further, we consider the possibility of constraining it with existing data from LIGO-VIRGO-KAGRA (LVK) experiments taken during their first three observing runs (O1, O2, O3). There have already been tight constraints on isotropic GW background from LVK observations [229]. The same constraints have been used in the context of particle physics models with first order phase transition capable of generating stochastic GW in the LVK ballpark [218, 230–232]. Firstly, we focused on a low scale FOPT or low scale leptogenesis such that the resulting GW spectrum remains within the sensitivities of future experiments. Next, we consider a high scale version of this setup which can already be constrained by existing GW experiments like LIGO-VIRGO-KAGRA. We show the parameter space consistent with successful leptogenesis which is disfavoured by LVK data. The minimal version of this model also predicts non-thermal fermion singlet DM with mass in the sub-GeV ballpark.

This chapter 3 is organised as follows. In section 3.1 we briefly discuss the conformal version of the scotogenic model followed by the details of first order phase transition in section 3.2. In section 3.3, we discuss the details of gravitational wave production followed by discussion of leptogenesis and dark matter in section 3.4. Further we discuss the constraints on dark matter and leptogenesis at high scale in section 3.5. Finally, we conclude in section 3.6.

3.1 Conformal Scotogenic Model

In order to realise a supercooled phase transition along with leptogenesis and dark matter, we first consider the conformal or scale invariant version [233] of the minimal scotogenic model [228] where the SM is extended by three gauge singlet right handed neutrinos N_i (with $i = 1, 2, 3$), one additional scalar doublet η . An additional Z_2 symmetry is imposed under which these newly added particles are odd while all SM particles are even. In order to preserve the conformal nature and generate masses, an additional Z_2 even singlet scalar S is introduced.

The leptonic Yukawa Lagrangian relevant for light neutrino mass is

$$\mathcal{L} \supset \frac{1}{2} Y'_{ij} S N_i N_j + (Y_{ij} \bar{L}_i \tilde{\eta} N_j + \text{h.c.}) . \quad (3.1)$$

Clearly, there is no coupling of neutrinos to the SM Higgs doublet Φ_1 due to the unbroken Z_2 symmetry. However, light neutrino masses arise at radiative level with Z_2 odd particles taking part in the loop.

The scalar potential of the model can be written as

$$V(\Phi_1, \eta, S) = \frac{\lambda_1}{4} |\Phi_1|^4 + \frac{\lambda_2}{4} |\eta|^4 + \lambda_3 |\Phi_1|^2 |\eta|^2 + \frac{1}{4} \lambda_S S^4 + \lambda_4 |\Phi_1^\dagger \eta|^2 + \left[\frac{\lambda_5}{2} (\Phi_1^\dagger \eta)^2 + \text{h.c.} \right] + \lambda_6 |\Phi_1|^2 S^2 + \lambda_7 |\eta|^2 S^2 . \quad (3.2)$$

where Φ_1 is the SM Higgs doublet. Light neutrino masses which arise at one loop level can be evaluated as [228, 234]

$$\begin{aligned} (m_\nu)_{ij} &= \sum_k \frac{Y_{ik} Y_{jk} M_k}{32\pi^2} \left(\frac{m_{H^0}^2}{m_{H^0}^2 - M_k^2} \ln \frac{m_{H^0}^2}{M_k^2} - \frac{m_{A^0}^2}{m_{A^0}^2 - M_k^2} \ln \frac{m_{A^0}^2}{M_k^2} \right) \\ &\equiv \sum_k \frac{Y_{ik} Y_{jk} M_k}{32\pi^2} [L_k(m_{H^0}^2) - L_k(m_{A^0}^2)] , \end{aligned} \quad (3.3)$$

where M_k is the mass eigenvalue of the mass eigenstate N_k in the internal line and the indices $i, j = 1, 2, 3$ run over the three neutrino generations as well as three copies of N_i . Also, A^0, H^0 are the neutral pseudoscalar and scalar respectively contained in η . The function $L_k(m^2)$ is defined as

$$L_k(m^2) = \frac{m^2}{m^2 - M_k^2} \ln \frac{m^2}{M_k^2} . \quad (3.4)$$

Using the physical scalar mass expressions [235], one can write $m_{H^0}^2 - m_{A^0}^2 = \lambda_5 v_{\text{ew}}^2$ where v_{ew} is the vacuum expectation value of SM Higgs doublet. Thus, light neutrino mass is directly proportional to the parameter λ_5 . In upcoming discussions, we will discuss the effects of λ_5 in detail.

3.2 First order phase transition

Here we are interested in the phase transition driven by the singlet scalar field at a scale above the electroweak scale as preferred from leptogenesis point of view. For electroweak phase transition in minimal scotogenic model without any conformal symmetry or singlet scalar, please refer to earlier work [236].

In order to study the details of FOPT in conformal scotogenic model, we first write down the full one-loop potential which can be schematically divided into following form:

$$V_{\text{eff}} = V_{\text{tree}} + V_{\text{CW}} + V_{\text{th}}, \quad (3.5)$$

where V_{tree} , V_{CW} and V_{th} denote the tree level scalar potential, the one-loop Coleman-Weinberg potential, the thermal effective potential, respectively. The tree level scalar potential is given by Eq. (3.2). In finite-temperature field theory, the effective potential, V_{CW} and V_{thermal} , are calculated by using standard background field method [93, 97]. In the following calculations, we take Landau gauge for simplicity.ⁱⁱ⁾ The Coleman-Weinberg potential [92] with $\overline{\text{DR}}$ regularisation is given by

$$V_{\text{CW}} = \sum_i (-)^{n_f} \frac{n_i}{64\pi^2} m_i^4(\phi) \left(\log \left(\frac{m_i^2(\phi)}{\mu^2} \right) - \frac{3}{2} \right), \quad (3.6)$$

where suffix i represents particle species, and n_i , $m_i(\phi)$ are the degrees of freedom (dof) and field dependent masses of i 'th particle. In addition, μ is the renormalisation scale, and $(-)^{n_f}$ is +1 for bosons and -1 for fermions, respectively. Since we are tracking the singlet scalar field for FOPT, we consider its VEV, denoted by M as the renormalisation scale as $\mu = M = \langle S \rangle$. We denote the singlet scalar as $S = (\phi + M)/\sqrt{2}$. The relevant field dependent masses along with their dof are

$$m_\eta^2 = \lambda_7 \phi^2 / 2 \ (n_\eta = 4), m_s^2 = 3\lambda_s \phi^2 \ (n_s = 1), m_{y_i}^2 = 2y_i^2 \phi^2 \ (n_y = 2) \quad (3.7)$$

ⁱⁱ⁾The gauge dependence of the thermal effective potential is discussed by many authors. See. e.g. Refs. [237, 238] and references therein.

Thermal contributions to the effective potential are given by

$$V_{\text{th}} = \sum_i \left(\frac{n_{B_i}}{2\pi^2} T^4 J_B \left[\frac{m_{B_i}}{T} \right] - \frac{n_{F_i}}{2\pi^2} T^4 J_F \left[\frac{m_{F_i}}{T} \right] \right), \quad (3.8)$$

where n_{B_i} and n_{F_i} denote the dof of the bosonic and fermionic particles, respectively. In this expressions, J_B and J_F functions are defined by following functions:

$$J_B(x) = \int_0^\infty dz z^2 \log \left[1 - e^{-\sqrt{z^2+x^2}} \right], \quad (3.9)$$

$$J_F(x) = \int_0^\infty dz z^2 \log \left[1 + e^{-\sqrt{z^2+x^2}} \right]. \quad (3.10)$$

In the calculation of the thermal potential, we also consider the contribution from the daisy diagrams such that the total thermal potential reads $V_T(\phi, T) = V_{\text{th}} + V_{\text{daisy}}(\phi, T)$. This has to be done in order to improve the perturbative expansion during the FOPT [101, 102, 239]. Such corrections can be implemented in two ways by inserting thermal masses into the zero-temperature field dependent masses. In one of these resummation prescriptions, known as the Parwani method [101], thermal corrected field dependent masses are used. In the other prescription, known as the Arnold-Espinosa method [102], the effect of the daisy diagram is included only for Matsubara zero-modes inside J_B function defined above. In our work, we use the Arnold-Espinosa method. The thermal part of the potential, including the daisy contributions can now be written as

$$V_T(\phi, T) = V_{\text{th}} + V_{\text{daisy}}(\phi, T), \quad (3.11)$$

$$V_{\text{daisy}}(\phi, T) = - \sum_i \frac{g_i T}{12\pi} [m_i^3(\phi, T) - m_i^3(\phi)],$$

wherein, V_{th} is the thermal correction and V_{daisy} is the daisy subtraction [101, 102, 239]. Denoting $m_i^2(\phi, T) = m_i^2(\phi) + \Pi_i(T)$, the relevant thermal masses can be written as [240]

$$\Pi_\eta(T) = \left(\frac{g_2^2}{8} + \frac{g_1^2 + g_2^2}{16} + \frac{\lambda_2}{2} + \frac{\lambda_3 + \lambda_4}{12} \right) T^2$$

$$\Pi_s(T) = \left(\frac{\lambda_s}{4} + \frac{\lambda_6}{3} + \frac{\lambda_7}{3} + \frac{y_1^2}{8} + \frac{y_2^2}{8} + \frac{y_2'^2}{8} \right) T^2.$$

The FOPT proceeds via tunnelling, and the corresponding spherical symmetric field configurations known as bubbles are nucleated followed by expansion and coalescence. For recent reviews of FOPT in cosmological context, please refer to [241, 242]. The tunnelling rate per unit time per unit volume can be estimated as

$$\Gamma(T) = \mathcal{A}(T)e^{-S_3(T)/T}, \quad (3.12)$$

where $\mathcal{A}(T) \sim T^4$ and $S_3(T)$ are determined by the dimensional analysis and given by the classical configurations, called bounce, respectively. At finite temperature, the $O(3)$ symmetric bounce solution [122] can be obtained by solving the following equation

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = \frac{\partial V_{\text{eff}}}{\partial \phi}. \quad (3.13)$$

The boundary conditions required to solve the above differential equation are

$$\phi(r \rightarrow \infty) = \phi_{\text{false}}, \quad \left. \frac{d\phi}{dr} \right|_{r=0} = 0, \quad (3.14)$$

where ϕ_{false} denotes the position of the false vacuum. Using ϕ governed by the above equation and boundary conditions, the bounce action can be written as

$$S_3 = \int_0^\infty dr 4\pi r^2 \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V_{\text{eff}}(\phi, T) \right]. \quad (3.15)$$

The temperature at which the bubbles are nucleated is called the nucleation temperature T_n . This can be calculated by comparing the tunnelling rate to the Hubble expansion rate as

$$\Gamma(T_n) = \mathbf{H}^4(T_n). \quad (3.16)$$

Here, assuming the usual radiation dominated universe, the Hubble parameter is given by $\mathbf{H}(T) \simeq 1.66\sqrt{g_*}T^2/M_{\text{Pl}}$ with g_* being the dof of the radiation component. Thus, the rate comparison equation above leads to

$$\frac{S_3(T_n)}{T_n} \simeq 140, \quad (3.17)$$

	v_c (GeV)	T_c (GeV)	v (GeV)	v_c/T_c	$\lambda_7(0)$	$Y'(0)$	$\lambda_s(0)$
BP1	9540.78	2374	9901.77	4.01	1.5	0.5	0.02
BP2	9599.59	2407	9968.24	3.98	1.6	0.4	0.02
BP3	9734.10	2422	9997.27	4.01	1.2	0.2	0.02
BP4	9648.57	2391	9988.00	4.03	1.4	0.3	0.02

Table 3.1: Benchmark parameters of the model and other details of the FOPT in conformal scotogenic model.

for $g_* \sim 100$ and $T_n \sim 100$ GeV while for lower temperature near MeV where $g_* \sim 10$, the above ratio becomes larger. For higher nucleation temperature, as we have in the present scenario, the ratio $\frac{S_3(T_n)}{T_n}$ becomes smaller than the one quoted above. If $\phi(T_n)/T_n > 1$ is satisfied, where $\phi(T_n)$ is the singlet scalar VEV at the nucleation temperature, $T = T_n$, the corresponding phase transition is conventionally called *strong* first order. Alternatively, the ratio at critical temperature namely $\phi(T_c)/T_c \equiv v_c/T_c > 1$ is also used as the strength of the FOPT. The critical temperature T_c corresponds to the temperature where the two minima of the potential are degenerate.

In order to simplify the bounce calculation, we write the zero temperature one-loop effective potential as [243, 244]

$$\begin{aligned} V_0 &= V_{\text{tree}} + V_{\text{CW}}, \\ &= \frac{1}{4} \lambda_S(t) G^4(t) \phi^4 \end{aligned} \quad (3.18)$$

where $t = \log(\phi/\mu)$ with $\mu = M$ being the scale of renormalisation. $G(t)$ is given by

$$G(t) = e^{-\int_0^t dt' \gamma(t')}, \quad \gamma(t) = \frac{1}{32\pi^2} \text{Tr}[Y'^\dagger Y'], \quad (3.19)$$

The Yukawa couplings and quartic coupling at the renormalisation scale are calculated by solving the renormalisation group evolution (RGE) equations of the model given in Appendix A. Taking the renormalisation scale μ to be M , the condition $\frac{dV}{d\phi}|_{\phi=M} = 0$ leads us to the relation,

$$10\lambda_s^2(0) + 32\pi^2\lambda_s(0) + 3Y'^2(0)\lambda_s(0) - 3Y'^4(0) + \lambda_6^2(0) + \lambda_7^2(0) = 0 \quad (3.20)$$

assuming all three Yukawa couplings to be identical for simplicity. In order to get

the required potential profile, the relative magnitude of the couplings Y' and λ_7 are important as we will discuss below. The other quartic coupling λ_6 needs to be small in order to get the desired electroweak symmetry breaking at later stages.

Apart from finding the nucleation and critical temperature, it is also required to estimate the epoch when the FOPT is completed. The corresponding temperature is known as the percolation temperature T_p , typically defined as the temperature at which significant volume of the universe is converted from the symmetric phase (false vacuum) to the broken phase (true vacuum). Adopting the prescription given in [245, 246], the percolation temperature T_p is obtained from the probability of finding a point still in the false vacuum given by

$$\mathcal{P}(T) = e^{-\mathcal{I}(T)},$$

where

$$\mathcal{I}(T) = \frac{4\pi}{3} \int_T^{T_c} \frac{dT'}{T'^4} \frac{\Gamma(T')}{\mathbf{H}(T')} \left(\int_T^{T'} \frac{d\tilde{T}}{\mathbf{H}(\tilde{T})} \right)^3. \quad (3.21)$$

The percolation temperature is then calculated by using $\mathcal{I}(T_p) = 0.34$ [245] (implying that at least 34% of the comoving volume is occupied by the true vacuum).

We then implement the model in **PhaseTracer** [247] to find the parameter space consistent with a FOPT. For a few benchmark points given in table 3.1, we show the potential profile in Fig. 3.1. For all these benchmarks, one can clearly see a barrier between the two minima at the critical temperature, indicating a FOPT. Such degenerate minima lead to the formation of bubbles which subsequently produce gravitational waves. We also perform a numerical scan to show the model parameter space in new scalar masses in Fig. 3.2. As shown in the colour code, the strength of the FOPT can be large $v_c/T_c \geq 3$ for certain region of the parameter space. This, along with the supercooled nature of the FOPT helps in enhancing the strength of the resulting gravitational waves emitted, as we discuss below.

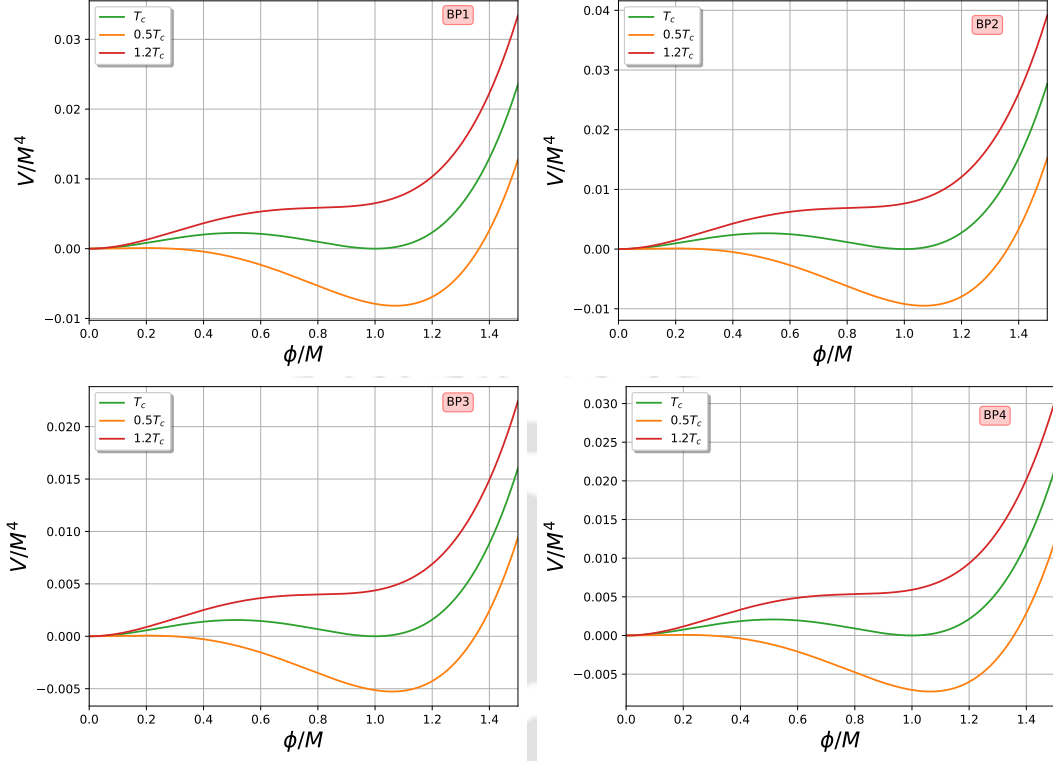


Figure 3.1: Shape of the potential at, above and below the critical temperature T_c for chosen benchmark points shown in Table 3.1.

3.3 Stochastic Gravitational Waves from FOPT

A FOPT can lead to the formation of stochastic gravitational waves background primarily due to three distinct mechanisms: the bubble collisions [123, 124, 140, 248, 249], the sound wave of the plasma [130, 131, 134, 149] and the turbulence of the plasma [135, 136, 138, 146, 250, 251]. The amplitude of such GW signal crucially depends upon two quantities: the amount of vacuum energy (or latent heat) released during the transition as well as the duration of the transition.

In order to calculate the energy released during the FOPT, we first find the free energy difference between the true and the false vacuum as

$$\Delta V_{\text{eff}} \equiv V_{\text{eff}}(\phi_{\text{false}}, T) - V_{\text{eff}}(\phi_{\text{true}}, T). \quad (3.22)$$

As a result of the bubble nucleation, the amount of vacuum energy released during the FOPT, in the units of radiation energy density of the universe, $\rho_{\text{rad}} = g_* \pi^2 T^4 / 30$,

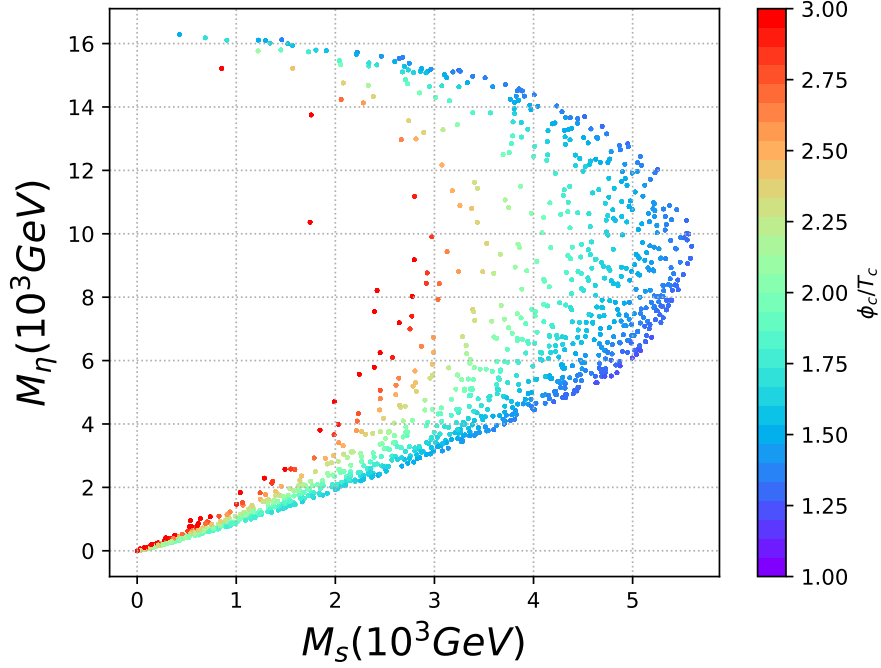


Figure 3.2: Parameter space in inert doublet Mass M_η versus scalar singlet mass M_S plane consistent with a FOPT in conformal scotogenic model. The colour code indicates the strength of the FOPT.

is given by

$$\alpha_* = \frac{\epsilon_*}{\rho_{\text{rad}}}, \quad (3.23)$$

with

$$\epsilon_* = \left[\Delta V_{\text{eff}} - \frac{T}{4} \frac{\partial \Delta V_{\text{eff}}}{\partial T} \right]_{T=T_n}, \quad (3.24)$$

which is also related to the change in the trace of the energy-momentum tensor across the bubble wall [151, 236].

On the other hand, the duration of the FOPT, denoted by the parameter β , is defined as [145]

$$\frac{\beta}{\mathbf{H}(T)} \simeq T \frac{d}{dT} \left(\frac{S_3}{T} \right). \quad (3.25)$$

Here, α_* and $\beta/\mathbf{H}(T)$ are evaluated at the nucleation temperature $T = T_n$ with S_3

	T_c (GeV)	v_c/T_c	T_n (GeV)	T_p (GeV)	$(\beta/\mathbf{H}_*)$	v_J	α_*
BP1	2374	4.01	810.86	808.35	77.43	0.93	0.99
BP2	2407	3.98	1098.88	1084.30	168.18	0.86	0.35
BP3	2422	4.01	834.79	823.52	125.87	0.91	0.61
BP4	2391	4.03	975.59	962.33	150.66	0.88	0.44

Table 3.2: Benchmark parameters consistent with FOPT in conformal scotogenic model, along with the FOPT related parameters calculated for GW spectrum estimation.

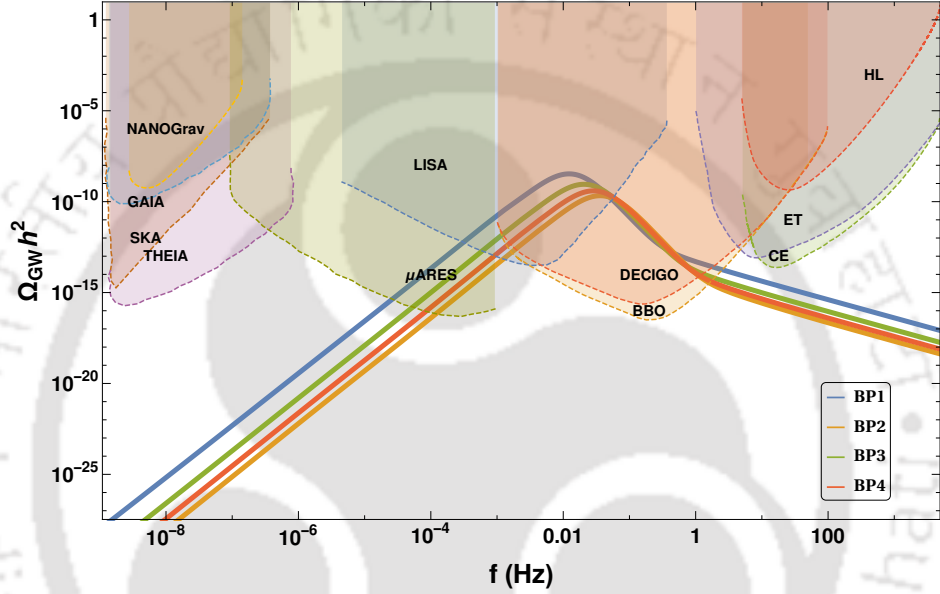


Figure 3.3: Gravitational wave spectrum from FOPT in conformal scotogenic model for four different benchmark points given in table 3.2. Different coloured curves show the sensitivities from GW search experiments like LISA, BBO, DECIGO, HL (aLIGO), ET, CE, NANOGrav, SKA, GAIA, THEIA and μ ARES.

being evaluated using Eq. (3.15). In order to calculate the action numerically, we use a fit for the actual potential which matches very well with the actual potential, as shown in Appendix B. For the benchmark points discussed earlier, we calculate these key parameters and show them in Table 3.2 along with other relevant parameters.

The stochastic GW spectrum, considering all the sources, can be written as [145, 252]

$$\Omega_{\text{GW}}(f) = \Omega_{\phi}(f) + \Omega_{\text{sw}}(f) + \Omega_{\text{turb}}(f), \quad (3.26)$$

where Ω_{ϕ} , Ω_{sw} , Ω_{turb} correspond to the individual contributions from bubble collisions, sound wave of the plasma and turbulence in the plasma respectively. In

general, each of these contributions has a peak type feature with peak frequency f_{peak} . The spectrum can be parametrised as

$$\Omega h^2(f) = \mathcal{R} \Delta(v_w) \left(\frac{\kappa \alpha_*}{1 + \alpha_*} \right)^p \left(\frac{\mathbf{H}_*}{\beta} \right)^q \mathcal{S}(f/f_{\text{peak}}) \quad (3.27)$$

where the pre-factor $\mathcal{R}$ takes into account the red-shift of the GW energy density, $\mathcal{S}(f/f_{\text{peak}})$ parametrises the shape of the spectrum and $\Delta(v_w)$ is the normalisation factor which depends on the bubble wall velocity v_w . κ is the efficiency parameter which denotes the fraction of latent heat driving that particular source of GW in a first order phase transition. The numerical values of the parameters p, q depend upon the source. For bubble collision as the source, the spectrum has been re-estimated in several recent works [133, 245, 246, 253, 254] and can be written as [133, 141, 252]

$$\Omega_\phi h^2 = 1.67 \times 10^{-5} \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{\mathbf{H}_*}{\beta} \right)^2 \left(\frac{\kappa_\phi \alpha_*}{1 + \alpha_*} \right)^2 \frac{A(a+b)^c}{\left[b(f/f_{\text{peak}}^\phi)^{-a/c} + a(f/f_{\text{peak}}^\phi)^{b/c} \right]^c}, \quad (3.28)$$

whereⁱⁱⁱ⁾, $a = 1.03, b = 1.84, c = 1.45$ and $A = 5.93 \times 10^{-2}$ and the peak frequency being [133, 141, 252]

$$f_{\text{peak}}^\phi = 1.65 \times 10^{-5} \text{Hz} \left(\frac{g_*}{100} \right)^{1/6} \left(\frac{T_n}{100 \text{ GeV}} \right) \frac{0.64}{2\pi} \left(\frac{\beta}{\mathbf{H}_*} \right). \quad (3.29)$$

The efficiency factor κ_ϕ for bubble collision is given by [146]

$$\kappa_\phi = \frac{1}{1 + 0.715\alpha_*} \left(0.715\alpha_* + \frac{4}{27} \sqrt{\frac{3\alpha_*}{2}} \right). \quad (3.30)$$

The GW spectrum generated from the sound wave in the plasma has been studied through large hydrodynamical simulations [134] which has also been updated in several recent works [132, 141, 151]. The corresponding spectrum can be written as [141]

$$\Omega_{\text{sw}} h^2 = 2.59 \times 10^{-6} \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{\mathbf{H}_*}{\beta} \right) \left(\frac{\kappa_{\text{sw}} \alpha_*}{1 + \alpha_*} \right)^2 v_w \frac{7^{3.5} (f/f_{\text{peak}}^{\text{sw}})^3}{(4 + 3(f/f_{\text{peak}}^{\text{sw}})^2)^{3.5}} \Upsilon. \quad (3.31)$$

ⁱⁱⁱ⁾The values of these parameters can change if the phase transition leads to breaking of gauge symmetry [133]. In our setup, the singlet scalar induced phase transition does not lead to any gauge symmetry breaking which is consistent with the chosen values of a, b, c .

The corresponding peak frequency is given by

$$f_{\text{peak}}^{\text{sw}} = 8.9 \times 10^{-6} \text{Hz} \left(\frac{g_*}{100} \right)^{1/6} \frac{1}{v_w} \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{\beta}{\mathbf{H}_*} \right) \left(\frac{z_p}{10} \right). \quad (3.32)$$

The efficiency factor for sound waves, applicable for relativistic bubble wall velocity $v_w \sim 1$ in our model, is [152]

$$\kappa_{\text{sw}} = \frac{\alpha_*}{0.73 + 0.083\sqrt{\alpha_* + \alpha_*}}. \quad (3.33)$$

Here, $\Upsilon = 1 - \frac{1}{\sqrt{1+2\tau_{\text{sw}}H_*}}$ is a suppression factor which depends on the lifetime of sound wave τ_{sw} [132] and it can be written as $\tau_{\text{sw}} \sim R_*/\bar{U}_f$ with mean bubble separation, $R_* = (8\pi)^{1/3}v_w\beta$ and rms fluid velocity, $\bar{U}_f = \sqrt{3\kappa_{\text{sw}}\alpha_*/4(1+\alpha_*)}$; $z_p \sim 10$. Finally, the spectrum generated by the turbulence in the plasma is given by [141, 145, 252]

$$\Omega_{\text{turb}}h^2 = 3.35 \times 10^{-4} \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{\mathbf{H}_*}{\beta} \right) \left(\frac{\kappa_{\text{turb}}\alpha_*}{1+\alpha_*} \right)^{1.5} v_w \frac{(f/f_{\text{peak}}^{\text{turb}})^3}{(1+f/f_{\text{peak}}^{\text{turb}})^{3.6}(1+8\pi f/h_*)} \quad (3.34)$$

with the peak frequency being [145]

$$f_{\text{peak}}^{\text{turb}} = 2.7 \times 10^{-5} \text{Hz} \left(\frac{g_*}{100} \right)^{1/6} \frac{1}{v_w} \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{\beta}{\mathbf{H}_*} \right). \quad (3.35)$$

The efficiency factor for turbulence is $\kappa_{\text{turb}} \simeq 0.1\kappa_{\text{sw}}$ [145] and the inverse Hubble time at the epoch of GW production, redshifted to today is

$$h_* = 1.65 \times 10^{-5} \text{Hz} \left(\frac{g_*}{100} \right)^{1/6} \left(\frac{T_n}{100 \text{ GeV}} \right). \quad (3.36)$$

In order to calculate the bubble wall velocity, we first calculate the Jouguet velocity [146, 152, 255]^{iv)}:

$$v_J = \frac{1/\sqrt{3} + \sqrt{\alpha_*^2 + 2\alpha_*/3}}{1 + \alpha_*}. \quad (3.37)$$

^{iv)}Also see Refs. [256–259], for the discussion of the bubble wall velocity v_w .

The bubble wall velocity is then calculated as [260]

$$v_w = \begin{cases} \sqrt{\frac{\Delta V_{\text{eff}}}{\alpha_* \rho_{\text{rad}}}} & \text{if } \sqrt{\frac{\Delta V_{\text{eff}}}{\alpha_* \rho_{\text{rad}}}} < v_J \\ 1 & \text{if } \sqrt{\frac{\Delta V_{\text{eff}}}{\alpha_* \rho_{\text{rad}}}} \geq v_J. \end{cases} \quad (3.38)$$

The total GW spectrum after summing over the contributions from all three sources is shown in Fig. 3.3 for the benchmark points shown in Table 3.2. The experimental sensitivities of NANOGrav [261], SKA [154], GAIA [155], THEIA [155], μ ARES [156], LISA [262], DECIGO [263], BBO [88], ET [213], CE [214] and aLIGO [264] are shown as shaded regions of different colours. Since the FOPT is occurring at a scale above the electroweak scale, the peak frequencies as well as the amplitudes are around the LISA sensitivity and hence remain verifiable in near future.

