

Cosmological Signatures of Some Beyond Standard Model Physics Scenarios

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A thesis submitted for the degree of

Doctor of Philosophy

Supervisor

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A thesis submitted by

Nayan Das

to

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Supervisor

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“Nothing in life is to be feared; it is only to be understood.”

-Marie Curie.



This thesis is dedicated

to

my Parents and my Sister



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I hereby declare that works presented in the thesis entitled **“Cosmological Signatures of Some Beyond Standard Model Physics Scenarios”** have been carried out by me under the supervision of **Dr. Debasish Borah** at the Department of Physics, Indian Institute of Technology Guwahati, India. The thesis has not been submitted anywhere else for any degree. Works presented in the thesis are all my own unless referenced to the contrary in the thesis.

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Signature



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Abstract

This thesis investigates a class of well-motivated beyond the Standard Model scenarios whose signatures are difficult to probe through conventional laboratory experiments but can be accessed through cosmological and astrophysical observations. In particular, the thesis focuses on the imprints of dark sector physics on the cosmic microwave background, the role of primordial black holes in generating dark radiation, which also comes with complementary detection prospects like gravitational waves, and high-energy astrophysical messengers. These observables provide powerful and complementary probes of new physics operating in the early Universe and at energy scales far beyond the reach of terrestrial experiments. A central theme of this thesis is the study of light relic species in the early Universe, such as right-handed Dirac neutrinos and Goldstone bosons, and their connection to well-motivated extensions of the Standard Model. In the context of light Dirac-neutrino portal dark matter, both thermal and non-thermal production mechanisms are systematically investigated, together with the associated dark radiation component. Particular emphasis is placed on their impact on the effective number of relativistic species and on structure formation, which provide robust and complementary observational constraints. The analysis is carried out first within a minimal, phenomenological framework and subsequently extended to an ultraviolet-complete model. In addition, the thesis examines the role of ultra-light primordial black holes in the early Universe and their possible implications for thermalised dark radiation, as well as for the co-genesis of axion dark matter and baryon asymmetry via leptogenesis. The associated production of stochastic gravitational waves due to density fluctuations and hot dark radiation from ultra-light primordial black holes provides promising avenues for probing such scenarios through current and future observations. Finally, the thesis investigates possible interpretations of two recently observed ultra-high-energy events in terms of the decay of superheavy dark matter. Adopting a multi-messenger approach that combines constraints from ultra-high-energy cosmic rays, neutrinos, and gamma rays, updated bounds on superheavy dark matter parameter space are provided. The non-observation of the corresponding gamma-ray counterparts places the dark matter interpretation of these events under significant tension. Overall, this thesis demonstrates that several beyond the Standard Model scenarios can be robustly constrained using cosmological and astrophysical probes, highlighting the crucial role of next-generation cosmic microwave background observations,

gravitational-wave detectors, and high-energy observatories in uncovering the nature of dark matter and the physics of the early Universe.



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List of Acronyms

ΛCDM	standard model of cosmology.
BAO	Baryonic Acoustic Oscillations.
BBN	Big Bang Nucleosynthesis.
BSM	beyond the Standard Model.
CDM	cold dark matter.
CMB	cosmic microwave background.
DM	dark matter.
DNPDM	Dirac neutrino portal dark matter.
DR	dark radiation.
FIMP	feebly interacting massive particle.
GW	gravitational wave.
LHC	Large Hadron Collider.
NFW	Navarro-Frenk-White.
PBHs	primordial black holes.
PQ	Peccei-Quinn.
RHN	right-handed neutrino.
RHS	right hand side.
SGBW	stochastic gravitational wave background.
SHDM	superheavy dark matter.
SM	Standard Model.
UHECR	ultra-high-energy cosmic ray.
WIMP	weakly interacting massive particle.



Introduction

The quest to understand nature drives scientific inquiry from the smallest constituents of matter to the largest structures in the Universe. Since the discovery of the electron in 1897, particle physics has made remarkable progress, culminating in the establishment of the Standard Model (SM) of particle physics [1–5]. The discovery of the last missing particle of the model, the Higgs boson, at the Large Hadron Collider (LHC) in 2012 [6, 7] marked a major milestone, confirming the mechanism responsible for the generation of masses of elementary particles.

In parallel, cosmological observations have led to the development of the standard model of cosmology (Λ CDM) model, which is based on General Relativity and supported by measurements of the cosmic microwave background (CMB)[8–11], large-scale structure [12–14], and type Ia supernovae [15, 16]. According to this framework, the present-day energy budget of the Universe consists of approximately 68% dark energy, 27% dark matter (DM), and 5% baryonic matter, while radiation, composed of photons and neutrinos, contributes less than 0.01%. Remarkably, the Λ CDM model, described by only six cosmological parameters, successfully explains the evolution of the Universe over the past 13.8 billion years. Together, the SM of particle physics and the Λ CDM cosmological model provide a coherent description of the fundamental constituents of matter and the large-scale evolution of the Universe.

Despite their extraordinary successes, both the SM and the Λ CDM frameworks face several important theoretical and observational challenges. Astrophysical and cosmological observations strongly indicate the existence of a non-luminous and weakly interacting DM component, yet no viable dark matter candidate exists within the Standard Model. Furthermore, the observation of tiny but nonzero neutrino masses [17] cannot be accommodated in the minimal SM. Additional unresolved issues include the strong CP problem, the origin of the matter–antimatter asymmetry, and the hierarchy problem. Addressing these puzzles generally requires extensions of the SM that introduce new particles and interactions. For example, models of leptogenesis [18, 19] that explain the baryon asymmetry often invoke heavy right-handed neutrino (RHN), while the axion is a well-motivated particle proposed to solve the strong CP problem [20–22]. On the cosmological side, the Λ CDM model also faces challenges, including the persistent Hubble tension [23–27], which refers to the 3σ – 4σ discrepancy between early- and late-time determinations of the present expansion rate of the Universe.

Although many beyond the Standard Model (BSM) scenarios have been proposed to address these problems, probing them in terrestrial experiments is often extremely challenging, particularly when the new particles are either very heavy or very weakly interacting. In this context, cosmology provides a powerful and complementary avenue for testing new physics. Particles present in the early Universe can leave characteristic imprints on cosmological observables, which can be probed through precision measurements. Rapid advances in observational technology, together with current and upcoming ground-based and satellite missions, are expected to achieve sub-percent level precision, enabling stringent constraints on even feebly interacting species. For instance, precise measurements of the CMB can probe the radiation energy density during recombination and thus constrain additional light relativistic species, while the detection of primordial gravitational waves would open a direct observational window to the earliest epochs of cosmic history. Such cosmological probes have the potential to test both SM and BSM physics, extending sensitivity to energy scales far beyond those accessible in laboratory experiments.

The primary goal of this thesis is to investigate several well-motivated BSM scenarios that are difficult to probe through collider or direct-detection experiments and to study their cosmological signatures, with particular emphasis on observables

associated with the CMB, gravitational waves, and high-energy astrophysical measurements. Our focus will mainly be on scenarios involving dark matter, Dirac and Majorana neutrinos, and ultra-light primordial black holes (PBHs). In the following sections, we briefly review the foundations of the Standard Model of particle physics and the standard cosmological framework, along with the theoretical motivations for dark matter and neutrino mass generation, which set the stage for the detailed studies presented in the subsequent chapters.

Throughout this thesis, we work in natural units in which the speed of light c , the reduced Planck constant $\hbar$, and the Boltzmann constant k_B are set to unity, $c = \hbar = k_B = 1$. Planck mass and the reduced Planck mass are defined as $M_{\text{Pl}}^2 = (G)^{-1}$ and $M_{\text{P}}^2 = (8\pi G)^{-1}$ respectively, where G is Newton's gravitational constant.

1.1 Review of Λ CDM Cosmology

Modern cosmology is founded on *Cosmological Principle*, which states that : *On sufficiently large scales (typically of order 300 Mpc or so), the Universe can be well approximated as homogeneous and isotropic.* Within the framework of General Relativity, the dynamics of such a universe are governed by Einstein's equations with a cosmological constant [28]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}, \quad (1.1)$$

where $R_{\mu\nu}$ is the Ricci tensor, $T_{\mu\nu}$ is the energy-momentum tensor, G is the Newton's constant and Λ represents the cosmological constant. R is the Ricci scalar defined as $R \equiv g^{\mu\nu} R_{\mu\nu}$. The solutions of Eq. (1.1), describe the evolution of the universe where the metric $g_{\mu\nu}$ is specified using the *Cosmological principle* as

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]. \quad (1.2)$$

This metric is known as Friedmann-Lemaître-Robertson-Walker (FLRW) metric. The parameter a is the *scale factor*, which characterizes the expansion of the Universe and is conventionally normalized such that $a_0 = 1$ today. The coordinates (t, r, θ, ϕ) are the comoving coordinates, and curvature parameter k determines the spatial geometry: $k = -1, 0, +1$ correspond to open (negative curvature), flat,

and closed (positive curvature) spatial slices, respectively. Solving Eq. (1.1) for FLRW metric and considering the energy-momentum tensor as that of a perfect fluid $T^\mu_\nu = \text{diag}(-\rho(t), P(t), P(t), P(t))$, one obtains the following equations known as Friedmann equations [29]

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho(t), \quad \left(\frac{\dot{a}}{a}\right)^2 + 2\frac{\ddot{a}}{a} + \frac{k}{a^2} = -8\pi G P(t). \quad (1.3)$$

Here, ρ and P are the energy density and pressure of various components in the reference frame of a comoving observer, and $\dot{}$ represents the time derivative. Using the Eq. (1.3), and parameterizing the relation between energy density and pressure by constant equation of state $\omega = \frac{P(t)}{\rho(t)}$, the continuity equation can be obtained

$$\dot{\rho} + 3\mathcal{H}(1 + \omega)\rho = 0. \quad (1.4)$$

We introduce the Hubble Parameter here as $\mathcal{H} \equiv \frac{\dot{a}}{a}$. The standard cosmological model, the Λ CDM model, describes the Universe as being composed primarily of three components: radiation ($\omega = 1/3$), matter ($\omega = 0$), which includes both baryonic matter and dark matter, and dark energy, modeled as a cosmological constant with $\omega = -1$. Solving Eq. (1.4) for these components yields

$$\rho^m = \rho_0^m \left(\frac{a_0}{a}\right)^3, \quad \rho^r = \rho_0^r \left(\frac{a_0}{a}\right)^4, \quad \rho^d = \rho_0^d, \quad (1.5)$$

where the subscript $_0$ denotes the present-day value of the corresponding quantity. Thus, the matter energy density scales as a^{-3} , the radiation energy density scales as a^{-4} , while the dark energy density remains constant in time. In the very early Universe, the energy budget was dominated by radiation. Since radiation redshifts faster than matter, the Universe eventually reached an epoch at which $\rho^m = \rho^r$, known as the *matter-radiation equality* epoch. At later times, because matter continues to dilute as a^{-3} while dark energy remains constant, the Universe entered the *matter-dark energy equality* epoch in the recent past. Today, the Universe is dominated by dark energy, leading to the observed phase of accelerated expansion. In fact, a wide range of cosmological observations indicate that the dark energy equation-of-state parameter is consistent with $\omega \simeq -1$, thereby motivating the inclusion of a cosmological constant Λ in the Λ CDM model [30]¹.

¹Recent results from the DESI collaboration indicate a preference for models with dynamical dark energy; see, e.g., [31–33].

It is often convenient to characterize the energy content of the Universe in terms of critical energy density, defined as $\rho_c(t) \equiv \frac{3\mathcal{H}(t)^2}{8\pi G}$. The relative contribution of each component i is then quantified by the density parameter $\Omega_i(t) = \frac{\rho_i(t)}{\rho_c(t)}$. The total density parameter can then be expressed as $\Omega_{\text{tot}}(t) = \sum_i \Omega_i(t)$. Using these definitions, the first Friedmann equation can be rewritten as

$$\Omega_{\text{tot}} - 1 = \frac{k}{a^2 \mathcal{H}^2}. \quad (1.6)$$

Therefore, $\Omega_{\text{tot}} = 1$ corresponds to a spatially flat Universe, while $\Omega_{\text{tot}} < 1$ and $\Omega_{\text{tot}} > 1$ describe open and closed geometries, respectively. Measurements of the CMB acoustic peaks strongly indicate that Ω_{tot} is very close to unity, implying that the Universe is consistent with being spatially flat.

1.1.1 Evolution and thermal history of the Universe

In the early Universe, particle species were kept in thermal equilibrium because interaction rates were much larger than the Hubble expansion rate. This thermal picture is strongly supported by the observed near-perfect blackbody spectrum of the CMB. The energy of a particle in the plasma is $E = \sqrt{p^2 + m^2} \approx T$, with T denoting the temperature of the thermal bath. At sufficiently high temperatures all species were relativistic ($p \gg m$), implying a radiation-dominated Universe. The number density n , energy density ρ , and pressure P of a species are obtained from phase-space integrals over the equilibrium distribution function $f(p, \mu, T)$. In equilibrium, the distribution is $f(p, \mu, T) = [e^{(E-\mu)/T} \pm 1]^{-1}$, with the plus sign for fermions (Fermi–Dirac statistics) and the minus sign for bosons (Bose–Einstein statistics). The chemical potential μ , quantifies the response of the system to changes in particle's numbers. In the early Universe, it is negligible compared to the temperature, $\mu \ll T$, and can be ignored for most cosmological applications.

Although the general expressions for n , ρ and P must be evaluated numerically, simple analytic results are obtained in the relativistic and non-relativistic limits. In the relativistic limit ($T \gg m$),

$$n = \begin{cases} \frac{\zeta(3)}{\pi^2} g T^3 & \text{for boson,} \\ \frac{3}{4} \frac{\zeta(3)}{\pi^2} g T^3 & \text{for fermion,} \end{cases} \quad \rho = \begin{cases} \frac{\pi^2}{30} g T^4 & \text{for boson,} \\ \frac{7}{8} \frac{\pi^2}{30} g T^4 & \text{for fermion,} \end{cases} \quad P = \frac{\rho}{3}. \quad (1.7)$$

$\zeta(3)$ here denotes the Riemann zeta function of order 3. In the non-relativistic regime ($T \ll m$), the number density is exponentially suppressed, $n \propto g(mT/2\pi)^{3/2}e^{-m/T}$, while the energy density is dominated by rest mass, $\rho \simeq mn$, and the pressure becomes negligible compared to ρ . This exponential “Boltzmann suppression” reflects the fact that once a species becomes non-relativistic, particle–antiparticle annihilation depletes its abundance and pair production from the thermal bath is no longer efficient. This behavior has important implications in early-Universe cosmology, particularly for the thermal history and production mechanisms of dark matter.

For a plasma containing multiple relativistic species, the total energy density can be written in the compact form

$$\rho = \frac{\pi^2}{30} g_*(T) T^4, \quad (1.8)$$

where $g_*(T)$ is the “effective number of relativistic degrees of freedom” at temperature T , and is obtained by summing over the contributions from all bosons and fermions

$$g_*(T) = \sum_{i=F} \frac{7}{8} g_i \left(\frac{T_i}{T} \right)^4 + \sum_{j=B} g_j \left(\frac{T_j}{T} \right)^4. \quad (1.9)$$

At a temperature above the electroweak scale, $T \gg 100$ GeV, all species of the SM are relativistic and share a common temperature T , giving an effective number of relativistic degrees of freedom $g_* = 106.75$. As the Universe cools, heavier particles become non-relativistic and gradually disappear from the thermal bath or equivalently, stop contributing to g_* . A commonly used analytical estimate for the decoupling temperature is $T \sim m/6$. Around $T \sim 100$ GeV, the electroweak symmetry of the Standard Model is spontaneously broken, after which particles acquire masses through the Higgs mechanism, although many of them remain relativistic at that epoch. A more pronounced change in the effective number of relativistic degrees of freedom occurs during the Quantum Chromodynamics (QCD) phase transition (or crossover) at $T \sim 150$ MeV, when quarks and gluons become confined into hadronic bound states, resulting in a rapid decrease in g_* .

Another closely related quantity is the entropy density $s = (\rho + P)/T$, which can be expressed as (for all species)

$$s = \frac{2\pi^2}{45} g_{*s}(T) T^3, \quad \text{with} \quad g_{*s}(T) = \sum_{i=F} \frac{7}{8} g_i \left(\frac{T_i}{T} \right)^3 + \sum_{j=B} g_j \left(\frac{T_j}{T} \right)^3. \quad (1.10)$$

The quantity $g_{*s}(T)$ is the “effective number of degrees of freedom in entropy” at temperature T . In thermal equilibrium, the entropy per comoving volume is conserved, $sa^3 = \text{const}$, implying

$$g_{*s}(T) T^3 a^3 = \text{const}. \quad (1.11)$$

Therefore, if g_{*s} remains constant, the temperature scales as $T \propto a^{-1}$. However, when a species becomes non-relativistic and annihilates, its entropy is transferred to the remaining relativistic particles, changing g_{*s} and causing the temperature to decrease slightly more slowly than a^{-1} across such transitions.

A standard example occurs around the MeV scale. At $T \sim 10$ MeV, the thermal bath consists of photons, electrons/positrons, and neutrinos, giving $g_* = g_{*s} = 10.75$. Neutrinos were decoupled from the thermal bath (due to weak interaction rate) at around $T \approx 0.8$ MeV and subsequently free-stream. But even after the decoupling, the temperatures of neutrino and photon remain the same till the electron positron annihilation, as both species are relativistic. The electron-positron pair annihilates after neutrino decoupling, resulting in entropy transfer to the thermal bath. As neutrinos do not receive any entropy, the temperatures of neutrinos and photons after electron-positron annihilation no longer remain the same. In the instantaneous decoupling limit of neutrinos, the ratio of neutrino temperature and photon temperature becomes (using Eq. (1.11) for entropy conservation)

$$\frac{T_\nu}{T_\gamma} = \left(\frac{4}{11} \right)^{1/3}. \quad (1.12)$$

The above relation remains valid from $e^\pm$ annihilation until today. Using the observed CMB temperature $T_0 = 2.7255$ K, this gives a present-day neutrino temperature $T_{0,\nu} \simeq 1.95$ K. Consequently, the cosmic neutrino background has a large number density, $n_{0,\nu} \simeq 112 \text{ cm}^{-3}$ per neutrino species, although its direct detection remains extremely challenging due to the very small neutrino energies at present.

The formation of light nuclei in the early Universe is described by Big Bang Nucleosynthesis (BBN) which also takes place at around MeV scale (time, $t \sim 1-10^3$ s). The important quantity for the formation of light elements is the neutron-to-proton ratio, n_n/n_p . When protons and neutrons are in equilibrium due to weak interactions, we have $n_n/n_p \simeq e^{(m_n-m_p)/T}$, where $m_n - m_p \simeq 1.293$ MeV. At a temperature below $T \sim 0.7$ MeV, the weak interaction rates become slower than the Hubble expansion, and hence the ratio n_n/n_p freezes out. After freeze-out, free neutrons decay until nuclear reactions start efficiently. Once the deuterium bottleneck was overcome, nuclear reactions proceeded quickly and produced helium-4, ${}^4\text{He}$ as the dominant product with the helium fraction $Y_P \equiv 4 \frac{n_{{}^4\text{He}}}{n_p} \simeq 0.25$. The other light elements are produced in a smaller amount $n_{\text{D}}/n_p \sim \mathcal{O}(10^{-5})$ and $n_{{}^7\text{Li}}/n_p \sim \mathcal{O}(10^{-10})$. Because these abundances depend sensitively on the baryon density and the relativistic energy content of the Universe, BBN provides one of the earliest quantitative tests of the standard cosmological model and yields powerful constraints on additional relativistic species or other new physics that could modify the expansion history at early times.

Following BBN, two major events in the thermal history of the Universe are the epoch of matter–radiation equality and the subsequent formation of the first neutral atoms, known as recombination, which is closely followed by photon decoupling. The Universe becomes matter dominated at a temperature of approximately $T \sim 0.75$ eV, and as it cools further to $T \sim 0.25-0.30$ eV, free electrons and protons combine to form neutral hydrogen. Once the temperature drops below $T \sim 0.4$ eV, photons are no longer energetic enough to efficiently ionize hydrogen, allowing recombination to proceed rapidly and causing a sharp decrease in the free-electron number density. As a result, the Thomson scattering rate, $\Gamma_\gamma \simeq n_e \sigma_T$, falls below the Hubble expansion rate, leading to the decoupling of photons from the baryonic plasma. From this point onward, photons propagate freely with mean free paths larger than the horizon scale. This transition occurs at a redshift $z \sim 1090$, and the photons released during this epoch are observed today as the CMB, whose spectrum has been redshifted into the microwave region of the electromagnetic spectrum.

1.1.2 The perturbative Universe

The small perturbations in the early Universe are responsible for the structures that exist today. In the early Universe, the perturbations in the photons, electrons, baryons, neutrinos, and dark matter are coupled tightly with the perturbations in the gravitational field, and hence need to be treated simultaneously. Due to cooling and expansion of the Universe, species get decoupled from one another with time. This leads to the non-equilibrium phenomena which can be described by the Boltzmann equations. The Boltzmann equations in its simplest form can be written as

$$\frac{\partial f}{\partial t} - \mathcal{H}p \frac{\partial f}{\partial p} = C[f] \quad (1.13)$$

Here, f is the distribution function and right hand side (RHS) denotes the collision term. This equation helps to get the evolution of different species in a perturbative Universe. The choice of perturbed metric is not unique due to the coordinate independence of general relativity. The perturbed metric in the conformal Newtonian gauge is written as

$$ds^2 = a^2(\eta) [-(1 + 2\Psi(\vec{x}, \eta)) dt^2 + (1 + 2\Phi(\vec{x}, \eta)) \delta_{ij} dx^i dx^j], \quad (1.14)$$

where $\eta = \int_0^t dt'/a(t')$ represents the conformal time. Although the metric above contains only the scalar perturbations, vector and tensor perturbations can also be present. However, the scalar perturbations only couple to matter perturbations and hence the most important for the purpose of this thesis. Moreover, at the linear order, the scalar, vector and tensor components evolve independently and hence, do not affect one another.

For writing down the Boltzmann equation for photons, it is convenient to write the distribution function of photon as

$$f(\vec{x}, \vec{p}, t) = \left(e^{\frac{p}{T[1+\Theta(\vec{x}, \vec{p}, t)]}} - 1 \right)^{-1}, \quad (1.15)$$

where Θ is the small fluctuations around background temperature T , p and $\hat{p}$ denote the magnitude and direction of photon momentum, respectively. The dependence of Θ on $\vec{x}$ and $\hat{p}$ describes the small spatial inhomogeneities and directional anisotropies around an otherwise smooth background. With this distribution function along with the perturbed metric and considering photon scattering off electrons, the perturbed

equation in Fourier space can be expressed as (neglecting the angular dependence of Compton scattering) [34, 35]

$$\frac{d\Theta_k}{d\eta} + ik\mu\Theta_k + \frac{d\Phi_k}{d\eta} + ik\mu\frac{d\Psi_k}{d\eta} = -n_e\sigma_T a[\Theta_{0,k} - \Theta_k + \mu u_k]. \quad (1.16)$$

Here, n_e , σ_T and u are number density of electrons, Thompson scattering cross-section and velocity of combined proton-electron fluid respectively. The subscript k indicates the representation of a quantity in its Fourier space. The parameter $\mu \equiv \frac{\vec{k} \cdot \hat{p}}{k}$ is the cosine of the angle between the wavenumber $\vec{k}$ and the photon direction $\hat{p}$. The l -th multipole moment of the temperature field is denoted by

$$\Theta_{l,k} = \frac{1}{2(-i)^l} \int_{-1}^1 d\mu \mathcal{P}_l(\mu) \Theta_k(\mu), \quad (1.17)$$

with $\mathcal{P}_l$ being the Legendre polynomial of order l .

The Eq. (1.16) is one of the most important equations in obtaining the CMB anisotropy spectra. Similar to Eq. (1.16), the equations for cold dark matter (CDM) are read as

$$\frac{d\delta_k}{d\eta} + ikv_k + 3\frac{d\Phi_k}{d\eta} = 0, \quad (1.18)$$

$$\frac{dv_k}{d\eta} + \frac{da/d\eta}{a} v_k + ik\Psi_k = 0, \quad (1.19)$$

where v is the velocity of CDM and δ is the perturbation in its energy density defined as $\delta = \frac{\delta\rho_{\text{dm}}}{\rho_{\text{dm}}^0}$ with ρ^0 being the average energy density of CDM. The RHS of both the equations are zero, which implies that cold dark matter do not interact with other species physically. They affect other species only via gravitational potential.

The equations for proton-electron fluid are

$$\frac{d\delta_{k,b}}{d\eta} + iku_k + 3\frac{d\Phi_k}{d\eta} = 0, \quad (1.20)$$

$$\frac{dv_{k,b}}{d\eta} + \frac{da/d\eta}{a} u_k + ik\Psi_k = -n_e\sigma_T a \frac{4\rho_\gamma}{3\rho_b} [3i\Theta_{1,k} + u], \quad (1.21)$$

with δ_b being the perturbation in the energy density for proton-electron fluid ρ_b represents its background energy density. The role of Boltzmann Solver like CLASS [36] and

CAMB [37] is to precisely solve the above Boltzmann equations. (for a comprehensive treatment of cosmological perturbation theory, one may refer to Refs. [35, 38–41]).

The above equations define how the perturbations evolve in presence of the metric in (1.2). These equations are accompanied by two other equations. These two equations encode how the perturbations affect the gravitational field. These two equations are [34]

$$k^2\Phi_k + 3\frac{da/d\eta}{a}\left(\frac{d\Phi_k}{d\eta} - \Psi_k\frac{da/d\eta}{a}\right) = 4\pi Ga^2[\rho_{dm}\delta_k + \rho_b\delta_{b,k} + 4\rho_\gamma\Theta_{0,k}], \quad (1.22)$$

$$k^2(\Phi_k + \Psi_k) = -32\pi Ga^2[\rho_\gamma\Theta_{2,k}]. \quad (1.23)$$

Solving these equations requires the initial condition for each of these variables. Fortunately, at very early times, when all the perturbations of interest have wavelengths $(1/k)$ much larger than the comoving horizon $(k\eta \ll 1)$, many of the variables can be related to one another. Thus, knowing the initial condition for one of these variables (say Φ) is sufficient to solve the equations. This initial condition is obtained from inflation, a phase of extremely rapid expansion believed to have occurred in the very early Universe, prior to the radiation-dominated era. During Inflation, the scale factor increased nearly exponentially, stretching quantum fluctuations generated on microscopic scales to super-horizon, cosmological scales, where their amplitudes became effectively frozen. These primordial perturbations later re-entered the horizon during the evolution and provided the seeds for the formation of cosmic structures. A gauge-invariant curvature perturbation, ζ , can be constructed, for which the primordial perturbation spectrum can be written as

$$\mathcal{P}_\zeta(k) = A_s \left(\frac{k}{k_0}\right)^{n_s-1}, \quad (1.24)$$

where, A_s is the power-spectrum amplitude and n_s is the spectral index. k_0 is an arbitrary pivot scale and is commonly set to $k_0 = 0.05 \text{ Mpc}^{-1}$.

The above equations lay the foundations for obtaining the required observables and matching with the data. Two main observables required for the purpose of our discussion are the matter-power spectrum and the temperature (anisotropy) spectrum. The matter power spectrum, $P(k)$ is defined as

$$\langle \delta_m^*(k')\delta_m(k) \rangle = (2\pi)^3 P(k)\delta^3(k - k'). \quad (1.25)$$

The anisotropy spectrum is obtained by writing Θ in terms of spherical harmonics, Y_{lm} , and then taking the two-point correlation function of the coefficient b_{lm} :

$$\Theta(\vec{x}, \hat{p}, \eta) = \sum_{l=1}^{\infty} \sum_{m=-l}^l b_{lm}(\vec{x}, \eta) Y_{lm}(\hat{p}) \quad (1.26)$$

$$\langle b_{lm} b_{l'm'}^* \rangle = \delta_{ll'} \delta_{mm'} C_l^{TT}. \quad (1.27)$$

Here, the expansion coefficients b_{lm} are called the multipole moments, and C_l^{TT} is the anisotropy (temperature) power spectrum. In the next section, we discuss the six-parameter Λ CDM model and its implications for the temperature anisotropy power spectrum.

1.1.3 The six-parameter Λ CDM model

It is remarkable that the vast array of modern cosmological measurements can be consistently explained within the compact six-parameter Λ CDM model [25, 42]. These six parameters, however, do not encompass all possible cosmological degrees of freedom, such as the sum of neutrino masses, a dynamical dark energy equation of state, or the amplitude of primordial tensor perturbations. Nevertheless, it is reasonable to omit such extensions, since these additional parameters are not directly constrained by the CMB temperature power spectrum.

The six free parameters are

- The two parameters A_s and n_s from the primordial power spectrum.
- The baryon density, $\omega_b = \Omega_b h^2$.
- The total matter density $\omega_M = (\Omega_b + \Omega_{DM}) h^2$.
- The current Hubble constant, $H_0 = 100 h \text{ km s}^{-1} \text{ Mpc}^{-1}$. h or H_0 can be traded with Ω_Λ as we assume the Universe to be spatially flat. The parameters Ω_Λ and h are related by $h = \sqrt{\omega_M / (1 - \Omega_\Lambda)}$.
- The optical depth to reionization, τ_{reion} , which quantifies the integrated probability that a CMB photon has been Thomson scattered by free electrons produced during the epoch of reionization.

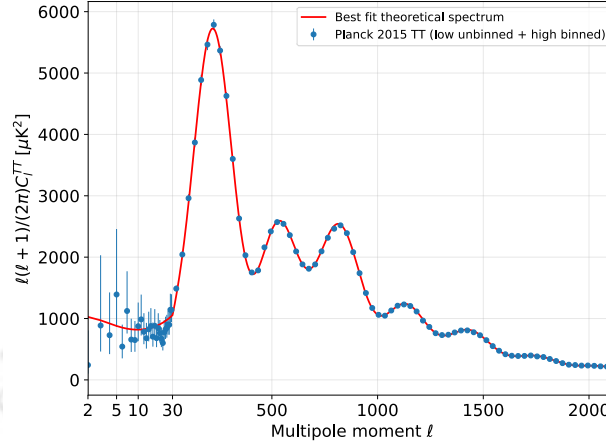


Figure 1.1: Temperature power spectrum of Planck 2015 data [43] (shown in the blue color). The red best fit theoretical spectrum of the Λ CDM model is shown by the red line.

The photon energy density, $\omega_\gamma = \Omega_\gamma h^2$ today is taken to be a fixed parameter as CMB temperature is precisely measured, $T_0 = 2.7255$ K. The other parameters like effective number of neutrino species, N_{eff} and primordial Helium fraction, Y_P , are kept fixed and standard values $N_{\text{eff}} = 3.044$ [44] and $Y_P = 0.2471$ [25, 45]. However, in the presence of extra relativistic species in the early Universe, N_{eff} should be used as the seventh parameter. The Fig. 1.1 shows the best-fit theoretical CMB temperature anisotropy power spectrum (red curve), obtained by numerically solving the complete Boltzmann equations discussed in section 1.1.2 using CLASS [36], together with a comparison to the Planck 2015 measurements [43].

The primary motivation for assuming a spatially flat Universe arises from the observed position of the first acoustic peak in the CMB temperature power spectrum. The position of the first acoustic peak is highly sensitive to the spatial curvature of the Universe through its dependence on the angular diameter distance to the surface of last scattering. Precise measurements of this peak led to the conclusion that the geometry of the Universe is very close to being spatially flat. Moreover, the epoch of matter–radiation equality is primarily set by ω_M , since the photon density ω_γ is measured with high precision. Likewise, the onset of matter–dark energy equality is fixed by Ω_Λ or h .

Let us now briefly discuss some important changes in the CMB temperature power spectrum as a result of change in parameters. These will be helpful in the

next chapter where we will discuss the variation of CMB spectrum with N_{eff} [46].

1. The locations of the acoustic peaks are determined by the angular size of the comoving sound horizon. This depends on the expansion history of the Universe and on the sound speed of the photon–baryon fluid, and is therefore sensitive to ω_b , ω_M , and h .
2. The relative height (contrast) between the odd and even peaks is primarily set by the baryon density, ω_b .
3. The overall amplitude of the peaks depends on the amount of expansion between matter–radiation equality and recombination. Since the redshift of recombination is largely fixed by thermodynamics, the peak amplitudes mainly depend on ω_M through its effect on the epoch of matter–radiation equality.
4. The damping of peaks at large l limits is controlled by the angular size of the photon diffusion (damping) scale, and is therefore also affected by ω_b , ω_M , and h .

Having established the cosmological framework and the parameter dependence of the CMB observables, we now turn to the underlying particle physics description that determines the microscopic particle content of the Universe, namely the Standard Model of particle physics.

1.2 The Standard Model of Particle Physics

In this section, we present a concise overview of the Standard Model of particle physics, emphasizing its particle spectrum, underlying gauge symmetries, and conceptual limitations. This brief summary establishes the theoretical framework necessary for the subsequent discussion of physics beyond the Standard Model, which forms the central theme of this thesis.

The Standard Model describes the fundamental interactions of nature (excluding gravity) within the framework of quantum field theory. It provides a unified description of the strong, weak, and electromagnetic forces and incorporates all

Spin	Fields		$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
$\frac{1}{2}$	Quarks	$Q_{L\alpha} = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$	$(3, 2, 1/6)$
		$U_{R\alpha} = u_R, c_R, t_R$	$(3, 1, 2/3)$
		$D_{R\alpha} = d_R, s_R, b_R$	$(3, 1, -1/3)$
	Leptons	$l_{L\alpha} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$	$(1, 2, -1/2)$
		$E_{R\alpha} = e_R, \mu_R, \tau_R$	$(1, 1, -1)$
1	Gauge bosons	G_μ^a	$(8, 1, 0)$
		$W_\mu^{1,2,3}$	$(1, 3, 0)$
		B_μ	$(1, 1, 0)$
0	Scalar	Higgs: H	$(1, 2, 1/2)$

Table 1.1: Particle content and their charges under $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ gauge symmetry. Electromagnetic charge is obtained by $Q = I_3 + Y$, where I_3 denotes the third component of isospin.

presently known elementary fermions, gauge bosons, and the Higgs scalar [1, 4, 47–50]. Mathematically, the theory is constructed from the gauge symmetry

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y,$$

where the subscripts C , L , and Y denote color charge, left-handed weak isospin, and weak hypercharge, respectively.

The electroweak interactions arise from the subgroup $SU(2)_L \otimes U(1)_Y$ and are mediated by four vector bosons, W_μ^i ($i = 1, 2, 3$) and B_μ . The strong interaction is governed by $SU(3)_C$ and is transmitted by eight gluon fields G_μ^a ($a = 1, \dots, 8$). In total, the gauge sector therefore contains twelve spin-1 mediators. The complete set of fields and their quantum numbers under the SM gauge group are summarized in Table 1.1.

The matter content of the theory consists of three generations of spin- $\frac{1}{2}$ fermions. Because the weak interaction is chiral, the left- and right-handed components transform differently under $SU(2)_L$: left-handed fermions assemble into weak doublets, whereas right-handed fermions are singlets. The fermionic spectrum includes both

quarks and leptons, together with their corresponding antiparticles. The quark sector comprises three generations of up-type quarks (u, c, t) and down-type quarks (d, s, b).

Organizing the fields according to their $SU(2)_L$ transformation properties, the quarks are written as

$$\begin{aligned} Q_{L\alpha} &= \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L, \\ U_{R\alpha} &= (u_R, c_R, t_R), \quad D_{R\alpha} = (d_R, s_R, b_R), \end{aligned} \quad (1.28)$$

where $Q_{L\alpha}$ denotes the left-handed quark doublets and $U_{R\alpha}$, $D_{R\alpha}$ represent the right-handed singlet states for the up- and down-type quarks, respectively. The index $\alpha = 1, 2, 3$ labels the three generations.

An analogous structure appears in the leptonic sector:

$$\begin{aligned} \ell_{L\alpha} &= \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, \\ E_{R\alpha} &= (e_R, \mu_R, \tau_R), \end{aligned} \quad (1.29)$$

where ν_e , ν_μ , and ν_τ denote the neutrinos and e^- , μ^- , and τ^- the corresponding charged leptons. Within the minimal Standard Model, neutrinos appear only as left-handed fields and no right-handed neutrino states are included.

Finally, the scalar sector contains a single weak doublet, the Higgs field, which is a spin-0 boson responsible for electroweak symmetry breaking,

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}, \quad (1.30)$$

whose charged and neutral components are denoted by H^+ and H^0 , respectively.

1.2.1 The Standard Model Lagrangian

With all the particles and the symmetry introduced, the SM Lagrangian is written as $\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Yukawa}} + \mathcal{L}_{\text{Scalar}}$ with different sub-parts explained below.

Gauge sector : The kinetic terms for the gauge fields are given by

$$\mathcal{L}_{\text{Gauge}} = -\frac{1}{4}G_{\mu\nu}^a G^{\mu\nu,a} - \frac{1}{4}W_{\mu\nu}^i W^{\mu\nu,i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}. \quad (1.31)$$

The associated field-strength tensors read

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_3 f^{abc} G_\mu^b G_\nu^c, \quad (1.32)$$

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g_2 \epsilon^{ijk} W_\mu^j W_\nu^k, \quad (1.33)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \quad (1.34)$$

where G_μ^a , W_μ^i , and B_μ correspond to the gauge bosons of $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$, respectively. Here g_3 and g_2 denote the non-Abelian gauge couplings of $SU(3)_C$, $SU(2)_L$ respectively with f^{abc} and ϵ^{ijk} being the corresponding structure constants. As $U(1)_Y$ is Abelian, the hypercharge field has no self-interaction terms.

Fermion sector: Quarks and leptons are spin- $\frac{1}{2}$ fields whose kinetic terms share a universal form,

$$\mathcal{L}_{\text{Fermion}} = i\bar{\psi}\gamma^\mu D_\mu\psi, \quad (1.35)$$

where the covariant derivative introduces gauge interactions,

$$D_\mu = \partial_\mu - ig_1 Y B_\mu - ig_2 \frac{\tau^i}{2} W_\mu^i - ig_3 \frac{\lambda^a}{2} G_\mu^a. \quad (1.36)$$

Here g_1 is the hypercharge coupling, while τ^i and λ^a represent the generators of $SU(2)_L$ and $SU(3)_C$ (Pauli and Gell-Mann matrices) respectively. For a field which is a singlet under $SU(3)_C$ or $SU(2)_L$, the covariant derivative does not contain the corresponding gauge fields.

Scalar and Yukawa sector: The gauge-invariant Yukawa couplings between the Higgs and fermions are written as

$$\mathcal{L}_{\text{Yukawa}} = -\bar{Q}_{L\alpha}(Y_u)_{\alpha\beta}\tilde{H}U_{R\beta} - \bar{Q}_{L\alpha}(Y_d)_{\alpha\beta}HD_{R\beta} - \bar{\ell}_{L\alpha}(Y_\ell)_{\alpha\beta}HE_{R\beta} + \text{h.c.}, \quad (1.37)$$

where $\tilde{H} = i\sigma_2 H^*$. After spontaneous symmetry breaking, these interactions generate all charged fermion masses.

Finally, the scalar part of the Lagrangian is described by

$$\mathcal{L}_{\text{Scalar}} = (D_\mu H)^\dagger (D^\mu H) - V(H), \quad V(H) = \mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2. \quad (1.38)$$

Here, μ_H^2 and λ_H are the quadratic and quartic couplings of the Higgs field, respectively.

1.2.2 Generation of masses

The masses of all charged fermions and weak vector bosons in the SM are generated via *Spontaneous Symmetry Breaking*. For $\mu_H^2 < 0$, the Higgs potential develops a non-zero minimum and the field acquires the vacuum expectation value (VEV)

$$\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu_H^2/\lambda_H}. \quad (1.39)$$

This spontaneously breaks $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{EM}}$. As a result, three gauge bosons become massive,

$$M_W = \frac{g_2 v}{2}, \quad M_Z = \frac{g_2 v}{2 \cos \theta_W}, \quad (1.40)$$

while the orthogonal combination remains massless and is identified with the photon. The physical fields are

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \quad Z_\mu = \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu, \\ A_\mu &= \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu, \quad \text{where } \theta_W = \tan^{-1} \left(\frac{g_1}{g_2} \right). \end{aligned} \quad (1.41)$$

The angle θ_W is known as the Weinberg angle. The eight gluons of $SU(3)_C$ remain massless since color symmetry is unbroken.

Expanding around the vacuum in the unitary gauge, $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$, reveals the physical Higgs boson with mass $m_h = \sqrt{2\lambda} v$. Substituting this vacuum configuration into the Yukawa terms yields fermion mass matrices,

$$(m_u)_{\alpha\beta} = \frac{v}{\sqrt{2}}(Y_u)_{\alpha\beta}, \quad (m_d)_{\alpha\beta} = \frac{v}{\sqrt{2}}(Y_d)_{\alpha\beta}, \quad (m_\ell)_{\alpha\beta} = \frac{v}{\sqrt{2}}(Y_\ell)_{\alpha\beta}. \quad (1.42)$$

Within the minimal formulation of the SM, neutrinos are predicted to be massless due to the absence of right-handed neutrino states.

1.3 Beyond the Standard Model

The Standard Model has proven to be remarkably successful, providing an exceptionally accurate description of particle interactions at the electroweak scale. Nevertheless, it remains incomplete and leaves several fundamental questions unresolved. Below, we highlight some of the issues that are particularly relevant to the scope of this thesis.

- **Dark matter** : Multiple observational evidences, including the CMB power spectrum as discussed in section 1.1, provide strong evidence for a non-luminous and non-baryonic DM with $\Omega_{\text{DM}} h^2 \simeq 0.12$ [51–53]. However, SM does not offer a potential DM candidate capable of explaining this observed abundance. In this thesis, our aim is to explore the particle nature of DM in different BSM scenarios and investigate their potential signatures through CMB observations and indirect detection.
- **Neutrino Mass** : Neutrino oscillation data confirm that neutrinos have tiny but non-zero masses, in conflict with the minimal Standard Model prediction of massless neutrinos [17, 54–56]. This provides a strong motivation for physics beyond the Standard Model [57, 58], a possibility explored in this thesis.
- **Strong CP problem** : The SM predicts CP violation of order $\bar{\theta} \simeq \mathcal{O}(1)$ in strong interaction. However, no CP violation is observed in the strong sector putting an upper bound $\bar{\theta} \lesssim 10^{-10}$ [59]. The problem can be solved by Peccei-Quinn (PQ) theory that introduces a new pseudo Nambu–Goldstone bosons known as axions [20–22, 60–63]. Interestingly, axions can behave as DM as well as dark radiation (DR) depending on their production mechanism and coupling strength [64, 65]. We discuss such a scenario and its detection prospects in one of the chapters of the thesis.
- **Matter anti-matter asymmetry** : Evidence from cosmological and astrophysical observations suggests an excess of matter over antimatter, quantified

by the baryon-to-photon ratio $\eta_B \simeq 6.1 \times 10^{-11}$ [66]. The generation of this matter-antimatter asymmetry requires the three Sakharov conditions [67] to be satisfied. Although these conditions can, in principle, be realized within the SM, it fails to produce the observed asymmetry in sufficient amount [68]. In one of the chapters, we discuss the generation of baryon asymmetry from heavy Majorana neutrinos [18, 19, 69] and their possible phenomenological signatures.

Apart from these shortcomings, the SM also faces several additional issues, including the hierarchy problem, gauge coupling unification, and Higgs vacuum stability. These limitations provide strong motivation to consider scenarios beyond the SM. In the following, we discuss mechanisms for dark matter production and neutrino mass generation, which play an important role in most of the subsequent chapters in this thesis.

1.3.1 Dark Matter

One of the earliest indications of dark matter dates back to 1933, when Fritz Zwicky studied the Coma Cluster and found that the visible mass of its galaxies was insufficient to account for the observed velocity dispersion, leading him to propose the existence of unseen matter [70]. More convincing evidence emerged in the 1970s from measurements of galactic rotation curves. Observations by Vera Rubin and collaborators showed that the rotational velocities of stars in spiral galaxies remain nearly constant at large radii, as shown in the left panel of Fig. 1.2, rather than decreasing as expected from Newtonian dynamics [71]. These flat rotation curves point to the presence of an extended, invisible mass component surrounding galaxies, now identified as dark matter.

Gravitational lensing offers an independent and robust probe of dark matter. In general relativity, massive objects curve spacetime and deflect light from background sources, allowing the total mass distribution to be mapped. Observations of both strong and weak lensing in galaxy clusters reveal significantly more mass than can be accounted for by visible matter alone. A notable example is the Bullet Cluster (1E0657-558), where lensing measurements show a clear separation between the baryonic gas observed in X-rays and the dominant gravitational mass component,

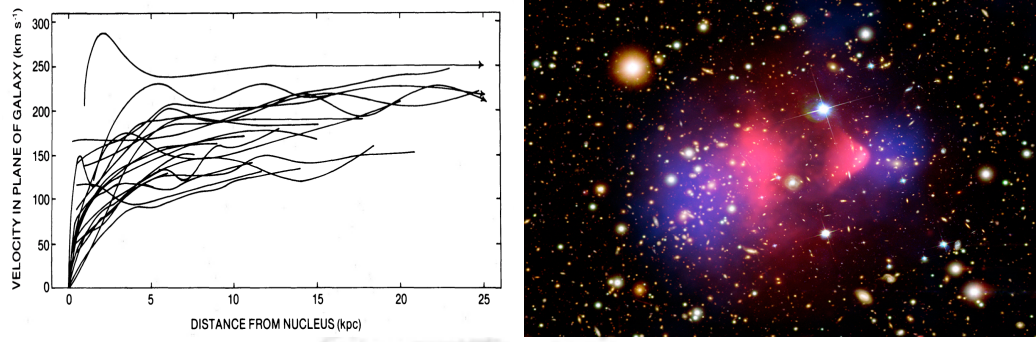


Figure 1.2: Left panel: Rotation curves of 21 Sc galaxies as a function of radial distance from the galactic center. Right panel: The Bullet Cluster (1E0657-558), indicating the collisions between two galaxy clusters; X-ray emission (pink) traces hot baryonic gas, while gravitational lensing (blue) maps the dominant dark matter distribution. The left and right panels are taken from Ref. [73] and [72], respectively.

providing direct evidence for a collisionless dark matter halo [72]. These results provide direct evidence of the existence of DM and also strongly constrain its interaction strength.

Another prominent piece of evidence for the existence of DM comes from the CMB temperature and matter power spectra. Precision measurements of the temperature anisotropies in the CMB by both Wilkinson Microwave Anisotropy Probe (WMAP) [74] and Planck [25] hint towards the presence of a significant amount of DM at cosmological scales. The acoustic peak structure of the temperature power spectrum is highly sensitive to the total matter content of the Universe. As discussed in section 1.1, the observed relative heights and positions of these peaks cannot be explained by baryonic matter alone and instead require a substantial non-baryonic matter component. This is shown in Fig 1.3. Analyses of CMB data therefore indicate that DM constitutes a significant fraction of the cosmic energy density at present, far exceeding the baryonic contribution. Recent Planck 2018 data reports the present value of DM density as [25]

$$\Omega_{\text{DM}} h^2 = 0.120 \pm 0.001 \quad (1.43)$$

at 68% CL. Similarly, observations of large-scale structure [75], together with N-body simulations [76] of structure formation, provide strong evidence for the presence of dark matter.

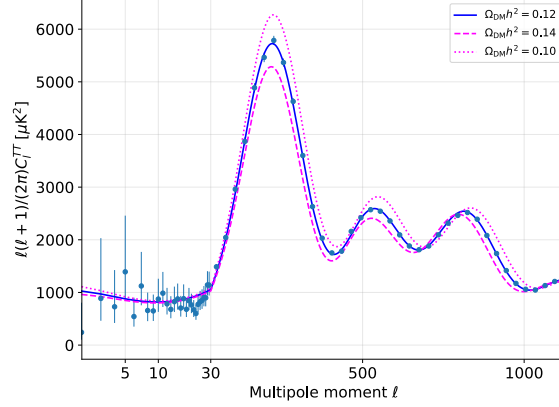


Figure 1.3: Variation of the CMB temperature anisotropy power spectrum for different values of $\Omega_{\text{DM}}h^2$. The best-fit spectrum is shown in blue. In each case, the total energy density is fixed to $\Omega_{\text{tot}} = 1$ by adjusting Ω_{Λ} to compensate for changes in $\Omega_{\text{DM}}h^2$.

Although a wide range of astrophysical and cosmological observations firmly establish the existence of dark matter, its particle nature remains unknown. Within the SM, neutrinos are the only neutral and weakly interacting particles that could resemble dark matter; however, their tiny masses make them relativistic in the early Universe, causing them to behave as hot dark matter and preventing the formation of small-scale structures. Moreover, their relic abundance is insufficient to explain the present observed density. In the following, we discuss two well-known BSM DM candidates, namely, weakly interacting massive particle (WIMP) and feebly interacting massive particle (FIMP), together with their production mechanisms.

WIMP DM:

The WIMP paradigm has long been one of the most well-motivated and extensively studied frameworks to explain the origin of DM [77–80]. In this scenario, a WIMP particle has non-gravitational interactions with the SM sector such that it can be produced in thermal equilibrium in the early Universe. As the Universe expands and cools, the DM interaction rate eventually drops below the Hubble expansion rate. At this stage, the DM particles decouple from the thermal bath and *freeze out*, after which their comoving number density, defined as $Y \equiv n/s$ (where n and s denote the number density and entropy density, respectively), becomes effectively constant.

Using the freeze-out value of the comoving number density, $Y_{\text{DM}}^{\text{fo}}$, the present-day DM abundance is given by

$$\Omega_{\text{DM}} h^2 = 2.755 \times 10^8 Y_{\text{DM}}^{\text{fo}} \left(\frac{m_{\text{DM}}}{\text{GeV}} \right), \quad (1.44)$$

where m_{DM} denotes the DM mass. The quantity Y_{DM} is obtained by solving the Boltzmann equation governing the evolution of the DM number density. For a typical annihilation process of the form $\text{DM DM} \rightarrow \text{SM SM}$, the Boltzmann equation can be written as

$$\frac{dY_{\text{DM}}}{dx} = -\frac{s \langle \sigma v \rangle}{\mathcal{H} x} (Y_{\text{DM}}^2 - (Y_{\text{DM}}^{\text{eq}})^2), \quad (1.45)$$

where $x \equiv m_{\text{DM}}/T$, $\mathcal{H}$ is the Hubble expansion rate, and $Y_{\text{DM}}^{\text{eq}}$ denotes the equilibrium comoving number density. The thermally averaged annihilation cross-section is related to the invariant annihilation cross-section $\sigma(\mathfrak{s})$ through [81]

$$\langle \sigma v \rangle = \frac{g_{\text{DM}}^2}{8m_{\text{DM}}^4 T K_2^2(m_{\text{DM}}/T)} \int_{\mathfrak{s}_0}^{\infty} \sigma(\mathfrak{s}) \sqrt{\mathfrak{s}(\mathfrak{s} - 4m_{\text{DM}}^2)} K_1\left(\frac{\sqrt{\mathfrak{s}}}{T}\right) d\mathfrak{s}, \quad (1.46)$$

where $\mathfrak{s}$ is the center-of-mass energy squared, $\mathfrak{s}_0 = 4m_{\text{DM}}^2$, and K_i are the modified Bessel functions of order i .

The left panel of Fig. 1.4 illustrates the freeze-out mechanism. Initially, the DM abundance closely tracks the equilibrium distribution. As the temperature falls below the DM mass, the equilibrium number density becomes Boltzmann suppressed, leading to a rapid decrease in the interaction rate. Around $x \sim 20$ – 30 , the interaction rate drops below the Hubble expansion rate, causing the comoving number density to freeze out. A larger annihilation cross-section $\langle \sigma v \rangle$ keeps the DM particles in equilibrium for a longer period, resulting in a smaller relic abundance. Remarkably, for DM masses in the range $\sim \mathcal{O}(100 \text{ GeV})$ – $\mathcal{O}(10 \text{ TeV})$, the annihilation cross-section required to reproduce the observed relic density naturally corresponds to interactions of electroweak strength, a coincidence commonly referred to as the *WIMP miracle*. Typically, the same interactions responsible for bringing WIMP into thermal equilibrium in the early Universe can also lead to observable signatures at terrestrial experiments [82].

Despite these appealing features of WIMP, a wide range of direct and indirect detection experiments, including XENONnT [83], LUX-ZEPLIN [84], PandaX-II [85], and Fermi-LAT [86], have not yet reported any confirmed WIMP signals.

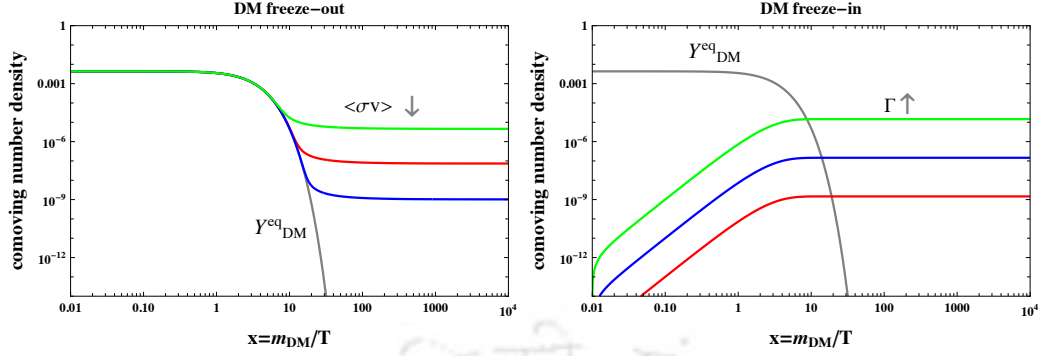


Figure 1.4: Left panel: Freeze-out evolution of WIMP dark matter for three different values of the interaction strength. The gray curve denotes the equilibrium comoving number density. As the coupling strength increases, the dark matter remains in thermal equilibrium for a longer duration, leading to a smaller freeze-out abundance. Right panel: Freeze-in evolution of FIMP dark matter. In contrast to the freeze-out case, the final freeze-in abundance increases with increasing interaction strength.

This absence of detection has motivated the exploration of alternative production mechanisms, such as the freeze-in scenario involving FIMPs.

FIMP DM:

If the interaction strength between the DM particle and the SM sector is extremely weak, the DM particles never attain thermal equilibrium in the early Universe. This situation typically arises when the DM–SM couplings, parametrized by a dimensionless coupling constant, are smaller than $\mathcal{O}(10^{-7})$ [87]. In such a scenario, DM cannot be produced via the freeze-out mechanism discussed above and must instead originate from a non-thermal production process.

A well-motivated possibility is the production of DM from particles in the thermal bath through decays or scatterings. This production continues as long as the bath particles remain abundant and effectively ceases once their number density becomes Boltzmann suppressed. Consequently, the comoving number density of DM approaches a constant value, and the DM abundance is said to *freeze-in* [87, 88]. Dark matter produced through this mechanism is commonly referred to as a *Feebly Interacting Massive Particle*.

The Boltzmann equation governing the evolution of the DM comoving number density, written in terms of Y_{DM} , can be expressed as follows. For freeze-in production of DM through the decay of a thermal bath particle, $\text{SM} \rightarrow \text{DM DM}$, the

evolution equation becomes

$$\frac{dY_{\text{DM}}}{dx} = \frac{2\Gamma_{\text{SM} \rightarrow \text{DM DM}}}{\mathcal{H}x} \frac{K_1(x)}{K_2(x)} Y_{\text{SM}}^{\text{eq}}, \quad (1.47)$$

where $\Gamma_{\text{SM} \rightarrow \text{DM DM}}$ denotes the decay rate of the bath particle, $Y_{\text{SM}}^{\text{eq}}$ is its equilibrium comoving number density, and $K_i(x)$ are the modified Bessel functions of order i .

The evolution of the comoving DM number density for different values of the decay rate is illustrated in the right panel of Fig. 1.4. In contrast to the freeze-out scenario, a larger interaction strength results in a larger final freeze-in abundance of DM, as shown in the figure. Owing to their extremely feeble interactions with the SM sector, FIMPs are typically challenging to probe in conventional direct detection experiments; however, they may still be tested indirectly through signatures of long-lived particles at colliders [89–91] or through cosmological and astrophysical observations [92, 93] sensitive to their production history.

