



INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

The Dominating Set Problem (DSP) is a classical combinatorial optimization problem that aims to find the smallest dominating set (DS) in a given graph $G=(V,E)$. Due to its numerous real-world applications across various domains, such as facility location, social networks, network design, security, and resource allocation, the dominating set and its variants have been extensively studied. In this thesis, we explore several variants of the dominating set, specifically the semi-total dominating set, total dominating set, and total Roman dominating set. All the problems listed here are NP-hard, and none of them admit a constant factor approximation algorithm for general graphs unless $P=NP$. This challenge motivated us to study these problems within special graph classes.

Unit disk graphs (UDGs) are geometric intersection graphs, where nodes correspond to disks, and there is an edge between two nodes if their corresponding disks intersect. Exploring domination and its variants within unit disk graphs (UDGs) is particularly significant due to their pivotal role as models for wireless communication networks, mobile Ad-Hoc networks (MANETs), and sensor networks.

Given the relation of UDGs with domination theory, we investigate most of these problems in UDGs and a few of them in grid graphs (GGs), and trees.

Firstly, we study the semi-total dominating set (T2DS) and we are the first to introduce the semi-total dominating set problem in unit disk graphs. We establish that the T2DS problem is NP-complete in UDGs and propose a 6-factor approximation algorithm. The running time of the algorithm is $O(n \log(k))$, where k is the size of the maximal independent set. We also propose a $2 + \ln(\Delta + 1)$ -factor approximation algorithm for the semi-total domination problem in general graphs, where Δ is the degree of the graph. Our proposed approximation algorithm is an improvement

over the approximation factor $2+3\ln(\Delta+1)$ given in the literature. We have also observed that the problem admits a PTAS with approximation factor $(2+\epsilon)$.

Secondly, we study the total dominating set (TDS) problem and propose a 7.79-factor approximation algorithm and a 6.29-factor approximation algorithm for the TDS problem in UDGs, with running time $O(n^2)$ and $O(n^2m)$, respectively, where n is the number of vertices and m is the number of edges of the graph. We have also shown that the TDS problem is NP-complete when restricted to grid graphs, a subclass of UDGs.

Next, we study the total Roman dominating set (TRDS), a well-known variant of Roman domination. We establish that the TRDS problem is NP-complete in UDGs and propose a 10.5-factor approximation algorithm and a 6.15-factor approximation algorithm for the same with time complexities $O(n\log(k))$ and $O(n^2m)$, respectively, where n is the number of vertices, m is the number of edges and k is the size of the maximal independent set of the graph.

Finally, we delved into the algorithmic aspect of the TRDS problem in trees and introduced a dynamic programming-based algorithm to yield an optimal solution. Our algorithm operates in linear time, offering an efficient approach to tackle the problem in tree structures.