3.4 Dark Matter and Leptogenesis via FOPT

In order to realise leptogenesis from decay, one needs to ensure that at least one of the RHNs remain heavier than the scalar doublet η . Since both η and RHNs acquire masses during the FOPT, this helps in realising DM and leptogenesis simultaneously. However, the desired profile of the scalar potential of singlet scalar S as well as the minimisation condition given in Eq. (3.20) pose a problem. As understood from the FOPT, the scalar singlet potential has one unique minima $\phi = 0$ at very high temperature $T \gg M$ with the effective self-quartic coupling $\lambda_s > 0$. However, for low temperature $T \ll M$, the self quartic coupling turns negative and $\phi = 0$ should become a false vacuum. This is however, not possible unless we have $\lambda_7^2(0) > 3Y'^2(0)$. This can be seen from Fig. 3.4 where the zero-temperature effective potential is shown for two different values of $\lambda_7(0)$ while keeping singlet-RHN Yukawa fixed $Y'(0) = 0.8$. Clearly, for smaller $\lambda_7(0)$, we can not achieve the desired potential profile at zero temperature.

In order to circumvent this problem, we consider a hybrid of scotogenic and type I seesaw model without increasing the number of fields. Out of the three RHNs in conformal scotogenic model, we consider two of them to be Z_2 even such that they couple to the SM lepton doublets via usual Higgs doublet as $y_D \bar{L} \tilde{\Phi}_1 N_{2,3}$. The other RHN namely, N_1 is Z_2 -odd and couple to the SM lepton doublets via η as before.

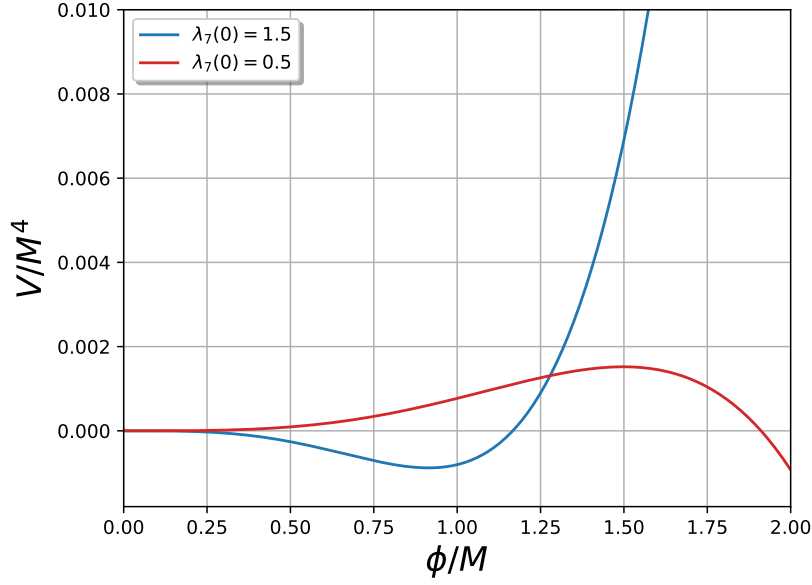


Figure 3.4: Zero temperature effective potential at one-loop for two different values of singlet- η coupling λ_7 , considering the singlet-RHN Yukawa coupling $Y'(0) = 0.8$.

Thus, two of the active neutrinos will receive non-zero mass from type I seesaw while the third one will receive scotogenic contribution at one-loop [228]. The scalar potential as well as singlet scalar coupling to RHNs remain same as before and hence we still require η to be heavier than the RHNs. Therefore N_1 is our DM candidate and $N_{2,3}$ can decay into $L\Phi_1$ to generate the required lepton asymmetry. So, the relevant part of the Yukawa Lagrangian is given by

$$\mathcal{L} \supset \frac{1}{2} Y'_{ij} S N_i N_j + \left(Y_{\alpha 1} \bar{L}_\alpha \tilde{\eta} N_1 + \sum_{j=2,3} (y_D)_{\alpha j} \bar{L}_\alpha \tilde{\Phi}_1 N_j + \text{h.c.} \right). \quad (3.39)$$

We first identify a few benchmark points consistent with the FOPT and the mass hierarchy among RHN and scalar doublet η required to have successful leptogenesis and DM phenomenology. The benchmark points along with other details calculated for the GW spectrum are shown in table 3.3. The corresponding GW spectrum is shown in Fig. 3.5. Clearly, choosing one RHN lighter and making the heavier RHNs Z_2 even does not change the FOPT details significantly and hence we obtain similar benchmark parameters and GW spectrum like before.

Now we implement the baryogenesis via relativistic bubble wall mechanism proposed

	v_c (GeV)	T_c (GeV)	v (GeV)	v_c/T_c	$\lambda_7(0)$	$Y_2'(0)$ $\approx Y_3'(0)$	$\lambda_s(0)$	T_n (GeV)	T_p (GeV)	(β/H_*)	v_J	α_*
BP1	9634.17	2521	9934.17	3.82	1.5	0.5	0.02	988.32	974.98	151.06	0.89	0.46
BP2	9553.88	2416	9757.60	3.95	1.6	0.7	0.02	896.89	887.67	110.66	0.91	0.66
BP3	9698.63	2370	9988.01	4.09	1.2	0.3	0.02	779.49	770.72	103.96	0.92	0.80
BP4	9692.84	2391	9978.08	4.05	1.3	0.4	0.02	1207.09	1190.58	204.51	0.84	0.24

Table 3.3: Benchmark parameters and other details involved in the GW spectrum calculation of the hybrid model.

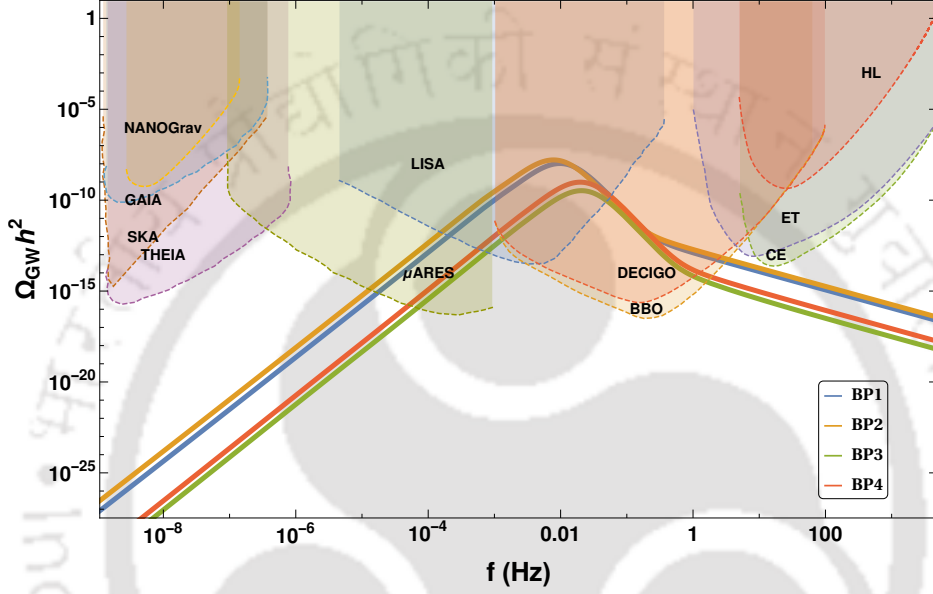


Figure 3.5: Gravitational wave spectrum from FOPT in the hybrid model for four different benchmark points given in table 3.3. Different coloured curves show the sensitivities from GW search experiments like LISA, BBO, DECIGO, HL (aLIGO), ET, CE, NANOGrav, SKA, GAIA, THEIA and μ ARES.

	ϵ_N	T_{RH} (GeV)	T_n (GeV)	$M_{N_2} \approx M_{N_3}$ (GeV)	y_D	ΔV_{tot} (GeV) ⁴
BP1	6.22×10^{-8}	988.32	988.32	4966.67	5.12×10^{-8}	1.75464×10^{13}
BP2	6.22×10^{-8}	896.89	896.89	6826.16	6×10^{-8}	1.69737×10^{13}
BP3	6.22×10^{-8}	779.49	779.49	2996.39	3.98×10^{-8}	1.15697×10^{13}
BP4	6.22×10^{-8}	1207.58	1207.58	3991.16	4.59×10^{-8}	2.15494×10^{13}

Table 3.4: CP asymmetry and other relevant details involved in leptogenesis calculation for the hybrid model.

in [223] to achieve leptogenesis in the hybrid model mentioned above. A first order phase transition in the singlet (S) sector will create bubbles such that the particles like N_i, η entering the bubble will become massive due to $\langle S \rangle \neq 0$ inside the bubble. This is followed by $N_{2,3}$ decays into leptons and SM Higgs creating the leptonic asymmetry. On the other hand N_1 being lighter than η become stable due to unbroken Z_2 symmetry and hence act like a DM candidate.

For the leptogenesis we closely follow Ref. [223, 225] i.e., the *mass-gain* mechanism. Let us briefly mention the *mass-gain* mechanism employed in our work. Firstly, we need to ensure that the Lorentz boost of the bubble wall should be more than the Lorentz factor of the particle in the plasma frame

$$\gamma_w > \gamma_N \sim \frac{M_N}{T_n} \quad (3.40)$$

where T_n is the nucleation temperature and $M_N = y_N v$ is the mass of the RHN coupling to the singlet scalar. Now, the above condition (3.40) pushes the RHN into the bubble while maintaining the equilibrium co-moving number density.

$$Y_N = \frac{135}{8\pi^4} \xi(3) \frac{g_N}{g_*} \quad (3.41)$$

where g_N and g_* are the degrees of freedom of RHN N and the total relativistic degrees of freedom in the energy density of the universe, respectively.

The final baryonic asymmetry is then written as follows

$$Y_B = \epsilon_N \kappa_{\text{sph}} Y_N \left(\frac{T_n}{T_{\text{RH}}} \right)^3. \quad (3.42)$$

where $\epsilon_N \simeq \sin(2\delta)/(16\pi)$ [51, 265] is the CP-asymmetry and δ is the relative CP phase between the RHNs (for resonant regime), $\kappa_{\text{sph}} = 8/23$ is the sphaleron conversion factor in the presence of two Higgs doublets [19], and T_{RH} is the reheating temperature after the FOPT. T_{RH} is defined as $T_{\text{RH}} = \text{Max}[T_n, T_{\text{inf}}]$ [223] where T_{inf} can be obtained from the following relation

$$\frac{g_* \pi^2}{30} T_{\text{inf}}^4 = \Delta V_{\text{eff}}. \quad (3.43)$$

Using ΔV_{eff} for the benchmark parameters given in table 3.4, we can calculate the corresponding T_{inf} and hence T_{RH} . The Y_B obtained in Eq. (3.42) should then be compared with the observed baryon asymmetry normalised over the entropy density: $Y_B^{\text{obs}} = (8.61 \pm 0.05) \times 10^{-11}$ [7].

The above asymmetry is feasible after satisfying two conditions

1. The feasibility of decay $N_{2,3} \rightarrow LH$

2. The wash-out from the dominant inverse decay to be suppressed.

For the first condition we will need to consider the thermally corrected masses for the SM Higgs and lepton doublets at the reheating temperature [266]

$$\begin{aligned} M_H^2(T) &= \left(\frac{3}{16}g_2^2 + \frac{1}{16}g_1^2 + \frac{1}{4}y_t^2 \right) T^2, \\ M_L^2(T) &= \left(\frac{3}{32}g_2^2 + \frac{1}{32}g_1^2 \right) T^2, \end{aligned} \quad (3.44)$$

where g_1 and g_2 are the $U(1)_Y$ and $SU(2)_L$ gauge couplings respectively, and y_t is the top quark Yukawa coupling. Therefore, at the reheating temperature after considering the coupling values at the electroweak scale^{v)} we get

$$M_H(T_{\text{RH}}) + M_L(T_{\text{RH}}) \simeq 0.77T_{\text{RH}}, \quad (3.45)$$

Hence for the feasibility of the decay we need the mass of RHN at the reheating temperature to be $M_N/T_{\text{RH}} \gtrsim 0.77$.

As for the second condition we have taken the Dirac Yukawa coupling y_D , which is responsible for the wash-out, to be parametrised by the Casas-Ibarra parametrisation [267] for type I seesaw with two RHNs given by

$$Y_D = \Lambda^{-1/2} \mathcal{O} \hat{m}_\nu^{1/2} U_{\text{PMNS}}^\dagger, \quad (3.46)$$

where $\Lambda = v_{\text{ew}}^2/M_N$, $\mathcal{O}$ is an arbitrary complex orthogonal matrix, $\hat{m}_\nu$ is the diagonal light neutrino mass matrix and U_{PMNS} is the light neutrino mixing matrix. Since only two RHNs contribute to type I seesaw, we consider the lightest active neutrino mass to be vanishing. Using the best-fit values of the light neutrino oscillation data [268] for normal hierarchy and assuming $\mathcal{O}$ to be the identity matrix, we obtain

$$y_D \equiv \sum_\alpha y_{D1\alpha} \sim 2.3 \times 10^{-8} \left(\frac{M_N}{1 \text{ TeV}} \right)^{1/2}. \quad (3.47)$$

^{v)}It should be noted that the values of these couplings do not change much between the electroweak scale and the reheating temperature for (multi) TeV-scale symmetry breaking considered here.

And proceeding with the above Dirac Yukawa we need to satisfy the following condition [223]

$$\frac{M_N}{T_{\text{RH}}} \gtrsim \ln \left[\frac{y_D^2 M_{\text{Pl}}}{24\pi T_{\text{RH}}} \left(\frac{M_N}{T_{\text{RH}}} \right)^{5/2} \right], \quad (3.48)$$

ensuring the inverse decay width to be suppressed. We calculate the required CP asymmetry and the Dirac Yukawa couplings for the four benchmark points and quote them in table 3.4. All these benchmark points satisfy the required baryon asymmetry due to the appropriate choice of Yukawa couplings and CP phase. As can be seen from the smallness of the Dirac Yukawa couplings, the decay width of the RHN remains small, also required from the resonant leptogenesis condition $M_3 - M_2 \sim \Gamma_2/2$. This also justifies the semi-degenerate nature of RHNs $N_{2,3}$ in the benchmark choice of parameters. We also check that the benchmark points satisfy the above mentioned conditions to ensure the viability of the leptogenesis scenario we are implementing.

For the standard vanilla leptogenesis scenario where the *massive* RHNs are in equilibrium, the baryon asymmetry in the weak washout regime $\Gamma/\mathbf{H}(T = M_N) \ll 1$ can be written as

$$Y_B = \epsilon_N \kappa_{\text{sph}} Y_N^{\text{eq}}(T = M_N); \quad (3.49)$$

$$Y_N^{\text{eq}}(T) = \frac{45}{4\pi^4} \frac{g_N}{g_*} \left(\frac{M_N}{T} \right)^2 K_2(M_N/T), \quad (3.50)$$

with K_2 being the modified Bessel function of 2nd order. Comparing with Eq. (3.42), we can see that in FOPT scenario there arises an extra dilution factor $(T_n/T_{\text{RH}})^3$ compared to the standard case. This was also noticed in earlier works [224, 225]. However, due to a different structure of our model in the absence of any additional gauge symmetry, we have $T_{\text{inf}} < T_n$ resulting in $T_{\text{RH}} = T_n$. Therefore, the final baryon asymmetry in our setup remains same as the standard one, but with the added advantage of detection prospects via stochastic GW observations. In other words, the model without FOPT is also consistent with successful leptogenesis for same set of parameters. However, the presence of FOPT increases the detection prospects in terms of future observations of stochastic GW.

However, as we increase the scale of FOPT, we get deviations from $T_{\text{RH}} = T_n$ resulting in dilution of lepton asymmetry compared to the standard one. To illustrate this, we show three such benchmark points for high scale leptogenesis in table 3.5 and 3.6 which are also consistent with a strong FOPT criteria. As can be seen from table 3.6, for such high scale FOPT, we have $T_{\text{inf}} > T_n$ resulting in $T_{\text{RH}} > T_n$. This leads to the dilution of lepton asymmetry by a factor $(T_n/T_{\text{RH}})^3$ which is as large as $\sim 10^3$ for the last benchmark point in table 3.6. Accordingly, the required CP asymmetry parameter needs to be enhanced for such scenarios. Even though we are in weak washout regime, FOPT can also lead to sizeable washout at the end of the phase transition due to the large latent heat released. Such washout effects can be significant if $T_{\text{RH}} \gg M_N$. In the benchmark points we have considered, $T_{\text{RH}} < M_N$ and hence such washout effects are expected to be smaller. Therefore, in the weak washout regime, high scale FOPT scenario can give successful leptogenesis for the parameter space leading to overproduction of asymmetry in standard leptogenesis.

On the other hand, if we are in the strong washout regime of standard leptogenesis, the FOPT triggered leptogenesis can, in principle, enhance the production of asymmetry if the dilution effects are under control and $T_{\text{RH}} \ll M_N$ in order keep the washout processes like inverse decay suppressed. The validity of baryon asymmetry estimate given by Eq. (3.42) also depends upon the assumption that the washout processes are absent or negligible. In particular, the dominant wash-out coming from inverse decay should be suppressed to validate Y_B estimated in Eq. (3.42). While this is true due to $M_N > T_n$, presence of L, Φ_1 with energy more than the mean thermal energy can induce such washout. As the RHNs $N_{2,3}$ enter the bubble, they receive a boost in the plasma frame with energy $\gamma_N M_N$, where $\gamma_N \sim M_N/T_n$. Consequently, the decay products L, Φ_1 and L^c, Φ_1^* also appear to be boosted, with the energy of the order $\frac{M_N^2}{T_n}$. These boosted particles can interact with SM particles in the plasma of energy T_n and go through cascade scattering with particles in the plasma while redistributing their energies. At the same time, these energetic particles can also lead to washout asymmetry through on-shell processes such as $L \Phi_1 \rightarrow L^c \Phi_1^*$ via $N_{2,3}$. With the simplest assumption of energy distribution, the energy of boosted particles in n -th step cascade scattering is $\frac{M_{N_{2,3}}^2}{2^n T_n}$. Following the procedure of [223, 224], the rate of the washout in the above process can be estimated as $\Gamma_{\text{wash}} \approx 2^{2n} T_n^3 \Gamma_N e^{-2^n/4} / (4M_N^3)$ where $\Gamma_N \sim (y_D^\dagger y_D)_{ii} M_N / 8\pi$ is total

v_c (GeV)	T_c (GeV)	v (GeV)	v_c/T_c	$\lambda_7(0)$	$Y'_2(0)$ $\approx Y'_3(0)$	$\lambda_s(0)$
9.68×10^7	3.14×10^7	9.93×10^7	3.07	2.2	0.6	0.02
9.52×10^8	2.05×10^8	9.76×10^8	4.63	1.2	0.6	0.02
9.61×10^9	2.62×10^9	9.89×10^9	3.66	1.7	0.6	0.02

Table 3.5: Benchmark parameters and other details for high scale leptogenesis scenario.

ϵ_N	T_{RH} (GeV)	T_n (GeV)	$M_{N_2} \approx M_{N_3}$ (GeV)	y_D	ΔV_{eff} (GeV) ⁴
8.93×10^{-7}	9.87×10^6	4.09×10^6	5.96×10^7	5.62×10^{-6}	3.62×10^{29}
1.81×10^{-7}	7.04×10^7	4.97×10^7	5.85×10^8	1.76×10^{-5}	9.96×10^{32}
4.40×10^{-5}	8.59×10^8	9.71×10^7	5.93×10^9	5.60×10^{-5}	2.07×10^{37}

Table 3.6: CP asymmetry and other relevant details involved in high scale leptogenesis calculation for the hybrid model.

decay width of RHN. Similarly, the thermalisation rate can be estimated [223, 224] as $\Gamma_{th} \approx \ln\left(\frac{3M_N^2}{5\pi 2^n \alpha_W T_n^2}\right) \frac{\zeta_3 g_{EW} 2^n \alpha_W^2 T_n^3}{4\pi M_N^2}$ where $g_{EW} = 46$, $\alpha_W = 1/137$. We check that for a typical benchmark point considered in our analysis, $\Gamma_{th} \gg \Gamma_{wash}$ such that all the boosted particles get thermalised quickly via cascade interactions with the SM bath, before inducing any washout effects via inverse decay. Therefore, for our scenario, washout processes are negligible and the baryon asymmetry estimate given by Eq. (3.42) remains valid.

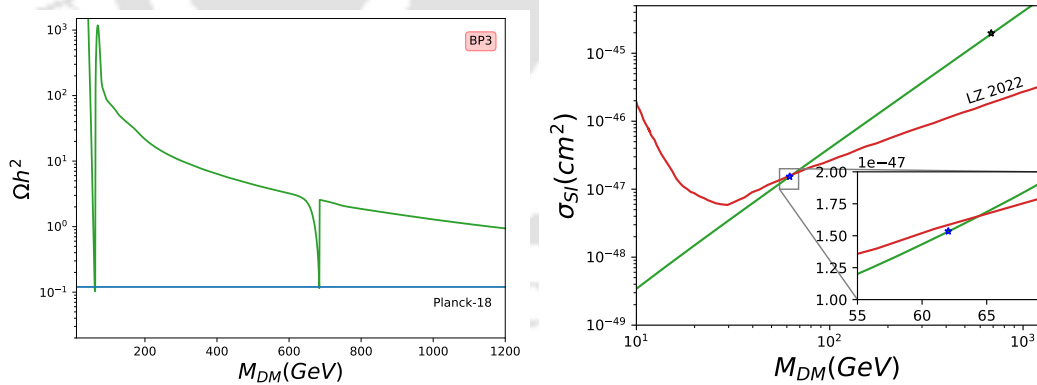


Figure 3.6: Left panel: WIMP DM relic as a function of its mass for BP3, while considering singlet-SM Higgs mixing to be $\sin \theta_{hS} = 0.23$. Right panel: Spin-independent DM-nucleon cross-section as a function of DM mass for the same benchmark choices of parameters considered in left panel.

As mentioned earlier, the Z_2 odd RHN namely, N_1 is the DM candidate which can have light masses due to its small Yukawa couplings with the singlet scalar. It is

possible, in principle, to realise either thermal WIMP or non-thermal DM scenario with the latter being popularly known as feebly interacting massive particle [62]. As $M_{\text{DM}} < T_n$, DM can be in equilibrium inside the bubble and undergo thermal freeze-out at a temperature $T_f \sim M_{\text{DM}}/20$, if its coupling to the SM bath is sizeable enough. DM can annihilate into SM particles via two possible processes: Yukawa interactions with SM leptons via inert scalar doublet η and singlet scalar mediated annihilations into SM particles via singlet-Higgs mixing. Since the scalar doublet η is much heavier than the RHNs, the corresponding DM annihilation cross-section remains suppressed compared to the singlet scalar mediated one. Since the singlet scalar mass is small, we can get the desired relic of DM by appropriate tuning of singlet-Higgs mixing. In order to calculate the thermally averaged annihilation cross-sections and solve the Boltzmann equation for DM numerically, we use `micrOMEGAS` [269]. We consider one particular benchmark point namely, BP3 such that the singlet scalar mass is fixed. The relic abundance as a function of DM mass is shown on the left panel of Fig. 3.6. Even for a considerably large mixing between singlet scalar and the SM Higgs $\sin \theta_{hS} = 0.23$, the relic can barely be satisfied at the resonances. Similarly, the stringent direct detection bounds [270] barely allows the relic satisfying point at the SM Higgs resonance while ruling out the heavier DM mass at singlet scalar resonance. This is due to the fact that, DM Yukawa coupling with singlet scalar is $\langle S \rangle / M_{\text{DM}} = v / m_{N1}$ which is very small for this mass range. It should also be noted that the actual singlet-SM Higgs mixing will be $\sin \theta_{hS} \sim \lambda_6 v_{\text{ew}} / v \ll 0.23$ thereby ruling out the WIMP possibility in this minimal setup.

We finally consider the FIMP DM possibility by considering singlet scalar decay after the phase transition. The corresponding Boltzmann equations can be written as

$$\frac{dY_{\text{DM}}}{dz} = \frac{2}{z\mathbf{H}} \Gamma_{sN1} Y_S, \quad \frac{dY_S}{dz} = -\frac{1}{z\mathbf{H}} (\Gamma_{sN1} + \Gamma_{sh} + \Gamma_{sN2} + \Gamma_{sN3}) Y_S, \quad (3.51)$$

with $z = M_s/T$ and assuming the relativistic dof to be constant, which is valid at temperatures above electroweak scale. While DM is produced only from the decay of singlet scalar, the latter can decay into SM Higgs as well. The corresponding

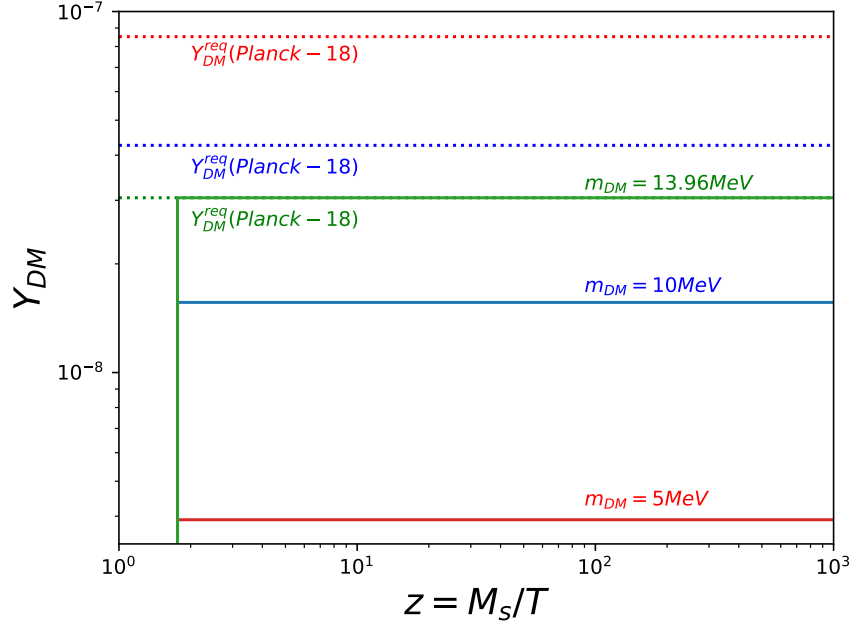


Figure 3.7: Comoving density of FIMP DM for BP3

decay widths are

$$\Gamma_{sNi} = \frac{1}{16\pi} y_i^2 M_s \left(1 - \frac{4M_{Ni}^2}{M_s^2}\right)^{3/2}, \quad \Gamma_{sh} = \frac{y_{shh}^2}{16\pi M_s} \left(1 - \frac{4M_h^2}{M_s^2}\right)^{1/2}, \quad (3.52)$$

where $y_{shh} \sim \lambda_6 v$. Similar to the case of heavy RHNs $N_{2,3}$, the singlet scalar also acquires a large abundance inside the bubble at nucleation temperature, close to the equilibrium comoving number density without Boltzmann suppression. Considering this to be the initial comoving abundance of S and taking appropriate partial decay width of S into SM Higgs, we solve the above Boltzmann equations for different DM masses and find that for DM mass around 14 MeV, the correct relic is satisfied if other parameters are fixed as in BP3 discussed before. Since singlet decay width into SM Higgs is substantial, we require somewhat large FIMP mass to satisfy the correct DM relic. This also leads to instantaneous freeze-in shortly after nucleation temperature $T \sim T_n$ as larger mass corresponds to larger Yukawa coupling of DM with singlet.

3.5 Constraints on dark matter and leptogenesis at high scale

As discussed earlier in Chapter 2 and Section 3.3, the production of gravitational waves is closely linked to the nucleation temperature of a first-order phase transition, with higher nucleation temperatures leading to gravitational wave spectra peaking at higher frequencies. Notably, several ground-based GW detectors currently in operation are sensitive to such high-frequency ranges, making them suitable for probing potential imprints of high-scale phase transitions. In Fig. 3.8, we show the parameter space in terms of physical masses of the heavier quasi-degenerate RHNs denoted by M_N , neutral real component of scalar doublet η and scalar singlet denoted by M_N, M_η, M_S respectively. The parameter space is consistent with a high scale FOPT with the colour code showing the nucleation temperature T_n . We also identify the points with $M_N < T_n$ where one has to consider thermal production of heavy neutrinos into account while estimating the lepton asymmetry produced. The total contribution to the stochastic GW spectrum is shown in Fig. 3.9 for a few

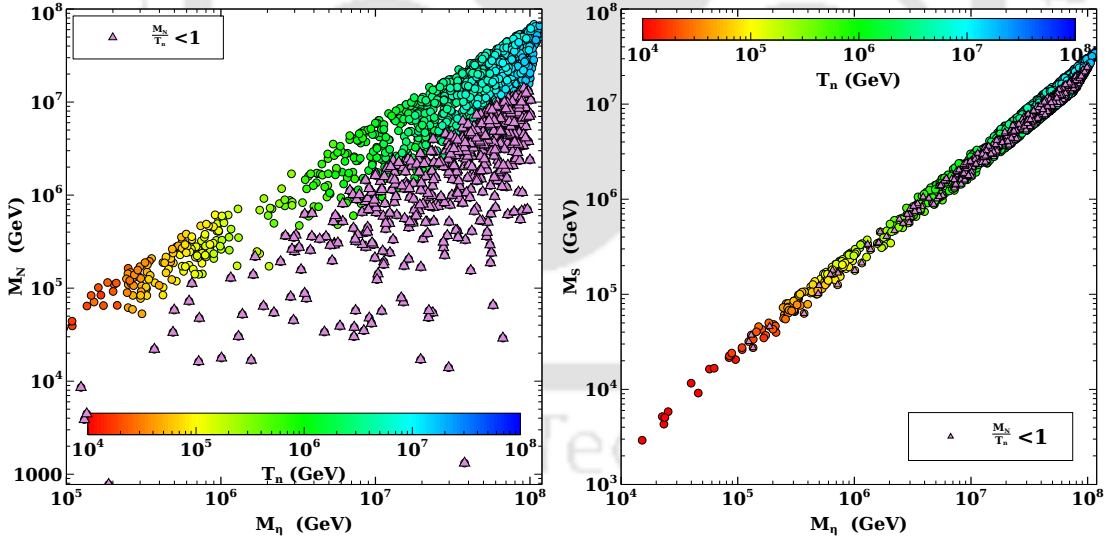


Figure 3.8: Parameter space in M_η versus M_N plane (left panel) and M_η versus M_S plane (right panel) consistent with a high scale FOPT $v_c/T_c > 1$ with the colour code showing the corresponding nucleation temperature T_n . In this scan, the scale of phase transition (M) is varied from 10^4 to 10^8 GeV, $\lambda_7(0)$ is varied from 1 to 3, $Y'_{22} \sim Y'_{33}$ is varied from 0 to 1 and Y'_{11} is varied upto 10^{-2} .

benchmark points shown in table 3.7. Since we are focusing on high scale FOPT,

	M (GeV)	v_c (GeV)	T_c (GeV)	$\frac{v_c}{T_c}$	$\lambda_7(0)$	$Y'_{22}(0)$ $\approx Y'_{33}(0)$	$\lambda_2(0)$	T_n (GeV)	T_p (GeV)	$\frac{\beta}{H_*}$	v_J	α_*
BP1	5.54×10^7	5.01×10^7	1.37×10^7	3.65	1.17	0.091	0.02	3.55×10^6	1.99×10^6	13.97	0.95	1.69
BP2	7.85×10^7	6.97×10^7	1.93×10^7	3.61	1.18	0.090	0.02	5.00×10^6	2.76×10^6	9.71	0.95	1.76
BP3	9.47×10^7	8.38×10^7	2.34×10^7	3.56	1.19	0.096	0.02	5.95×10^6	3.25×10^6	3.56	0.95	1.89
BP4	7.95×10^7	2.37×10^7	1.96×10^7	1.20	1.19	0.097	0.02	5.29×10^6	3.10×10^6	21.24	0.95	1.51
BP5	2.80×10^6	1.54×10^6	6.57×10^5	2.36	1.06	0.086	0.02	1.70×10^5	9.78×10^4	20.81	0.95	1.75

Table 3.7: Benchmark parameters and other details involved in the GW spectrum calculation of the model.

we are showing the relevant sensitivities of future experiments like DECIGO [263], ET [213] and ongoing LIGO-VIRGO (HVO3) [229, 264] as shaded regions of different colours.

Table 3.8 shows the required CP asymmetry along with other relevant parameters involved in calculating baryon asymmetry for the benchmark points discussed before.