1.3.2 Neutrino Mass

Neutrino oscillation experiments have conclusively demonstrated that neutrinos possess nonzero, albeit very small, masses [94–99]. Moreover, measurements of the CMB by the Planck satellite along with Baryonic Acoustic Oscillations (BAO) observations constrain the sum of neutrino masses to $\sum_i m_{\nu_i} < 0.12$ eV [25, 100] at 95% CL. Similarly, KATRIN, a terrestrial experiment, provides the limit on the kinematical mass from beta decay as $(\sum_i^3 |U_{ei}|^2 m_{\nu_i}^2)^{1/2} < 0.8$ eV at 90% CL [56, 101], where U_{ei} is the mixing matrix element of the mass state i with the electron neutrino. In the SM, however, neutrinos remain massless because a renormalizable mass term cannot be written using the existing particle content while maintaining gauge invariance. Therefore, the origin of neutrino masses necessarily points toward physics beyond the SM.

Unlike other SM fermions, whose masses arise from the Higgs mechanism and correspond to Dirac fermions, neutrinos may instead be Majorana particles due to their electrical neutrality, allowing their masses to originate from mechanisms beyond the conventional Higgs framework. In the following, we briefly review several representative neutrino mass-generation mechanisms.

In the SM, the masses of all fermions, except neutrinos, originate from the Higgs mechanism. This mechanism cannot generate neutrino masses within the minimal SM because the right-handed counterpart of the left-handed neutrino, ν_L , is absent. Furthermore, owing to their electrical neutrality, neutrinos may be Majorana particles (i.e., they are their own antiparticles), suggesting that their masses may arise from mechanisms beyond the conventional Higgs framework. In the following, we briefly review several well-known mechanisms for neutrino mass generation.

Dirac masses:

A simple possibility is that neutrinos are Dirac fermions. This scenario requires the introduction of right-handed neutrino fields, ν_R , which allow a Yukawa interaction with the SM lepton doublet and the Higgs field,

$$-\mathcal{L}_{\text{Dirac}} = (Y_\nu)_{\alpha i} \bar{\ell}_{L\alpha} \tilde{H} \nu_{Ri} + \text{h.c.} \quad (1.48)$$

Following electroweak symmetry breaking, the Higgs field acquires a vacuum expectation value, generating the Dirac mass term

$$-\mathcal{L}_{\text{Dirac}} = \bar{\nu}_{L\alpha} (m_D)_{\alpha i} \nu_{Ri} + \text{h.c.}, \quad (1.49)$$

where $(m_D)_{\alpha i} = (Y_\nu)_{\alpha i} \langle H \rangle$ and $\langle H \rangle \simeq 174$ GeV. The physical neutrino masses are then obtained by diagonalizing the resulting mass matrix through a bi-unitary transformation.

Because this mass originates from a Yukawa interaction analogous to those of the charged fermions, explaining the observed sub-eV neutrino masses requires an exceptionally small coupling, $Y_\nu \sim 10^{-12}$, which is often viewed as theoretically unnatural. In this framework, total lepton number is conserved, while lepton flavor violation can still arise through neutrino mixing. The experimentally observed neutrino oscillations, therefore, remain consistent with neutrinos being Dirac particles, although they do not exclude the Majorana possibility. Moreover, being light, RH Dirac neutrinos can behave as a dark radiation in the early Universe, and hence can have signatures in the CMB. We discuss such possibilities in the subsequent chapters.

Majorana masses: If neutrinos are Majorana particles, their masses can be generated through the minimal Type-I seesaw mechanism [102–104] by introducing

right-handed Majorana neutrinos N_R . The relevant Lagrangian is given by

$$-\mathcal{L}_{\text{Type-I}} = (Y_\nu)_{\alpha i} \bar{\ell}_{L\alpha} \tilde{H} N_{Ri} + \frac{1}{2} (M_R)_{ij} \overline{(N_{Ri})^c} N_{Rj} + \text{h.c.}, \quad (1.50)$$

where the first term corresponds to the Yukawa interaction among the left-handed lepton doublet, the Higgs doublet, and the right-handed neutrinos, while the second term represents the Majorana mass term for the right-handed neutrinos, which violates total lepton number by two units.

After electroweak symmetry breaking, the neutrino mass terms can be written in matrix form as

$$\mathcal{L}_{\text{mass}} = \frac{1}{2} (\bar{\nu}_L \quad \overline{N_R^c}) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.}, \quad (1.51)$$

where $m_D = Y_\nu \langle H \rangle$ denotes the Dirac mass matrix. With three right-handed neutrinos, both m_D and M_R are 3×3 matrices, with M_R symmetric, resulting in a 6×6 symmetric neutrino mass matrix.

Assuming the hierarchy $m_D \ll M_R$, block diagonalization of the full mass matrix separates the light and heavy neutrino sectors, yielding

$$m_\nu^{\text{I}} \simeq -m_D M_R^{-1} m_D^T, \quad (1.52)$$

$$M_N \simeq M_R, \quad (1.53)$$

which correspond to the effective mass matrix of the light neutrinos and the heavy Majorana neutrino mass matrix, respectively. The physical light neutrino masses are obtained by diagonalizing m_ν^{I} ,

$$U_\nu^\dagger m_\nu^{\text{I}} U_\nu^* = \text{diag}(m_1, m_2, m_3) \equiv D_m, \quad (1.54)$$

where U_ν denotes the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) leptonic mixing matrix.

Unlike the Dirac mass scenario, the Type-I seesaw allows the generation of sub-eV neutrino masses even for Yukawa couplings of order $\mathcal{O}(1)$, provided the right-handed neutrino masses are very large, typically $M_R \sim 10^{14}$ GeV. Such a high mass scale makes direct experimental tests challenging; however, indirect probes may arise

in extended frameworks, such as scenarios connecting the seesaw mechanism with axion physics, which will be explored in a later chapter.

1.4 Conclusion

In this thesis, we investigate cosmological signatures of well-motivated scenarios of physics beyond the SM. This chapter has presented a brief overview of the Λ CDM cosmological framework and the Standard Model, emphasizing both their successes and the open questions that motivate the search for new physics. We reviewed the primary mechanisms of dark matter production, focusing on the WIMP freeze-out and FIMP freeze-in paradigms, and discussed theoretical frameworks for generating Dirac and Majorana neutrino masses. These topics provide the theoretical foundation for the subsequent chapters, where the cosmological and phenomenological implications of these scenarios are explored in detail.

Cosmological Signatures

In this chapter, we explore several cosmological signatures of the BSM scenarios introduced in Chapter 1 that are particularly relevant to the goals of this thesis. In Sec. 2.1, we examine the imprints of dark radiation or additional relativistic degrees of freedom in the early Universe, emphasizing how future CMB experiments can probe a variety of thermally produced relativistic species. In Sec. 2.2, we investigate the cosmological consequences of the presence of PBHs and discuss their potential observational signatures through precision CMB measurements and observation of stochastic gravitational wave (GW). Finally, in Se. 2.3, we discuss possible indirect signatures of dark matter decay, focusing on high-energy neutrino and cosmic-ray observations.

2.1 Cosmological Signatures of Additional Relativistic Species

In this section, we discuss the cosmological imprints of additional relativistic species in the early Universe. Moreover, we will see how the constraints from various cosmological experiments can be used to constrain BSM physics scenarios with additional light degrees of freedom.

Let us start with the Eq. (1.12) of Chapter 1. The equation describes the difference between the neutrino and photon temperatures after $e^\pm$ annihilation in the instantaneous neutrino decoupling limit. Following the equation, the neutrino energy density for each species $\rho_{\nu i}$ in terms of the photon energy density, ρ_γ can be written as

$$\rho_{\nu i} = \frac{7}{8} \left(\frac{T_\nu}{T_\gamma} \right)^4 \rho_\gamma = \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \rho_\gamma \equiv a_\nu^{-1} \rho_\gamma, \text{ where } a_\nu^{-1} = \frac{7}{8} \left(\frac{4}{11} \right)^{4/3}. \quad (2.1)$$

This relationship holds from the time of electron-positron annihilation till the time when the neutrino becomes non-relativistic. Although today, the photon energy density is negligible compared to the total energy density ($\Omega_\gamma h^2 \sim 2.5 \times 10^{-5}$), at the time of recombination, the radiation density contributed significantly to the total energy density. Consequently, the contribution from three neutrino species was also high: $\rho_\nu/\rho_r \sim 41\%$. Thus, the gravitational effects of neutrinos are significant at early times, and that is why we can observe their signatures via CMB or BAO.

In the discussion above, we assumed that neutrinos decouple instantaneously and therefore do not receive any entropy from $e^\pm$ annihilation. However, in reality, neutrino decoupling occurs at temperatures of the order ~ 1 MeV, which is close to the epoch of $e^\pm$ annihilation (~ 0.5 MeV). Because of this proximity, neutrinos are not completely decoupled during $e^\pm$ annihilation, and a small fraction of the entropy is transferred to the neutrino sector. To account for this effect, it is convenient to parameterize the total radiation energy density as

$$\rho_r = \rho_\gamma + \rho_\nu = (1 + a_\nu^{-1} N_\nu) \rho_\gamma, \quad (2.2)$$

where we introduce a new parameter N_ν , the *effective number of neutrinos*. This parameter encapsulates the contribution of neutrinos to the radiation content of the early Universe. In the limit of instantaneous neutrino decoupling, one obtains exactly $N_\nu = 3$, corresponding to three different species of neutrinos. However, in the Standard Model, effects such as finite-temperature (plasma) corrections of quantum electrodynamics, neutrino flavor oscillations, and the non-instantaneous decoupling of neutrinos, lead to a small increase in this value, giving $N_\nu = 3.044$ [105–113].¹ Therefore, any measured deviation from $N_\nu = 3.044$ would provide a sensitive probe

¹Although the state-of-the-art value is $N_\nu = 3.044$ [108, 111–113], some dispute remains about the last significant digit. The references [105–107, 110] report slightly higher value of N_ν as 3.045. A very recent paper [109] reports $N_\nu = 3.043$ by taking into account of the NLO correction to

of non-standard neutrino properties or additional interactions beyond the Standard Model.

The parameter N_ν can be replaced with a more general parameter, *effective number of relativistic species* N_{eff} , which will not only include information of unknown properties of neutrinos but will also have information of extra light BSM degrees of freedom. With this, the Eq. (2.2) takes the form

$$\rho_r = \rho_\gamma + \rho_\nu = \left(1 + \frac{7}{8} \left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right) \rho_\gamma. \quad (2.3)$$

The parameter N_{eff} has two parts

$$N_{\text{eff}} = N_{\text{eff}}^{\text{SM}} + \Delta N_{\text{eff}} = 3.044 + \Delta N_{\text{eff}}. \quad (2.4)$$

This is the central cosmological parameter of interest for studying the presence of extra radiation in the early Universe. Any non-zero value of ΔN_{eff} would provide a direct probe of physics beyond the Standard Model. The current constraint from the CMB, including BAO data, is [25]

$$N_{\text{eff}} = 2.99^{+0.34}_{-0.33}, \quad (2.5)$$

at 2σ (or 95% C.L.). This translates into an approximate upper bound $\Delta N_{\text{eff}} \lesssim 0.285$. The latest DESI 2024 results give an updated constraint, $N_{\text{eff}} = 3.10 \pm 0.17$ at 68% C.L. Complementary bounds also arise from BBN, which constrains the effective number of relativistic species to $2.3 < N_{\text{eff}} < 3.4$ at 95% C.L. [114].

Future CMB experiments are expected to reach sub-percent precision on this parameter. For instance, CMB Stage IV (CMB-S4) is projected to achieve a sensitivity $\Delta N_{\text{eff}} \simeq 0.06$ [115], while the proposed CMB-HD experiment could further improve this to $\Delta N_{\text{eff}} \simeq 0.014$ at 1σ [116]. Similar precision measurements are also expected from other planned future experiments like SPT-3G [117] and Simons Observatory [118]. Such measurements will enable stringent tests of a wide class of BSM scenarios that are difficult or impossible to access through collider experiments and other terrestrial probes.

$e^+e^- \leftrightarrow \nu_L \bar{\nu}_L$ interactions along with finite temperature QED corrections to the electromagnetic plasma density and effect of neutrino oscillations.

2.1.1 Probing thermal BSM species with N_{eff}

In this section, we discuss how cosmological bounds on N_{eff} can be used to constrain light BSM species, in particular scenarios in which such particles were in thermal equilibrium with the primordial plasma at some stage in the early Universe. Let us consider an additional relativistic species X that decouples from the thermal bath at a temperature T_{dec} . The total radiation energy density at recombination then receives contributions from photons, neutrinos, and this extra species, $\rho_r = \rho_\gamma + \rho_\nu + \rho_X$. The corresponding contribution to the effective number of relativistic species can be written as $\Delta N_{\text{eff}} = \frac{\rho_X}{\rho_{\nu 1}}$, where $\rho_{\nu 1}$ denotes the energy density of a single neutrino species.

Using entropy conservation (see Eq. (1.11)), the energy density of X at the time of recombination can be related to its decoupling temperature through the change in the effective number of relativistic degrees of freedom in the plasma. This leads to the approximate expression

$$\Delta N_{\text{eff}} \approx \begin{cases} 0.027 g_X \left(\frac{106.75}{g_*(T_{\text{dec}})} \right)^{4/3}, & \text{if } X \text{ is a boson,} \\ \frac{7}{8} \times 0.027 g_X \left(\frac{106.75}{g_*(T_{\text{dec}})} \right)^{4/3}, & \text{if } X \text{ is a fermion,} \end{cases} \quad (2.6)$$

where g_X denotes the internal degrees of freedom of the species X . In deriving this result, we have assumed $T_{\text{dec}} \gtrsim 10$ MeV, so that X decouples before neutrino decoupling. This assumption is motivated by the fact that a fully thermalized relativistic species that decouples after neutrino decoupling (or after $e^\pm$ annihilation) would typically generate a much larger ΔN_{eff} , which is already strongly constrained by the Planck 2018 measurements. For example, one thermalized sterile neutrino with $\Delta N_{\text{eff}} = 1$ is excluded at about 6σ [25]. Note that ΔN_{eff} decreases for earlier decoupling, since the decoupled species does not share in the subsequent entropy release from heavier SM ($m < T_{\text{dec}}$) particles. As a result, its temperature (and hence energy density) gets diluted relative to the photon bath.

To illustrate this more concretely, let us consider the case of right-chiral parts of light Dirac neutrinos (ν_R) that were initially in thermal equilibrium with the primordial plasma and later decouple [119, 120]. Their contribution to ΔN_{eff} as a function

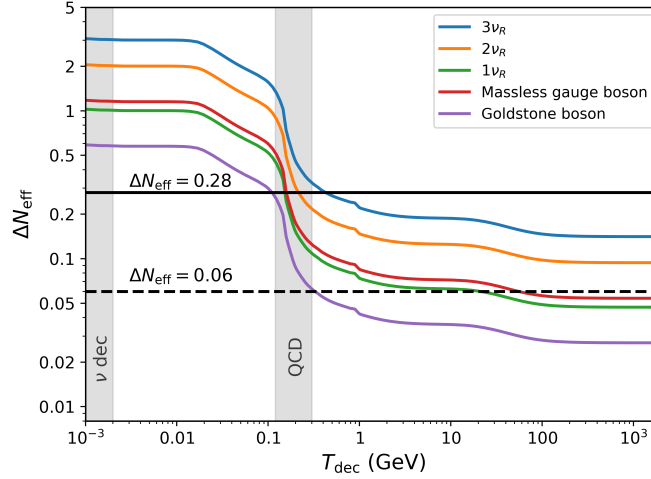


Figure 2.1: ΔN_{eff} as a function of the decoupling temperature for different light thermal species. The contributions from three, two, and one species of right-handed Dirac neutrinos are shown by the blue, orange, and green curves, respectively. The contributions from a massless gauge boson and a Goldstone boson are shown by the red and violet curves, respectively. The solid black horizontal line denotes the current Planck 2018 constraint, and the dashed black horizontal line indicates the projected sensitivity of future CMB-S4 measurements.

of the decoupling temperature is shown in Fig. 2.1. As discussed, a species contributes more to the radiation density if it decouples at a later time. Current CMB measurements imply a lower bound on the decoupling temperature, $T_{\text{dec}} \gtrsim 500$ MeV for 3 species of ν_R . Therefore, if the right-handed neutrinos decouple at temperatures well above this scale, the present constraints become increasingly insensitive. However, the anticipated sub-percent sensitivity of future CMB experiments will allow much smaller values of ΔN_{eff} to be tested, thereby probing a significantly wider range of T_{dec} . In particular, Fig. 2.1 shows that future experiments can probe essentially the entire range of decoupling temperatures for which two or three right-handed neutrino species could have *ever been in thermal equilibrium*. Moreover, the proposed CMB-HD experiment is expected to reach a sensitivity of $\Delta N_{\text{eff}} \simeq 0.014$, which would effectively rule out the entire range of decoupling temperatures for any fully thermalized relativistic species. This example highlights the significance of upcoming cosmological measurements in probing well-motivated particle physics scenarios beyond the Standard Model.

For a fully thermalized relativistic species that decouples sufficiently early, the minimal contribution to the effective number of relativistic degrees of freedom is

$\Delta N_{\text{eff}} \simeq 0.027$, as follows from Eq. (2.6) for a single bosonic degree of freedom. This estimate, however, relies on the assumption that the maximal number of relativistic degrees of freedom in the thermal plasma is given by the SM value, $g_*^{\text{SM}} = 106.75$. In non-minimal scenarios, such as supersymmetric extensions of the SM, the particle content of the plasma can be significantly enlarged at high temperatures, leading to $g_* > 106.75$. Since ΔN_{eff} scales as $g_*(T_{\text{dec}})^{-4/3}$, a larger g_* implies a stronger entropy dilution of the decoupled species and therefore a smaller late-time contribution to ΔN_{eff} . Nevertheless, achieving a suppression by a factor of two already requires $g_* \sim 200$, which typically demands a substantial number of additional heavy BSM degrees of freedom to be relativistic at the time of decoupling [121].

Another way to reduce the late-time contribution of a decoupled species is through additional entropy injection into the visible thermal bath after X decouples [122]. Such an entropy release can occur in extensions of the Λ CDM framework in which the energy density of the Universe prior to BBN is temporarily dominated by an extra component, for example, a coherently oscillating scalar field or a population of PBHs. The decay or evaporation of this dominant component reheats the plasma and dilutes the relative energy density of X , thereby lowering the inferred ΔN_{eff} . We will discuss such a scenario in Chapter 4.

Apart from the thermal contribution of dark radiation to ΔN_{eff} , a non-thermal contribution is also possible. In this case, the additional relativistic species never attain thermal equilibrium with the Standard Model plasma, but are instead produced through processes such as the decay or annihilation of other particles [120, 123]. Unlike the thermal scenario, where the minimum contribution from a fully thermalized species is typically of order $\mathcal{O}(10^{-2})$, non-thermal species can give rise to significantly smaller contributions, depending on their production mechanism and final abundance. A consistent computation of the non-thermal contribution requires solving the relevant Boltzmann equations that govern the evolution of the number and energy densities of these species, and we follow this general approach in this thesis.

A nonzero value of ΔN_{eff} is not only informative from a particle-physics perspective but can also significantly affect the ongoing Hubble tension problem [23–27]. It has been pointed out that the CMB-inferred value of the present-day Hubble constant can be increased if the expansion rate $\mathcal{H}(T)$ prior to recombination is enhanced [27]. One way to realize such a scenario is through $\Delta N_{\text{eff}} > 0$, arising from additional relativistic species. Several studies have explored mechanisms that

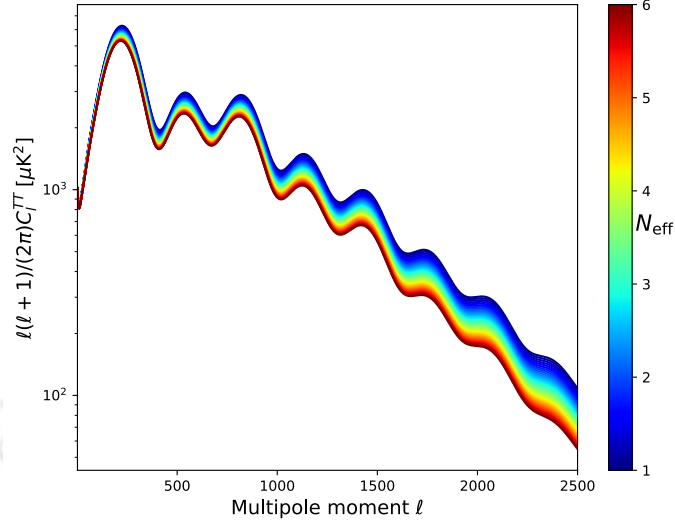


Figure 2.2: Variation of CMB temperature anisotropy spectrum due to a change of N_{eff} . Here, we fix the angular size of sound horizon.

can generate these effects, including self-interacting neutrinos [124, 125], interacting dark matter [126], and neutrino–majoron interactions [127]. It is important to note, however, that although a positive ΔN_{eff} can help alleviate the Hubble tension, it is generally insufficient on its own to fully resolve the discrepancy [128, 129].

So far, our discussion has focused on scenarios that lead to a positive contribution, $\Delta N_{\text{eff}} > 0$. However, it is also possible to obtain $\Delta N_{\text{eff}} < 0$, and such a signal would indicate non-standard dynamics in the neutrino–photon plasma, thereby pointing towards specific classes of BSM models [130–132]. One example arises when a heavy BSM particle decays into high-energy neutrinos at a time when neutrinos are still partially coupled to the electromagnetic plasma [132]. These high-energy neutrinos can transfer their energy both to the thermal neutrino background and to the electromagnetic plasma. If the energy transferred to the electromagnetic plasma exceeds that transferred to the thermal neutrinos, the inferred value of ΔN_{eff} becomes negative. Similar effects can also arise in models with non-standard neutrino interactions or in scenarios involving late-time energy injection into photons, both of which can drive ΔN_{eff} to negative values.

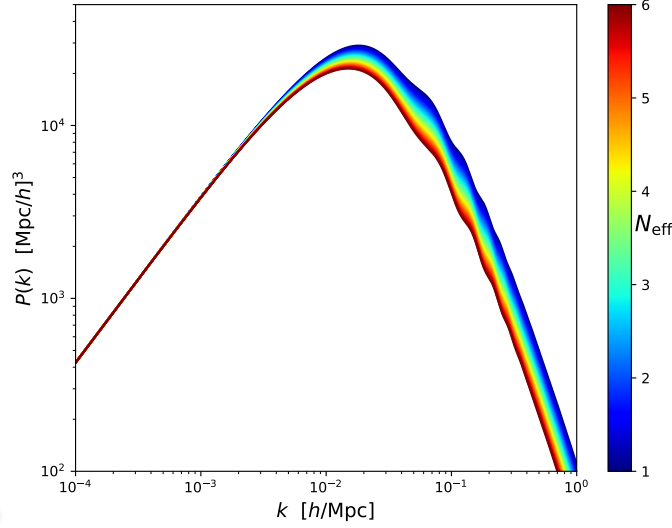


Figure 2.3: Variation of matter power spectrum due to a change of N_{eff} .

2.1.2 CMB spectrum and N_{eff}

Next, we briefly discuss the variations of CMB spectrum as a function of N_{eff} . At the background level, increasing N_{eff} increases the total radiation energy density and therefore raises the expansion rate, as described by the Friedmann equation $3M_P^2\mathcal{H}^2 = \rho_\gamma(1 + a_\nu^{-1}N_{\text{eff}})$, where M_P denotes the reduced Planck mass. If the other cosmological parameters are kept fixed, a change in N_{eff} shifts the redshift of matter–radiation equality according to [46]

$$z_{\text{eq}} = \frac{\omega_M}{\omega_\gamma(1 + a_\nu^{-1}N_{\text{eff}})}. \quad (2.7)$$

From the above discussion in section 1.1.3, we know that a change in matter–radiation equality not only alters the height of the peaks (Effect 3 in 1.1.3), but also alters the location of the peaks (Effect 1 in 1.1.3) and a tiny change in the damping scale (Effect 4 in 1.1.3). Since the location of the first acoustic peak is measured very precisely, varying N_{eff} alone would generally shift the peak positions, and one must simultaneously adjust other parameters in order to keep the angular size of the sound horizon fixed. In particular, maintaining matter–radiation equality at the same redshift requires rescaling ω_M by a factor $\alpha = (1 + a_\nu^{-1}N_{\text{eff}})/(1 + a_\nu^{-1}N_{\text{eff}}^{\text{SM}})$. As a consequence, h must be reduced by a factor $\sqrt{\alpha}$ in order to preserve the

matter–dark energy equality. With this choice, the peak locations remain approximately unchanged. Now, keeping the remaining parameters, $\omega_b, A_s, n_s, \tau_{reion}$ fixed and varying ω_M, h and N_{eff} , we obtain the temperature power spectra shown in Fig. 2.2. From the figure, it can be seen that for an increase in N_{eff} , the damping kicks at a smaller multipoles with an overall amplitude suppression in the damping tail [46, 119]. Similarly, a variation of matter power spectrum as a function of N_{eff} is shown in Fig. 2.3. We use the publicly available package CLASS to generate Figs. 2.2 and 2.3.

2.2 PBHs and its cosmological signatures

PBHs have attracted renewed interest following the first direct images of black holes [133] and the rapidly expanding catalog of gravitational-wave detections from black hole mergers by LIGO [134, 135]. PBHs are hypothetical black holes that are believed to have formed in the early Universe, well before the formation of stars and galaxies. The possibility of their formation was first proposed by Zeldovich and Novikov [136], and later explored by Hawking [137]. PBHs differ fundamentally from astrophysical black holes, which originate from the gravitational collapse of massive stellar objects. In contrast to astrophysical black holes, whose masses are restricted by stellar evolution, PBHs can span an extremely broad mass range, from as light as a few grams to masses several orders of magnitude larger than the solar mass.

Primordial black holes may form through a variety of processes operating in the early Universe, where the extremely high energy density provides favorable conditions for their production over a wide range of masses. In many scenarios, PBHs originate from the gravitational collapse of sufficiently large overdensities present in the primordial plasma [138]. PBHs form if the overdensity δ exceeds the critical value δ_c , which is estimated to be around 0.3–0.5 [139]. These overdense regions can arise from enhanced primordial inhomogeneities generated during inflation, which subsequently collapse when they re-enter the Hubble horizon. While observations of the CMB power spectrum tightly constrain the amplitude of primordial perturbations on large scales [140], they allow for the possibility of significant amplification on much smaller scales. Achieving such small-scale enhancement typically requires deviations from simple inflationary dynamics, and several mechanisms have been proposed, including inflationary models with a running spectral index [141] and inflection point

inflation scenarios [142] that feature a transient ultra-slow-roll phase. In addition to inflation-based scenarios, several non-inflationary mechanisms for PBH formation have been proposed. These include the gravitational collapse of cosmic string loops [143], scalar field instabilities [144], bubble collisions [145], phase transitions [146], etc. Comprehensive discussions of the various PBH formation mechanisms can be found in Refs. [147, 148].

For the purposes of this thesis, we remain agnostic about the specific formation mechanisms of PBHs and instead focus on their phenomenological consequences, particularly in probing scenarios beyond the Standard Model. We assume that all PBHs are formed with a monochromatic mass function, such that they share the same mass, and for simplicity, we further take them to be of Schwarzschild type, having no spin and charge. While more realistic scenarios may involve extended mass functions and non-zero spins, these introduce additional parameters. In order to isolate and highlight the essential physical effects associated with PBHs, we adopt this minimal and simplified framework.

Assuming PBHs to be formed from gravitational collapse in a radiation-dominated Universe, their initial mass is typically close to the mass enclosed by the post-inflationary particle horizon

$$M_{\text{in}} = \gamma \frac{4\pi}{3\mathcal{H}(T_{\text{in}})^3} \rho_{\text{R}}(T_{\text{in}}), \quad (2.8)$$

where T_{in} is the initial formation temperature and the factor γ is associated with the uncertainty of PBH formation with typical value of $\simeq 0.2$ [138]. The corresponding formation time and temperature can be obtained as

$$t_{\text{in}} = \frac{M_{\text{in}}}{8\pi\gamma M_P^2}, \quad (2.9)$$

$$T_{\text{in}} = \left(\frac{1440\gamma^2}{g_*(T_{\text{in}})} \right)^{1/4} \sqrt{\frac{M_P}{M_{\text{in}}}} M_P, \quad (2.10)$$

with M_P denoting the reduced Planck mass. Consequently, PBHs formed at the Planck scale can have masses as small as $\sim 10^{-5}$ g, while those formed at the MeV scale may reach masses of order $\sim 10^5 M_{\odot}$, where $M_{\odot}$ denotes the solar mass. The upper bound on inflationary scale [149, 150] provides a lower bound on the PBH

mass

$$\mathcal{H} \lesssim 6 \times 10^{13} \text{GeV} \implies M_{\text{in}}(T_{\text{in}}) \gtrsim 0.1 \text{g}. \quad (2.11)$$

Another important related quantity is the initial fraction of PBHs energy density, defined as

$$\beta = \frac{\rho_{\text{BH}}(T_{\text{in}})}{\rho_r(T_{\text{in}})}. \quad (2.12)$$

For the purposes of this thesis, the PBH mass at formation M_{in} and the initial energy fraction β are the two key parameters. In the next section, we discuss the evolution and Hawking evaporation of PBHs.

2.2.1 PBHs evolution and Hawking evaporation

Hawking, in his seminal work [151], showed that quantum mechanical effects cause black holes to emit particles with a thermal spectrum, behaving as black bodies characterized by a well-defined temperature²

$$T_{\text{BH}} = \frac{1}{8\pi G M_{\text{BH}}}. \quad (2.13)$$

The emission rate of a particle species i with mass m_i , spin s_i and momentum between p and $p + dp$ from a black hole of mass M_{BH} is obtained as [152, 153]

$$\frac{d^2 N_i}{dp dt} = \frac{g_i}{2\pi} \frac{\sigma_{si}(M_{\text{BH}}, m_i, p)}{e^{E_i(p)/T_{\text{BH}}} - (-1)^{2s_i}} \frac{p^3}{E_i(p)}. \quad (2.14)$$

The absorption cross-section σ_{si} is related to the greybody factor Γ_{si} , which describes the possible back-scattering of a particle by the effective gravitational potential, by $\Gamma_{si} = \sigma_{si} p^2 / \pi$. The greybody factor describes the possible back-scattering of a particle by the effective gravitational potential. In the high energy limit $G M_{\text{BH}} p \gg 1$, the absorption cross-section is same for all particle spins and is given by *geometrical-optics* limit $\sigma_{si} = 27\pi G^2 M_{\text{BH}}^2$. Black hole evaporation leads to the emission of a wide variety of particle species; however, the production of particles with masses $m_i > T_{\text{BH}}$ is exponentially suppressed. As the black hole loses mass during the evaporation process, its temperature T_{BH} increases, thereby allowing progressively

²In this thesis, we use M_{in} to denote the initial mass of the black hole at formation, while M_{BH} denotes the black hole mass at a later stage of its evolution.

heavier particles to be emitted. Since particle emission proceeds through gravitational interactions, black hole evaporation produces all particle species irrespective of their couplings to SM or BSM fields. As discussed in the next section, this feature enables the potential production and detection of BSM particles that interact only very feebly with Standard Model particles. A widely used public code, **BlackHawk** [154], is available for computing Hawking evaporation spectra for black holes with arbitrary mass and spin distributions.

The net mass loss rate of a black hole is given by integrating over momentum and summing over all the particles [155–157]

$$\begin{aligned}\frac{dM_{\text{BH}}}{dt} &= -\sum_i \int_0^\infty E_i \frac{d^2 N_i}{dp dt} dp \\ &\simeq -\epsilon \frac{M_P^4}{M_{\text{BH}}^2},\end{aligned}\quad (2.15)$$

where ϵ carries the information of the emitted particles and is given as

$$\epsilon = \frac{3.8 \pi g_{*,H}(T_{\text{BH}})}{480}, \quad g_{*,H}(T_{\text{BH}}) = \sum_i \omega_i g_{i,H}, \quad g_{i,H} = \begin{cases} 1.82 & \text{for } s = 0, \\ 1.0 & \text{for } s = 1/2, \\ 0.41 & \text{for } s = 1, \\ 0.05 & \text{for } s = 2, \end{cases} \quad (2.16)$$

with $\omega_i = 2s_i + 1$ for massive particles of spin s_i , $\omega_i = 2$ for massless species with $s_i > 0$, and $\omega_i = 1$ for spinless species. Integrating Eq. (2.15), PBH mass at an epoch $t > t_{\text{in}}$ can be obtained as (for $t \gg t_{\text{in}}$)

$$M_{\text{BH}}(t) = M_{\text{in}} \left(1 - \frac{3 \epsilon M_P^4}{M_{\text{in}}^3} t \right)^{1/3}. \quad (2.17)$$

The corresponding evaporation lifetime becomes

$$t_{\text{ev}} = \frac{M_{\text{in}}^3}{3 \epsilon M_P^4}. \quad (2.18)$$

Hence, a black hole of mass $M_{\text{in}} \lesssim 10^{15}$ g would have evaporated by now. Since PBHs with masses above 10^{15} g survive until the present epoch, they can potentially account for the dark matter abundance. The left side of Fig. 2.4 shows the

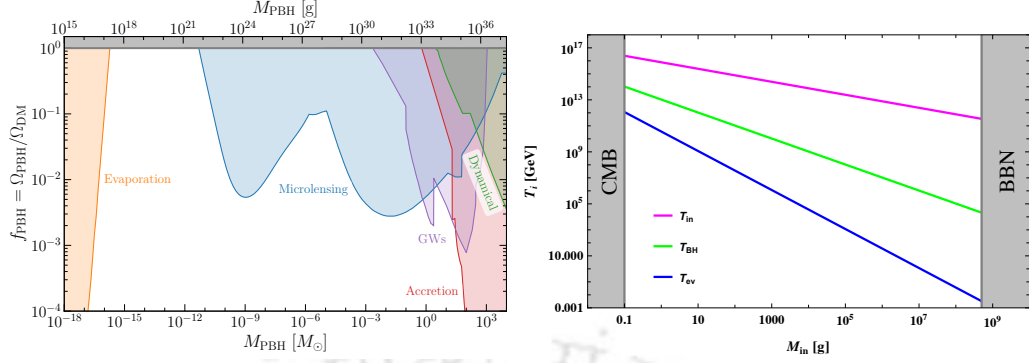


Figure 2.4: Left panel: Constraints on the PBH dark matter fraction f_{PBH} for PBH masses larger than $\sim 10^{15}$ g, taken from Ref [158]. Right panel: Associated formation, evaporation, and black hole temperatures for ultra-light PBHs. The grey shaded regions indicate the upper and lower bounds derived from BBN and CMB.

constraint on PBHs fraction $f_{\text{PBH}} = \Omega_{\text{PBH}}/\Omega_{\text{DM}}$ of total DM coming from evaporation, microlensing, gravitational waves, PBH accretion and dynamical constraints. Primordial black holes with initial masses in the range $4 \times 10^8 \text{ g} \lesssim M_{\text{in}} \lesssim 10^{15} \text{ g}$ evaporate between BBN and the present epoch. Similar constraints on the primordial black hole abundance are derived from Big Bang nucleosynthesis, cosmic microwave background observations, and galactic and extragalactic gamma-ray measurements.³

In this thesis, we focus on PBHs with initial masses in the range $0.1 \text{ g} \lesssim M_{\text{in}} \lesssim 4 \times 10^8 \text{ g}$. These PBHs form at very early times in the evolution of the Universe and evaporate well before the onset of Big Bang nucleosynthesis. The right panel of Fig. 2.4 illustrates the corresponding formation temperature, evaporation temperature, and black hole temperature. Primordial black holes with smaller initial masses form earlier and also evaporate at earlier epochs. Since such ultra-light PBHs evaporate prior to BBN, their parameter space is difficult to constrain using conventional indirect or astrophysical observations. In the following sections, we show that gravitational waves and the effective number of relativistic species, N_{eff} , can nevertheless impose significant constraints on a large portion of the parameter space of these ultra-light PBHs.

Before proceeding to the next section, we discuss the possibility that primordial black holes temporarily dominate the energy budget of the Universe prior to Big Bang nucleosynthesis. After their formation, the energy density of PBHs scales as

³See Ref. [147, 158] for a comprehensive review of these constraints.

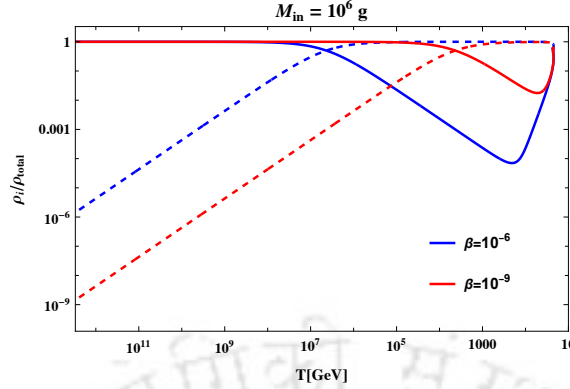


Figure 2.5: Evolution of the fractional energy densities of radiation (solid lines) and PBHs (dashed lines) as functions of temperature for $\beta = 10^{-6}$ (blue) and $\beta = 10^{-9}$ (red). The initial PBH mass is fixed to $M_{\text{in}} = 10^6$ g.

a^{-3} , whereas the radiation energy density redshifts as a^{-4} . As a consequence, the relative contribution of PBHs to the total energy density increases with time, and PBHs may dominate the energy budget of the Universe for a brief period before completely evaporating. The condition for PBH domination is $\beta > \beta_c$, where the critical value β_c is given by

$$\beta_c = \left(\frac{3\epsilon}{8\pi\gamma} \right)^{1/2} \frac{M_P}{M_{\text{in}}} \simeq 2.5 \times 10^{-14} \gamma^{-1/2} \left(\frac{10^8 \text{ g}}{M_{\text{in}}} \right). \quad (2.19)$$

Figure 2.5 illustrates the evolution of the fractional PBH energy density for two different values of β , for a fixed initial PBH mass $M_{\text{in}} = 10^6$ g. If PBHs evaporate while the Universe remains radiation-dominated, the evaporation temperature is given by

$$T_{\text{ev}} = \left(\frac{90 M_P^2}{4\pi^2 g_*(T_{\text{ev}}) t_{\text{ev}}^2} \right)^{1/4}. \quad (2.20)$$

However, if PBHs dominate the total energy density of the Universe prior to their evaporation, the Standard Model temperature immediately after complete evaporation is enhanced and is given by $\bar{T}_{\text{ev}} = \frac{2}{\sqrt{3}} T_{\text{ev}}$ [159].

Throughout this thesis, we assume that the evaporation of PBHs is well described by the semi-classical Hawking framework [151, 160] until the end of their lifetime. The semi-classical treatment neglects the backreaction of the emitted radiation on the black hole, an effect that can become important when the energy carried by the emitted quanta approaches the remaining PBH energy. It has recently been

argued that including such backreaction gives rise to a so-called *memory burden* effect, which can slow down the evaporation process and thereby prolong the lifetime of PBHs [161–163]. This modification becomes increasingly relevant once the PBHs mass drops below a certain critical scale, potentially leading to significant phenomenological implications. In this thesis, however, we do not consider these effects and restrict ourselves to the standard semi-classical evaporation picture.

2.2.2 PBHs and its signatures via N_{eff}

As discussed in the previous section, black hole evaporation proceeds through gravitational interactions and, therefore, produces all particle species, irrespective of their couplings to the Standard Model. Consequently, both SM and BSM particles can be emitted during the evaporation of a PBH. The total number of particles of species i produced during the complete evaporation of a PBH can be estimated as

$$N_i \simeq \frac{27}{128} \frac{\xi g_i \zeta(3)}{\pi^3 \epsilon} \begin{cases} \left(\frac{M_{\text{in}}}{M_P}\right)^2, & \text{for } m_i < T_{\text{BH}}^{\text{in}}, \\ \left(\frac{M_P}{m_i}\right)^2, & \text{for } m_i > T_{\text{BH}}^{\text{in}}, \end{cases} \quad (2.21)$$

where $T_{\text{BH}}^{\text{in}} = \frac{1}{8\pi G M_{\text{in}}}$ denotes the initial Hawking temperature of the black hole. In the absence of any BSM degrees of freedom, the Standard Model particles produced during the evaporation of ultra-light PBHs before BBN thermalize rapidly with the surrounding plasma and therefore do not lead to any additional contribution to N_{eff} . However, a significant contribution to N_{eff} may arise in the presence of light BSM species that either do not interact or interact only very feebly with the Standard Model [157, 164–171].

For PBH masses below $4 \times 10^8 \text{ g}$, the initial Hawking temperature exceeds 10^5 GeV . As a consequence, any light beyond-the-Standard-Model species X with vanishing or sub-eV mass is produced from the very beginning of the PBH evaporation process. To evaluate its contribution to $\Delta N_{\text{eff}} = \rho_X / \rho_{\nu_1}$, the energy density ρ_X at the time of recombination must be determined. A complete treatment requires solving the coupled Boltzmann equations governing the evolution of the energy densities of radiation, PBHs, and the species X . We present a full numerical analysis in Chapter 4, where we study an extension of this scenario. Here, instead, we provide

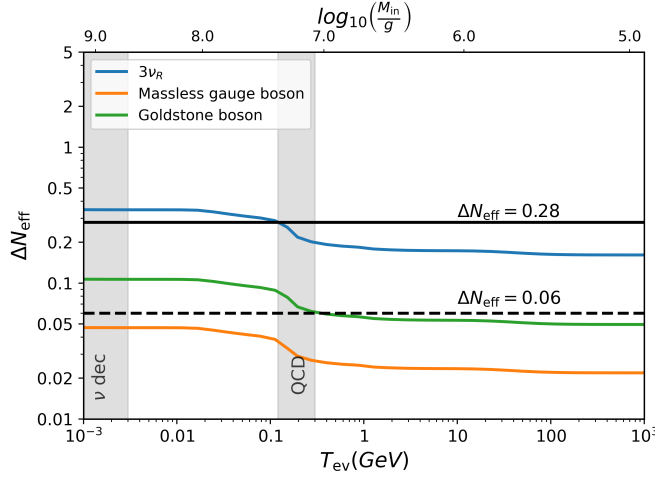


Figure 2.6: Contribution of different light BSM species to ΔN_{eff} as a function of PBH evaporation temperature (lower x axis) and PBH mass (upper x axis). The solid black horizontal line denotes the current Planck 2018 constraint, and the dashed black horizontal line indicates the projected sensitivity of future CMB-S4 measurements.

an approximate estimate of ΔN_{eff} for the case in which PBHs temporarily dominate the energy density of the early Universe. In this regime, the contribution to ΔN_{eff} is given by [157, 164, 172, 173]

$$\Delta N_{\text{eff}}^{\text{BH}} = 13.714 \times \frac{\epsilon_X(M_{\text{BH}})}{\epsilon(M_{\text{BH}})} \frac{g_*(T_{\text{ev}})}{g_{*s}(T_{\text{ev}})^{4/3}}. \quad (2.22)$$

Fig. 2.6 shows the contribution to ΔN_{eff} from three different light BSM species: namely a Goldstone boson, a massless gauge boson, and three species of light right-handed neutrinos (ν_R) as functions of the PBH mass and the evaporation temperature. Fig. 2.7 presents the resulting constraints in the β – M_{in} plane for three species of ν_R . PBHs can dominate the energy density of the Universe for the region above the solid blue line. As seen, the current bound on ΔN_{eff} already excludes a significant portion of the parameter space. Moreover, all the parameter space above the blue line can be probed by future CMB observations.

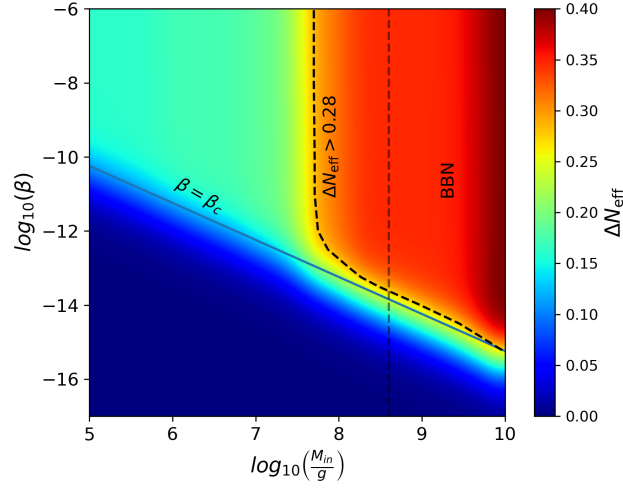


Figure 2.7: ΔN_{eff} for 3 species of ν_R in β vs M_{in} plane. The solid blue line separates the regions corresponding to PBH domination and radiation domination. The current observational bound on ΔN_{eff} is indicated by the black dashed line. The vertical grey dashed line denotes the upper PBH mass limit beyond which evaporation would affect Big Bang nucleosynthesis.

2.2.3 PBHs and its signatures via gravitational waves

Another possible signature of primordial black holes arises from observations of the stochastic gravitational wave background (SGBW) associated with PBHs. Primordial black holes may act as sources of a SGWB through several underlying physical processes. These mechanisms include: (i) the production of gravitons through Hawking evaporation, resulting in GWs at extremely high frequencies [174]; (ii) gravitational waves generated at second order from large primordial scalar fluctuations that also seed PBH formation during inflation [175]; and (iii) gravitational waves sourced by density perturbations arising from spatial clustering of PBHs after their formation [176–180].

In this work, we restrict our attention to the final mechanism, namely the generation of gravitational waves from PBH-induced scalar density fluctuations in the post-formation era. This framework is particularly compelling for several reasons. To begin with, the corresponding gravitational-wave spectrum falls within the sensitivity ranges of future space-based interferometers as well as currently operating experiments, rendering the signal potentially detectable in the near term. Furthermore, this mechanism does not depend sensitively on the detailed dynamics of PBH formation, but instead relies on the generic emergence of PBH clustering and

the resulting matter inhomogeneities. Most notably, gravitational-wave production sourced by these scalar perturbations is intimately tied to an early matter-dominated epoch triggered by PBH domination. Consequently, the associated SGWB provides a distinctive observational window into an early matter-dominated phase of the Universe. In the following, we provide a concise overview of the production of such GWs, following the analyses presented in [179, 180].

The basic mechanism of gravitational-wave production sourced by density perturbations can be summarized as follows. After their formation, PBHs are approximately randomly distributed in space and, to a good approximation, follow Poisson statistics. Consequently, although the PBH population behaves as a homogeneous matter component on large scales, significant density fluctuations arise on small scales due to their discrete nature. These PBH density fluctuations can be interpreted as early isocurvature perturbations. As the PBH abundance grows and eventually dominates the energy density, the Universe undergoes a transition from an early radiation-dominated epoch to an early matter-dominated phase. During this transition, the early isocurvature perturbations are converted into curvature perturbations. The resulting curvature perturbations then act as a source for gravitational waves at second order in cosmological perturbation theory, leading to the generation of a stochastic gravitational-wave background. It turns out that gravitational waves generated immediately after evaporation, due to the sudden jump in the time derivative of the curvature perturbation, constitute the dominant contribution.

The mean physical separation between PBHs at the time of formation can be expressed as

$$d_{\text{in}} = \frac{\gamma^{1/3}}{\beta^{1/3}\mathcal{H}(T_{\text{in}})}. \quad (2.23)$$

The density perturbation of PBHs, $\langle \delta\rho_{\text{BH}}(k)\delta\rho_{\text{BH}}(k') \rangle$, occur at a scale larger than the comoving ultra-violet (UV) scale

$$k_{\text{UV}} \equiv \frac{a}{d} = \frac{a}{d_{\text{in}}(a/a_{\text{in}})} = a_{\text{in}}\mathcal{H}(T_{\text{in}})\beta^{1/3}\gamma^{-1/3}. \quad (2.24)$$

Denoting the scale of early PBHs-radiation equality as $k_{\text{eq}}^{\text{eMD}}$, the hierarchy of the relevant scales are $k_{\text{UV}} > k_{\text{eq}}^{\text{eMD}} > k_{\text{ev}}$. The dominant contribution to the gravitational waves' amplitude is obtained from the modes $k \gg k_{\text{eq}}^{\text{eMD}}$ [179], i.e., at the small scales, the amplitudes are enhanced and peaks around the resonant scale

$k \sim k_{UV}$. Hence, the resonant amplitude at a characteristic temperature T_c during the radiation-dominated epoch after PBH evaporation, when the GW spectrum becomes constant, can be written as [179]

$$\Omega_{GW,c}^{\text{res}}(k \sim k_{UV}) \simeq \frac{1}{27648\pi\sqrt{3}} \left(\frac{k_{\text{eq}}^{\text{eMD}}}{k_{UV}} \right)^8 \left(\frac{k_{UV}}{k_{\text{ev}}} \right)^7. \quad (2.25)$$

The ratio of UV cut-off scale to the evaporation scale is

$$\frac{k_{UV}}{k_{\text{ev}}} \simeq 2.3 \times 10^6 \left(\frac{108}{g_{*,H}(T_{\text{BH}})} \right)^{1/3} \left(\frac{M_{\text{in}}}{10^4} \right)^{2/3}, \quad (2.26)$$

which increases with M_{in} . Even for PBH mass of 0.1 g, this enhancement factor can reach ~ 1000 . The ratio $\frac{k_{\text{eq}}^{\text{eMD}}}{k_{UV}}$ can be expressed in terms of β as $\frac{k_{\text{eq}}^{\text{eMD}}}{k_{UV}} \simeq \sqrt{2}\gamma^{1/3}\beta^{2/3}$. With these, the amplitude in Eq. (2.25) can be re-expressed as

$$\Omega_{GW,c}^{\text{res}}(k \sim k_{UV}) \simeq 10^{30} \beta^{16/3} \left(\frac{\gamma}{0.2} \right)^{8/3} \left(\frac{108}{g_{*,H}(T_{\text{BH}})} \right)^{17/9} \left(\frac{M_{\text{in}}}{10^4} \right)^{34/9}. \quad (2.27)$$

The total GW amplitude today in terms of frequency can then be expressed as

$$\Omega_{GW}(t_0, f) \simeq \Omega_{GW}^{\text{peak}} \left(\frac{f}{f_{\text{peak}}} \right)^{11/3} \Theta(f_{\text{peak}} - f), \quad (2.28)$$

with the peak amplitude

$$\Omega_{GW}^{\text{peak}} \simeq 2 \times 10^{-6} \left(\frac{\beta}{10^{-8}} \right)^{16/3} \left(\frac{M_{\text{in}}}{10^7 \text{g}} \right)^{34/9}, \quad (2.29)$$

and the peak frequency related to k_{UV}

$$f_{\text{peak}} \simeq 1.7 \times 10^3 \text{ Hz} \left(\frac{108}{g_{*,H}(T_{\text{BH}})} \right)^{1/6} \left(\frac{g_*(T_{\text{ev}})}{106.75} \right)^{1/4} \left(\frac{g_{*s}(T_{\text{ev}})}{106.75} \right)^{-1/3} \left(\frac{10^4 \text{g}}{M_{\text{in}}} \right)^{5/6}. \quad (2.30)$$

For PBH masses $M_{\text{in}} \gtrsim 10^4 \text{ g}$, the corresponding GWs frequencies fall within the projected sensitivity bands of next-generation detectors such as ET [181], LISA [182], Advanced LIGO [183, 184], DECIGO [185, 186], BBO [187], and CE [188]. The left panel of Fig. 2.8 displays the resulting GW spectra for several benchmark PBH

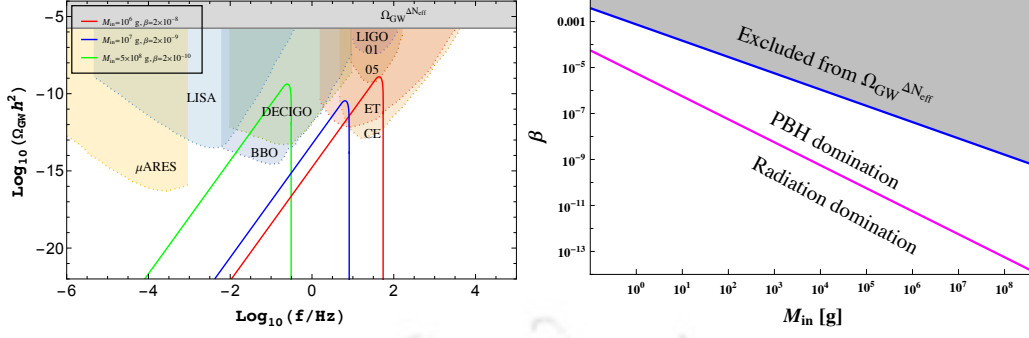


Figure 2.8: Left panel: the GWs amplitude generated by PBH density fluctuations is shown as a function of frequency for three representative PBH masses: 10^6 g (red), 10^7 g (blue), and 4×10^8 g (green). The sensitivities of various GW experiments are indicated by the shaded regions. The horizontal grey band denotes the amplitude excluded by the constraint arising from the GW contribution to ΔN_{eff} . Right panel: The parameter space excluded by the ΔN_{eff} constraint from GWs is displayed in grey. The magenta line separates the regions of PBH domination and radiation domination.

masses together with the projected experimental sensitivities. The values of β are chosen such that the predicted spectra lie within the reach of these future detectors. As indicated by Eq. (2.30), the peak frequency decreases as the PBH mass increases.

Since GWs redshift as radiation, they contribute to the total radiation energy density of the Universe and therefore to ΔN_{eff} , particularly during the epochs BBN and CMB formation. The corresponding constraint on the GWs energy density fraction at BBN (or CMB) is given by

$$\Omega_{\text{GW, BBN}} < \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \Delta N_{\text{eff}} \simeq 0.07. \quad (2.31)$$

This bound translates into the following upper limit on the present-day GW amplitude:

$$\Omega_{\text{GW}}^{\Delta N_{\text{eff}}} h^2 \simeq 1.75 \times 10^{-6}. \quad (2.32)$$

The horizontal grey band in the left panel of Fig. 2.8 indicates the region excluded by this ΔN_{eff} constraint. Combining Eqs. (2.29) and (2.32), we obtain an upper bound on the initial PBH abundance,

$$\beta \lesssim 1.1 \times 10^{-6} \left(\frac{10^4 \text{ g}}{M_{\text{in}}} \right)^{17/24}. \quad (2.33)$$

The resulting excluded parameter space is shown by the grey shaded region in the right panel of Fig. 2.8. Hence, contributions to ΔN_{eff} from both dark radiation and gravitational waves provide two complementary and powerful observational probes of ultra-light PBHs with masses $M_{\text{in}} \lesssim 4 \times 10^8 \text{ g}$.

2.3 Signatures from high-energy neutrino and cosmic-ray events

One of the promising avenues for detecting DM is via the observation of the secondary SM particles produced from dark matter annihilation or decay in astrophysical environments [189–192]. These secondary particles, which include cosmic rays, high-energy neutrinos, and gamma rays, can travel over astronomical distances and potentially be detected by terrestrial or space-based experiments, as illustrated in Fig. 2.9.

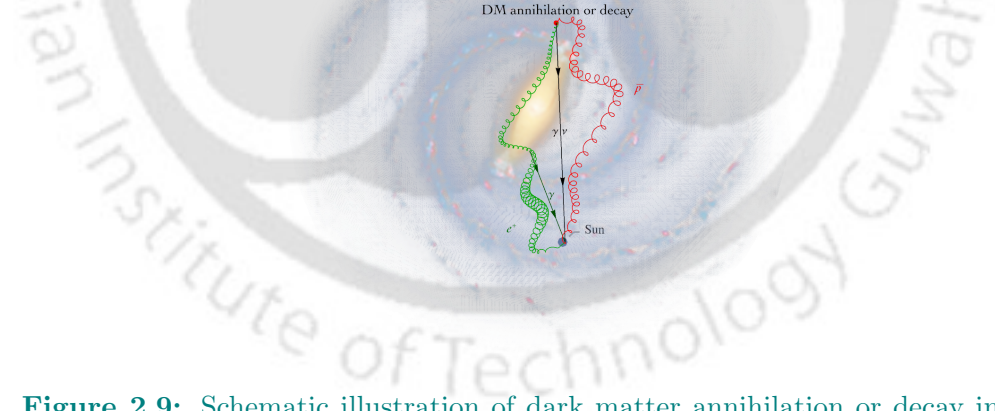


Figure 2.9: Schematic illustration of dark matter annihilation or decay in the Milky Way halo, producing Standard Model (SM) particles that propagate to the Earth as detectable messengers. Figure adapted from Ref. [192].

