INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

Name of the Student	:	SWARUP KUMAR PANDA
Roll Number	:	11612306
Programme of Study	:	Ph.D.

Thesis Title: On the Inverse of Bipartite Graphs with Unique Perfect Matchings and Reciprocal Eigenvalue Properties.

Name of Thesis Supervisor(s)	:	Prof. Sukanta Pati
Thesis Submitted to the Department/ Center	:	Mathematics
Date of completion of Thesis Viva-Voce Exam	:	24.6.2016
Key words for description of Thesis Work	:	Graph, Adjacency matrix, Inverse graph, Property (R) , Property (SR)

SHORT ABSTRACT

We consider only simple graphs. A graph G is said to be nonsingular (resp. singular) if its adjacency matrix A(G)is nonsingular (resp. singular). The inverse of a nonsingular graph G is the unique weighted graph whose adjacency matrix is similar to the inverse of the adjacency matrix A(G) via a diagonal matrix with each diagonal entry is either 1 or -1. The study of inverse graphs has been a subject of interest in the past few years (see [15, 16, 17]). In [15] , Godsil posed the question of characterizing the bipartite graphs with unique perfect matchings possess inverses. In this thesis, we introduce the `even'ness of a nonmatching edge and show that under certain conditions a connected bipartite graph with a unique perfect matching has an inverse. Our results extend some known results, providing us with a larger class of graphs possessing inverses.

A graph G is said to be self-inverse if G is isomorphic to its inverse. We consider the problem of characterizing the self-inverse bipartite graphs with unique perfect matchings. This question, for the class of bipartite graphs G with unique perfect matchings such that the graph obtained by contracting all matching edges is also bipartite was asked by Godsil in 1985 and has already been answered by Simion and Cao in [26]. In this thesis, we enlarge the subclass of self-inverse bipartite graphs with unique perfect matchings containing the class of bipartite graphs with unique perfect matchings such that the graph obtained by contracting all matching edges is also bipartite.

A nonsingular graph G has the <u>reciprocal eigenvalue property</u> (property (R)), that is, the reciprocal of each eigenvalue of the adjacency matrix A(G) is also an eigenvalue of A(G). A nonsingular graph G has the <u>strong reciprocal eigenvalue property</u> (property (SR)), that is, the reciprocal of each eigenvalue of the adjacency matrix A(G) is also an eigenvalue of A(G) and they both have the same multiplicities. On the class of such nonsingular trees T the equivalence of the following statements are known. 1) The reciprocal of the spectral radius of the adjacency matrix A(T) is the least positive eigenvalue of the adjacency matrix. 2) The graph T is self- inverse. 3) The graph T has the property (R). 4) The graph T has the property (SR). 5) The graph is a corona graph, that is, it is obtained by taking a bipartite graph and

then by inserting a new adjacent vertex of degree one at each vertex. In this thesis, we supply a class of graphs of bipartite graphs with a unique perfect matchings (larger than the nonsingular trees) where the statements 1)--4) are equivalent. We supply examples to show that no two of 1)--4) are equivalent, in general.