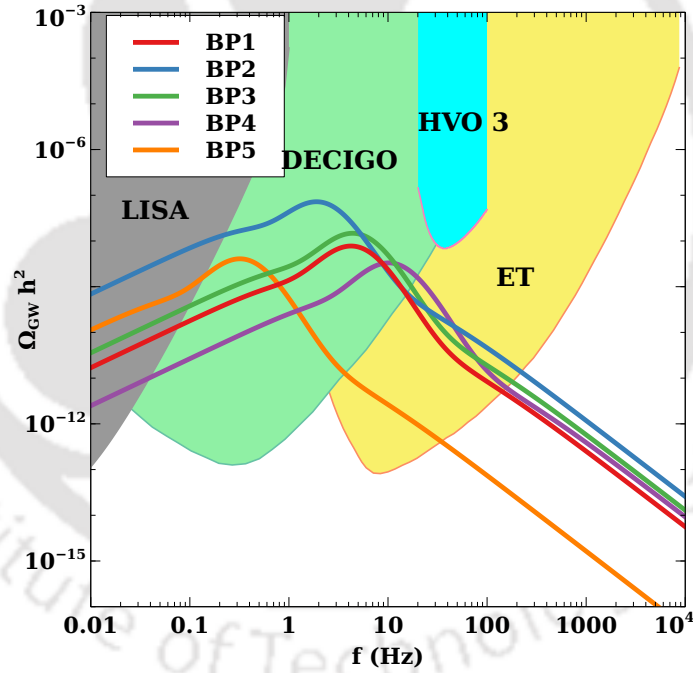


Figure 3.9: GW spectrum for different benchmark points of our model shown in table 3.7.

Adopting a conservative approach to apply the GW constraints on the parameter space of our model consistent with a high scale FOPT and leptogenesis, we define

	ϵ_N	T_{RH} (GeV)	T_n (GeV)	$M_{N_2} \approx M_{N_3}$ (GeV)	y_D	ΔV_{eff} (GeV) ⁴	M_{DM} (MeV)
BP1	2.51×10^{-8}	4.05×10^6	3.55×10^6	3.58×10^6	4.35×10^{-5}	1.02×10^{28}	12.54
BP2	2.60×10^{-8}	5.77×10^6	5.00×10^6	5.04×10^6	5.16×10^{-5}	4.22×10^{28}	14.24
BP3	2.73×10^{-8}	6.98×10^6	5.95×10^6	6.43×10^6	5.83×10^{-5}	9.05×10^{28}	7.86
BP4	2.31×10^{-8}	5.87×10^6	5.29×10^6	5.50×10^6	5.39×10^{-5}	4.53×10^{28}	3.90
BP5	2.56×10^{-8}	1.95×10^5	1.70×10^5	1.71×10^5	9.51×10^{-6}	5.51×10^{22}	4.06

Table 3.8: CP asymmetry and other relevant details involved in leptogenesis and dark matter calculation for the model.

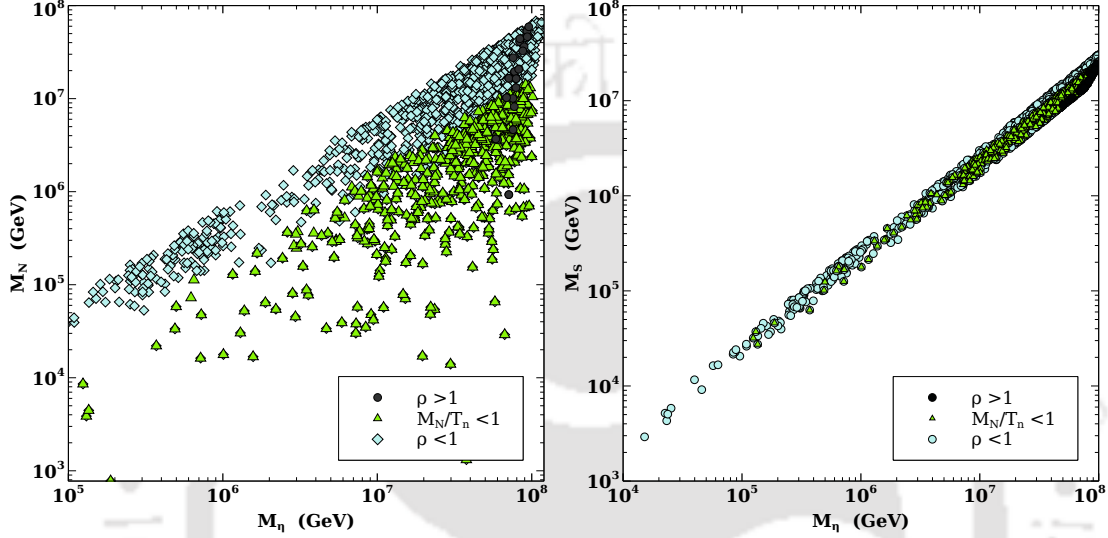


Figure 3.10: Parameter space in M_η versus M_N plane (left panel) and M_η versus M_S plane (right panel) consistent with a high scale FOPT showing the SNR with respect to LIGO-VIRGO O3. In this scan, the scale of phase transition (M) is varied from 10^4 to 10^8 GeV, $\lambda_7(0)$ is varied from 1 to 3, $Y'_{22} \sim Y'_{33}$ is varied from 0 to 1 and Y'_{11} is varied upto 10^{-2} .

the signal-to-noise ratio (SNR) for ongoing LIGO-VIRGO experiment as [271]

$$\rho = \sqrt{\tau \int_{f_{\min}}^{f_{\max}} df \left[\frac{\Omega_{GW}(f) h^2}{\Omega_{\text{expt}}(f) h^2} \right]^2}, \quad (3.53)$$

with τ being the observation time for a particular detector, which we consider to be 1 yr. To register a detection SNR needs to be more than a threshold value $\rho > \rho^{\text{th}}$. In this work, we consider $\rho^{\text{th}} = 1$, such that the parameter space giving rise to SNR $\rho > 1$ at LIGO-VIRGO latest run namely O3 is disfavoured due to non-detection of such stochastic GW, as per our conservative selection criteria.

In Fig. 3.10, we show the parameter space of our model in terms of physical masses of heavier quasi-degenerate RHNs, singlet and doublet scalar masses in a way similar

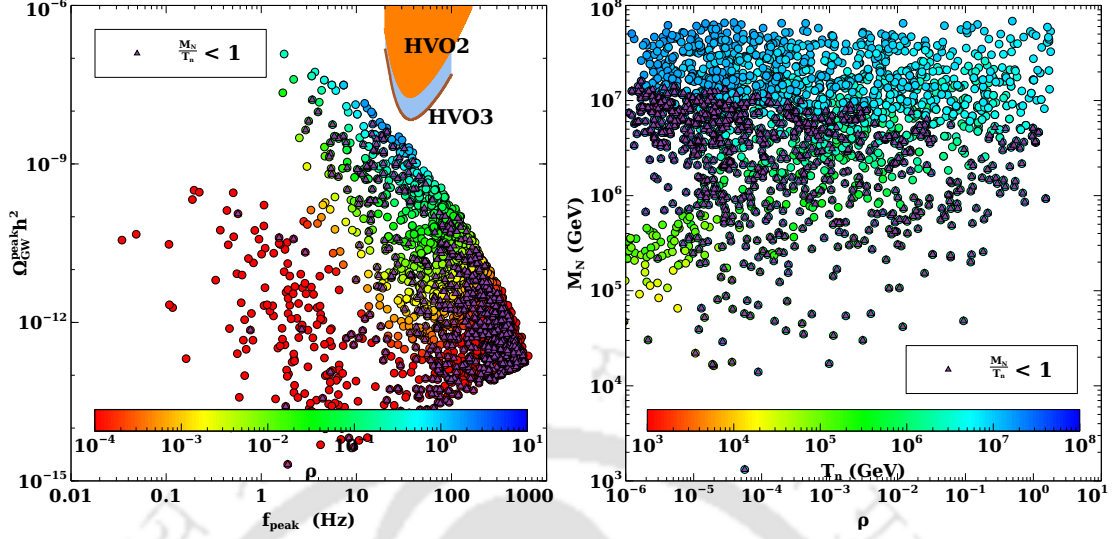


Figure 3.11: Left panel: GW spectrum peak vs its corresponding Peak frequency with the colour code showing the SNR. Right panel: Heavy RHN mass versus SNR with colour code indicating the corresponding nucleation temperature.

to the ones in Fig. 3.8 but without any colour bar. The points corresponding to large SNR namely $\rho > 1$ are shown with a different colour. Clearly, a few points in the larger mass range $\gtrsim 10^7$ GeV with SNR > 1 get disfavoured. We also indicate the points with $T_n > M_N$ where heavy RHNs can be thermally produced. In such a case, the lepton asymmetry needs to be calculated in a way similar to thermal leptogenesis while solving the relevant Boltzmann equations explicitly. We also calculate the peak amplitudes of GW for the parameter space consistent with FOPT requirements and show them against LIGO-VIRGO sensitivities on the left panel of Fig. 3.11. We show the sensitivity curves of run 2 (HVO2) [272] as well as run 3 (HVO3) [229] for comparisons. While the peak frequencies of the points with SNR > 1 lie outside the sensitivity curves, they can still be detected if we consider the entire GW spectrum. On the right panel of Fig. 3.11, we show the heavy quasi-degenerate RHN mass M_N versus SNR with colour code indicating the nucleation temperature. Clearly, some of the points with $T_n \in (10^6 - 10^7)$ GeV get disfavoured.

Dark matter or the Z_2 -odd RHN can, in principle, be produced either thermally or non-thermally. For TeV scale FOPT in this model, studied earlier [273], it was found that thermal DM parameter space is almost entirely ruled out due to stringent direct detection bounds. This is precisely due to the constraint on quartic coupling between

singlet scalar and the SM Higgs from the requirement of successful electroweak symmetry breaking. This coupling, namely λ_6 in our notation then decides Higgs portal coupling through which fermion singlet thermal DM typically annihilate. While DM has Yukawa coupling with leptons and η , Yukawa portal annihilations typically remain sub-dominant especially due to large η mass required by FOPT criteria and often require large Yukawa couplings to satisfy relic. Such Yukawa coupling faces tight constraints from neutrino mass as well as charged lepton flavour violation [274]. If we go to high scale where some part of the parameter space gets constrained by LIGO-VIRGO data, thermal DM is likely to hit the unitarity limit [55]. While DM mass need not be same as the scale of FOPT, making it much lighter suppresses the annihilation rates due to heavy mediators in the form of η as well as singlet scalar. In addition to mediator suppression, DM coupling with singlet scalar also gets smaller for lighter DM masses. Therefore, thermal DM gets overproduced in such a scenario due to insufficient annihilation rates. While entropy dilution at the end of FOPT could lower this thermal abundance, we find such entropy dilution to be negligible as discussed above, in the context of leptogenesis. Therefore, we consider non-thermal DM which remains feasible for some part of the parameter space under study.

Such non-thermal or feebly interacting massive particle type DM can be produced dominantly from singlet scalar (S) decay inside the bubble. Similar to $N_{2,3}$, the singlet scalar with mass $M_S > T_n$ is also out-of-equilibrium inside the bubble having a large initial abundance, given by

$$Y_S = \frac{45}{2\pi^4} \xi(3) \frac{1}{g_*}. \quad (3.54)$$

Since singlet scalar coupling with the SM Higgs is very small, one requires a dominant decay channel of S such that it decays at least before BBN without overproducing DM. Due to $M_S > T_n$ applicable to the entire parameter space, dilution of singlet scalar abundance via annihilation is also suppressed. In the minimal model we are studying, DM overproduction from singlet scalar decay can be avoided only when $M_S > 2M_{N_{2,3}} \sim 2M_N$ keeping the branching ratio $\text{BR}(S \rightarrow \text{DM DM})$ small. We have chosen our benchmark points in table 3.7, 3.8 such that this condition is satisfied. The corresponding DM masses, consistent with correct relic abundance, are shown in the last column of table 3.8. On the left panel of Fig. 3.12, we show the parameter space in $M_S - M_N$ plane while indicating the region consistent with DM

relic criteria ($\diamond$ shaped points) and both DM and non-thermal leptogenesis criteria ($\star$ shaped points). On the right panel of the same figure, we show the mass of DM as a function of singlet scalar mass, which satisfy correct DM relic criteria (blue coloured points) and both DM and non-thermal leptogenesis criteria (red coloured points). We also check the contribution of SM Higgs decay to FIMP DM relic and find it to be negligible, due to the smallness of singlet-SM Higgs mixing. Thus, incorporating DM relic criteria puts tight constraints on the model parameter space.

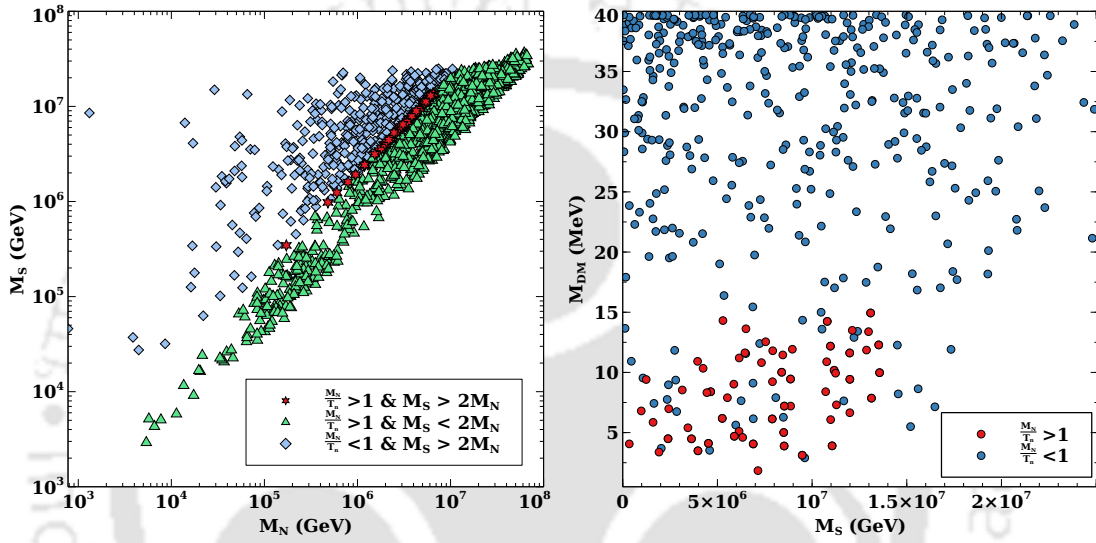


Figure 3.12: Left panel: The parameter space in M_N vs M_S plane where $\diamond$ shaped points are consistent with dark matter relic while $\triangle$ shaped points are not, and $\star$ shaped points satisfy dark matter relic as well as non-thermal leptogenesis criteria. Right panel: The parameter space in $M_S - M_{DM}$ plane which satisfies DM relic criteria. In both the plots, the points labelled as $M_N/T_n > 1$ are consistent with non-thermal leptogenesis criteria via mass-gain mechanism.

3.6 Conclusion

In this chapter, we have explored the simultaneous generation of dark matter and baryon asymmetry through leptogenesis, both originating from first-order phase transitions in the early Universe. We first considered a low-scale, supercooled FOPT driven by a TeV-scale singlet scalar field. In this scenario, the right-handed neutrinos (RHNs), responsible for generating lepton asymmetry, as well as the dark matter candidate, acquire their masses upon crossing relativistic bubble walls formed during

the phase transition. This also leads to a large abundance of RHN in true vacuum inside the bubble sufficient for generating the required lepton asymmetry without washout or Boltzmann suppression. Meanwhile, the dark matter particle, being lighter than the nucleation temperature, remains in thermal equilibrium after the transition and freezes out at later stages to yield the correct relic abundance. But, this possibility is excluded by large singlet scalar and Higgs mixing. We first studied a conformal version of the scotogenic model, which not only accounts for radiative light neutrino masses and a viable dark matter candidate, but also enables a strongly supercooled phase transition capable of generating gravitational waves within the sensitivity reach of future experiments such as LISA. While a strong supercooled FOPT is possible, the hierarchy of the additional field content of the model does not allow the realisation of leptogenesis from RHN decay. We then consider a hybrid scenario with the same field content but different seesaw realisation to show correct DM phenomenology from the lightest RHN while the heavier two RHNs can lead to successful TeV scale resonant leptogenesis. Light neutrino masses emerge through a hybrid mechanism combining both type-I and radiative seesaw contributions. Since the nature of the FOPT remains similar to the conformal case, the associated gravitational wave signal remains within reach of near-future detectors, while the TeV-scale RHNs and additional scalars offer complementary signatures at both energy and intensity frontier experiments.

We also considered a setup involving a high-scale FOPT, potentially producing a stochastic gravitational wave background detectable by ongoing experiments such as LIGO-VIRGO-KAGRA. In this high-scale framework, dark matter is naturally produced via a non-thermal mechanism, consistent with the so-called freeze-in or FIMP paradigm. The simultaneous requirements of successful leptogenesis and correct dark matter relic density impose significant constraints on the model's parameter space, including the mass spectrum of BSM particles. Interestingly, certain regions of this space predict a large signal-to-noise ratio in gravitational wave experiments and may already be under pressure from non-observation. Although probing such high-scale FOPT directly is experimentally challenging, the indirect signatures in gravitational waves, along with cosmological relic constraints, provide valuable tests of these scenarios. As more data from current and future gravitational wave detectors become available, they will play a crucial role in further constraining and

refining these unified frameworks addressing the origin of both dark matter and baryon asymmetry.



Dark matter and baryogenesis with PBH formed during FOPT

In this Chapter 4, we explore primordial black holes formed during a first-order phase transition in the early Universe, which can leave distinct cosmological imprints, depending on their masses. In particular, PBH arising from the collapse of non-topological solitons, such as Fermi-balls, represent a mechanism that links high-energy particle physics to observable cosmological phenomena. The mass of the resulting PBH plays a critical role in determining their impact: ultra-light PBH, with masses below 10^{15} g, evaporate via Hawking radiation well before the present epoch, and their evaporation can lead to the production of present day dark matter relic abundance through purely gravitational processes. On the other hand, PBH in the asteroid-mass range (10^{17} – 10^{23} g) can be long-lived and potentially constitute a substantial fraction or even the entirety of dark matter. In this chapter, we present both possibilities. The section 4.1 focuses on scenarios where ultra-light PBH evaporate and leave behind a dark matter relic, while the section 4.2 investigates the simultaneous generation of asteroid-mass PBH dark matter and baryon asymmetry through a cogenesis framework, establishing a novel connection between FOPT induced PBH formation and the matter content of the Universe.

4.1 Dark matter from PBH evaporation with gravitational wave signatures

Primordial black holes originally proposed in [172] and later by Hawking [175, 275] can have several interesting cosmological consequences [65, 173]. A recent review of PBH can be found in [72]. Depending upon the initial mass (m_{in}) and initial energy fraction (β), PBH can face different astrophysical and cosmological constraints. Among them, the ultra-light PBH mass window is relatively less constrained as such PBH evaporate by emitting Hawking radiation [175, 275] before the big bang nucleosynthesis (BBN) epoch. Comparing the evaporation temperature with the BBN temperature $T_{\text{ev}} > T_{\text{BBN}} \simeq 4 \text{ MeV}$ leads to the upper bound on this ultra-light PBH mass window. On the other hand, a lower bound can be obtained by using the upper limit on the scale of inflation from cosmic microwave background (CMB) measurements. Using these BBN and CMB bounds simultaneously leads to the allowed initial mass for ultra-light PBH that reads $0.1 \text{ g} \lesssim m_{\text{in}} \lesssim 3.4 \times 10^8 \text{ g}$. As mentioned above, the range of PBH masses in this window is relatively unconstrained [72]. It should be noted that the upper bound is obtained only for PBH which evaporates on a timescale smaller than the age of the universe and have a large initial fraction β to dominate the energy density at some epoch. Such evaporation of ultra-light PBH can also lead to the production of dark matter (DM) in the universe [276–292]. As suggested by observations in astrophysics and cosmology based experiments, approximately 26% of the present universe is made up of DM, the particle origin of which is not yet known. If DM has only gravitational interactions as observations indicate, the conventional thermal freeze-out mechanism fails and PBH evaporation provides a very good alternative.

While DM with only gravitational interactions may not have much detection prospects, the ultra-light PBH and their formation mechanisms in the early universe can lead to promising detection prospects. In spite of this relatively less constrained mass window, it is still possible to probe an ultra-light PBH dominated epoch in the early universe via detection of stochastic gravitational waves background ⁱ⁾. Interestingly, detection of stochastic GW can also probe the formation mechanism of PBH in the early universe. In the early universe, PBH can be formed in a variety of ways

ⁱ⁾See Ref. [166] for a recent review on stochastic gravitational wave background.

like, from inflationary perturbations [67, 171, 176–178], first-order phase transition [68, 69, 179–189], the collapse of topological defects [190, 191] etc. Such mechanisms often come up with their own unique GW spectrum. On the other hand, PBH can lead to generation of GW in a variety of ways. The Hawking evaporation of PBH itself can produce gravitons which might constitute an ultra-high frequency GW spectrum [201]. PBH can also lead to GW emission by forming successive mergers [293]. In addition, the scalar perturbations leading to the formation of PBH can induce GW at second-order [203], which can be further enhanced during the PBH evaporation [294]. Finally, the inhomogeneity in the distribution of PBH may also induce GW at second order, as recently studied in [207, 295, 296]. In the present work, we consider the last possibility which, for ultra-light PBH mass window, can lead to stochastic GW with peak frequencies in the ballpark of ongoing and planned future experiments.

We study the possibility of simultaneously probing the signatures of a formation mechanism of PBH and their evaporation via detection of stochastic GW background with the PBH evaporation being the dominant production channel for dark matter. Recently, FOPT origin of heavy PBH was studied by the authors of [297–302]. Here, we consider the ultra-light PBH mass window, not considered in earlier works and study the possibility of forming them in a FOPT. The combined GW spectrum arising out of the FOPT and density perturbations of ultra-light PBH consists of a unique doubly-peaked feature. For the PBH mass range of interest, the peak in GW spectrum due to FOPT occurs at a frequency higher than the one due to PBH density perturbations. Depending upon the model parameters, the peak frequencies and corresponding blue and red tilted parts of the GW spectrum remain within the sensitivities of several planned experiments like DECIGO [263], BBO [88], ET [213], CE [214], UDECIGO (UDECIGO-corr) [303, 304] etc. While doubly peaked GW spectrum arises in a scenario with PBH domination, the DM parameter space is typically restricted to the superheavy ballpark. In sub-dominant PBH scenario, DM mass range gets much wider whereas the GW spectrum generated by FOPT comes within the sensitivities of several future experiments. Depending upon the specific scenario, DM masses from GeV to superheavy regime and PBH mass as low as a few kg remains within sensitivities of such GW experiments. It is worth mentioning that in most of the earlier works studying DM production from PBH evaporation, the formation mechanism of the latter is not included. The inclusion

of the PBH formation mechanism not only leads to a tighter parameter space but also to complementary detection prospects.

This section is organised as follows. In section 4.1.1, we outline the particle physics setup responsible for FOPT followed by discussion of PBH formation in section 4.1.2. In section 4.1.3 and section 4.1.4, we discuss the stochastic GW generated by FOPT and PBH density fluctuations respectively. In section 4.1.5, we discuss the combined GW spectrum arising from FOPT and PBH followed by implications for DM phenomenology in section 4.1.6. We briefly comment on other FOPT origin of PBH not discussed in this work and the corresponding GW signatures in section 4.1.7. We finally conclude in section 4.1.8.

4.1.1 The setup

Among different scenarios relating FOPT to PBH formation [68, 69, 179–189], we consider the false vacuum collapse as the origin of PBH formation. In particular, we consider the collapsing Fermi-ball scenario [69, 184, 188, 189, 298, 305–308]. In this scenario, a dark sector fermion χ with an asymmetry between χ and $\bar{\chi}$ leads to the formation of Fermi-balls after getting trapped in the false vacuum. Due to the strong Yukawa force among the dark fermions, such Fermi-balls can collapse into PBH. As the dark sector is asymmetricⁱⁱ⁾ and carries a conserved charge, we consider a global dark $U(1)_D$ symmetry which remains unbroken as the Universe goes through a FOPT. The dark sector fermion χ has a charge q_χ under this $U(1)_D$ global symmetry while all other fields are neutral. We consider the FOPT to be driven by a singlet scalar Φ . The dark sector fermion χ is coupled to Φ through Yukawa interaction. Since fermion and bosons contribute with opposite signs to the effective potential, and fermion χ has large Yukawa coupling with Φ , additional bosonic degrees of freedom are required to have an effective potential consistent with a strong FOPT while keeping all dimensionless couplings within perturbative limits. We consider an additional singlet scalar Φ' and doublet scalar H_2 for this purpose. However, this choice is arbitrary. For example, if we embed this setup within a new gauge symmetry under which Φ transforms non-trivially, such extra scalar fields are

ⁱⁱ⁾Here, we remain agnostic about the origin of such an asymmetry. Example of a UV complete model can be found in [184]. The GW phenomenology discussed here is generic and independent of such UV completions.

no longer required. These additional scalars are also neutral under $U(1)_D$ global symmetry and their couplings with Φ are the only relevant ones for our analysis.

The relevant Lagrangian of these newly introduced fields is given by

$$\mathcal{L} \supset -g_\chi \Phi \bar{\chi} \chi - m_\chi \bar{\chi} \chi - V(\Phi, \Phi', H_2) \quad (4.1)$$

where, the scalar potential (suppressing a few allowed terms for simplicity) is

$$\begin{aligned} V(\Phi, \Phi', H_2) = & \lambda_D \left(|\Phi|^2 - \frac{v_D^2}{2} \right)^2 + m_{\Phi'}^2 |\Phi'|^2 + \lambda_{\Phi'} |\Phi'|^4 + \lambda_{\Phi\Phi'} |\Phi|^2 |\Phi'|^2 + m_{H_2}^2 |H_2|^2 \\ & + \lambda_{H_2} |H_2|^4 + \lambda_{\Phi H_2} |\Phi|^2 |H_2|^2 + \lambda_{\Phi H} |\Phi|^2 |H|^2. \end{aligned} \quad (4.2)$$

Here, v_D is the vacuum expectation value of the singlet scalar $\Phi (\equiv \phi/\sqrt{2})$. In order to realise the electroweak vacuum, the coupling $\lambda_{\Phi H}$ with the Standard Model Higgs H is suppressed. The one-loop correction to the tree-level potential is computed considering the couplings of all the particles to the singlet scalar Φ and this correction is known as Coleman-Weinberg potential [92], which can be written as

$$V_{\text{CW}}(\phi) = \frac{1}{64\pi^2} \sum_{i=\phi, \phi', H_2, \chi} n_i M_i^4(\phi) \left\{ \log \left(\frac{M_i^2(\phi)}{v^2} \right) - C_i \right\} \quad (4.3)$$

where, $n_\Phi = 1$, $n_{\Phi'} = 1$, $n_{H_2} = 4$ and $n_\chi = 4$ and $C_{\Phi, \Phi', H_2, \chi} = \frac{3}{2}$. The physical field-dependent masses of all particles are

$$\begin{aligned} M_\Phi^2(\phi) &= \lambda_D (3\phi^2 - v_D^2), \quad M_{\Phi'}^2(\phi) = m_{\Phi'}^2 + \frac{\lambda_{\Phi\Phi'} \phi^2}{2}, \\ M_{H_2}^2(\phi) &= m_{H_2}^2 + \frac{\lambda_{\Phi H_2} \phi^2}{2}, \quad M_\chi^2(\phi) = (m_\chi + g_\chi \phi)^2. \end{aligned}$$

Now, the thermal contributions to the effective potential [93, 97] can be expressed as

$$V_T(\phi, T) = \sum_{i=\Phi, \Phi', H_2} \frac{n_i T^4}{2\pi^2} J_B \left(\frac{M_i^2(\phi)}{T^2} \right) - \frac{n_\chi T^4}{2\pi^2} J_F \left(\frac{M_\chi^2(\phi)}{T^2} \right) \quad (4.4)$$

where

$$J_F(x) = \int_0^\infty dy y^2 \log[1 + e^{-\sqrt{y^2 + x^2}}], \quad J_B(x) = \int_0^\infty dy y^2 \log[1 - e^{-\sqrt{y^2 + x^2}}].$$

Besides this, the Daisy corrections [101, 102, 239] are also incorporated in thermal contribution to improve perturbative expansion considering Arnold-Espinosa method [102] during FOPT such that $V_{\text{thermal}}(\phi, T) = V_T(\phi, T) + V_{\text{daisy}}(\phi, T)$. The Daisy contribution can be written as

$$V_{\text{daisy}}(\phi, T) = -\frac{T}{2\pi^2} \sum_{i=\Phi', H_2} n_i [M_i^3(\phi, T) - M_i^3(\phi)] \quad (4.5)$$

where, $M_i^2(\phi, T) = M_i^2(\phi) + \Pi_i(T)$ and the relevant thermal masses are

$$M_{\Phi'}^2(\phi, T) = m_{\Phi'}^2(\phi) + \left(\frac{\lambda_{\Phi'}}{4} + \frac{\lambda_{\Phi\Phi'}}{6} \right) T^2, \quad M_{H_2}^2(\phi, T) = m_{H_2}^2(\phi) + \left(\frac{\lambda_{H_2}}{4} + \frac{\lambda_{\Phi H_2}}{12} \right) T^2.$$

So, at finite temperature, the effective potential can be written as

$$V_{\text{eff}}(\phi, T) = V_{\text{tree}}(\phi) + V_{\text{CW}}(\phi) + V_{\text{thermal}}(\phi, T). \quad (4.6)$$

At high temperatures, the Universe lies in a vacuum $\langle\phi\rangle=0$. When the temperature decreases to a critical temperature T_c , two degenerate vacua are created which are separated by a barrier. Below this critical temperature, the Universe tunnels through the barrier from false vacuum $\langle\phi\rangle=0$ to true vacuum $\langle\phi\rangle \neq 0$. The rate of tunnelling is estimated by calculating the bounce solution [122]. The tunnelling rate per unit volume is defined in terms of O(3) symmetric bounce action $S_3(T)$ as

$$\Gamma(T) \sim T^4 e^{-S_3(T)/T}. \quad (4.7)$$

The nucleation temperature T_n is calculated by comparing the tunnelling rate with the Hubble expansion rate as

$$\Gamma(T_n) = \mathbf{H}^4(T_n) = \mathbf{H}_*^4. \quad (4.8)$$

The volume fraction of false vacuum of the Universe is defined by $p(T) = e^{-\mathcal{I}(T)}$ [245, 246], where

$$\mathcal{I}(T) = \frac{4\pi}{3} \int_T^{T_c} \frac{dT'}{T'^4} \frac{\Gamma(T')}{\mathbf{H}(T')} \left(\int_T^{T'} \frac{d\tilde{T}}{\mathbf{H}(\tilde{T})} \right)^3. \quad (4.9)$$

The percolation temperature T_p is then calculated by using $\mathcal{I}(T_p) = 0.34$ [245]

indicating that at least 34% of the comoving volume is occupied by the true vacuum. After the nucleation, an amount of energy is released as latent heat related to the change of energy-momentum tensor across the bubble wall during the FOPT [151, 236] given by

$$\alpha_* = \frac{\epsilon}{\rho_{\text{rad}}} \quad (4.10)$$

where, $\epsilon = (\Delta V_{\text{eff}} - \frac{T}{4} \frac{\partial \Delta V_{\text{eff}}}{\partial T})_{T=T_n}$, $\Delta V_{\text{eff}} = V_{\text{eff}}(\phi_{\text{false}}, T) - V_{\text{eff}}(\phi_{\text{true}}, T)$ and $\rho_{\text{rad}} = g_* \pi^2 T^4/30$. Here, g_* denotes the total relativistic degrees of freedom. The time span of FOPT is denoted by the parameter $\beta/\mathbf{H}(T) \simeq T \frac{d}{dT} (\frac{S_3}{T})$, whereas, the bubble wall velocity v_w , in general, is related to the Jouguet velocity $v_J = \frac{1/\sqrt{3} + \sqrt{\alpha_*^2 + 2\alpha_*/3}}{1 + \alpha_*}$. For the type of FOPT we are considering, $v_w \approx v_J$ [146, 152, 255] whereas in the presence of supercooling they are related non-trivially [260]. In these analyses, the calculation for the action $S_3(T)$ can be performed numerically by fitting the effective potential given by Eq. (4.6) to a generic potential [309]. The details of this procedure is described in Ref. [273].