For the purpose of this thesis, we focus on the observable signatures such as ultra-high-energy cosmic ray (UHECR), neutrinos ν and gamma-rays arising from the decay of DM. Experiments such as IceCube [193], the Pierre Auger Observatory [194], and the Telescope Array [195] are sensitive to such extreme-energy events. The non-observation (or measured flux limits) of anomalous UHE spectra from regions

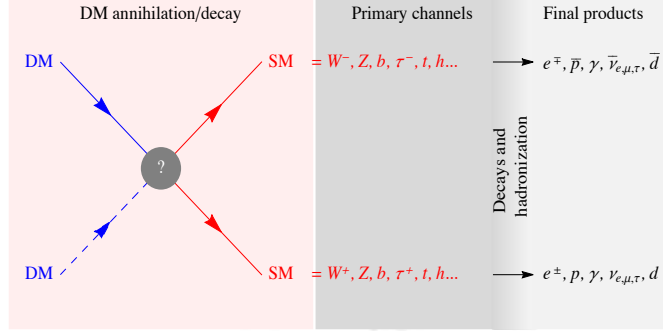


Figure 2.10: Schematic representation of DM decay/annihilation and the production of primary and secondary particles. Figure adapted from Ref. [192].

with large DM density, such as the Galactic center, places stringent constraints on the DM annihilation cross-section $\langle\sigma v\rangle$ or decay lifetime τ as a function of the DM mass m_{DM} . In particular, for superheavy and decaying DM ($m_{\text{DM}} \gtrsim 10^6\text{--}10^{10}\text{ GeV}$), the absence of excess UHE neutrinos, gamma-rays and cosmic rays implies lower bounds on the decay lifetime typically exceeding $\sim 10^{26}\text{--}10^{28}\text{ s}$ [196–199], significantly restricting the viable parameter space. UHE observations thus provide powerful and complementary probes of DM models, especially those beyond the reach of collider or direct-detection experiments. Although ultra-high-energy events can arise from both superheavy dark matter (SHDM) decay and annihilation, the annihilation flux scales with the square of the DM density and is therefore strongly suppressed for superheavy DM. For this reason, we primarily focus on the decay scenario. In this thesis, we discuss recent observations of two ultra-high-energy events and explore their possible origin in superheavy DM decay.

The decay of dark matter particles in astrophysical environments can produce Standard Model states through two-body processes of the form $1 \rightarrow 2$, e.g., $\text{DM} \rightarrow \bar{X}X$. The primary decay products subsequently undergo a sequence of secondary processes, including further decays, parton showering, and hadronization, leading to the production of stable SM particles such as protons, gamma rays, and neutrinos. A schematic representation of this sequence is shown in Fig. 2.10. To predict the observable signals at Earth, a key ingredient is the differential energy spectrum of each stable species, dN_i/dE , which characterizes the number of particles produced per decay per unit energy. Here, the index i labels the final-state species, in particular protons (p), gamma rays (γ), and neutrinos (ν). These spectra serve as the primary inputs for calculating the corresponding fluxes. The expression for

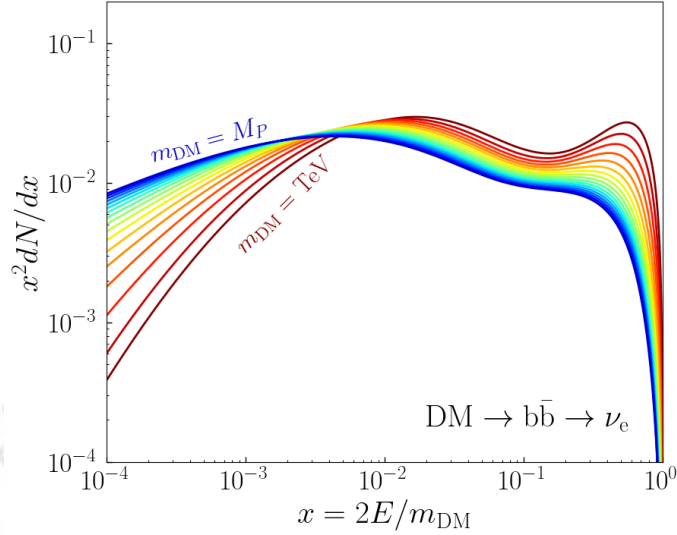


Figure 2.11: Energy spectra $x^2 dN/dx$ for ν_e as a function of x produced from DM decay to b quark. DM mass is varied from TeV scale to the Planck scale.

differential energy spectra for DM mass m_{DM} is given by [200]

$$\frac{dN_i}{dx} = \frac{1}{\Gamma_0} \frac{d\Gamma}{dx} (\text{DM} \rightarrow i + \dots) , \quad (2.34)$$

where $x = 2E/m_{\text{DM}}$ and $\Gamma_0 = 1/\tau_{\text{DM}}$ with τ_{DM} being the lifetime of DM. Γ is the inclusive decay rate of $\text{DM} \rightarrow \bar{X}X \rightarrow i + \dots$ [200]. The calculation of differential energy spectra requires careful treatment of primary particles' decays, showering, and hadronization, which are dictated by SM physics. Several public Monte Carlo event generators, such as **Pythia** [201] and **Herwig** [202], are widely used for this purpose. In this thesis, we compute the differential energy spectra using the **HDMSpectra** package [200], which is based on **Pythia** and is specifically designed for high-energy dark matter decay and annihilation processes and provides reliable predictions for the stable final-state yields. The **HDMSpectra** package enables the computation of particle energy spectra over a broad dark matter mass range, extending from the electroweak scale up to the grand unification (GUT) scale. The spectra are obtained using fragmentation functions, $D_a^b(x; \mu_Q, \mu_0)$, which describe the probability that an initial state a produced at the hard scale μ_Q evolves into a final state particle b at the lower scale μ_0 . For example, an initial partonic state such as $X\bar{X}$ can fragment into stable hadronic products like $p + \bar{p}$ (see [200] for details).

Fig. 2.11 shows the scaled energy spectra, $x^2 dN/dx$, as a function of x , for several dark matter masses. The spectra correspond to a specific decay channel in

which the dark matter particle decays into a $b\bar{b}$ final state. In order to calculate the flux of i^{th} secondary particle, one needs to sum over the differential spectra for all the possible decay channels e.g. $\text{DM} \rightarrow b\bar{b}, W^+W^-, \mu^+\mu^-, \nu\bar{\nu}$. The different flux of i^{th} secondaries from a given angular direction $d\Omega$ from galactic DM is given as

$$\frac{d^2\phi_{i,G}(E_i)}{dE_i d\Omega} = \frac{\mathcal{D}}{4\pi m_{\text{DM}}\tau_{\text{DM}}} \frac{dN_i(E_i)}{dE_i}, \quad (2.35)$$

where

$$\mathcal{D} = \frac{1}{\Delta\Omega} \int_{\Delta\Omega} d\Omega \int_0^{s_{\text{max}}} ds \rho_{\text{DM}}(s, b, l). \quad (2.36)$$

Here ρ_{DM} is the DM density in the Milky Way, l, b are the Galactic longitude and latitude, and s denotes the line-of-sight coordinate. The differential solid angle, $d\Omega$ is given by $d\Omega = d(\sin b)dl$. These coordinates are related as $r(s, b, l) = \sqrt{s^2 + r_{\odot}^2 - 2sr_{\odot} \cos b \cos l}$, where $r_{\odot} = 8.3$ kpc is the distance to Milky Way center from the Sun.

The DM density in the Milky Way is characterized by several different DM profiles available in the literature [203–207]. In this thesis, we primarily consider the standard Navarro-Frenk-White (NFW) density profile, which as a function of the Galactocentric radius, R_{GC} given by,

$$\rho_{\text{NFW}}(R_{\text{GC}}) = \frac{\rho_{\text{ch}}}{(R_{\text{GC}}/R_{\text{ch}})(1 + R_{\text{GC}}/R_{\text{ch}})^2}. \quad (2.37)$$

Here ρ_{ch} is the DM density at the characteristic scale $R_{\text{c}} = 11$ kpc, and its value is obtained by normalizing ρ_{NFW} with the DM density in the solar neighbourhood, $\rho_{\odot} = 0.43 \text{ GeV cm}^{-3}$. The other DM profiles like Einasto [205] and Burkert [207] also predict a similar DM density except at the Galactic center.

In addition to the Galactic component, DM decay in the extra-galactic medium also produces secondary SM particles through the same decay channels, giving rise to a diffuse cosmological flux. For gamma rays, this contribution is largely suppressed due to absorption by the extra-galactic background light (EBL) and CMB. In contrast, neutrinos propagate essentially without attenuation over cosmological distances, making the extra-galactic component particularly relevant for the neutrino flux. The differential flux of extra-galactic secondaries of species i is given

by

$$\frac{d\phi_{i,\text{EG}}(E)}{dE} = \frac{\rho_{\text{DM}}}{4\pi M_{\text{DM}} \tau_{\text{DM}}} \int dz \left| \frac{dt}{dz} \right| \frac{dN(E')}{dE'} e^{-\tau(E',z)}, \quad (2.38)$$

where $E' = E(1+z)$ accounts for redshift effects, $\rho_{\text{DM}} = \Omega_{\text{DM}}\rho_c$ is the present DM density, and $\rho_c = 4.7 \times 10^{-6} \text{ GeV cm}^{-3}$ is the critical density today. The attenuation factor $e^{-\tau(E',z)}$ incorporates gamma-ray absorption, with $\tau(E',z)$ denoting the total optical depth due to interactions with the EBL and CMB [208].

The total fluxes of UHE cosmic rays, gamma rays, and neutrinos arising from both Galactic and extra-galactic DM decay can be directly compared with observational data to constrain the DM mass m_{DM} and lifetime τ_{DM} . In the following, we briefly review the capabilities of past, current, and future experiments relevant for the detection of UHECRs, UHE gamma rays, and UHE neutrinos.

UHECR telescopes and measurements: UHECRs were first observed by the Akeno Giant Air Shower Array (AGASA) [209] and the High Resolution Fly’s Eye (HiRes) experiments [210, 211] several decades ago. In particular, HiRes reported events with energies up to 320 EeV [212], which remain among the most energetic cosmic rays detected to date. More recent measurements of the UHECR spectrum above 0.1 EeV by the Pierre Auger Observatory (PAO) [213] and the Telescope Array (TA) [214] are broadly consistent with these earlier results, although some differences persist in the inferred mass composition and spectral features at the highest energies. Future experiments such as the Giant Radio Array for Neutrino Detection (GRAND) [215] are expected to further improve the sensitivity and provide complementary information on UHECR sources.

UHE gamma-ray telescopes: So far, no confirmed detection of UHE gamma rays in the PeV–EeV energy range has been reported, apart from a few Galactic sources slightly above PeV energies [216]. Instead, several experiments, including Haverah Park [217], AGASA [218], Yakutsk [219, 220], AGASA+Yakutsk [221, 222], TA [223, 224], and PAO [225, 226], have placed upper limits on the diffuse UHE gamma-ray flux. Future facilities such as GRAND 200k and the proposed upgrade of the PAO surface detector (SD) [227] are expected to significantly enhance the sensitivity, enabling either detection or more stringent constraints.

UHE neutrino telescopes: Apart from the recent KM3-230213A event [228], no other UHE neutrinos have been conclusively detected by operating instruments such as the Antarctic Impulsive Transient Antenna (ANITA) [229], IceCube [230], TA [195], and PAO [194]. Both theoretical predictions [231–242] and current experimental limits [194, 243–245] often lie below the sensitivities of several existing detectors, including the Askaryan Radio Array (ARA) [246], the Radio Ice Cherenkov Experiment (RICE) [247], and the Square Kilometer Array (SKA) [248, 249]. Nevertheless, next-generation experiments such as IceCube-Gen2 [230] and GRAND 200k [215, 250] will achieve substantially improved sensitivities and offer promising prospects for the detection of UHE neutrinos.

2.4 Objective of the thesis

The thesis examines a class of well-motivated BSM frameworks that remain largely inaccessible to conventional collider and direct-detection searches, focusing instead on their cosmological imprints. We analyze the resulting signatures in observables such as the CMB, stochastic gravitational wave backgrounds, and high-energy astrophysical measurements, with special emphasis on models involving dark matter, Dirac and Majorana neutrinos, and ultra-light PBHs.

In Chapter 3, we study the simultaneous production of dark radiation in the form of Dirac neutrinos and dark matter originating from a common source within a simple BSM scenario. We investigate the implications of this framework for ΔN_{eff} and the free-streaming length of dark matter. The scenario is subsequently extended to a gauged $B - L$ model in order to construct a UV-complete realization and to explore its phenomenological consequences.

In Chapter 4, we extend the discussion of Sec. 2.2 by incorporating thermalized dark radiation in the presence of ultra-light PBHs and examine their potential signatures in future CMB and GW observations. We further consider a scenario in which dark radiation is in the form of axions, a hypothetical particle arising from the Peccei-Quinn solution to the strong CP problem discussed earlier. The same framework can also explain matter-antimatter asymmetry via leptogenesis. The otherwise untestable vanilla leptogenesis framework can become experimentally accessible by

linking its origin to the PQ symmetry-breaking scale. This setup is analyzed in the presence of ultra-light PBHs, and its possible detection prospects are explored.

In Chapter 5, we shift our focus from early-Universe signatures to late-time observables and investigate dark matter decay and its possible role in explaining recently observed high-energy events such as the Amaterasu event [251] and KM3-230213A [228]. We also derive updated constraints on the decaying dark matter parameter space using these observations.

Finally, in Chapter 6, we summarize the main results of the thesis and outline several directions for future research.



Cosmological Signatures of Dirac Neutrino Portal Dark Matter

In this chapter, we discuss a unified framework that simultaneously addresses the origin of neutrino masses and DM, together with its cosmological implications. Although neutrinos can, in principle, be either Dirac or Majorana particles, our discussion focuses on the Dirac neutrino scenario. In the Dirac neutrino portal dark matter (DNPDM) framework, light right-handed Dirac neutrinos mediate the interactions between the DM and the SM thermal bath. Since these right-handed neutrinos remain relativistic in the early Universe, they can contribute to N_{eff} , thereby providing a potential cosmological probe. Light Dirac neutrino models often lead to enhanced ΔN_{eff} , some recent works on which can be found in [120, 123, 173, 252–267]. Moreover, as DM is produced through the same processes that generate ν_R , the requirement of obtaining the correct relic abundance is directly correlated with the resulting contribution to N_{eff} . We also discuss the implications of structure-formation constraints on the DM parameter space and discuss how these limits affect the overall detectability of the scenario.

The chapter is mainly divided into two sections. In the first section, we discuss a minimal DNPDM model where we have a fermionic singlet DM ψ , right-handed neutrinos ν_R , and a complex scalar singlet ϕ . The details of this study are presented in Section 3.1 and are based on the published paper [268]. In the next section 3.2,

we discuss the UV completer of the above scenario in a gauged $B - L$ framework, which is based on the published paper [269].

3.1 Freeze-in Dark Matter via Light Dirac Neutrino Portal

3.1.1 The minimal setup

Here we consider only three types of BSM particles sufficient to highlight the interesting phenomenology. They are namely, right handed neutrinos ν_R , fermion singlet DM ψ and a complex scalar singlet ϕ transforming non-trivially under an unbroken discrete $\mathbb{Z}_4$ symmetry. The right handed neutrinos couple to left handed lepton doublets via SM Higgs with fine-tuned Dirac Yukawa couplings to generate sub-eV Dirac neutrino masses. All SM leptons as well as ν_R have $\mathbb{Z}_4$ charge i which keep the Majorana mass terms away. The $\mathbb{Z}_4$ charges of ψ, ϕ are chosen to be $-1, i$ respectively which ensures DM has only one tree level coupling of the form $y_\phi \bar{\psi} \nu_R \phi$. On the other hand, ν_R, ϕ can have other couplings as well. For example ν_R couples to SM lepton doublet ℓ_L and Higgs H as $y_H \bar{\ell}_L \tilde{H} \nu_R$. On the other hand, the scalar singlet ϕ can have quartic interactions with the SM Higgs as $\lambda_{H\phi} (H^\dagger H)(\phi^\dagger \phi)$. Thus, the Lagrangian involving the newly introduced fermions can be written as

$$\mathcal{L}_F = i \bar{\nu}_R \gamma^\mu \partial_\mu \nu_R + i \bar{\psi} \gamma^\mu \partial_\mu \psi - m_\psi \bar{\psi} \psi - \left(y_H \bar{\ell}_L \tilde{H} \nu_R + y_\phi \bar{\psi} \nu_R \phi + \text{h.c.} \right). \quad (3.1)$$

Similarly, the scalar Lagrangian of the model is

$$\begin{aligned} \mathcal{L}_S = & (D_{H\mu} H)^\dagger (D_H^\mu H) + (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \left[-\mu_H^2 (H^\dagger H) + \lambda_H (H^\dagger H)^2 + \mu_\phi^2 (\phi^\dagger \phi) \right. \\ & \left. + \lambda_\phi (\phi^\dagger \phi)^2 + \lambda_{H\phi} (H^\dagger H)(\phi^\dagger \phi) + \lambda'_\phi (\phi^4 + (\phi^\dagger)^4) \right], \end{aligned} \quad (3.2)$$

where, the covariant derivative for H is defined as

$$D_{H\mu} H = \left(\partial_\mu + i \frac{g_2}{2} \tau_a W_\mu^a + i \frac{g_1}{2} B_\mu \right) H. \quad (3.3)$$

Since $\mathbb{Z}_4$ needs to remain unbroken, the singlet scalar does not acquire any VEV. After spontaneous symmetry breaking, the physical masses of the scalars can be written as

$$m_h^2 = 2\lambda_H v^2, \quad m_\phi^2 = \mu_\phi^2 + \frac{1}{2}v^2\lambda_{H\phi}. \quad (3.4)$$

While Dirac Yukawa coupling y_H remains suppressed from neutrino mass criteria, without much relevance to the phenomenology of DM and ΔN_{eff} , the two other couplings namely, $y_\phi, \lambda_{H\phi}$ play crucial roles along with the masses of ϕ, ψ denoted by m_ϕ, m_ψ respectively. Therefore, the relevant free parameters of this model are the following couplings and the masses,

$$m_\phi, m_\psi, y_\phi, \lambda_{H\phi}. \quad (3.5)$$

Since both DM and ν_R will be dominantly produced from ϕ , it is important to track the evolution of ϕ in the early universe. Depending upon coupling of ϕ with SM Higgs and its mass m_ϕ , production of DM, ν_R can occur while ϕ is either in equilibrium or out of equilibrium. In order to discuss our results in details, we consider three different scenarios and write the corresponding Boltzmann equations as follows. For the detailed derivations of the Boltzmann equations for each of these scenarios, please refer to appendix A.1.

Case I: (ϕ in equilibrium) In this case, ϕ remains in equilibrium with the SM bath during DM and ν_R production from ϕ decay. Thus ϕ abundance can be considered to be its equilibrium abundance throughout while for the other two species ψ, ν_R , the relevant Boltzmann equations, in terms of comoving number densities of ϕ and ψ , and comoving energy density of ν_R , are given by

$$\frac{dY_\psi}{dx} = \frac{\beta'}{x\mathcal{H}} \Gamma_\phi \frac{K_1(x)}{K_2(x)} Y_\phi^{\text{eq}}, \quad (3.6)$$

$$\frac{d\tilde{Y}}{dx} = \frac{\beta'}{\mathcal{H}s^{1/3}x} \langle E\Gamma \rangle Y_\phi^{\text{eq}}, \quad (3.7)$$

where $x = m_\phi/T$ and

$$\begin{aligned}
 \beta' &= \left[1 + \frac{T dg_{*s}/dT}{3g_{*s}} \right], \\
 \Gamma_\phi &= \frac{g_\psi g_{\nu_R}}{16\pi m_\phi} |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 \left(1 - \frac{m_\psi^2}{m_\phi^2} \right) \\
 \langle E\Gamma \rangle &= g_\psi g_{\nu_R} \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 (m_\phi^2 - m_\psi^2)^2}{32\pi m_\phi^4}, \\
 |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 &= \frac{1}{g_\phi g_\psi g_{\nu_R}} y_\phi^2 (m_\phi^2 - m_\psi^2).
 \end{aligned} \tag{3.8}$$

While the comoving number density $Y_\psi = n_\psi/s$, the comoving energy density of ν_R which remains relativistic during the CMB formation, is defined in terms of its energy density as $\tilde{Y} = \rho_{\nu_R}/s^{4/3}$.

Case II: (freeze-out of ϕ) For certain choices of model parameters, one can have a scenario where ϕ gets thermally produced first followed by its freeze-out and only after that dominant production of DM and ν_R take place from decay of ϕ . Since ϕ can no longer be taken to be in equilibrium throughout, we need to track its evolution using the corresponding Boltzmann equation. The system of Boltzmann equations in this case is given by

$$\frac{dY_\phi}{dx} = \frac{\beta' s}{\mathcal{H}x} \left(-\langle \sigma v \rangle_{\phi\phi^\dagger \rightarrow X\bar{X}} ((Y_\phi)^2 - (Y_\phi^{\text{eq}})^2) - \frac{\Gamma_\phi}{s} \frac{K_1(m_\phi/T)}{K_2(m_\phi/T)} Y_\phi \right), \tag{3.9}$$

$$\frac{dY_\psi}{dx} = \frac{\beta'}{x\mathcal{H}} \Gamma_\phi \frac{K_1(x)}{K_2(x)} Y_\phi, \tag{3.10}$$

$$\frac{d\tilde{Y}}{dx} = \frac{\beta'}{\mathcal{H}s^{1/3}x} \langle E\Gamma \rangle Y_\phi. \tag{3.11}$$

Here $\langle \sigma v \rangle_{\phi\phi^\dagger \rightarrow X\bar{X}}$ is the thermally averaged annihilation cross-section [81, 270] of ϕ into the SM particles via Higgs portal interactions. These include the contact interaction of ϕ with the Higgs (h) along with all other Higgs portal interactions $\phi\phi^\dagger \rightarrow f\bar{f}, VV, hh$, where f denotes the SM fermions (quarks and leptons) and V denotes the SM gauge bosons. The definition of other parameters remain same as in case I discussed earlier.

Case III: (non-thermal ϕ) Finally, we consider the remaining possibility where ϕ can be out-of-equilibrium throughout due to tiny couplings with the SM Higgs. Thus, the initial abundance of ϕ remains negligible, like FIMP DM, and then it starts to populate the universe due to decay or annihilation of SM bath particles. Since ϕ has only Higgs portal couplings, the relevant production mechanism is from Higgs decay or Higgs annihilation depending upon m_ϕ . The distribution function for ϕ can be calculated by solving the following equation

$$\frac{\partial f_\phi}{\partial t} - \mathcal{H} p_1 \frac{\partial f_\phi}{\partial p_1} = C^{h \rightarrow \phi \phi^\dagger} + C^{hh \rightarrow \phi \phi^\dagger} + C^{\phi \rightarrow \bar{\nu}_R \psi}, \quad (3.12)$$

the details of the collision terms on the RHS are given in Appendix A.1. Once the distribution function f_ϕ is evaluated, it can be used to find the evolution of DM and ν_R densities by solving the following Boltzmann equations

$$\begin{aligned} \frac{dY_\psi}{dr} &= \frac{g_\phi \beta'}{r \mathcal{H} s} \frac{\Gamma_\phi m_\phi}{2\pi^2} \int \frac{(\mathcal{A}_{\frac{m_0}{r}})^3 \xi^2 f_\phi(\xi, r)}{\sqrt{(\xi \mathcal{A}_{\frac{m_0}{r}})^2 + m_\phi^2}} d\xi, \\ \frac{d\tilde{Y}}{dr} &= \frac{g_\phi \beta'}{r \mathcal{H} s^{4/3}} \langle E \Gamma \rangle \frac{1}{2\pi^2} \int_0^\infty \left(\mathcal{A}_{\frac{m_0}{r}} \right)^3 \xi^2 f_\phi(\xi, r) d\xi, \end{aligned} \quad (3.13)$$

where $r = m_0/T$ with m_0 being an arbitrary mass scale and details of $\mathcal{A}, \xi$ are given in Appendix A.1.

3.1.2 Structure formation constraints

Fermion DM with mass roughly below a keV is ruled out from galactic phase space arguments [271, 272]. This implies that a fermion DM with mass above a keV can still allow, in principle, the formation of structures as we observe in the universe. However, such generic lower bound on fermion DM mass based on phase space arguments, can become more stringent depending upon the production mechanism of DM. Such bounds can be imposed on a particular DM scenario by calculating the free-streaming length (FSL) of DM. While hot DM is already ruled out, warm DM with FSL $\lambda_{\text{FSL}} < 0.1$ Mpc is still allowed, and can be favourable over cold DM of FSL $\lambda_{\text{FSL}} < 0.01$ Mpc due to the small-scale structure problems associated with the latter [273]. Dark matter free-streaming length can be estimated from matter power spectrum inferred from the Lyman- α forest data [274, 275]. This has been

done in several earlier works including [276]. Quasar data have also been used to for studying free-streaming properties of DM [277]. For theoretical and simulation based studies of dark matter free-streaming properties, one may refer to [278–281]. For some recent discussions on structure formation constraints on DM production mechanisms, please see [282–284] and references therein.

The free-streaming length can be defined as

$$\lambda_{\text{FSL}} = \int_{T_{\text{prod}}}^{T_{\text{eq}}} \frac{\langle v(T) \rangle}{a(T)} \frac{dt}{dT} dT, \quad (3.14)$$

where T_{eq} is the temperature of the universe at the time of matter-radiation equality while T_{prod} denotes the temperature during maximum production of DM. The average velocity of DM ($\langle v(T) \rangle$) at a temperature T can be expressed as

$$\langle v(T) \rangle = \frac{\int \frac{p_1}{E_1} \frac{d^3 p_1}{(2\pi)^3} f_\psi(p_1, T)}{\int \frac{d^3 p_1}{(2\pi)^3} f_\psi(p_1, T)}. \quad (3.15)$$

Here p_1 is the momentum of DM particle ψ having energy E_1 . The above integration over p_1 are for all possible values of the momentum (p_1) of ψ . In terms of two new variables ξ_ψ and r , the above definition of $\langle v \rangle$ becomes

$$\langle v(r) \rangle = \frac{\mathcal{A}(r)}{\int \xi_\psi^2 f_\psi(\xi_\psi, r) d\xi_\psi} \times \int \frac{\xi_\psi^3 f_\psi(\xi_\psi, r) d\xi_\psi}{\sqrt{(\mathcal{A}(r)\xi_\psi)^2 + (\frac{r}{m_0} m_\psi)^2}}. \quad (3.16)$$

The function $\mathcal{A}(r)$ is defined in Appendix A.1 with m_0 being a reference mass scale, considered to be 125 GeV in our analysis. Now, in terms of r the above definition of FSL takes the following form

$$\lambda_{\text{FSL}} = \left(\frac{11}{43} \right)^{1/3} r_0 \int_{r_{\text{prod}}}^{r_{\text{eq}}} \langle v(r) \rangle g_{*s}^{1/3} \frac{\beta'}{H(r)} \frac{dr}{r^2}. \quad (3.17)$$

Therefore, in order to calculate the free-streaming length of dark matter ψ , we first need to find the distribution function $f_\psi(\xi_\psi, r)$. The non-thermal distribution function of ψ depends mostly on two factors. One of the factors is the momentum distribution of the parent particle ϕ while the rest is the production mechanism of ψ from the parent ϕ . In our case, ψ can be produced from the decay of ϕ as the decay is always kinematically allowed. The Boltzmann equation for f_ψ due to the

process $\phi(K_1) \rightarrow \psi(P_1) + \bar{\nu}_R(P_2)$ is given by

$$\frac{\partial f_\psi}{\partial t} - \mathcal{H} p_1 \frac{\partial f_\psi}{\partial p_1} = \frac{1}{16\pi E_{p_1} p_1} \int_{k_1^{\min}}^{k_1^{\max}} \frac{k_1 dk_1}{E_{k_1}} |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 f_\phi(k_1). \quad (3.18)$$

where,

$$\begin{aligned} k_1^{\min} &= \frac{1}{2m_\psi^2} \left[-p_1(m_\phi^2 + m_\psi^2) + \sqrt{p_1^2(m_\phi^2 + m_\psi^2)^2 - m_\psi^2 \{4p_1^2 m_\phi^2 - (m_\phi^2 - m_\psi^2)^2\}} \right], \\ k_1^{\max} &= \frac{1}{2m_\psi^2} \left[p_1(m_\phi^2 + m_\psi^2) + \sqrt{p_1^2(m_\phi^2 + m_\psi^2)^2 - m_\psi^2 \{4p_1^2 m_\phi^2 - (m_\phi^2 - m_\psi^2)^2\}} \right], \end{aligned} \quad (3.19)$$

and when $m_\phi \gg m_\psi$, the above limits on k_1 reduce to the following simplified forms

$$k_1^{\min} \simeq \frac{m_\phi^2}{2m_\psi^2} \left(-p_1 + \sqrt{p_1^2 - 4 \frac{m_\psi^2}{m_\phi^2} p_1^2 + m_\psi^2} \right), \quad (3.20)$$

$$k_1^{\max} \simeq \frac{m_\phi^2}{2m_\psi^2} \left(p_1 + \sqrt{p_1^2 - 4 \frac{m_\psi^2}{m_\phi^2} p_1^2 + m_\psi^2} \right). \quad (3.21)$$

Now for

- Case I: $f_\phi(k_1) = e^{-E_{k_1}/T}$,
- Case II: we can find $f_\phi(k_1)$ after the freeze-out of ϕ by using

$$\frac{\partial f_\phi}{\partial t} - \mathcal{H} k_1 \frac{\partial f_\phi}{\partial k_1} = C^{\phi \rightarrow \psi \bar{\nu}_R}, \quad (3.22)$$

- Case III: we can find $f_\phi(k_1)$ by using

$$\frac{\partial f_\phi}{\partial t} - \mathcal{H} k_1 \frac{\partial f_\phi}{\partial k_1} = C^{h \rightarrow \phi \phi^\dagger} + C^{hh \rightarrow \phi \phi^\dagger} + C^{\phi \rightarrow \bar{\nu}_R \psi}. \quad (3.23)$$

Once we find f_ϕ from the above equations, we can use that to find f_ψ which we can use again to find thermal average velocity and free-streaming length. We can also cross-check the numerical calculations by obtaining $n_\phi = g_\phi \int \frac{d^3 k_1}{(2\pi)^3} f_\phi$ and $n_\psi = g_\psi \int \frac{d^3 p_1}{(2\pi)^3} f_\psi$ and comparing it with the previous sub-section's results. Note that the same expression for n_ψ also appear in the denominator for the expression of $\langle v(T) \rangle$.

We will discuss the results for free-streaming length for each case together with DM and ΔN_{eff} results in the upcoming sub-section.

3.1.3 Numerical Results

In this sub-section, we discuss our numerical results for all the three cases mentioned above. After solving the Boltzmann equations for comoving densities of dark sector species, we can find the observable quantities like DM abundance $\Omega_{\text{DM}} h^2$ and ΔN_{eff} . Since the region of validity for these three cases crucially depends upon the parameters involving complex scalar singlet ϕ , we first show the parameter space in terms of its mass and Higgs portal couplings in left panel of Fig. 3.1 indicating the region excluded by the constraints from the LHC on invisible decay of the SM Higgs boson into a pair of ϕ . The ATLAS and the CMS collaboration have put the limit on invisible Higgs branching ratio as $\text{BR}_{h \rightarrow \text{inv}} < 14.6\%$ [285] and $\text{BR}_{h \rightarrow \text{inv}} < 18\%$ [286] respectively, of which we use the stronger ATLAS bound in the left panel of Fig. 3.1. In the right panel of Fig. 3.1, we show the interaction rate of ϕ (Γ) in comparison to the Hubble expansion rate for three benchmark values of $m_\phi, \lambda_{H\phi}$ to indicate typical Higgs portal couplings required to consider thermal production of ϕ in the early universe. Clearly, for Higgs portal coupling $\lambda_{H\phi} \leq 10^{-8}$ validates the non-thermal nature of ϕ as we consider while discussing details of case III. In the following, we will choose the benchmark points as well as the scan range while keeping Fig. 3.1 in mind.

In addition to bounds on $\Omega_{\text{DM}} h^2, \Delta N_{\text{eff}}$ and $(m_\phi, \lambda_{H\phi})$ plane mentioned above, we also note the model independent bounds on DM mass. If DM is very light, it can remain relativistic for a long time after being produced from ϕ decay resulting in large free-streaming length. While hot dark matter is ruled out, a warm dark matter (WDM) component is still allowed provided certain bounds are satisfied. Depending upon the details of production mechanism, warm dark matter mass below a few keV is ruled out as shown in several works incorporating different observations [279, 287, 288]. Coincidentally, similar lower bound exists on fermion DM mass from galactic phase space arguments [271, 272]. While these lower bounds can vary slightly depending upon the production scenario and observational constraint imposed, we consider a lower bound of $\mathcal{O}(1)$ keV in our analysis. We also consider a conservative upper bound on ϕ lifetime such that its decay is complete before the

BBN epoch $T_{\text{BBN}} \sim \mathcal{O}(10)$ MeV. This ensures the production of dark matter as well as dark radiation before the onset of BBN epoch.

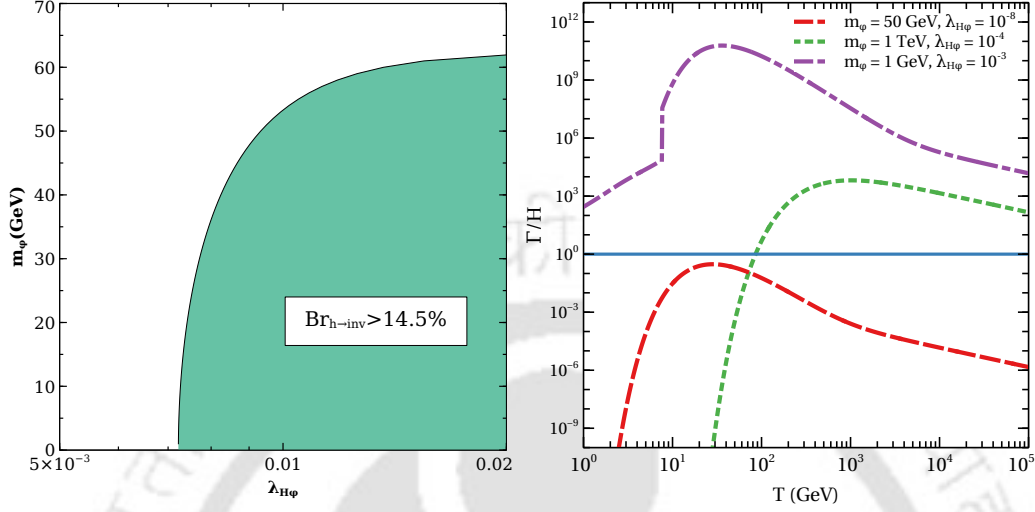


Figure 3.1: Left panel: LHC constraint in $m_\phi - \lambda_{H\phi}$ plane showing the region excluded by upper limit on invisible decay width of the SM Higgs. Right panel: Interaction rates of ϕ in comparison to the Hubble expansion rate for benchmark choices of $m_\phi - \lambda_{H\phi}$ used in our analysis.

Case I In this case, ϕ remains in equilibrium while DM and ν_R production takes place. This is the simplest scenario where we need to solve only two coupled Boltzmann equations for ψ, ν_R while using equilibrium abundance for ϕ throughout. Left panel of Fig. 3.2 shows the evolution of dark sector particles as functions of temperature for different sets of parameters. The magenta, blue and green lines correspond to the comoving number densities of ϕ (in equilibrium) and ψ , and comoving energy density of ν_R respectively. The three free parameters m_ϕ, y_ϕ and m_ψ are taken in such a way that DM abundance, $\Omega_{\text{DM}} h^2$ is always satisfied. While ϕ abundance follows the equilibrium abundance as shown by the magenta line, DM and ν_R freeze in from decay of ϕ and get saturated after ϕ abundance gets Boltzmann suppressed for $T \lesssim m_\phi$.

Now, let us discuss the phenomenology for this situation with respect to the parameters m_ϕ, y_ϕ and m_ψ govern by Eqs. (3.6) and (3.7). The approximate analytical solutions of these two equations are given in the Appendix A.2. Equations in (A.69) say that for $m_\phi \gg m_\psi$, both Y_ψ and $\tilde{Y}$ depend on m_ϕ and y_ϕ only, making ΔN_{eff} independent of m_ψ . Eq. (A.72) that gives a relation between ΔN_{eff} and $\Omega_{\text{DM}} h^2$,

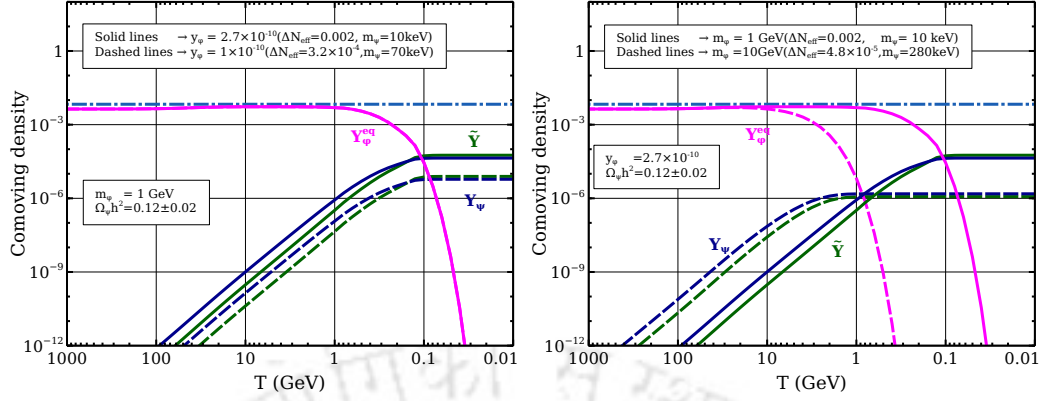


Figure 3.2: Evolution of dark sector particles (ϕ, ψ, ν_R) in case I considering ϕ to be in equilibrium throughout. All the lines denote the total comoving number/energy densities of dark sector particles. The left and right panel plots show the change in evolution for two different choices of y_ϕ, m_ϕ respectively. Chosen sets of points keep the DM abundance within the Planck limit.

carries DM mass as an independent parameter. For correct relic abundance, a minimum value of DM mass will provide a maximum contribution to extra radiation energy density. Keeping this in mind, we plot the solid line in left panel of Fig. 3.2, where we keep DM mass to be 10 keV. We see that the corresponding ΔN_{eff} value is 0.002. This is the maximum value of effective relativistic degrees of freedom and it is out of the reach of both Planck 2018 and CMB-S4 limit. An approximate analytical approximation also gives the same value $\Delta N_{\text{eff}} \approx 0.0016$ (from Eq. (A.71)). For the dashed line in the left panel, we have changed y_ϕ and observed its effects on ΔN_{eff} . In order to satisfy the DM abundance, m_ψ has to be increased accordingly for the dashed lines. As expected, ΔN_{eff} is reduced further. The right panel in Fig. 3.2 has been plotted for a different value of m_ϕ . Here, due to a larger mass, ϕ gets Boltzmann suppressed earlier resulting in a smaller Y_ψ and $\tilde{Y}$. In both the plots, we show a horizontal line denoting the comoving energy density of a single species of right-handed neutrino that corresponds to the 2σ upper bound from the Planck 2018 data. In conclusion, for this situation when ϕ is always in bath, the contribution of dark radiation to effective relativistic degrees of freedom is beyond the reach of future CMB experiments.

Structure formation constraints: For case I, where the particle ϕ is always in thermal equilibrium, we have calculated the free-streaming length for three different benchmark points. Here, we already know the distribution function of ϕ using which the distribution function of ψ can be calculated. The Eq. (3.14) tells that the free-streaming length is mainly dependent on two factors: the production temperature

(T_{prod}) and the injected energy to the DM from the decaying particle which will determine the average thermal velocity of the latter. In this section, we will see that if the production temperature is same and injected energy to DM is more, one can expect a larger free-streaming length as the dark matter particle will be relativistic for a longer duration. If the production temperature is high but the injected energy is same, one can expect a smaller FSL due to higher red-shift of DM momentum which will make the DM to be non-relativistic at an earlier epoch.

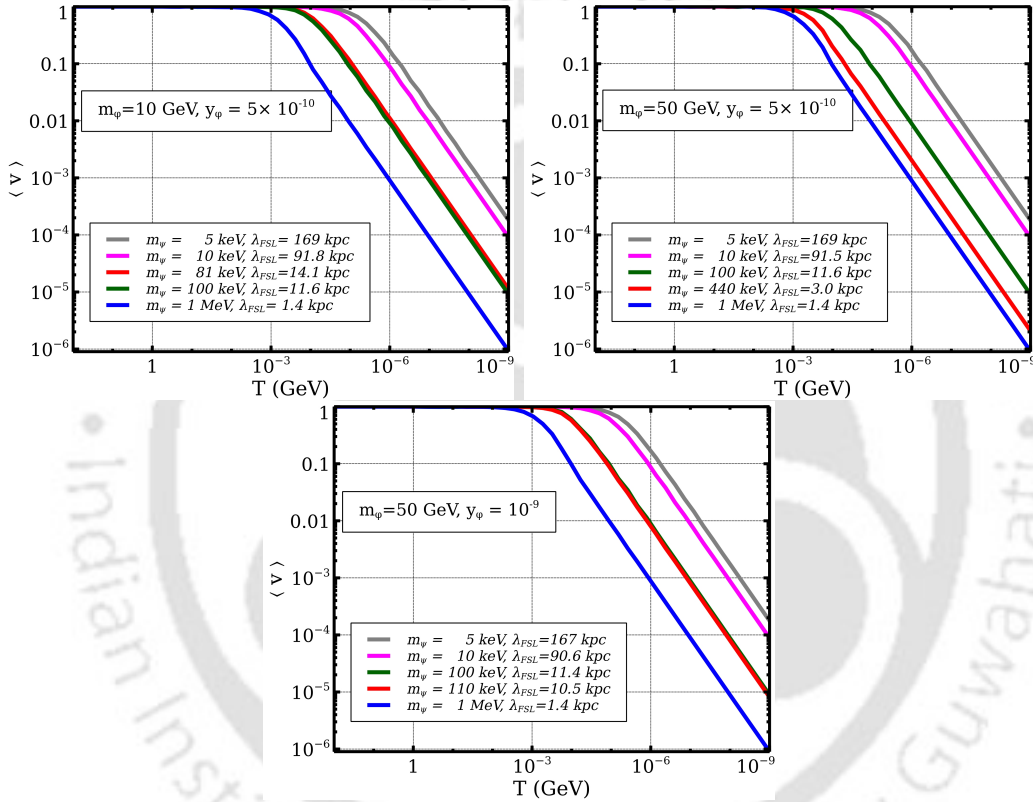


Figure 3.3: Average velocity of DM as a function of temperature in case I for different benchmark combinations of relevant parameters.

In Fig. 3.3, we have shown the average velocity of DM as a function of temperature for two different values of ϕ , $m_\phi = 10$ GeV and $m_\phi = 50$ GeV. The values of dark sector coupling, y_ϕ is same in the upper panel plots of Fig. 3.3. In all the figures, the red lines show the average velocity of ψ for which m_ψ gives the correct DM relic. As can be seen from all the three plots, for $m_\psi \gtrsim 10$ keV, the free-streaming length is less than 0.1 Mpc, that is, they are in the warm DM region. From the first two plots, we see that for a particular value of m_ψ (e.g. $m_\psi = 1000$ keV), both the plots give very similar values for free-streaming length. This is contrary

Table 3.1: Table for case I

Parameters			$\Omega_{\text{DM}} h^2$	ΔN_{eff}	FSL(Mpc)
$m_\phi(\text{GeV})$	y_ϕ	$m_\psi(\text{keV})$			
10	5×10^{-10}	81	0.12	1.6×10^{-4}	0.0141
50	5×10^{-10}	440	0.12	2.9×10^{-5}	0.0030
50	10^{-9}	110	0.12	1.2×10^{-4}	0.0105

to the expectation as for higher mass of decaying particle, the injected energy to the DM should be more. The reason why the FSL is still small for higher decaying particle mass is that the production of DM from ϕ also occurs at a earlier epoch (see benchmark plot in Fig. 3.2). As a result although the DM has higher momentum, its momentum gets red-shifted more. These two different phenomena compete with each other and as a result, we get similar FSL values in both the plots. In the top right panel plot and in the bottom plot, we have kept ϕ mass to be same and have changed the dark sector coupling, y_ϕ . Due to same m_ϕ , the initial energy of DM will be the same. Also we have already seen that changing y_ϕ does not change T_{prod} , the production temperature of dark matter. Hence we can expect same FSL for same DM mass. This is exactly what we can see from the top right plot and the bottom plot. The only difference in these two plots is that the ψ mass satisfying correct DM relic is different. From the above analysis, we have found that the FSL for DM mass corresponding to correct DM relic falls under the warm dark matter region.

We summarize our FSL results for case I in table 3.1, by including only those benchmark points from above analysis which satisfy correct DM relic. Clearly, the constraints on DM mass from FSL criteria can be as severe as $\mathcal{O}(100 \text{ keV})$ keeping the $\Delta N_{\text{eff}} \leq \mathcal{O}(10^{-3})$.

Case II We now discuss the results for the intermediate scenario where ϕ gets produced thermally followed by its freeze-out. This requires solving the Boltzmann equation for ϕ as well together with the ones for ψ, ν_R . Therefore, in addition to m_ϕ, y_ϕ, m_ψ , the Higgs portal coupling $\lambda_{H\phi}$ can play crucial role in deciding DM abundance as well as ΔN_{eff} . We show the evolution of dark sector particles for case II in Fig. 3.4. The top left, top right and bottom panels in this figure show the comparisons for two different choices of $y_\phi, \lambda_{H\phi}, m_\phi$ respectively. Similar to case I, the magenta, blue and green lines correspond to the comoving number densities of ϕ (in equilibrium) and ψ , and comoving energy density ν_R respectively. The red line corresponds to the actual comoving number density of ϕ which undergoes thermal

freeze-out at an intermediate epoch followed by complete decay at later epochs. In all these plots, one can clearly see the production of ψ, ν_R to be taking place during equilibrium as well as frozen out phases of ϕ separated by a kink in between, as seen from the blue and green lines. The Higgs portal coupling of ϕ is chosen in such a way that the freeze-out abundance of ϕ is non-negligible in order to play substantial role in ψ, ν_R production. This is clearly visible from the plots shown in Fig. 3.4, where the production of ψ, ν_R from frozen out ϕ appear to be significant. Another significant improvement from case I is that mass of DM can satisfy the lower limits discussed earlier even when ΔN_{eff} saturates Planck upper bound.

In the top left panel plot of Fig. 3.4, we show the evolution for two different values of y_ϕ while keeping other parameters fixed. Since y_ϕ dictates the decay width of ϕ , a lower value of y_ϕ delays the decay of frozen out ϕ . Change in y_ϕ , however, keeps DM density same as the number of ϕ gets transferred to number of ψ , both of which behave as non-relativistic particles. On the other hand, a lower value of y_ϕ or delayed production of ν_R from frozen out ϕ increases the comoving energy density of ν_R which behaves as radiation with comoving energy density defined as $\tilde{Y} = \frac{\rho_{\nu_R}}{s^{4/3}}$. This can be understood if we solve the coupled Boltzmann equations given in Eqs. (3.9), (3.10), (3.11) analytically after the freeze-out of ϕ . Eqs. (A.73) and (A.74) give the approximate analytical expressions for $Y_\phi^{\text{fo}}, Y_\psi$ and $\tilde{Y}$. As evident from Fig. 3.4, the freeze-out abundance of ϕ namely, Y_ϕ^{fo} gets converted to Y_ψ ; whereas, $\tilde{Y} \propto \frac{\langle E \Gamma \rangle}{f_2} \propto y_\phi^{-1}$.

In top right panel plot of Fig. 3.4, we show the evolution for two different choices of Higgs portal coupling $\lambda_{H\phi}$. As expected from freeze-out mechanism of WIMP type particles, a larger value of $\lambda_{H\phi}$ leads to smaller freeze-out abundance of ϕ and hence smaller yield of ψ, ν_R at later epochs. On the other hand, for larger benchmark value of $\lambda_{H\phi}$ resulting in smaller yield of Y_ψ , we choose a heavier DM mass in order to keep $\Omega_{\text{DM}} h^2$ within Planck bounds. Finally, in the bottom panel plot of Fig. 3.4, we show the evolution of dark sector particles for two different choices of ϕ mass. Due to change in Boltzmann suppression, the equilibrium evolution also changes for these two values. Since annihilation cross section decreases with increase in mass, we see larger freeze-out abundance for heavier ϕ . Naturally, a larger freeze-out abundance for heavier ϕ leads to enhancement in comoving densities of DM and ν_R as well. The benchmark values of m_ϕ, m_ψ are chosen in such a way that DM abundance $\Omega_{\text{DM}} h^2$ remains within Planck limit while heavier (lighter) benchmark of m_ϕ keep ΔN_{eff}

close to Planck upper bound (CMB-S4 sensitivity). It should also be noted that increasing ϕ mass also increases its decay width (for $m_\psi \ll m_\phi$) and hence we notice a delay in production of ψ, ν_R for lighter ϕ mass. Although we noticed enhanced $\tilde{Y}$ from such delayed production in top left panel plot of Fig. 3.4, in bottom panel plot of the same figure, this effect remains sub-dominant. The expected increase in $\tilde{Y}$ for lighter m_ϕ due to delayed production remains subdominant compared to decrease in $\tilde{Y}$ for lighter m_ϕ due to reduced freeze-out abundance of the latter. Therefore, we only notice an overall increase in $\tilde{Y}$ for heavier ϕ having larger freeze-out abundance. In each of these plots shown in Fig. 3.4, the two benchmark parameter values (that is, y_ϕ in top left, $\lambda_{H\phi}$ in top right, m_ϕ in bottom) are chosen in such a way that one of them leads to ΔN_{eff} close to Planck 2σ upper limit while the other pushes it close to CMB-S4 sensitivity limit.

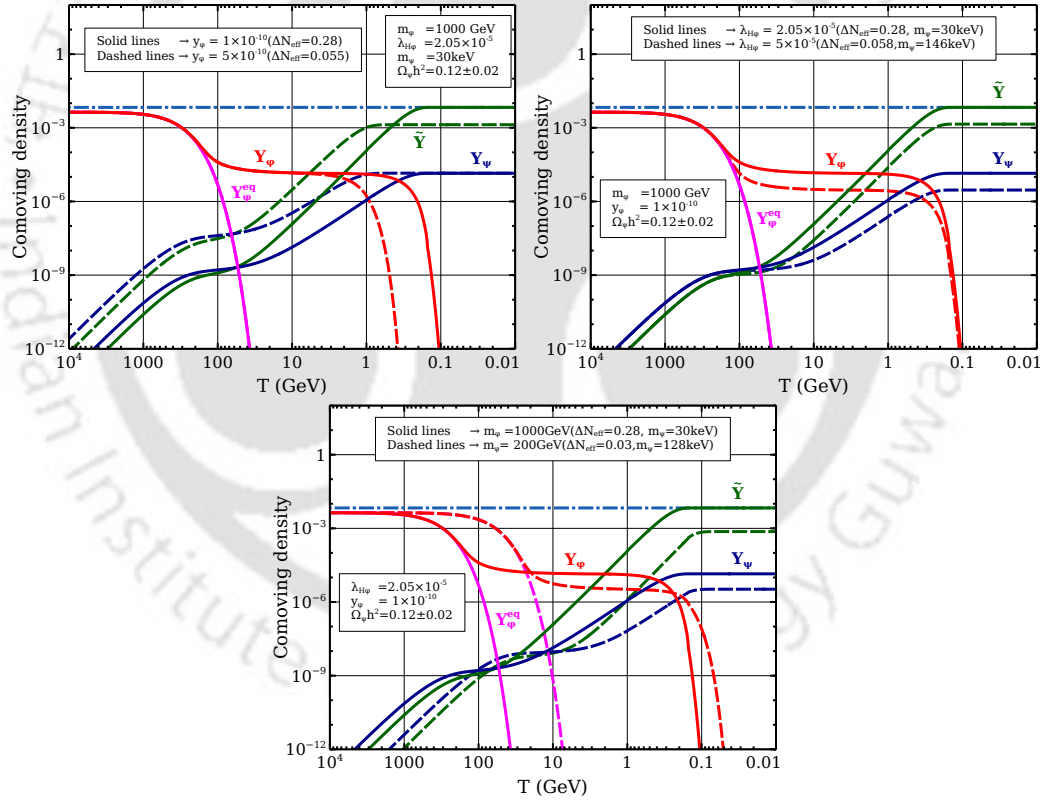


Figure 3.4: Evolution of dark sector particles (ϕ, ψ, ν_R) in case II considering ϕ to freeze out from the bath while decaying into (ψ, ν_R) . Top left, top right and bottom panel plots show the change in evolution for two different choices of $y_\phi, \lambda_{H\phi}, m_\phi$ respectively. Chosen sets of points keep the DM abundance within the Planck limit.

As seen from the evolution plots of case I and case II discussed above, case II becomes similar to case I if the maximum production of ψ from the decay of ϕ happens before the freeze-out of latter from the thermal bath. This requires either late freeze-out of ϕ (due to large portal coupling $\lambda_{H\phi}$) or a short lived ϕ (due to large Yukawa coupling y_ϕ). Unless we consider such regimes of couplings, these two cases need to be considered separately, yielding distinct result and phenomenology.

After highlighting the interesting features of case II with benchmark choices of key parameters, we perform a numerical scan over the parameter space. The relevant parameters are varied in the following range:

$$\begin{aligned} 200 \text{ GeV} &\leq m_\phi \leq 2000 \text{ GeV}, \\ 10^{-5} &\leq \lambda_{H\phi} \leq 10^{-3.5}, \\ 1 \text{ keV} &\leq m_\psi \leq 10 \text{ MeV}. \end{aligned}$$

The value of y_ϕ is kept constant and remains fixed at 10^{-10} , which also ensures that the decay of ϕ occurs before the BBN epoch. The resulting parameter space is shown in ΔN_{eff} vs m_ϕ plane in Fig. 3.5. The colour bar in left and right panel plots show the variation in $\lambda_{H\phi}$ and m_ψ respectively. While all the points satisfy the Planck bound on DM relic abundance, the corresponding upper bound on ΔN_{eff} is shown by magenta shaded region. The future sensitivity of CMB-S4 experiment is shown as grey shaded region. From the left panel of Fig. 3.5, we can clearly see that for decrease in $\lambda_{H\phi}$, while keeping m_ϕ constant, ΔN_{eff} decreases. This is expected as a smaller value of Higgs portal coupling $\lambda_{H\phi}$ leads to a larger freeze-out abundance of ϕ followed by enhanced production of ν_R from ϕ decay. Since the same decay also produces DM, we need to choose lower values of DM masses in order to keep its relic abundance within Planck limits. This can be noticed from the right panel plot of Fig. 3.5 where the points with large ΔN_{eff} correspond to smaller DM masses. Additionally, for fixed $\lambda_{H\phi}$, if we increase m_ϕ , the corresponding ΔN_{eff} increases. Once again, this is due to larger freeze-out abundance of ϕ for heavier masses, as noticed while discussing the evolution plots in Fig. 3.4. Accordingly, for heavier m_ϕ with fixed $\lambda_{H\phi}$, we need to choose lighter DM masses in order to keep its relic abundance within observed limits, as seen from the right panel plot of Fig. 3.5. Thus, FIMP type DM candidate in our setup with masses all the way up to a few tens of keV can already get disfavoured by Planck 2018 limit (2σ) on ΔN_{eff} . As we will see in the next sub-section, this lower bound on DM mass gets pushed to

hundreds of keV after imposing the structure formation bounds. Accordingly, as these Fig. 3.5 suggests, ΔN_{eff} gets pushed down to second or third decimal places.

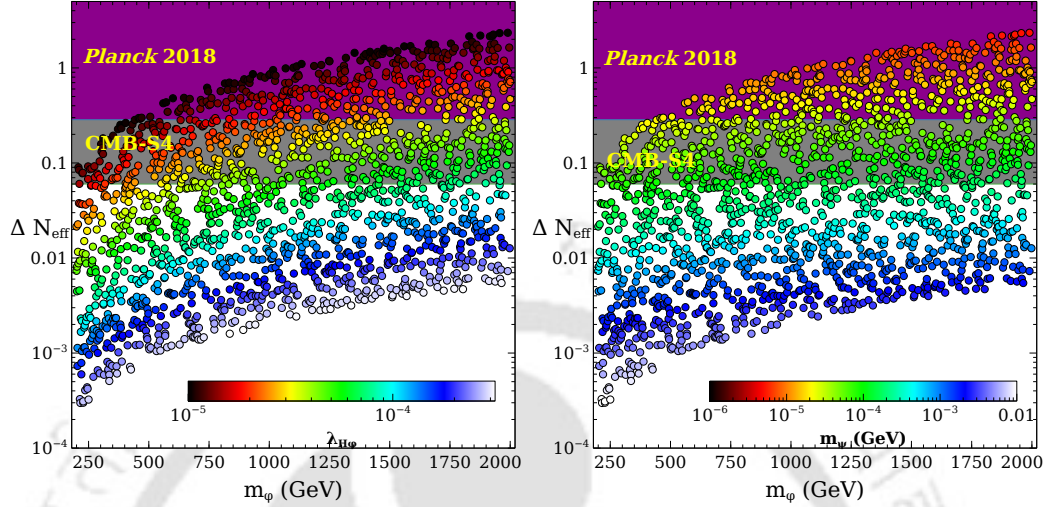


Figure 3.5: Parameter space plot for case II obtained from numerical scans, shown in terms of ΔN_{eff} vs m_ϕ while $\lambda_{H\phi}$ (left panel) and m_ψ (right panel) are shown in colour code. The other relevant parameter y_ϕ is kept fixed at 10^{-10} . The magenta and grey shaded regions indicate the current and future bound on ΔN_{eff} from Planck 2018 (2σ) and CMB-S4 respectively.

