INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

The primary objective of the thesis titled "A Study of Torsion of Elliptic Curves and Fundamental Units over Number Fields" is to study the torsion subgroup of elliptic curves over certain quadratic number fields and to establish certain congruence relations for the fundamental unit of totally imaginary biquadratic number fields of odd class number.

The first part of the work is devoted to finding possible torsion structures over given quadratic fields. In 1922, Louis Mordell proved that the group of rational points of an elliptic curve over the rational field is finitely generated and in 1928, Andre´ Weil generalized this result for abelian varieties over algebraic number fields. In 1977, Barry Mazur opened up a vast area of research by listing all possible torsion subgroups of all possible elliptic curves over the rational field. He showed that only 15 possible groups could occur as the torsion subgroup of elliptic curves over Q. After extensive collaborations among various mathematicians, Kenku, Momose and Kamienny to name a few, a complete list of 26 groups was finally published for the torsion subgroup of elliptic curves over all possible quadratic number fields.

In 2011, Najman computed the torsion subgroups of all elliptic curves over the imaginary quadratic fields $Q(\sqrt{-1})$ and $Q(\sqrt{-3})$ separately. In 2012, Kamienny and Najman outlined a method to study the possible torsion structure over a given quadratic field. We follow their method in determining the possible torsion structures over the remaining imaginary quadratic fields of class number 1. We also compute the possible torsion structures over the real quadratic fields $Q(\sqrt{2})$ and $Q(\sqrt{5})$, which have the smallest discriminants among all real quadratic fields $Q(\sqrt{d})$ with $d \equiv 2,3 \pmod 4$ and $d \equiv 1 \pmod 4$ respectively.

In the latter half of our work, we prove certain congruence relations for the fundamental unit of totally imaginary biquadratic fields of odd class number. In 2014, Zhang and Yue obtained certain congruence relations for the fundamental unit of real quadratic fields of odd class number by using 2-adic analysis. In 2016, Chakraborty and Saikia obtained the same congruences by elementary methods. In this work, we refine these congruences by using ramification of primes in quadratic fields. We then use these congruences to establish certain congruence relations for the fundamental unit of totally imaginary biquadratic fields of odd class number.