4.1.2 PBH formation from Fermi-ball

As the FOPT proceeds, the true vacuum expands to cover the whole Universe, while the false vacuum gradually shrinks to a smaller region. At a later stage, the false vacuum region contracted into smaller disconnected volumes, which again split to negligible size at percolation temperature [245]. Within these final pockets of false vacuum, some pre-existing asymmetry of χ fermions can get trapped and the symmetric part annihilates via $\chi\chi \rightarrow S$ and $\chi\chi \rightarrow SS$, to final state particles S (either SM particles or ϕ) through decay or annihilation. Now, the trapped χ 's create a degeneracy pressure. If such a pressure is able to balance the vacuum pressure, these χ remnants form a bound state which is known as Fermi-ball [69, 199, 310, 311]. These Fermi-balls formed at T_* defined by $p(T_*)=0.29$. The energy of a Fermi-ball can be written as

$$E_{\text{FB}} = \frac{3\pi}{4} \left(\frac{3}{2\pi} \right)^{2/3} \frac{Q_{\text{FB}}^{4/3}}{R} + \frac{4\pi}{3} U_0(T_*) R_{\text{FB}}^3, \quad (4.11)$$

where, $Q_{\text{FB}} = F_{\chi}^{\text{trap}} \frac{n_{\chi} - n_{\bar{\chi}}}{n_{\text{FB}}^*}$ with n_X denoting number density of X , $F_{\chi}^{\text{trap}} \approx 1$ for maximum trapping of χ inside false vacuum and U_0 is vacuum energy. Now, minimizing

the Fermi-ball energy, we can get the mass and radius as

$$M_{\text{FB}} = Q_{\text{FB}}(12\pi^2 U_0(T_*))^{1/4}, \quad R_{\text{FB}}^3 = \frac{3}{16\pi} \frac{M_{\text{FB}}}{U_0(T_*)}. \quad (4.12)$$

Within the Fermi-balls, the χ 's interact through attractive Yukawa interaction $g_\chi \phi \bar{\chi} \chi$ and the interaction range is $L_\phi(T) = \left(\frac{d^2 V_{\text{eff}}}{d\phi^2} \Big|_{\phi=0} \right)^{-1/2}$ [305]. The Yukawa potential energy contribution to the Fermi-ball energy can be expressed as

$$E_Y \simeq -\frac{3g_\chi^2}{8\pi} \frac{Q_{\text{FB}}^2}{R_{\text{FB}}} \left(\frac{L_\phi}{R_{\text{FB}}} \right)^2. \quad (4.13)$$

At a later stage, when the Yukawa potential energy becomes larger than the Fermi-ball energy, the Fermi-ball becomes unstable and collapses to form PBH [69]. We consider that the Fermi-ball formation temperature is close to the nucleation temperature, $T_* \sim T_n$. The initial PBH mass during formation can be estimated in terms of FOPT parameters as [69]

$$M_{\text{in}} \sim 1.4 \times 10^{21} \times v_w^3 \left(\frac{\eta_\chi}{10^{-3}} \right) \left(\frac{100}{g_*} \right)^{1/4} \left(\frac{100 \text{ GeV}}{T_*} \right)^2 \left(\frac{100}{\beta/\mathbf{H}_*} \right)^3 \alpha_*^{1/4} \text{ g}. \quad (4.14)$$

The initial temperature of the black hole is related to its mass as $T_{\text{BH}}^{\text{in}} \simeq 10^{13} \text{ GeV} \left(\frac{1 \text{ g}}{M_{\text{in}}} \right)$. The initial abundance of PBH which is characterised by the parameter β_{PBH} , defined as the ratio of the energy density of PBH to the total energy density, is obtained to be [69]

$$\beta_{\text{PBH}} \sim 1.4 \times 10^{-15} \times v_w^{-3} \left(\frac{g_*}{100} \right)^{1/2} \left(\frac{T_*}{100 \text{ GeV}} \right)^3 \left(\frac{\beta/\mathbf{H}_*}{100} \right)^3 \left(\frac{M_{\text{in}}}{10^{15} \text{ g}} \right)^{3/2}. \quad (4.15)$$

Now, if the initial abundance of PBH is large enough, PBH can dominate the energy density of the Universe over radiation. This puts a lower bound on β_{PBH} for PBH domination, given by

$$\beta_{\text{PBH}} \gtrsim \beta_{\text{crit}} \simeq \frac{T_{\text{evap}}}{T_n}, \quad (4.16)$$

where $T_{\text{evap}} \simeq \left(\frac{9g_*(T_{\text{BH}}^{\text{in}})}{10240} \right)^{1/4} \left(\frac{M_{\text{P}}^5}{M_{\text{in}}^3} \right)^{1/2}$ indicates the temperature of the thermal bath when PBH evaporates. Solving Eq. (4.14) and (4.15), we can connect the FOPT

parameters T_n and $\beta/\mathbf{H}_*$ to the PBH parameters M_{in} and β_{PBH} as

$$T_n = 1.61 \times 10^{18} \beta_{\text{PBH}} \left(\frac{100}{g_*} \right)^{1/4} \left(\frac{M_{\text{in}}}{1 \text{ g}} \right)^{-1/2} \left(\frac{\eta_\chi}{10^{-3}} \right)^{-1} \frac{1}{\alpha_*^{1/4}} \text{ GeV}, \quad (4.17)$$

$$\beta/\mathbf{H}_* = 1.75 \times 10^{-2} \alpha_*^{1/4} \left(\frac{\eta_\chi}{10^{-3}} \right) v_w \left(\frac{g_*}{100} \right)^{1/12} \beta_{\text{PBH}}^{-2/3}. \quad (4.18)$$

It should be noted that, we require to maintain $g_\chi v_D > T_n$ to ensure that the fermion χ 's get trapped inside the false vacuum and its penetration to the true vacua is kinematically disfavoured.

4.1.3 GW from FOPT

The stochastic GW spectrum from a FOPT can be estimated by considering all the relevant contributions from bubble collisions [123, 124, 140, 248, 249], the sound wave [130, 131, 134, 149] and the turbulence [135, 136, 138, 146, 250, 251] of the plasma medium. So, the stochastic GW spectrum can be written as [145]

$$\Omega_{\text{GW}}^{\text{PT}}(f) = \Omega_\phi(f) + \Omega_{\text{sw}}(f) + \Omega_{\text{turb}}(f), \quad (4.19)$$

where Ω_ϕ , Ω_{sw} , Ω_{turb} are individual contributions from bubble collisions, sound wave of the plasma and turbulence in the plasma respectively. The GW spectrum for bubble collision can be written as [145]

$$\Omega_\phi h^2 = 1.67 \times 10^{-5} \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{\mathbf{H}_*}{\beta} \right)^2 \left(\frac{\kappa \alpha_*}{1 + \alpha_*} \right)^2 \frac{0.11 v_w^3}{0.42 + v_w^2} \frac{3.8 (f/f_{\text{peak}}^{\text{PT},\phi})^{2.8}}{1 + 2.8 (f/f_{\text{peak}}^{\text{PT},\phi})^{3.8}}, \quad (4.20)$$

with the peak frequency [145] given by

$$f_{\text{peak}}^{\text{PT},\phi} = 1.65 \times 10^{-5} \text{ Hz} \left(\frac{g_*}{100} \right)^{1/6} \left(\frac{T_n}{100 \text{ GeV}} \right) \frac{0.62}{1.8 - 0.1 v_w + v_w^2} \left(\frac{\beta}{\mathbf{H}_*} \right). \quad (4.21)$$

The efficiency factor κ_ϕ for bubble collision can be expressed as [146]

$$\kappa_\phi = \frac{1}{1 + 0.715 \alpha_*} \left(0.715 \alpha_* + \frac{4}{27} \sqrt{3 \alpha_*/2} \right). \quad (4.22)$$

The GW spectrum produced from the sound wave in the plasma can be written as [132, 145, 151]

$$\Omega_{\text{sw}} h^2 = 2.65 \times 10^{-6} \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{\mathbf{H}_*}{\beta} \right) \left(\frac{\kappa_{\text{sw}} \alpha_*}{1 + \alpha_*} \right)^2 v_w (f/f_{\text{peak}}^{\text{PT,sw}})^3 \left(\frac{7}{4 + 3(f/f_{\text{peak}}^{\text{PT,sw}})^2} \right)^{7/2} \Upsilon \quad (4.23)$$

and the corresponding peak frequency is given by [145]

$$f_{\text{peak}}^{\text{PT,sw}} = 1.65 \times 10^{-5} \text{Hz} \left(\frac{g_*}{100} \right)^{1/6} \left(\frac{T_n}{100 \text{ GeV}} \right) \left(\frac{\beta}{\mathbf{H}_*} \right) \frac{2}{\sqrt{3}}. \quad (4.24)$$

The efficiency factor for sound wave can be expressed as [152]

$$\kappa_{\text{sw}} = \frac{\sqrt{\alpha_*}}{0.135 + \sqrt{0.98 + \alpha_*}}. \quad (4.25)$$

The suppression factor $\Upsilon = 1 - \frac{1}{\sqrt{1 + 2\tau_{\text{sw}} H_*}}$ depends on the lifetime of sound wave τ_{sw} [132] given by $\tau_{\text{sw}} \sim R_*/\bar{U}_f$ where mean bubble separation is $R_* = (8\pi)^{1/3} v_w \beta^{-1}$ and rms fluid velocity is $\bar{U}_f = \sqrt{3\kappa_{\text{sw}} \alpha_*/4}$.

Finally, the GW spectrum generated by the turbulence in the plasma is given by [145]

$$\Omega_{\text{turb}} h^2 = 3.35 \times 10^{-4} \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{\mathbf{H}_*}{\beta} \right) \left(\frac{\kappa_{\text{turb}} \alpha_*}{1 + \alpha_*} \right)^{3/2} v_w \frac{(f/f_{\text{peak}}^{\text{PT,turb}})^3}{(1 + f/f_{\text{peak}}^{\text{PT,turb}})^{11/3} (1 + 8\pi f/h_*)} \quad (4.26)$$

with the peak frequency as [145]

$$f_{\text{peak}}^{\text{PT,turb}} = 1.65 \times 10^{-5} \text{Hz} \left(\frac{g_*}{100} \right)^{1/6} \left(\frac{T_n}{100 \text{ GeV}} \right) \frac{3.5}{2} \left(\frac{\beta}{\mathbf{H}_*} \right). \quad (4.27)$$

The efficiency factor for turbulence is $\kappa_{\text{turb}} \simeq 0.1\kappa_{\text{sw}}$ and the inverse Hubble time at the epoch of GW emission, redshifted to today is

$$h_* = 1.65 \times 10^{-5} \frac{T_n}{100 \text{ GeV}} \left(\frac{g_*}{100} \right)^{1/6}. \quad (4.28)$$

It is clear from the above expressions that the contribution from sound waves turns out to be the most dominant one and the peak of the total GW spectrum corresponds to the peak frequency of sound waves contribution.

4.1.4 GW from PBH density fluctuations

While there are several ways in which PBH can give rise to GW [201, 203, 293, 312], for our purpose of ultralight PBH, the GW spectrum generated by most of these mechanisms lies at very high frequencies. Hence, we rather focus on GW generated because of the inhomogeneity in the PBH distribution after they are formed [205–207, 295, 296]. Such a spectrum is independent of the formation mechanism of PBH, and is within reach of near-future GW detectors.

After PBH are formed, they are inhomogeneously distributed in space following Poisson statistics [295]. These inhomogeneities can induce GW at second order once PBH dominates the energy density of the Universe [207, 295], which gets further enhanced during PBH evaporation [207]. The dominant contribution to the GW spectrum observed today can be written as [207, 290, 296, 313]

$$\Omega_{\text{GW}}^{\text{PBH}} \simeq \Omega_{\text{peak}}^{\text{PBH}} \left(\frac{f}{f_{\text{peak}}^{\text{PBH}}} \right)^{11/3} \Theta(f_{\text{peak}}^{\text{PBH}} - f). \quad (4.29)$$

Note that there exists an ultraviolet cutoff that corresponds to length scales smaller than the mean separation between PBH, below which the PBH cannot be treated as a continuous fluid. The associated frequency is estimated to be

$$f_{\text{peak}}^{\text{PBH}} \simeq 1.7 \times 10^3 \text{ Hz} \left(\frac{M_{\text{in}}}{10^4 \text{ g}} \right)^{-5/6}. \quad (4.30)$$

The peak amplitude, on the other hand, is given by

$$\Omega_{\text{peak}}^{\text{PBH}} \simeq 9.67 \times 10^{-11} \left(\frac{k_{\text{UV}}}{k_{\text{eva}}} \right)^{17/3} \left(\frac{k_{\text{eq}}}{k_{\text{UV}}} \right)^8, \quad (4.31)$$

where k_{UV} corresponds to the comoving wavenumber corresponding to the mean separation between PBH, whereas k_{eq} , k_{eva} indicates the comoving wavenumber of the modes entering the horizon when PBH begins to dominate and when they evaporate respectively. The ratio of the comoving wavenumbers are found using [207] $k_{\text{UV}}/k_* = (\frac{3M_{\text{in}}}{4\pi\rho_{\text{PBH}}^{\text{in}}})^{-1/3} \mathbf{H}_*^{-1}$, $k_{\text{eq}}/k_* = \sqrt{2}\beta_{\text{PBH}}^{2/3}$ and $k_{\text{eva}}/k_* = (\beta_{\text{PBH}} H_{\text{eva}}/\mathbf{H}_*)^{1/3}$, where $\rho_{\text{PBH}}^{\text{in}}$, k_* and H_{eva} ($\mathbf{H}_*$) denotes the initial energy density of PBH, wavenumber at the time of PBH formation, Hubble parameter at the time of PBH evaporation

(formation) respectively. Expressing explicitly in terms of the PBH formation temperature $T_* \simeq T_n$, we get

$$\frac{k_{\text{UV}}}{k_{\text{eva}}} \simeq 10^6 \left(\frac{M_{\text{in}}}{10^4 \text{g}} \right)^{2/3}, \quad (4.32)$$

$$\frac{k_{\text{eq}}}{k_{\text{UV}}} \simeq 3.18 \times 10^{-11} (\beta_{\text{PBH}})^{2/3} \left(\frac{M_{\text{in}}}{1 \text{g}} \right)^{1/3} \left(\frac{T_n}{1 \text{GeV}} \right)^{2/3}. \quad (4.33)$$

Using the above ratio of wavenumbers in Eq. (4.31), we can write the GW amplitude as

$$\Omega_{\text{peak}}^{\text{PBH}} \simeq 4.75 \times 10^{-20} (\beta_{\text{PBH}})^{16/3} \left(\frac{M_{\text{in}}}{10^7 \text{g}} \right)^{58/9} \left(\frac{T_n}{100 \text{GeV}} \right)^{16/3}. \quad (4.34)$$

Note that for PBH formed through Fermi-ball collapse, T_n in turn depends on the PBH parameters and the asymmetry η_χ through Eq. (4.17).

Now, since GW behave as radiation, they can contribute to extra relativistic degrees of freedom during the epoch of BBN. GW from PBH density fluctuations can violate the BBN bound from Planck, if the GW amplitude is very large. This translates into an upper bound on the initial PBH fraction β_{PBH} , which we find to be given by

$$\beta_{\text{PBH}} \lesssim \beta_{\text{max}} \simeq 1.8 \times 10^{-4} \left(\frac{M_{\text{in}}}{10^4 \text{g}} \right)^{-17/32} \left(\frac{\alpha}{0.1} \right)^{3/16} \left(\frac{\beta/\mathbf{H}_*}{10} \right)^{3/4}. \quad (4.35)$$

4.1.5 The combined spectra and detection prospects

If PBH are formed during the phase transition through Fermi-ball collapse, they can leave their indirect imprints in the GW spectrum. Considering the dominant sound wave contribution and using Eq. (4.17) and (4.18), we can analytically relate the FOPT GW spectrum to the PBH mass and initial abundance approximately as

$$f_{\text{peak}}^{\text{PT}} \simeq \frac{5.33 \times 10^9}{S^{1/3}} \beta_{\text{PBH}}^{1/3} \left(\frac{M_{\text{in}}}{1 \text{g}} \right)^{-1/2} \text{Hz}, \quad (4.36)$$

$$\Omega_{\text{peak}}^{\text{PT}} h^2 \simeq \frac{1.42 \times 10^{-7}}{S^{4/3}} \beta_{\text{PBH}}^{2/3} \alpha_*^{-1/4} \eta_\chi^{-1} \left(\frac{\kappa_{\text{sw}} \alpha_*}{1 + \alpha_*} \right)^2. \quad (4.37)$$

Here, the factor S accounts for the extra entropy dilution after the gravitational waves from FOPT are produced, which is because of PBH domination and their subsequent evaporation. This makes the frequency and the FOPT generated GW

amplitude more red-shifted, compared to the usual scenario without PBH domination. The factor S can be approximated as

$$S \simeq \frac{T_{\text{dom}}}{T_{\text{eva}}} = \beta \frac{T_n}{T_{\text{eva}}}, \quad (4.38)$$

where T_{dom} indicates the temperature of the Universe when PBH starts to dominate. Now, note that the peak frequency depends explicitly both on the PBH mass as well as the initial abundance of PBH, but not on the asymmetry η_χ . On the other hand, the peak amplitude is not dependent on the PBH mass explicitly, but is sensitive to the initial abundance of PBH as well as the asymmetry η_χ .

Considering the contribution of GW from PBH density perturbations, the combined spectrum exhibits a unique doubly peaked feature. While the peak at the higher frequency corresponds to the contribution from FOPT, the sharp peak at the lower frequency is the contribution from PBH density fluctuations. In Fig. 4.1, we show the combined GW spectrum varying the initial PBH mass M_{in} (left panel) and initial PBH abundance β_{PBH} for a fixed PBH mass (right panel). Note that changing M_{in} also changes β_{PBH} in the left panel (cf. Eq. (4.15)). The other details of these benchmark points are given in table 4.1. In all these plots, the experimental sensitivities of SKA [154], GAIA [155], THEIA [155], μARES [156], LISA [262], AEDGE [314], DECIGO [263], UDECIGO (UDECIGO-corr) [303, 304], BBO [88], ET [213], CE [214] and aLIGO (HL) [264] are shown as shaded regions of different colours. The horizontal dashed red-coloured line corresponds to the BBN limit on effective relativistic degrees of freedom from GW.

The value of the initial asymmetry η_χ depends on the details of the χ dynamics in the early Universe before the phase transition, and is generally unconstrained. However, the constraints on the PBH abundance β_{PBH} can constrain η_χ , since for PBH formation from Fermi-ball collapse, η_χ is related to β_{PBH} through Eq. (4.17), (4.18). In Fig. 4.2, we show the allowed η_χ values in the $M_{\text{PBH}} - \eta_\chi$ plane, along with corresponding values of the temperature T_n in colour code for two different values of $\beta/\mathbf{H}_*$. The grey shaded upper triangular region is ruled out since here $\beta_{\text{PBH}} > \beta_{\text{max}}$, leading to overproduction of GW violating BBN bounds, as discussed earlier. On the other hand, in the grey shaded lower triangular region, β_{PBH} turns out to be less than β_{crit} , which does not lead to PBH domination. In such a case,

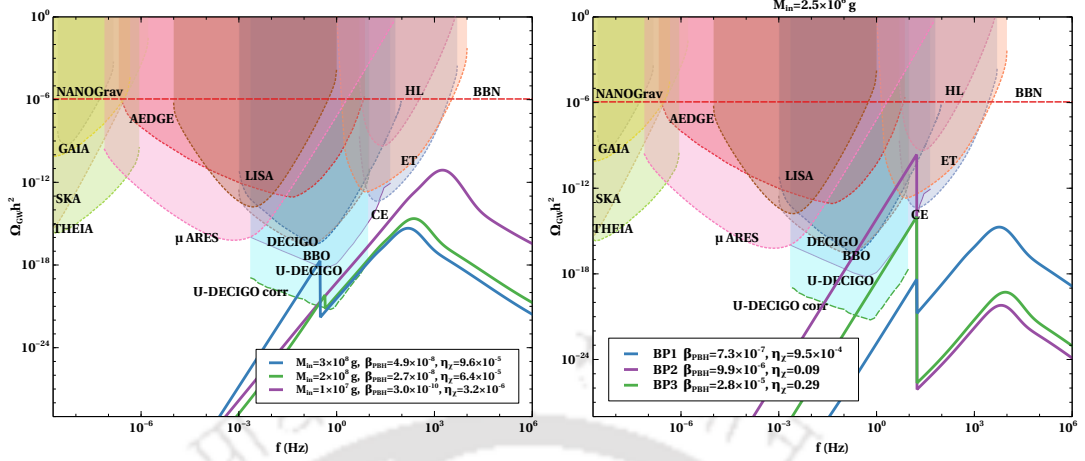


Figure 4.1: Combined GW spectrum from phase transition and PBH density fluctuations for BP4 (see Table 4.1), considering different values of M_{in} (left panel), and for BP1, BP2 and BP3 for a fixed M_{in} (right panel) but different values of β_{PBH} .

	v_D (GeV)	g_χ	$\lambda_{\Phi\Phi'}$	$\lambda_{\Phi H_2}$	T_c (GeV)	T_n (GeV)	T_p (GeV)	T_* (GeV)	β/H_*	α_*	v_J
BP1	10^{10}	1.4	1.8	2.5	2.8×10^9	1.6×10^9	1.3×10^9	1.3×10^9	95	0.12	0.78
BP2	2×10^9	1.5	1.9	2.3	3.3×10^8	2.6×10^8	2.6×10^8	2.6×10^8	1014	0.03	0.70
BP3	10^9	1.4	1.8	2.3	2.9×10^8	2.3×10^8	2.2×10^8	2.2×10^8	1729	0.04	0.71
BP4	10^9	1.4	1.8	2.5	2.1×10^8	1.2×10^8	7.9×10^7	7.9×10^7	58	0.12	0.78

Table 4.1: Benchmark parameters considered in the analysis.

GW from density fluctuations do not exist, leading to only a single peak feature in the GW spectra, arising from the phase transition. The double peak feature is also absent if the peak of GW contribution from PBH density fluctuation falls below the corresponding GW amplitude from FOPT, i.e. $\Omega_{\text{peak}}^{\text{PBH}} < \Omega_{\text{GW}}^{\text{PT}}(f_{\text{peak}}^{\text{PBH}})$. As the GW peak arising from the FOPT remains outside the sensitivities of low frequency experiments for most part of the parameter space, we show the parameter space and sensitivities to the GW peak arising from PBH density perturbations in Fig. 4.3. However, for sub-dominant PBH, the GW peak from FOPT does not suffer any entropy dilution and can be brought within sensitivities of several experiments which we discuss below along with the corresponding implications for DM phenomenology.

4.1.6 Implications for DM phenomenology

As seen in the previous section, if PBH dominates the energy density of the Universe we observe a unique doubly-peaked spectrum. However, the second peak at

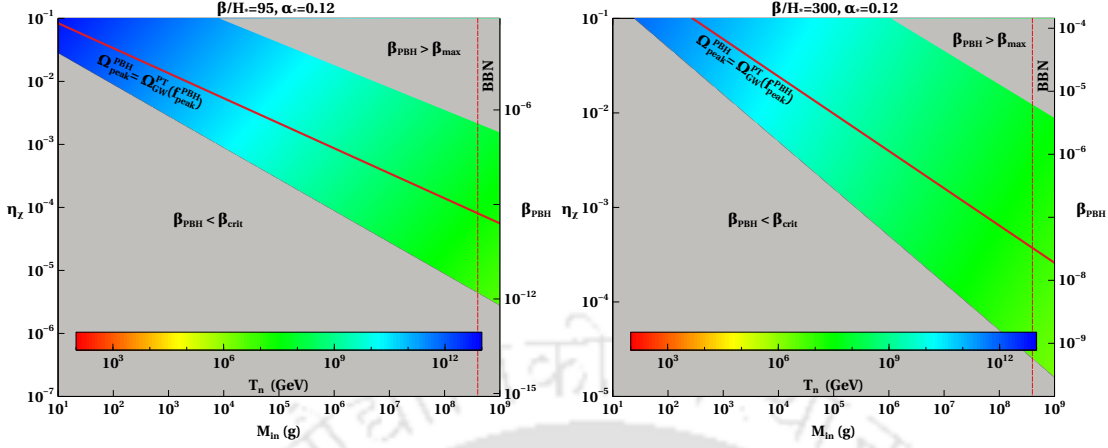


Figure 4.2: $M_{\text{in}} - \eta_\chi$ parameter space with $\beta/H_* = 95$, $\alpha_* = 0.12$ (left panel) and $\beta/H_* = 300$, $\alpha_* = 0.12$ (right panel).

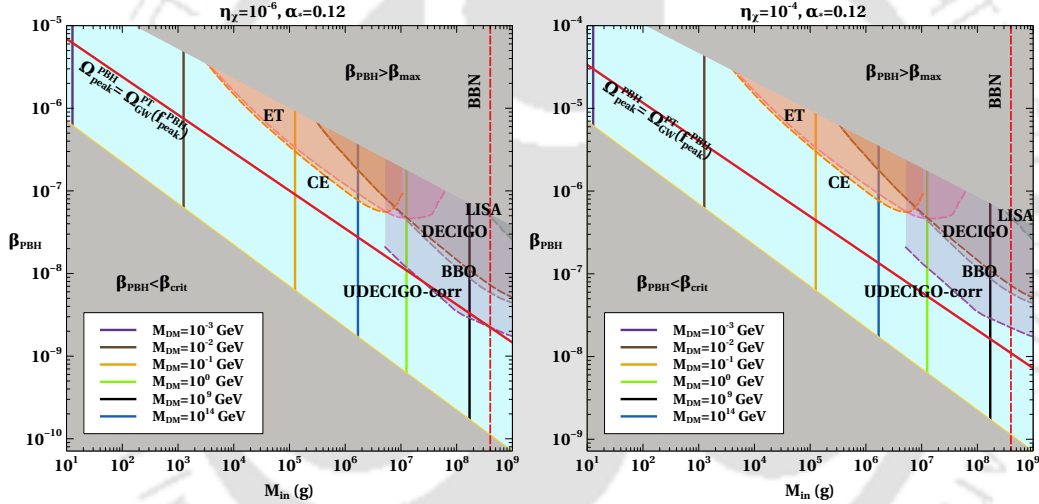


Figure 4.3: $M_{\text{in}} - \beta_{\text{PBH}}$ parameter space for $\eta_\chi = 10^{-6}$, $\alpha_* = 0.12$ (left panel) and $\eta_\chi = 10^{-4}$, $\alpha_* = 0.12$ (right panel) showing the sensitivity of different experiments to the GW peak from PBH density perturbations.

the higher frequency sourced by the FOPT remains out of reach of future GW experiments like ET, CE, DECIGO, BBO. This is primarily because of the extra redshift and dilution arising because of PBH domination and subsequent evaporation (cf. Eq. (4.36), (4.37)). If PBH do not dominate the energy density, the single peak can be within reach of future GW sensitivity. This can also have interesting consequences in terms of producing the observed DM relic abundance, which otherwise is overproduced in the case of PBH domination, unless superheavy [289, 315].

The dark sector fermion χ in our setup can not lead to the observed DM abundance,

	v_D (GeV)	g_χ	$\lambda_{\Phi\Phi'}$	$\lambda_{\Phi H_2}$	T_c (GeV)	T_n (GeV)	T_p (GeV)	T_* (GeV)	β/H_*	α_*	v_J	η_χ
BP5	10^4	1.3	1.3	2.2	2.7×10^3	1.6×10^3	1.4×10^3	1.4×10^3	206	0.13	0.79	10^{-12}
BP6	10^5	1.4	1.3	2	3.2×10^4	2.1×10^4	1.8×10^4	1.7×10^4	140	0.09	0.76	10^{-11}
BP7	10^6	1.4	1.3	2.2	3.1×10^5	1.8×10^5	1.2×10^5	1.0×10^5	78	0.12	0.78	10^{-10}

Table 4.2: Benchmark parameters considered in the analysis for sub-dominant PBH.

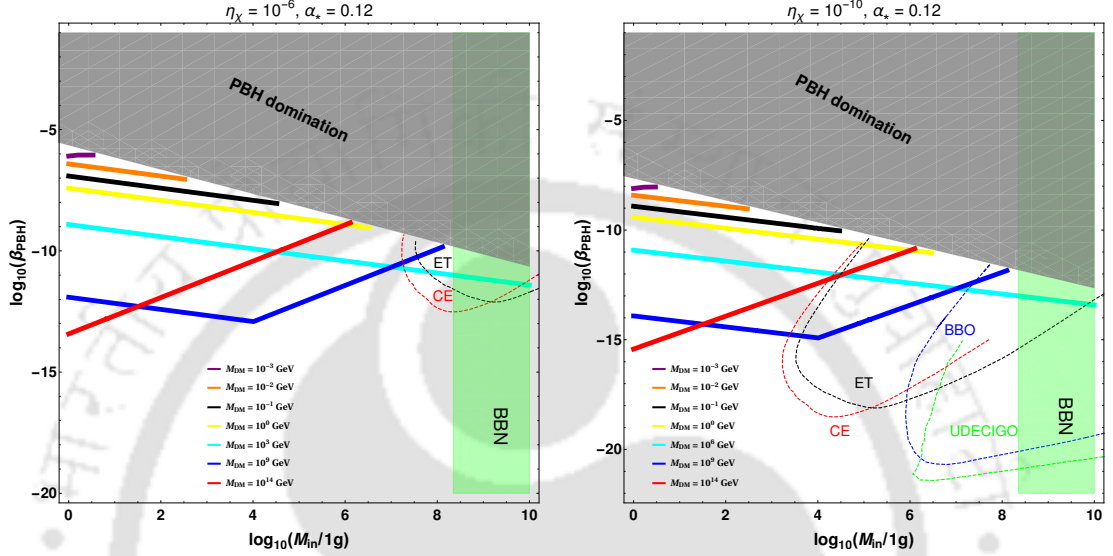


Figure 4.4: Contours satisfying the observed DM relic abundance for different values of DM mass, considering $\eta_\chi = 10^{-6}$ (left panel) and $\eta_\chi = 10^{-10}$ (right panel).

because of being trapped in the false vacuum. However χ can still be produced from PBH evaporation [188, 189] or from the SM bath. Also, some small fraction of χ can cross from the false vacuum to the true vacuum [316]. While a detailed calculation of such trapping and crossing is beyond the scope of our present work, simple conservative estimates, shown in Appendix C justifies maximum trapping. In all such cases where χ appears in the true vacuum, the interaction of χ with the SM bath can play crucial role in deciding its final relic. Since we remain agnostic about the UV completion of the dark sector, we do not consider the possibility of χ being DM. The amount of χ produced from the above possible ways can finally decay into the SM bath, before the BBN epoch. As long as χ is long-lived enough and decay at a temperature $T_{\text{decay}} \ll T_*$, the formation temperature of PBH, the phenomenology discussed here remain unaffected. Such a decay of χ can be introduced via explicit $U(1)_D$ global symmetry breaking higher dimensional operators like $\bar{\ell}_L \tilde{H} \chi \Phi / \Lambda$ where ℓ_L denotes the lepton doublet of the SM. Since χ is lighter in the false vacuum,

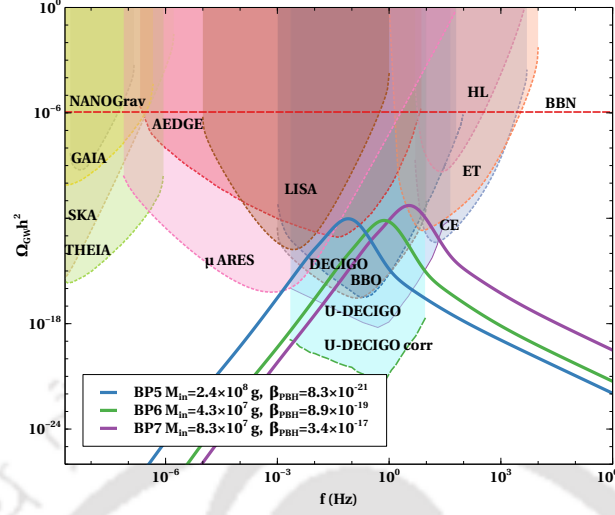


Figure 4.5: GW spectrum from FOPT with sub-dominant PBH for different benchmark points.

depending upon the UV completion of the dark sector, it is also possible to ensure DM decay in the true vacuum while preventing the same in the false vacuum due to kinematical restrictions.