Structure formation constraints: For case II, we have estimated the free-streaming length of dark matter for some benchmark points. The free-streaming length for dark matter when $m_\phi = 1000$ GeV, $\lambda_{H\phi} = 5 \times 10^{-5}$ and $y_\phi = 10^{-10}$ are shown in the left side of Fig. 3.6 for different values of m_ψ . Except the red colored lines, the other lines do not satisfy the current DM abundance. As expected, for lower mass, the dark matter remains relativistic for a longer period and hence its free-streaming length is higher. Even for the maximum m_ψ in the figure, i.e. for $m_\psi = 1000$ keV, the free-streaming length is greater than 0.1 Mpc, which is roughly the boundary between warm and hot dark matter. Thus for all m_ψ in the figure, the free-streaming lengths are found to be higher than 0.1 Mpc. By decreasing the injected energy to dark matter from the particle ϕ , the dark matter can be made to become non-relativistic at an earlier epoch. This can be obtained by decreasing m_ϕ . The top right panel plot of Fig. 3.6 shows the free-streaming length for a smaller $m_\phi = 500$ GeV with $\lambda_{H\phi}$ and y_ϕ having the same value as the top left panel plot. We can see that although the free-streaming length now has a smaller value, but still all the points give hot dark-matter. Another effective way to make dark-matter non-relativistic at an earlier time is to increase the dark sector coupling y_ϕ . This will give a higher decay

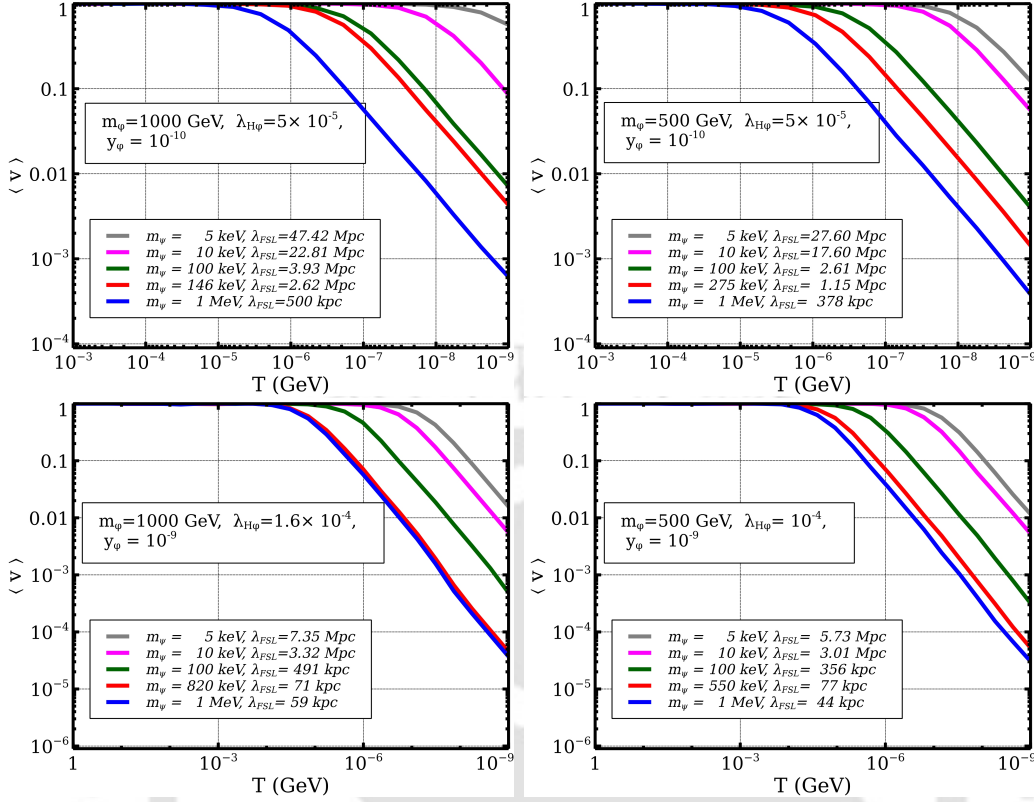


Figure 3.6: Average velocity of DM as a function of temperature in case II for different benchmark combinations of relevant parameters.

Table 3.2: Table for case II

Parameters				$\Omega_{\text{DM}} h^2$	ΔN_{eff}	FSL(Mpc)
m_ϕ (GeV)	$\lambda_{H\phi}$	y_ϕ	m_ψ (keV)			
1000	5×10^{-5}	10^{-10}	146	0.12	5.8×10^{-2}	2.625
500	5×10^{-5}	10^{-10}	275	0.12	2.2×10^{-2}	1.146
1000	1.6×10^{-4}	10^{-9}	820	0.12	7.2×10^{-4}	0.071
500	10^{-4}	10^{-9}	550	0.12	6.5×10^{-4}	0.077

rate Γ_ϕ , leading to a higher dark-matter production temperature. The results can be seen from the bottom plots of Fig. 3.6. The left hand side is for $m_\phi = 1000$ GeV and the right hand side is for $m_\phi = 500$ GeV. As increasing $\lambda_{H\phi}$ will also increase m_ψ for correct DM abundance, the other parameters are tuned in both the figures so that we get DM mass in order of hundred of keV mass, satisfying the relic density constraint. We summarize our FSL results for case II in Table 3.2, by including only those benchmark points from above analysis which satisfy correct DM relic density.

Case III In this subsection, we discuss the results for the last subclass of scenarios mentioned earlier where the mother particle ϕ never enters equilibrium due to feeble Higgs portal coupling. In order to simplify the analysis, we consider ϕ production to be taking place dominantly from the SM Higgs, either via decay or via annihilation. For $m_\phi < m_h/2$, the decay process ($h \rightarrow \phi\phi$) dominates while in the other limit only annihilation ($hh \rightarrow \phi\phi$) can contribute to ϕ production. To show the roles of decay and annihilation separately, we discuss these two limits separately.

For $m_\phi < m_h/2$: In this case, ϕ freezes in from Higgs decay and then decays into ψ and ν_R . Similar to earlier cases, we first show the evolution of dark sector particles for suitable choices of model parameters such that both DM abundance as well as ΔN_{eff} remain within Planck 2σ limits. The corresponding evolution plots are shown in Fig. 3.7. We maintain similar colour codes as before namely, magenta, red, blue, green to show the evolution of comoving number densities of ϕ (equilibrium), ϕ (actual), DM ψ , ν_R respectively. In sharp contrast to case I, II discussed earlier, here we see that the initial abundance of ϕ remains negligible and then it slowly freezes in from decay of SM Higgs. In the top left panel of Fig. 3.7, we show the differences in these evolution for two different choices of y_ϕ . As usual, a smaller value of y_ϕ delays the decay of ϕ . While final DM density remains same for both the values of y_ϕ , the smaller value of y_ϕ leads to enhancement in ν_R density. Similar observation was noted in case II as well. In the top right panel of Fig. 3.7, we show the variation due to two different choices of Higgs portal coupling $\lambda_{H\phi}$. In sharp contrast to case II, here we get smaller abundance of ϕ for smaller value of $\lambda_{H\phi}$ which also highlights the generic difference between freeze-in and freeze-out production mechanisms [87]. Consequently, smaller $\lambda_{H\phi}$ leads to smaller yields in ψ, ν_R as clearly seen from the same plot in top right panel. Finally, in the bottom panel plot of Fig. 3.7, we show the variation due to two different choices of m_ϕ . We see a marginal decrease in freeze-in abundance of ϕ for larger m_ϕ due to the fact that as m_ϕ approaches $m_h/2$, the corresponding partial decay width $\Gamma_{h \rightarrow \phi\phi^\dagger}$ decreases suppressing the production of ϕ slightly. On the other hand, a larger m_ϕ corresponds to larger decay width of ϕ in the limit $m_\psi \ll m_\phi$ leading to depletion in ϕ abundance earlier. The increase in ϕ decay width for larger m_ϕ also results in increased initial production of ψ and ν_R . While final DM abundance decreases slightly for larger m_ϕ due to smaller freeze-in abundance of heavier ϕ , the abundance of ν_R gets slightly enhanced for larger m_ϕ due to larger decay width. Thus, there exists a competition between two effects: (i)

decrease in ν_R production due to decrease in freeze-in production of ϕ for larger m_ϕ and (ii) increase in ν_R production due to increase in ϕ decay width for larger m_ϕ and the final results will be decided by the dominance of either of these, to be discussed below. In all the plots shown in Fig. 3.7, we notice an intermediate plateau region for ϕ abundance. This arises when the freeze-in production rate of ϕ from Higgs decay and decay rate of ϕ into ψ, ν_R remain comparable.

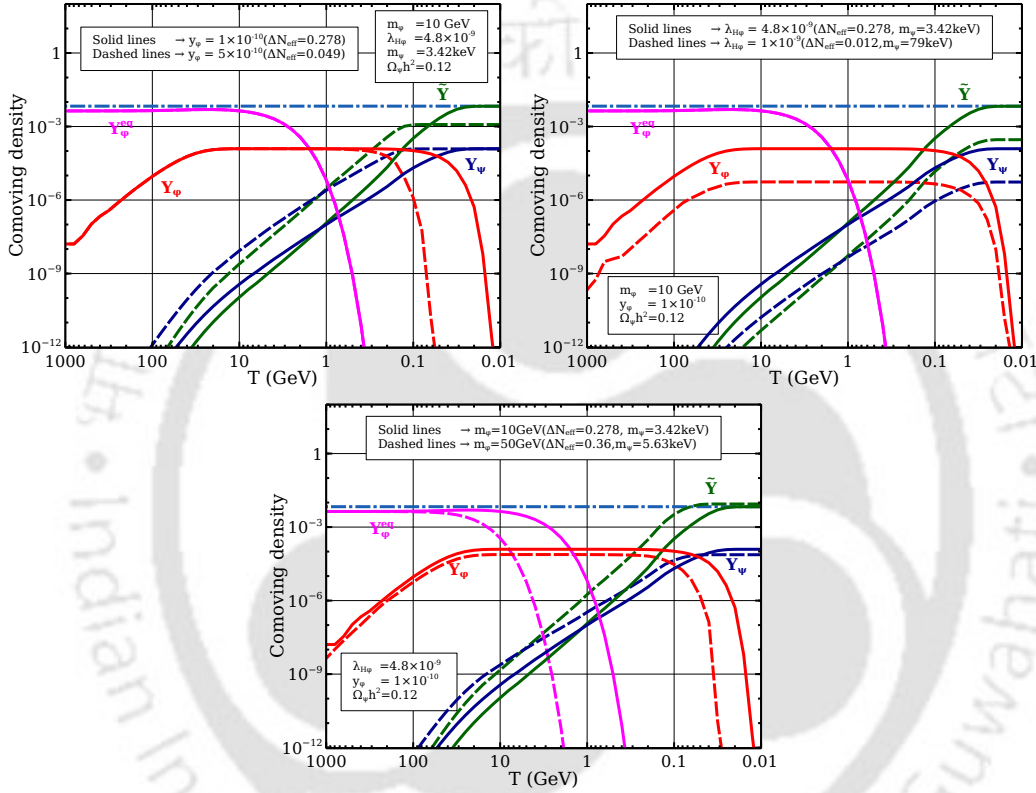


Figure 3.7: Evolution of dark sector particles (ϕ, ψ, ν_R) in case III considering ϕ to freeze in from Higgs decay and then decaying into (ψ, ν_R). Top left, top right and bottom panel plots show the change in evolution for two different choices of $y_\phi, \lambda_{H\phi}, m_\phi$ respectively. Chosen sets of points keep the DM abundance within the Planck limit.

We then perform a numerical scan to show the parameter space assuming ϕ to be out-of-equilibrium throughout which freezes in only from the SM Higgs decay. In the scan, we vary the relevant parameters in the following range:

$$\begin{aligned} 5 \text{ GeV} &\leq m_\phi \leq 60 \text{ GeV}, \\ 10^{-9} &\leq \lambda_{H\phi} \leq 10^{-8}, \\ 1 \text{ keV} &\leq m_\psi \leq 1 \text{ MeV}. \end{aligned}$$

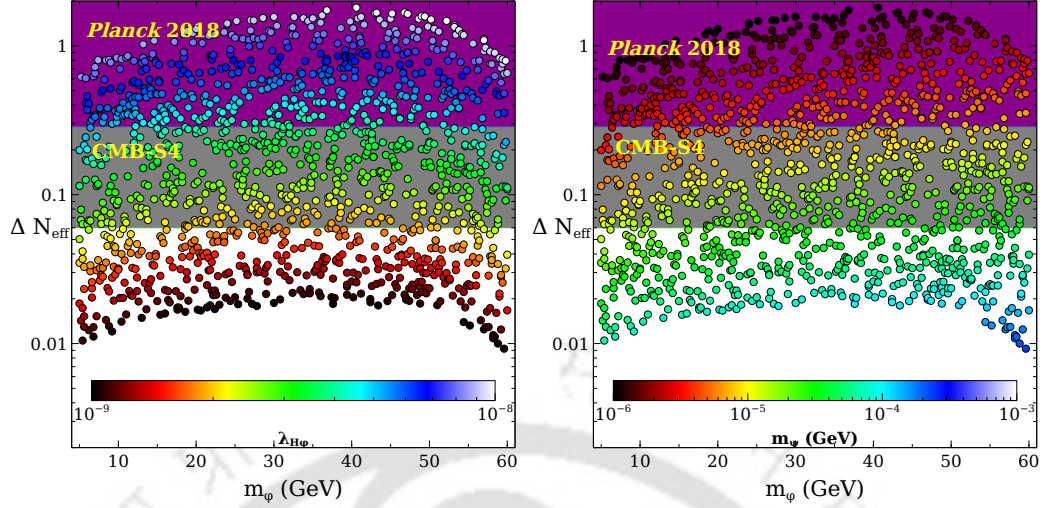


Figure 3.8: Parameter space plot for case III (considering ϕ to freeze in from Higgs decay) obtained from numerical scans, shown in terms of ΔN_{eff} vs m_ϕ while $\lambda_{H\phi}$ (left panel) and m_ψ (right panel) are shown in colour code. The other relevant parameter y_ϕ is kept fixed at 10^{-10} . The magenta and grey shaded regions indicate the current and future bound on ΔN_{eff} from Planck 2018 (2σ) and CMB-S4 respectively.

Here also y_ϕ is kept fixed at 10^{-10} . The resulting parameter space is shown in ΔN_{eff} vs m_ϕ plane in Fig. 3.8 with the colour bars in left and right panel plots showing the variation in $\lambda_{H\phi}$ and m_ψ respectively. Similar to case II, here also the scattered points satisfy the Planck bound on DM relic abundance while the corresponding upper bound (future sensitivity) on ΔN_{eff} is shown by magenta (grey) shaded region. With an increase in $\lambda_{H\phi}$ while keeping m_ϕ fixed, we get enhancement in ΔN_{eff} as seen from the left panel plot of Fig. 3.8, in sharp contrast with the corresponding results in case II. As discussed above, this trend is expected as increase in $\lambda_{H\phi}$ leads to increased freeze-in production of ϕ . Since DM number density also increases from the same ϕ decay, we need to choose lighter DM masses for larger $\lambda_{H\phi}$ in order keep $\Omega_{\text{DM}} h^2$ within observed limits, as seen from the right panel plot of Fig. 3.8. On the other hand, if ϕ mass increases for fixed $\lambda_{H\phi}$, we first see an increase in ΔN_{eff} followed by decrease for m_ϕ closer to $m_h/2$. The initial rise in ΔN_{eff} can be explained by noting the increase in ϕ decay width for larger m_ϕ . However, if we continue to increase m_ϕ , taking it closer to $m_h/2$, the partial decay width of the SM Higgs $\Gamma_{h \rightarrow \phi\phi^\dagger}$ decreases leading to suppression in freeze-in abundance of ϕ . Consequently, this leads to decrease in ν_R, ψ densities. Correct DM abundance can be obtained by choosing heavier DM masses in the high m_ϕ regime, as seen from the right panel plot of Fig. 3.8. Similar to case II discussed before, here also the

bounds on DM mass become more severe, after imposing the structure formation constraints, as we discuss in the next sub-section.

For $m_\phi > m_h/2$: We now briefly discuss the essential features of the non-thermal ϕ scenario where its freeze-in production is dominated by annihilations only and decay is forbidden kinematically due to $m_\phi > m_h/2$. The evolution of dark sector particles in this case are shown in Fig. 3.9. Once again, the choice of benchmark parameters is made in such a way that the final DM abundance and ΔN_{eff} remain within Planck 2018 limits. In top left panel of Fig. 3.9, we show the variation in evolution for two different choices of y_ϕ . As expected, this only alters the decay width of ϕ and hence the production of ν_R, ψ . While final DM density remains same for both the choices, late production of ν_R due to smaller y_ϕ leads to an enhancement in $\tilde{Y}$, an observation which was also made in other scenarios discussed above. In top right panel of Fig. 3.9, we show the difference in evolution due to variation in Higgs portal coupling $\lambda_{H\phi}$. Naturally, a smaller $\lambda_{H\phi}$ results in smaller freeze-in abundance of ϕ from annihilation and hence smaller yields in ν_R, ψ . Variation due to change in m_ϕ is shown in the bottom panel plot of Fig. 3.9. We do not see much difference between the two values except for the fact that a larger m_ϕ increase ϕ decay width leading to early depletion. Since the overall features in this case remains similar to the earlier case where ϕ is produced from decay only, we expect the parameter space to remain similar. Therefore, we do not perform any numerical scan in this case.

Structure formation constraints: For case III, we have considered the situation when $m_\phi < m_h/2$. Here, we have considered the same benchmark point as in the bottom plot of Fig. 3.7 for two different ϕ mass, $m_\phi = 10$ GeV and $m_\phi = 50$ GeV. The production temperature for both the situation is around 10 MeV (production temperature of DM for $m_\phi = 10$ GeV and $m_\phi = 50$ GeV are about 10 MeV and 30 MeV respectively). The Fig. 3.10 shows that the FSL for a particular dark matter mass is more in the right plot where m_ϕ is 50 GeV. This is expected as the production temperature is almost same, so an increase in mass of decaying particle injects more energy to the dark matter particles. For the left plot, the DM relic is satisfied when $m_\psi = 3.42$ keV and for the right side plot when $m_\psi \approx 5$ keV. For both the cases, the FSL when DM mass gives correct DM relic is larger than 0.1 Mpc making the DM “hot”. For these two benchmark points, the ΔN_{eff} is within

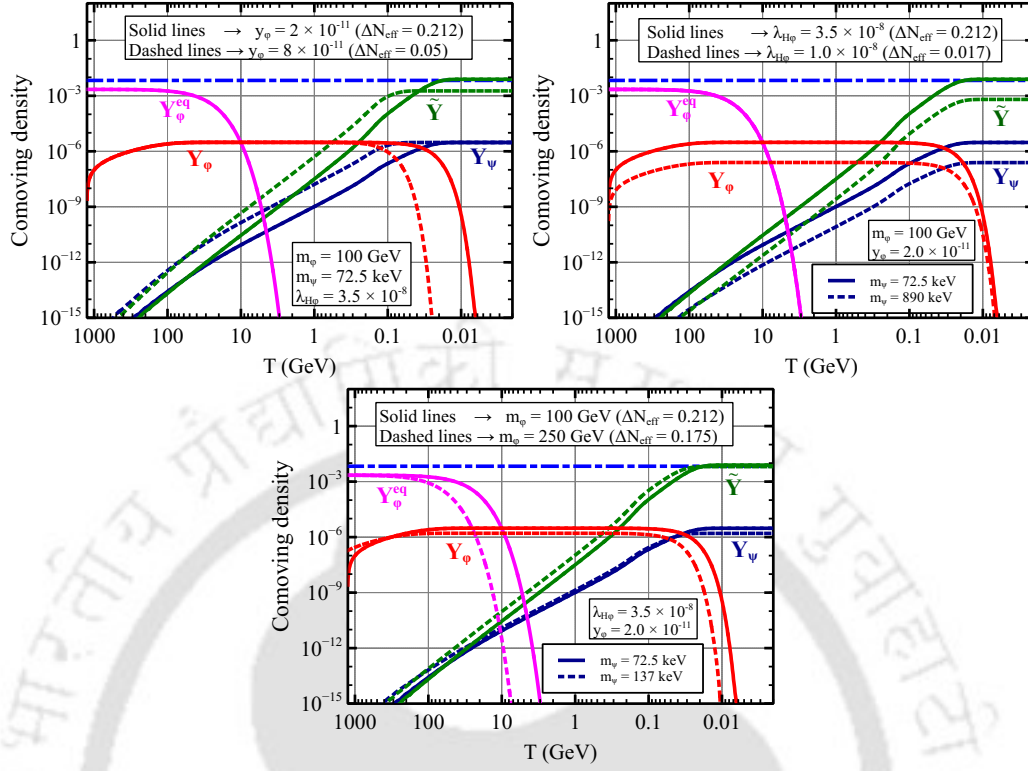


Figure 3.9: Evolution of dark sector particles (ϕ, ψ, ν_R) in case III considering ϕ to freeze in from Higgs annihilations and then decaying into (ψ, ν_R) . Top left, top right and bottom panel plots show the change in evolution for two different choices of $y_\phi, \lambda_{H\phi}, m_\phi$ respectively. Chosen sets of points keep the DM abundance within the Planck limit.

the current CMB bound. In principle, by increasing the dark sector coupling y_ϕ , the production temperature can be increased making the FSL small.

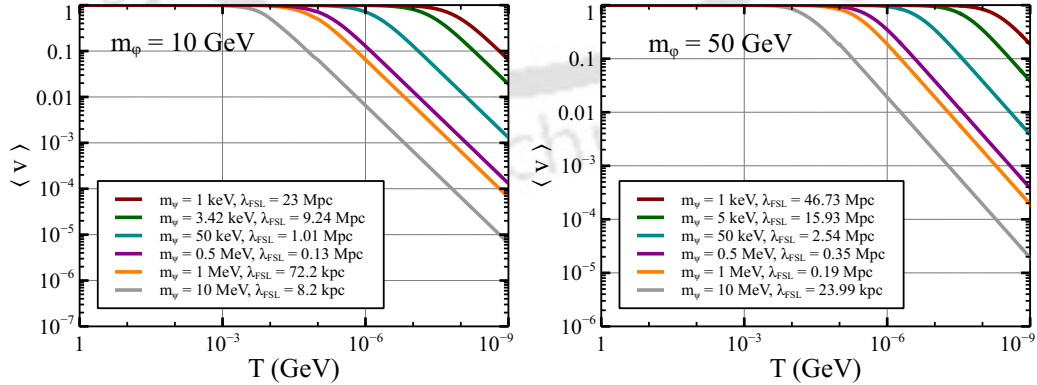


Figure 3.10: Average velocity of DM as a function of temperature in case III for different benchmark combinations of relevant parameters.

Table 3.3: Table for case III

Parameters				$\Omega_{\text{DM}} h^2$	ΔN_{eff}	FSL(Mpc)
$m_\phi(\text{GeV})$	$\lambda_{H\phi}$	y_ϕ	$m_\psi(\text{keV})$			
10	4.8×10^{-9}	10^{-10}	3.42	0.12	2.7×10^{-1}	9.42
50	4.8×10^{-9}	10^{-10}	5.63	0.12	3.6×10^{-1}	15.5

We summarize our FSL results for case III in Table 3.3, by including only those benchmark points from above analysis which satisfy correct DM relic density. Clearly, the constraints on DM mass from FSL criteria can be as severe as $\mathcal{O}(100 \text{ keV})$ keeping the $\Delta N_{\text{eff}} \leq \mathcal{O}(10^{-3})$. In the next sub-section, we briefly comment on possible UV completions which can bring the ΔN_{eff} within CMB-S4 sensitivity while keeping the DM phenomenology similar to above analysis.

3.1.4 Possible UV Completions

We have discussed a minimal scenario to illustrate the essential results of freeze-in DM via light Dirac neutrino portal. The minimal nature of this model with only three new BSM fields has led to strong predictions on DM mass as well as ΔN_{eff} allowed from experimental constraints. Possible UV completions of this model can, in principle, give rise to a *natural* origin of light Dirac neutrino masses, a gauge symmetric realisation of the discrete $\mathbb{Z}_4$ symmetry while also giving a flexibility to enhance ΔN_{eff} to bring it within future experimental sensitivity.

One simple possibility is to introduce an additional Higgs doublet H_2 , responsible for generating a light Dirac neutrino mass [289]. While the freeze-in contribution to ΔN_{eff} from Dirac Yukawa interaction with the SM Higgs doublet is negligibly small due to tiny Yukawa couplings [123, 257], the neutrinophilic Higgs doublet H_2 can have a larger Yukawa coupling leading to either thermalised ν_R or large freeze-in contribution to ΔN_{eff} . We can choose the $\mathbb{Z}_4$ charges of SM leptons, ν_R, ψ, ϕ, H_2 to be $i, -i, -1, i, -1$ respectively, so that the Yukawa interaction $\bar{\ell}_L \tilde{H}_2 \nu_R$ is responsible for light Dirac neutrino mass. This charge assignment leaves the dark sector interactions same as in the minimal model. The second Higgs doublet can have a tiny soft-breaking term with the SM Higgs $\mu_{12} H^\dagger H_2$ by virtue of which its neutral component can acquire a tiny VEV, leading to a larger Dirac Yukawa. Due to the presence of multiple sources of ΔN_{eff} , we can have correct FIMP DM phenomenology while enhancing ΔN_{eff} to remain within the sensitivity of next generation experiments.

Another possibility is to consider a gauge extension of the SM which naturally accommodates three right handed neutrinos required to realise a Dirac neutrino scenario. Perhaps the simplest possibility is to consider the gauged $B - L$ extension of the SM [290–295] where three right handed neutrinos arise a minimal possibility to keep the model anomaly free. Depending upon the scalar content, light neutrinos can be purely Dirac in this model [253, 258, 296–300]. The $B - L$ gauge charges of SM leptons, ν_R, ψ, ϕ are $-1, -1, 0, 1$ to realise the minimal possibility. The fermion singlet DM couples via the same portal $\bar{\psi}\nu_R\phi$ while light Dirac neutrino mass arise from the SM Higgs Yukawa couplings. Although the contribution to ΔN_{eff} from SM Higgs Yukawa interactions remain suppressed, there can be sizeable enhancement to it due to $B - L$ gauge interactions of ν_R . The DM phenomenology will remain similar to the minimal setup except for the fact that ϕ can now interact with the SM bath via Higgs as well as $B - L$ gauge portal interactions. Therefore, such non-minimal FIMP DM via light Dirac neutrino portal can lead to observable ΔN_{eff} which can be probed at CMB-S4 as well as other planned experiments like SPT-3G [117], Simons Observatory [118].

3.1.5 Summary

We have studied a minimal scenario where the origin of neutrino mass and dark matter remain connected with interesting observational prospects at CMB experiments. Assuming light neutrinos to be of Dirac nature necessitates the inclusion of right handed neutrinos ν_R which can also act like a portal to dark sector comprising of a fermion singlet DM and a scalar singlet ϕ . While the scalar singlet can be directly coupled to the SM bath via Higgs portal coupling, fermion singlet DM can couple only to ν_R via ϕ . We have studied in details, the freeze-in production of ψ and ν_R from ϕ decay, by considering three different possibilities with (i) ϕ in equilibrium, (ii) ϕ undergoing thermal freeze-out and (iii) ϕ getting produced via freeze-in. Since ν_R couples to SM leptons very feebly due to the requirement of generating sub-eV scale Dirac neutrino mass, the corresponding freeze-in production of ν_R directly from the SM bath remains suppressed. Since the same coupling with ϕ leads to freeze-in production of both DM and ν_R with the latter remaining relativistic throughout, we show the possibility of correlating DM parameter space with effective relativistic degrees of freedom ΔN_{eff} . We find that the scenario with ϕ in equilibrium throughout leads to tiny enhancement in ΔN_{eff} while being consistent with DM relic criteria.

However, for the other two scenarios, due to one additional free parameter in the form of Higgs portal coupling $\lambda_{H\phi}$ at play, we can have correct DM phenomenology while getting a sizeable enhancement in ΔN_{eff} at the same time. Additionally, depending upon the choice of parameters, existing bounds from the Planck experiment can also rule out DM mass up to a few tens of keV. However, structure formation constraints on such non-thermal DM rules out DM masses all the way up to a few hundred keV. Since DM and ν_R are produced from the same decay in this minimal model, the resulting ΔN_{eff} also gets reduced to $\leq \mathcal{O}(10^{-3})$ to be in agreement with required DM properties. We briefly discuss two possible UV completions which can disentangle the production of DM and ν_R while still maintaining the light Dirac neutrino portal scenario, such that correct DM properties can be realised even with enhanced ΔN_{eff} within experimental sensitivity.

Since the scalar singlet can be light in these scenarios opening up the possibility of SM Higgs decaying invisibly into a pair of ϕ , future LHC measurements will be able to constrain the Higgs portal coupling further from measurements of Higgs invisible decay rates. In addition to these specific signatures of our model keeping it very predictive, one can also pursue such neutrino portal dark matter scenarios from the point of view of easing cosmological tensions between early and late universe cosmological observations [301]. There have been a few works already in this direction [302, 303] which we plan to explore in future works.

3.2 Light Dirac neutrino portal dark matter with gauged $U(1)_{B-L}$ symmetry

In the previous work, discrete symmetry like Z_4 was considered to get the desired couplings, mass terms as well as the stability of DM. Here, we consider a UV completion with a gauged $B - L$ framework. The right-handed neutrinos are naturally part of the model providing the minimal anomaly-free setup. While RHNs are thermally produced due to gauged $B - L$ interactions, DM can be of WIMP or FIMP type depending upon Dirac neutrino portal couplings. The model not only gives rise to the desired DM phenomenology with observable ΔN_{eff} , but also leads to new constraints in the gauged $B - L$ parameter space not obtained previously.

3.2.1 The Model

Gauged $B - L$ extension of the SM [290–295] has been a popular BSM framework studied in the context of neutrino mass among others. The three right handed neutrinos ν_R having $B - L$ charge -1 each not only keep the model anomaly free¹ but also lead to massive Dirac neutrinos in combination with ν_L after electroweak symmetry breaking. A singlet fermion ψ is considered to be the DM candidate while two singlet scalars ϕ_1 and ϕ_2 with non-zero $B - L$ charges help in realising light Dirac neutrino portal DM and spontaneous $B - L$ symmetry breaking respectively. The relevant particle content is shown in Table 3.4.

	L	Φ	ν_R	ψ	ϕ_1	ϕ_2
$SU(2)$	2	2	1	1	1	1
$U(1)_Y$	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
$U(1)_{B-L}$	-1	0	-1	0	1	3

Table 3.4: Relevant particle content of the model with respective quantum numbers under the symmetry group.

The scalar Lagrangian for the model can be written as

$$\mathcal{L}_s = (D^\mu \Phi)^\dagger D_\mu \Phi + (D^\mu \phi_1)^\dagger D_\mu \phi_1 + (D^\mu \phi_2)^\dagger D_\mu \phi_2 - V(\Phi, \phi_1, \phi_2), \quad (3.24)$$

where

$$D_\mu \Phi = \left(\partial_\mu - i \frac{g_2}{2} \tau_a W_\mu^a - i \frac{g_1}{2} B_\mu \right) \Phi, \quad D_\mu \phi_1 = (\partial_\mu - i g_{BL} B'_\mu) \phi_1, \\ D_\mu \phi_2 = (\partial_\mu - i 3 g_{BL} B'_\mu) \phi_2, \quad (3.25)$$

denote the respective covariant derivatives and

$$V(\Phi, \phi_1, \phi_2) = -\mu^2 (\Phi^\dagger \Phi) + \mu_1^2 (\phi_1^\dagger \phi_1) - \mu_2^2 (\phi_2^\dagger \phi_2) + \lambda (\Phi^\dagger \Phi)^2 + \lambda_1 (\phi_1^\dagger \phi_1)^2 \\ + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_{H\phi_1} (\Phi^\dagger \Phi) (\phi_1^\dagger \phi_1) + \lambda_{H\phi_2} (\Phi^\dagger \Phi) (\phi_2^\dagger \phi_2) \\ + \lambda_{\phi_1 \phi_2} (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) + (\lambda'_{\phi_1 \phi_2} \phi_1^3 \phi_2^\dagger + \text{h.c.}) \quad (3.26)$$

denotes the scalar potential. The relevant part of the fermion Lagrangian is

$$-\mathcal{L}_Y \supset Y_\nu \bar{L} \tilde{\Phi} \nu_R + y_\phi \bar{\psi} \phi_1 \nu_R + m_\psi \bar{\psi} \psi. \quad (3.27)$$

¹See appendix A.3 for other solutions to anomaly cancellation conditions.

The singlet scalar field ϕ_2 acquires a non-zero VEV denoted by $\langle\phi_2\rangle = v_2$, leading to the spontaneous breaking of $U(1)_{B-L}$ gauge symmetry. The singlet field ϕ_1 does not acquire any VEV and remains heavier than DM ψ , ensuring latter's stability. We also consider the Higgs portal coupling $\lambda_{H\phi_2}$ to be negligible for simplicity.

Considering the free parameters in the model to be $\mu_1, \mu_2, \lambda_1, \lambda_2, \lambda_{H\phi_1}, \lambda'_{\phi_1\phi_2}, g_{BL}, y_{\phi_1}, m_\psi$, the physical masses of ϕ_1, ϕ_2 and Z' (the $U(1)_{B-L}$ gauge boson) can be written as

$$m_{\phi_1}^2 = \mu_1^2 + \frac{1}{2}\lambda_{H\phi_1}v^2, \quad (3.28)$$

$$m_{\phi_2}^2 = 2\lambda_2v_2^2, \quad (3.29)$$

$$m_{Z'}^2 = 9g_{B-L}^2v_2^2, \quad (3.30)$$

where $v \left(= \sqrt{\frac{\mu^2}{\lambda}} \right)$ and $v_2 \left(= \sqrt{\frac{\mu_2^2}{\lambda_2}} \right)$ are the VEVs of the neutral component of the SM Higgs Φ and ϕ_2 respectively. As a result, the parameters μ_1, μ_2 and λ_2 can be traded for the parameters m_{ϕ_1}, m_{ϕ_2} and $m_{Z'}$. This leads to the free parameters as $m_{\phi_1}, m_{\phi_2}, m_{Z'}, \lambda_1, \lambda_{H\phi_1}, \lambda'_{\phi_1\phi_2}, g_{BL}, y_{\phi_1}, m_\psi$, out of which the relevant parameters for the phenomenology to be discussed are ² $m_{\phi_1}, m_{Z'}, m_\psi, \lambda_{H\phi_1}, g_{BL}, y_{\phi_1}$.

While we have a gauged $B - L$ symmetry, DM is neutral under $U(1)_{B-L}$ and its relic depends on the Dirac neutrino portal couplings in the spirit of light Dirac neutrino portal DM [260, 268]. Now, depending on the value of this Dirac neutrino portal Yukawa coupling y_{ϕ_1} , the DM analysis can be broadly divided into two categories namely, (a) FIMP ($y_{\phi_1} < 10^{-7}$) and (b) WIMP ($y_{\phi_1} > 10^{-7}$). In the first case, due to small Yukawa coupling, the DM ψ is produced non-thermally and dominantly from ϕ_1 decay. In the second case, DM can attain equilibrium in the early universe due to sizeable interactions. We now discuss these two broad cases one by one.

3.2.2 FIMP type dark matter

In this scenario, the Yukawa coupling among ν_R , DM and singlet scalar is small $y_{\phi_1} < 10^{-7}$ keeping the associated processes out-of-equilibrium throughout. While DM never thermalises, the RHNs can thermalise with the SM bath by virtue of $U(1)_{B-L}$

²The scalar ϕ_2 is not expected to play any role in dark matter analysis. First of all, DM does not couple directly to ϕ_2 . Also, ϕ_2 is heavier than the rest of the particles and hence does not appear in final states.

gauge interactions. Similar to the active neutrinos, the RHNs also contribute to the effective relativistic degrees of freedom defined as

$$N_{\text{eff}} \equiv \frac{8}{7} \left(\frac{11}{4} \right)^{4/3} \left(\frac{\rho_{\text{rad}} - \rho_\gamma}{\rho_\gamma} \right), \quad (3.31)$$

where ρ_{rad} is the net radiation content of the universe³. As mentioned earlier, the SM prediction is $N_{\text{eff}}^{\text{SM}} = 3.045$ [105–107]. In our model for thermal right handed neutrinos (ν_R), ΔN_{eff} can be estimated by finding the decoupling temperature T_{ν_R} of ν_R using

$$\Gamma(T_{\nu_R}) = \mathcal{H}(T_{\nu_R}) \quad (3.32)$$

where $\Gamma(T)$ is the interaction rate and $\mathcal{H}(T)$ is the expansion rate of the universe.

Let us consider the thermalisation of ν_R with the SM bath via the Z' portal interactions. Sufficiently large Z' portal interactions can lead to thermalisation of ν_R via the s-channel process $f\bar{f} \leftrightarrow \nu_R \bar{\nu}_R$ (here f denotes SM fermions). As a result, ν_R enters the thermal bath and then, after a certain period, it decouples from the bath. This generates the thermal contribution of ν_R to ΔN_{eff} [120]. Fig. 3.11 shows the thermal ΔN_{eff} as a function of coupling g_{BL} and gauge boson mass $m_{Z'}$. Thus for $g_{BL} \gtrsim 10^{-8}$, ΔN_{eff} receives a thermal contribution from light Dirac neutrinos which we denote as $\Delta N_{\text{eff}}^{\text{th}}$. The region corresponding to large Z' portal interactions, labelled as $\Delta N_{\text{eff}} > 0.28$ is disfavoured by Planck 2018 limits at 2σ CL.

On the other hand, due to the tiny dark sector Yukawa coupling y_{ϕ_1} , it is also possible to get a non-thermal contribution to ΔN_{eff} [123, 268], which we denote by $\Delta N_{\text{eff}}^{\text{non-th}}$. In addition to the Yukawa coupling y_{ϕ_1} involved in $\phi_1 - \nu_R - \psi$ coupling, the $U(1)_{B-L}$ portal coupling and scalar portal coupling $\lambda_{H\phi_1}$ can also play a crucial role in deciding the strength of this non-thermal contribution. While the total contribution to ΔN_{eff} , in general, is a combination of thermal contribution and non-thermal contribution, the latter can occur only if the decoupling of RHNs precedes the non-thermal or freeze-in production.

³Due to the presence of three relativistic RHNs, change in N_{eff} can be written as $\Delta N_{\text{eff}} = 3 \frac{\rho_{\nu_R}}{\rho_{\nu_L}} \Big|_{\text{CMB}} = 3 \frac{\rho_{\nu_R}}{\rho_{\nu_L}} \Big|_{10\text{MeV}} = 3 \left(\frac{T_{\nu_R}}{T_{\nu_L}} \right)^4 \Big|_{10\text{MeV}}$. Here ρ_{ν_L} denotes energy density of one species of SM neutrinos. For non-thermal RHNs, ρ_{ν_R} should be calculated by solving appropriate Boltzmann equation whereas for thermal RHNs, ρ_{ν_R} can be written in terms of its decoupling temperature.

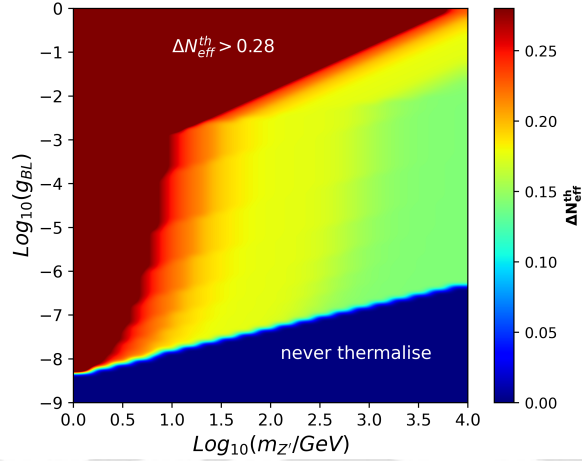


Figure 3.11: $\Delta N_{\text{eff}}^{\text{th}}$ as a function of $m_{Z'}$ and g_{BL} .

To study this non-thermal contribution $\Delta N_{\text{eff}}^{\text{non-th}}$ in details, let us first consider the situation where ϕ_1 remains in thermal equilibrium initially and then freezes out from the bath. The frozen out ϕ_1 later decays into ψ and ν_R , leading to non-thermal production of both dark matter and ν_R . The non-thermal contribution to ΔN_{eff} from ν_R can be calculated from the total energy density of frozen-in Dirac neutrinos.

The Boltzmann equation for comoving number density of ϕ_1 can be written as

$$\frac{dY_{\phi_1}}{dx} = \frac{\beta s}{\mathcal{H}x} \left(-\langle \sigma v \rangle_{\phi_1 \phi_1^\dagger \rightarrow X \bar{X}} ((Y_{\phi_1})^2 - (Y_{\phi_1}^{\text{eq}})^2) - \frac{\Gamma_{\phi_1}}{s} \frac{K_1(x)}{K_2(x)} Y_{\phi_1} \right), \quad (3.33)$$

where K_1 and K_2 are the modified Bessel functions of first and second kind respectively. The comoving abundance of species i is defined as $Y_i = n_i/s$ with n_i, s being number density of species i and entropy density of the universe respectively. $\mathcal{H}$ denotes the Hubble parameter. The variable x is defined as $x = m_{\phi_1}/T$ and $\beta' = 1 + \frac{T}{3g_*} \frac{dg_*}{dT}$ with g_*^s being the relativistic entropy degrees of freedom. Here $\langle \sigma v \rangle_{\phi_1 \phi_1^\dagger \rightarrow X \bar{X}}$ is the thermally averaged annihilation cross-section of ϕ_1 into the all allowed final state particles and Γ_{ϕ_1} denotes the decay rate of ϕ_1 into ν_R and ψ . The annihilation cross-section of ϕ_1 depends upon Higgs portal coupling $\lambda_{H\phi_1}$ as well as $U(1)_{B-L}$ portal coupling. Depending upon the strength of these individual couplings, the freeze-out abundance of ϕ_1 is either determined by scalar portal or gauge portal interactions. The freeze-in abundance of ψ and ν_R (from the decay of frozen out ϕ_1) can be obtained by solving the relevant Boltzmann equations given

by

$$\frac{dY_\psi}{dx} = \frac{\beta'}{x\mathcal{H}}\Gamma_{\phi_1}\frac{K_1(x)}{K_2(x)}Y_{\phi_1}, \quad (3.34)$$

$$\frac{d\tilde{Y}}{dx} = \frac{\beta'}{\mathcal{H}s^{1/3}x}\langle E\Gamma\rangle_{\phi_1}Y_{\phi_1}, \quad (3.35)$$

with

$$\Gamma_{\phi_1} = \frac{1}{32\pi}m_{\phi_1}y_{\phi_1}^2\left(1 - \frac{m_\psi^2}{m_{\phi_1}^2}\right)^2; \quad \langle E\Gamma\rangle_{\phi_1} = \frac{1}{64\pi}m_{\phi_1}^2y_{\phi_1}^2\left(1 - \frac{m_\psi^2}{m_{\phi_1}^2}\right)^3.$$

The term $\tilde{Y} = \frac{\rho_{\nu_R}}{s}$ represents the comoving energy density of RHNs. The preferred choice of energy density instead of number density of ν_R is based on the fact that the calculation of ΔN_{eff} requires comoving energy density of ν_R .

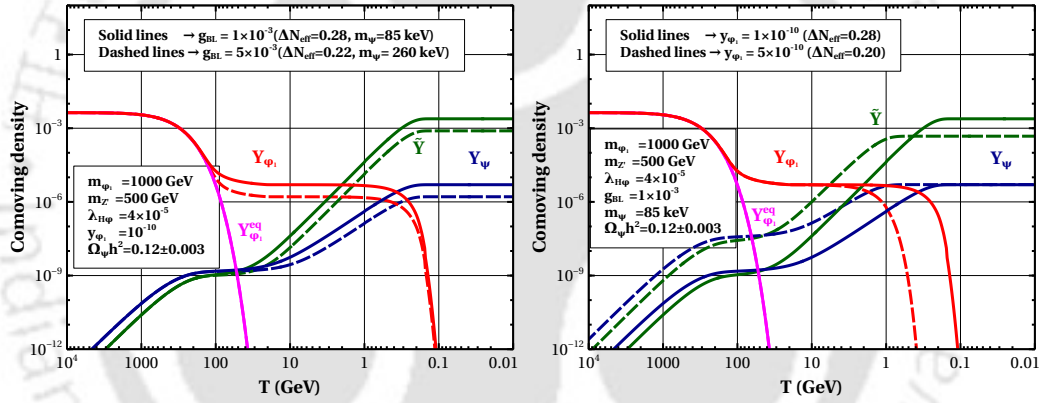


Figure 3.12: Evolution of comoving number density of ϕ_1 , ψ and comoving energy density of ν_R . The left (right) panel shows the variation with respect to g_{BL} (y_{ϕ_1}).

The non-thermal contribution to ΔN_{eff} namely, $\Delta N_{\text{eff}}^{\text{non-th}}$ depends on the parameters $m_{\phi_1}, m_{Z'}, \lambda_{H\phi_1}, g_{BL}$ and y_{ϕ_1} . While the parameters $m_{\phi_1}, m_{Z'}, \lambda_{H\phi_1}$ and g_{BL} determine the freeze-out abundance of ϕ_1 , the parameters that determine the decay width of ϕ_1 are m_{ϕ_1}, y_{ϕ_1} . Fig. 3.12 shows the evolution of comoving densities for chosen benchmark points clearly indicating the roles of g_{BL} and y_{ϕ_1} . While we only show the evolution of non-thermal contribution to ΔN_{eff} in this figure, the total ΔN_{eff} for the chosen benchmark values of parameters include both thermal and non-thermal contributions. In the left panel plot of Fig. 3.12, the solid line for $\tilde{Y}$ corresponds to an asymptotic non-thermal contribution $\Delta N_{\text{eff}}^{\text{non-th}} = 0.10$ while the same choice of parameters generates a thermal contribution $\Delta N_{\text{eff}}^{\text{th}} = 0.18$. Therefore, the total contribution of $\Delta N_{\text{eff}} = 0.28$, as indicated in the legends for the solid

line. For the dashed line, corresponding to a larger value of g_{BL} , the thermal contribution also comes out to be slightly larger $\Delta N_{\text{eff}}^{\text{th}} = 0.19$, as expected. However, a significant decrease in non-thermal contribution is observed for this benchmark point leading to $\Delta N_{\text{eff}}^{\text{non-th}} = 0.03$. Thus, for the dashed line in left panel plot of Fig. 3.12, we have total $\Delta N_{\text{eff}} = 0.22$. For the right panel plot of the same figure, the thermal contribution remains same $\Delta N_{\text{eff}}^{\text{th}} = 0.18$ as g_{BL} and $m_{Z'}$ are kept fixed and variation in Yukawa coupling y_{ϕ_1} in the non-thermal ballpark does not alter thermal contribution to ΔN_{eff} . The dashed line, with a larger y_{ϕ_1} , have a smaller $\Delta N_{\text{eff}}^{\text{non-th}} = 0.02$ and vice versa. As a result, for the dashed line in right panel plot, the total contribution is $\Delta N_{\text{eff}} = 0.20$. In both the left and right panel plots, the values of m_ψ are taken in such a way that it gives correct FIMP type dark matter relic. Also, we have seen that the value of m_ψ does not affect ΔN_{eff} as long as $m_\psi \ll m_{\phi_1}$.

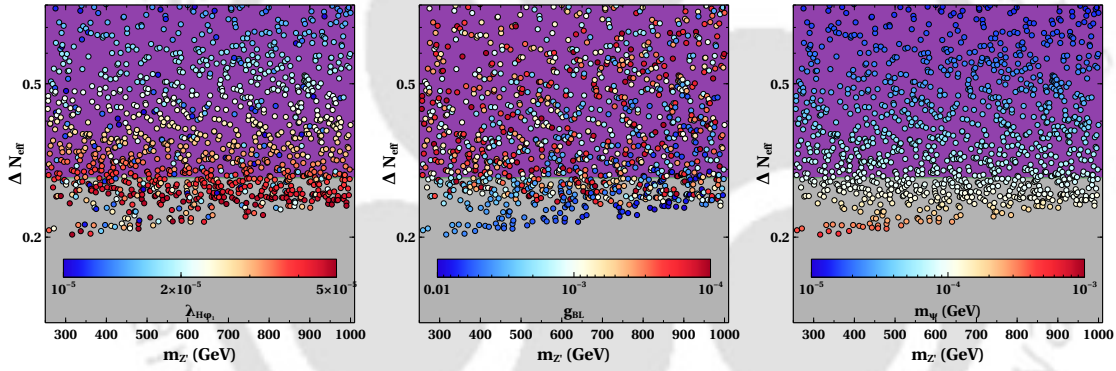


Figure 3.13: Scan plot showing the total ΔN_{eff} versus $m_{Z'}$ for different $\lambda_{H\phi_1}$, g_{BL} and m_ψ . The magenta coloured region is excluded by Planck 2018 bounds at 2σ CL. The region in grey color is within the reach of future experiments.

After studying the evolution of comoving densities for different benchmark points, we perform a numerical scan over key model parameters to find out the parameter space consistent with ΔN_{eff} and DM properties. We have kept m_{ϕ_1} fixed at 1000 GeV and y_{ϕ_1} at 10^{-10} . The rest of the parameters are varied in the following range:

$$\begin{aligned}
 250 \text{ GeV} &< m_{Z'} < 1000 \text{ GeV} \\
 10^{-5} &< \lambda_{H\phi_1} < 5 \times 10^{-5} \\
 10^{-4} &< g_{BL} < 10^{-2} \\
 10^{-5} \text{ GeV} &< m_\psi < 10^{-3} \text{ GeV}.
 \end{aligned} \tag{3.36}$$

The resulting parameter space in terms of ΔN_{eff} and $m_{Z'}$ is shown in Fig. 3.13. All the points shown in this figure satisfy the requirements of correct DM relic abundances. The magenta shaded region denotes the region excluded by Planck 2018 bounds at 2σ CL while the grey shaded region remains within the reach of future experiments like CMB-S4. The colour codes in the left, middle and right panel plots of Fig. 3.13 show the variation in $\lambda_{H\phi_1}$, g_{BL} , m_ψ respectively. As the left panel plot shows, decrease in $\lambda_{H\phi_1}$, while keeping m_{ϕ_1} constant, increases ΔN_{eff} . A smaller value of Higgs portal coupling $\lambda_{H\phi_1}$ leads to a larger freeze-out abundance of ϕ_1 followed by enhanced production of ν_R from ϕ_1 decay. Since the same decay is also responsible for freeze-in production of DM, we require smaller DM masses in order to keep its relic abundance within Planck limits, as seen from the right panel plot of Fig. 3.13. As the middle panel plot shows, small values of ΔN_{eff} typically correspond to smaller g_{BL} as the corresponding thermal contribution $\Delta N_{\text{eff}}^{\text{th}}$ decreases.

While we have incorporated the constraints from cosmological observations on ΔN_{eff} and DM relic abundance, there can be strong constraints on light dark matter from astrophysical structure formation. As discussed in Sec. 3.1, the FSL of DM depends primarily on the production temperature, DM mass and the production mechanism of DM or distribution function of DM. A higher production temperature gives a smaller FSL due to high momentum redshift making DM non-relativistic earlier and vice-versa. A different production mechanism also gives different FSL. In our case, the DM is produced due to decay of a frozen out scalar. Fig. 3.14 shows the average velocity of DM for four different benchmark parameters. The parameters are shown in Table 3.5 along with total ΔN_{eff} and FSL. The benchmark point (BP) II corresponds to a smaller ϕ_1 mass than BP I. Hence, the freeze-out abundance of ϕ_1 for BP II is smaller than that of BP I. This implies that a larger ψ mass is required to satisfy DM abundance. As a result, we obtain a smaller FSL for BP II. However, for both BP I and BP II, the computed FSL keeps DM in hot DM category and hence ruled out from structure formation constraints. The FSL can be reduced by increasing the production temperature as well as increasing DM mass. BP III and BP IV have a larger y_{ϕ_1} giving a higher production temperature. Similarly it has a larger $\lambda_{H\phi_1}$ coupling, giving smaller ϕ_1 freeze-out abundance. Hence for both BP III and BP IV, DM masses can be large while being consistent with relic abundance criteria. Combining these two effects, we get warm dark matter for BP III and BP IV. From the resulting on ΔN_{eff} , it can be seen that both BP III and BP IV remain within the sensitivity of future CMB experiments like CMB-S4.

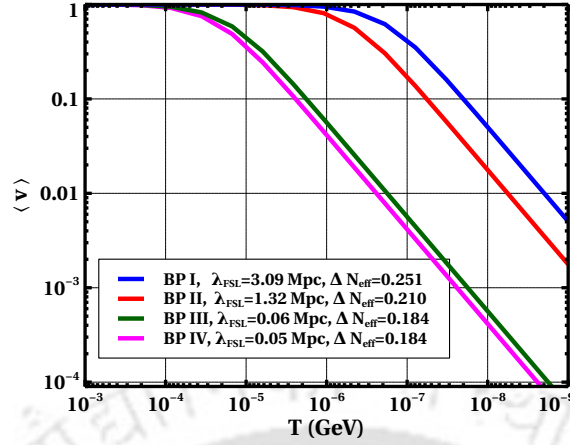


Figure 3.14: Thermal average velocity of FIMP dark matter as a function of temperature for four different benchmark points.

	Parameters						$\Omega_{\text{DM}} h^2$	ΔN_{eff}	FSL(Mpc)
	m_{ϕ_1} (GeV)	$\lambda_{H\phi_1}$	y_{ϕ_1}	$m_{Z'}$ (GeV)	g_{BL}	m_{ψ} (keV)			
BP I	1000	5×10^{-5}	10^{-10}	500	0.001	126	0.12	0.251	3.09
BP II	500	5×10^{-5}	10^{-10}	500	0.001	233	0.12	0.210	1.32
BP III	1000	2×10^{-4}	10^{-9}	500	0.001	970	0.12	0.184	0.06
BP IV	500	2×10^{-4}	10^{-9}	500	0.001	938	0.12	0.184	0.05

Table 3.5: Benchmark parameters for FIMP DM

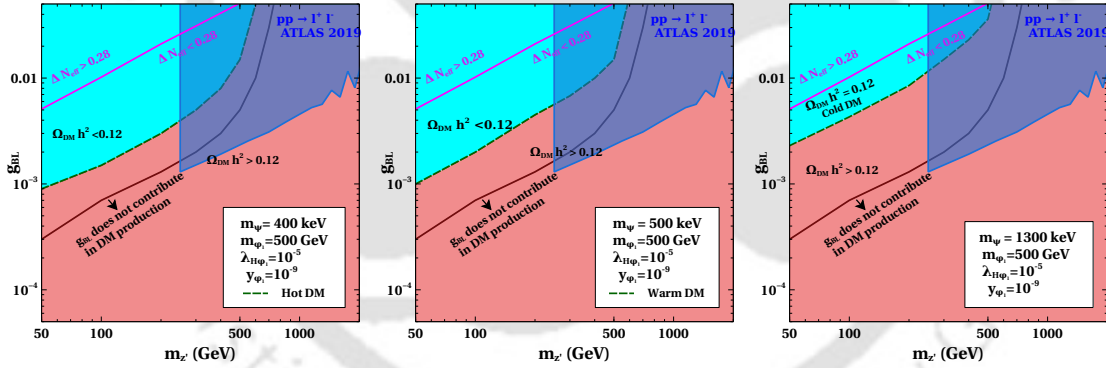


Figure 3.15: Parameter space in $m_{Z'}$ versus g_{BL} plane for the FIMP scenario considering three different values of DM mass. The other parameters are kept fixed as $m_{\phi_1} = 500$ GeV, $\lambda_{H\phi_1} = 10^{-5}$, $y_{\phi_1} = 10^{-9}$.

