INDIAN INSTITUTE OF TECHNOLOGY GUWAHATI
SHORT ABSTRACT OF THESIS

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SHORT ABSTRACT

This thesis studies arithmetic properties of certain partition functions, namely mex-related partition functions, Andrews' singular overpartitions, t-regular partitions and 3-regular color partitions. Firstly, we study the mex-related partition function which has been introduced by Andrews and Newman very recently. The minimal excludant, or "mex" function, on a set S of positive integers is the least positive integer not in S. Andrews and Newman extended the mex-function to integer partitions and found numerous surprising partition identities connected with these functions. We study two of the families of functions they introduced, and prove that thay too satisfy the Ramanujan's famous congruences for the integer partitions. We then use a result of Ono and Taguchi on nilpotency of Hecke operators to find infinite families of congruences modulo 2 satisfied by the mex-related partition functions.

Secondly, we study certain arithmetic properties of Andrews' singular overpartitions. We use arithmetic properties of modular forms and eta-quotients to study divisibility and distributions modulo arbitrary powers of 2 and 3 for infinite families of Andrews' singular overpartition functions.

Thirdly, we study arithmetic properties of certain t-regular partitions. In a recent paper, Keith and Zanello established infinite families of congruences and self-similarity results for t-regular partitions modulo 2 for certain values of t. Further, they proposed some conjectures on self-similarities of t-regular partitions modulo 2 for certain values of t. We prove their conjectures when t = 3, 25. We also establish infinite families of congruences modulo 2 for t-regular partitions when t = 3, 21 using an approach developed by Radu and the theory of Hecke operators.

Finally, we study a partition function which counts the number of 3-regular partitions in three colours. In a very recent paper, da Silva and Sellers studied certain arithmetic properties of this partition function, and they conjectured four Ramanujan-like congruences modulo 5. We confirm the conjectural congruences of da Silva and Sellers using the theory of modular forms.