If we consider the dark sector to be purely gravitational in agreement with all experimental evidences, it can still be produced from PBH evaporation since the process of Hawking radiation is based solely on gravitational interactionsⁱⁱⁱ⁾. For the case of sub-dominant PBH ($\beta < \beta_{\text{crit}}$), the DM relic density produced from PBH evaporation is found to be^{iv)} [277, 287, 318, 319]

$$\Omega_{\text{DM}} h^2 \simeq M_{\text{DM}} \beta_{\text{PBH}} \frac{T_n}{M_{\text{in}}} \begin{cases} \left(\frac{M_{\text{in}}}{M_P} \right)^2 & \text{for } M_{\text{DM}} < T_{\text{BH}}^{\text{in}}, \\ \left(\frac{M_P}{M_{\text{DM}}} \right)^2 & \text{for } M_{\text{DM}} > T_{\text{BH}}^{\text{in}}, \end{cases} \quad (4.39)$$

where M_{DM} indicates the DM mass. Note that T_n in turn, depends on the PBH mass and initial fraction (cf. Eq. (4.17)). On the other hand, in the case of PBH

ⁱⁱⁱ⁾ PBH can also leave Planck remnants after evaporation, which can act as DM and have their imprints on GW. See Ref. [317].

^{iv)} Here, for our demonstration, we consider the dark sector to consist of scalar particles. Similar results can be obtained for other particles depending on their degrees of freedom.

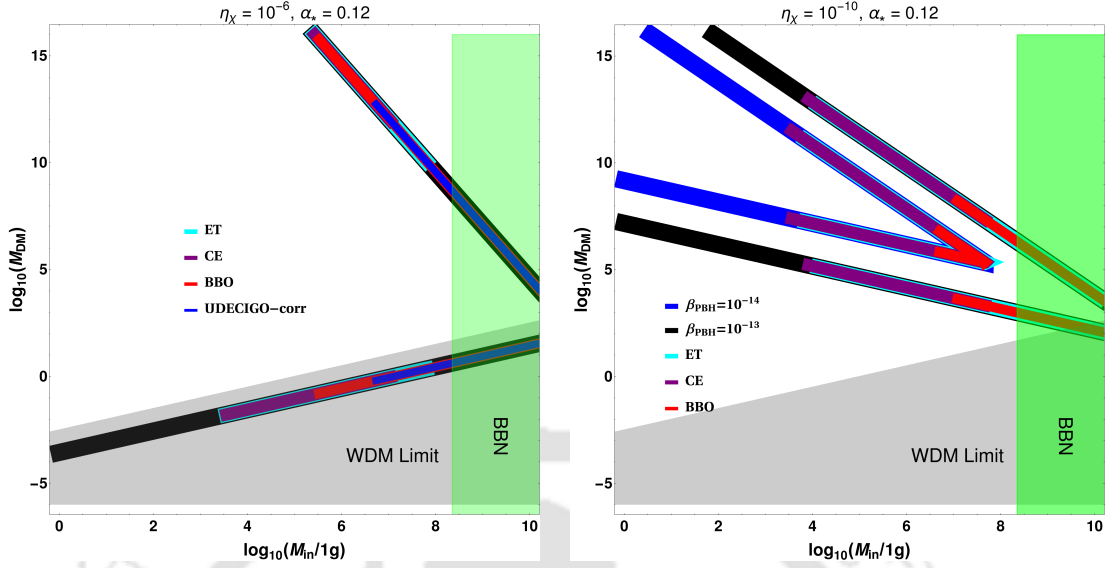


Figure 4.6: Contours satisfying the observed DM relic abundance in $M_{\text{in}} - M_{\text{DM}}$ plane, considering $\eta_\chi = 10^{-6}$ with PBH domination (left panel) and $\eta_\chi = 10^{-10}$ with PBH sub-domination (right panel), along with the future GW experiments that can probe our scenario. The grey-shaded region is constrained from warm dark matter bounds.

domination, the DM relic density is given by

$$\Omega_{\text{DM}} h^2 \simeq M_{\text{DM}} \frac{T_{\text{ev}}}{M_{\text{in}}} \begin{cases} \left(\frac{M_{\text{in}}}{M_P} \right)^2 & \text{for } M_{\text{DM}} < T_{\text{BH}}^{\text{in}}, \\ \left(\frac{M_P}{M_{\text{DM}}} \right)^2 & \text{for } M_{\text{DM}} > T_{\text{BH}}^{\text{in}}. \end{cases} \quad (4.40)$$

As mentioned above, for PBH domination, DM gets over-produced from PBH evaporation irrespective of their spins and only $M_{\text{DM}} \gtrsim 10^9$ GeV can lead to right abundance for $M_{\text{in}} \gtrsim 10^6$ g [285, 289, 315]. DM over-production can also be controlled by choosing sufficiently light DM mass, but it is likely to face constraints from structure formation.

In Fig. 4.4, we show the contours of observed DM abundance in the $M_{\text{in}} - \beta_{\text{PBH}}$ plane considering two different values of the initial asymmetry η_χ . As we can see, for PBH domination regime, the DM mass range gets restricted while for sub-dominant PBH scenario, a much wider DM mass range becomes viable. While the value of η_χ does not change the DM parameter space, the GW sensitivity changes as changing η_χ also amounts to changing the FOPT parameters in order to generate the PBH of same initial mass and energy fraction. Clearly, future GW experiments like CE, ET, BBO,

UDECIGO can probe a sizeable part of the parameter space in PBH mass ranging from $\mathcal{O}(1000 \text{ g})$ as well as DM masses ranging from GeV to superheavy ballpark. The detectable GW spectrum in this low frequency regime is purely generated by the FOPT in this case which is also free from any entropy dilution due to PBH evaporation. Fig. 4.5 shows the GW spectrum of a few such benchmark points, the details of which are given in table 4.2. Similarly, the PBH domination region also remains within GW experimental sensitivities as shown in Fig. 4.3. While DM is mostly in the superheavy ballpark for such a scenario, PBH mass $M_{\text{in}} \gtrsim \mathcal{O}(1000 \text{ g})$ can be probed by future GW experiments. While PBH mass range within GW sensitivities is similar in both the PBH dominance and sub-dominance cases, it is possible to distinguish these two scenarios at GW experiments due to different GW spectrum arising from PBH density fluctuations (PBH dominance) and FOPT (PBH sub-dominance).

Finally in Fig. 4.6, we show the contours of the observed DM relic density in $M_{\text{in}}-M_{\text{DM}}$ plane, for the two cases, i.e. PBH dominance (left panel) and subdominance (right panel). For a fixed PBH mass, the DM relic is satisfied at two DM mass values, either when DM is light ($M_{\text{DM}} < T_{\text{BH}}^{\text{in}}$) or when DM is superheavy ($M_{\text{DM}} > T_{\text{BH}}^{\text{in}}$) (cf. Eq. (4.39), (4.40)). The free-streaming length of DM is constrained from Lyman- α flux-power spectra [320–322], which puts a constraint on light DM depending on the PBH mass [285, 289, 318, 323]. This is due to the fact that light DM produced from such PBH evaporation can have relativistic speeds more than what is allowed for warm dark matter. This is shown by the grey-shaded region. For the PBH-dominance case, only the superheavy DM regime remains viable, while for subdominant PBH, we can satisfy the correct DM relic density at lighter DM mass as well while being consistent with Lyman- α bounds. The different coloured contours along the relic density contour indicate the reach of future GW experiments that can probe the corresponding peak frequencies of the GW spectrum relevant for the PBH and DM mass. The left and the right panels specify the GW peak from density fluctuations and FOPT respectively, both having a one-to-one correspondence with the PBH mass (cf. Eq. (4.30), (4.36)), considering other parameters to be fixed.

4.1.7 PBH from other mechanisms related to FOPT

Before concluding, we briefly comment on other FOPT origins of PBH and corresponding GW signatures. In other mechanisms without not relying on any dark sector asymmetry or Fermi-ball collapse mechanism, PBH are formed when horizon-sized perturbations become more than the critical overdensity $\delta > \delta_c \sim 0.45$ leading to gravitational collapse [197]. Such type of gravitational collapse as a consequence of FOPT have been studied in several works including [68, 186, 324, 325] and references therein. During FOPT, nucleation may get delayed in some Hubble-sized region, and the late decay of the false vacuum increases the total energy density. In an expanding Universe, the radiation energy density keeps on decreasing, whereas in some regions the vacuum energy density remains constant. This leads to total energy density higher than the threshold overdensity and the mass inside such Hubble-sized region collapses to form PBH. After solving the Friedmann equation and radiation energy density evolution equations [68] in late false vacuum decay region and background region, we can estimate the time, t_{PBH} when PBH formed and the initial PBH mass can be written as $M_{\text{PBH}} = \frac{4\pi}{3} \mathbf{H}^{-3}(t_{\text{PBH}}) \rho_c$, where ρ_c is critical energy density^{v)}. In such mechanisms, ultra-light PBH of mass $\sim 8 \times 10^8$ g can be formed from FOPT with nucleation temperature of order 10^{13} GeV. Thus, if PBH is subdominant, the GW peak from FOPT will be in the ultra-high frequency regime $\sim 10^7$ Hz which is far outside the reach of near-future GW experiments^{vi)}. For PBH domination, the GW peak from FOPT can get redshifted to a lower frequency (depending on β_{PBH}) because of entropy dilution during PBH evaporation. For instance, considering the PBH mass mentioned above and taking $\beta_{\text{PBH}} \sim \beta_{\text{max}}$, we find the FOPT peak in the frequency range of order 10^4 Hz, with a very small amplitude ($\Omega_{\text{peak}}^{\text{PT}} h^2 \lesssim 10^{-25}$) because of the entropy dilution. Moreover, note that a detailed analysis requires calculating the energy fraction PBH β_{PBH} through the probability of late false vacuum decay [324]. Therefore, the PBH formation mechanism adopted in this work offers more detection prospects irrespective of whether PBH dominates or remains subdominant.

^{v)}Similar relation is obtained for PBH formed through bubble collisions[326].

^{vi)}There have been several proposed ideas to look for these ultra-high frequency gravitational waves. See Ref. [327] for a review.

4.1.8 Conclusion

We have studied the detection prospects of an ultra-light primordial black hole formation mechanism in gravitational wave experiments and consequences for production of gravitational dark matter from such PBH evaporation. Considering the Fermi-ball collapse mechanism of PBH formation in the context of a first-order phase transition, we consider the combined GW spectrum arising from the FOPT as well as PBH density fluctuations with the latter requiring a PBH dominated phase. We show that such a PBH formation mechanism leads to a unique doubly peaked GW spectrum. While the GW spectrum generated from FOPT suffers from entropy dilution at later stages due to ultra-light PBH evaporation, the blue-tilted part of its arm for certain region of parameter space can remain within experimental sensitivity. Combined detection of both the GW peaks require new experiments in kHz frequency ballpark having sufficient sensitivity. The PBH domination scenario mostly produces superheavy DM whereas the sub-dominant PBH can generate a much wider range of DM. While the doubly peaked feature in sub-dominant PBH scenario disappears due to absence of GW generated by PBH density fluctuations, the FOPT generated GW spectrum including the peak can be brought within sensitivities of future GW experiments like CE, ET, BBO, UDECIGO corresponding to DM mass in GeV to superheavy regime and PBH mass as low as a few kg. The PBH domination regime remains within sensitivities of such experiments for PBH mass as low as a few kg and DM masses mostly in the superheavy ballpark. Due to the strong connection among PBH parameters and FOPT parameters which in turn affect DM mass from relic abundance criteria, we have interesting correlations and complementarity among different observables which can be tested in near future.

4.2 Filteredogenesis of PBH dark matter and baryons

While typical baryogenesis or leptogenesis scenarios involve very high scale physics making direct experimental probe difficult, null results at direct detection experiments [270] have pushed WIMP DM scenario to a tight corner. This has led to exploration of DM scenarios beyond the WIMP framework. One such promising alternative is primordial black hole, recent reviews of which can be found in [72, 328]. PBH with mass in the range $\sim 10^{17} - 10^{22}$ g can be sufficiently stable and free from astrophysical constraints to be a good DM candidate. While PBH can have their own astrophysical or cosmological signatures, their formation mechanism also have its own detection aspects. Among well-motivated production mechanisms like PBH from inflationary perturbations [67, 171, 176–178, 329], first-order phase transition [68, 69, 179–189, 330–332], the collapse of topological defects [190, 191] etc., we consider the FOPT as a source while also connecting it to the origin of baryon asymmetry of the Universe. This leads to a cogenesis mechanism indirectly testable via FOPT generated stochastic gravitational waves. In particular, we consider the collapsing Fermi-ball scenario [69, 184, 188, 189, 298, 305–308, 333] for PBH formation which also lead to the origin of BAU. A FOPT occurs due to a scalar field which gives a new contribution to a dark fermion mass. The same dark fermion χ with an initial asymmetry gets trapped in the false vacuum due to the mass gap, leading to the formation of Fermi-balls. These Fermi-balls can further collapse to black holes if there exists an attractive Yukawa force among dark fermions. Since dark fermion maintains equilibrium with the bath prior to the FOPT, its momenta follow an equilibrium distribution allowing some of them to leak into the true vacuum. The enhanced mass of dark fermion in the true vacuum allows its decay into the standard model particles thereby transferring the dark asymmetry into the observed baryon asymmetry. The conceptual visualisation of the idea is summarised in the schematic shown in Fig. 4.7. Depending upon the scale of the FOPT compared to the sphaleron decoupling temperature, BAU can be generated by direct transfer of dark asymmetry into baryons or via the leptogenesis route. By considering the initial dark asymmetry to be a free parameter, we find a small allowed parameter space consistent with the cogenesis of PBH dark matter and BAU. Coincidentally,

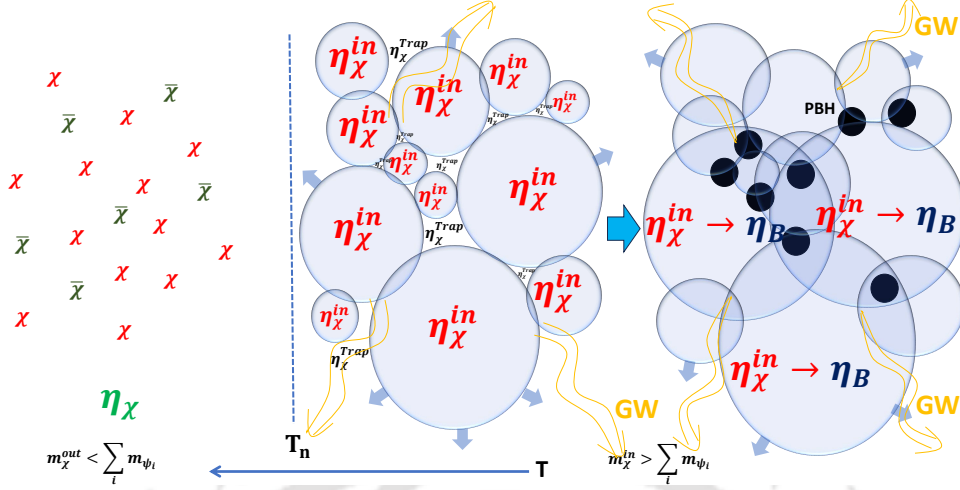


Figure 4.7: A schematic diagram of the proposedogenesis scenario.

this allowed region with nucleation temperature around a few tens of MeV to a few GeV remains within the sensitivity of μ ARES.

This section is organised as follows. In section 4.2.1, we discuss the particle physics framework leading to a high scale origin of dark asymmetry and a first-order phase transition at low scale. In section 4.2.2, we discuss the details of filtering or the number of dark fermions leaking into the true vacuum. In section 4.2.3 we discuss the main results related to cogenesis of PBH dark matter and baryon asymmetry of the Universe and finally conclude in section 4.2.4.

4.2.1 The framework

We first formulate the theoretical setup such that a non-zero dark sector asymmetry can be generated at high scale. There are various ways of generating dark sector asymmetries as studied in asymmetric dark matter scenarios [288, 334–350]. Here, for simplicity, we outline one such UV completion based on the Affleck-Dine (AD) mechanism [31].

Consider an Affleck-Dine field η singlet under the SM gauge symmetry but having a global $U(1)_B$ charge 2. The Dirac fermion χ has global charge 1. The relevant

terms involving η, χ can be written as

$$-\mathcal{L} \supset Y_D \overline{\chi} \chi \eta^\dagger + \epsilon m_\eta^2 \eta^2 + \text{h.c.} \quad (4.41)$$

While the Yukawa interaction in Eq. (4.41) preserves $U(1)_B$, the $\epsilon \mu^2 \eta^2$ term of the AD field explicitly breaks $U(1)_B$. Due to this explicit baryon number violating term, the cosmological evolution of the AD field leads to a generation of non-zero baryon number, which gets transferred to the dark fermion χ when η decays into them.

Additionally, the $U(1)_B$ preserving AD field potential can be expressed as $V(\eta) = m_\eta^2 |\eta|^2 + \lambda_\eta |\eta|^4$. For the field range $\eta^* = \frac{m_\eta}{\sqrt{\lambda_\eta}} \lesssim \eta$, the quartic term $\lambda_\eta |\eta|^4$ dominates the potential, and the scalar field η evolves as $\eta \propto 1/a$. As η approaches the value η^* , a difference between the real and imaginary components of η emerges, leading to an asymmetry in the η condensate. The resulting comoving asymmetry generated for $t > t^*$ can then be expressed as [348, 351]

$$N_{\Delta\chi}(t) \simeq 4Q_\eta A \eta_{1I} \eta_{2I} \left(\frac{\eta_I}{\eta^*} \right) \int_{t^*}^t dt' \cos[m_1(t' - t^*)] \cos[m_2(t' - t^*)] e^{-\Gamma_\eta(t' - t^*)}. \quad (4.42)$$

Here, Γ_η denotes the total decay rate of the scalar field η into χ particles. The quantities η_{1I} and η_{2I} represent the initial values of the real and imaginary components of the field η , and $\eta_I = \sqrt{(\eta_{1I})^2 + (\eta_{2I})^2}$. The mass parameters are given by $m_1^2 = m_\eta^2 - 2A$ and $m_2^2 = m_\eta^2 + 2A$, with $A = \epsilon m_\eta^2$. The asymmetry generated in this process is subsequently transferred to the dark sector via the decay channel $\eta \rightarrow \chi\chi$. Under the assumption $2A \gg \Gamma_\eta m_\eta$, the integral simplifies for $t \gtrsim 1/\Gamma_\eta$ to:

$$N_{\Delta\chi}(t) \simeq C \frac{\gamma}{8\epsilon^2 m_\eta}, \quad (4.43)$$

where $\gamma = \Gamma_\eta/m_\eta$ and $C = 4Q_\eta A \eta_{1I} \eta_{2I} \left(\frac{\eta_I}{\eta^*} \right)$. In Fig. 4.8, we show the evolution of the comoving asymmetry $N_{\Delta\chi}(t)$, which initially grows from zero, oscillates due to mass splitting, and eventually settles to the constant value given by Eq. (4.43) as the amplitude is exponentially damped for $t \gtrsim 1/\Gamma_\eta$. Assuming the decay of the AD field also to reheat the Universe to a temperature $T_{\text{RH}} \simeq \sqrt{\Gamma_\eta M_{\text{Pl}}}$, the dark fermion

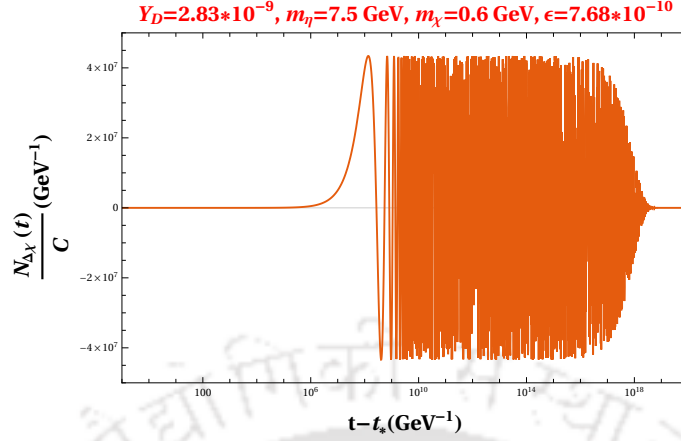


Figure 4.8: The evolution of the dark sector asymmetry over time is shown for the chosen benchmark values, with the final asymmetry reaching $Y_{\Delta\chi}^{\text{in}} \sim 10^{-10}$.

asymmetry can be estimated as [348, 351, 352]

$$Y_{\Delta\chi}^{\text{in}} = \frac{(n_\chi - n_{\bar{\chi}})}{s} \simeq \frac{T_{\text{RH}}^3}{\epsilon m_\eta^2 M_{\text{Pl}}} \quad (4.44)$$

where m_η is the mass of the AD field. The dominant washout process is the one which breaks $U(1)_B$ by 4 units: $\chi\chi \leftrightarrow \bar{\chi}\bar{\chi}$ mediated by η . Keeping this washout out-of-equilibrium leads to the following condition

$$T_{\text{RH}}^3 \frac{Y_D^4 \epsilon^2 T_{\text{RH}}^2}{64\pi m_\eta^4} \lesssim \sqrt{\frac{\pi^2}{90} g_*} \frac{T_{\text{RH}}^2}{M_{\text{Pl}}}, \quad (4.45)$$

which can be ensured by suitable choices of couplings and masses. Fig. 4.9 shows the parameter space in $\epsilon - Y_D$ plane consistent with different dark sector asymmetries represented by contours. The grey shaded region is disfavoured as the dark sector asymmetry is less than the observed baryon asymmetry. The magenta meshed region on the left hand side is disfavoured as reheat temperature is less than the scale of FOPT $T_{\text{RH}} < T_c$.

For the first-order phase transition and transfer of dark fermion asymmetry to visible sector in the true vacuum, we consider a minimal framework consisting of a Dirac singlet fermion χ , two singlet real scalars $\Phi_{1,2}$ as the new BSM degrees of freedom present at the scale of FOPT. Another heavy scalar ζ is introduced in order to allow the coupling of χ with a fermion ψ in the SM. The low energy effective Yukawa

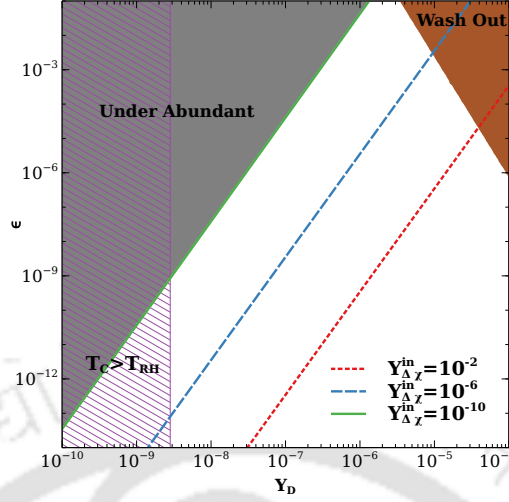


Figure 4.9: Parameter space in the Y_D vs. ϵ plane, with contour lines indicating different values of the final dark sector asymmetry for $m_\eta = 7.5$ GeV.

Lagrangian relevant for our study is

$$-\mathcal{L} \supset (y_1 \bar{\chi} \Phi_1 \chi + y_2 \bar{\psi} \zeta \chi + \text{h.c.}) + m_\chi \bar{\chi} \chi. \quad (4.46)$$

While the first term in the above equation leads to a new contribution to dark fermion's mass in true vacuum, the second term paves the way of transferring dark sector asymmetry into SM. As discuss later, the Yukawa coupling y_2 and scalar ζ remains much heavier compared to the FOPT scale and hence do not play any role in low energy phenomenology. The quantum numbers of ζ, ψ and their roles in cogenesis will be discussed in section 4.2.3.

While the singlet scalar Φ_1 drives a first-order phase transition acquiring a non-zero vacuum expectation value, the other singlet scalar Φ_2 assists in generating the required effective potential while keeping all dimensionless couplings within perturbative limits. We, however, keep the coupling of the SM-like Higgs doublet H with Φ_1 negligible such that the required electroweak symmetry breaking can be achieved independently of this singlet-driven FOPT. With these assumptions, the relevant part of the tree level scalar potential is given by

$$V_{\text{tree}} \supset \lambda_1 \left(\Phi_1^2 - \frac{v_D^2}{2} \right)^2 + \frac{\lambda_{\Phi_1 \Phi_2}}{2} \Phi_1^2 \Phi_2^2. \quad (4.47)$$

The other possible terms allowed by the symmetries are ignored for simplicity. The dark fermion receives a new contribution to its mass in the true vacuum $\langle \Phi_1 \rangle = v_D$. This splits the zero-temperature mass of χ inside and outside the bubbles of true vacuum as

$$m_\chi^{\text{in}} = m_\chi + y_1 \frac{v_D}{\sqrt{2}}, \quad m_\chi^{\text{out}} = m_\chi. \quad (4.48)$$

The one-loop correction to the tree-level potential is calculated by considering the couplings of all particles to the singlet scalar $\Phi_1 = \phi/\sqrt{2}$. This correction, known as the Coleman-Weinberg potential [92], can be expressed as

$$V_{\text{CW}}(\phi) = \frac{1}{64\pi^2} \sum_{i=\Phi_2, \chi} n_i m_i^4(\phi) \left\{ \log \left(\frac{m_i^2(\phi)}{v_D^2} \right) - C_i \right\} \quad (4.49)$$

where, $n_{\Phi_2} = 1$, $n_\chi = 4$ are the respective degrees of freedom and $C_{\Phi_2, \chi, \zeta} = \frac{3}{2}$. The field dependent masses are

$$m_{\Phi_2}^2(\phi) = \mu_{\Phi_2}^2 + \frac{\lambda_{\Phi_1 \Phi_2} \phi^2}{2}, \quad m_\chi^2(\phi) = (m_\chi + y_1 \phi/\sqrt{2})^2.$$

Now, the thermal contributions to the effective potential [93, 97] can be written as

$$V_T(\phi, T) = \sum_{i=\Phi_2, \zeta} \frac{n_i T^4}{2\pi^2} J_B \left(\frac{m_i^2(\phi)}{T^2} \right) - \frac{n_\chi T^4}{2\pi^2} J_F \left(\frac{m_\chi^2(\phi)}{T^2} \right) \quad (4.50)$$

where,

$$J_F(x) = \int_0^\infty dy y^2 \log[1 + e^{-\sqrt{y^2+x^2}}], \quad J_B(x) = \int_0^\infty dy y^2 \log[1 - e^{-\sqrt{y^2+x^2}}].$$

In addition, the Daisy corrections [101, 102, 239] need to be included in the thermal contribution to enhance the perturbative expansion following the Arnold-Espinosa method [102]. These Daisy contributions can be expressed as

$$V_{\text{daisy}}(\phi, T) = -\frac{T}{2\pi^2} \sum_{i=\Phi_2} n_i [m_i^3(\phi, T) - m_i^3(\phi)] \quad (4.51)$$

where, $m_i^2(\phi, T) = m_i^2(\phi) + \Pi_i(T)$ and the relevant thermal masses are

$$m_{\Phi_2}^2(\phi, T) = m_{\Phi_2}^2(\phi) + \left(\frac{\lambda_2}{4} + \frac{\lambda_{\Phi_1 \Phi_2}}{12} \right) T^2.$$

The complete effective finite-temperature potential is

$$V_{\text{eff}}(\phi, T) = V_{\text{tree}}(\phi) + V_{\text{CW}}(\phi) + V_T(\phi, T) + V_{\text{daisy}}(\phi, T). \quad (4.52)$$

At high temperatures, the Universe stays in the false vacuum $\langle \Phi_1 \rangle = 0$. At the critical temperature T_c another degenerate minimum appears at $\langle \Phi_1 \rangle \neq 0$ with a barrier between them. Subsequently, at a lower temperature known as the nucleation temperature T_n , the Universe tunnels through the barrier to go from the false vacuum to the true vacuum $\langle \Phi_1 \rangle \neq 0$. The tunnelling rate per unit volume defined in terms of $O(3)$ symmetric bounce action $S_3(T)$ is $\Gamma(T) \sim T^4 e^{-S_3(T)/T}$ [122]. The nucleation temperature T_n is determined by comparing the tunnelling rate with the Hubble expansion rate as

$$\Gamma(T_n) = \mathcal{H}^4(T_n) = \mathcal{H}_*^4. \quad (4.53)$$

The volume fraction of false vacuum of the Universe is defined by $p(T) = e^{-\mathcal{I}(T)}$ [245, 246], where

$$\mathcal{I}(T) = \frac{4\pi}{3} \int_T^{T_c} \frac{dT'}{T'^4} \frac{\Gamma(T')}{\mathcal{H}(T')} \left(\int_T^{T'} \frac{d\tilde{T}}{\mathcal{H}(\tilde{T})} \right)^3. \quad (4.54)$$

The percolation temperature T_p is then calculated by using $\mathcal{I}(T_p) = 0.34$ [245] indicating that at least 34% of the comoving volume is occupied by the true vacuum.

The inverse of the FOPT duration is given by $\beta/\mathcal{H} = T \frac{d}{dT} \left(\frac{S_3}{T} \right)$ while $\alpha_* = \frac{\epsilon}{\rho_{\text{rad}}}$ is amount of latent heat released at nucleation temperature where, $\epsilon = \left(\Delta V_{\text{eff}} - \frac{T}{4} \frac{\partial \Delta V_{\text{eff}}}{\partial T} \right)_{T=T_n}$, $\Delta V_{\text{eff}} = V_{\text{eff}}(\phi_{\text{false}}, T) - V_{\text{eff}}(\phi_{\text{true}}, T)$ and $\rho_{\text{rad}} = g_* \pi^2 T^4 / 30$ [151]. The bubble wall velocity v_w , in general, is related to the Jouguet velocity $v_J = \frac{1/\sqrt{3} + \sqrt{\alpha_*^2 + 2\alpha_*/3}}{1 + \alpha_*}$. For the type of FOPT considered here, we can assume $v_w \approx v_J$ [146, 152, 255]. The calculation for the bounce action $S_3(T)$ is performed numerically by fitting the effective potential to a generic potential [309], the details of which is described in [273].

We assume the dark fermion to have an initial asymmetry with subsequent annihilation of the symmetric part due to $\chi - \Phi_1$ interactions. In order to trap most of the dark fermions in the false vacuum, we impose the condition $T_n \ll m_\chi^{\text{in}} - m_\chi^{\text{out}}$, such that χ moving to the true vacuum is kinematically disfavoured. However, as the momenta of χ follows a distribution, some of them can penetrate to the true

vacuum. We discuss the trapping of χ in false vacuum and cogenesis in the next two sections.

4.2.2 Filtering of dark fermions

During a first-order phase transition, bubbles of the true vacuum nucleate and expand. Within the bubbles, the symmetry is broken, whereas it remains unbroken outside. Consequently, this results in the mass of the particle χ taking different values inside and outside the bubbles, denoted as m_χ^{in} and m_χ^{out} , respectively. As the bubble expands, χ particles penetrate the bubble wall depending on their energy. Due to the restriction $T_n \ll m_\chi^{\text{in}} - m_\chi^{\text{out}}$, only the high momentum modes of χ can penetrate the walls resulting its filtering. The idea of bubble filtering has been used in the context of dark matter and baryogenesis in several earlier works [188, 227, 316, 353–357].