Fig. 3.15 shows the parameter space in $m_{Z'}$ versus g_{BL} parameter space for the FIMP scenario. The left, middle and right panel plots correspond to dark matter mass of 400 keV, 500 keV and 1300 keV respectively. The other relevant parameters are fixed as $m_{\phi_1} = 500$ GeV, $\lambda_{H\phi_1} = 10^{-5}$, $y_{\phi_1} = 10^{-9}$. For the region below the solid brown line in each of these plots, $U(1)_{B-L}$ -portal coupling does not play any role in FIMP DM production. In that region, the freeze-out abundance of ϕ_1 , responsible for DM production, is determined by the Higgs-portal coupling $\lambda_{H\phi_1}$. In other words,

the cross section $\langle\sigma v\rangle_{\phi_1\phi_1^\dagger\rightarrow SM\overline{SM}}$ dominates over $\langle\sigma v\rangle_{\phi_1\phi_1^\dagger\rightarrow Z'Z'}$. In the region above the solid brown line, the freeze-out abundance of ϕ_1 is determined by the $U(1)_{B-L}$ -portal coupling. The dashed line in the left and middle panel plots, separating the pink and cyan shaded regions, represent the parameter space satisfying correct DM relic for $m_\psi = 400$ keV and $m_\psi = 500$ keV respectively. However, for DM mass $m_\psi = 400$ keV, the free-streaming length turns out to be very large ~ 0.1 Mpc, keeping it in hot DM ballpark and hence ruled out. In the middle panel plot, due to a higher DM mass $m_\psi = 500$ keV, the dashed line corresponds to an intermediate FSL keeping it in the warm DM regime which is still allowed by structure formation constraints. In the region below the dashed line, the freeze-out abundance of ϕ_1 is more than that for the dashed line, leading to a larger FIMP DM abundance $\Omega_{\text{DM}}h^2 > 0.12$, produced from ϕ_1 decay. On the other hand, the region above the dashed line have $\Omega_{\text{DM}}h^2 < 0.12$ due to smaller freeze-out abundance of ϕ_1 . This is understood from the fact that a larger $U(1)_{B-L}$ -portal interaction leads to smaller freeze-out abundance of ϕ_1 and vice versa. While the total ΔN_{eff} can have both thermal and non-thermal contributions, for the choice of parameters in Fig. 3.15, the non-thermal contribution is suppressed compared to the thermal contribution. The magenta solid line corresponds to $\Delta N_{\text{eff}} = 0.28$. The region above this line has $\Delta N_{\text{eff}} > 0.28$ and hence ruled out from Planck 2018 limits. In the right panel plot, the dashed line corresponds to relic satisfying region for DM mass $m_\psi = 1300$ keV. The region below the dashed line correspond to over-abundance like before. However, the region above the dashed line is no longer under-abundant, but satisfies the current DM abundance. For this region of parameter space and chosen DM mass, the production of FIMP DM while ϕ_1 is in equilibrium dominates compared to the production after ϕ_1 freeze-out. This can also be understood from the evolution plots shown in Fig. 3.12. From the left panel plot of Fig. 3.12, we see that there is a non-zero yield of DM Y_ψ from the decay of ϕ_1 when the latter is in equilibrium. Additionally, the freeze-out abundance of ϕ_1 ($Y_{\phi_1}(x \rightarrow \infty)$) decreases as we increase g_{BL} . Combining these two, we can have a situation where for sufficiently large $U(1)_{B-L}$ -portal couplings, Y_ψ from the decay of ϕ_1 in equilibrium is larger compared to the post freeze-out production. Hence, $U(1)_{B-L}$ -portal couplings do not determine the comoving abundance of Y_ψ and as a result, the region above the dashed line satisfies the correct DM relic in the right panel plot of Fig. 3.15. In all the plots, the region below the magenta solid line has $0.28 > \Delta N_{\text{eff}} > 0.14$ keeping it within reach of future CMB experiments like CMB-S4. The blue shaded region towards

the upper right corner indicates the region ruled out from the large hadron collider (LHC) bounds, for specifically from ATLAS experiments at 13 TeV centre of mass energy [304, 305]. Similar bounds also exist from the CMS experiment at the LHC [306]. The details of the LHC bound on $g_{BL} - m_{Z'}$ plane is given in Appendix A.4. A relatively weaker bound exists from the large electron positron (LEP) collider disfavouring the region $m_{Z'}/g_{BL} < 7$ TeV [307, 308]. The parameter space shown in Fig. 3.15 already satisfies the LEP bound.

When ϕ_1 is always in equilibrium:

Before moving onto the WIMP DM scenario, we briefly comment on the possibility of FIMP DM production from ϕ_1 when the latter remains in equilibrium throughout the production. When ϕ_1 is in equilibrium during the production of DM, the comoving number density of ψ and comoving energy density of ν_R , respectively, are given as

$$\frac{dY_\psi}{dx} = \frac{\beta}{x\mathcal{H}} \Gamma_{\phi_1} \frac{K_1(x)}{K_2(x)} Y_{\phi_1}^{\text{eq}}, \quad (3.37)$$

$$\frac{d\tilde{Y}}{dx} = \frac{\beta}{\mathcal{H}s^{1/3}x} \langle E\Gamma \rangle_{\phi_1} Y_{\phi_1}^{\text{eq}}. \quad (3.38)$$

These two equations can be analytically solved to get the asymptotic abundances (in the limit $m_\psi \ll m_{\phi_1}$)

$$\begin{aligned} Y_\psi(x \rightarrow \infty) &= \frac{135}{16\pi \times 1.66 \times 8\pi^3 g_*^s \sqrt{g_*^\rho}} M_{\text{pl}} \frac{y_{\phi_1}^2}{m_{\phi_1}}, \\ \tilde{Y}(x \rightarrow \infty) &= \frac{675}{32\pi \times 1.66 \times 8\pi^3 g_*^s \sqrt{g_*^\rho}} \left(\frac{45}{2\pi^2 g_*^s} \right)^{1/3} M_{\text{pl}} \frac{y_{\phi_1}^2}{m_{\phi_1}}, \end{aligned} \quad (3.39)$$

where M_{pl} is the Planck mass. With this, the DM abundance and contribution to extra radiation energy density can be calculated as -

$$\begin{aligned} \Omega_{\text{DM}} h^2 &= 2 \frac{m_\psi s^0}{\rho_c^0} Y_\psi(x \rightarrow \infty) h^2 \\ \Delta N_{\text{eff}}^{\text{non-th}} &= 2 \times 3 \times \left(\frac{s^{4/3}}{\rho_{\nu_1}} \right)_{T=10 \text{ MeV}} \tilde{Y}(x \rightarrow \infty) \simeq 0.61 \tilde{Y}(x \rightarrow \infty), \end{aligned} \quad (3.40)$$

where s^0 and ρ_c^0 denote the entropy density and critical energy density, respectively, at the present epoch. Here, m_{ϕ_1}, y_{ϕ_1} and m_ψ determine the DM abundance and

$\Delta N_{\text{eff}}^{\text{non-th}}$ is determined by m_{ϕ_1} and y_{ϕ_1} only. The thermal contribution to ΔN_{eff} is determined by the parameters $m_{Z'}$ and g_{BL} . When DM abundance is satisfied, it turns out that the non-thermal contribution to ΔN_{eff} is way below the minimum thermal contribution. So, the total ΔN_{eff} is only determined by the thermal contribution (shown in Fig. 3.11).

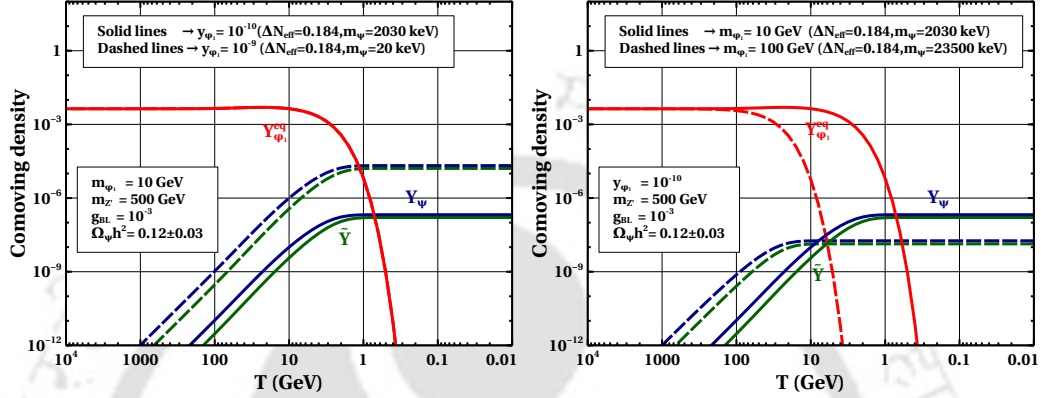


Figure 3.16: Evolution of comoving number densities of ϕ_1 and ψ and comoving energy density of ν_R for two different values of y_{ϕ_1} (left panel) and m_{ϕ_1} (right panel).

	Parameters					$\Omega_{\text{DM}} h^2$	ΔN_{eff}	FSL(Mpc)
	m_{ϕ_1} (GeV)	y_{ϕ_1}	$m_{Z'}$ (GeV)	g_{BL}	m_{ψ} (keV)			
BP I	10	10^{-10}	500	0.001	2030	0.12	0.184	0.0009
BP II	10	10^{-9}	500	0.001	20	0.12	0.184	0.06
BP III	100	10^{-10}	500	0.001	23500	0.12	0.184	0.0001
BP IV	100	10^{-9}	500	0.001	235	0.12	0.184	0.0064

Table 3.6: Table for FIMP DM (equilibrium ϕ_1)

Fig. 3.16 shows evolutions of comoving abundances of ϕ_1 , ψ and ν_R for two different values of y_{ϕ_1} (left panel) and m_{ϕ_1} (right panel). For $m_{\psi} \ll m_{\phi_1}$ the evolutions for comoving number density of ψ and comoving energy density of ν_R are independent of m_{ψ} . So in all the evolution plots, the dark matter masses are chosen in such a way that observational DM abundance is satisfied. The solid lines in the left plot have $y_{\phi_1} = 10^{-10}$ and DM mass $m_{\psi} = 2030$ keV. The non-thermal contribution to ΔN_{eff} from frozen-in ν_R is 6.5×10^{-6} . For the dashed lines, the Yukawa coupling is larger by a factor of 10. Consequently, we have higher $Y_{\psi}(x \rightarrow \infty)$ and $\tilde{Y}(x \rightarrow \infty)$. From Eq. (3.40), we get DM mass to be 20 keV and $\Delta N_{\text{eff}}^{\text{non-th}} = 6.5 \times 10^{-4}$. The solid lines in the right plot have the same parameters as those of in the left panel plot. The dashed lines in the right panel plot have $m_{\phi_1} = 100$ GeV. Due to higher mass of ϕ_1 , the Boltzmann suppression occurs early on, leading to smaller $Y_{\psi}(x \rightarrow \infty)$ and $\tilde{Y}(x \rightarrow \infty)$. As a result DM mass for the solid lines is about 23.5 MeV and

$\Delta N_{\text{eff}}^{\text{non-th}} = 5.5 \times 10^{-7}$. We have kept $B - L$ gauge coupling and mass fixed for all the plots, $g_{BL} = 10^{-3}$ and $m_{Z'} = 500$ GeV. This gives a thermal contribution to $\Delta N_{\text{eff}} = 0.184$. As in all the plots non-thermal contribution is way smaller than the thermal counterpart, so the total ΔN_{eff} is determined by the thermal contribution as shown in the plot. Since $U(1)_{B-L}$ -portal parameters do not decide DM abundance in this case, we do not show any summary plot like before.

Table 3.6 shows four benchmark points (BP) and their corresponding contribution to effective number of relativistic species and structure formation. BP I and BP II have same $m_{\phi_1} = 10$ GeV whereas BP III and BP IV have $m_{\phi_1} = 100$ GeV. As the Yukawa coupling is ten times larger in BP II compared to BP I, the corresponding DM mass is 100 times smaller in BP II (see Eq. (3.39)). The similar trend is followed between BP III and BP IV. Due to smaller DM mass, λ_{FSL} is larger in both the BP II (compared to BP I) and BP IV (compared to BP III). Again from Eq. (3.40), we get that m_{ϕ_1} and m_{ψ} are correlated for constant DM abundance. Hence a larger m_{ϕ_1} gives a larger m_{ψ} (BP III, BP IV vs BP I, BP II). The DM production temperature ($T \sim m_{\phi_1}$) is also larger for larger m_{ψ_1} . Combining these two, we get a smaller λ_{FSL} for BP III (BP IV) compared to BP I (BP II). Among the four benchmark points mentioned in the table, only BP II gives warm DM whereas rest points give cold DM. As all the points have same $B - L$ gauge coupling and mass, we have same ΔN_{eff} .

3.2.3 WIMP type dark matter

If the Yukawa coupling among the dark sector particles i.e. ϕ_1 , ψ and ν_R is sufficiently strong ($y_{\phi_1} \gg 10^{-7}$), the dark sector can be in thermal equilibrium among themselves even after their decoupling from the SM bath. Since the thermalisation of ϕ_1, ν_R with the SM bath relies on Higgs portal and $U(1)_{B-L}$ portal couplings instead of Yukawa y_{ϕ_1} , we have two different sub-cases depending upon whether the Yukawa interactions in dark sector go out of equilibrium before or after the dark sector decouples from the thermal bath. Among the dark sector particles, ϕ_1 and ν_R can have interactions with other particles in the bath via the processes - $\phi_1 X \rightarrow \phi_1 X$, $\phi_1 Z' \rightarrow \phi_1 Z'$ and $\nu_R X \rightarrow \nu_R X$, here X denotes SM particles. Since Z' has sizeable interactions with all the SM fermions, we consider it to be in the bath while dark sector particles decouple. While the first two processes can arise via

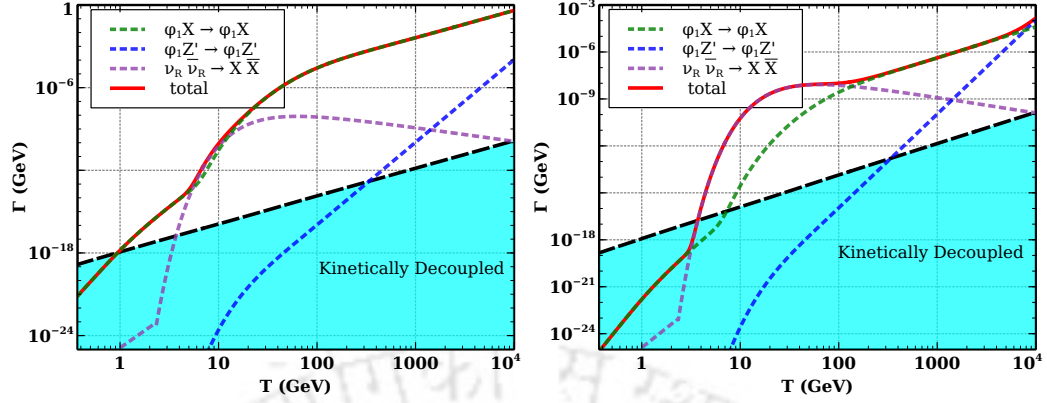


Figure 3.17: Interaction rate as a function of temperature for processes responsible for keeping dark sector in equilibrium with the SM bath. The benchmark parameters chosen are $\lambda_{H\phi_1} = 10^{-2}$ (left panel), 10^{-4} (right panel), $m_{\phi_1} = 100$ GeV, $m_{Z'} = 100$ GeV, $g_{BL} = 10^{-4}$.

contact interactions too, the ν_R scattering arises via mediation of heavy Z' boson only. Unlike other interactions, ν_R interactions with the bath rely significantly on resonant enhancement and hence we consider $\nu_R \bar{\nu}_R \rightarrow X \bar{X}$ via resonantly enhanced s-channel mediation of Z' to calculate its decoupling [120]. Therefore, the ratio of interaction rate of dark sectors with SM bath to the expansion rate, $\mathcal{H}$, can be written as [309, 310]

$$\frac{\Gamma_{\text{total}}}{\mathcal{H}} = \frac{1}{\mathcal{H}} \left[n_X^{\text{eq}} (\langle \sigma v \rangle_{\phi_1 X \rightarrow \phi_1 X} + \langle \sigma v \rangle_{\phi_1 Z' \rightarrow \phi_1 Z'}) + n_{\nu_R}^{\text{eq}} \langle \sigma v \rangle_{\nu_R \bar{\nu}_R \rightarrow X \bar{X}} \right]. \quad (3.41)$$

The decoupling temperature of dark sector is approximately calculated by comparing the total interaction rate with the Hubble expansion rate i.e. $\frac{\Gamma_{\text{total}}}{\mathcal{H}} \approx 1$. Fig. 3.17 shows interaction rates of various processes responsible for keeping the dark sector in bath with the SM as a function of temperature. In the left panel plot, a large scalar portal coupling $\lambda_{H\phi} = 10^{-2}$ dictates the decoupling temperature whereas in the right panel plot, the resonantly enhanced process $\nu_R \bar{\nu}_R \rightarrow X \bar{X}$ determines the decoupling temperature due to smaller scalar portal coupling $\lambda_{H\phi} = 10^{-4}$.

To track the evolution of dark sector, one has solve the Boltzmann equation in terms of comoving number densities Y_{ϕ_1} and Y_{ψ} . Using the approximation $\frac{Y_i}{Y} \approx \frac{Y_i^{\text{eq}}}{Y^{\text{eq}}}$ ($i = \phi_1, \psi$), with $Y = Y_{\phi_1} + Y_{\psi}$, we can write a single Boltzmann equation as

$$\frac{dY}{dx} = \frac{1}{2} \frac{\beta s}{\mathcal{H} x} \langle \sigma v \rangle_{\text{eff}} ((Y^{\text{eq}})^2 - Y^2), \quad (3.42)$$

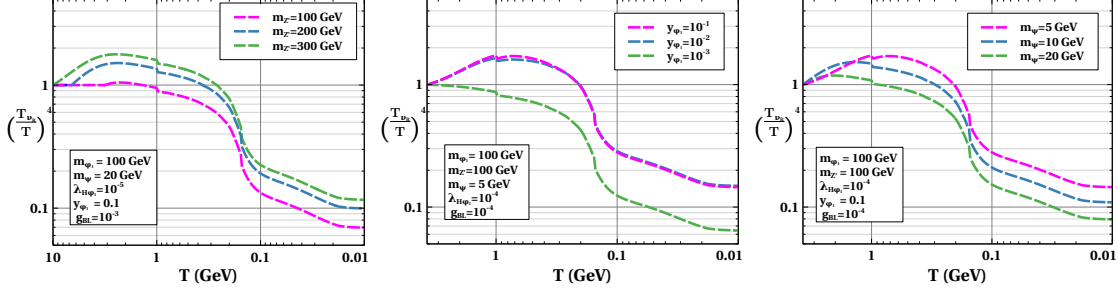


Figure 3.18: Evolution of dark sector temperature after its decoupling from SM bath ($T < T_{\text{dec}}$) for different values of $m_{Z'}$ (left panel), dark sector Yukawa coupling y_{ϕ_1} (middle panel) and dark matter mass m_ψ (right panel). The rest of the parameters are kept fixed and are shown in respective plots.

where

$$\langle \sigma v \rangle_{\text{eff}} = \frac{(Y_{\phi_1}^{\text{eq}})^2 \langle \sigma v \rangle_{\phi_1 \phi_1^\dagger \rightarrow X \bar{X}, \nu_R \bar{\nu}_R, Z' Z'} + (Y_\psi^{\text{eq}})^2 \langle \sigma v \rangle_{\psi \bar{\psi} \rightarrow \nu_R \bar{\nu}_R}}{(Y_{\phi_1}^{\text{eq}} + Y_\psi^{\text{eq}})^2}. \quad (3.43)$$

The Eq. (3.42) can be solved numerically from a sufficiently high temperature to the decoupling temperature, T_{dec} . If the Yukawa coupling y_{ϕ_1} is large enough, even after $T < T_{\text{dec}}$, the dark sector particles ϕ_1 , ψ and ν_R can be in dark sector equilibrium among themselves. We can track the dark sector temperature $T_{\nu_R} (\equiv T_{\text{DS}})$ by solving the following equation.

$$\frac{d\xi}{dx} = \frac{1}{x} \left(-\frac{1}{2} \frac{\beta x^4 s^2}{4\alpha \xi^3 H m_{\phi_1}^4} \langle E \sigma v \rangle_{\text{eff}} ((Y^{\text{eq}})^2 - Y^2) - (\beta - 1) \xi \right), \quad (3.44)$$

where

$$\langle E \sigma v \rangle_{\text{eff}} = \frac{(Y_{\nu_R}^{\text{eq}})^2 \langle E \sigma v \rangle_{\nu_R \bar{\nu}_R \rightarrow \psi \bar{\psi}} + (Y_{\nu_R}^{\text{eq}})^2 \langle E \sigma v \rangle_{\nu_R \bar{\nu}_R \rightarrow \phi_1 \phi_1^\dagger}}{(Y_{\phi_1}^{\text{eq}} + Y_\psi^{\text{eq}})^2}, \quad (3.45)$$

and ξ is the ratio of dark sector temperature to that of the SM bath, $\frac{T_{\nu_R}}{T}$ and $\alpha = 6 \times \frac{7}{8} \frac{\pi^2}{30}$. After decoupling, $\langle \sigma v \rangle_{\text{eff}}$ in Eq. (3.42) can depend upon both x as well as ξ . We solve Eq. (3.42) and Eq. (3.44) simultaneously from $T = T_{\text{dec}}$ to the onset of the BBN era namely, $T \sim 10$ MeV and estimate the corresponding DM relic and ΔN_{eff} . Fig. 3.18 shows the evolution of T_{ν_R} for different benchmark parameters. Different T_{ν_R} can lead to changes in ΔN_{eff} . The first term on the RHS of Eq. (3.44) is a positive term as $Y \geq Y^{\text{eq}}$ for $T < T_{\text{dec}}$. Thus, this term indicates the increase in T_{ν_R} over T due to annihilation of heavy dark sector particles ϕ_1 and

ψ into ν_R . On the other hand, the second term on the RHS of the same equation indicates the decrease in T_{ν_R} over T due to changes in entropy degrees of freedom g_*^s . The combined effects of these two terms can be seen in Fig. 3.18. For an analytical description of the behaviour of the plots shown in Fig. 3.18, please refer to Appendix A.5.

The left panel plot in Fig. 3.18 shows the variation of ξ^4 with respect to bath temperature T for different choices of $m_{Z'}$. The rest of the parameter are kept fixed as shown in the plot. Increasing $m_{Z'}$ gives a higher decoupling temperature, T_{dec} . As long as $T_{\text{dec}} \lesssim m_{\phi_1}$, a higher decoupling temperature leads to a larger conversion of ϕ_1 and ψ to ν_R resulting in a larger $\frac{T_{\nu_R}}{T}$. Another way to understand it is from entropy conservation (see Appendix A.5). For a higher T_{dec} , we have a larger dark sector entropy degrees of freedom g_{*s}^{DS} resulting in a larger $\frac{T_{\nu_R}}{T}$. From an analytical estimation, we get a maximum value of ξ^4 to be 2.6 provided g_{*s} remains same from the epoch of dark sector decoupling to DM freeze-out, as shown in Appendix A.5. This is consistent with the numerical results shown in left panel of Fig. 3.18. The reason why maximum value of ξ^4 has not reached 2.6 in the figure is that g_{*s} changes from the period of kinetic decoupling of dark sector to DM freeze-out. The middle panel plot shows the results for different values of dark sector Yukawa coupling. As the dark sector Yukawa coupling does not affect the DM-SM decoupling temperature, so all the coloured lines correspond to the same decoupling temperature. For a larger Yukawa coupling, conversion of ϕ_1 and ψ to ν_R is possible and as a result, temperature of ν_R with respect to SM bath increases. For the green line in the middle panel plot of Fig. 3.18, the dark sector freeze-out occurs before T_{dec} due to small Yukawa coupling. Hence the first term on the RHS of Eq. (3.44) ceases and due to the second term, the ratio $\frac{T_{\nu_R}}{T}$ decreases. To see the effect of m_ψ , in the right panel plot, we vary m_ψ keeping the other parameter constant. For a smaller m_ψ , freeze-out occurs late, hence we get a higher $\frac{T_{\nu_R}}{T}$ ratio.

To see the complete picture in WIMP dark matter scenario, we perform a numerical scan by varying the following parameters as

$$\begin{aligned} 50\text{GeV} &< m_{\phi_1} < 150\text{GeV}, & 100\text{GeV} &< m_{Z'} < 500\text{GeV}, & (3.46) \\ 10\text{GeV} &< m_\psi < 100\text{GeV}, & 10^{-5} &< \lambda_{H\phi_1} < 10^{-2}, \\ 10^{-5} &< g_{BL} < 10^{-3}, & 0.2 &< y_{\phi_1} < 0.3. \end{aligned}$$

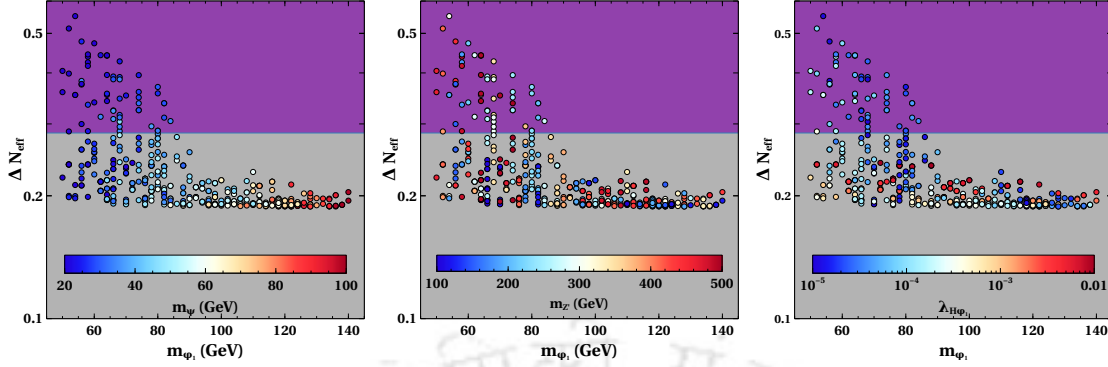


Figure 3.19: Scan plot showing ΔN_{eff} vs m_{ϕ_1} for different dark matter mass m_ψ and different $m_{Z'}$ and g_{BL} .

The corresponding results are shown in Fig. 3.19 with y-axis representing ΔN_{eff} whereas x-axis showing the mass of ϕ_1 . The relevant parameters m_ψ , $m_{Z'}$ and g_{BL} are shown in colour bars of left, middle and right panel plots respectively of Fig. 3.19. All the points shown in this figure satisfy the requirements of correct WIMP DM relic abundance. The magenta shaded region denotes the region excluded by Planck 2018 bounds at 2σ CL while the grey shaded region remains within the reach of future experiments like CMB-S4. From the left panel plot, we can see that for some points with dark matter mass $m_\psi \lesssim 50$ GeV are already excluded by Planck 2018 data at 2σ CL. Similarly, from the middle and right panel plots, we get some points having $m_{Z'}$ between 100 GeV and 500 GeV and g_{BL} between 10^{-3} and 10^{-5} GeV, that are already excluded. All the others points in these plots can be probed in the future CMB experiments like CMB-S4 keeping the detection prospects promising.

As we have discussed above, the kinetic decoupling of the dark sector (ϕ , ψ and ν_R) from its bath is determined by both the Higgs-portal coupling and B-L gauge coupling. After the kinetic decoupling, due to large Yukawa coupling, dark sector maintain a dark equilibrium. An increase in $\lambda_{H\phi_1}$ coupling or g_{BL} coupling keeps the dark sector in bath for longer. Increasing the g_{BL} coupling to a very high value gives us a scenario where the dark matter freezes out before the decoupling of ν_R . In this case, we do not have a separate dark sector evolution different from the SM bath. Hence, we have two regimes namely, regime 1: DM freezes out when $\xi_{\text{fo}} \neq 1$ (i.e. separate dark sector); regime 2: DM freezes out when $\xi_{\text{fo}} = 1$ (no separate dark sector). Depending upon whether kinetic decoupling temperature is determined by gauged B-L coupling (sub-case 1) or Higgs-portal coupling (sub-case 2) we can have two further sub-cases.

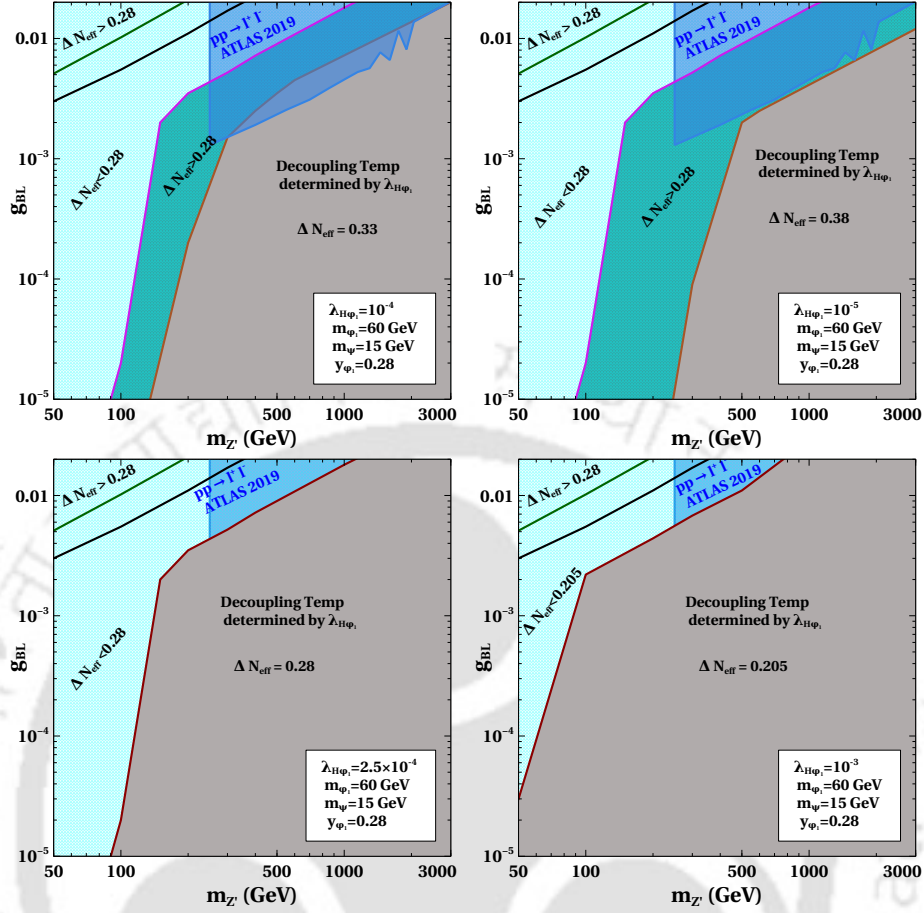


Figure 3.20: Bound on $m_{Z'}$ vs g_{BL} for the WIMP scenario with different values of $\lambda_{H\phi_1}$. The solid black line separates regime 1 (below the solid black line) and regime 2 (above the solid black line). In all the plots, $m_{\phi_1} = 60$ GeV, $m_{\psi} = 15$ GeV, $y_{\phi_1} = 0.28$.

In the plots shown in Fig. 3.20, we show the parameter space in $m_{Z'}$ versus g_{BL} plane for WIMP type DM. Each of these plots correspond to a fixed value of $\lambda_{H\phi_1}$. The other parameters fixed in all the plots are, $m_{\phi_1} = 60$ GeV, $m_{\psi} = 15$ GeV, $y_{\phi_1} = 0.28$. The solid black line separates regime 1 (upper left triangular region) from regime 2. The solid brown line separates the regime where kinetic decoupling temperature is determined by Higgs portal coupling from the one where it is determined by the gauge portal coupling. In the grey coloured region below the solid brown line, g_{BL} and $m_{Z'}$ have no role in the kinetic decoupling of dark sector. In the upper panel plots, the region below the magenta line is where we have $\Delta N_{\text{eff}} > 0.28$, disfavoured by Planck 2018 limits. In the grey coloured region, due to constant decoupling temperature, we have constant ΔN_{eff} . Due to smaller Higgs-portal coupling in the upper right panel plot compared to the upper left one,

the grey coloured region shrinks. In the lower left panel plot, we increase the Higgs-portal coupling compared to upper left panel plot to 2.5×10^{-4} . For this particular choice of $\lambda_{H\phi_1}$, the region between the black and brown line has $\Delta N_{\text{eff}} < 0.28$ and the region below the brown line has $\Delta N_{\text{eff}} = 0.28$, which is the maximum allowed value from Planck 2018 limits. In the lower right panel plot, $\lambda_{H\phi_1}$ is increased even further to 10^{-3} giving $\Delta N_{\text{eff}} < 0.28$ in the whole regime 1. For regime 2, as DM freezes out before RHN decoupling, the bound from ΔN_{eff} (denoted by region above the green line) is same as the bound shown in Fig. 3.11. The entire parameter space shown in these plots satisfy the criteria of DM relic abundance. This is because DM abundance has very weak dependence on the kinetic decoupling temperature. For the WIMP type DM scenario, we also check the constraints from direct detection and find them to be very weak due to radiative suppression. The details are given in Appendix A.6.

3.2.4 Summary

We have studied a possible UV completion of the light Dirac neutrino portal dark matter scenario. In such a scenario, right chiral parts of light sub-eV Dirac neutrinos act like a portal between dark and visible sectors responsible for the production of dark matter. A gauged $U(1)_{B-L}$ symmetry provides one possible UV completion by naturally accommodating right chiral parts of neutrinos from anomaly cancellation requirements while also preventing direct coupling of DM, a gauge singlet Dirac fermion with the SM required for its stability. Keeping $U(1)_{B-L}$ symmetry breaking scale upto a few TeV ballpark, we study the details of dark matter production together with additional relativistic degrees of freedom ΔN_{eff} brought in by right chiral parts of light Dirac neutrinos. While the chosen values of $U(1)_{B-L}$ gauge couplings ensures a non-zero thermal contribution to ΔN_{eff} , DM production can be either purely thermal or non-thermal depending upon the light Dirac neutrino portal Yukawa coupling. Although dark matter does not face stringent direct detection bounds due to loop-suppressed couplings with the SM quarks and charged leptons, the parameter space can be tightly constrained from other constraints related to structure formation, CMB constraints on ΔN_{eff} , collider constraints on $U(1)_{B-L}$ gauge bosons. We show interesting correlations in the parameter space from simultaneous requirement of correct DM phenomenology and ΔN_{eff} indicating the region within reach of future CMB experiments.

Primordial black holes and their role in probing new physics

In this chapter, we investigate the role of PBHs as probes of BSM scenarios, with particular emphasis on dark radiation, axions, and leptogenesis. Our analysis focuses on the ultra-light PBH mass regime, where the initial PBH mass lies within the range $0.1 \text{ g} \lesssim M_{\text{in}} \lesssim 4 \times 10^8 \text{ g}$.

In Section 4.1, we examine a hybrid production scenario in which dark radiation is generated not only through PBH evaporation but also from the thermal bath due to sizeable interactions between dark radiation and the Standard Model plasma. We demonstrate that, in certain regions of parameter space, the prospects for detection through measurements of ΔN_{eff} in CMB are significantly enhanced compared to the purely non-thermal scenario in which dark radiation originates exclusively from PBH evaporation. On the other hand, for certain part of the parameter space, DR which initially decouples from the bath followed by its production from PBH evaporation, can re-enter the thermal bath leading to much tighter constraints on the PBH parameter space. Furthermore, we discuss complementary detection prospects through the observation of stochastic gravitational waves sourced by PBH-induced density perturbations. The results presented in Section 4.1 are based on the published work [173].

In Section 4.2, we present a scenario in which the axion scale, f_a , coincides with the leptogenesis scale within the framework of a seesaw mechanism. The fundamental parameter in QCD axion models is the scale of the PQ phase transition, f_a , commonly referred to as the axion decay constant. In models of light QCD axions, astrophysical observations impose a lower bound on this scale, $f_a \gtrsim 10^8$ GeV [311, 312]. Furthermore, the misalignment mechanism provides a well-motivated region around $f_a \sim 10^{11}$ GeV in which the observed dark matter relic abundance can be naturally reproduced [313]. The PQ mechanism therefore not only provides an elegant solution to the strong CP problem but also offers a viable dark matter candidate while remaining consistent with astrophysical constraints, particularly for high-scale PQ symmetry breaking. This naturally motivates the question of whether other cosmological phenomena associated with high-energy scales may share a common origin with axion physics. In this context, Section 4.2 explores a scenario in which the generation of the baryon asymmetry of the Universe (BAU) through leptogenesis is intrinsically linked to the PQ symmetry breaking scale.

We demonstrate that the presence of PBHs can render this framework observationally testable through the stochastic gravitational wave background, in addition to conventional axion searches. The results presented in Section 4.2 are based on the published work [314].

4.1 Probing thermalised dark radiation in the presence of PBHs

4.1.1 Dark radiation from PBHs

Considering PBH to be formed in the early radiation-dominated universe at a temperature, say T_{in} , the initial mass of PBH is related to the mass enclosed in the particle horizon and is given by [147, 172, 315]

$$M_{\text{in}} = \frac{4\pi}{3} \gamma \frac{\rho_{\text{R}}(T_{\text{in}})}{\mathcal{H}^3(T_{\text{in}})}, \quad (4.1)$$

where $\gamma \approx 0.2$ [147], $\rho_{\text{R}}(T_{\text{in}})$ is the initial radiation density and $\mathcal{H}$ is the Hubble expansion rate. The temperature of a black hole can be related to its mass as [316]

$$T_{\text{BH}} = \frac{1}{8\pi G M_{\text{BH}}} \approx 1.06 \left(\frac{10^{13} \text{ g}}{M_{\text{BH}}} \right) \text{ GeV}. \quad (4.2)$$

PBH may dominate the energy density of the universe depending on their initial abundance, characterized by the dimensionless parameter

$$\beta = \frac{\rho_{\text{BH}}(T_{\text{in}})}{\rho_{\text{R}}(T_{\text{in}})}. \quad (4.3)$$

Once PBH form¹, they lose mass through Hawking evaporation at a rate given by [155]

$$\frac{dM_{\text{BH}}}{da} = -\frac{\epsilon(M_{\text{BH}})\kappa}{a\mathcal{H}} \left(\frac{1 \text{ g}}{M_{\text{BH}}} \right)^2. \quad (4.4)$$

Here a is the scale factor and $\kappa = 5.34 \times 10^{25} \text{ g s}^{-1}$ [164]. Because of only gravitational effects, production of all particles takes place, regardless of their interaction with other particles. The evaporation function $\epsilon(M_{\text{BH}})$ [164] contains contributions from both SM and BSM particles. The temperature of the thermal plasma when the PBH have completely disappeared can be found by integrating Eq. (4.4), and can be written as [159]

$$T_{\text{ev}} \simeq \left(\frac{9g_*(T_{\text{BH}})}{10240} \right)^{\frac{1}{4}} \left(\frac{M_{\text{P}}^5}{M_{\text{in}}^3} \right)^{\frac{1}{2}}, \quad (4.5)$$

where M_{P} denotes the reduced Planck mass.

For the early universe to be black hole dominated, the initial energy density of PBH should satisfy [164]

$$\beta \geq \beta_{\text{crit}} \equiv 2.5 \times 10^{-14} \gamma^{-\frac{1}{2}} \left(\frac{M_{\text{BH}}(T_{\text{in}})}{10^8 \text{ g}} \right)^{-1} \left(\frac{\epsilon(M_{\text{BH}}(T_{\text{in}}))}{15.35} \right)^{\frac{1}{2}}. \quad (4.6)$$

Since PBH evaporation produces all particles, including radiation that can disturb the successful predictions of BBN, we require $T_{\text{ev}} > T_{\text{BBN}} \simeq 4 \text{ MeV}$. This can be

¹Since our primary motive is to explore the effect of PBH on dark radiation, in this work we remain agnostic about the formation mechanism of PBH. PBH with our desired mass range and initial energy density can be formed through several mechanisms, say from inflationary perturbations [137, 317, 318], Fermi-ball collapse [319], loop quantum gravity [320] etc.

translated into an upper bound on the PBH mass. On the other hand, a lower bound on PBH mass can be obtained from the CMB bound on the scale of inflation [149] : $H_I \equiv \mathcal{H}(T_{\text{in}}) \leq 2.5 \times 10^{-5} M_P$, where $\mathcal{H}(T_{\text{in}}) = \frac{1}{2t_{\text{in}}}$ with $t(T_{\text{in}}) \propto M_{\text{in}}$. Using these BBN and CMB bounds together, we have a window² for allowed initial mass for ultra-light PBH that reads $0.1 \text{ g} \lesssim M_{\text{in}} \lesssim 4 \times 10^8 \text{ g}$. We consider this allowed mass range of ultra-light PBH in the context of dark radiation. For simplicity, we consider a monochromatic mass function of PBHs implying all PBHs to have identical masses. Additionally, the PBHs are assumed to be of Schwarzschild type without any spin and charge.

Now, if there exist any light BSM degrees of freedom or dark radiation, they can be produced directly from evaporating PBH, contributing to the effective number of relativistic degrees of freedom N_{eff} defined as

$$N_{\text{eff}} = \frac{8}{7} \left(\frac{11}{4} \right)^{4/3} \left(\frac{\rho_R - \rho_\gamma}{\rho_\gamma} \right), \quad (4.7)$$

where ρ_R, ρ_γ denote total radiation and photon densities respectively. In order to track the evolution of the energy densities, we need to consider the following set of Boltzmann equations [164]

$$\frac{d\rho_{\text{BH}}}{da} + 3\frac{\rho_{\text{BH}}}{a} = \frac{1}{M_{\text{BH}}} \frac{dM_{\text{BH}}}{da} \rho_{\text{BH}}, \quad (4.8)$$

$$a\mathcal{H} \frac{d\rho_R}{da} + 4\mathcal{H}\rho_R = -\frac{\epsilon_R(M_{\text{BH}})}{\epsilon(M_{\text{BH}})} \frac{a\mathcal{H}}{M_{\text{BH}}} \frac{dM_{\text{BH}}}{da} \rho_{\text{BH}}, \quad (4.9)$$

$$a \frac{d\rho_X^{\text{BH}}}{da} + 4\rho_X^{\text{BH}} = -\frac{\epsilon_X(M_{\text{BH}})}{\epsilon(M_{\text{BH}})} \frac{a}{M_{\text{BH}}} \frac{dM_{\text{BH}}}{da} \rho_{\text{BH}}. \quad (4.10)$$

Here, ρ_X^{BH} denotes energy density of a light species X produced solely from the evaporation of PBH. The ϵ_R and ϵ_X are the evaporation functions of SM particles and X species respectively. The combined evaporation function is denoted by $\epsilon = \epsilon_R + \epsilon_X$. The Hubble parameter $\mathcal{H}$ entering in the above equations is given by

$$\mathcal{H} = \sqrt{\frac{\rho_{\text{BH}} + \rho_{\text{SM}} + \rho_X}{3M_P^2}}. \quad (4.11)$$

Since entropy is not conserved due to PBH evaporation, we track the evolution of the thermal bath separately through the equation given by

²The range of PBH masses in this window remains typically unconstrained [147].

$$\frac{dT}{da} = -\frac{T}{\Delta} \left(\frac{1}{a} + \frac{\epsilon_{SM}(M_{BH})}{\epsilon(M_{BH})} \frac{1}{M_{BH}} \frac{dM_{BH}}{da} \frac{\rho_{BH}}{4(\rho_{SM} + \rho_X)} \right), \quad (4.12)$$

where

$$\Delta = 1 + \frac{T}{3g_{*s}(T)} \frac{dg_{*s}(T)}{dT}. \quad (4.13)$$

The extra relativistic degrees of freedom, ΔN_{eff} is defined as

$$\Delta N_{\text{eff}} = \frac{\rho_X(T_{\text{eq}})}{\rho_{\nu_L,1}(T_{\text{eq}})}. \quad (4.14)$$

Here $\rho_{\nu_L,1}(T_{\text{eq}})$ denotes the energy density of one species of SM neutrinos at the time of matter-radiation equality. In terms of energy densities at the time of PBH evaporation, the above expression turns out to be [164]

$$\Delta N_{\text{eff}}^{\text{BH}} = \left\{ \frac{8}{7} \left(\frac{4}{11} \right)^{-\frac{4}{3}} + N_{\text{eff}}^{\text{SM}} \right\} \frac{\rho_X(T_{\text{ev}})}{\rho_R(T_{\text{ev}})} \left(\frac{g_*(T_{\text{ev}})}{g_*(T_{\text{eq}})} \right) \left(\frac{g_{*s}(T_{\text{eq}})}{g_{*s}(T_{\text{ev}})} \right)^{\frac{4}{3}}, \quad (4.15)$$

where $N_{\text{eff}}^{\text{SM}} = 3.044$ and $\rho_X(T_{\text{ev}}), \rho_R(T_{\text{ev}})$ denote energy density of species X and SM after PBH evaporation respectively. The relativistic degrees of freedom in energy and entropy densities are denoted by g_*, g_{*s} respectively.

Now, in the case when PBH dominates the energy density of the universe at some epoch, Eq. (4.15) simplifies into

$$\Delta N_{\text{eff}}^{\text{BH}} = 13.714 \times \frac{\epsilon_X(M_{BH})}{\epsilon(M_{BH})} \frac{g_*(T_{\text{ev}})}{g_{*s}(T_{\text{ev}})^{\frac{4}{3}}}. \quad (4.16)$$

From the above equation, we can see that ΔN_{eff} remains constant for those values of PBH mass for which PBH evaporate before the electroweak scale since g_*, g_{*s} remains constant at high temperatures. As we keep on increasing the PBH mass further, ΔN_{eff} keeps on increasing. In the left panel of Fig. 4.1, we show the variation of ΔN_{eff} with the initial PBH mass M_{in} , for three different types of DR which includes right chiral part of Dirac active neutrinos ν_R , massless gauge boson (MGB) and Goldstone boson (GB). The current 2σ bound from Planck 2018 [25] and the future sensitivity of CMB-S4 [115] are also shown in the same figure.

4.1.2 Thermalised dark radiation in the presence of PBH

If we consider the extra relativistic species to have interactions with the SM bath, ΔN_{eff} would receive a thermal contribution which depends on the temperature at which the thermal species decouple from the bath namely T_{dec} . It can be written as [120]

$$\Delta N_{\text{eff}}^{\text{th}} = 0.0267 \times f n_X g_X \left(\frac{106.75}{g_{*s}(T_{\text{dec}})} \right)^{4/3}, \quad (4.17)$$

where g_X, n_X is the internal spin degree of freedom of X and number of different species of type X respectively. f takes the value of $7/8$ for fermion and 1 for boson. T_{dec} denotes the decoupling temperature of the thermalised species. In the right panel of Fig. 4.1, we show the corresponding ΔN_{eff} by considering DR to be of thermal origin only. The x-axis denotes the decoupling temperature T_{dec} of DR from the thermal bath. As expected, a lower decoupling temperature leads to a larger contribution to ΔN_{eff} .

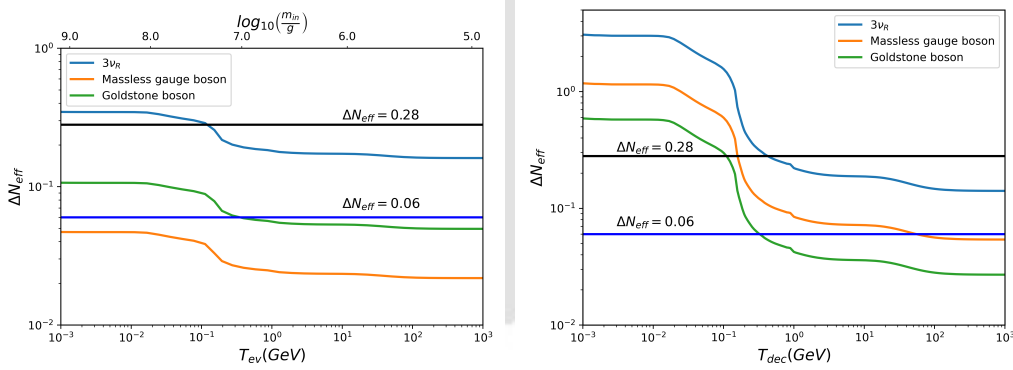


Figure 4.1: Left panel: Variation of non-thermal contribution to ΔN_{eff} coming from PBH, with T_{ev} (or M_{in}), for different species. Right panel: Variation of thermal contribution to ΔN_{eff} with T_{dec} for different species.

As we will see, in the presence of PBH, the total contribution to ΔN_{eff} from thermalised DR is decided by the interplay between the thermal and the non-thermal contribution. Interestingly, the total contribution depends on when the PBH evaporates relative to the decoupling temperature of the thermalised species. Depending on the initial fractional energy density (β) and the initial mass (M_{in}) of PBH at the time of formation, we can broadly divide our study into two categories: (A) PBH dominates the energy density of the universe at some epoch, and (B) PBH

never dominates the energy density of the universe. These two possibilities lead to different observational consequences, which we discuss below.

PBH domination

The evaporation of PBH will produce all the SM particles along with DR. In the hybrid scenario considered here, DR can be produced gravitationally from PBH as well as from the SM bath by virtue of non-standard DR-SM interactions. Therefore, we can think of the total contribution to the extra number of relativistic species, ΔN_{eff} as the contribution from DR emitted from PBH and the already existing thermal counterpart, which was thermalised followed by decoupling from the thermal bath.

The decoupling temperature of extra light species, X depends on its coupling with SM particles. A smaller value of coupling can lead to an early decoupling. In such a scenario, the duration between decoupling and PBH evaporation can be quite long. Thus, to begin with, we neglect the interaction between the decoupled X and X produced from PBH. So, both decoupled and light species from PBH can be treated as different species while considering their individual contribution to ΔN_{eff} . In a follow-up sub-section, we explicitly consider a specific type of DR namely Dirac type active neutrinos with four-Fermi type interactions of right chiral part with the SM and study the validity of this assumption. On the other hand, for a larger value of DR-SM coupling, the PBH evaporation can occur much before the decoupling of light species, X . In that case, the emitted X from PBH will be thermalised completely with the SM bath, and both emitted and preexisting thermal X will behave as a single species.

Here, we make an estimate of ΔN_{eff} in the two limits discussed above. The evaporation temperature of PBH is connected to the initial PBH mass M_{in} through Eq. (4.5). Assuming that the species X decouples at the same temperature as PBH evaporation, we can draw the solid red line in Fig. 4.2 corresponding to $T_{\text{ev}} = T_{\text{dec}}$. Now, depending upon the values of its coupling with SM and M_{in} , we can have the two different scenarios, i.e. either $T_{\text{ev}} > T_{\text{dec}}$ (above the red line) or $T_{\text{ev}} < T_{\text{dec}}$ (below the red line). We discuss the dynamics for these two cases below.

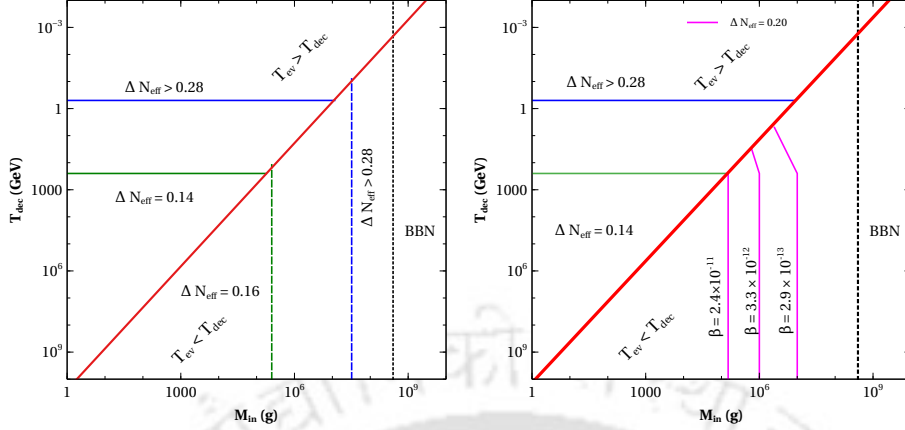


Figure 4.2: ΔN_{eff} contours in the plane of initial PBH mass versus decoupling temperature. Here, we consider the extra light species X to be ν_R . The left plot is for the case where PBH dominate the energy density at the time of evaporation, whereas in the right panel, PBH energy density is always less than that of radiation. The black vertical dashed line indicates the upper bound on PBH mass from BBN limit.

PBH evaporation before decoupling of thermalised DR We first consider the scenario, where the PBH evaporates earlier than the decoupling of thermalised species X . In such a case, the species X produced from PBH evaporation will also get thermalised. Therefore, the contribution to ΔN_{eff} comes only from the thermalised X , and is given by Eq. (4.17). For example, if X is considered to be the right chiral parts of Dirac active neutrinos, this equation reduces to

$$\Delta N_{\text{eff}} \simeq 0.027 \times 2 \times 3 \times \frac{7}{8} \left(\frac{106.75}{g_*(T_{\text{dec}})} \right)^{4/3} \simeq 0.14175 \times \left(\frac{106.75}{g_*(T_{\text{dec}})} \right)^{4/3}. \quad (4.18)$$

Thus, ΔN_{eff} becomes independent of m_{in} . For decoupling temperature $T_{\text{dec}} \lesssim 500$ MeV, ΔN_{eff} is more than 0.28, the current Planck bound at 2σ level. This is indicated by the region above the solid blue line in Fig. 4.2. A larger decoupling temperature reduces the contribution to N_{eff} . Note that this also requires a smaller value of PBH mass to make sure that PBH evaporates earlier compared to the decoupling epoch. Above a certain value of T_{dec} , the contribution to ΔN_{eff} saturates at a value of 0.14, indicated by the triangular region between solid green and solid red lines of Fig. 4.2.

PBH evaporation after decoupling of thermalised DR Next, we consider the scenario where PBH evaporates after the thermal decoupling of species X . In

this case, ΔN_{eff} can, in principle, get contributions from both sources. We can write

$$\Delta N_{\text{eff}} = \Delta N_{\text{eff}}^{\text{BH}} + \Delta N_{\text{eff}}^{\text{th}}, \quad (4.19)$$

where the first term on the right hand side (RHS) denotes the contribution from PBH and the second term indicates the thermal contribution. Now, for PBH domination, evaporation of PBH will lead to entropy injection to the thermal bath. Thus the thermal contribution from X to ΔN_{eff} gets diluted. Taking into account the effect of dilution, the above equation for Dirac active neutrinos can be written as

$$\Delta N_{\text{eff}} = 0.772 \frac{g_*(T_{\text{ev}})}{(g_{*s}(T_{\text{ev}}))^{4/3}} + 0.14175 \times \left(\frac{106.75}{g_*(T_{\text{dec}})} \right)^{4/3} \xi^{-4/3}, \quad (4.20)$$

where the parameter ξ quantifies decrease in ΔN_{eff} due to entropy dilution. It can be written as the ratio of the comoving entropy density at PBH evaporation (T_{ev}) to that at thermal decoupling of X (T_{dec})

$$\xi = \frac{S_{\text{ev}}}{S_{\text{dec}}} = \frac{g_{*s}(T_{\text{ev}})a^3(T_{\text{ev}})T_{\text{ev}}^3}{g_{*s}(T_{\text{dec}})a^3(T_{\text{dec}})T_{\text{dec}}^3}. \quad (4.21)$$

Without any entropy injection, conservation of entropy gives $\xi = 1$. For entropy injection due to PBH evaporation, we have $\xi > 1$. It turns out that for PBH dominated region, $\xi \gg 1$, diluting the contribution from thermal ν_R or X in general. Thus, one can safely neglect the second term in the RHS of Eq. (4.19). Hence, only the X from PBH will contribute to ΔN_{eff} . PBH mass greater than around 4×10^7 g leads to ΔN_{eff} greater than the Planck 2σ bound, as shown earlier (cf. left panel of Fig. 4.1).

Radiation domination

If the ratio of the initial energy density of PBH to that of radiation energy density is less than β_{crit} , the early universe remains dominated by radiation only. Similar to the PBH-dominated universe, we can also have the same two scenarios, namely $T_{\text{ev}} > T_{\text{dec}}$ and $T_{\text{ev}} < T_{\text{dec}}$. The solid red line in the right panel plot of Fig. 4.2 corresponds to $T_{\text{dec}} = T_{\text{ev}}$. We discuss the two scenarios corresponding to either side of this solid red line below.

PBH evaporation before decoupling of thermalised DR Similar to the case of PBH domination, here the thermal contribution is the only contributing factor to ΔN_{eff} . Hence the region and bounds are same as that of PBH domination, as already discussed in 4.1.2.