Let $\tilde{v}$ denote the velocity of χ particles relative to the bubble wall along z direction, while v_w represents the velocity of the bubble wall in the plasma rest frame. The number of particles per unit area going inside the bubble in time Δt along the z direction can be written in the bubble wall frame following energy conservation as [188, 353]

$$\frac{\Delta N_{\text{in}}}{\Delta A} = n_\chi \int \frac{d^3 p}{(2\pi)^3} \int_{r_o}^{r_o - \frac{p_z \Delta t}{|p|}} dr \mathcal{I}(p) \Theta(-p_z) f(p, x) \quad (4.55)$$

where ΔA is the area and the energy conservation is ensured by $\mathcal{I}(p) = \Theta(-p_z - m_d)$, $m_d = m_\chi^{\text{in}} - m_\chi^{\text{out}}$, n_χ denotes the degrees of freedom of the particle. Near the bubble wall, χ particles will follow equilibrium distribution locally and hence $f(p, x) \simeq f_{\text{eq}}(p, \tilde{v}, T)$. So, the distribution function outside the bubble is

$$\begin{aligned} f(p, x) &= \frac{1}{e^{\tilde{\gamma}(E - \tilde{v} p_z)/T} + 1} \\ &= \frac{1}{e^{\tilde{\gamma}(\sqrt{p^2 + (m_\chi^{\text{out}})^2} - \tilde{v} p \cos \theta)/T} + 1} \end{aligned} \quad (4.56)$$

with $\tilde{\gamma}$ being the Lorentz factor. After performing the integration over r , the flux of particles entering the bubble $J_w = \Delta N_{\text{in}}/(\Delta A \Delta t)$ is given by

$$\begin{aligned}
 J_w &= \frac{n_\chi}{(2\pi)^3} \int d^3p \Theta(-p_z - m_d) \Theta(-p_z) f(p, x) \left(-\frac{p_z}{|p|} \right) \\
 &= \frac{n_\chi}{(2\pi)^3} \int_0^\pi p d\theta \int_0^{2\pi} p \sin \theta d\phi \int_0^\infty dp (-\cos \theta) \Theta(-p \cos \theta - m_d) \Theta(-\cos \theta) f(p) \\
 &= \frac{n_\chi}{(2\pi)^2} \int_0^{-1} d(\cos \theta) \cos \theta \int_{-\frac{m_d}{\cos \theta}}^\infty dp \frac{p^2}{e^{\tilde{\gamma}(\sqrt{p^2 + (m_\chi^{\text{out}})^2} - \tilde{v} p \cos \theta)/T} + 1} \\
 &= \frac{n_\chi}{(2\pi)^2} \int_0^{-1} dx x \int_{-\frac{m_d}{x}}^\infty dp \frac{p^2}{e^{\tilde{\gamma}(\sqrt{p^2 + (m_\chi^{\text{out}})^2} - \tilde{v} p x)/T} + 1}.
 \end{aligned} \tag{4.57}$$

Now, the number of particles going inside the bubble with respect to the plasma frame is $n_{\text{in}} = J_w/(\gamma_w v_w)$. Hence, the fraction of particles trapped outside the bubble can be written as

$$F(m_\chi^{\text{out}}, m_\chi^{\text{in}}, T, \tilde{v}, v_w) = 1 - \frac{n_{\text{in}}}{n_{\text{eq}}} \tag{4.58}$$

From Fig. 4.10 left panel, it is evident that particle trapping is more efficient when

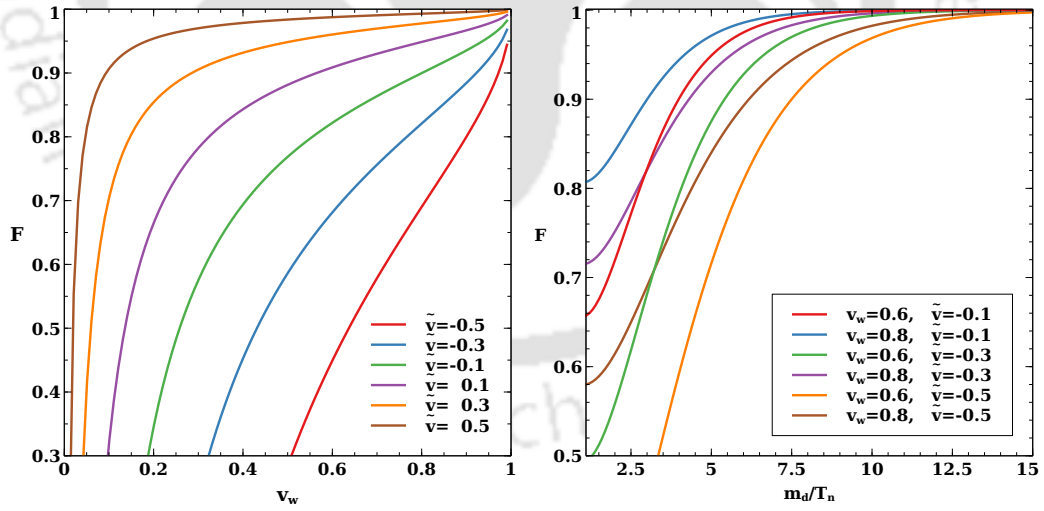


Figure 4.10: Left panel: Trapping fraction F versus bubble wall velocity v_w for different values of $\tilde{v}$, where $m_\chi^{\text{out}} \sim T_n$ and $m_d/T_n \sim 2$. Right panel: Trapping fraction F versus m_d/T_n for different values of $\tilde{v}$ and v_w , where $m_\chi^{\text{out}} \sim T_n$.

the particles move away from the wall, corresponding to a positive $\tilde{v}$. Conversely, the trapping efficiency decreases for negative $\tilde{v}$. However, the trapping function increase with wall velocity, as seen from the right panel plot of Fig. 4.10. The right

panel plot also shows the increase of the trapping fraction with the mass gap m_d/T_n as expected. These also agree with the earlier works [188, 353]. For a complete hydrodynamical treatment of bubble and fluid velocities, refer to Appendix D.

4.2.3 Cogenesis of PBH and Baryon

As the phase transition proceeds via bubble nucleation, the bubbles containing true vacuum grows to cover the entire Universe while the false vacuum regions shrink. Eventually, the false vacuum after shrinking to small disconnected volumes, split further to negligible size at percolation temperature [245]. These squeezed pockets of false vacuum enhance the annihilation of dark fermions [226, 358] leaving only the asymmetric part η_χ . An approximate global charge at low scale ensures the survival of asymmetric dark fermion part in these false vacuum pockets leading to the degeneracy pressure. If the Fermi degeneracy pressure balances the vacuum pressure, it leads to the formation of Fermi-balls at T_* defined by $p(T_*) = 0.29$ [69, 199, 310, 311].

The energy of a Fermi-ball can be written as

$$E_{\text{FB}} = \frac{3\pi}{4} \left(\frac{3}{2\pi} \right)^{2/3} \frac{Q_{\text{FB}}^{4/3}}{R} + \frac{4\pi}{3} U_0(T_*) R_{\text{FB}}^3, \quad (4.59)$$

where, $Q_{\text{FB}} = F_\chi^{\text{trap}} \frac{n_\chi - n_{\bar{\chi}}}{n_{\text{FB}}^*}$ with n_χ denoting number density of χ , $F_\chi^{\text{trap}} \approx 1$ for maximum trapping of χ inside false vacuum and U_0 is vacuum energy. Now, minimising the Fermi-ball energy, we can get the mass and radius as

$$M_{\text{FB}} = Q_{\text{FB}} (12\pi^2 U_0(T_*))^{1/4}, \quad R_{\text{FB}}^3 = \frac{3}{16\pi} \frac{M_{\text{FB}}}{U_0(T_*)}. \quad (4.60)$$

Within the Fermi-balls, the χ 's interact through attractive Yukawa interaction $g_\chi \phi \bar{\chi} \chi$ and the interaction range is $L_\phi(T) = \left(\frac{d^2 V_{\text{eff}}}{d\phi^2} |_{\phi=0} \right)^{-1/2}$ [305]. The Yukawa potential energy contribution to the Fermi-ball energy can be expressed as

$$E_Y \simeq -\frac{3g_\chi^2}{8\pi} \frac{Q_{\text{FB}}^2}{R_{\text{FB}}} \left(\frac{L_\phi}{R_{\text{FB}}} \right)^2. \quad (4.61)$$

At a later stage, when the Yukawa potential energy becomes larger than the Fermi-ball energy, the Fermi-ball becomes unstable and collapses to form PBH [69]. We consider that the Fermi-ball formation temperature is close to the nucleation temperature, $T_* \sim T_n^{\text{vii}}$. The initial PBH mass during formation can be estimated in terms of FOPT parameters as [69]

$$M_{\text{PBH}} \sim 1.4 \times 10^{21} v_w^3 \left(\frac{\eta_\chi^{\text{trap}}}{10^{-3}} \right) \left(\frac{100}{g_*} \right)^{1/4} \left(\frac{100 \text{ GeV}}{T_n} \right)^2 \left(\frac{100}{\beta/\mathcal{H}_*} \right)^3 \alpha_*^{1/4} \text{ g} \quad (4.62)$$

where $\eta_\chi^{\text{trap}} = F\eta_\chi$ denotes the asymmetric part trapped inside the false vacuum pockets. Primordial black holes, if sufficiently heavy, can contribute to dark matter and its contribution is parametrised as [69]

$$f_{\text{PBH}} = 1.3 * 10^3 v_w^{-3} \left(\frac{g_*}{100} \right)^{1/2} \left(\frac{T_n}{100 \text{ GeV}} \right)^3 \left(\frac{\beta/\mathcal{H}_*}{100} \right)^3 \left(\frac{M_{\text{PBH}}}{10^{15} \text{ g}} \right). \quad (4.63)$$

For PBH constituting entire DM in the Universe, we have $f_{\text{PBH}} = 1$. PBH can fully constitute dark matter within the mass range of $9.15 \times 10^{16} \text{ g}$ and $6.70 \times 10^{21} \text{ g}$. The lower bound on mass comes from the evaporation constraint from the cosmic microwave background (CMB) observations [70, 359] while the upper bound on mass comes from the micro-lensing constraint[71].

In a co-genesis scenario, the initial dark sector asymmetry helps in the formation of PBH dark matter, while a portion of it filtered through the bubble walls is transferred to baryons or visible sector generating the observed baryon asymmetry of the Universe. As the bubble expands in the Universe, a fraction of the dark asymmetry remains outside the bubble, while the rest passes through the wall and subsequently decays into the visible sector. If the initial asymmetry is partitioned into two components, one contributing to the formation of PBH, while the other fully converting into the baryon asymmetry, we can write $\eta_\chi = \eta_\chi^{\text{trap}} + \eta_\chi^{\text{in}}$ or $F + \frac{\eta_\chi^{\text{in}}}{\eta_\chi} = 1$.

In Fig. 4.11, we present the parameter space in $T_n - \eta_\chi$ plane which satisfies the requirements of successful cogenesis of PBH dark matter and baryon asymmetry. We fix $\beta/\mathcal{H}_* = 200$, $\alpha_* = 0.3$, $v_w = 0.85$ for the left panel and $\beta/\mathcal{H}_* = 50$, $\alpha_* = 0.3$, $v_w = 0.85$ for the right panel of Fig. 4.11. The coloured band denotes the region

^{vii)}We also find that the trapping function F remains almost same at T_n and T_* justifying these temperatures to be used interchangeably.

where PBH can account for entire DM. This can be found by using Eq. (4.62) in Eq. (4.63) to obtain

$$f_{\text{PBH}} = 5.75 \times 10^8 \alpha_*^{1/4} g_*^{1/4} \left(\frac{F\eta_\chi}{10^{-3}} \right) \left(\frac{T_n}{100} \right) \quad (4.64)$$

and then setting $f_{\text{PBH}} = 1$. We also calculate the trapping function $F \equiv F(m_\chi^{\text{out}}, m_\chi^{\text{in}}, T_n, \tilde{v}, v_w)$ numerically using Eq. (4.58), where m_d/T_n is varied from 1 to 50 for each of these points in order to get one-to-one relation between η_χ and T_n in deflagration regime. The black dashed contours are plotted by fixing dark fermion asymmetry leaked to the true vacuum via bubble filtering η_χ^{in} . This allows using $F\eta_\chi = \eta_\chi - \eta_\chi^{\text{in}}$ in Eq. (4.64) leading to a relation between η_χ and T_n . This is clear from the asymptotic behaviour of the black dashed contours as $\eta_\chi \rightarrow \eta_\chi^{\text{in}}$. Only where black dashed contours meet the coloured band, however, the trapping function F corresponds to the exact value calculated in terms of the relevant model parameters $F(m_\chi^{\text{out}}, m_\chi^{\text{in}}, T_n, \tilde{v}, v_w)$. To calculate the trapping fraction which is a crucial parameter for cogenesis, we consider the explicit dependence of F on parameters related to the FOPT: $m_\chi^{\text{out}}, m_\chi^{\text{in}}, T_n$, and v_w . The remaining parameter is $\tilde{v}(v_f)$, the local fluid velocity outside the bubble wall in the wall frame, which is related to the fluid velocity inside the wall, v_t , through the energy-momentum conservation equations. We solve the hydrodynamical profile of the fluid velocity inside the wall for a given wall velocity in plasma frame, and then use a Lorentz transformation to relate v_t with v_w . This allows us to determine $\tilde{v}$ as a function of v_w the details of which is given in Appendix D. Since the leftmost dashed contour corresponds to $\eta_\chi^{\text{in}} = \eta_B$, the observed baryon asymmetry, the grey meshed region towards the left of the point where it touches the coloured band is disfavoured from cogenesis point of view. For this analysis, we find the local fluid velocity as a function of v_w , considering the deflagration regime as discussed in Appendix D and Fig. D.2 therein. In Fig. 4.12, we show the parameter space in $T_n - \eta_\chi$ plane for the same set of parameters as in Fig. 4.11, considering the detonation regime discussed in Appendix D. The detonation regime results in a high local fluid velocity relative to the wall, leading to a wider range of the trapping function F ranging from 0.08 to 1. Consequently, the coloured region, where PBH constitute the entire DM, becomes broader compared to the deflagration regime of Fig. 4.11.

The orange meshed region in Fig. 4.11, 4.12 correspond to the disallowed PBH

mass window on lighter side which affects big bang nucleosynthesis (BBN) and the extragalactic photon background [70, 359]. Additionally, the light-green meshed region corresponding to heavier PBH mass is disfavoured by HSC-M31 microlensing constraints [71]. All these constraints finally allow only the white coloured regions in Fig. 4.11, 4.12 with the cogenesis favoured points lying on the green coloured band. Different coloured points on this band refer to the benchmark points given in table 4.3 and table 4.4. The two blue dashed contours represent the μ ARES sensitivity for PBH mass fixed at the minimum and maximum of the allowed range for it to be DM. Interestingly, all these benchmark points remain within the sensitivities of future GW experiments like μ ARES [156]. The stochastic GW spectrum from a FOPT is estimated by considering all the relevant contributions from bubble collisions [123, 124, 140, 248, 249], the sound wave [130, 131, 134, 149] and the turbulence [135, 136, 138, 146, 250, 251] of the plasma medium.

Clearly, F varies over a wider range in the detonation regime. In the deflagration scenario, the benchmark points BP3 and BP6 satisfy both baryon asymmetry and PBH dark matter scenario. However, in detonation regime, BP4 satisfy both baryon asymmetry and PBH dark matter scenario, for other BPs baryon asymmetry is overproduced requiring additional washout in the true vacuum. For completeness, we perform a random scan satisfying the FOPT condition by varying v_D from 10 MeV to 50 GeV, $m_\chi \leq 0.1v_D$ and $10^{-11} < \eta_\chi < 10^{-6}$, while keeping $y_1 = 1.4$ and $\lambda_{\Phi_1\Phi_2} = 3.5$. The resulting points, correlating visible matter and dark matter abundances, are shown in left and right panels of Fig. 4.13 for the deflagration and detonation regimes respectively. Since detonation regime can have wider allowed values of $F \leq 1$, it leads to smaller DM abundances compared to the deflagration. The black dashed line corresponds to DM abundance five times of baryons $\Omega_{\text{DM}} = 5\Omega_B$, suggested by observations. Thus we observe a proportionate rise in DM abundance with baryon density while also accommodating the cogenesis motivated parameter space. Fig. 4.14 shows the GW spectra for the chosen benchmark points given in table 4.3 and 4.4. The sensitivities of planned future experiment like LISA [262], μ ARES [156], THEIA [155], GAIA [155], SKA [154] are shown as shaded regions together with the recent NANOGrav data [79] as orange coloured violin-shaped points. Due to the restrictions of having the FOPT below the electroweak scale, all the benchmark points remain within the sensitivities of future experiments like μ ARES [156].

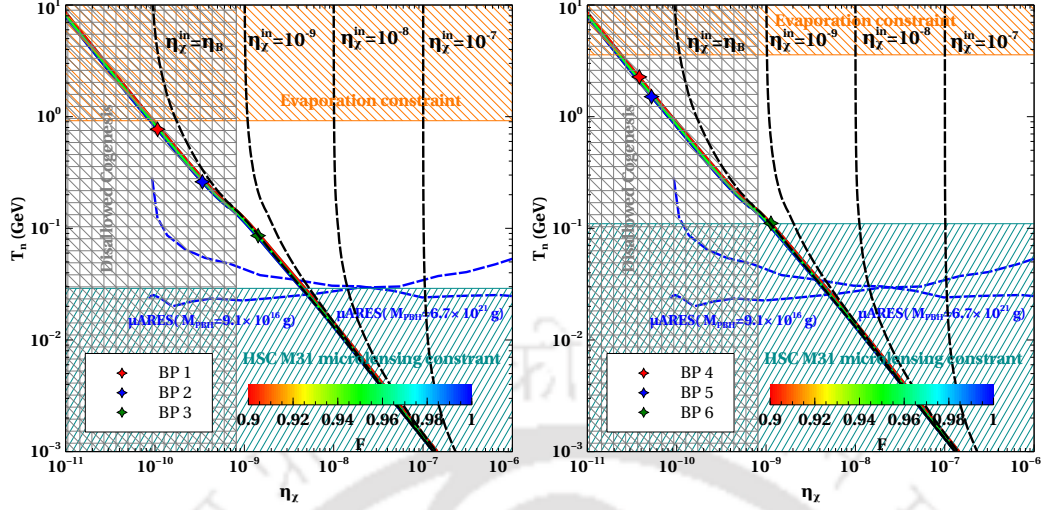


Figure 4.11: The parameter space in the η_χ - T_n plane where the solid green line represents condition where PBH fully constitute dark matter ($f_{\text{PBH}} = 1$) consistent with appropriate bubble filtering. The black dashed lines correspond to the parameter space with fixed dark asymmetry filtered to the true vacuum. For left panel, $\beta/\mathcal{H}_* = 200$, $m_\chi^{\text{out}} \sim T_n$, $\alpha_* = 0.3$ and $v_w = 0.85$ considering deflagration regime. Same for right panel with $\beta/\mathcal{H}_* = 50$, $m_\chi^{\text{out}} \sim T_n$, $\alpha_* = 0.3$ and $v_w = 0.85$.

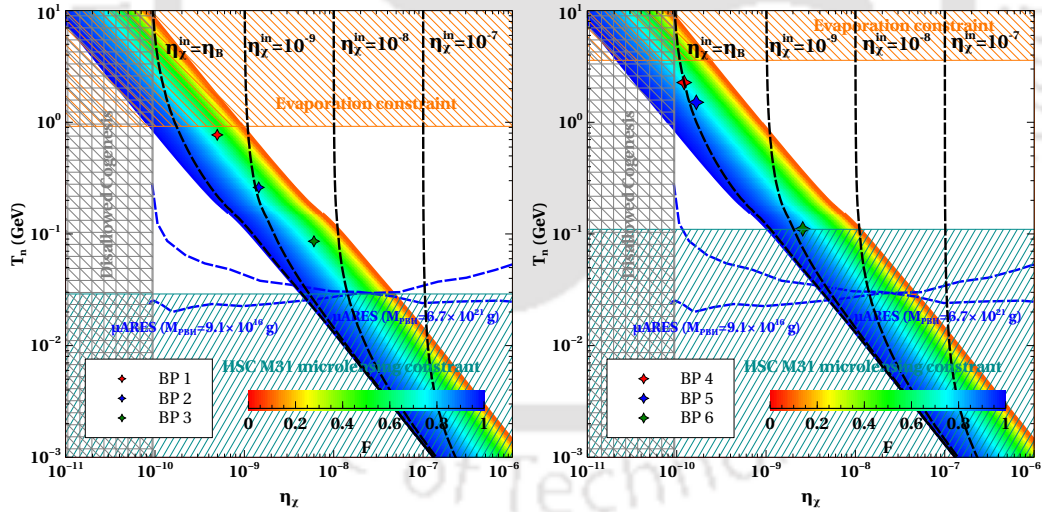


Figure 4.12: The parameter space in the η_χ - T_n plane where the solid green line represents condition where PBH fully constitute dark matter ($f_{\text{PBH}} = 1$) consistent with appropriate bubble filtering. The black dashed lines correspond to the parameter space with fixed dark asymmetry filtered to the true vacuum. For left panel, $\beta/\mathcal{H}_* = 200$, $m_\chi^{\text{out}} \sim T_n$, $\alpha_* = 0.3$ and $v_w = 0.85$ considering detonation regime. Same for right panel with $\beta/\mathcal{H}_* = 50$, $m_\chi^{\text{out}} \sim T_n$, $\alpha_* = 0.3$ and $v_w = 0.85$.

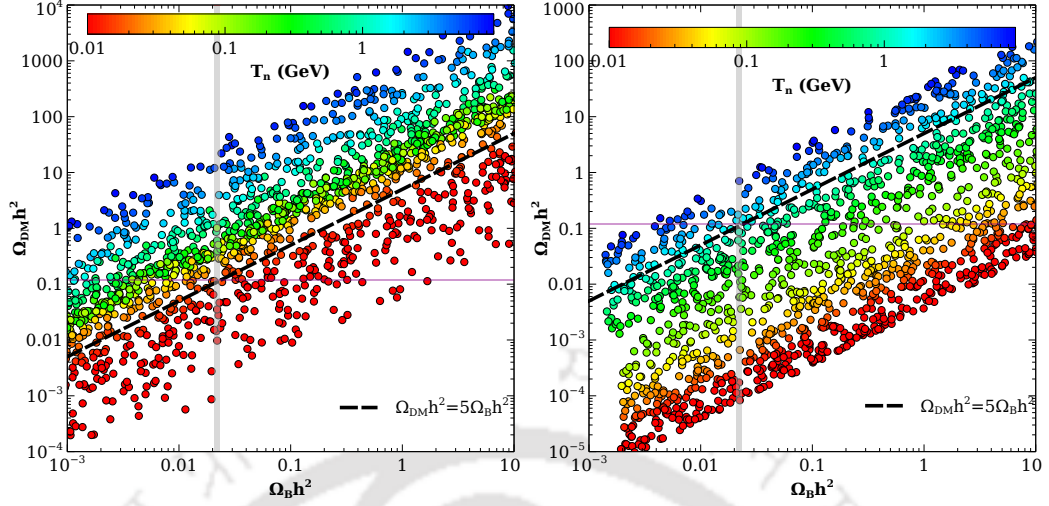


Figure 4.13: Variation of DM abundance $\Omega_{\text{DM}} h^2$ with baryon abundance $\Omega_{\text{B}} h^2$. The colour bar indicates the nucleation temperature in the deflagration (left) and detonation (right) regimes. The horizontal and vertical bands correspond to the observational limits on DM and baryon densities respectively.

	v_D (GeV)	m_{Φ_2} (GeV)	m_χ (GeV)	m_χ^{in} (GeV)	T_c (GeV)	T_n (GeV)	$\beta/\mathcal{H}_*$	α_*	M_{PBH} (g)
BP1	3.5	3.61	0.50	3.46	1.12	0.77	200	0.05	9.9×10^{16}
BP2	1.2	1.24	0.20	1.21	0.38	0.26	200	0.08	3.3×10^{18}
BP3	0.4	0.42	0.06	0.39	0.12	0.08	200	0.26	2.7×10^{20}

Table 4.3: Benchmark points for first-order phase transition.

	v_D (GeV)	m_{Φ_2} (GeV)	m_χ (GeV)	m_χ^{in} (GeV)	T_c (GeV)	T_n (GeV)	$\beta/\mathcal{H}_*$	α_*	M_{PBH} (g)
BP4	12	15.91	0.60	12.47	3.75	2.27	50	0.09	9.2×10^{17}
BP5	8	10.62	0.40	8.31	2.50	1.51	50	0.09	2.9×10^{18}
BP6	0.6	0.79	0.10	0.69	0.18	0.11	50	0.42	1.8×10^{22}

Table 4.4: Benchmark points for first-order phase transition.

If the phase transition and bubble filtering of χ occurs before sphaleron decoupling, χ can transfer the asymmetry into leptons which can then get converted into baryon asymmetry via electroweak sphalerons. However, as shown in Fig. 4.11 and Fig. 4.12, the allowed region consistent with PBH DM corresponds to nucleation temperature only upto a few GeV. This requires the dark fermion asymmetry to be transferred directly into baryons in the true vacuum. This is possible by identifying ψ in Eq. (4.46) as a right-handed down-type quark d_R with ζ being a coloured scalar with electromagnetic charge $-1/3$. This lead to the following interactions relevant for transferring dark fermion asymmetry:

$$-\mathcal{L} \supset y_2 \overline{d_R} \zeta \chi + y_3 \zeta^* \overline{u_R^c} d_R + \text{h.c.} \quad (4.65)$$

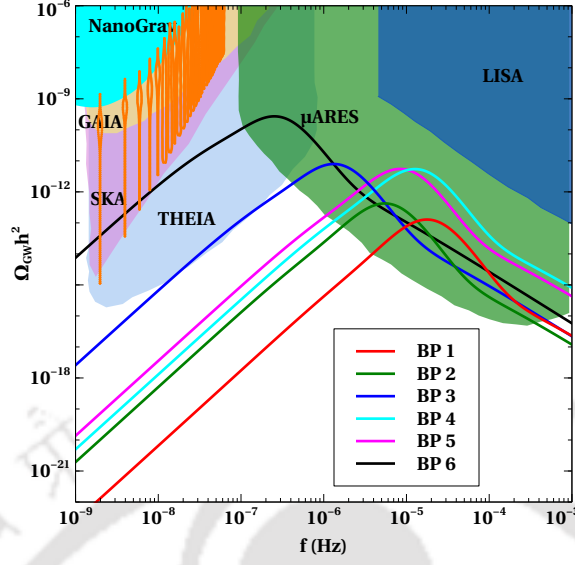


Figure 4.14: Gravitational wave spectrum corresponding to benchmark points in Table 4.3 and 4.4.

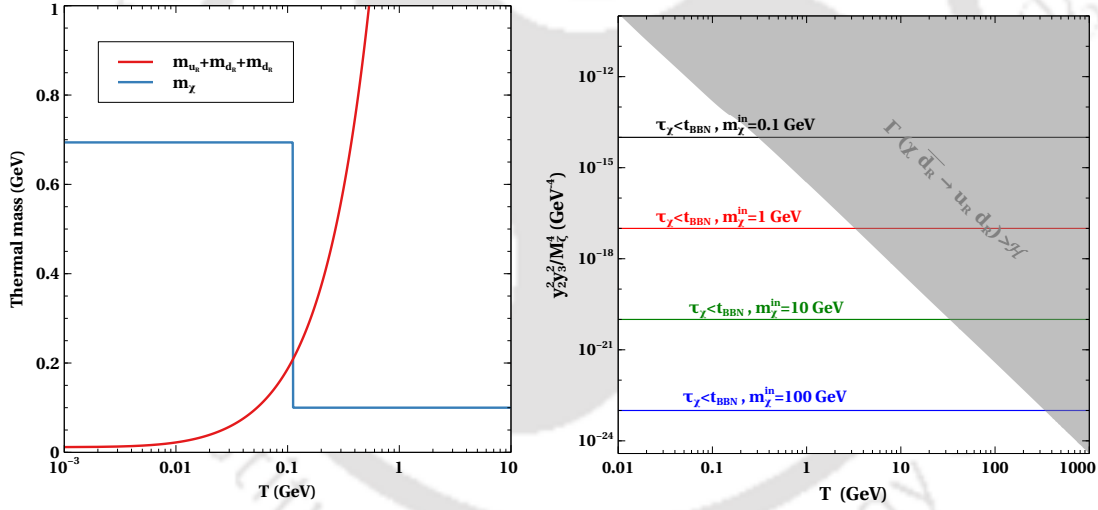


Figure 4.15: Left panel: Thermal mass profiles of dark fermion χ and quarks corresponding to the benchmark point BP6 in Table 4.4. Right panel: Allowed parameter space from the criteria of lifetime $\tau_\chi < t_{\text{BBN}}$ and inefficient transfer of dark fermion asymmetry to baryons via scattering. The white region above a horizontal line is allowed for a specific value of χ mass in true vacuum.

Similar interactions have been considered in low scale baryogenesis models [360]. While ζ is much heavier than χ or SM particles due to collider bounds on such a coloured scalar [360], we consider three body decay $\chi \rightarrow u_R d_R d_R$ mediated by off-shell ζ to transfer the dark fermion asymmetry to the baryons in the true vacuum. While χ is stable in the false vacuum prior to FOPT or the formation of PBH, it

can decay into three SM quarks due to the sudden change in mass after the phase transition. In the left panel of Fig. 4.15, we show the variation of finite-temperature masses of χ and combination of three right-handed quarks. The thermal mass of quarks can be written as [99, 361]

$$m_{u_R, d_R}^2(T) = m_{u_R, d_R}^2(T=0) + \frac{1}{6}g_s^2 T^2 + \frac{1}{8}4\pi\alpha_{\text{em}}T^2. \quad (4.66)$$

Clearly, before the FOPT, χ decay is kinematically forbidden while in a true vacuum, χ can decay such that it transfers the remnant asymmetry to baryons. The corresponding three-body decay width is given by

$$\Gamma_\chi \equiv \Gamma(\chi \rightarrow u_R d_R d_R) \approx \frac{y_2^2 y_3^2 (m_\chi^{\text{in}})^5}{2038\pi^3 M_\zeta^4}, \quad (4.67)$$

where M_ζ denotes the mass of the coloured scalar ζ . Since dark fermion stores an asymmetry much larger than the observed baryon asymmetry, it is also required to ensure that they do not transfer the asymmetry to quarks via scattering processes like $\chi \bar{d}_R \rightarrow u_R d_R$ mediated by ζ . Keeping this conversion process out-of-equilibrium at a temperature T leads to

$$T^5 \frac{y_2^2 y_3^2}{64\pi M_\zeta^4} \lesssim \sqrt{\frac{\pi^2}{90} g_*} \frac{T^2}{M_{\text{Pl}}}, \quad (4.68)$$

assuming massless initial and final states appropriate for high temperatures. The out-of-equilibrium condition of this process can be satisfied for suitable choices of Yukawa couplings y_2, y_3 and coloured scalar mass M_ζ . However, the combination of this out-of-equilibrium criterion and lifetime of χ : $\tau_\chi = \Gamma_\chi^{-1} < t_{\text{BBN}}$ being shorter compared to the BBN timescale restrict the reheat temperature of the Universe (or the production temperature of dark fermion asymmetry $T_{\Delta\chi} < T_{\text{RH}}$) to be $\lesssim \mathcal{O}(10)$ GeV depending upon the mass of χ in true vacuum. This requires the creation of dark fermion asymmetry just before the FOPT without leaving much window to transfer the asymmetry to quarks via scatterings. In the right panel of Fig. 4.15, we show $\frac{y_2^2 y_3^2}{M_\zeta^4}$ as a function of T indicating the disfavoured region (grey-shaded) where dark fermion asymmetry can be transferred to baryons via scattering. The region above the horizontal line for a particular value of χ mass in true vacuum corresponds to required χ lifetime being shorter than the BBN epoch $\tau_\chi < t_{\text{BBN}}$ for successful

baryogenesis. Clearly, depending upon χ mass in true vacuum, the parameter space of the model and the production temperature of dark fermion asymmetry are tightly constrained.

4.2.4 Conclusion

We have proposed a novel cogenesis mechanism of producing dark matter and baryon asymmetry in the Universe by utilising a first-order phase transition in the early Universe. The FOPT driven by a singlet scalar also leads to a sharp rise in a dark fermion mass. While the dark fermion remains in equilibrium prior to the FOPT, a large mass gap across the bubble wall $m_d \gg T_n$ prevents most of these dark fermions to get trapped in the false vacuum. An initial dark sector asymmetry prevents the dark fermions from being annihilated away in the squeezed pockets of false vacuum. While such false vacuum pockets lead to the formation of Fermi-balls when Fermi degeneracy pressure balances the vacuum pressure, a strong attractive Yukawa type interaction among dark fermions can induce further collapse to primordial black holes. Since the dark fermion momenta follow equilibrium distribution prior to the FOPT, its high momentum modes are energetic enough to leak into the true vacuum through the bubble walls, leading to filtering of dark fermions. The filtered dark fermion can also decay into the visible sector due to the mass gain in the true vacuum, thereby transferring the filtered dark sector asymmetry to visible sector asymmetry. We constrain the model from the requirement of producing the observed dark matter relic in the form of PBH and observed baryon asymmetry from filtered dark sector asymmetry in the true vacuum. We find a small allowed region of parameter space with nucleation temperature ranging from a few tens of MeV to a few GeV which is consistent with the cogenesis requirements and experimental bounds on PBH mass. Interestingly, for this allowed region of parameter space, the FOPT generated stochastic gravitational wave remains within reach of future GW experiments like μ ARES [156]. Additionally, such low scale FOPT scenario has several light new degrees of freedom accessible at terrestrial experiments. The PBH in this asteroid mass range can also have their own observational signatures related superradiance, Hawking evaporation or microlensing [362]. While we have performed our detailed numerical analysis assuming an initial dark sector asymmetry, we also outline one possible UV completion where such an asymmetry can be generated at a scale above

the phase transition scale via the Affleck-Dine mechanism. If the FOPT occurs prior to the sphaleron decoupling epoch, the dark sector asymmetry can be transferred to leptons opening up the leptogenesis route. However, this will overproduce PBH in the desired mass range requiring additional entropy dilution. Alternately, we can have such high scale FOPT and production of heavier planetary mass PBH with $f_{\text{PBH}} \ll 1$ which can show up in gravitational microlensing observations [363]. While this allows PBH to be a small fraction of DM, the remaining DM can originate from leftover dark fermion asymmetry after partial transfer to baryon asymmetry. This requires different model building approach which we leave for future works.