PBH evaporation after decoupling of thermalised DR Here, the total contribution to ΔN_{eff} can be written in the form of Eq. (4.19), with the non-thermal contribution $\Delta N_{\text{eff}}^{\text{BH}}$ given by Eq. (4.15). $\Delta N_{\text{eff}}^{\text{BH}}$ is evaluated by solving the Boltzmann equations (4.8)-(4.10). Unlike the case of PBH domination, here the contribution from the thermalised DR can not be neglected as the entropy injection is negligible (i.e. $\xi \sim 1$) for $\beta < \beta_{\text{crit}}$. Hence, ΔN_{eff} will bear contribution from both thermalised as well as PBH generated DR. Also, for PBH domination, $\Delta N_{\text{eff}}^{\text{BH}}$ is independent of the parameter β , whereas in the present scenario β plays a significant role. The role of β can be seen from the magenta coloured lines shown in the right panel plot of Fig. 4.2. The magenta coloured lines denote a total $\Delta N_{\text{eff}} = 0.20$. As the initial mass of PBH increases, its contribution to ΔN_{eff} also increases. As a result, one needs to reduce the value of β to get same ΔN_{eff} . Let us consider the magenta line with $M_{\text{in}} = 10^7$ g. The straight vertical portion of the line indicates a thermal contribution of 0.14 and a PBH contribution of 0.06. The contribution from PBH can be set to the required value by adjusting β . Decreasing T_{dec} in the right panel plot up to ~ 200 GeV leads to no changes in the thermal contribution as g_{*s} remain constant, giving a constant contribution to $\Delta N_{\text{eff}}^{\text{th}}$. Decreasing T_{dec} below ~ 200 GeV, the thermal contribution increases due to the reasons discussed earlier. So, one requires a smaller contribution of $\Delta N_{\text{eff}}^{\text{BH}}$ in order to maintain a total contribution of 0.20. This can be done by either decreasing PBH mass or by decreasing β . Here we decrease the PBH mass keeping β constant. This leads to the bending of magenta lines for $T_{\text{dec}} < 200$ GeV.

Summary of results

Now let us look at the combined results of PBH and radiation domination in the β - M_{in} plane. In the PBH dominated case, the magenta-shaded region in Fig. 4.3 shows the portion of parameter space where $\Delta N_{\text{eff}} > 0.28$, corresponding to PBH initial mass $M_{\text{in}} \gtrsim 4 \times 10^7$ g (cf. blue-dashed contour of left panel plot in Fig. 4.2). Now, this constraint is only valid if ν_R (or DR, in general) decouples from the bath

before PBH evaporation i.e. if $T_{\text{dec}} > 150$ MeV (intersection of the blue dashed contour with the solid red line in left panel plot of Fig. 4.2). If ν_R decouples after PBH evaporation, then ΔN_{eff} will bear contribution from thermalised ν_R (existing thermal $\nu_R + \nu_R$ from PBH that will also get thermalised). In such a case, no constraint on the PBH parameters β or M_{in} can be obtained, since the contribution is purely thermal.

In the radiation-dominated region, if the thermalised ν_R decouples earlier, then even for very small β , one expects minimum contribution of $\Delta N_{\text{eff}}^{\text{th}} = 0.14$ to ΔN_{eff} . For example, let us fix $T_{\text{dec}} \approx 35$ GeV. So, for PBH mass greater than $\sim 7 \times 10^5$ g, the decoupling of ν_R occurs before PBH evaporation. The thermal contribution to ΔN_{eff} is about ≈ 0.17 . So, if the PBH contribution to ΔN_{eff} is more than 0.11, the total ΔN_{eff} would exceed 0.28. The blue shaded region in Fig. 4.3 denotes the area for which $\Delta N_{\text{eff}} > 0.28$. Note that there is a discontinuity between the magenta and blue shaded regions. This is because, we have assumed that in the PBH dominated era, the thermal contribution to ΔN_{eff} will be totally diluted away due to subsequent PBH evaporation and hence can be neglected. However, in the vicinity of the solid red line, i.e. $\beta \sim \beta_{\text{crit}}$, the dilution would be smaller leading to a non-negligible thermal contribution which should be counted for. At the same time, for the radiation-dominated case, we have assumed no entropy dilution for the thermal contribution, which is not completely true in the vicinity of $\beta \sim \beta_{\text{crit}}$. This is due to the fact that even a sub-dominant but sizeable abundance of PBH can lead to some entropy dilution, howsoever small. All these intricacies will be taken care of when we explicitly solve the relevant Boltzmann equations by properly taking into account the entropy dilution factor near the transition region from PBH to radiation domination, which we discuss below.

We perform a numerical analysis to evaluate the total contribution to ΔN_{eff} . For this purpose, we use the publicly available code FRISBHEE [169] to calculate the non-thermal contribution $\Delta N_{\text{eff}}^{\text{BH}}$, and to properly include the entropy dilution factor (cf. Eq. (4.21)), which goes into the calculation of the thermal contribution $\Delta N_{\text{eff}}^{\text{th}}$. For illustrative purposes, we consider the DR or the extra light species X to be of three types namely, (i) ν_R , (ii) Goldstone boson and (iii) massless gauge boson.

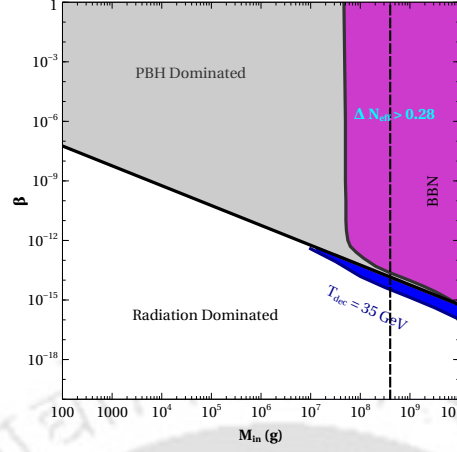


Figure 4.3: ΔN_{eff} contours in the β - M_{in} plane. The magenta and blue region are the portion for PBH and radiation domination respectively where $\Delta N_{\text{eff}} > 0.28$. The black vertical dashed line indicates the upper bound on PBH mass from BBN limit.

Dirac neutrino Here, we consider 3 species of ν_R as the extra light degrees of freedom, as in usual Dirac active neutrino scenarios. In Fig. 4.4, we show the total contribution to ΔN_{eff} in the $M_{\text{in}} - \beta$ plane for four different sub-cases. In the top left panel, we show the results where ν_R is produced purely from PBH due to negligible interactions with the SM bath. The black dashed contour corresponds to $\Delta N_{\text{eff}} = 0.28$, the maximum allowed by Planck 2018 data at 2σ level. This plot can be taken as a reference to compare the results in the presence of extra interactions. In the top right panel of Fig. 4.4, we show the results when the contribution from thermal ν_R is $\Delta N_{\text{eff}}^{\text{th}} \sim 0.147$. For this particular value of $\Delta N_{\text{eff}}^{\text{th}}$, the decoupling temperature should be $T_{\text{dec}} \gtrsim 100$ GeV corresponding to PBH mass $M_{\text{in}} \sim 4 \times 10^5$ g. PBH masses greater than $\sim 4 \times 10^5$ g, will evaporate later satisfying $T_{\text{ev}} < T_{\text{dec}}$. The vertical dashed line distinguishes these two regions. For $\beta \gg \beta_{\text{crit}}$, there is no change on the Planck 2018 bound compared to the results without interactions as the contribution of thermally decoupled species is negligible because of entropy dilution from PBH evaporation. However, near $\beta \sim \beta_{\text{crit}}$, i.e. the transition region between PBH and radiation domination, we can see a decrease albeit small in the values of β required to produce the same value of ΔN_{eff} as in the case without interactions. This is because near the boundary of the transition region, the effect of the thermal contribution starts appearing, and hence we need a lower value of β such that the non-thermal contribution $\Delta N_{\text{eff}}^{\text{BH}}$ decreases, resulting in a similar value of the total ΔN_{eff} . This explains slight lowering of the black-dashed contour corresponding to

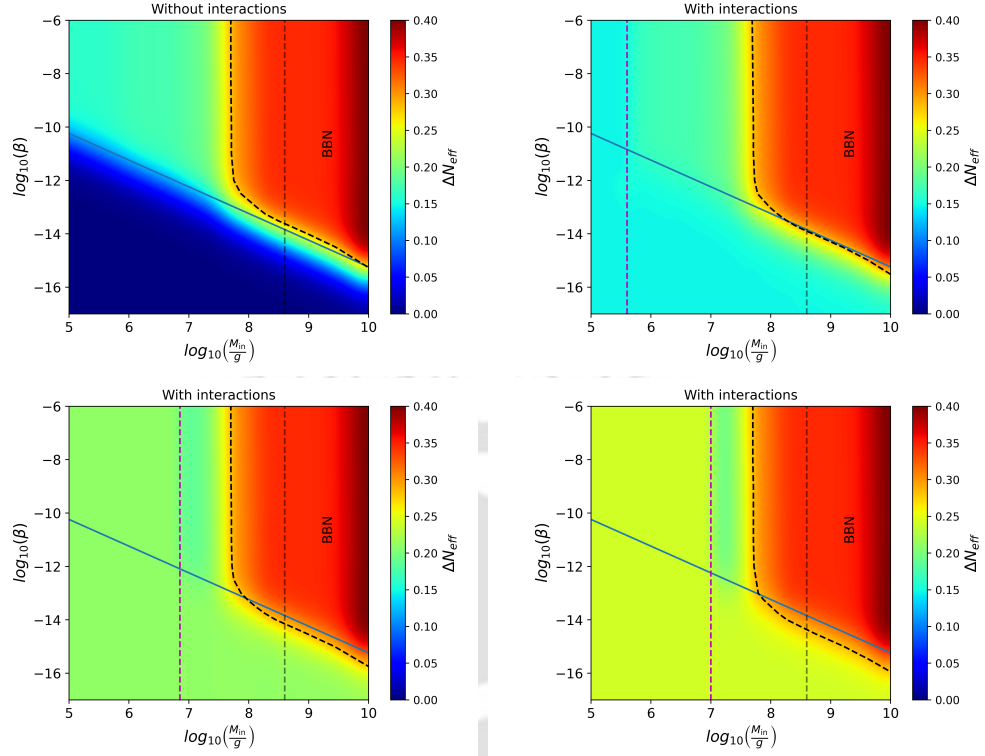


Figure 4.4: *Dirac neutrino* : Variation of ΔN_{eff} in the M_{in} versus β plane for four different scenarios related to Dirac neutrinos: without any extra interactions of ν_R (top left panel), with interactions giving $\Delta N_{\text{eff}}^{\text{th}} = 0.14$, $T_{\text{dec}} = 100$ GeV (top right panel), $\Delta N_{\text{eff}}^{\text{th}} = 0.21$, $T_{\text{dec}} = 1$ GeV (bottom left panel) and $\Delta N_{\text{eff}}^{\text{th}} = 0.24$, $T_{\text{dec}} = 850$ MeV (bottom right panel) respectively. The regions right to the black dashed line indicate $\Delta N_{\text{eff}} > 0.28$ ruled out by Planck 2018 data at 2σ level. The dashed blue vertical line separates the regions $T_{\text{dec}} < T_{\text{ev}}$ and $T_{\text{dec}} > T_{\text{ev}}$ (not applicable to the top left panel due to no DR-SM interactions). The grey vertical dashed line indicates the upper bound on PBH mass from BBN limit.

$\Delta N_{\text{eff}} = 0.28$. Note that the minimum contribution of the thermal species is 0.147, which is visible as we depart from the transition region. This is in sharp contrast to the results without interactions, where the value of ΔN_{eff} fades away to zero as we move deeper into the radiation domination ballpark. In the bottom panel, we have $\Delta N_{\text{eff}}^{\text{th}} = 0.21$ (left) and 0.24 (right). Due to the higher values of $\Delta N_{\text{eff}}^{\text{th}}$, and hence a lower decoupling temperature, the dashed vertical line shifts to the right corresponding to higher values of PBH mass (lower T_{ev}). Here, the change in the transition region is more apparent, due to a decrease in the contribution from PBH namely, $\Delta N_{\text{eff}}^{\text{BH}}$. We can see that for $\Delta N_{\text{eff}}^{\text{th}} = 0.24$, much more parameter space near

the transition region is ruled out by current Planck 2018 data compared to that in the case without interactions (cf. top left panel of Fig. 4.4).

Goldstone boson Let us consider the extra light species to be a Goldstone boson, a massless scalar. From Fig. 4.1 discussed before, we can see that the maximum contribution of Goldstone boson produced solely from evaporating PBH to ΔN_{eff} is 0.10, which is below the Planck 2018 limit. However, it comes under the sensitivity of future experiments like CMB-S4. The top left panel plot of Fig. 4.5 shows the bound on ΔN_{eff} without any interaction. Unlike the case of ν_R , here the region to the right of the black dashed contour represents $\Delta N_{\text{eff}} > 0.06$, the CMB-S4 sensitivity as Planck 2018 data do not rule out any region of the parameter space at 2σ level. The top right panel and the bottom panel plots show the corresponding bounds with interactions between GB and SM. In the top right panel plot, the decoupling temperature is $T_{\text{dec}} = 100$ GeV and in the bottom panel plot, decoupling temperature is $T_{\text{dec}} = 1$ GeV. These correspond to $\Delta N_{\text{eff}}^{\text{th}} = 0.028$ and 0.042 respectively. Similar to the previous situation, a lower decoupling temperature gives a stronger constraint on the β versus M_{in} plane.

Massless gauge boson From Fig. 4.1 discussed earlier, it can be seen that in the absence of PBH, the contribution of thermalised massless gauge boson to ΔN_{eff} is within the future CMB-S4 sensitivities for decoupling temperature $T_{\text{dec}} \lesssim 100$ GeV. As a result, a wide range of interactions of MGB with SM for which $T_{\text{dec}} \lesssim 100$ GeV can be probed by future CMB experiment like CMB-S4 (cf. right panel of Fig. 4.1). On the other hand, if PBH dominates in the early universe, the contribution to ΔN_{eff} always remains beyond the reach of future sensitivities, as shown in Fig. 4.1 (left panel). Thus, even if CMB-S4 rules out large coupling between MGB and SM which would produce large ΔN_{eff} , such large couplings (or equivalently lower T_{dec}) would still be allowed if we consider PBH domination in the early universe. This would be possible if we consider that PBH evaporates after the decoupling of MGB and sufficiently dilutes the thermal contribution.

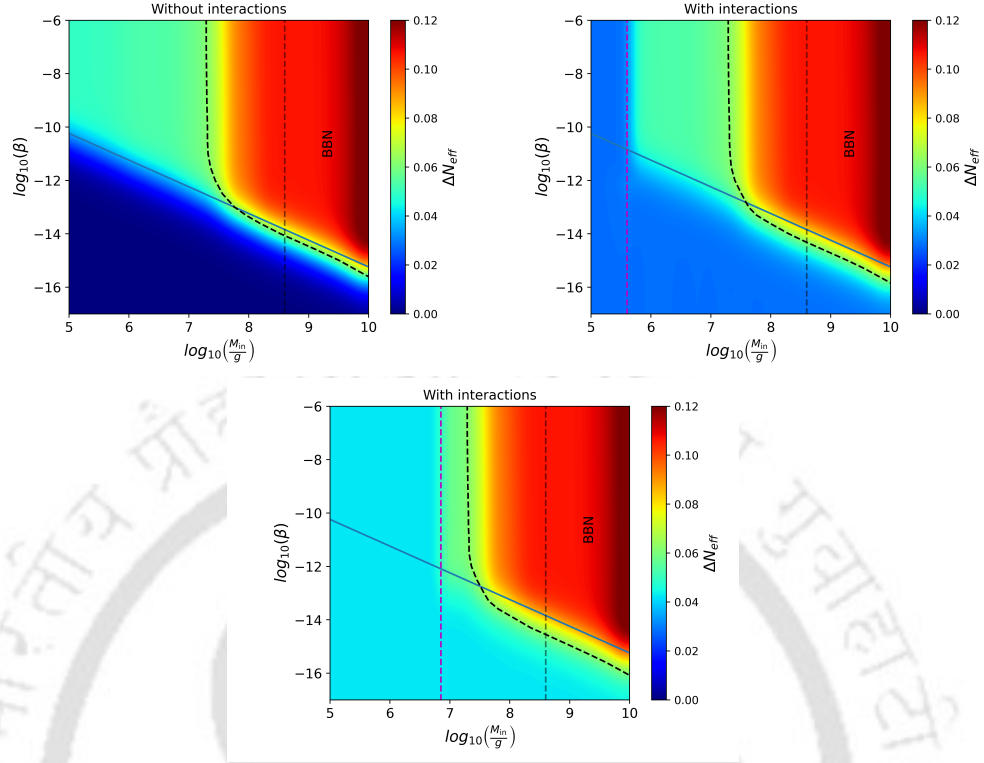


Figure 4.5: *Goldstone boson* : Variation of ΔN_{eff} in the M_{in} versus β plane for three different scenarios related to Goldstone boson: without any extra interactions (top left panel), with interactions giving $\Delta N_{\text{eff}}^{\text{th}} = 0.028$, $T_{\text{dec}} = 100$ GeV (top right panel) and $\Delta N_{\text{eff}}^{\text{th}} = 0.042$, $T_{\text{dec}} = 1$ GeV (bottom panel) respectively. The regions right to the black dashed contour indicate $\Delta N_{\text{eff}} > 0.06$, the CMB-S4 sensitivity. The dashed blue vertical line separates the regions $T_{\text{dec}} < T_{\text{ev}}$ and $T_{\text{dec}} > T_{\text{ev}}$. The grey vertical dashed line indicates the upper bound on PBH mass from BBN limit.

4.1.3 An example: thermalised Dirac neutrino in the presence of PBHs

In the previous sub-sections, we have studied the consequence of thermalised DR in the presence of PBH while considering different examples of DR as well as PBH parameters corresponding to both PBH and radiation domination in the early universe. However, we discussed our results only in terms of decoupling temperatures of thermalised DR while being agnostic about the type of interactions with the SM bath. In this sub-section, we consider a specific type of DR namely, light Dirac neutrinos and discuss our results for effective four-Fermi type interactions with the SM parametrised by the coupling parameter G_{eff} . Enhancement of ΔN_{eff}

in Dirac neutrino models (without PBH) have been studied in several recent works [120, 123, 252–261, 263, 264, 267, 268].

The SM extended by three right-handed singlet Dirac neutrinos with Yukawa interactions $\mathcal{L}_Y = -Y_\nu^{ab} \bar{\ell}_L^a \tilde{\Phi} \nu_{bR}$, can explain tiny neutrino mass of $\mathcal{O}(\text{eV})$, with $Y_\nu \sim 10^{-12}$ as a result of electroweak symmetry breaking induced by the SM Higgs doublet Φ . However, such small interactions can not thermalise the ν_R . Due to the smallness of Yukawa coupling, the non-thermal or freeze-in contribution to ΔN_{eff} from ν_R also remains negligible [123]. Thermalisation of ν_R may be possible in the presence of non-standard interactions which we consider to be of four-Fermi type interactions with the SM neutrinos ν_L [255] (see Appendix A.7 for details). This allows us to work in a model-independent manner while constraining the effective interactions as well as PBH parameters. The total interaction rate of ν_R arising from such four-fermion operators can be written as

$$\begin{aligned} \Gamma &= n_R^{\text{eq}} \langle \sigma v \rangle \\ &\simeq \frac{3}{4} \frac{\zeta(3)}{\pi^2} 2T^3 \frac{1}{4\pi} 4(3.151T)^2 G_{\text{eff}}^2, \end{aligned} \quad (4.22)$$

where G_{eff} is given by

$$G_{\text{eff}}^2 = \frac{4}{3} (G_S - 12G_T)^2 + \frac{5}{12} (\tilde{G}_S - 2G_V)^2, \quad (4.23)$$

where $G_S, \tilde{G}_S, G_T, G_V$ are dimensionful couplings involved in different four-fermion operators involving ν_R and ν_L as shown in appendix A.7. The equilibrium number density is denoted by n_R^{eq} . Comparing this interaction rate Γ with the Hubble rate of expansion (given by Eq. (4.11) and obtained by solving the Boltzmann equations (4.8)-(4.10)), we can find the decoupling temperature of ν_R . Note that the Hubble parameter contains contributions from both SM radiation and PBH, which can lead to a change in the ν_R decoupling temperature compared to the standard case without PBH.

With this, we redraw the Fig. 4.2 by replacing T_{dec} in the y-axis with the equivalent G_{eff} . The resulting plots are shown in Fig. 4.6 and Fig. 4.7 for PBH and radiation domination respectively. In the left panel plot of Fig. 4.6, the coupling G_S is non-zero while the rest (i.e. $G_T, \tilde{G}_S$ and G_V) are zero. In the right panel plot, G_T is non-zero and the rest are set to zero. For $G_{\text{eff}} \gtrsim 10^{-3} G_F$ with G_F being the Fermi coupling, the decoupling temperature of ν_R turns to be $\lesssim 500$ MeV and ΔN_{eff}

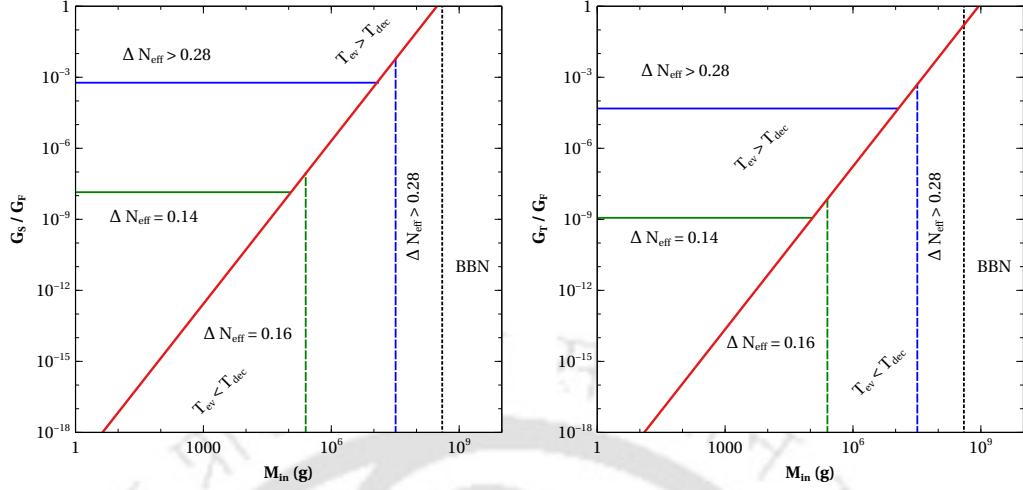


Figure 4.6: ΔN_{eff} contours in the plane of initial PBH mass versus the effective neutrino coupling for the case of PBH domination. In the left panel, $G_T = \tilde{G}_S = G_V = 0$ with non-zero G_S , whereas in the right panel, we consider $G_S = \tilde{G}_S = G_V = 0$ with non-zero G_T . The black vertical dashed line indicates the upper bound on PBH mass from BBN limit.

is more than 0.28. Similarly, below a certain value of coupling, $G_{\text{eff}} \sim 2 \times 10^{-7} G_F$, the contribution to ΔN_{eff} saturates at a value of 0.14. The corresponding parameter space is shown for the case of radiation domination in Fig. 4.7, which can be understood in a way analogous to the right panel plot of Fig. 4.2 discussed earlier.

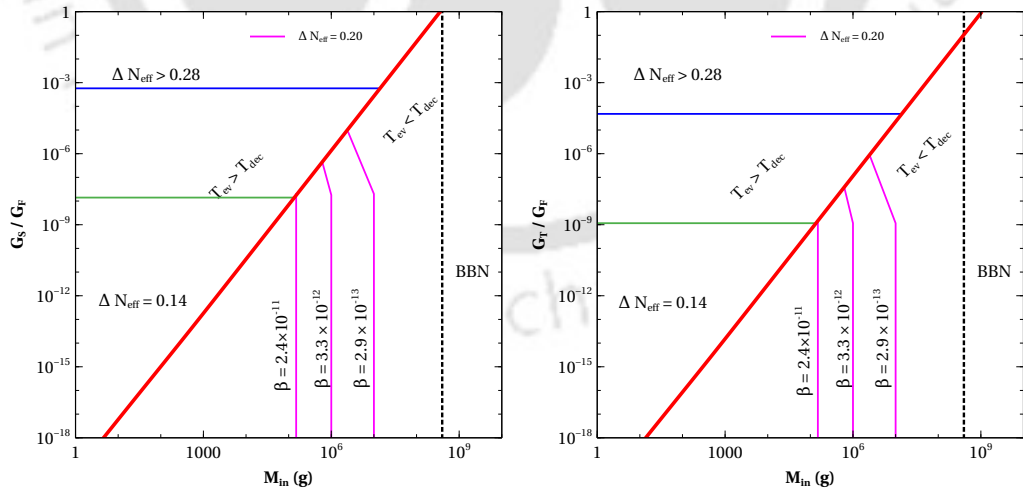


Figure 4.7: ΔN_{eff} contours in the plane of initial PBH mass versus the effective neutrino coupling for the case of radiation domination. In the left panel, $G_T = \tilde{G}_S = G_V = 0$ with non-zero G_S , whereas in the right panel, we consider $G_S = \tilde{G}_S = G_V = 0$ with non-zero G_T . The black vertical dashed line indicates the upper bound on PBH mass from BBN limit.

Now, in order to check whether the right-handed neutrinos produced after PBH evaporation enters into thermal equilibrium with the SM bath, we calculate the interaction of ν_L present in the bath and non-thermal ν_R produced from PBH evaporation. Following [321], the thermal averaged cross-section for two massless species having different temperatures is found to be

$$\langle\sigma v\rangle_{T_{\text{ev}}T_{\text{BH}}} = \frac{1}{32(T_{\text{ev}}T_{\text{BH}})^{5/2}} \int_0^\infty \sigma s^{3/2} K_1\left(\frac{\sqrt{s}}{\sqrt{T_{\text{ev}}T_{\text{BH}}}}\right) ds, \quad (4.24)$$

with K_1 being the modified Bessel's function of order 1. For $\sigma = \frac{1}{4\pi} s G_{\text{eff}}^2$, this gives

$$\langle\sigma v\rangle_{T_{\text{ev}}T_{\text{BH}}} = \frac{6}{\pi} G_{\text{eff}}^2 T_{\text{ev}}T_{\text{BH}} \quad (4.25)$$

Comparing the interaction rate corresponding to the above cross-section with the Hubble leads to the thermalisation condition

$$\frac{n_{\nu_L}^{\text{eq}}(T_{\text{ev}})\langle\sigma v\rangle_{T_{\text{ev}}T_{\text{BH}}}}{\mathcal{H}(T_{\text{ev}})} = C \frac{6}{\pi} G_{\text{eff}}^2 T_{\text{ev}}^2 T_{\text{BH}} = 1, \quad (4.26)$$

where $C = \frac{\frac{3}{4}\zeta(3) \times 2 \times \sqrt{45} \times M_P}{\pi^2 \sqrt{4\pi^3 g_*(T_{\text{ev}})}}$.

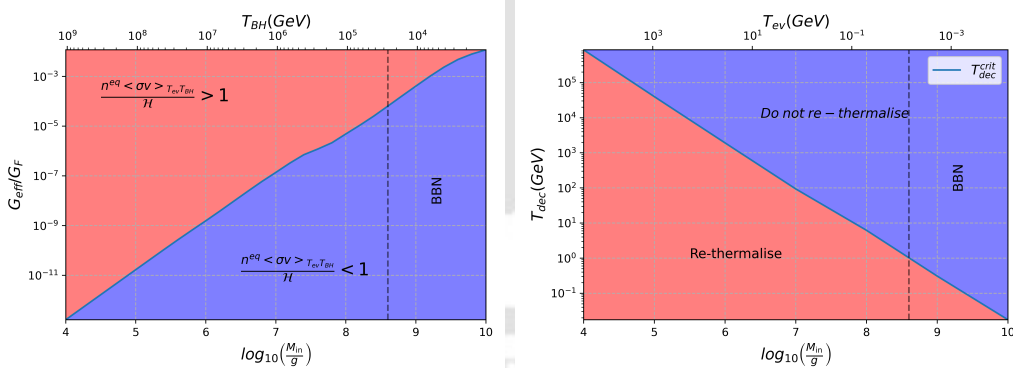


Figure 4.8: Left panel: M_{in} versus G_{eff}/G_F parameter space showing the possibility of re-thermalisation. The red (blue) coloured region denotes the parameter space for which the PBH generated ν_R can (can not) come into thermal equilibrium. Right panel: M_{in} versus T_{dec} parameter space showing the possibility of re-thermalisation. The red and blue coloured regions have the same meaning as the ones in left panel plot. The vertical dashed line indicates the upper bound on PBH mass from BBN limit.

Both T_{ev} and T_{BH} are functions of M_{in} as can be seen from Eq.(4.5) and Eq.(4.2). In the left panel of Fig. 4.8, we show the parameter space in the $M_{\text{in}} - G_{\text{eff}}$

plane, where re-thermalisation of the PBH generated ν_R takes place (red-shaded), along with the region which does not lead to such re-thermalisation (blue-shaded). We can see that for a fixed PBH mass, if G_{eff} is smaller than a particular critical value, the non-thermally generated ν_R from PBH never thermalise again with the bath. A smaller value of G_{eff} also implies a higher initial decoupling temperature of thermal ν_R . Hence, it is also possible to relate this critical value of G_{eff} below which re-thermalisation does not occur, with the initial decoupling temperature of ν_R . In the right panel of Fig. 4.8, we show the parameter space in the $M_{\text{in}} - T_{\text{dec}}$ plane, separating the region of re-thermalisation from the rest. For a particular PBH mass, a decoupling temperature higher than $T_{\text{dec}}^{\text{crit}}$ leads to the situation where ν_R from PBH will never re-thermalise with the bath ³.

To summarise, Fig. 4.8 shows the constraints on the parameter space for which the ν_R produced from PBH evaporation will never re-thermalise. Once, we identify the parameter space where re-thermalisation occurs, the next step is to find the new ΔN_{eff} for these regions. However, this requires a careful treatment of complete Boltzmann equations to track the phase-space distribution function of evaporated ν_R , which is numerically expensive and out of the scope of the current work. Here, we assume that the new decoupling temperature of ν_R after re-thermalisation is $\sim T_{\text{ev}}$ for a particular PBH mass. This can be justified by the fact that PBH generated ν_R having a large initial energy and a sufficiently large interaction rate with the bath responsible for re-thermalisation, also suffer from instant dissipation of the energy to the bath. Due to this instant dissipation of energy, ν_R can not maintain equilibrium for a longer period after re-thermalisation leading to decoupling of ν_R once again at a temperature very close to T_{ev} .

With this, it is possible to find new constraint on G_{eff} and M_{in} . In the left panel of Fig. 4.9, constraints are shown in $M_{\text{in}} - G_{\text{eff}}$ plane. The black dashed lines denote $\Delta N_{\text{eff}} = 0.28$. Taking into account re-thermalisation, it is possible to obtain a stronger constraint on $G_{\text{eff}}/G_{\text{F}}$. In the region, $T_{\text{dec}} < T_{\text{ev}}$, $\Delta N_{\text{eff}} > 0.28$ implies $G_{\text{eff}}/G_{\text{F}} \gtrsim 5 \times 10^{-4}$. However, in the re-thermalisation region, $G_{\text{eff}}/G_{\text{F}}$ up to $\sim 5 \times 10^{-7}$ can be excluded, for $M_{\text{in}} \gtrsim 10^{7.2}$ g. In the bottom triangular region, which

³Considering a radiation dominated universe during decoupling, the value of this critical decoupling temperature is found to be

$$T_{\text{dec}}^{\text{crit}} \simeq (T_{\text{ev}}^2 T_{\text{BH}})^{1/3}. \quad (4.27)$$

shows the parameter space where PBH generated ν_R do not re-thermalise, the results correspond to the one discussed in Fig. 4.4. In this region, we have shown the ΔN_{eff} values for a particular value of $\beta = 10^{-8}$. In the right panel of Fig. 4.9, we show the constraint in the M_{in} versus β plane for a particular $G_{\text{eff}}/G_F = 10^{-6}$. Here, the region between the white dashed lines shows the parameter space leading to re-thermalisation. As expected from the left plot, here a stronger constraint on PBH mass can be seen.

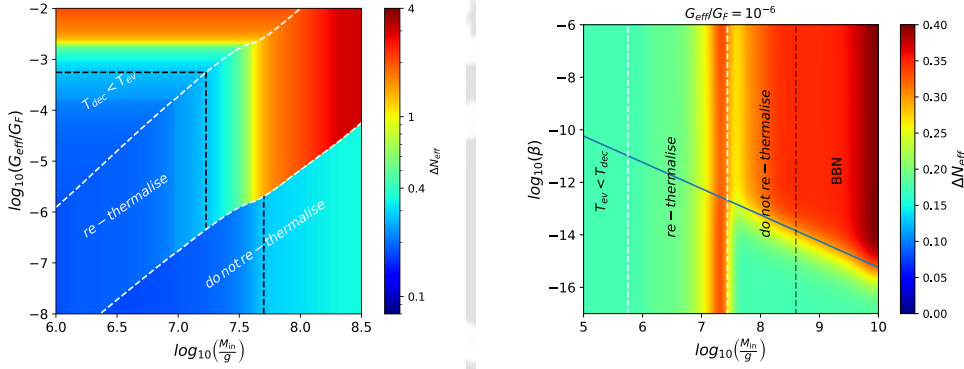


Figure 4.9: *Left panel:* Variation of ΔN_{eff} in $M_{\text{in}}-G_{\text{eff}}/G_F$ plane for three different regions where (i) $T_{\text{dec}} < T_{\text{ev}}$, (ii) PBH generated ν_R re-thermalise and (iii) PBH generated ν_R do not re-thermalise, separated by white dashed line. The dashed black coloured line distinguishes the regions with $\Delta N_{\text{eff}} > 0.28$ and $\Delta N_{\text{eff}} < 0.28$. *Right panel:* Variation of ΔN_{eff} in the $M_{\text{in}}-\beta$ plane for a particular $G_{\text{eff}}/G_F = 10^{-6}$. A new constraint is obtained in the re-thermalised region. The black vertical dashed line indicates the upper bound on PBH mass from BBN limit.

4.1.4 Gravitational waves from PBHs' density perturbations

PBH can be involved in the generation of gravitational waves in several ways [174, 175, 322]. In this work, we focus on the gravitational waves induced by the density fluctuations of PBH after their formation [179, 180, 323]. This is primarily because for the PBH mass range we are working with, the GW spectrum produced through this route can be within the sensitivity of near-future GW experiments. Moreover, these induced gravitational waves are independent of the formation mechanism of PBH.

After the formation of PBH, they are distributed randomly in space following Poissonian statistics [180]. These inhomogeneities in the distribution of PBH induce

curvature perturbations when PBH begin to dominate the energy density of the universe, which at second order can source gravitational waves. The amplitude of these gravitational waves is further enhanced during the evaporation of PBH. The dominant contribution to the present-day GW amplitude can be written as⁴ [179, 324, 325]

$$\Omega_{\text{GW}}(t_0, f) \simeq \Omega_{\text{GW}}^{\text{peak}} \left(\frac{f}{f^{\text{peak}}} \right)^{11/3} \Theta(f^{\text{peak}} - f), \quad (4.28)$$

where $\Omega_{\text{GW}}^{\text{peak}}$ indicates the peak amplitude and is given by

$$\Omega_{\text{GW}}^{\text{peak}} \simeq 2 \times 10^{-6} \left(\frac{\beta}{10^{-8}} \right)^{16/3} \left(\frac{m_{\text{in}}}{10^7 \text{g}} \right)^{34/9}. \quad (4.29)$$

Now, for length scales smaller than the mean separation between PBH, the assumption of PBH as a continuous fluid ceases to hold true. This imposes an ultraviolet cutoff to the GW spectrum, with f^{peak} corresponding to comoving scales representing the mean separation between PBH. The peak frequency is found to be

$$f^{\text{peak}} \simeq 1.7 \times 10^3 \text{ Hz} \left(\frac{m_{\text{in}}}{10^4 \text{g}} \right)^{-5/6}. \quad (4.30)$$

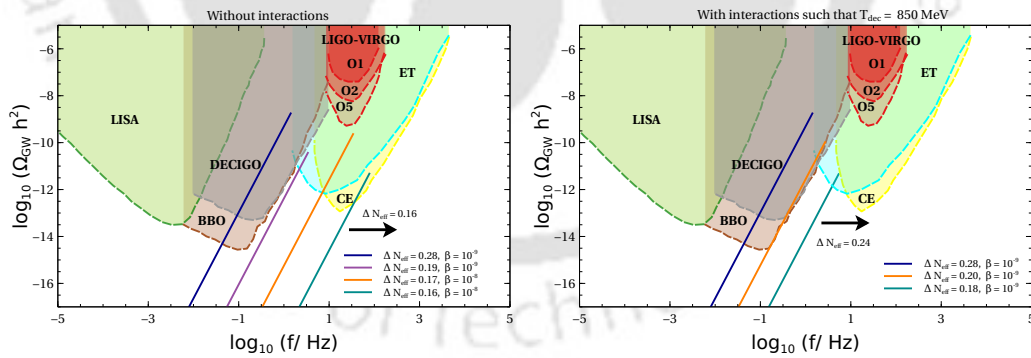


Figure 4.10: GW spectra from PBH density fluctuations for different values of ΔN_{eff} , without extra interactions (left panel) and in the presence of extra interactions (right panel) of right-handed neutrinos. The various shaded regions indicate the future sensitivities and current bounds of GW detectors which includes LISA, BBO, DECIGO, LIGO-VIRGO, ET, CE. The arrows indicate a constant ΔN_{eff} beyond a particular frequency (see text).

⁴The amplitude of the induced GW spectrum is sensitive to the PBH mass distribution [177]. Here, we consider a monochromatic mass spectrum for simplicity, as mentioned earlier.

Since GW behave like radiation, they can contribute to extra relativistic degrees of freedom during BBN. This gives an upper bound on β , depending on PBH mass, which is given by [179]

$$\beta \lesssim 1.1 \times 10^{-6} \left(\frac{M_{\text{in}}}{10^4 g} \right)^{-17/24}. \quad (4.31)$$

This upper bound on β for ultralight PBH is stronger than other bounds, obtained for eg. in Ref. [180] to avoid the backreaction problem.

Now, if PBH play a role in sourcing the extra relativistic degrees of freedom ΔN_{eff} , then the peak amplitude and the peak frequency of the GW spectrum discussed above would depend on the value of ΔN_{eff} , since M_{in} in Eq. (4.29), (4.30) is connected to ΔN_{eff} , as we have already seen in earlier discussions. Hence, observation of such a GW spectrum would provide a complementary probe to ΔN_{eff} observations at CMB experiments. In the left panel of Fig. 4.10, we show the GW spectrum arising from PBH density fluctuations, for different values of ΔN_{eff} , in the absence of extra interactions of the right-handed neutrinos. Note that such a GW spectrum is absent for the radiation-dominated case. For PBH domination, a higher value of ΔN_{eff} corresponds to a higher value of M_{in} (cf. Fig. 4.1), which shifts the peak frequency given by Eq. (4.30) to lower values. Any spectral line to the right of the one coloured in dark cyan, would correspond to the same value of $\Delta N_{\text{eff}} \simeq 0.16$, since PBH masses in that range evaporate above the electroweak scale, giving the same value of ΔN_{eff} (see Fig. 4.1). Here, β remains a free parameter as for PBH domination, ΔN_{eff} is independent of β . In the right panel of Fig. 4.10, we show the GW spectra in the presence of extra interactions of right-handed neutrinos, with the strength of the interactions taken such that the thermal ν_R decouple at a temperature of ~ 850 MeV. This case corresponds to the bottom right panel of Fig. 4.4. Here, any spectral line to the right of the one coloured in dark cyan, would correspond to the same value of $\Delta N_{\text{eff}} = 0.24$, since PBH in this mass range evaporate at a temperature higher than 850 MeV. Thus, the thermal contribution would dominate as discussed earlier, contributing to a total ΔN_{eff} of 0.24, similar to the bottom right panel of Fig. 4.4. In both the plots shown in Fig. 4.10, the experimental sensitivities of relevant GW detectors namely, LISA [326], DECIGO [327], BBO [185], ET [181], CE [188] and LIGO-VIRGO [183] are shown.

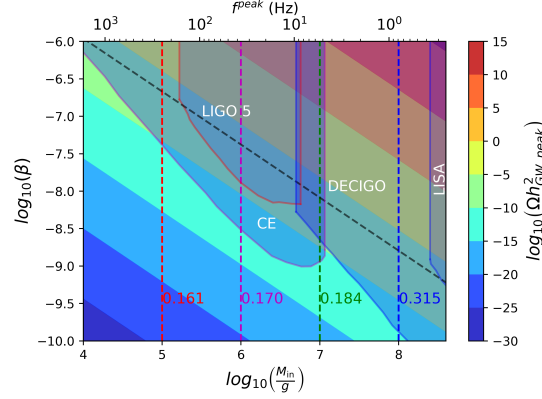


Figure 4.11: Contours of peak amplitude of GW induced by PBH density perturbations. The corresponding ΔN_{eff} due to PBH generated ν_R are shown assuming no (or very feeble) ν_R -SM interactions. The black dashed line indicates the upper bound on β (cf. Eq. (4.31)).

As we have seen already, the value of ΔN_{eff} is independent of β for PBH domination. Thus, future observations of N_{eff} from CMB observations will not be able to tell us anything about β provided $\beta > \beta_{\text{crit}}$. However, the value of β can be inferred from future GW detection with the spectral shape similar to the one generated by PBH density perturbations, since the peak of the GW amplitude is determined by β . In Fig. 4.11, we show the contours of the peak amplitude of GW in the $M_{\text{in}} - \beta$ plane, indicating the values of f_{peak} and ΔN_{eff} , considering no extra interactions of ν_R with the SM. The future sensitivities of several GW experiments are also shown in the same figure. In the presence of extra interactions, the values of ΔN_{eff} would be different as discussed above (cf. Fig. 4.10). ΔN_{eff} values would also be different for other light species. Finally, we summarise the GW and CMB complementarities in Table 4.1 and 4.2 by choosing a few benchmark points. PBH mass and initial energy fraction are fixed at benchmark values and the corresponding implications for ΔN_{eff} and GW observations for different types of dark radiation namely, ν_R , Goldstone boson, massless gauge boson are shown in Table 4.1 and 4.2. Note that we work with β values consistent with the upper bound given by Eq. (4.31), and higher than the critical value β_{crit} (cf. Eq. (4.6)) required for PBH to dominate. While in Table 4.1, no DR-SM interactions are assumed, in Table 4.2, thermalised DR is considered with specific decoupling temperatures. While BP1 and BP2 in both the cases remain within future sensitivities, BP3 for some specific type of DR is already ruled out by Planck 2σ limits.

Table 4.1: Without DR-SM interaction

BP	M_{in} (g)	β	ΔN_{eff}			CMB experiment (2σ)			GW experiment
			ν_R	GB	MGB	ν_R	GB	MGB	
BP1	10^6	1.5×10^{-8}	0.167	0.052	0.023	CMB-S4	CMB-HD	None	CE, ET, LIGO-VIRGO
BP2	10^7	6×10^{-9}	0.183	0.056	0.024	CMB-S4	CMB-HD	None	CE, ET, DECIGO, BBO
BP3	10^8	6×10^{-10}	0.313	0.096	0.042	Planck	CMB-S4	CMB-HD	DECIGO, BBO

Table 4.2: With DR-SM interaction

BP	M_{in} (g)	β	T_{dec} (GeV)	ΔN_{eff}			CMB experiment (2σ)			GW experiment
				ν_R	GB	MGB	ν_R	GB	MGB	
BP1	10^6	1.5×10^{-8}	10	0.188	0.035	0.072	CMB-S4	CMB-HD	CMB-S4	CE, ET, LIGO-VIRGO
BP2	10^7	6×10^{-9}	0.8	0.242	0.046	0.092	CMB-S4	CMB-HD	CMB-S4	CE, ET, DECIGO, BBO
BP3	10^8	6×10^{-10}	0.01	3.02	0.575	1.15	Planck	Planck	Planck	DECIGO, BBO

4.1.5 Summary

We revisit the prospects of generating dark radiation from primordial black holes in the early universe, by considering sizeable DR-SM interactions. We first reproduce the results by considering PBH and thermal interactions separately and the consequences for ΔN_{eff} observations at CMB experiments. We then consider the hybrid scenario with both thermal interactions and PBH to be source of DR and put new constraints on the PBH parameters as well as DR-SM interactions from the requirement of satisfying BBN and CMB limits on ΔN_{eff} . Compared to the scenario with no DR-SM interactions discussed in earlier works, our present scenario puts tighter constraints on the PBH parameter space namely, initial PBH mass and energy density.

We also find an interesting region of parameter space involving PBH as well as DR-SM interactions where DR initially decouples from the bath followed by PBH evaporation leading to re-thermalisation of DR with the SM bath. This leads to a lower decoupling temperature of DR enhancing the ΔN_{eff} and putting new constraints in the PBH parameter space. Though we have considered a specific type of DR namely, light Dirac neutrinos for the numerical analysis, the generic conclusions are applicable to any thermalised DR.

We also find the gravitational wave complementarity of this scenario by considering PBH density perturbations to be the source of such stochastic GW. In the ultra-light PBH window considered in our work, GW sourced from PBH this way not only remains within current and planned experiment's sensitivity but also remains independent of PBH formation mechanism. We show the complementarity between GW and CMB observations for different types of DR with and without DR-SM interactions. Interestingly, some of the benchmark points can lead to GW

peak amplitude, frequency in LIGO-VIRGO ballpark while keeping ΔN_{eff} within the reach future CMB experiments like CMB-S4. On the contrary, some part of the parameter space can keep GW prospects within future experiment's reach while saturating Planck 2018 limits on ΔN_{eff} . Such complementary detection prospects at CMB and GW experiments of thermalised dark radiation is particularly interesting due to limited prospects of detecting such light degrees of freedom at particle physics experiments.

Before we end, we briefly comment on the impact of Kerr PBH and a non-monochromatic mass function of PBH. First of all, considering Kerr PBH does not significantly alter DR contribution to ΔN_{eff} . This is true for DR with spin 0, 1/2 and 1, but significant enhancement is observed for spin 2 DR particles [169, 328, 329]. At the same time, our results on the analysis of GW spectrum would almost be unchanged. This is mainly because the evaporation temperature of a spinning BH changes only slightly compared to the non-spinning case [330, 331]. On the other hand, ΔN_{eff} can depend significantly on the mass distribution of PBH. In Ref. [170], it was shown that the contribution to ΔN_{eff} might substantially change for some choices of the mass spectrum, compared to the monochromatic case. Similarly, the amplitude of induced GW is also sensitive to the PBH mass spectrum [177, 178]. It would be interesting to study the combined effect in the situation where DR is produced both from thermal bath (due to sizeable DR-SM couplings) and from PBH with non-monochromatic mass distribution and non-zero spin distribution. We leave such a complete study for future works.

4.2 Cogenesis of baryon and dark matter with PBH and QCD axion

In this section, we discuss one such possibility, where the generation of BAU through thermal leptogenesis [18] shares a common origin with axion physics. While there are several ways in which one couples the axion physics to the baryogenesis [332–335], our interest aligns with the SMASH-like [336] scenarios (and similar extensions [337, 338]). In this scenario, the RHN introduced in the SM to generate light neutrino mass via seesaw mechanism and to provide leptogenesis, get mass from the PQ

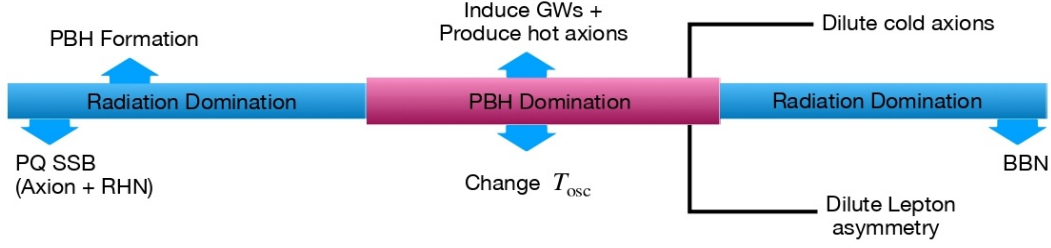


Figure 4.12: Schematic diagram showing a possible timeline of the scenario.

phase transition, i.e., $f_a \equiv M \equiv \Lambda_{\text{lepto}}$, where $M(\Lambda_{\text{lepto}})$ is the RHN mass, equivalent to the scale of seesaw and thermal leptogenesis. Such a unification therefore addresses neutrino mass, strong-CP problem, DM, and BAU at one stroke. Unlike Ref.[336] for SMASH-like frameworks, the main motivation to unify the leptogenesis scale Λ_{lepto} and the PQ scale f_a in this work is to explore the testability of $\Lambda_{\text{lepto}} \equiv f_a \gtrsim 10^{12}$ GeV—also referred to as the vanilla regime of leptogenesis. On the other hand, for $f_a \gtrsim 10^{12}$ GeV, axion dark matter gets overproduced via vacuum misalignment mechanism with standard cosmological expansion. In order not to lose the interesting connection among axion DM, strong-CP problem and vanilla leptogenesis, we explore the possibility that before the BBN, the Universe underwent an early matter-dominated phase releasing entropy while reproducing the standard radiation dominated Universe [339, 340].

Here, we show that if the matter domination is provided by ultralight evaporating PBHs [147, 151, 160] with initial mass $M_{\text{in}} \lesssim 4 \times 10^8$ g, testability enhances significantly. This is because, in addition to predicting axion parameter space in the $g_{\alpha\gamma} - m_a$ plane, one can have stochastic GWs signatures via PBH density fluctuations [177–180, 323]. Remarkably, the mass range of PBH required to generate correct DM and at the same time BAU, produces GWs within the frequency bands of current and planned GW detectors. A schematic diagram of this scenario showing a possible timeline is presented in Fig. 4.12. Two additional features in this figure namely, the axion oscillation temperature T_{osc} and the production of hot axions in the form of dark radiation within reach of future CMB experiments are discussed later in this section.

4.2.1 The Framework

We consider a type-I seesaw framework [102, 103, 341–343] extended by Peccei-Quinn symmetry [20, 21]. A recent review of QCD axion models can be found in [64]. In addition to the heavy RHNs (here we consider three RHNs) required for the seesaw, the other field content and PQ charges are kept similar to a KSVZ type model [60, 61] in which $v_{\text{PQ}} = f_a$, where v_{PQ} is the vacuum expectation value (VEV) of the PQ scalar field σ and f_a is the axion decay constant⁵. Thus, the BSM field content is constituted by a complex PQ scalar field $\sigma \equiv \frac{v_{\text{PQ}} + \rho}{\sqrt{2}} e^{ia/f_a}$ ($\sigma \sim (1, 1, 0)$), a heavy quark $Q \sim (3, 1, 0)$ and three right-handed neutrinos $N_R \sim (1, 1, 0)$ where the numbers in brackets denote their charges under the SM gauge symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$. The transformation of these fields and leptons under PQ symmetry are

$$\sigma \rightarrow e^{i\alpha} \sigma; \quad N_R \rightarrow e^{-i\alpha/2} N_R; \quad l_R, L \rightarrow e^{-i\alpha/2} l_R, L; \quad Q_L \rightarrow e^{i\alpha/2} Q_L; \quad Q_R \rightarrow e^{-i\alpha/2} Q_R.$$

The SM quarks have vanishing PQ charges. The Yukawa coupling part of the PQ invariant Lagrangian can be written as

$$\mathcal{L}_Y = - \left[y \bar{Q}_L \sigma Q_R + G_{ij} \bar{L}_i H l_{jR} + F_{ij} \bar{L}_i \tilde{H} N_{jR} + \frac{1}{2} y_{ij} \bar{N}_{iR}^c \sigma N_{jR} \right] + \text{h.c.} \quad (4.32)$$

with H being the SM Higgs. The scalar potential of the model is given by

$$V(H, \sigma) = \lambda_H \left(H^\dagger H - \frac{v^2}{2} \right)^2 + \lambda_\sigma \left(|\sigma|^2 - \frac{v_{\text{PQ}}^2}{2} \right)^2 + \lambda_{H\sigma} \left(H^\dagger H - \frac{v^2}{2} \right) \left(|\sigma|^2 - \frac{v_{\text{PQ}}^2}{2} \right) \quad (4.33)$$

with v being the VEV of the SM Higgs H . After PQ symmetry breaking the first and the last terms in $\mathcal{L}_Y$ give rise to

$$\begin{aligned} y \bar{Q}_L \sigma Q_R &\rightarrow \frac{y}{\sqrt{2}} \rho \bar{Q}_L Q_R e^{ia/f_a} + \frac{y}{\sqrt{2}} v_{\text{PQ}} \bar{Q}_L Q_R e^{ia/f_a} \\ \frac{1}{2} y_{ij} \bar{N}_{iR}^c \sigma N_{jR} &\rightarrow \frac{1}{2} \frac{y_{ij}}{\sqrt{2}} \rho \bar{N}_{iR}^c N_{jR} e^{ia/f_a} + \frac{1}{2} \frac{y_{ij}}{\sqrt{2}} v_{\text{PQ}} \bar{N}_{iR}^c N_{jR} e^{ia/f_a}. \end{aligned} \quad (4.34)$$

⁵It should be noted that our particle content is same as the SMASH framework [336]. While SMASH considers Higgs portal inflation via non-minimal coupling to gravity, we remain agnostic about the origin of inflation.

The phase part can be absorbed by the transformation $Q_R \rightarrow e^{-i\frac{a}{2f_a}} Q_R$ and $N_R \rightarrow e^{-i\frac{a}{2f_a}} N_R$. As the chiral transformation on Q is anomalous under QCD, this gives the term $\frac{g_s^2}{32\pi^2} \frac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu}$ with $G_{\mu\nu} (\tilde{G}^{\mu\nu})$ being (dual) field strength tensor of QCD. Simultaneously, from the kinetic term $\bar{Q} i \gamma^\mu \partial_\mu Q$, after transformation one gets the term $-\frac{\partial_\mu a}{2f_a} \bar{Q} \gamma^\mu \gamma_5 Q$. Similarly, from the kinetic term of RHN $\frac{i}{2} \bar{N} \gamma^\mu \partial_\mu N$, one gets the term $-\frac{1}{4} \frac{\partial_\mu a}{f_a} \bar{N} \gamma^\mu \gamma_5 N$. Now using the Dirac equation and the fact that the total derivative is zero at the boundary, we get

$$\begin{aligned} -\frac{\partial_\mu a}{2f_a} \bar{Q} \gamma^\mu \gamma_5 Q &= i \frac{M_Q}{f_a} a \bar{Q} \gamma_5 Q \\ -\frac{1}{4} \frac{\partial_\mu a}{f_a} \bar{N} \gamma^\mu \gamma_5 N &= \frac{i}{2} \frac{M_i}{f_a} a \bar{N} \gamma_5 N \end{aligned} \quad (4.35)$$

which define axion couplings to RHN and heavy quark Q . We have also dropped the chirality index from RHN notation and will denote them only by N_i with mass M_i hereafter.

4.2.2 Axion dark matter, PBH, and Leptogenesis

For QCD axions, the zero temperature mass ($T \leq T_{\text{QCD}} \simeq 150 \text{ MeV}$) is related to PQ symmetry breaking scale, f_a as

$$m_a \simeq 5.7 \left(\frac{10^{12} \text{ GeV}}{f_a} \right) \mu\text{eV}. \quad (4.36)$$

Above $T > T_{\text{QCD}}$, the temperature-dependent axion mass is given as

$$m_a(T) = m_a \left(\frac{T_{\text{QCD}}}{T} \right)^4. \quad (4.37)$$

As indicated by lattice simulation, the power of 4 is not precise [64], but it does not affect our overall results significantly. If the PQ symmetry breaks in post inflationary era, the initial misalignment angle, $\theta_i = a_i/f_a$ takes the average value $\theta_i = \frac{\pi}{\sqrt{3}} \sim 1.81$. At high temperatures, the axion evolution is friction-dominated and frozen at θ_i . It then starts to oscillate at a temperature T_{osc} , when the Hubble rate becomes comparable to axion mass: $\mathcal{H}(T_{\text{osc}}) \sim 3m_a(T_{\text{osc}})$. From the onset of oscillations temperature, axion starts behaving as matter, and from conservation of comoving

number density, its number density at a later epoch can be written as

$$n_a(T) = n_a(T_{\text{osc}}) \frac{s(T)}{s(T_{\text{osc}})}, \quad (4.38)$$

where $s(T)$ is the comoving entropy density at a temperature T . The axion behaves as cold matter and can make all the observed dark matter abundance with total abundance

$$\Omega_a h^2 = \frac{\rho_a(T_0)}{\rho_c} h^2 = \frac{m_a(T_0)}{\rho_c} \left(\frac{\rho_a(T_{\text{osc}})}{m_a(T_{\text{osc}})} \frac{s(T)}{s(T_{\text{osc}})} \right) h^2. \quad (4.39)$$

Here ρ_c represents the critical energy density of the Universe today. If axion constitutes all the dark matter, it should have a mass around $\sim 14 \mu\text{eV}$, when produced via the misalignment mechanism [64]. The presence of PBH modifies the axion dark matter abundance in two different ways: (1) PBH change the oscillation temperature T_{osc} , because ρ_{PBH} contributes to Hubble expansion rate $\mathcal{H}$; (2) PBH evaporation dilutes the existing abundance of axions via entropy injection. The latter effect is the dominant one. In addition to affecting the axion dark matter abundance, PBH evaporation can also produce axions. However, these axions behave as dark radiation contributing to N_{eff} . We numerically study the evolution of the PBH-axion system, closely following Ref. [344] (see Appendix A.1).