Gravitational wave signatures of leptogenesis with discrete symmetry breaking

Recently, four different pulsar timing array experiments namely NANOGrav [79], EPTA together with the first data release from InPTA [364], PPTA [81], all part of the consortium called International Pulsar Timing Array have released their latest findings hinting at a significant evidence for stochastic gravitational waves background at nano-Hz frequencies. Similar evidence with larger statistical significance has also been reported by the CPTA collaboration [82]. While such a signal can be generated by supermassive black hole binary (SMBHB) mergers though with a mild tension, presence of exotic new physics alone or together with SMBHB can make the fit better [85]ⁱ⁾. Several follow-up papers have also studied the possible origin or implications of this observation from the point of view of dark matter [365, 366], axions or axion-like particles [367, 368], SMBHB [84], first-order phase transition [369–372]ⁱⁱ⁾ and associated challenges [374], primordial black holes [375], primordial magnetic field [376], domain walls [377, 378], inflation [379, 380], cosmic strings [381, 382], scalar induced gravitational waves [85] including earlier works [383], astrophysical neutrino oscillation [384] and QCD crossover [385]. New physics

ⁱ⁾ Similar conclusions can also be found in [80].

ⁱⁱ⁾ See [373] for earlier works.

possibilities leading to primordial GW in the nano-Hz regime can also be found in [386].

While GW from domain walls (DW) has already been studied as a possible new physics explanation for PTA results [85, 377, 378], we consider the consequence for two popular beyond standard model (BSM) scenarios namely, the minimal Dirac leptogenesis and the left-right symmetric model (LRSM). The first model is a type I seesaw realisation for light Dirac neutrino mass with the heavy vector-like neutral fermions being responsible for generating baryogenesis via leptogenesis [34] with light Dirac neutrinos, known as the Dirac leptogenesis scenario [387, 388]. GW probe of high scale leptogenesis models have received considerable attention in recent times. In most of these works [389–392], cosmic string (CS) origin of GW has been studied by considering a $U(1)_{B-L}$ framework with in-built heavy Majorana fermions responsible for generating Majorana mass of light neutrinos as well as for leptogenesis. The scale of leptogenesis or $U(1)_{B-L}$ breaking scale then decides the amplitude of the CS generated GW spectrum. However, in view of the latest PTA results preferring a positive slope of the GW spectrum, stable CS in such models no longer provide a good fit [85]. This raises the prospects for a Dirac leptogenesis model whose minimal version must have a softly broken Z_2 symmetry leading to formation of DW followed by generation of GW due to annihilation or collapse. While a general study related to GW probe of minimal Dirac leptogenesis was carried out in [393] (and subsequently in [394] for Dirac seesaw), here we consider the implications of recent PTA findings on the scale of Dirac leptogenesis. On the other hand, GW probe of LRSM considering DW as the source have been studied in earlier works [395, 396]. DW arise due to spontaneous breaking of parity in such models. While earlier works considered the detection aspects of this model, we now constrain the scale of left-right symmetry considering the latest PTA data. While both the models can explain the latest PTA data, the allowed parameter space remains squeezed to a tiny window, which should face more scrutiny with future data.

In Chapter 5, we explore the generation of gravitational waves from domain wall dynamics and derive constraints from measurements of the effective number of relativistic species, ΔN_{eff} , in Section 5.1. In Section 5.2, we investigate the implications of these signals for constraining Dirac leptogenesis scenarios, while in Section 5.3, we

extend the analysis to constrain the symmetry-breaking scale in left-right symmetric models. Finally we conclude in Section 5.4.

5.1 Domain walls as source of GW

Domain wall is a two-dimensional topological defect arising from spontaneous breaking of discrete symmetries [161, 163, 397]. With the expansion of the universe, the energy density of DW falls slower compared to that of radiation or ordinary matter, having the potential to start dominating the energy density of the universe and ruin the successful predictions of standard cosmology. Such a disastrous situation can be prevented if DW are made unstable or diluted or if they have asymmetric initial field fluctuations [398, 399].

In minimal model of Dirac leptogenesis [393] as well as left-right symmetric models [396], such DW arise due to the spontaneous breaking of a Z_2 symmetry. If we consider a Z_2 -symmetric potential of a scalar field φ , it is straightforward to show the existence of two different vacua $\langle \varphi \rangle = \pm u$. It is also possible to find a static solution of the equation of motion given the two vacua to be realised at $x \rightarrow \pm\infty$,

$$\varphi(\mathbf{x}) = u \tanh \left(\sqrt{\frac{\lambda_\varphi}{2}} u x \right), \quad (5.1)$$

representing a domain wall extended along the $x = 0$ plane. Here λ_φ is the quartic self-coupling of the scalar field. The DW width is $\delta \sim m_\varphi^{-1} = (\sqrt{2\lambda_\varphi} u)^{-1}$. Another key parameter, known as the DW tension is given by

$$\sigma_w = \int_{-\infty}^{\infty} dx \rho_\varphi = \frac{2\sqrt{2}}{3} \sqrt{\lambda_\varphi} u^3 = \frac{2}{3} m_\varphi u^2 \sim u^3, \quad (5.2)$$

where ρ_φ denotes (static) energy density of φ and in the last step, $m_\varphi \sim u$ is used.

Assuming the walls to be formed after inflation, the simplest way to make them disappear is to introduce a small pressure difference [159–161, 163, 400], a manifestation of a soft Z_2 -breaking term. Such a pressure difference or equivalently, a bias term in the potential ΔV needs to be sufficiently large to ensure DW disappearance prior to the epoch of big bang nucleosynthesis (BBN) that is, $t_{\text{BBN}} > t_{\text{dec}} \approx \sigma_w / \Delta V$.

It is also important to take care of the fact that the DW disappear before dominating the universe, requiring $t_{\text{dec}} < t_{\text{dom}}$, where $t_{\text{dom}} \sim M_{\text{Pl}}^2/\sigma_w$ and M_{Pl} is the *reduced* Planck mass. Both of these criteria lead to a lower bound on the bias term ΔV . However, ΔV can not be arbitrarily large as it would otherwise prevent the percolation of both the vacua separated by DW. Such decaying DW therefore can emit GW [164, 167, 170, 401–407]. At peak frequency f_{peak} , the spectral energy density can be estimated as [167, 170]

$$\Omega_{\text{GW}} h^2(t_0) \Big|_{\text{peak}} \simeq 5.2 \times 10^{-20} \tilde{\epsilon}_{\text{gw}} A_w^4 \left(\frac{10.75}{g_*} \right)^{1/3} \left(\frac{\sigma_w}{1 \text{ TeV}^3} \right)^4 \left(\frac{1 \text{ MeV}^4}{\Delta V} \right)^2, \quad (5.3)$$

with t_0 being the present time. Away from the peak, the amplitude varies asⁱⁱⁱ⁾

$$\Omega_{\text{GW}} \simeq \Omega_{\text{GW}} \Big|_{\text{peak}} \times \begin{cases} \left(\frac{f_{\text{peak}}}{f} \right) & \text{for } f > f_{\text{peak}} \\ \left(\frac{f}{f_{\text{peak}}} \right)^3 & \text{for } f < f_{\text{peak}} \end{cases}, \quad (5.4)$$

where the peak frequency is given by

$$f_{\text{peak}}(t_0) \simeq 4 \times 10^{-9} \text{ Hz } A_w^{-1/2} \left(\frac{1 \text{ TeV}^3}{\sigma_w} \right)^{1/2} \left(\frac{\Delta V}{1 \text{ MeV}^4} \right)^{1/2}.$$

In the above expressions, A_w is the area parameter [409, 410] $\simeq 0.8$ for DW arising from Z_2 breaking, and $\tilde{\epsilon}_{\text{gw}}$ is the efficiency parameter $\simeq 0.7$ [167]. Note that the above spectrum can be obtained from a general parametrisation $S(f/f_{\text{peak}})$

$$S(x) = \frac{(a+b)^c}{(bx^{-a/c} + ax^{b/c})^c} \quad (5.5)$$

for $a = 3$ (required by causality [411, 412]) and $b \approx c \approx 1$ (suggested by simulation [167]). Utilising these values of a, b and c , together with $x \gg 1$ (or $x \ll 1$), we can produce Eq. (5.4). However, as noted in [85, 405], the values of b, c may depend upon the specific DW annihilation mechanism or regime all of which have not been

ⁱⁱⁱ⁾The low-frequency spectrum of GW from melting DWs [404, 408] characterised by a *time-dependent* tension, contrary to the constant tension DWs as discussed here, behaves as f^2 , without violating the causality.

explored in numerical simulations yet. This allows one to vary b, c to get a better fit with the PTA data [85].

When the GW production is ceased after the annihilation of the domain walls, the energy density of GW redshifts mimicking that of the SM radiation. As a result, GW itself acts as an additional source of radiation with the potential to alter the prediction of BBN. Thus, an excess of the GW energy density around $T \lesssim \mathcal{O}(\text{MeV})$, can be restricted by considering the limits on the number of relativistic degrees of freedom from CMB and BBN, encoded in ΔN_{eff} . This, in turns, puts a bound on the amplitude of GW spectrum, demanding [413, 414]

$$\Omega_{\text{GW}} h^2 \lesssim 5.6 \times 10^{-6} \Delta N_{\text{eff}}. \quad (5.6)$$

Here we consider several projected limits on ΔN_{eff} on top of the existing limit from Planck: $\Delta N_{\text{eff}} \lesssim 0.28$ at 95% CL [7]. This bound is shown by the solid grey horizontal line in right panel of Fig. 5.1. Once the baryon acoustic oscillation (BAO) data are included, the measurement becomes more stringent: $N_{\text{eff}} = 2.99 \pm 0.17$. A combined BBN+CMB analysis shows $N_{\text{eff}} = 2.880 \pm 0.144$, as computed in Ref. [415]. This constraint is denoted by the dashed horizontal line. On the other hand, upcoming CMB experiments like CMB-S4 [416] and CMB-HD [417] will be able to probe ΔN_{eff} as small as ~ 0.06 and ~ 0.027 , respectively. These are indicated by dot-dashed and dotted lines respectively. The next generation of satellite missions, such as CORe [418] and Euclid [419], leads to $\Delta N_{\text{eff}} \lesssim 0.013$, as shown by the large dashed line.

It should be noted that we are ignoring the friction effects between the domain walls and the thermal plasma [402, 420, 421]. Such friction effects can be important when the field responsible for symmetry breaking or constituting the wall has large couplings with the SM bath particles, leading to smaller GW amplitude than that without friction discussed here. Since the scalar fields responsible for symmetry breaking has tiny couplings with the SM plasma in the models discussed here, such effects can be ignored [404].

In the left panel of Fig. 5.1 we summarise bounds on the VEV u and the bias term ΔV , where all the shaded regions are disallowed from (i) decay of the DWs post BBN (darker grey), where $t_{\text{dec}} > 1$ sec, (ii) DW domination (lighter grey) or $t_{\text{dom}} < t_{\text{dec}}$

	u (GeV)	$\Delta V(\text{MeV})^4$
BP1	2×10^5	10^8
BP2	3×10^5	10^8
BP3	4×10^5	10^8

Table 5.1: Details of the benchmark points (BPs) used in right panel of Fig. 5.1 and Fig. 5.2.

and (iii) ΔN_{eff} bound from PLANCK on excessive GW energy density (light grey). This leaves us with the white region in-between that is allowed, from where we choose our benchmark points (BP), as indicated in Tab. 5.1. It is important to note here, we also consider $\Delta V \ll u^4$ to prevent the percolation of both the vacua separated by DW. However, such a condition is trivially satisfied in the regime we are interested in.

The GW spectrum corresponding to the BPs in Tab. 5.1 is illustrated in right panel of Fig. 5.1. As explained before, we distinctly see a blue-tilted pattern for $f < f_{\text{peak}}$, while the spectrum is red-tilted in the opposite limit. Here we project limits from BBO [88], LISA [422], DECIGO [263], ET [213], CE [214], THEIA [155], HL (aLIGO) [264], μARES [156] and SKA [154]. In this plot, the range of GW spectrum from NANOGrav results [79] is shown by the red points. The grey-shaded region is completely disallowed from ΔN_{eff} bound on overproduction of Ω_{GW} as discussed before, depending on the sensitivity of a particular experiment. As one can already notice, BP3 is already ruled out from PLANCK bound, while BP1 is beyond the reach of any future experiments proposed so far.

Corresponding to the two epochs t_{dom} and t_{ann} stated before, we define two temperatures T_{dom} and T_{ann} . We are typically interested in the regime $T_{\text{ann}} > T_{\text{dom}}$, i.e., the DWs disappear before they dominate the Universe. Following [378], T_{ann} reads

$$T_{\text{ann}} \simeq 120 \text{ MeV} \sqrt{\frac{\Delta V/\text{MeV}^4}{10^8}} \left(\frac{A_w}{0.8}\right)^{-1/2} \left(\frac{\sigma_w/\text{GeV}^3}{10^{16}}\right)^{-1/2} \left(\frac{g_*(T_{\text{ann}})}{10}\right)^{-1/4}, \quad (5.7)$$

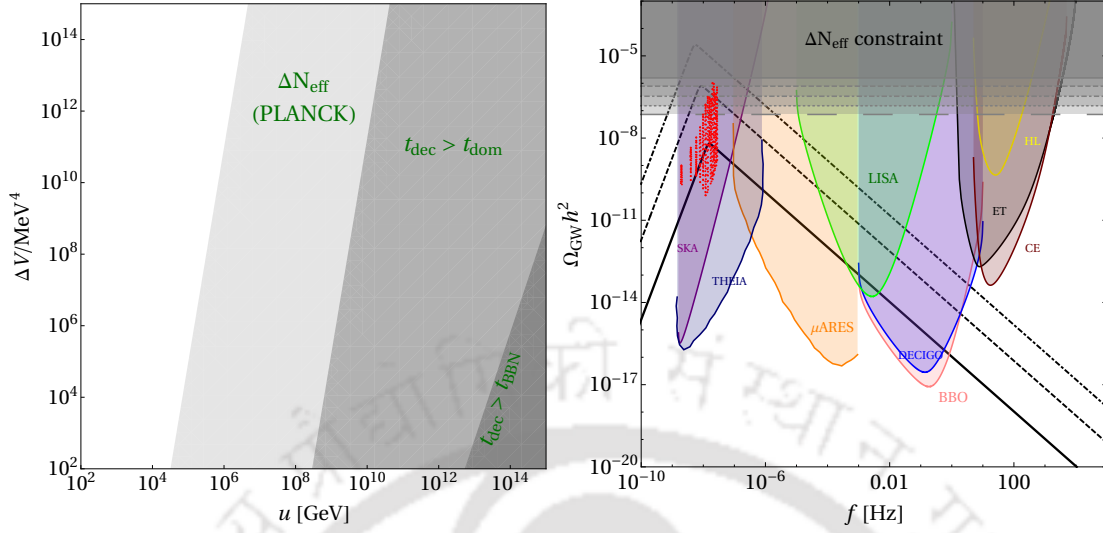


Figure 5.1: Left panel: Bound on the size of VEV and the bias term. All shaded regions are disallowed (see text for details), Right panel: Spectrum of gravitational wave from DW decay, where we show sensitivities of several GW experiments. The black curves correspond to the chosen benchmark points for $a = 3, b = 1, c = 0.3$. The grey region marked as “ ΔN_{eff} constraint” is disallowed from overproduction of GW energy density (see text for details). The red points correspond to the NanoGrav 15 yr observation [79].

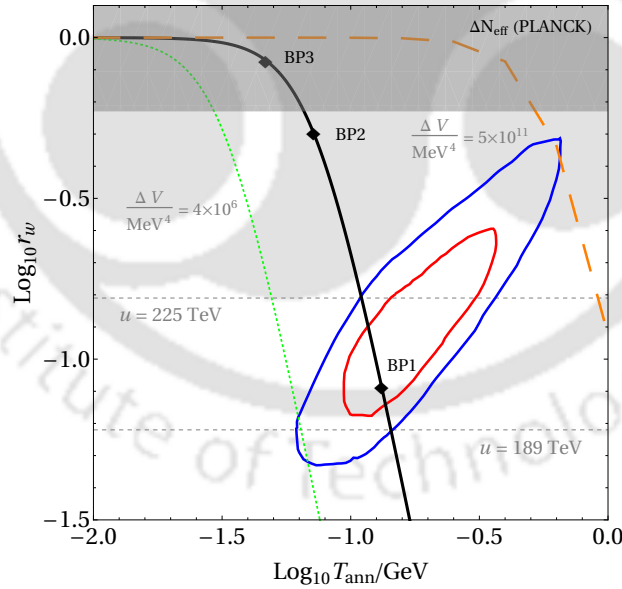


Figure 5.2: Viable parameter space in the bi-dimensional plane of $T_{\text{ann}}-r_w$, based on the NanoGrav 15 yr dataset [79]. The black solid contour corresponds to the ΔV -value relevant for the BPs in Tab. 5.1. The red and blue contours correspond to 1- and 2σ CL respectively. The grey shaded region is disallowed from ΔN_{eff} bound from Planck.

implying, for larger surface tension, it takes longer for the walls to collapse, while for larger bias the opposite happens. We also define another quantity

$$r_w = \frac{\rho_r}{1 + \rho_r}, \quad (5.8)$$

where

$$\rho_r = \frac{\rho_w(T_{\text{ann}})}{\rho_R(T_{\text{ann}})} \simeq 0.14 \left(\frac{A_w}{0.8} \right)^2 \left(\frac{\sigma_w/\text{GeV}^3}{10^{16}} \right)^2 \left(\frac{10^8 \text{ MeV}^4}{\Delta V} \right), \quad (5.9)$$

which quantifies the energy density contained within the DW compared to that of radiation.

We show the compatibility of our relevant model parameters, namely, the bias term ΔV and the VEV u (equivalently, the strain σ) in Fig. 5.2 with the NANOGrav data, utilizing Eq. (5.7) and Eq. (5.8). We superimpose the 1 and 2σ contours (shown by red and blue solid curves) provided by the NANOGrav result [79]. As one can see, BP1 lies well within the 1σ contour, while the other two BPs are well off. For BP1, $a = 3, b \in [0.5, 1], c \in [0.3, 3]$ is needed to be in compliance within 1σ of NANOGrav data for frequency $f \in [2 \times 10^{-9}, f_{\text{yr}}]$ Hz, where $f_{\text{yr}} = 1 \text{ yr}^{-1} \approx 3 \times 10^{-8}$ Hz. It is possible to derive a lower and upper bound on the VEV for a fixed ΔV , as denoted by the grey dashed horizontal lines for $\Delta V = 10^8 \text{ MeV}^4$. Thus, for $\Delta V = 10^8 \text{ MeV}^4$, we find $189 \lesssim u/\text{TeV} \lesssim 225$, in compliance with NANOGrav 2σ contour. On the other hand, the viable range of bias term, that lies within 2σ CL of NANOGrav result, turns out to be $4 \times 10^6 \lesssim \Delta V/\text{MeV}^4 \lesssim 5 \times 10^{11}$, as shown by the green dotted and orange dashed curves. Depending on the choice of ΔV , the upper limit on u can be pushed to larger values. One can also project the ΔN_{eff} bound on the same plane (by trading $\{\sigma, \Delta V\}$ with $\{T_{\text{ann}}, r_w\}$) ruling out the region of the parameter space that results in overproduction of GW [cf. Eq. (5.6)]. This is shown by the grey shaded region, where we have used the $2\text{-}\sigma$ bound from Planck. This rules out BP3, as already seen in left panel of Fig. 5.1. Note that, the limits obtained on VEV u together with that on the bias ΔV , satisfy $t_{\text{dom}} < t_{\text{dec}}$ and $t_{\text{dec}} < t_{\text{BBN}}$, obeying $\Delta V \ll u^4$.

5.2 Consequence for Dirac leptogenesis

In the minimal model of Dirac leptogenesis or Dirac neutrino seesaw [393], the Standard Model is extended by three copies of vector-like neutral singlet fermions $N_{L,R}$ and three copies of right chiral part (ν_R) of light Dirac neutrinos. A real singlet scalar field φ is introduced to couple ν_R with N . A Z_2 symmetry under which φ, ν_R are odd, prevents direct coupling of the SM lepton doublet L with ν_R via SM Higgs H . The relevant part of the Yukawa Lagrangian can be written as

$$-\mathcal{L}_Y \supset Y_L \bar{L} \tilde{H} N_R + M_N \bar{N} N + Y_R \bar{N}_L \varphi \nu_R + \text{h.c.} \quad (5.10)$$

After the neutral components of H and φ acquire VEV v, u respectively, light Dirac neutrino mass arises from the Type-I seesaw equivalent for Dirac neutrino as

$$m_\nu = \frac{1}{\sqrt{2}} Y_L M_N^{-1} Y_R v u \quad (5.11)$$

with M_N being the scale of Dirac seesaw. The same heavy fermions $N_{L,R}$ can have out-of-equilibrium decay to achieve successful Dirac leptogenesis. Although no net lepton asymmetry is produced due to total lepton number conservation, it is possible to create equal and opposite lepton asymmetries in left and right sectors due to CP violating out-of-equilibrium decays $N \rightarrow L H$ and $N \rightarrow \nu_R \varphi$ respectively. The CP asymmetry parameter is given as [423]

$$\epsilon \simeq -\frac{1}{8\pi} \frac{M_1}{uv} \frac{\text{Im}[(Y_R m_\nu^\dagger Y_L)_{11}]}{(Y_R Y_R^\dagger) + (Y_L Y_L^\dagger)}, \quad (5.12)$$

where $v = 246$ GeV and M_1 is the lightest heavy fermion mass. If a net lepton asymmetry is generated before the sphaleron decoupling epoch, it is possible to create a net baryon asymmetry. However, in order to prevent the left sector lepton asymmetry from being washed out, it is important to prevent the equilibration of left and right sectors, leading to a condition

$$\Gamma_{L-R} \sim \frac{|Y_L|^2 |Y_R|^2}{M_1^2} T^3 < \mathcal{H}(T), \quad (5.13)$$

where $\mathcal{H}(T) = \frac{\pi}{3} \sqrt{\frac{g_*}{10}} \frac{T^2}{M_{\text{Pl}}}$ is the Hubble parameter. We do not compute the lepton asymmetry here and refer the reader to earlier works [393, 423, 424] where explicit Boltzmann equations were solved, and corresponding parameter space has been obtained. Depending upon the scale of leptogenesis M_1 , one can consider either quasi-degenerate or hierarchical heavy fermions to achieve the desired CP asymmetry while being consistent with sub-eV Dirac neutrino mass.

As the Z_2 -odd scalar φ acquires a non-zero VEV u , it leads to the formation of domain walls^{iv}. One can introduce a bias term ΔV (which breaks Z_2 symmetry softly) in the scalar potential which eventually lead to the disappearance of domain walls. Now, for $u \gtrsim 190$ TeV, preferred from NANOGrav 2023 data as discussed before, and considering light neutrino mass $m_\nu \leq 0.1$ eV, we get $M_N > y^2 \times 10^{17}$ GeV. This implies that, for order one Yukawa couplings, the scale of Dirac leptogenesis is even above the upper limit on reheating temperature, disfavouring the possibility of thermal Dirac leptogenesis. If the Yukawa couplings are made as low as electron Yukawa coupling, we have $M_N > 10^7$ GeV, keeping it at intermediate scale. To summarise, the possibility of low-scale Z_2 -symmetric Dirac leptogenesis is disfavoured unless we tune Yukawa couplings involved in the Dirac seesaw more than what we have in the SM.

It is worth stressing that Dirac leptogenesis can be realised without Z_2 symmetry too and in those setups there will not be any DW formation. One simple alternative is to consider a $U(1)$ global symmetry under which ϕ , now a complex scalar, and ν_R transform non-trivially. Soft breaking of such global $U(1)$ symmetry, required to generate light Dirac neutrino mass, leads to a light pseudo-Goldstone boson with interesting phenomenological consequences. On the other hand, for gauged $U(1)$ symmetry, an additional massive gauge boson will arise in the spectrum. If such additional neutral bosons couple to the heavy fermions (N) responsible for generating lepton asymmetry, they can lead to dilution of asymmetry while keeping N in equilibrium for longer epochs [425]. As far as topological defects are concerned, such models with $U(1)$ symmetries can lead to cosmic strings which can have their own GW signatures. Since the number of new degrees of freedom in Z_2 -symmetric

^{iv}) Because of the non-zero VEV, φ can mix with the SM Higgs doublet (H) via a portal interaction $\lambda_p |\varphi|^2 |H|^2$, that leads to its decay into the SM. One can always tune the mixing parameter such that φ decays efficiently before the onset of BBN.

Dirac leptogenesis is less than scenarios with $U(1)$ or higher symmetries, we have referred to it as the minimal Dirac leptogenesis.

5.3 Consequence for left-right symmetry

Left-right symmetric models [426–438] have been one of the most well-studied BSM frameworks where the SM gauge symmetry is extended to $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In addition to the gauge symmetry, the model also has an in-built Z_2 symmetry or parity symmetry ($\mathbb{P}$) under which the left and right sector fields are interchanged, keeping the framework parity or left-right symmetric. While a detailed discussion related to GW probe of LRSM can be found in [395, 396, 439], here we briefly summarise the implications of 2023 PTA data on the scale of left-right symmetry.

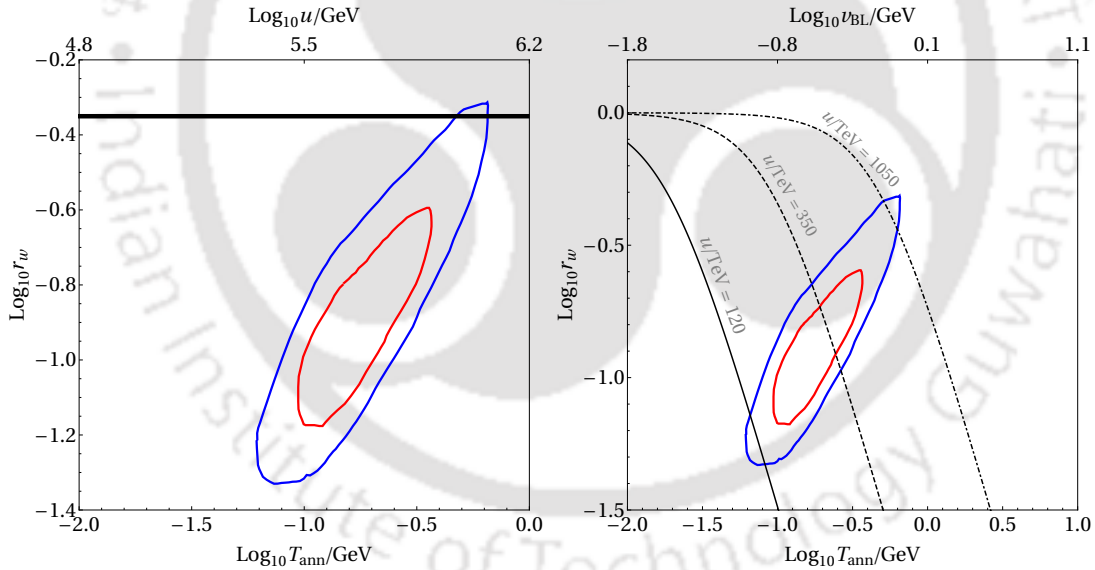


Figure 5.3: Left panel: Same as Fig. 5.2, but for the minimal LRSM. Along the black solid line the left-right symmetry breaking scale u is varying. Right panel: Same as Fig. 5.2, but for the non-minimal LRSM. Different black curves correspond to different choices of left-right symmetry breaking scale u , as denoted.

Unlike in the Z_2 -odd scalar singlet model discussed above, the LRSM gauge symmetry does not allow arbitrary bias terms. It is possible to generate such bias term via higher dimensional operators invariant under gauge symmetry but explicitly

breaking the parity symmetry. While Planck scale effects are expected to break any global symmetries like parity [175, 440, 441], the corresponding bias term can lead to DW disappearance [442, 443]. The minimal LRSM has three different types of scalars namely, $\Phi \equiv (1, 2, 2, 0)$, $\Delta_L \equiv (1, 3, 1, 2)$, $\Delta_R \equiv (1, 1, 3, 2)$ where the numbers in the brackets are the quantum numbers corresponding to the LRSM gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Left and right handed fermions transform as doublets under $SU(2)_L, SU(2)_R$ respectively. Quark and lepton fields are represented as $Q_L \equiv (3, 2, 1, 1/3)$, $Q_R \equiv (3, 1, 2, 1/3)$, $\ell_L \equiv (1, 2, 1, -1)$, $\ell_R \equiv (1, 1, 2, -1)$. Under parity $\mathbb{P}$, left and right sector fields get interchanged as

$$Q_L \leftrightarrow Q_R, \ell_L \leftrightarrow \ell_R, \Delta_L \leftrightarrow \Delta_R, \Phi \leftrightarrow \Phi^\dagger.$$

This not only ensures the equality of left and right sector gauge couplings $g_L = g_R$, but also relates the Yukawa and scalar potential couplings of these two sectors.

The neutral component of the scalar triplet Δ_R acquires a non-zero VEV breaking $SU(2)_R \times U(1)_{B-L} \times \mathbb{P}$ into $U(1)_Y$ of the SM. At a later stage, the electroweak gauge symmetry gets spontaneously broken to $U(1)_{\text{em}}$ by the neutral components of scalar bidoublet Φ . Thus, the symmetry breaking pattern is

$$\begin{aligned} &SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times \mathcal{P} \xrightarrow{\langle \Delta_R \rangle} \\ &SU(2)_L \times U(1)_Y \xrightarrow{\langle \Phi \rangle} U(1)_{\text{em}}. \end{aligned} \quad (5.14)$$

While this is the desired symmetry breaking pattern, it is equally probable for left sector scalar field Δ_L to acquire non-zero VEV. This leads to left and right sector vacua separated by domain walls. It is also possible to replace the pair of triplets $\Delta_{L,R}$ by a pair of doublets $H_{L,R}$ while achieving the same symmetry breaking pattern. In either of these minimal models, the bias term or soft $\mathbb{P}$ breaking term can arise from dimension six operators given by

$$V_{\text{NR}} \supset f_L \frac{(\Sigma_L^\dagger \Sigma_L)^3}{M_{\text{Pl}}^2} + f_R \frac{(\Sigma_R^\dagger \Sigma_R)^3}{M_{\text{Pl}}^2} \quad (5.15)$$

where $\Sigma_{L,R} \equiv \Delta_{L,R}, H_{L,R}$ depending upon the type of LRSM. This leads to a bias term in the minimal model given by $\Delta V \sim u^6/M_{\text{Pl}}^2$, where u is the $SU(2)_R \times U(1)_{B-L}$ as well as parity breaking scale. Due to the dependence of the bias term on the

scale of left-right breaking, the constraint on left-right symmetry breaking scale is stronger than what we had on the scale of Dirac leptogenesis. As shown in left panel of Fig. 5.3, the scale of left-right symmetry should be approximately around $\sim 10^6$ GeV (shown by the black solid horizontal line), in order to be in agreement with NANOGrav 15 yr data at 2σ .

Similarly, one can also check the status of non-minimal LRSM frameworks in the light of the recent PTA results. Unlike the minimal LRSM, in the non-minimal scenario, the symmetry of LRSM is broken down to that of the SM in more than one steps. For illustrative purpose, we consider a two-step symmetry breaking chain leading to

$$\begin{aligned} SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes \mathbb{P} &\xrightarrow{u} \\ SU(2)_L \otimes U(1)_R \otimes U(1)_{B-L} &\xrightarrow{v_{BL}} \\ SU(2)_L \otimes U(1)_Y &\xrightarrow{v_{EW}} U(1)_{\text{em}}, \end{aligned} \quad (5.16)$$

where v_{BL} is the intermediate symmetry breaking scale and v_{EW} is the electroweak symmetry breaking scale. For example, the first stage of the above symmetry breaking chain can be achieved by $SU(2)_R$ triplet of vanishing $B - L$ charge while the second stage can be taken care of by a triplet of non-zero $B - L$ charge. In such a scenario, the bias term can be written as dimension five operator involving both types of triplet scalars. Following [396], one can show that the bias term is related to the two scales of symmetry breaking via $\Delta V \simeq u^3 v_{BL}^2 / M_{\text{Pl}}$. In that case, one finds, $T_{\text{ann}} \propto v_{BL}$ [cf.Eq. (5.7)]. We show constraint on the LR symmetry breaking scale u , that allows to be within $2\text{-}\sigma$ of the NANOGrav 15 yr data in right panel of Fig. 5.3. Corresponding to each T_{ann} , the v_{BL} is fixed, as mentioned in the upper axis label. We find, in order to be compatible with $1\text{-}\sigma$ contour of the NANOGrav 15 yr data, $0.04 \text{ GeV} \lesssim v_{BL} \lesssim 1 \text{ GeV}$, while $10^2 \text{ TeV} \lesssim u \lesssim 10^3 \text{ TeV}$. However, such low scale v_{BL} will lead to light Z' gauge bosons, already ruled out by the Large Hadron Collider (LHC) data [444, 445]. Although we do not explore leptogenesis in the LRSM explicitly (more details of which can be found in [446, 447]), the bounds on symmetry breaking scales derived here could provide valuable insight into the parameter space consistent with successful leptogenesis.

5.4 Conclusion

We have investigated the consequence of the recent PTA results on the scale of Dirac leptogenesis and left-right symmetric model. In minimal version of both these scenarios, domain walls arise due to spontaneous breaking of a discrete Z_2 symmetry. While the bound on the scale of Dirac leptogenesis from PTA data depend upon the size of Dirac Yukawa couplings, for conservative choice of such couplings with fine-tuning at the level of the SM, we find a lower bound $M_N > 10^7$ GeV, keeping leptogenesis at intermediate scales. However, for order one Yukawa coupling, this bound is much stronger $M_N > 10^{17}$ GeV keeping only non-thermal Dirac leptogenesis option viable [cf. Fig. 5.2]. Due to the constrained structure of the minimal LRSM, we get much tighter constraint on the scale of left-right breaking namely $M_{LR} \sim 10^6$ GeV [cf. left panel of Fig. 5.3], in order to satisfy NANOGrav 15 yr data, keeping the model out of reach from direct search experiments like the LHC. For non-minimal LRSM, we can fit NANOGrav 15 yr data for $M_{LR} \simeq \{120 - 1050\}$ TeV [cf. right panel of Fig. 5.3] but with a very low $B - L$ breaking scale ruled out by the LHC data. Future data from PTA or other GW experiments are expected to shed more light on the parameter space of this model, by constraining the spectrum at higher frequencies.

Summary and Future Outlook

This thesis addresses two longstanding puzzles in particle physics and cosmology: the origin of the baryon asymmetry and dark matter in the Universe, through some beyond standard model frameworks having detectable stochastic gravitational waves (GW) signatures arising from symmetry-breaking phenomena in the early Universe. The persistent null results from dark matter searches, including direct-detection experiments, indirect search experiments, collider-based production attempts, and inability to directly probe typical baryogenesis models with high new physics scales, strongly motivate the pursuit of complementary, non-conventional detection avenues. By indirectly probing energy scales and dynamics inaccessible to traditional particle detectors, gravitational wave astronomy provides a novel and promising observational window into the mechanisms responsible for the genesis of dark matter and baryon asymmetry in the Universe. Particular emphasis is placed on the role of FOPT, where the dynamics of spontaneous symmetry breaking are not only responsible for mass generation but also leave imprints in the form of stochastic GW backgrounds.

A key feature explored in Chapter 3 of this thesis is the *mass-gain* mechanism, where particles remain abundant in the symmetric phase and acquire mass upon crossing the bubble wall into the broken phase during a FOPT, maintaining relativistic number density. This mechanism plays a crucial role in enhancing the efficiency of leptogenesis while also determining the final dark matter relic abundance. Throughout

this thesis, we study several models in which the interplay among FOPT or similar symmetry-breaking dynamics, mass generation, and gravitational wave signatures provides a testable framework for jointly addressing the origin of both baryon asymmetry and dark matter in the Universe.

Further, in Chapter 4 we discuss that particles passing the expanding bubble wall during a first-order phase transition can experience a filtering effect, wherein not all particles successfully enter the broken phase. This selective transmission can result in the accumulation or trapping of certain species in false vacuum remnants, potentially leading to the formation of primordial black holes. The cosmological implications of such PBH depend critically on their mass: low-mass PBH evaporate via Hawking radiation and may produce dark matter particles gravitationally, while PBH in the asteroid-mass range can survive until the present day and serve as viable dark matter candidates. These processes can also be inherently connected to the dynamics of baryon asymmetry generation, particularly in scenarios where the particle distributions across the bubble wall influence asymmetry production. Moreover, the formation and evolution of PBH during a first-order phase transition can generate gravitational waves, making such scenarios potentially testable through GW observations across a wide range of frequencies.

In Chapter 5, we explore the cosmological consequences of discrete symmetry breaking in leptogenesis models like minimal type-I Dirac seesaw or left-right symmetric model. Such symmetry breaking gives rise to unstable domain wall networks in the early Universe, whose annihilation can generate a stochastic gravitational wave background. The recent observations of nanoHz range GW by Pulsar Timing Array experiments may be interpreted as signatures of this mechanism. These signals offer a unique opportunity to place constraints on the scale of leptogenesis, linking gravitational wave phenomenology to neutrino mass generation and baryon asymmetry.

This thesis opens several promising directions for future research. An intriguing observation is that the present day abundances of visible and dark matter are of the same order, $\Omega_{\text{DM}} \approx 5 \Omega_{\text{B}}$. This coincidence motivates the idea of *cogenesis*, the simultaneous generation of visible and dark matter, which remains an active and compelling topic, particularly in the context of first-order phase transitions that bridge the two sectors and generate correlated gravitational wave signatures. At

the electroweak scale or below, the *mass-gain mechanism* in the presence of finite-temperature effects can produce distinctive phenomenological features. In particular, it can enable *forbidden decays* of dark matter at finite temperature [349, 448], leaving potential imprints on cosmological observables such as the effective number of relativistic degrees of freedom, ΔN_{eff} , near BBN [449]. Further study is warranted to determine whether such dark matter could generate an observable ΔN_{eff} and correlated gravitational wave signature. Progress in this field will benefit greatly from improved modelling of GW production during FOPT. Both high resolution simulations and refined analytical approaches, capturing bubble collisions, sound wave propagation, and plasma turbulence can sharpen theoretical predictions of the GW spectrum for various symmetry breaking patterns. A confirmed detection of a primordial stochastic GW background from a cosmological phase transition would provide a strong indirect indication of new scalar degrees of freedom, offering valuable guidance for identifying their properties prior to direct discovery at colliders or intensity frontier experiments. Notably, baryogenesis scenarios invoking the mass-gain mechanism below the sphaleron decoupling scale can yield GW signals at lower frequencies, potentially accessible at GW experiments around or below mHz frequencies. Presence of new coloured particles in typical baryogenesis models also keep the collider detection prospects more promising. We also plan to investigate electroweak baryogenesis models assisted by a singlet scalar with an additional CP-violating source, accompanied by detectable GW signatures. Considering very high scale phase transition, on the other hand, takes the GW spectrum away from reach of the typical GW detectors discussed in this thesis. However, proposed resonant cavity type experiments can probe such high frequency GW arising from such high scale phase transitions. While terrestrial experiments can not probe such scales directly, consequences of such high scale symmetry breaking can still be probed at experiments in a way similar to axions or axion-like particles. Further, such phase transition dynamics in axion models can also find connections to the origin of baryon asymmetry via axiogenesis or similar setups. Another avenue is *primordial black hole* formation from false vacuum collapse, a mechanism that does not require particle degrees of freedom and can contribute to dark matter in multi-component scenarios alongside particle candidates. Such models open the possibility of multiple, complementary GW detection channels. In addition, other topological defects

from symmetry breaking, such as cosmic strings, carry substantial cosmological significance and need deeper exploration. Connection of these new physics scenarios to dark matter and baryogenesis together with GW, cosmology and astrophysics signatures can lead to interesting future research directions. Overall, the frameworks and mechanisms explored in this thesis not only address key gaps in our understanding of dark matter and baryogenesis but also lay the groundwork for a broader range of theoretical developments and observational probes in early Universe cosmology.



Renormalisation Group Evolution Equations

In this appendix, we present the renormalisation group equations (RGEs) relevant for obtaining the symmetry-broken scalar potential in the conformal model discussed in Chapter 3. These RGEs govern the running of the model parameters and are essential for studying the radiative symmetry breaking mechanism. The explicit expressions for the RGEs are taken from [450]. The RGEs are written as

$$\frac{d\lambda_s}{dt} = \frac{1}{16\pi^2} (20\lambda_s^2 + 2\lambda_6^2 + 2\lambda_7^2 + 8\lambda_s \text{Tr}[Y'^\dagger Y'] - \text{Tr}[Y'^\dagger Y' Y'^\dagger Y']),$$

$$\frac{d\lambda_2}{dt} = \frac{1}{16\pi^2} (12\lambda_2^2 + 2\lambda_7^2 + 3g_1^2/4 + 9g_2^2/4 + 3g_1^2 g_2^2/2),$$

$$\frac{d\lambda_7}{dt} = \frac{1}{16\pi^2} (4\lambda_7^2 + 6\lambda_2\lambda_7 + 8\lambda_s\lambda_7 + 4\lambda_7 \text{Tr}[Y'^\dagger Y']),$$

$$\frac{d\lambda_6}{dt} = \frac{1}{16\pi^2} (4\lambda_6^2 + 6\lambda_6 g_t^2 + 8\lambda_s\lambda_6 + 4\lambda_6 \text{Tr}[Y'^\dagger Y']),$$

$$\frac{dY'}{dt} = \frac{1}{16\pi^2} (4Y'^3 + 2Y' \text{Tr}[Y'^\dagger Y']),$$

$$\frac{dg_1}{dt} = \frac{1}{16\pi^2} (7g_1^3),$$

$$\frac{dg_2}{dt} = \frac{1}{16\pi^2} (-3g_2^3),$$

$$\frac{dy_t}{dt} = \frac{1}{16\pi^2}(9y_t^3/2 - y_t(17g_1^2/12 + 9g_2^2/4)).$$



Action calculation

In this appendix, we present the action calculation approach that we extensively use in Chapters 3 and 4 to evaluate the gravitational wave parameters associated with first-order phase transitions (FOPTs). In general, the tunnelling rate per unit time per unit volume between two vacua separated by a potential barrier can be expressed as

$$\Gamma_V = \mathcal{A}e^{-B}, \quad (\text{B.1})$$

where, the coefficients $\mathcal{A}$ and B depend on the potential. In Euclidean space or imaginary time formalism, the coefficient B is S_3/T with S_3 being the Euclidean action at finite temperature in three dimensions. As described in [122], the dimensional estimation of pre-exponential factor $\mathcal{A}$ is $T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2}$ considering three zero modes of solution for thermal tunnelling. So, the probability of thermal tunnelling per unit time per unit volume can be written as

$$\Gamma(T) = \mathcal{A}(T)e^{-S_3(T)/T}, \quad (\text{B.2})$$

where $\mathcal{A}(T) \sim T^4 \left(\frac{S_3(T)}{2\pi T} \right)^{3/2}$. In order to calculate the action S_3 , we need to solve the $O(3)$ symmetric equation of motion i.e. bounce equation

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = \frac{\partial V(\phi, T)}{\partial \phi}, \quad (\text{B.3})$$

satisfying the boundary conditions

$$\phi(r \rightarrow \infty) = \phi_{\text{false}} \quad \text{and} \quad \left. \frac{d\phi}{dr} \right|_{r=0} = 0. \quad (\text{B.4})$$

The action corresponding to the solution of the above equation is

$$S_3(T) = 4\pi \int_0^\infty r^2 dr \left[\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V(\phi(r), T) \right]. \quad (\text{B.5})$$

Here, we used a fitting method to calculate the action. We fitted the effective potential to a generic potential for which the action calculations are done in semi-analytical way [309]. The generic quartic and logarithmic potential is

$$V(\phi) = (2A - B)\mu^2\phi^2 - A\phi^4 + B\phi^4 \ln \left(\frac{\phi^2}{\mu^2} \right). \quad (\text{B.6})$$

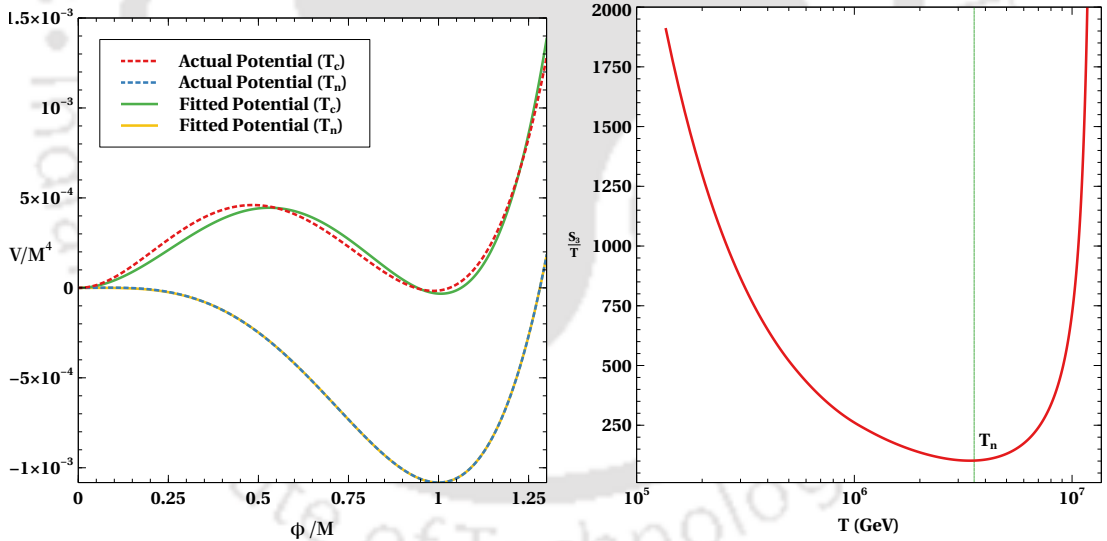


Figure B.1: Left Panel: Comparison between actual potential and fitted generic potential at critical temperature T_c and nucleation temperature T_n for BP1 of table 3.7. Right Panel: S_3/T as a function of temperature for BP1 of table 3.7.

We show that the actual potential and the fitted generic potential overlap quite well, depicted in left panel of Fig. B.1 for BP1 of table 3.7. The bounce equation can be simplified with dependence on just one parameter using scaling transformation (

$\Phi = \phi/\mu$ and $a = 2\mu\sqrt{Br}$ as

$$\frac{d^2\Phi}{da^2} + \frac{2}{a} \frac{d\Phi}{da} = \delta(\Phi - \Phi^3) + \Phi^3 \ln \Phi^2, \quad (\text{B.7})$$

where $\delta \equiv \frac{2A-B}{2B}$. As discussed in [309], the desired Euclidean action is calculated in semi-analytical fashion and can be written as

$$S_3 = \frac{16\pi\sigma I^3}{3(1-2\delta)^2} \left(\frac{2}{B}\right)^{1/2} (2\delta)^{n_\mu} \left\{1 + \mu_1\delta + \mu_2\delta^2 + \mu_3\delta^3\right\}, \quad (\text{B.8})$$

where, $I=0.4199$, $n_\mu = 0.557$, $\mu_1 = 4.2719$, $\mu_2 = -14.5908$ and $\mu_3 = 12.0940$. In our analysis, we fitted the effective potential using the above generic expression of potential for a wide range of temperatures such that we have a profile of action over temperature shown in right panel of Fig. B.1 for BP1 of table 3.7. That plays an important in tunnelling rate and $\beta/\mathcal{H}$ calculations.

Particle trapping in false vacuum

In this appendix, we estimate the number of fermions trapped in the false vacuum, which is relevant to the discussions in Chapter 4, and in particular, Section 4.1 of that chapter. Initially, all particles are in the false vacuum state. As the first order phase transition proceeds, particles gain mass in the true vacuum. The bubble of true vacuum expands and tries to capture the particles in the plasma. Due to the dynamical change of mass in the true vacuum, only sufficiently energetic particles are able to enter the bubble while conserving energy. Here, we show a simple quantitative analysis of particles getting trapped or reflected from bubble wall to false vacuum following the works of [188, 353]. We can estimate the particle flux going to true vacuum in the bubble wall rest frame. The number of particles passing through the bubble wall along -z direction (considered for simplicity) per area ΔS in Δt time can be written as

$$\frac{\Delta N_{in}}{\Delta S} = g_i \int \frac{d^3 p}{(2\pi)^3} \int_{r_0}^{r_0 - \frac{p_z \Delta t}{p}} dr \mathcal{T}(p) \Theta(-p_z) f(p, x), \quad (\text{C.1})$$

where, r_0 is bubble radius, $\Theta(-p_z)$ is ensuring particles going inside and $\mathcal{T}(p) = \Theta(-p_z - m_i)$ satisfied energy conservation inside bubble. Also, g_i is degrees of freedom of particle with mass m_i in true vacuum. Here, $f(p, x)$ is the distribution function of particles in false vacuum and it will be Bose-Einstein (BE) or Fermi-Dirac (FD) distribution. The distribution can be approximated to be Maxwell-Boltzmann

distribution as $f(p, x) = e^{-\tilde{\gamma}(p+\tilde{v}p_z)/T}$, where $\tilde{v}$ is the velocity of fluid or plasma with respect to bubble wall and $\tilde{\gamma}$ is corresponding Lorentz factor. The flux of particles in bubble wall frame can be calculated from above Eq. (C.1) as

$$J_w = g_i T^3 \left(\frac{\tilde{\gamma}(1-\tilde{v})m_i/T + 1}{4\pi^2 \tilde{\gamma}^3 (1-\tilde{v})^2} \right) e^{-\frac{\tilde{\gamma}(1-\tilde{v})m_i}{T}}. \quad (\text{C.2})$$

Since, the bubble is expanding with a wall velocity v_w , the average number density inside true vacuum in global plasma frame can be expressed as

$$n_{\text{in}} = \frac{J_w}{\gamma_w v_w}. \quad (\text{C.3})$$

So, the fraction of particle which gets trapped in false vacuum can be written as

$$F^{\text{trap}} = 1 - \frac{n_{\text{in}}}{n_{\text{eq}}}, \quad (\text{C.4})$$

where, n_{eq} is equilibrium number density of particles. In general, the velocity of fluid or plasma $\tilde{v}$ depends on fluid dynamics. From Eq. (C.4), it can be seen that F^{trap} is maximum (~ 1) for fluid velocity $\tilde{v} \rightarrow 0$.

Hydrodynamical treatment of velocities

In this appendix, we study the fluid velocity profile across the bubble wall using a hydrodynamical treatment, which is relevant for understanding the trapping kinematics across the wall. This analysis is discussed in detail in Section 4.2 of Chapter 4. As the first-order phase transition progresses, the expanding bubble releases latent heat into the radiation bath of the Universe, generating velocity and temperature profiles of the fluid across the bubble wall. In this section, we discuss the dynamics of wall velocity. The energy-momentum tensor of the scalar field that drives the phase transition is

$$T_{\alpha\beta}^{\phi} = \partial_{\alpha}\phi\partial_{\beta}\phi - g_{\alpha\beta} \left[\frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - V_{\text{eff}}(\phi) \right]. \quad (\text{D.1})$$

On the other hand, the energy-momentum tensor of the plasma is given by

$$T_{\alpha\beta}^{\text{pl}} = \sum_i \int \frac{d^3k}{(2\pi)^3 E_i} k_{\alpha} k_{\beta} f_i(k, x), \quad (\text{D.2})$$

where, the i index corresponds to each species in the plasma. Assuming the plasma to be locally in thermal equilibrium, its energy-momentum tensor can be expressed as that of a perfect fluid,

$$T_{\alpha\beta}^{\text{pl}} = w_{\text{pl}} u_{\alpha} u_{\beta} - g_{\alpha\beta} p_{\text{pl}}, \quad (\text{D.3})$$

where w_{pl} , p_{pl} and $u_\alpha \equiv \gamma(1, \mathbf{v})$ are plasma enthalpy, pressure and four-velocity respectively. With the ϕ background contributing to total pressure, the total fluid energy-momentum tensor can be written as

$$T_{\alpha\beta}^{\text{fl}} = T_{\alpha\beta}^{\phi} + T_{\alpha\beta}^{\text{pl}} = w u_\alpha u_\beta - g_{\alpha\beta} p, \quad (\text{D.4})$$

where, the enthalpy $w = T \frac{\partial p}{\partial T}$ and the energy density $e = T \frac{\partial p}{\partial T} - p = w - p$. The conservation of energy-momentum leads to $\partial^\alpha T_{\alpha\beta}^{\text{fl}} = 0$. For a constant bubble wall velocity, the time-independent components of the energy-momentum conservation equations in bubble wall frame can be expressed as [152, 357]

$$\partial_z T^{zz} = \partial_z T^{z0} = 0, \quad (\text{D.5})$$

where z -axis is chosen to be direction of bubble and fluid velocities. From the above equation, we get the matching equations across the bubble wall as follows

$$w_f v_f^2 \gamma_f + p_f = w_t v_t^2 \gamma_t + p_t \quad \text{and} \quad w_f v_f \gamma_f^2 = w_t v_t \gamma_t^2, \quad (\text{D.6})$$

here, subscripts t and f represent inside the bubble (true vacuum) and outside the bubble (false vacuum), respectively. In the relativistic gas approximation of plasma, the equation of state outside the bubble can be written as

$$p_f = \frac{1}{3} a_f T_f^4 - \epsilon, \quad e_f = a_f T_f^4 + \epsilon, \quad (\text{D.7})$$

where, ϵ is the false vacuum energy and a correspond to degrees of freedom across the wall. While inside the bubble

$$p_t = \frac{1}{3} a_t T_t^4, \quad e_t = a_t T_t^4. \quad (\text{D.8})$$

After substituting Eq. (D.7) and Eq. (D.8) in Eq. (D.6), we get

$$v_f v_t = \frac{1 - (1 - \alpha_*)\rho}{3 - 3(1 + \alpha_*)\rho}, \quad \frac{v_f}{v_t} = \frac{3 + (1 - 3\alpha_*)\rho}{1 + 3(1 + \alpha_*)\rho}, \quad (\text{D.9})$$

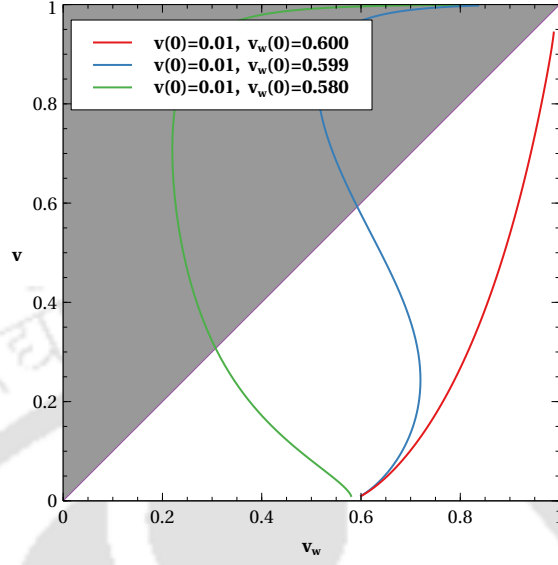


Figure D.1: Generic fluid velocity profile with corresponding wall velocity $v_w = \xi$.

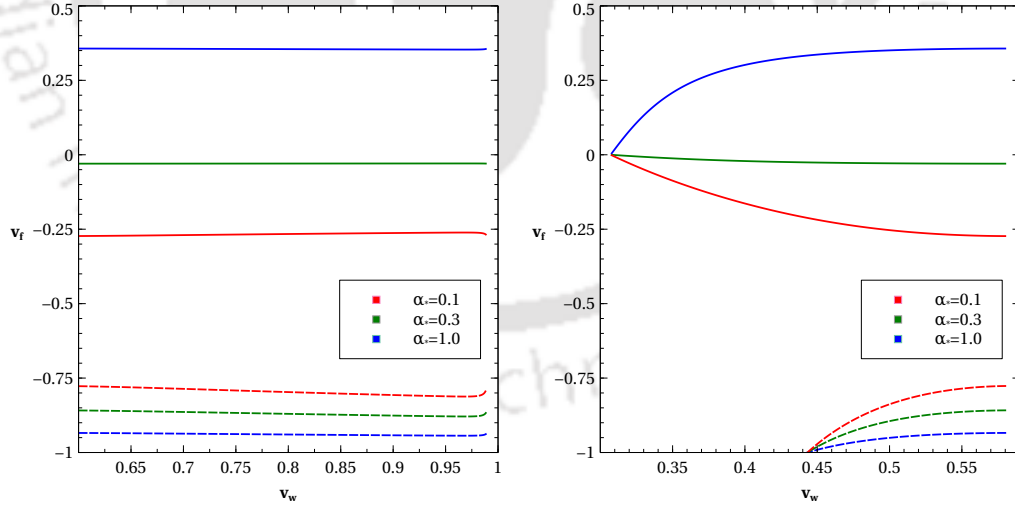


Figure D.2: Left panel: local fluid velocity in front of the wall, v_f versus bubble wall velocity for different values of α_* corresponding to the red coloured profile of Fig. D.1. Right panel: same as the left panel but corresponding to the green coloured profile of Fig. D.1. The solid and dashed lines correspond to deflagration and detonation regimes, respectively.

where, $\alpha_* = \epsilon/(a_f T_f^4)$, $\rho = a_f T_f^4/(a_t T_t^4)$. Then, combining the above two equations, we get

$$v_f = \frac{1}{1 + \alpha_*} \left[\left(\frac{v_t}{2} + \frac{1}{6v_t} \right) \pm \sqrt{\left(\frac{v_t}{2} + \frac{1}{6v_t} \right)^2 + \alpha_*^2 + \frac{2}{3}\alpha_* - \frac{1}{3}} \right]. \quad (\text{D.10})$$

There are two branches of solutions corresponding to the $\pm$ signs on the right hand side of Eq. (D.10). The positive branch solution belongs to detonation, $v_f > v_t$ and the negative branch solution gives deflagration, $v_f < v_t$. Connecting with the discussion in section 4.2.2, v_f represents the velocity of the fluid in front of the wall i.e. $\tilde{v}$, while v_t denotes the velocity of the fluid behind the wall, both measured in the wall rest frame. We now calculate the bubble wall velocity and the corresponding velocity of the fluid moving behind the wall from plasma frame of reference. The conservation equation can be written as

$$\partial_\alpha T^{\alpha\beta} = u^\beta \partial_\alpha (u^\alpha w) + u^\alpha w \partial_\alpha u^\beta - \partial^\beta p. \quad (\text{D.11})$$

Taking the projection along the flow, leads to

$$\partial(u^\alpha w) - u_\alpha \partial^\alpha p = 0. \quad (\text{D.12})$$

Now, taking the projection perpendicular to the flow with $\bar{u}_\alpha \equiv \gamma(v, v/v)$, we get

$$\bar{u}^\alpha u^\beta w \partial_\alpha u_\beta - \bar{u}^\beta \partial_\beta p = 0. \quad (\text{D.13})$$

In the absence of a characteristic distance scale, the solution is expected to depend on the self-similar parameter $\xi = r/t$, where r is the distance from the centre of the bubble, and t is the time elapsed since nucleation. The gradients can be defined as

$$u_\alpha \partial^\alpha = -\frac{\gamma}{t}(\xi - v) \partial_\xi, \quad \bar{u}_\alpha \partial^\alpha = \frac{\gamma}{t}(1 - \xi v) \partial_\xi. \quad (\text{D.14})$$

From Eq. (D.12) and Eq. (D.13), we get

$$(\xi - v) \frac{\partial_\xi e}{w} = 2 \frac{v}{\xi} + \{1 - \gamma^2 v(\xi - v)\} \partial_\xi v, \quad (1 - v\xi) \frac{\partial_\xi p}{w} = \gamma^2 (\xi - v) \partial_\xi v. \quad (\text{D.15})$$

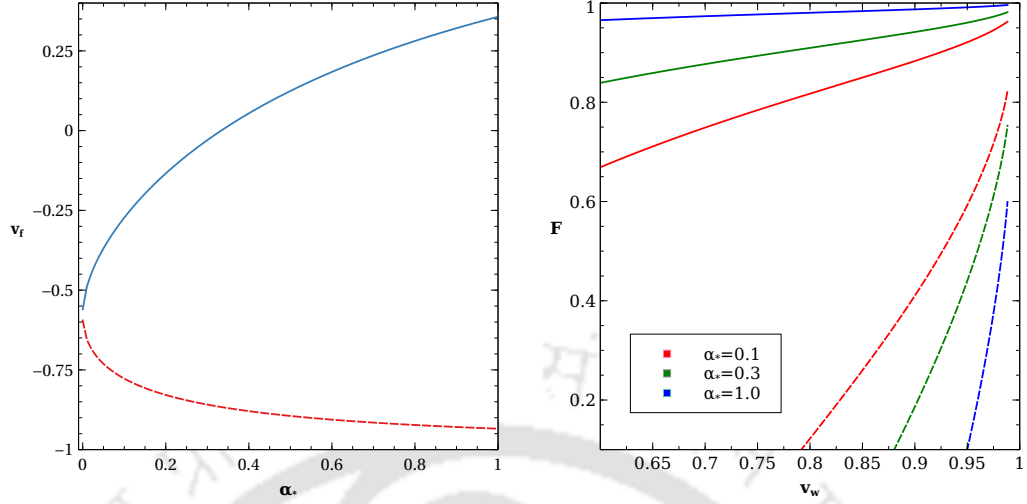


Figure D.3: Left panel: Local fluid velocity in front of wall, v_f versus α_* , Right panel: Fraction of trapped particles F versus bubble wall velocity, v_w corresponding to the left panel of Fig. D.2. The solid and dashed lines correspond to deflagration and detonation regimes, respectively.

Using the definition of the sound speed of the plasma $c_s^2 \equiv (dp/dT)/(de/dT)$, the above two equations can be expressed as

$$2\frac{v}{\xi} = \gamma^2(1 - v\xi)\left(\frac{\mu^2}{c_s^2} - 1\right)\partial_\xi v, \quad (\text{D.16})$$

where, $\mu = (\xi - v)/(1 - \xi v)$. The above equation can be solved by introducing an auxiliary parameter τ , which helps to write the equation as

$$\frac{dv}{d\tau} = 2vc_s^2(1 - v^2)(1 - \xi v), \quad \frac{d\xi}{d\tau} = \xi((\xi - v)^2 - c_s^2(1 - \xi v)^2), \quad (\text{D.17})$$

and the solutions for various initial conditions are illustrated in Fig. D.1. Here, we show three types of profiles corresponding to the green, blue and red coloured contours respectively. The grey shaded upper triangular region in the does not provide physically consistent solutions.

The fluid velocity behind the wall is v in the plasma frame, whereas it is v_t in the wall's rest frame. We can write $v = (v_t + v_w)/(1 + v_t v_w)$ using Lorentz transformation of velocities. Thus, v_w can be expressed as a function $f(v_f)$ in terms of the local plasma velocity in front of the wall, measured in the wall's rest frame. In Fig. D.2, we connect the local fluid velocity with wall velocity, with solid and dashed lines

corresponding to deflagration and detonation regimes, respectively. From the left panel of Fig. D.2, we can see that v_f is nearly independent of the wall velocity v_w , while it depends on α_* shown in the left panel of Fig. D.3. In the right panel of Fig. D.3, we show the fraction of particles trapped outside the bubble as a function of wall velocity, where $m_\chi^{\text{out}} \sim T_n$, $m_\chi^{\text{in}} \sim 2T_n$ corresponding to the left panel of Fig. D.2. We use these dependence of fluid and bubble velocities while calculating the trapping function F relevant forogenesis discussed in section 4.2.3.



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