Lepton asymmetry can be generated thermally (with the thermal production of RHNs mediated by Yukawa couplings) at a high scale via the CP-violating decays of RHNs, which can be converted to the baryon asymmetry through the electroweak sphaleron processes [18]. At a temperature $T \sim M_i$, lepton asymmetry produced by i th RHN freezes out; therefore, M_i is typically the scale of thermal leptogenesis; $M_i \equiv \Lambda_{\text{lepto}}$. In the high-scale leptogenesis scenario, there are three distinct regimes of leptogenesis. The RHNs produce lepton doublets as a coherent superposition of flavors: $L_i = A_{i\alpha} L_{\alpha i}$, where $\alpha = e, \mu, \tau$ and $A_{i\alpha}$ is the corresponding amplitude determined by $F_{i\alpha}$. For temperatures $T \gtrsim 10^{12}$ GeV, all the charged lepton Yukawa couplings are out of equilibrium (the interaction strength is given by $\Gamma_\alpha \sim 5 \times 10^{-3} T h_\alpha$, with h_α being the charged lepton Yukawa coupling) and do not participate in the leptogenesis process. This is the unflavor or the vanilla regime of leptogenesis. In this regime, leptogenesis is not sensitive to the neutrino mixing matrix, e.g., mixing angles and low-energy CP phases [345]. When the temperature drops down to $T \lesssim 10^{12}$ GeV, the τ flavored charged lepton interaction

comes into equilibrium and breaks the coherent L_i state into τ and a composition of $e + \mu$ flavor (two flavor regime). Likewise, the later composition is broken once $T \lesssim 10^9$ GeV, when the μ flavored charged lepton interaction comes into equilibrium (three flavor regime). In both these flavored regimes, leptogenesis may be sensitive to the low-energy neutrino parameters because the neutrino mixing matrix cannot be eliminated from the CP asymmetry parameter. Therefore, for $T \sim M_i \gtrsim 10^{12}$ GeV, practically there are no ways to test high-scale thermal leptogenesis, barring some recent effort with gravitational waves from topological defects [346–350]. As mentioned in the introduction, here we show, in the unified setup of axion and see-saw, the vanilla regime is testable with axion physics as well as gravitational waves from PBH density fluctuations.

A crucial aspect of this work is to obtain the correct DM density for large $f_a \gtrsim 10^{12}$ GeV, requiring PBH evaporation after axion oscillation so that the overdensity of DM can be diluted. As we shall see, the required range of PBH mass for this does not affect the lepton asymmetry production (happens around $T \sim f_a$), however, dilutes it similarly to the DM density, and therefore correlates both of them.

For hierarchical RHNs, the CP asymmetry parameter can be estimated as [19, 351]

$$\epsilon_1 = \frac{3M_1 \sqrt{(\Delta m_{\text{atm}})^2}}{4\pi v^2}, \quad (4.40)$$

with $v/\sqrt{2} = 174$ GeV is the VEV of the SM Higgs and $(\Delta m_{\text{atm}})^2 \simeq 2.4 \times 10^{-3} \text{eV}^2$ is the active neutrino atmospheric mass-squared difference. The resulting baryon asymmetry from the decay of N_1 can be approximately written in terms of baryon-to-photon ratio as [324]

$$\eta_B \approx 10^{-2} \kappa_1 \frac{\epsilon_1}{\xi}. \quad (4.41)$$

Here ξ denotes the entropy dilution due to PBH evaporation, estimated as [324]

$$\xi = 233 \beta \left(\frac{M_{\text{in}}}{M_{\text{Pl}}} \right) \left(\frac{\gamma}{g_{*,B}(T_{\text{BH}}) \mathcal{G}} \right)^{1/2}, \quad (4.42)$$

and $\kappa_1 \simeq 10^{-2}$ is the efficiency of lepton asymmetry production. $\mathcal{G} \sim 3.8$ is the greybody factor and $M_{\text{Pl}} = \sqrt{8\pi} M_{\text{P}}$ is the Planck mass. The extra factor 10^{-2} on the right-hand side of Eq.(4.41) accounts for the combined effects of sphaleron conversion of the leptons to baryons plus the photon dilution [351]. Because of our

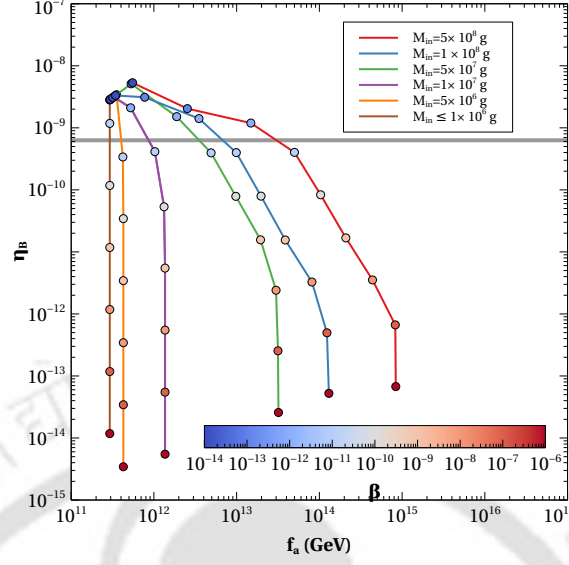


Figure 4.13: Baryon to photon ratio, η_B versus f_a for different PBH mass and initial fractional abundance, β . Here, the contours satisfy the correct DM relic. Initial misalignment angle is taken to be $\theta_i = \frac{\pi}{\sqrt{3}} \sim 1.81$. The horizontal gray line denotes the observed value of η_B . Here, we consider $M_1 = f_a$ which gives us Eq.(4.43).

proposed unification $f_a \simeq M_1$ ⁶, Eq.(4.40), (4.41), and (4.42) impose the following condition on β :

$$\beta \simeq 5.7 \times 10^{-12} \left(\frac{M_{\text{Pl}}}{M_{\text{in}}} \right) \frac{f_a}{\text{GeV}} \simeq 32.3 \left(\frac{M_{\text{Pl}}}{M_{\text{in}}} \right) \left(\frac{1\mu\text{eV}}{m_a} \right). \quad (4.43)$$

Eq.(4.43) is the most important analytical relation we derive. Because of this stringent relation among β , M_{in} , and $f_a(m_a)$, concurrent generation of correct DM and BAU occurs for an extremely constrained parameter space as analyzed below in detail.

In Fig.4.13, we show the baryon asymmetry as a function of the PQ scale f_a , for different values of initial PBH mass. The contours shown are consistent with the observed DM relic from axion oscillation. If PBH evaporates before the axion oscillation temperature T_{osc} , they do not affect the axion dynamics. For $M_{\text{in}} \lesssim 10^6$ g, the PBH evaporation temperature remains higher than T_{osc} leading to the standard

⁶Note that here we use the formula for the CP asymmetry parameter valid only for a hierarchical mass spectrum of RHNs, and we consider a N_1 -dominated leptogenesis scenario. In this case, although the heavier RHNs do not have any significant role to play, their masses are constrained as $M_{i=2,3} < f_a \sqrt{4\pi}$.

axion results with $f_a \simeq 3 \times 10^{11}$ GeV, generating the correct DM relic. This is indicated by the left-most brown-colored contour in Fig.4.13. Along this contour, β is varied, as per the color bar, which changes the baryon asymmetry because of entropy dilution, following Eq.(4.42). Increasing the PBH mass above 10^6 g decreases the oscillation temperature and reduces axion abundance because of entropy dilution, resulting in a higher f_a to obtain the observed DM abundance. With a decrease in β , this cumulative effect of PBH starts to diminish and hence the contours move towards the standard case of $f_a \simeq 3 \times 10^{11}$ GeV.

On the other hand, decreasing f_a and β has opposite effects on the baryon asymmetry η_B , with the former decreasing the asymmetry by decreasing M_1 (cf. Eq.(4.40)), while the latter increasing the asymmetry because of lesser entropy dilution. The latter effect turns out to be more dominant, leading to an increase in the value of η_B .

The lepton asymmetry can be significantly enhanced if two of the RHNs are quasi-degenerate, i.e., $\Delta M = M_2 - M_1 \ll \bar{M} = (M_1 + M_2)/2$ leading to the resonant leptogenesis scenario [352, 353]. For $\Delta M \simeq \Gamma$, the CP asymmetry parameter can reach up to $\mathcal{O}(1)$, where Γ indicates the decay width of the RHN. For the resonant scenario, the expression for β can be derived as

$$\beta = 2.9 \times 10^4 \epsilon_1 \left(\frac{M_{\text{Pl}}}{M_{\text{in}}} \right). \quad (4.44)$$

We quantify the results for both, the hierarchical and the resonant leptogenesis scenarios on the $M_{\text{in}} - \beta$ plane in Fig.4.14. The variation of axion mass is represented by the color gradient (see also the contours). For PBH mass $M_{\text{in}} \lesssim 10^{6.5}$ g, the evaporation of PBH occurs before the onset of axion oscillation, thereby not changing the axion abundance. Similarly, in the radiation domination region (below the blue line), PBH do not affect the axion abundance. For $M_{\text{in}} \gtrsim 10^{6.5}$ g, PBH delay the oscillation temperature plus dilute the axion density. Therefore, an ample amount of axions is required before PBH evaporation so that the correct DM relic is satisfied. This is achieved by reducing the axion mass (or increasing the PQ symmetry breaking scale f_a). The larger the M_{in} , the stronger the entropy production and therefore smaller the axion mass (larger f_a).

The observed baryon asymmetry constraints for the hierarchical (left) and resonant scenario (right) are shown with the red curves. For the PBH mass range,

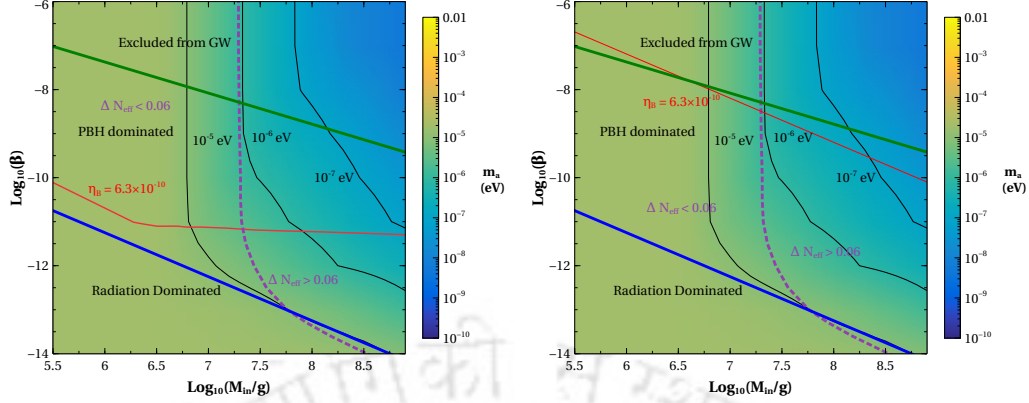


Figure 4.14: Constraint on M_{in} versus β plane considering a hierarchical N_1 -dominated leptogenesis scenario (left plot) and a resonant leptogenesis scenario (right plot). The color bar represents the axion mass that satisfies the observed DM abundance. The red color lines in both plots denote the parameter space that is consistent with both, DM relic and baryon asymmetry. The region excluded from GW is separated by the solid green line. The magenta dashed line separates the regions that can be probed by future CMB-S4 experiments due to the contribution of hot axion on N_{eff} .

$10^{5.5}\text{g} \lesssim M_{\text{in}} \lesssim 10^{6.5}\text{g}$, the CP asymmetry parameter remains constant as $M_1 \sim f_a \sim 10^{11}$ as the axion DM production is not affected by PBH. Therefore, for a fixed f_a , the correlation between β and M_{in} determined according to Eq.(4.43) (β decreases with the increase of M_{in}). For $M_{\text{in}} \gtrsim 10^{6.5}\text{g}$, an increase in PBH mass increases the axion scale f_a owing to the interplay of entropy dilution and producing correct DM abundance. Because the CP asymmetry increases monotonically with f_a , in this case, a sharp decrease in β with the increase of M_{in} gets partially compensated, resulting in a plateau in the red curve (left). The region below (above) the red curve corresponds to $\eta_B > (<) 6.3 \times 10^{-10}$. As we shall discuss in sec.4.2.3, the allowed values of β in this hierarchical leptogenesis scenario produce GWs with weaker amplitude (the maximum value of β allowed by N_{eff} constraint on GWs is shown with the green line) compared to its resonant version.

In the right panel of Fig.4.14, we show the results for the resonant leptogenesis scenario. Instead of Eq.(4.40), here we have considered a maximal CP asymmetry $\epsilon_1 = 0.1$, achievable with resonant enhancement. Due to this enhancement in the value CP asymmetry parameter value, strong entropy dilution and therefore for a given PBH mass, a larger value of β is required to produce the observed BAU. This is evident from the upward shift of the solid red line compared to the one in the

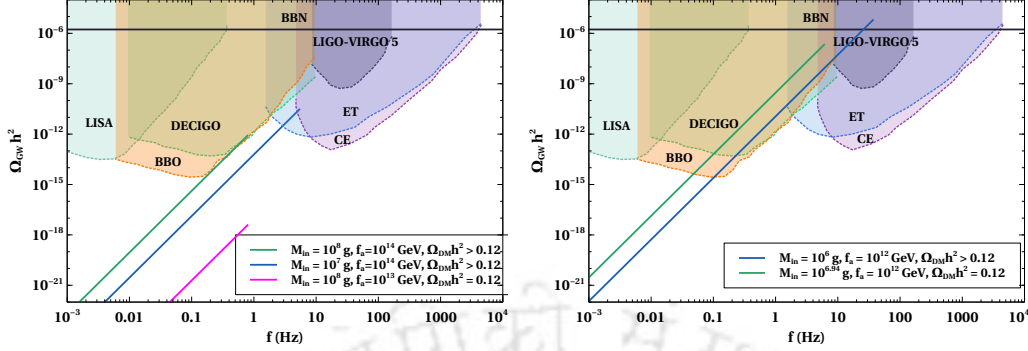


Figure 4.15: Left panel: GW spectrum for hierarchical scenario. Right panel: GW spectrum for the resonant scenario.

left panel. In addition, due to a larger value of β , the resonant leptogenesis scenario produces much stronger GWs compared to the hierarchical case.

For completeness, in both the plots, we also indicate the parameter space within reach future CMB experiments like CMB-S4 [115] which can measure additional relativistic degrees of freedom ΔN_{eff} . Such non-zero ΔN_{eff} or dark radiation arises in our setup due to the production of hot axions from PBH evaporation. Note remarkably that in both the plots, the $T \sim M_i \sim f_a \gtrsim 10^{12} \text{ GeV}$ is potentially testable, with either axion detectors or in combination with GW experiments.

4.2.3 Detection prospects

Signatures in the form of GW from PBH density fluctuation

As discussed in Section 2.2, GWs arise from the inhomogeneities in the PBH distribution after they are formed. Here, using Eq.(4.43) and (4.44), we can relate leptogenesis scale and $\Omega_{\text{gw}}^{\text{peak}}$ as

$$\Omega_{\text{GW}}^{\text{peak}} \simeq 6.29 \times 10^{-22} \left(\frac{f_a}{10^{12} \text{ GeV}} \right)^{16/3} \left(\frac{10^7 \text{ g}}{M_{\text{in}}} \right)^{14/9} \quad (4.45)$$

for hierarchical leptogenesis, and

$$\Omega_{\text{GW}}^{\text{peak}} \simeq 1.71 \times 10^{-7} \left(\frac{\epsilon_1}{0.1} \right)^{16/3} \left(\frac{10^7 \text{ g}}{M_{\text{in}}} \right)^{14/9} \quad (4.46)$$

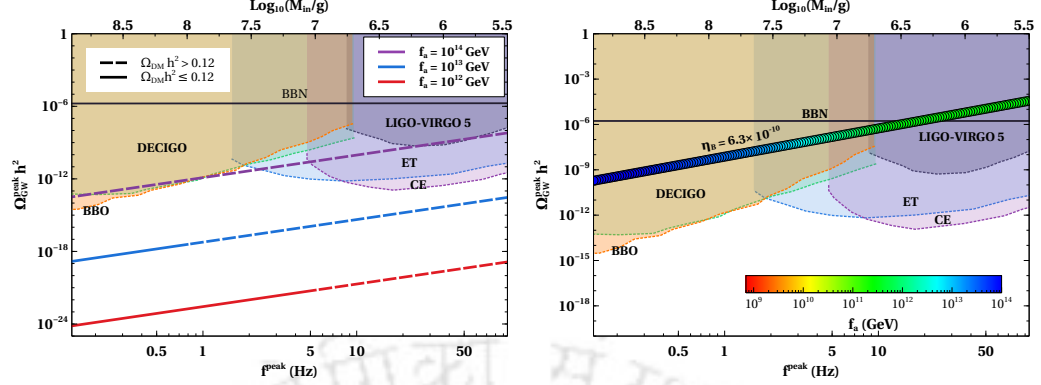


Figure 4.16: Left panel: $\Omega_{\text{GW}}^{\text{peak}} h^2$ versus f^{peak} for hierarchical scenario with three different f_a . Right panel: $\Omega_{\text{GW}}^{\text{peak}} h^2$ versus f^{peak} for the resonant scenario with maximal CP asymmetry for correct axion DM abundance.

for resonant leptogenesis, where for the latter scenario ϵ_1 is expressed as a function of f_a and mass splitting $\sim (M_2 - M_1)M_2^{-1}$ between two degenerate RHNs. To relate GW with a successfulogenesis, Eq.(4.45) and (4.46) are required to be analysed with the constraint from correct axion DM production. Similar to the previous section, this has been implemented by solving the set of Boltzmann equations for axion and PBH (cf. Appendix A.1).

In Fig.4.15, we show the GW spectra for the hierarchical (left panel) and the resonant scenario (right panel), along with the sensitivity of different GW experiments such as advanced LIGO-VIRGO/LIGO 5 [183], LISA [326], DECIGO [327] and CE [188]. The corresponding $\{M_{\text{in}}, f_a\}$ values are marked in the legend. Each spectrum corresponds to a different DM relic density but is consistent with the observed baryon asymmetry. Note that the signal strength is relatively weaker for the hierarchical leptogenesis case (left panel) when the model produces the correct DM relic (the magenta) curve. We however would like to mention that future planned detectors such as UDECIGO (UDECIGO-corr) [354, 355] can test such signal. On the other hand, for the resonant case, one obtains much stronger signal strength (right) without overproducing the DM. The reason being, as discussed in the previous section, resonant leptogenesis scenario requires a large value of β , and the peak amplitude goes as $\Omega_{\text{gw}}^{\text{peak}} \sim \beta^{16/3}$.

Plots in Fig.4.16, represent an enlarged parameter space covering $\Omega_{\text{GW}}^{\text{peak}} h^2$ and f^{peak} for a wide range of M_{in} . The left panel plot represents the hierarchical leptogenesis leading to successful BAU. The solid (dashed) segment in the curves represents

the correct (overproduced) DM abundance. As discussed earlier, because resonant cases allow strong amplitude GW, we find it sufficient to show the GW spectra only for correct DM abundance (right panel).

Note also that, although arguable [356], one can potentially have GWs from axion-strings [357] in this model. Due to the PBH domination, our scenario can marginally access the values $f_a \gtrsim 10^{14}$ GeV to produce GWs from axion cosmic strings. However, we do not discuss this spectrum because, in the presence of PBH, a string-PBH network evolution requires numerical simulation [358].

Signatures on $g_{a\gamma} - m_a$ plane

The axion detection prospects in most experiments primarily rely on axion-photon coupling. The axion-photon coupling leading to $\frac{1}{4}g_{a\gamma}aF\tilde{F}$ is given by (same as KSVZ model) [359]

$$g_{a\gamma} = -\frac{\alpha}{2\pi f_a} \left(\frac{2}{3} \frac{4m_d + m_u}{m_u + m_d} \right) = -1.92 \frac{\alpha}{2\pi f_a}, \quad (4.47)$$

with α being the fine-structure constant. Fig.4.17 shows the axion-photon coupling with axion mass including the bounds and sensitivities of different experiments. The current experimental bound on the axion-photon coupling from various experiments or observables are shown by the solid color lines (CAST [360, 361], SN87A [362–364], NGC 1275 [365], ADMX [366–368]) whereas future experimental sensitivities are shown by the dashed lines (CASPER [369], KLASH [370–372], ABRACADABRA [373, 374], CULTASK [375, 376], MADMAX [377], IAXO [378, 379], Fermi-LAT [380], BH superradiance [381]).

The left panel of Fig.4.17 shows the constraints on $g_{a\gamma}$ in the presence of PBH for three different axion mass associated with three different sets of $\{M_{\text{in}}, \beta\}$. These three points are indicated by green, cyan, and magenta colored star-shaped point (★). For all three points, both DM relic and correct baryon asymmetry (with resonant leptogenesis) are satisfied. While the green point comes within the sensitivity of future CULTASK experiment, the cyan and magenta points can be probed by future KLASH and ABRACADABRA experiments. Apart from their signatures in axion detection experiments, the three sets of $\{M_{\text{in}}, \beta\}$ also lie within the sensitivity of future DECIGO. This provides a complementary probe ofogenesis in terms of

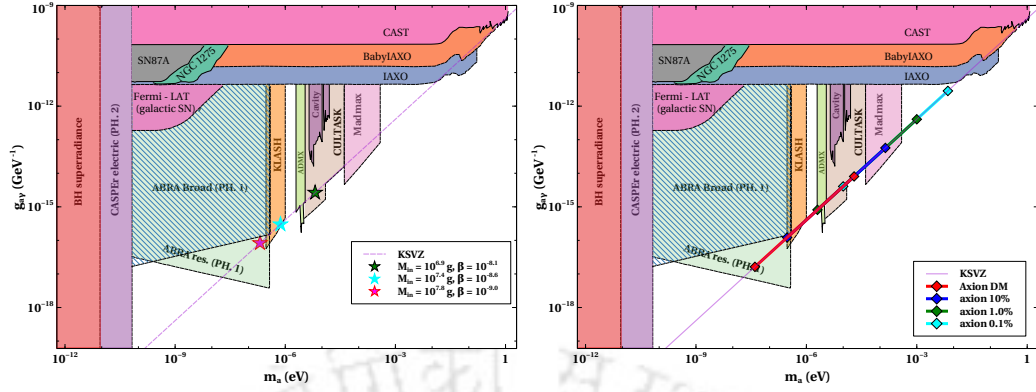


Figure 4.17: Left panel: Axion-photon coupling versus axion mass in presence of PBH for three different sets of $\{M_{in}, \beta\}$ denoted by green, cyan, and magenta colored star-shaped points. Cogenesis with resonant leptogenesis is possible for all three points. Also, all three sets of $\{M_{in}, \beta\}$ come within future GW experiments (see related text for more details). Right panel: Constraints on axion-photon coupling in the presence of PBH for four different axion contributions to DM.

GWs and axion detection experiments. Note that for a smaller value of β (cf. left panel of Fig.4.14), the scenario of hierarchical leptogenesis can also be within the sensitivity of axion experiments, however with a weaker GW signal.

We conclude with the following remark: Because we consider axions to constitute the entire DM density, the overall scenario is highly constrained. This framework, however, can naturally allow one of the RHNs to make up the dominant dark matter density while keeping axion abundance subdominant. In which case, the predictions of this scenario would be very different. Interested readers may look up to a detailed discussion presented on this in Appendix A.9, A.10, and A.11. Here for example, in the right panel of Fig.4.17, we show the constraint for four different situations where axion contribution to DM is 100%, 10%, 1% and 0.1% denoted by red, blue, green, and cyan line respectively. The minimum f_a is obtained for PBH mass with $T_{ev} > T_{osc}$ (i.e. $M_{in} \lesssim 10^{6.5} \text{ g}$). The maximum f_a is obtained for PBH mass with $T_{ev} \sim T_{BBN}$ and for maximum allowed β (i.e. $M_{in} \sim 10^{8.6} \text{ g}$ and $\beta \sim 6 \times 10^{-10}$). While in this scenario the vanilla leptogenesis parameter space is still accessible, a striking difference would be to have only resonant leptogenesis for successful baryogenesis.

4.2.4 Summary

The presence of an early matter-dominated epoch in the early Universe allows a larger axion decay constant f_a consistent with correct DM abundance. We implement this idea in a SMASH-like setup where the f_a and seesaw or leptogenesis scale are equivalent. We show that contrary to the EMD provided, e.g., by a long-lived field, a PBH-domination scenario is markedly testable with gravitational waves alongside the axion experiments that intend to detect low-mass axions ($m_a \lesssim 10^{-5}$ eV). In particular, this setup opens up a novel opportunity to test unflavored leptogenesis scales $f_a \gtrsim 10^{12}$ GeV, which are otherwise neither testable from measurements of neutrino mixing parameters like Dirac CP phase nor accessible directly at terrestrial experiments. A hierarchical (resonant) leptogenesis scenario produces weaker (stronger) gravitational waves within the frequency band of planned GW detectors. A successfulogenesis renders extremely constrained model parameter space and thereby robust predictions. In addition to GW and axion detection prospects, we can also have observable dark radiation in future CMB experiments due to the axions produced from PBH evaporation. This keeps our cogenesis setup verifiable at gravitational wave, axion detectors as well as CMB experiments.

Signatures of BSM physics in high energy events

In Chapters 3 and 4, we examined the early-Universe signatures of DNPDM scenarios and the production of thermalized dark radiation, as well as theogenesis of the baryon asymmetry and axion dark matter in the presence of ultra-light PBHs, respectively. In the present chapter, we shift our focus to late-time observables of BSM physics. In particular, we investigate recent observations of two UHE events and explore their possible interpretation in terms of the decay of super-heavy dark matter.

In Section 5.1, we consider the recent detection of an UHECR event by the TA experiment, known as the Amaterasu event, with an energy of 2.44×10^{11} GeV [251]. This event is among the highest-energy cosmic rays ever observed. The arrival direction (R.A.: $255.9 \pm 0.6^\circ$, Dec.: $16.1 \pm 0.5^\circ$) of this UHECR points back to a void in the local Universe rendering more questions than answers about its origin. As UHECRs above energy 5×10^{10} GeV propagating more than ~ 10 Mpc (mean free path) undergo energy losses due to the Greisen–Zatsepin–Kuzmin (GZK) effect [231, 382–388], the likely source of this event is expected to be located within a distance of ~ 30 Mpc [251]. The TA collaboration [251] analysed a catalogue of gamma-ray sources to identify the probable source and found that the known sources

were [389, 390] located at distances beyond 30 Mpc. One possibility could be that the primary UHECR is an iron nucleus at source, which might have undergone significant deflection in the extragalactic magnetic field (EGMF) during propagation to Earth [391–393]. Due to this, the actual source direction might be different than the arrival direction. Another possible explanation of the origin could be attributed to decaying SHDM in the Milky Way or any local galaxy along the arrival direction. In Section 5.1, we examine if such a possibility of decaying SHDM in the Milky Way could give rise to the TA Amaterasu event.

In Section 5.2, we examine another remarkable recent observation: the detection of an ultra-high-energy neutrino event, KM3-230213A, by the KM3NeT collaboration [228]. The deep-sea neutrino telescope has detected an UHE muon with energy 120^{+110}_{-60} PeV arriving from a near-horizontal direction (RA: 94.3° , Dec: -7.8°) which is most likely to be created by a neutrino of even higher energy in the vicinity of the detector. The required neutrino energy was found to be in the range 110-790 PeV at 1σ CL with median energy being 220 PeV. This is the highest energy neutrino event ever detected. Creation of such energetic neutrinos through cosmic ray interactions (pp or $p\gamma$) in conventional astrophysical sources would require protons to be accelerated to around EeV energies. However, such sources are not available in our Milky Way or any nearby Galaxies. The KM3NeT collaboration has analyzed several classes of source candidates such as blazars, gamma-ray bursts using different electromagnetic telescope catalogs and found no conclusive evidence [228]. The absence of any conclusive astrophysical source [394, 395] points towards a cosmic origin of the highest energy neutrino event detected so far [396]. Cosmogenic UHE neutrinos together with UHE gamma-rays are expected to be created in the interactions of UHE cosmic rays with the CMB photons, the so called GZK process. Several model estimates of these UHE neutrinos and gamma-rays flux can be found in the literature [e.g., see 231, 384, 385, 397]. The UHE neutrino flux required to produce the KM3-230213A event was found to overshoot not only these model estimates, but also the sensitivities of other presently running neutrino telescopes such as IceCube and PAO. This could be due to the large uncertainty (about 3 orders of magnitude) in the KM3-230213A flux, possibly created by an upward fluctuation [228]. On the other hand, non-observation of similar events at other experiments like IceCube and PAO leads to a $2.5\sigma - 3\sigma$ tension¹ with the cosmogenic origin hypothesis of the KM3-230213A event [399]. This tension is significant given

¹An independent analysis [398] finds a slightly larger tension.

the 1σ CL estimate of the neutrino energy being above 100 PeV. Several follow-up studies on implications of this neutrino event for UHE cosmic rays [400, 401], Lorentz invariance violation [402–404], possible astrophysical origin [405, 406], possible origin from primordial black hole evaporation [407], in-vacuo dispersion of neutrinos [408] have also appeared in the literature. In Section 5.2, we consider a DM origin of the KM3-230213A event flux at 3σ CL while also addressing the mild tension with IceCube and PAO for such diffuse, isotropic origin of the event.

Section 5.1 is based on the published work [409], while Section 5.2 is based on the published work [410].

5.1 Possible origin of the Amaterasu event from dark matter decay

In this section, we consider the origin of the recent TA Amaterasu event in light of SHDM decay. We put multi-messenger constraints on the lifetime and mass of the SHDM using UHECR, gamma-ray and neutrino telescopes. We also account for the low energy UHECR spectrum detected by the TA. We consider different decay modes for SHDM as well as a wide range ($10^{11} - 10^{15}$ GeV) of SHDM mass. The constraints on the lifetime from gamma-ray non-detection limit of TA and PAO are found to be stronger than that of the UHECRs, discrediting the possibility of the TA Amaterasu event originating from SHDM decay. Furthermore, we also analyse the potential of the future telescopes mentioned above for probing SHDM lifetime through UHE gamma-ray and UHE neutrino detection. While future UHE neutrino detection are found to yield constraints comparable to that from UHECRs, UHE gamma-ray detection will lead to a few order of magnitudes improved constraints than that from UHECRs. Our constraints based on the TA Amaterasu event are consistent with those existing in the literature [197, 199, 411–426, 426–431]. These works are based on various observations of UHECRs, UHE gamma-rays, and UHE neutrinos, but not the recent TA Amaterasu event.

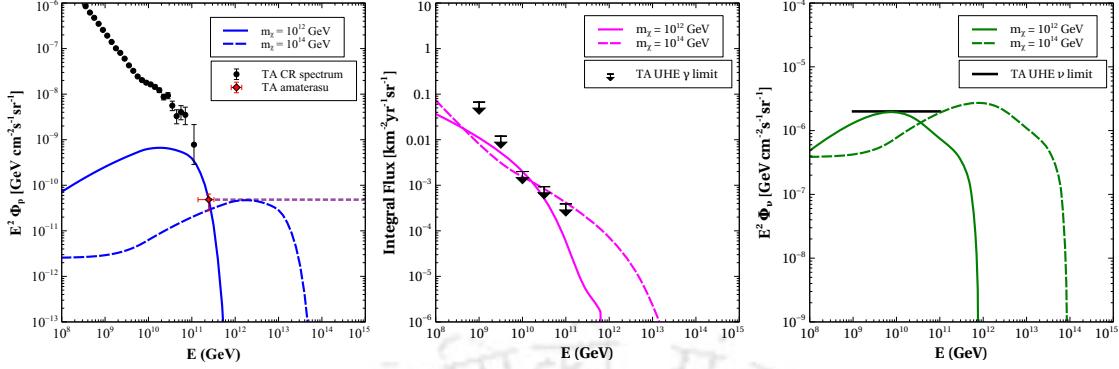


Figure 5.1: UHE flux fitted to different observations of TA for $m_\chi = 10^{12}$ GeV (solid) and $m_\chi = 10^{14}$ GeV (dashed). The decay channel is taken to be $\chi \rightarrow b\bar{b}$. *Left plot:* UHECR proton flux (blue curves) from SHDM decay fitted to the TA Amaterasu event (red data point) and the UHECR spectrum of TA (black data point). The UHECR flux above the energy of the TA Amaterasu event is taken to be constant (purple dotted). The flux fitting resulted in $\tau_\chi = 5.5 \times 10^{28}$ s and 7.5×10^{29} s for $m_\chi = 10^{12}$ GeV and $m_\chi = 10^{14}$ GeV, respectively. *Middle plot:* Integral UHE gamma-ray flux (magenta curves) from SHDM decay for the two m_χ values fitted to the upper limit on UHE gamma-ray flux from TA (black downward arrow). The resultant values of τ_χ are 3×10^{30} s and 8×10^{29} s, respectively. *Right plot:* UHE neutrino flux (green curves) from SHDM decay for the two m_χ values fitted to the upper limit on UHE neutrino flux from TA (black line). The corresponding values of τ_χ are 7×10^{25} s and 4.8×10^{25} s, respectively. The UHE neutrino flux constraint infers significantly small τ_χ values compared to that of UHE protons and UHE gamma-rays.

5.1.1 UHE flux from SHDM decay

Here, we discuss the production of UHECRs, UHE gamma-rays and UHE neutrinos from the decay of SHDM and estimate the flux of these daughter particles. The SHDM particle, χ is considered to decay via the $1 \rightarrow 2$ process ($\chi \rightarrow X\bar{X}$) into various SM particles-antiparticle pairs. These SM particles and antiparticles can further produce other lighter particle-antiparticle pairs. In particular, we focus on the secondaries convenient for detection and further phenomenology, i.e., the UHECR protons (p), the UHE gamma-rays (γ) and the UHE neutrinos (ν).

To estimate the total diffuse secondary particle flux from these SHDM decays one needs to include the contribution from all the SHDM distributed all over the Universe. In principle, the SHDM abundance over the Universe could be crucial. However, it turns out that the extra-galactic contribution is negligible in comparison to the secondary flux from SHDM decay in the Milky Way [199]. Thus, we focus

on the diffuse flux of UHE protons, UHE gamma-rays and UHE neutrinos produced from the Galactic abundance of SHDM. The DM density in the Milky Way is expected to be the largest at the Galactic center and decreases as a function of the Galactocentric distance (R_{GC}). There are various models and simulations for the density profile of DM in the Milky Way [203–207]. In this work, we consider the NFW profile [203].

Thus, given mass (m_χ) and lifetime (τ_χ) of the SHDM particle, the total flux, $\Phi_i(E, \theta)$ of the i^{th} secondary UHE particle ($i = p, \gamma$, and ν) along the line of sight can be expressed as,

$$\begin{aligned}\Phi_i(E, \theta) &= \frac{1}{4\pi m_\chi \tau_\chi} \frac{dN_i(E)}{dE} \int_0^{l_{\max}(\theta)} \rho_\chi(l) dl \\ &= \frac{\rho_\odot R_{SGC}}{4\pi m_\chi \tau_\chi} \frac{dN_i(E)}{dE} \mathcal{J}(\theta) ,\end{aligned}\tag{5.1}$$

where,

$$\mathcal{J}(\theta) = \frac{1}{\rho_\odot R_{SGC}} \int_0^{l_{\max}(\theta)} \rho_\chi(l) dl .\tag{5.2}$$

Here, R_{SGC} is the distance between the Sun and the Galactic center and is taken to be 8.34 kpc. This allows changing the line of sight coordinate (l) to the convenient Galactocentric coordinate (R_{GC}). The coordinate transformation from l to R_{GC} is given by $R_{GC} = \sqrt{R_{SGC}^2 - 2lR_{SGC} \cos \theta + l^2}$.

The flux over a solid angle, $\Omega = 2\pi(1 - \cos \theta)$ can be obtained by integrating out Eq. (5.2) from $\theta = 0$ to $\theta = \theta_{\max}$. Thus the angle integrated total flux of the i^{th} secondary is,

$$\Phi_i(E) = \frac{\rho_\odot R_{SGC}}{4\pi m_\chi \tau_\chi} \frac{dN_i(E)}{dE} \mathcal{J}^{\theta_{\max}} ,\tag{5.3}$$

where,

$$\mathcal{J}^{\theta_{\max}} = \frac{2\pi}{\Omega} \int_0^{\theta_{\max}} \sin \theta d\theta \mathcal{J}(\theta) .\tag{5.4}$$

As we are interested in the diffuse flux of UHECRs, UHE neutrinos and UHE gamma-rays, we integrate Eq. (5.4) over the entire solid angle, i.e., $\theta_{\max} = \pi$ and obtain $\mathcal{J}^{\theta_{\max}} \approx 2$. The dependence of the fluxes of UHE secondaries on a specific DM density profile can be estimated by calculating the $\mathcal{J}^{\theta_{\max}}$ factor. Note that, a different choice of DM density profile other than the NFW profile leads to a negligible change in the UHE secondary fluxes, see Appendix A.12.

Now, this diffuse flux of the secondary particles can be obtained for different decay modes of χ , i.e., for different quarks, leptons and bosons for the X and $\bar{X}$ combination (eg., $b\bar{b}$, W^+W^- , $\mu^+\mu^-$, $\nu\bar{\nu}$) in the decay process $\chi \rightarrow X\bar{X}$. In particular, the differential energy spectra $dN_i(E)/dE$ is estimated for these different decay modes. To demonstrate the flux estimation for the UHECRs, UHE gamma-rays and UHE neutrinos, we consider the example of $\chi \rightarrow b\bar{b}$ mode using Eq. (5.3) with 100% branching fraction. The results are depicted in Fig. 5.1 together with the observational data and constraints of these UHE fluxes from the TA experiment. We consider two specific choices of m_χ : 10^{12} GeV and 10^{14} GeV. The fluxes of all these UHE particles are inversely proportional (see Eq. (5.3)) to τ_χ . Rather than fixing the parameter τ_χ arbitrarily, we constrain the value of τ_χ in a phenomenological manner. τ_χ is estimated by normalising the SHDM derived flux of UHECRs, UHE gamma-rays and UHE neutrinos with the observed data or experimental upper limits on these fluxes. Since these fluxes as well as the observed flux/limits are all different from each other, they give rise to different values of τ_χ .

The left plot of Fig. 5.1 shows the observed UHECR spectrum data by TA in black dots. The red data point represents the flux of the recently observed TA Amaterasu event of energy 2.44×10^{11} GeV [251]. The flux for this TA Amaterasu event is obtained by dividing the total number of events by the total exposure of the telescope [251]. The uncertainties in the flux correspond to the propagated uncertainties. Importantly, for the energies higher than 244 EeV, we consider a conservative estimate that the maximum possible CR flux is not larger than the observed TA flux at the 244 EeV event, shown by the purple dashed line. This is contrary to the harder CR spectra suggestions at the highest energies [227, 432]. For these maximum limits, we show the upper limit of $p + \bar{p}$ flux for two different DM masses ($m_\chi = 10^{12}$ GeV and $m_\chi = 10^{14}$ GeV) considering the decay of DM into $b\bar{b}$ channel. This establishes the lower bound on DM lifetime, as indicated by Eq. (5.3). For the DM masses of 10^{12} GeV and 10^{14} GeV, the observational constraints fix the τ_χ at 5.5×10^{28} s and 7.5×10^{29} s, respectively. Note that the required energy for the TA Amaterasu event can not be created from SHDM with $m_\chi \lesssim 10^{12}$ GeV. This can be understood from the flux corresponding to $m_\chi = 10^{12}$ GeV in the left plot of Fig. 5.1 as the TA Amaterasu event lies on the high energy tail of this flux. For $m_\chi \lesssim 10^{12}$ GeV, the flux will fall off at energies smaller than the energy of the TA Amaterasu event. Similarly, there exists an upper limit for the SHDM mass. For example, the flux corresponding to $m_\chi = 10^{14}$ GeV is falling below the TA flux for

our assumption of constant UHECR limit beyond the energy of the TA Amaterasu event. In particular, we find an upper limit on m_χ for this specific channel to be $m_\chi \sim 4 \times 10^{13}$ GeV. However, later in Sec. 5.1.2, we show that this range of m_χ is in significant conflict with the UHE gamma-ray constraints from different telescopes.

In the middle plot of Fig. 5.1, we show the upper limit on the integral UHE gamma-ray flux from the TA experiment [223, 224]. This upper limit comes from the non-detection of UHE gamma-rays. The solid and dashed magenta lines represent the gamma-ray flux for DM masses of 10^{12} GeV and 10^{14} GeV, the resultant τ_χ to maintain the TA gamma-ray upper limits are 3×10^{30} s and 8×10^{29} s, respectively. Again, the decay of DM is considered to be into $b\bar{b}$ final states. These values of τ_χ are significantly larger than those obtained from UHECR constraints. This is due to the fact that production of gamma-rays from $b\bar{b}$ annihilation is significantly enhanced compared to that of protons [200]. Similarly, in the right panel plot of Fig. 5.1, we show the TA's upper limit (black continuous line) on the UHE neutrino flux from the non-detection of UHE neutrinos in TA [195]. The solid and dashed green lines correspond to the flux of UHE neutrinos for the DM masses of 10^{12} GeV and 10^{14} GeV, respectively. The lifetimes for these two masses respecting the TA neutrino limits turn out to be 7×10^{25} s and 4.8×10^{25} s, respectively. Due to the poorer sensitivity of the neutrino detectors, the τ_χ limiting values from UHE neutrinos are at least 3 – 4 orders of magnitude smaller than the gamma-ray and UHECR constraints. Thus, the three UHE messengers give rise to different constraints on the SHDM lifetime and mass.

In all the three plots of Fig. 5.1, we do not take into account the contribution from the conventional astrophysical as well as extragalactic SHDM sources. For UHECRs, the contribution of the extragalactic SHDM component is found to be negligible [197, 199]. Whereas, the contribution of the conventional astrophysical component might be significant in the UHECR spectral energy range below the energy of the TA Amaterasu event, i.e., $\lesssim 10^{11}$ GeV [199]. However, considering a dominant fraction of UHECRs from the conventional astrophysical origin allows only a little fraction of UHECRs from SHDM. This results in stringent constraints on the production of UHECRs via SHDM decay in the DM mass range $\lesssim 10^{12}$ GeV and do not yield a conservative description of the constraints on SHDM lifetime. In order to avoid this, we consider that the UHECRs in this energy range are solely due to SHDM decay. This assumption gives rise to a lower bound on the SHDM lifetime.

For UHE gamma-rays, the contribution of the extragalactic component is negligible in the energy range of our interest in comparison to the Galactic SHDM component. This is because UHE gamma-rays from extragalactic sources are attenuated severely due to pair production losses in the CMB and the extragalactic radio background during propagation to Earth [199, 231]. Note that, the attenuation of Galactic UHE gamma-rays due to the pair production losses is negligible [199, 431]. For UHE neutrinos, the contribution from extragalactic sources can be important as unlike gamma-rays, neutrinos do not suffer any propagation losses. Inclusion of extragalactic sources will result in lowering of the UHE neutrino flux from SHDM decay in the Milky Way which, in turn, will give rise to larger SHDM lifetime. In order to be consistent with the assumption for UHECRs above, we do not include the extragalactic neutrino flux in our analysis. Due to these assumptions, the constraints obtained on the SHDM lifetime from different messengers can be comprehended as the most conservative constraints. In the following, we discuss these constraints for present and future telescopes in detail and expand the description to different decay channels.

5.1.2 Multi-messenger constraints on SHDM lifetime

Here, we analyse the possibility that the TA Amaterasu event was produced via the decay of SHDM in the Milky Way. Following this, we place constraints on the lifetime/decay rate of SHDM. In order to derive the constraints in the energy band lower to the detected energy of the TA Amaterasu event, we consider the UHECR data detected by the TA. We also carry out a multi-messenger analysis on the SHDM decay rate considering the non-detection of UHE gamma-rays and neutrinos at various telescopes such as TA and PAO [225, 226]. Furthermore, we look into the SHDM lifetime probing capabilities of the various proposed future UHE telescopes.

Present UHE telescopes

In this sub-section, we present the constraints on SHDM lifetime from different observations of UHECRs, UHE gamma-rays and UHE neutrinos and analyse the origin of the TA Amaterasu event. The constraints are obtained by normalising the flux of these particles with the experimental data or limits from non-observation. For

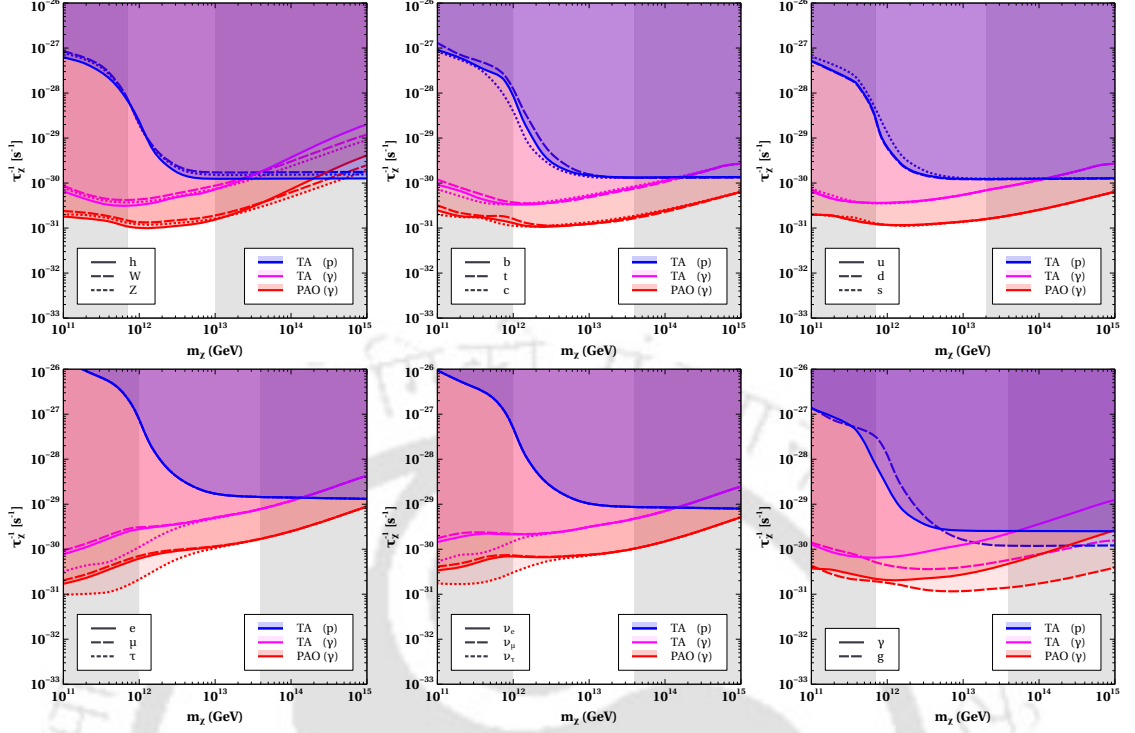


Figure 5.2: Constraints on SHDM decay rate for all possible two-body decay channels into SM particles derived from present UHECR data and UHE gamma-ray non-observation limits. Constraints from UHE neutrino non-observation limits are weak, hence not shown. The shaded regions above the curves in all the plots represent the excluded part of the parameter space. The blue curves show the constraints from the TA Amaterasu event together with the low energy UHECR data. The magenta (TA) and red (PAO) curves depict the constraints from non-observation of UHE gamma-rays. Constraints from different decay modes are distinct. The gamma-ray constraints are in general stronger than that of UHECRs. The vertically shaded grey regions depict the excluded range of m_χ for the TA Amaterasu event.

UHECRs, we consider the spectrum measured by TA together with the recent TA Amaterasu event as depicted in the left plot of Fig. 5.1. Including the UHECR data from other experiments such as PAO, AGASA and HiRes will yield similar values of DM lifetime as the CR spectral shapes from these experiments are also very similar to that of TA. For the UHE gamma-rays, we take the upper limits on the flux from TA [223, 224] and PAO [225, 226]. Whereas for the UHE neutrinos, we consider the upper limits on the flux from PAO [194] and IceCube [230]. We consider a wide range of $m_\chi = (10^{11} - 10^{15})$ GeV and different two-body decay modes. For the decay modes, we take all the quarks (u, d, s, b, t, c), the gauge bosons (W, Z, γ , gluon), and the Higgs boson (h). The lower limit of m_χ is chosen to be 10^{11} GeV as our primary

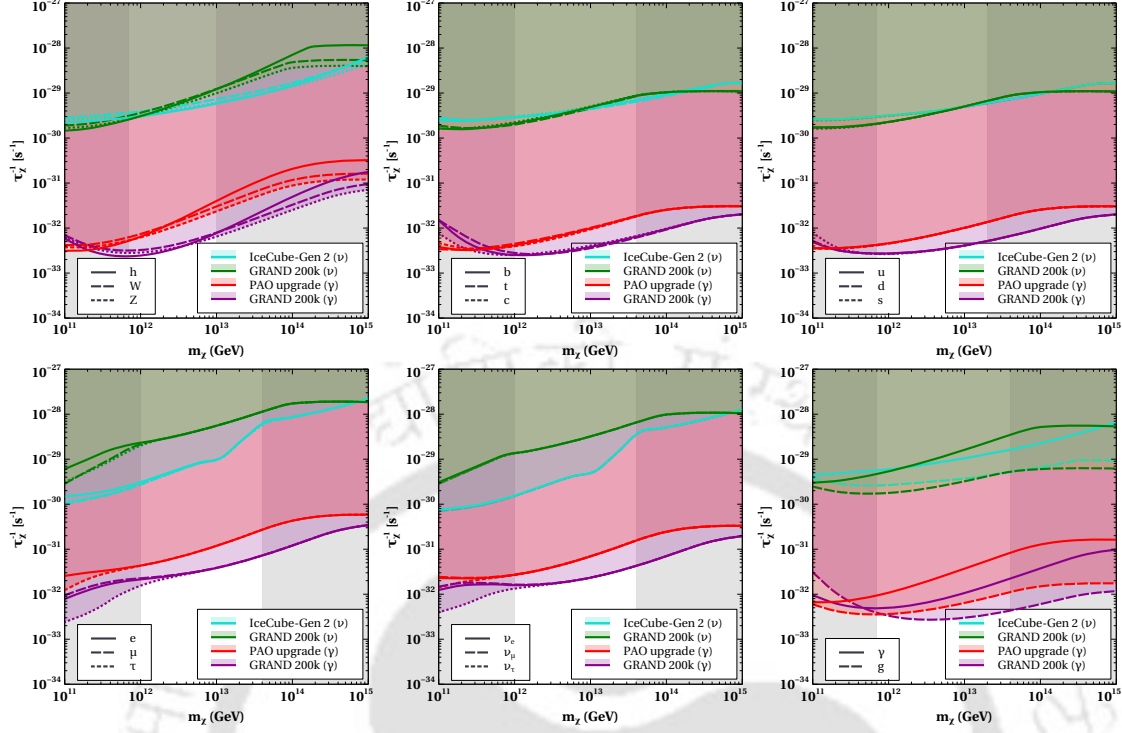


Figure 5.3: Projected constraints on SHDM decay rate from future observation of UHE gamma-rays and UHE neutrinos for all possible two-body decay channels into SM particles. The shaded regions above the curves depict the parameter space which can be probed by the future UHE gamma-ray (PAO upgrade: red and GRAND 200k: purple) and UHE neutrino (IceCube-Gen2: cyan and GRAND 200k: green) telescopes. UHE gamma-ray observations will put stringent constraints on the SHDM lifetime. The vertically shaded grey regions depict the excluded range of m_χ for the TA Amaterasu event.

goal is to analyse the origin of the TA Amaterasu event at 2.44×10^{11} GeV. On the other hand, the upper limit of $m_\chi = 10^{15}$ GeV is taken to be close to the GUT scale. For the given m_χ , we constrain the lifetime such that the UHE secondary particle observations, limits and sensitivities are satisfied. In particular, we follow convention of exclusion plots and constrain the inverse of the lifetime, i.e., τ_χ^{-1} rather than the lifetime (τ_χ). The constraints on τ_χ^{-1} as a function of m_χ for different decay modes are depicted in Fig. 5.2. The shaded regions above the curves depict the excluded part of the parameter space. The vertically shaded grey regions depict the excluded range of m_χ for the TA Amaterasu event as discussed in Sec. 5.1.1.

In Fig. 5.2, the constraints on τ_χ^{-1} from TA's UHECR data, TA's gamma-ray flux upper limit and PAO's gamma-ray flux upper limit are shown by the blue, magenta and red curves, respectively. As the bounds from non-observation of UHE neutrinos

are orders of magnitude smaller than the bounds from UHECRs and gamma-rays, the neutrino bounds are not shown in Fig. 5.2. In the top panel, the left plot shows the bounds on SHDM lifetime obtained from $\chi \rightarrow hh, W^+W^-, ZZ$, the middle plot shows the bounds from $\chi \rightarrow b\bar{b}, t\bar{t}, c\bar{c}$, and the right plot shows the bounds from $\chi \rightarrow u\bar{u}, d\bar{d}, s\bar{s}$. Similarly, in the bottom panel, the left plot displays the bounds from the decay modes $\chi \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$, the middle plot from $\chi \rightarrow \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau$, and the right plot from $\chi \rightarrow \gamma\gamma, gg$. The excluded regions of the τ_χ^{-1} vs m_χ parameter in the plots correspond to flux larger than the experimental data or limits and hence disallowed.

There are some general features of all these exclusion plots coming for different decay channels. The bounds on τ_χ^{-1} from UHECR data for $m_\chi \lesssim 10^{13}$ GeV mimic the spectral shape of the UHECR spectrum for all decay modes, whereas above this energy they remain almost constant due to the constant flux assumption above energy of the TA Amaterasu event. If we consider the TA Amaterasu event only, the resulting constraints remain almost constant for all values of m_χ and are same as the constant values of τ_χ^{-1} for $m_\chi \gtrsim 10^{13}$ GeV.

The different channels also show some distinctions. The constraints from leptons are in general weaker than that of quarks or the bosons by about an order of magnitude. Therefore, the bounds on SHDM lifetime from the TA Amaterasu event can vary depending on the decay mode. However, within a specific family (quarks, leptons, bosons), the bounds do not differ significantly.

Regarding the constraints from UHE neutrinos, the present UHE neutrino telescopes such as IceCube, TA and PAO are not sensitive to the flux corresponding to τ_χ inferred by the TA Amaterasu event and therefore remains consistent with non-detection of UHE neutrino counterpart to the TA Amaterasu event. Interestingly, the gamma-ray sensitivities of the TA and PAO are close to the TA Amaterasu constrained SHDM gamma-ray flux. However, no such correlated gamma-ray flux has been detected by the TA and PAO collaboration. This results in much stronger constraints from the gamma-ray upper limits of these telescopes than that of the UHECR data. Nevertheless, it is noteworthy that at the extreme values of m_χ , close to 10^{15} GeV for the quark channels the UHECR (Fig. 5.2, top middle and right plot) constraint is the strongest.

In all decay modes, for values of m_χ above $\sim 10^{13}$ GeV, the TA UHECR constraint curves cross the UHE gamma-ray constraint ones, opening a tiny parameter space supporting the SHDM origin of UHECRs. However, this inference is in conflict with the gamma-ray constraints from PAO except for the top left plot (bosonic decay modes) in Fig. 5.2. In this case, a small region above $m_\chi > 10^{14}$ GeV is still allowed for UHECRs of SHDM origin. However, the allowed range of m_χ for the TA Amaterasu event is found to be between $\sim (10^{11.8} - 10^{13.6})$ GeV. Note the slight differences in the allowed mass ranges for different decay modes in the vertically shaded grey bands. The gamma-ray constraints being the strongest in this m_χ range, the SHDM origin of the TA Amaterasu event is found to be in significant conflict with the gamma-ray constraints. Whereas, outside this range the SHDM decay can not explain both the TA Amaterasu event and the UHECR constant flux limit. Because, in this m_χ range, the constraints from the gamma-ray limits of TA and PAO disfavour the SHDM origin of TA Amaterasu event.

While considering the PAO limits one needs to be careful with the field of view (FOV) of the PAO (RA: 0 to 2π , Dec: $-\pi/2$ to $+\pi/4$). Indeed, the possible coordinate of the recent TA Amaterasu event (RA: 256° , Dec: 16°) is outside the FOV of PAO and can explain the non-detection of any correlated gamma-ray in PAO. In this scenario, the gamma-ray non-detection constraints from the TA turn out to be more crucial than the PAO constraint. Nevertheless, the values of τ_χ inferred by the TA Amaterasu event produce UHE gamma-ray flux larger than the present upper limit from the PAO. Hence, taking into account these multi-messenger constraints, we argue that the SHDM decay origin of the TA Amaterasu event is in severe tension.

The constraints extrapolated to higher m_χ values from the TA Amaterasu event allow a parameter space of $10^{-30} \text{ s}^{-1} \lesssim \tau_\chi^{-1} \lesssim 10^{-29} \text{ s}^{-1}$ for $m_\chi \gtrsim 10^{14}$ GeV, for a SHDM originated UHECR flux at the extreme energies of the spectrum. However, this UHE secondary flux being a diffuse flux, the PAO should also observe gamma-rays at similar energies. For instance, if we consider decay via the $b\bar{b}$ channel, the PAO can be expected to detect a UHE gamma-ray flux of about $(10^{-3} - 10^{-4}) \text{ km}^{-2} \text{ yr}^{-1} \text{ sr}^{-1}$ at $\sim 10^{20}$ GeV. Hence, future detection of UHECRs at or above this energy will be crucial for constraining UHECRs from SHDM decay. Finally, we also study the dependence of these constraints on the choice of Galactic DM density

profile and find that the results are nearly independent of density profile choice (see Appendix A.12).

Future UHE telescopes

The above analysis shows that the observed UHECR data together with the experimental upper limits on the UHE gamma-ray and UHE neutrino flux put a strong constraint on the SHDM origin for the recent TA Amaterasu event. However, these stringent constraints do not completely rule out the possibility of a SHDM component in the UHECR spectrum. It might be possible that the SHDM component of the UHECR flux is smaller than that of the observed spectrum, eradicating detection in current telescopes and, therefore motivates us to look at the possibilities of the future telescopes. Detection of such a small flux requires more sensitive telescopes. The upcoming telescopes such as GRAND 200k, IceCube-Gen2 and PAO upgrade having larger sensitivities compared to the existing ones provide us with better opportunities to probe the SHDM through detection of UHE gamma-rays and UHE neutrinos. In the following, we discuss the potential of these future telescopes for constraining τ_χ and m_χ for the various two-body decay modes as considered above.

The constraints on the τ_χ for various decay modes are obtained by normalising the computed fluxes from SHDM at the level of sensitivities (gamma-ray and neutrino) of the said telescopes. Fig. 5.3 shows the gamma-ray and neutrino bounds on the DM lifetime and mass for the different decay channels. For the gamma-ray constraints, we consider the sensitivities of the proposed PAO upgrade [227] and the GRAND 200k detector. The neutrino bounds are based on the UHE neutrino flux sensitivities of the IceCube-Gen 2 and the GRAND 200k detectors. The red and purple curves in Fig. 5.3 depict the constraints from gamma-rays in PAO upgrade and GRAND 200k, respectively. The constraints from neutrinos in IceCube-Gen2 and GRAND 200k are shown by the cyan and green curves, respectively. Overall, the constraints from the gamma-ray sensitivities are about two to three order of magnitudes stringent than that of the ones from neutrinos. Similar to bounds from the present experiments, the more stringent limits come from the quarks and gauge bosons decay channels as depicted in Fig. 5.3 for future telescopes. For gamma-rays, the constraints from GRAND 200k are found to be stronger than that of PAO for all decay modes. For neutrinos, the constraints from quarks and bosons channels

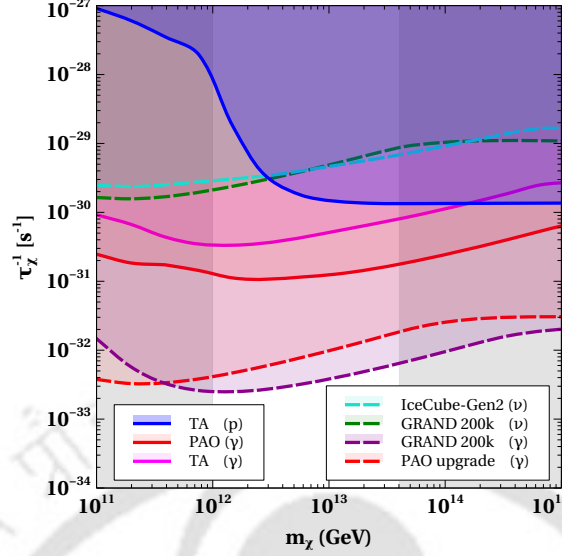


Figure 5.4: Constraints on DM lifetime/decay rate from both current and future experiments considering the DM decay via $b\bar{b}$ channel. The blue curve depicts the constraint from TA’s UHECR data. The cyan and green dashed curves represent the constraints from UHE neutrino sensitivities of IceCube-Gen2 and GRAND 200k, respectively. The magenta and red solid curves show the present constraints from UHE gamma-ray non-observation in TA and PAO, respectively, whereas, the red and purple dashed curves depict the projected reach of the upcoming telescopes PAO upgrade and GRAND 200k, respectively. While the constraints from current observations (blue, magenta, red solid curves) suggest $\tau_\chi \gtrsim 10^{30}$ s, future gamma-ray observations (red and purple dashed curves) will be able to probe τ_χ up to 10^{32} s. The vertically shaded grey regions depict the excluded range of m_χ for the TA Amaterasu event.

are found to be similar for both the telescopes. However, for the leptonic channels the constraints from IceCube-Gen2 are stronger than that of GRAND 200k. This is because the energy spectra of the leptonic channels give rise to a peak at the energy close to the value of m_χ [200]. The constraints anticipated from UHE neutrino detection in both the future telescopes will be comparable to that from current UHECR data for $m_\chi \gtrsim 2 \times 10^{12}$ GeV. Therefore, the results from the future experiments will be crucial to put further and stronger multi-messenger constraints on the parameter space of SHDM.

5.1.3 Discussion and Summary

In this work, we have analysed the possibility of the recently detected TA Amaterasu event, at 244 EeV to have originated from the decay of SHDM in the Milky Way.

The arrival direction of this UHECR being a void in the local Universe makes it hard to explain its origin from the conventional astrophysical shock acceleration and opens up possibility of a SHDM origin. The analysis is carried out with a multi-messenger approach considering UHECR data from TA and upper limits on UHE gamma-ray and UHE neutrino flux from different experiments such as TA and PAO. For this, we have estimated the UHECR, UHE gamma-ray and UHE neutrino flux produced by SHDM decay with the NFW DM density profile for various SHDM decay channels and a wide range of SHDM mass. The constraints on these fluxes from the experimental data/limits have resulted in constraints on the SHDM lifetime or decay rates. While we have found an allowed SHDM mass range of $\sim (10^{11.8} - 10^{13.6})$ GeV for the TA Amaterasu event, the multi-messenger gamma-ray constraints from TA and PAO are in severe tension with the SHDM origin for the said TA Amaterasu event. However, these constraints indicate the possibility of UHECR flux at higher energies originating from heavier SHDM particles, i.e., $m_\chi \gtrsim 10^{14}$ GeV. In order to test the viability of such a scenario one would need more sensitive future telescopes such as PAO upgrade, IceCube-Gen2 and GRAND 200k.

Therefore, we have also looked into the potential of these future telescopes to constrain the lifetime and mass of SHDM. All the results of our work are summarised in Fig. 5.4 for the decay mode $\chi \rightarrow b\bar{b}$. We show the bounds on τ_χ^{-1} from UHECRs (blue curve), UHE gamma-rays from TA (magenta curve), PAO (solid red curve), PAO upgrade (red dashed curve), GRAND 200k (purple dashed curve) and UHE neutrinos from IceCube-Gen2 (cyan dashed curve), GRAND 200k (green dashed curve). The constraints on lifetime from the TA UHECR data are found to be of the order of 10^{30} s for SHDM mass, $m_\chi \gtrsim 10^{13}$ GeV, whereas at lower masses the lifetime can be smaller. The constraints from gamma-ray upper limit of the TA data vary within an order of magnitude and found to be maximum of $\sim 3 \times 10^{30}$ s at around $m_\chi \sim 10^{12}$ GeV. The TA constraints (UHECR and UHE gamma-ray) alone allow for a small parameter space above 10^{14} GeV for the SHDM. However, the gamma-ray upper limit of PAO is about a factor of 3-4 larger and creates tension for the SHDM origin of the TA Amaterasu event. The future telescopes will be able to put much stronger constraints ($\gtrsim 10^{32}$ s) on the SHDM lifetime from gamma-ray observations as depicted by the purple dashed (GRAND 200k) and red dashed (PAO SD upgrade) curves. However, future UHE neutrino detectors such as IceCube-Gen2 (cyan dashed) and GRAND 200k (green dashed) will place constraints similar to

the present UHECR derived limits (blue solid). These results vary substantially between different decay modes, we have done a detailed analysis for the different decay channels and found that the lepton channels to have the weakest constraints (see, Fig. 5.2 and Fig. 5.3).

To conclude, we have provided an updated estimate of multi-messenger constraints on the lifetime of SHDM by considering the recent TA Amaterasu event in conjunction with the TA UHECR data and non-observation of UHE gamma-rays and UHE neutrinos. While doing so, we have ensured consistency with the existing constraints in the literature [197, 199, 411, 413, 414, 426, 431, 433]. Note that the constraints for heavier DM mass, i.e., $m_\chi > 10^{12}$ GeV are not very robust due to the lack of data (UHECRs, UHE gamma-rays and UHE neutrinos) at energies above $\sim 10^{11}$ GeV. For harder UHECR flux anticipated from SHDM [199] or conventional diffusive acceleration [227, 432], the resulting constraints on the lifetime for $m_\chi \gtrsim 10^{12}$ GeV will be smaller than the ones obtained in this work. Nevertheless, such shorter lifetimes for the SHDM are not supported by UHE gamma-ray observations and do not impact the conclusion of this work.

5.2 Possible origin of the KM3-230213A neutrino event from dark matter decay

Here, we consider a heavy right-handed neutrino to be the DM candidate whose late decay into neutrinos is responsible for the highest energy neutrino event mentioned at the beginning of this chapter. While the neutrino event can, in principle, originate from DM annihilation as well, the flux will be proportional to DM density squared and hence suppressed for such superheavy DM. Such a RHN appears in type-I seesaw scenario which explains the origin of light neutrino masses. While two RHNs are sufficient to generate light neutrino mass and mixing, the third RHN can be DM if sufficiently long-lived due to feeble Yukawa coupling with the SM leptons. Since RHN can decay into other SM particles due to the same Yukawa interactions, we also have secondary gamma-ray productions, tightly constrained from astrophysical observations. We show that the required flux and energy of the KM3-230213A neutrino event can be explained within the uncertainties from decay of a RHN DM having mass $M_{\text{DM}} \sim 440$ PeV while being in agreement with the gamma-ray bounds.

While the required relic of such superheavy DM can not be generated in the minimal type-I seesaw model, we consider a gauged $U(1)_{B-L}$ extension of the model where relic can be generated non-thermally. We also comment on possible alternative DM realisations in which the gamma-ray bounds can be weaker due to phase space suppression. Depending upon the specific realisation and DM mass, gauged $U(1)_{B-L}$ offers very interesting multi-messenger detection prospects at energy, intensity as well as cosmic frontiers [120, 434–437].

5.2.1 The Model

We consider heavy right handed neutrino DM in a gauged $B - L$ extension of the SM [290–295]. In its minimal realisation, the SM particle content is extended with three RHNs N_i ($i = 1, 2, 3$) and one complex singlet scalar (Φ) all of which are singlet under the SM gauge symmetry. The requirement of triangle anomaly cancellation in this minimal model requires each of the RHNs to have $B - L$ charge -1 . The complex singlet scalar having $B - L$ charge 2 not only leads to spontaneous breaking of gauge symmetry but also generate RHN masses or the seesaw scale dynamically. The Lagrangian involving RHN can be written as

$$\begin{aligned} \mathcal{L}_{\text{fermion}} = & i \sum_{\kappa=1}^3 \overline{N_{R\kappa}} D N_{R\kappa} - \sum_{\substack{j=1 \\ \alpha=e,\mu,\tau}}^3 Y_D^{\alpha j} \overline{l_L^\alpha} \tilde{H} N_R^j \\ & - \sum_{i,j=1}^3 Y_{N_{ij}} \Phi \overline{N_{Ri}^C} N_{Rj} + \text{h.c.}, \end{aligned} \quad (5.5)$$

where l_L, H denote the SM Lepton and Higgs doublets, respectively. The covariant derivative for $N_{R\kappa}$ is defined as

$$D N_{R\kappa} = \gamma^\mu (\partial_\mu - i g_{BL} Z'_\mu) N_{R\kappa}, \quad (5.6)$$

with g_{BL}, Z' being the coupling and gauge boson respectively for $U(1)_{B-L}$. After the singlet scalar Φ acquires a non-zero VEV denoted by v_{BL} , the RHNs and Z' acquire masses as,

$$M_{Z'} = 2g_{BL}v_{BL}, \quad M_i = \sqrt{2}Y_{N_i}v_{BL}, \quad (5.7)$$

where we have assumed the Yukawa coupling Y_N to be diagonal. We consider the lightest RHN namely N_1 to be the DM candidate such that the corresponding Yukawa coupling $Y_D^{\alpha 1}$ to ensure a long lifetime. Due to such Yukawa interactions which also mixes N_1 with the light neutrinos after electroweak symmetry breaking, we can have dominant tree-level decays like $N_1 \rightarrow \nu, h; \nu, Z; l^\mp, W^\pm$ with h being the SM-like Higgs. Ignoring the masses of SM particles in comparison to N_1 mass M_1 , these decay widths are given as [438]

$$\begin{aligned}\Gamma(N_1 \rightarrow \nu_\alpha h(Z)) &= \frac{|Y_D^{\alpha 1}|^2 M_1}{32\pi}, \\ \Gamma(N_1 \rightarrow l_\alpha^- W^+) &= \frac{|Y_D^{\alpha 1}|^2 M_1}{16\pi}.\end{aligned}\quad (5.8)$$

We can also have a radiative decay $N_1 \rightarrow \gamma \nu_\alpha$ as [439, 440]

$$\Gamma_{N_1 \rightarrow \gamma \nu_\alpha} \simeq \frac{9G_F^2 \alpha_{\text{EM}}}{1024\pi^4} \sin^2(\theta_{\alpha 1}) M_1^5, \quad (5.9)$$

where $\alpha_{\text{EM}} = 1/137$ is the fine structure constant, $G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant. The active-sterile mixing is given by $\sin(\theta_{\alpha 1}) \sim \frac{Y_D^{\alpha 1} v}{\sqrt{2} M_1}$ with v being the VEV of the SM Higgs doublet. The lifetime of N_1 , dictated by its tree-level two-body decays, can then be estimated as

$$\tau_{N_1} \approx 8 \times 10^{28} \text{ s} \left(\frac{440 \text{ PeV}}{M_1} \right) \left(\sum_\alpha \left| \frac{Y_D^{\alpha 1}}{4.7 \times 10^{-31}} \right|^2 \right)^{-1} \quad (5.10)$$

which is identified as the DM lifetime τ_{DM} in our setup. The dominant tree level decay widths given in Eq. (5.8) have 2 : 1 : 1 ratio for $l_L^\pm W^\mp, \nu Z, \nu h$ final states, respectively. Using the best-fit values of neutrino mixing angles for normal ordering (NO) [441], the branching ratio (BR) into different lepton flavours for a particular decay mode follows the ratio $\text{BR}(e) : \text{BR}(\mu) : \text{BR}(\tau) = 0.68 : 0.21 : 0.11$. The flavour ratio at source for inverted ordering (IO) turns out to be $\text{BR}(e) : \text{BR}(\mu) : \text{BR}(\tau) = 0.02 : 0.55 : 0.43$. We assume the Dirac CP phase to be zero for simplicity. The neutrino flavour ratio at the detector, after considering oscillation effects [442], turns out to be approximately 3 : 2 : 1 and 1 : 2 : 3 for NO and IO respectively.

While DM of required mass can not be of thermal origin due to the violation of the unitarity bound [443], it can not be produced non-thermally via Yukawa portal interactions as well due to the suppressed Yukawa couplings in Eq. (5.10) required

to ensure its longevity. However, due to additional interactions present in the $B - L$ model which are unrelated to DM lifetime, it is still possible to produce the desired relic via the freeze-in mechanism [87, 444–446].

Using the approach of [444, 445], we consider the production of superheavy N_1 of mass 440 PeV from decay of heavier particle. The required non-thermal criteria forces the gauge coupling g_{BL} to be tiny keeping Z' out-of-equilibrium as well. We consider the singlet scalar to be in equilibrium sourcing Z' with the latter acting as source of DM. The relic can be calculated by solving the coupled Boltzmann equations written in terms of comoving number densities $Y_X = \frac{n_X}{s}$ as follows:

$$\begin{aligned}\frac{dY_{Z'}}{dx} &= \frac{\beta}{x\mathcal{H}}\Gamma_{\phi \rightarrow Z'Z'}\frac{K_1(M_\phi/T)}{K_2(M_\phi/T)}Y_\phi^{\text{eq}} \\ &\quad - \frac{\beta}{x\mathcal{H}}\Gamma_{Z' \rightarrow \text{all}}\frac{K_1(M_{Z'}/T)}{K_2(M_{Z'}/T)}Y_{Z'}, \\ \frac{dY_{N_1}}{dx} &= \frac{\beta}{x\mathcal{H}}\Gamma_{\phi \rightarrow N_1N_1}\frac{K_1(M_\phi/T)}{K_2(M_\phi/T)}Y_\phi^{\text{eq}} \\ &\quad + \frac{\beta}{x\mathcal{H}}\Gamma_{Z' \rightarrow N_1N_1}\frac{K_1(M_{Z'}/T)}{K_2(M_{Z'}/T)}Y_{Z'}.\end{aligned}\tag{5.11}$$

Here $x = M_{Z'}/T$, $\beta = 1 + \frac{Tdg_s/dT}{3g_s}$, with g_s being the relativistic entropy degrees of freedom. $\mathcal{H}$ is the Hubble expansion parameter, Γ denotes the decay width and K_i 's denote the modified Bessel functions of the second kind.

5.2.2 Neutrino and Gamma-ray flux from DM decay

Here, we describe the method to compute the flux of high energy neutrinos and gamma-rays produced from DM decay. As discussed above, we consider the RHN N_1 decay via all possible two-body decay modes namely, $N_1 \rightarrow \nu, h(Z, \gamma)$ and $N_1 \rightarrow \ell^\pm, W^\mp$, emitting secondaries such as high energy neutrinos, gamma-rays and electrons. Note that we do not distinguish between particle and anti-particle final states here. The DM particle N_1 is considered to be the constituent of the Milky Way DM halo. Thus, the decay of N_1 in the Milky Way can give rise to backgrounds of high energy neutrinos and gamma-rays. The flux of these secondaries from galactic ($\phi_{i,G}(E_i)$) and extra-galactic ($\phi_{i,EG}(E_i)$) DM can be estimated using Eqs. (2.35) and (2.38). The total flux of secondary gamma-rays and neutrinos is given by $\phi_i = \phi_{i,G} + \phi_{i,EG}$.

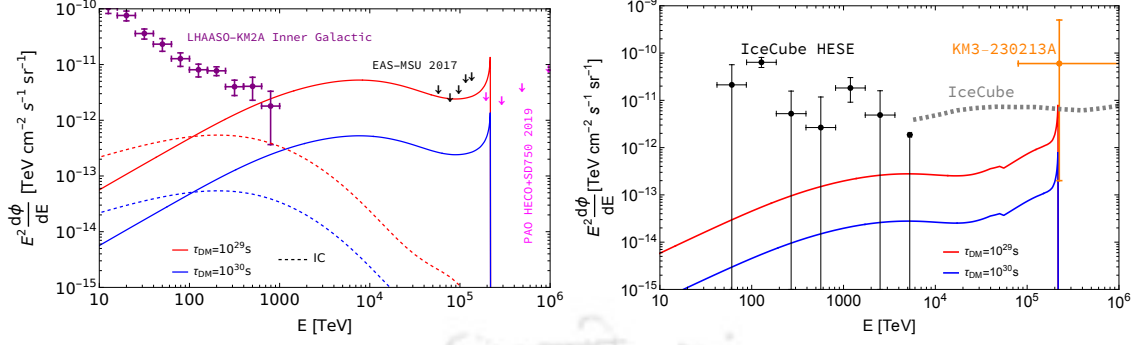


Figure 5.5: Gamma-ray (left-panel) and muon neutrino (right-panel) fluxes with DM mass $M_1 = 440$ PeV and two different values of DM lifetime $\tau_{\text{DM}} = 10^{29}$ s (solid red), $\tau_{\text{DM}} = 10^{30}$ s (solid blue). The dashed lines on the left-panel correspond to inverse Compton radiation. The fluxes are summed over the all possible decay modes and both Galactic and Extra-galactic contributions. The data points on the left panel plot correspond to gamma-ray bounds from LHAASO [447], EAS-MSU [448] and PAO [449]. The data points on the right panel plot correspond to UHE neutrino events at IceCube [450] and KM3NeT [228]. The gray dotted contour (right panel) denotes IceCube’s sensitivity to UHE neutrino flux[243].

5.2.3 Results and Discussion

In this sub-section, we discuss the results obtained by computing the fluxes of UHE neutrinos and gamma-rays from DM decay using Eq. (2.35) and analyze them in view of the KM3-230213A event. The mass of the DM particle N_1 is chosen to be 440 PeV in order to match the energy requirement (220 PeV) of the KM3-230213A event. For this analysis, we take into account the constraints from three different gamma-ray observations. For energies below 1 PeV, we take the diffuse gamma-ray measurements from the inner Galactic plane by the Large High Altitude Air Shower Observatory (LHAASO) [447], whereas for energies above 40 PeV, we consider the upper limits on UHE photon flux from Moscow State University Extensive Air Shower (EAS-MSU) array [448] and PAO (PAO HECO +SD750) [449]. In addition to these gamma-ray constraints, we also consider the IceCube’s High Energy Starting Events (HESE) [450].

We show the total gamma-ray (left) and muon neutrino (right) fluxes together with these constraints in Fig. 5.5. The fluxes are summed over all possible decay modes, and both Galactic and EG component. The gamma-ray fluxes shown here are computed for the inner Galactic plane ($15^\circ < l < 125^\circ$, $-5^\circ < b < 5^\circ$) to remain consistent with the LHAASO data (shown by the purple data points). This flux is

slightly larger (about a factor of ~ 1.25) than the overall diffuse flux of the DM halo. For comparison, we also include the high energy diffuse gamma-ray limits, ESU-MSU and PAO HECO +SD750 shown by the black and magenta downward arrows, respectively. Given these constraints, the gamma-ray flux can be appropriately normalized by tuning the DM lifetime, τ_{DM} (see Eq. (2.35)). To demonstrate this, the fluxes for two different values of τ_{DM} are plotted in the left panel of Fig. 5.5. The flux corresponding to $\tau_{\text{DM}} \approx 10^{29}$ s (red curve) remain consistent with the LHAASO data, however, it is in tension with the ESU-MSU and PAO HECO+SD750 limits. This tension can be relaxed by choosing a larger lifetime, $\tau_{\text{DM}} \approx 10^{30}$ s as shown by the blue curve. Note that both Galactic and EG gamma-ray fluxes can undergo absorption due to pair production losses on low energy background photons such as interstellar radiation field (ISRF), EBL and CMB. This absorption is negligible above 10 PeV [451] for the Galactic gamma-ray flux. However, the absorption of the EG gamma-ray flux in the EBL and CMB is found to be severe. Thus, the total gamma-ray flux is completely dominated by the Galactic component. Apart from absorption, the Galactic flux can also be affected by inverse Compton radiation from secondary electrons and positrons. We calculate the inverse Compton radiation for two specific values of τ_{DM} using the method outlined in [451], and show the results as dashed curves with the same color scheme. For the target photons, we consider both CMB and ISRF. However, the contribution of ISRF is found to be negligible in the energy range of interest. In computing the inverse Compton flux, we also account for the synchrotron energy losses of secondary electrons and positrons in the intergalactic magnetic field, assuming an average field strength of $1 \mu\text{G}$. Since the synchrotron power is proportional to the square of the electron energy, these losses become severe at higher energies. This, in turn, yields a softer inverse Compton flux at higher energies. Thus, in the energy range of our interest, the gamma-ray flux is solely dominated by the secondary photons from DM decay.

For the two specific values of τ_{DM} , we show the corresponding muon neutrino fluxes in the right panel of Fig. 5.5 following the same color coding as the left panel. The muon neutrino flux is computed for the NO scenario considering the flavour ratio $3 : 2 : 1$ at Earth. The IO scenario having a flavour ratio $1 : 2 : 3$ produces similar neutrino fluxes and are not shown here. The EG component is found to contribute slightly to the total muon neutrino flux below 50 PeV. We also show the flux of the KM3-230213A event by orange data point with 3σ CL. The black data points and the gray dotted curve, respectively, show the IceCube's HESE events [450] and

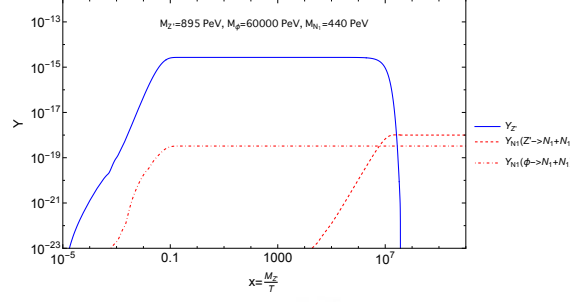


Figure 5.6: Evolution of Z' and DM N_1 as a function of $x = \frac{M_{Z'}}{T}$ for a benchmark point.

UHE neutrino flux sensitivity at 90% CL [243]. For the estimation of the neutrino fluxes, we consider the arrival direction (RA: 94.3° , Dec: -7.8°) of KM3-230213A with an angular uncertainty of 3° as reported by the KM3NeT collaboration [228]. The flux obtained with this conservative approach is about a factor of 2 smaller than the overall diffuse neutrino flux of the DM halo. Clearly, RHN DM with lifetime $\tau_{\text{DM}} = 10^{29}$ s and mass $M_1 = 440$ PeV can give rise to a flux close to the central value of the KM3-230213A event at 220 PeV energy. However, it also produces a large flux of secondary gamma-rays saturating the existing upper limits from LHAASO while being in tension with EAS-MSU and PAO data as discussed above. Lowering the decay width of DM or increasing the lifetime leads to agreement with gamma-ray bounds at the cost of reducing the neutrino flux while still being consistent at 3σ CL, as depicted by the flux corresponding to $\tau_{\text{DM}} \approx 10^{30}$ s. Interestingly, this neutrino flux can also alleviate the tension between KM3NeT and IceCube regarding the diffuse, isotropic origin of the KM3-230213A event. This tension arises as the central value of the KM3-230213A flux is larger than the IceCube's sensitivity. While the present DM model can not simultaneously explain all UHE neutrino events including IceCube HESE and KM3-230213A, the former can be explained with astrophysical sources [450].

Finally, we show the viability of generating the observed relic of RHN DM of mass 440 PeV in this model by solving the coupled Boltzmann equations in Eq. (5.11). Fig. 5.6 shows the non-thermal production of Z' and DM N_1 as a function of $x = M_{Z'}/T$ for one particular benchmark point corresponding to $M_{Z'} = 895$ PeV, $M_\phi = 60000$ PeV, and DM mass $M_{N_1} = 440$ PeV. We consider the $B - L$ symmetry breaking scale to be $v_{\text{BL}} = 4 \times 10^{19}$ GeV which implies $g_{\text{BL}} \sim 10^{-11}$ and $Y_{N_{11}} \sim 8 \times 10^{-12}$. For this particular benchmark point, the production of DM

from non-thermal Z' dominates over the production of DM from thermal ϕ . The asymptotic comoving abundance of N_1 gives rise to present DM relic $\Omega h^2 = 0.12$ consistent with PLANCK data [25]. The required lifetime of DM can be obtained by appropriate choice of Yukawa coupling $Y_D^{\alpha 1}$ without affecting DM relic. We can also produce DM non-thermally from scattering of SM bath particles into N_1 mediated by Z' , applicable when $Z' \rightarrow N_1 N_1$ is kinematically forbidden. As shown in [446], we can get correct DM relic in such a case for $v_{BL} \gtrsim 10^{13}$ GeV allowing sub-Planckian symmetry breaking scales.

We have chosen a minimal DM scenario which contains a long-lived RHN DM, produced non-thermally in the early Universe via scalar or Z' decays. DM mass, DM lifetime as well as $U(1)_{B-L}$ breaking scale are chosen in a way to fit the KM3-230213A event energy and flux. Such new physics scales at $\mathcal{O}(100)$ PeV can, however, be motivated in a variety of beyond standard model frameworks. For example, such a scale can arise as an intermediate scale in non-supersymmetric $SO(10)$ grand unified theories [452]. The framework discussed in [453, 454] has a stable RHN DM whose relic matches with observed DM relic only for mass ~ 480 PeV. On the other hand, if RHN DM is stabilised by a global symmetry like Z_2 forbidding its coupling to lepton and Higgs, Planck scale suppressed operators like $(H^\dagger H) \bar{l}_L \tilde{H} N_R / M_{Pl}^2$ violating Z_2 explicitly while being gauge invariant can lead to DM decay. Such operators arise naturally given the fact that any theory of quantum gravity is expected to violate global symmetries explicitly [455–457]. This can lead to an effective Yukawa coupling $Y_{\text{eff}} \sim 10^{-32}$ (for $v \sim 246$ GeV, $M_{Pl} = 2.4 \times 10^{18}$ GeV) keeping it in the required ballpark as given in Eq. (5.10).

5.2.4 Other dark matter scenarios

Since RHN DM decays into neutrino and other SM particles via two-body decay with the same Yukawa coupling controlling all these partial decay widths, it is difficult to reduce the gamma-ray flux without reducing the neutrino flux. This is observed while comparing fluxes for two different τ_{DM} values in Fig. 5.5 discussed in the previous section. One can make suitable changes to this model in order to decouple the gamma-ray and neutrino flux to some extent. Here we briefly outline two such possibilities.

Singlet Scalar DM

We can consider another complex singlet scalar Φ' added to the model such that $M_{\Phi'} < M_i$. It couples to RHNs as $y_{\phi'} \Phi' \bar{N}_i^c N_i$. Unlike Φ , this new singlet scalar does not acquire VEV and be a long-lived DM. Since RHNs mix with active neutrinos after electroweak symmetry breaking we can have scalar DM decay as $\Phi' \rightarrow \nu\nu$ [458]. In such a case DM decay to charged particles will arise only at three-body or higher final states, possibly giving a softer spectra for diffuse gamma-rays. Such charged final states arise via $\phi \rightarrow \nu \ell^\pm W^\mp$. One can also have production of other SM particles via similar three-body decay processes. Such a scalar singlet DM can be generated non-thermally via scalar portal or Z' portal interactions, similar to RHN DM discussed above.

Doublet Scalar DM

Instead of considering Majorana neutrino, one can consider RHNs to be the right chiral parts ν_R of sub-eV Dirac neutrinos. In this simplest version, they can acquire sub-eV neutrino masses from the SM Higgs itself with tiny Yukawa couplings. In such a case, the gauged $B - L$ symmetry needs to be broken by a singlet scalar with $B - L$ charge different from 2 such that Majorana mass terms are not dynamically generated.

Let us consider a neutrinophilic scalar doublet η which couples to neutrinos as $y_\eta \bar{l}_L \tilde{\eta} \nu_R$, where ν_R is the right chiral part of light Dirac neutrinos. Assuming a Z_2 symmetry in the scalar sector under which $\eta \rightarrow -\eta$, the scalar potential is similar to the inert doublet model given by

$$V = \mu_H^2 |H|^2 + \mu_\eta^2 |\eta|^2 + \lambda_1 |H|^4 + \lambda_2 |\eta|^4 + \lambda_3 |H|^2 |\eta|^2 + \lambda_4 |\eta^\dagger H|^2 + \lambda_5 [(\eta^\dagger H)^2 + \text{h.c.}]. \quad (5.12)$$

After electroweak symmetry breaking, the components of η split and the lightest neutral real scalar η_R can be DM. Since the scalar sector has a Z_2 symmetry and $\mu_\eta^2 > 0$, the neutral component of η does not acquire any VEV. Therefore, DM in this setup can decay only via the Yukawa interactions which break Z_2 symmetry. We consider the corresponding Yukawa coupling y_η to be tuned such that DM is still long-lived on cosmological scales. Its decay $\eta_R \rightarrow \nu_L \nu_R$ can give rise to the

observed neutrino event while its decay into charged final states appear only via three-body decay like $\eta_R \rightarrow \ell^\pm W^\mp \nu_R$ keeping the resulting secondary gamma-ray spectrum softer compared to the RHN DM studied earlier. While pure thermal scalar doublet DM will be overproduced for DM mass above the unitarity limit, incomplete thermalisation with suitable reheating temperature can generate the desired relic [459].

5.2.5 Summary

We have explored the possibility of generating the recently detected highest energy neutrino event of 220 PeV energy by the KM3NeT collaboration from decay of superheavy dark matter. Considering the simplest possibility of a right handed neutrino coupling with Yukawa coupling to leptons in type-I seesaw model to be the DM candidate, we calculate the flux of neutrino and gamma-ray by considering all possible two-body decay modes of DM. Considering the best-fit value of the neutrino flux reported by the KM3NeT collaboration, we find that the present gamma-ray constraints disfavour the DM decay origin of the event, in addition to the mild tension with non-observation of similar events at IceCube and PAO. Due to the large uncertainty associated with the KM3-230213A flux at present, it is still possible to explain the event at within 3σ CL while being consistent with the above-mentioned constraints. As we consider two-body decay of DM, the preferred DM mass is 440 PeV, the relic of which can be generated non-thermally, if we embed the type-I seesaw scenario in a gauged $U(1)_{B-L}$ setup where additional scalar and vector boson play the crucial role in DM production. We also outline the possibility of two more DM realisations where gamma-ray constraints can be made weaker. To conclude, the possibility of DM origin of the KM3-230213A event is marginal at present due to gamma-ray constraints and non observation of similar events at other experiments. More precise estimates of flux, multi-messenger observations or identification of astrophysical sources in future will be able to settle this hypothesis more concretely.

Conclusion and Future Outlook

This thesis explores a class of well-motivated BSM frameworks that are challenging to probe through conventional collider and direct-detection experiments, focusing instead on their cosmological and astrophysical signatures. Particular emphasis is placed on observables associated with the CMB, stochastic GW backgrounds, and high-energy astrophysical measurements. The work investigates several scenarios involving DM, dark radiation, Dirac and Majorana neutrinos, and ultra-light PBHs, highlighting the importance of cosmological measurements in testing such models. We begin the thesis with a brief overview of the Λ CDM model, the prevailing framework describing the evolution and composition of the Universe. This is followed by an introduction to the Standard Model of particle physics and a discussion of its shortcomings, highlighting the need for BSM physics, with particular focus on dark matter and the generation of neutrino masses.

In Chapter 2, we introduce several key cosmological signatures of BSM physics. We begin by discussing the *effective number of relativistic species*, N_{eff} , and its sensitivity to the presence of additional light degrees of freedom, illustrating how precision cosmological measurements can constrain thermal relativistic species. We then examine the phenomenological consequences of ultra-light PBHs in the early Universe, focusing on their evolution, Hawking evaporation, and resulting signatures in N_{eff} and stochastic gravitational waves. Finally, we discuss indirect detection

signatures of dark matter decay in the form of UHE neutrinos, gamma rays, and cosmic rays, along with a brief overview of past, current, and future experiments sensitive to these extreme-energy observables.

Chapter 3 investigates DNPDM frameworks in which DM and DR originate from a common source. In the minimal DNPDM scenario, we study the non-thermal production of both DM and DR through particle decay. We show that, depending on the choice of parameters, existing constraints from Planck 2018 can exclude DM masses up to a few tens of keV, while structure formation bounds impose even stronger limits, ruling out DM masses up to a few hundred keV. Furthermore, the resulting contribution to ΔN_{eff} is constrained to be $\lesssim \mathcal{O}(10^{-3})$ in order to remain consistent with the required dark matter properties. The framework is subsequently extended to a gauged $U(1)_{B-L}$ model, providing a UV-complete realization. Within this extended setup, we consider both thermal and non-thermal production mechanisms for dark matter and examine their correlation with parameter regions accessible to future CMB experiments. The combined constraints from dark matter relic abundance, CMB observations, structure formation, and terrestrial experiments render the scenario highly predictive, significantly restricting the viable $U(1)_{B-L}$ parameter space.

In Chapter 4, we investigate the cosmological implications of an early epoch dominated by PBHs. The evaporation of ultra-light PBHs serves as a source of DR, imprinting observable signatures on ΔN_{eff} . Moreover, they also produce stochastic GWs backgrounds providing complementary avenues for detection. We explore a hybrid scenario in which DR originates from both thermal interactions and PBH evaporation, and derive new constraints on PBH parameters as well as DR-SM couplings by demanding consistency with CMB bounds on ΔN_{eff} . Relative to earlier studies that did not consider DR-SM interactions, our framework yields more stringent bounds on the PBH parameter space, particularly on the initial PBH mass and energy density. We further discuss the possibility in which DR first decouples from the thermal bath and subsequently re-thermalises with the SM sector following PBH evaporation. We find that this re-thermalisation lowers the effective decoupling temperature of DR, amplifying ΔN_{eff} and imposing additional constraints on PBH parameters. We then discuss another scenario of early PBHs dominated epoch where the origin of axion dark matter and leptogenesis are connected. Within SMASH-like frameworks, this naturally gives rise to a unified and testable picture. In particular,

while maintaining the correct axion DM abundance, the regime $f_a \gtrsim 10^{12}$ GeV, corresponding to unflavored leptogenesis with hierarchical right-handed neutrinos, becomes accessible to GW detectors and axion searches, even though it remains out of reach for collider or neutrino experiments. Moreover, axions produced directly through PBH evaporation can contribute to dark radiation at levels detectable by forthcoming CMB experiments.

In the final Chapter 5, we turn our attention to the late-time observational signatures of decaying SHDM, with particular focus on two remarkable recent events: the Amaterasu UHECR event and the KM3-230213A neutrino event. For the Amaterasu event, we perform a multi-messenger analysis incorporating UHE cosmic rays, gamma rays, and neutrinos to derive updated constraints on the SHDM mass and lifetime. We find that existing gamma-ray observations already place the SHDM interpretation of this event under considerable tension. While current UHE neutrino bounds remain comparatively weaker than those from UHECRs, we show that next-generation neutrino detectors will eventually close this gap. Furthermore, future gamma-ray observatories are expected to improve the bounds on SHDM lifetime by at least an order of magnitude beyond current limits, substantially sharpening the probe of this scenario. We then turn to the KM3-230213A event and investigate whether it can be attributed to the decay of a heavy RHN acting as dark matter within a gauged $B - L$ framework. Our analysis reveals that present gamma-ray constraints disfavour DM decay interpretation of this event, compounded by a mild tension with the non-observation of comparable events at IceCube and the PAO. Nevertheless, given the sizable flux uncertainties currently associated with KM3-230213A, an explanation of the event remains viable within 3σ confidence level while still satisfying the aforementioned constraints. We close the chapter by sketching two alternative DM realisations in which the gamma-ray constraints can be relaxed, opening up viable parameter space for explaining the extreme-energy event through dark matter decay.

Overall, the thesis demonstrates that a wide range of otherwise elusive BSM particles and cosmological phenomena can be probed through precision cosmological observations, establishing strong connections between particle physics, early-Universe cosmology, and upcoming experimental searches.

The thesis also opens up several promising directions for future research. Beyond the Dirac-neutrino-portal dark matter scenarios studied here, a wide class of

BSM frameworks can be explored in connection with N_{eff} . For instance, although thermal dark matter with masses below the MeV scale is typically strongly constrained by current bounds on N_{eff} , it is possible to construct well-motivated models in which the contribution to N_{eff} is suppressed due to specific thermal histories or interactions within hidden sectors. Developing such scenarios within complete UV-consistent frameworks constitutes an important direction for future work. In addition, models involving light pseudo-Nambu–Goldstone bosons, such as majorons and ALPs, and their possible connections to baryogenesis, dark matter production, or unified dark-sector origins, provide further opportunities for exploration.

Another interesting avenue concerns the astrophysical and cosmological signatures of Hawking evaporation of primordial black holes. Future studies may investigate observational consequences of PBH evaporation, including the memory-burdened effects, to derive constraints on BSM physics. It would also be worthwhile to further examine the role of black-hole superradiance in probing new light degrees of freedom associated with BSM theories. Beyond these, high-scale physics can also leave its imprint through gravitational bremsstrahlung, which emerges as a potent probe of physics at very high energy scales. Taken together, these directions underscore the broad and promising potential of bridging cosmology and particle phenomenology in the pursuit of new physics beyond the Standard Model.

Appendix:

A.1 Chapter 3: Details of Boltzmann equations

Case I: ϕ in equilibrium

For ψ abundance:

For the process: $\phi(K) \rightarrow \psi(P_1) + \overline{\nu}_R(P_2)$, the integral form of the Boltzmann equation is

$$\int g_\psi \frac{d^3 p_1}{(2\pi)^3} \left[\frac{\partial f_\psi}{\partial t} - \mathcal{H} p_1 \frac{\partial f_\psi}{\partial p_1} \right] = \int g_\psi \frac{d^3 p_1}{(2\pi)^3} C[f_\psi]. \quad (\text{A.1})$$

Using the definition of n_ψ and integration by parts method for the term proportional to $\mathcal{H}$, the LHS of Eq. (A.1) becomes

$$\frac{dn_\psi}{dt} + 3\mathcal{H}n_\psi, \quad (\text{A.2})$$

where,

$$n_\psi = \int g_\psi \frac{d^3 p_1}{(2\pi)^3} f_\psi, \quad (\text{A.3})$$

with g_ψ being the internal degree of freedom of ψ . The RHS of Eq. (A.1) is

$$\int g_\psi \frac{d^3 p_1}{(2\pi)^3} C[f_\psi] = \int g_\psi \frac{d^3 p_1}{(2\pi)^3} \frac{1}{2E_1} \int g_{\nu_R} \frac{d^3 p_2}{(2\pi)^3 2E_2} g_\phi \frac{d^3 k}{(2\pi)^3 2E_k} \times (2\pi)^4 \delta^4(K - P_1 - P_2) |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 (f_\phi^{\text{eq}} - f_\psi f_{\nu_R}). \quad (\text{A.4})$$

We assume that the initial abundances of both ψ and ν_R are negligible, so both f_ψ and f_{ν_R} can be set to zero. Thus we can omit the back-reaction term in the above equation. Now using the definition of decay width of ϕ in the rest frame of ϕ i.e.

$$\Gamma_\phi = \frac{1}{2m_\phi} \int \frac{g_\psi d^3 p_1}{(2\pi)^3 2E_1} \frac{g_{\nu_R} d^3 p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(K - P_1 - P_2) |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2, \quad (\text{A.5})$$

we get

$$\text{RHS} = g_\phi \int \frac{d^3 k}{(2\pi)^3} \frac{2m_\phi}{2E_k} \Gamma_\phi f_\phi^{\text{eq}}. \quad (\text{A.6})$$

Here, the decay width Γ_ϕ is given by

$$\Gamma_\phi = \frac{g_\psi g_{\nu_R}}{16\pi m_\phi} |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 \left(1 - \frac{m_\psi^2}{m_\phi^2} \right)$$

and

$$|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 = \frac{1}{g_\phi g_\psi g_{\nu_R}} y_\phi^2 (m_\phi^2 - m_\psi^2). \quad (\text{A.7})$$

Using $f_\phi^{\text{eq}} = e^{-E_k/T}$, the Maxwell-Boltzmann distribution, we get,

$$\begin{aligned} \text{RHS} &= g_\phi \Gamma_\phi \int \frac{d^3 k}{(2\pi)^3} \frac{2m_\phi}{2E_k} e^{-E_k/T} \\ &= g_\phi \Gamma_\phi \frac{T}{2\pi^2} m_\phi^2 K_1(m_\phi/T). \end{aligned} \quad (\text{A.8})$$

Putting $n_\phi^{\text{eq}} = \frac{g_\phi}{2\pi^2} m_\phi^2 T K_2(m_\phi/T)$, the RHS becomes

$$\text{RHS} = \Gamma_\phi \frac{K_1(m_\phi/T)}{K_2(m_\phi/T)} n_\phi^{\text{eq}}. \quad (\text{A.9})$$

Finally, after equating LHS and RHS of Eq. (A.1), the Boltzmann equation for n_ψ becomes

$$\frac{dn_\psi}{dt} + 3\mathcal{H}n_\psi = \Gamma_\phi \frac{K_1(m_\phi/T)}{K_2(m_\phi/T)} n_\phi^{\text{eq}}. \quad (\text{A.10})$$

Now, instead of n_ψ , we can write the equation in terms of a new variable $Y_\psi = n_\psi/s$, known as comoving number density. Using the fact that $sa^3 = \text{constant}$ with s, a being the entropy density, cosmic scale factor of the FLRW metric, respectively, the LHS of Eq. (A.10) becomes

$$\begin{aligned} s \frac{dY_\psi}{dt} &= \frac{dn_\psi}{dt} + 3\mathcal{H}n_\psi \\ \Rightarrow \frac{dY_\psi}{dT} &= -\frac{1}{3\mathcal{H}s} \left[\frac{3}{T} + \frac{dg_{*s}/dT}{g_{*s}} \right] \left(\frac{dn_\psi}{dt} + 3\mathcal{H}n_\psi \right) \\ &= -\frac{1}{3\mathcal{H}} \left[\frac{3}{T} + \frac{dg_{*s}/dT}{g_{*s}} \right] g_\psi g_{\nu_R} g_\phi \Gamma_\phi \frac{K_1(m_\phi/T)}{K_2(m_\phi/T)} Y_\phi^{\text{eq}} \\ &= -\frac{1}{\mathcal{H}T} \left[1 + \frac{Tdg_{*s}/dT}{3g_{*s}} \right] \Gamma_\phi \frac{K_1(m_\phi/T)}{K_2(m_\phi/T)} Y_\phi^{\text{eq}}. \end{aligned} \quad (\text{A.11})$$

Now defining $x = m_\phi/T$, we can write the above equation in terms of dimensionless variables x

$$\frac{dY_\psi}{dx} = \frac{\beta}{x\mathcal{H}} \Gamma_\phi \frac{K_1(x)}{K_2(x)} Y_\phi^{\text{eq}}, \quad (\text{A.12})$$

where,

$$\beta = \left[1 + \frac{Tdg_{*s}/dT}{3g_{*s}} \right]. \quad (\text{A.13})$$

For ν_R energy density:

Let us start with the differential Boltzmann equation for ν_R

$$\frac{\partial f_{\nu_R}}{\partial t} - \mathcal{H}p_2 \frac{\partial f_{\nu_R}}{\partial p_2} = C[f_{\nu_R}]. \quad (\text{A.14})$$

Integrating both side with $\int g_{\nu_R} E_2 \frac{d^3 p_2}{(2\pi)^3}$, we get

$$\int g_{\nu_R} E_2 \frac{d^3 p_2}{(2\pi)^3} \left(\frac{\partial f_{\nu_R}}{\partial t} - \mathcal{H}p_2 \frac{\partial f_{\nu_R}}{\partial p_2} \right) = \int g_{\nu_R} E_2 \frac{d^3 p_2}{(2\pi)^3} C[f_{\nu_R}]. \quad (\text{A.15})$$

The LHS, after simplification becomes

$$\int g_{\nu_R} E_2 \frac{d^3 p_2}{(2\pi)^3} \left(\frac{\partial f_{\nu_R}}{\partial t} - \mathcal{H}p_2 \frac{\partial f_{\nu_R}}{\partial p_2} \right) = \frac{d\rho_{\nu_R}}{dt} + 4\mathcal{H}\rho_{\nu_R}, \quad (\text{A.16})$$

where,

$$\rho_{\nu_R} = \int g_{\nu_R} \frac{d^3 p_2}{(2\pi)^3} E_2 f_{\nu_R}. \quad (\text{A.17})$$

Expanding the collision term, the RHS becomes

$$\begin{aligned} \int g_{\nu_R} E_2 \frac{d^3 p_2}{(2\pi)^3} C[f_{\nu_R}] &= g_{\nu_R} \int \frac{d^3 p_2}{(2\pi)^3} \frac{1}{2E_2} \int g_\psi \frac{d^3 p_1}{(2\pi)^3 2E_1} g_\phi \frac{d^3 k}{(2\pi)^3 2E_k} \\ &\times E_2 (2\pi)^4 \delta^4(K - P_1 - P_2) |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 f_\phi^{\text{eq}}. \end{aligned} \quad (\text{A.18})$$

Let us do the following integral first.

$$\begin{aligned} I &= \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \frac{d^3 p_2}{(2\pi)^3 2E_2} E_2 (2\pi)^4 \delta^4(K - P_1 - P_2) |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 \\ &= \frac{1}{4(2\pi)^2} \int \frac{d^3 p_1}{E_1} d^3 p_2 \delta^4(K - P_1 - P_2) |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2. \end{aligned} \quad (\text{A.19})$$

We first do the integration over $\vec{p}_2$ using the Dirac delta function,

$$\begin{aligned} I &= \frac{1}{4(2\pi)^2} \int \frac{d^3 p_1}{E_1} \delta(E_k - E_1 - E_{k-1}) |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 \\ &= \frac{2\pi}{4(2\pi)^2} \int \frac{p_1^2 dp_1 d(\cos \theta)}{E_1} \delta(f(\theta)) |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2. \end{aligned} \quad (\text{A.20})$$

Here, θ is the angle between $\vec{k}$ and $\vec{p}_1$ and $f(\theta) = E_k - E_1 - E_{k-1}$ with $E_{k-1} = \sqrt{(\vec{k} - \vec{p}_1)^2 + m_\nu^2}$. Now to find the root of $f(\theta)$, we set

$$\begin{aligned} f(\theta) &= 0 \\ \Rightarrow E_k - E_1 - E_{k-1} &= 0 \\ \Rightarrow \cos \theta &= \frac{2E_k E_1 - (m_\phi^2 + m_\psi^2 - m_\nu^2)}{2|\vec{k}||\vec{p}_1|} \equiv \cos \theta_0. \end{aligned} \quad (\text{A.21})$$

Also,

$$\left. \frac{df}{d \cos \theta} \right|_{\cos \theta = \cos \theta_0} = \frac{|\vec{k}||\vec{p}_1|}{E_k - E_1}. \quad (\text{A.22})$$

Thus, the integral I reduces to

$$\begin{aligned}
 I &= \frac{1}{4(2\pi)} \int \frac{p_1^2 dp_1}{E_1} \int d(\cos \theta) \frac{\delta(\cos \theta - \cos \theta_0)}{\left| \frac{df}{d \cos \theta} \right|_{\theta=\theta_0}} |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 \\
 &= \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2}{8\pi} \int \frac{p_1^2 dp_1}{E_1} \frac{E_k - E_1}{|\vec{k}| |\vec{p}_1|} \\
 &= \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2}{8\pi |\vec{k}|} \int_{E_1^{\min}}^{E_1^{\max}} dE_1 (E_k - E_1). \tag{A.23}
 \end{aligned}$$

In the above, $|\mathcal{M}|'$ implies $|\mathcal{M}|$ at $\theta = \theta_0$. The limits of the integration will come from the condition

$$-1 \leq \cos \theta_0 \leq 1. \tag{A.24}$$

Working through it, we get

$$\begin{aligned}
 E_1^{\min} &= \frac{E_k(m_\phi^2 + m_\psi^2 - m_\nu^2) - \sqrt{E_k^2(m_\phi^2 + m_\psi^2 - m_\nu^2)^2 - m_\phi^2(\Lambda + 4E_k^2 m_\psi^2)}}{2m_\phi^2} \equiv g_1(E_k) \\
 E_1^{\max} &= \frac{E_k(m_\phi^2 + m_\psi^2 - m_\nu^2) + \sqrt{E_k^2(m_\phi^2 + m_\psi^2 - m_\nu^2)^2 - m_\phi^2(\Lambda + 4E_k^2 m_\psi^2)}}{2m_\phi^2} \equiv g_2(E_k), \tag{A.25}
 \end{aligned}$$

where,

$$\Lambda = (m_\phi^2 + m_\psi^2 - m_\nu^2)^2 - 4m_\phi^2 m_\psi^2. \tag{A.26}$$

Hence, I becomes

$$\begin{aligned}
 I &= \frac{g_2(E_k) - g_1(E_k)}{8\pi |\vec{k}|} |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 \left(E_k - \frac{g_2(E_k) + g_1(E_k)}{2} \right) \\
 &= \frac{\sqrt{E_k^2(m_\phi^2 + m_\psi^2 - m_\nu^2)^2 - m_\phi^2(\Lambda + 4E_k^2 m_\psi^2)}}{8\pi |\vec{k}| m_\phi^2} |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 \left(E_k - \frac{E_k(m_\phi^2 + m_\psi^2 - m_\nu^2)}{2m_\phi^2} \right) \\
 &= |\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2 \frac{\sqrt{E_k^2(m_\phi^2 + m_\psi^2 - m_\nu^2)^2 - m_\phi^2(\Lambda + 4E_k^2 m_\psi^2)}}{8\pi |\vec{k}| m_\phi^2} E_k \left(\frac{m_\phi^2 - m_\psi^2 + m_\nu^2}{2m_\phi^2} \right). \tag{A.27}
 \end{aligned}$$

Finally, the RHS becomes -

$$\begin{aligned}
 \text{RHS} &= g_\phi g_\psi g_{\nu_R} \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2}{32\pi^3} \frac{(m_\phi^2 - m_\psi^2 + m_\nu^2)}{2m_\phi^4} \\
 &\times \int_{m_\phi}^{\infty} E_k f_\phi^{\text{eq}} \sqrt{E_k^2 (m_\phi^2 + m_\psi^2 - m_\nu^2)^2 - m_\phi^2 (\Lambda + 4E_k^2 m_\psi^2)} dE_k \\
 &= g_\phi g_\psi g_{\nu_R} \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2}{32\pi^3} \frac{(m_\phi^2 - m_\psi^2)^2}{2m_\phi^4} \int_{m_\phi}^{\infty} E_k f_\phi^{\text{eq}} \sqrt{E_k^2 - m_\phi^2} dE_k \quad (\because m_\nu \simeq 0) \\
 &\quad \quad \quad (A.28)
 \end{aligned}$$

$$\begin{aligned}
 &= g_\phi g_\psi g_{\nu_R} \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2}{32\pi^3} \frac{(m_\phi^2 - m_\psi^2)^2}{2m_\phi^4} \int_{m_\phi}^{\infty} E_k e^{-E_k/T} \sqrt{E_k^2 - m_\phi^2} dE_k \\
 &= g_\phi g_\psi g_{\nu_R} \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2}{32\pi^3} \frac{(m_\phi^2 - m_\psi^2)^2}{2m_\phi^4} m_\phi^2 T K_2(m_\phi/T) \\
 &= \langle E\Gamma \rangle n_\phi^{\text{eq}}, \quad \quad \quad (A.29)
 \end{aligned}$$

where

$$\langle E\Gamma \rangle = g_\psi g_{\nu_R} \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2}{32\pi} \frac{(m_\phi^2 - m_\psi^2)^2}{m_\phi^4}. \quad (A.30)$$

So, the final form of the evolution equation of ρ_{ν_R} is

$$\begin{aligned}
 \frac{d\rho_{\nu_R}}{dt} + 4\mathcal{H}\rho_{\nu_R} &= \langle E\Gamma \rangle n_\phi^{\text{eq}} \\
 \Rightarrow \frac{d\tilde{Y}}{dT} &= -\frac{\beta}{\mathcal{H}T s^{4/3}} \langle E\Gamma \rangle n_\phi^{\text{eq}} \quad (\text{where } \tilde{Y} = \frac{\rho_{\nu_R}}{s^{4/3}}). \quad (A.31)
 \end{aligned}$$

In terms of $x = m_\phi/T$, the above equation becomes

$$\frac{d\tilde{Y}}{dx} = \frac{\beta}{\mathcal{H}s^{1/3}x} \langle E\Gamma \rangle Y_\phi^{\text{eq}}. \quad (A.32)$$

Case II

In this case, ϕ is not in equilibrium always. It is produced in equilibrium and at some epoch it goes out of equilibrium due to thermal freeze-out.

For ψ abundance:

The procedure to obtain the Boltzmann equation for ψ in this case is same as the above case from Eq. (A.1) to Eq. (A.6) except that f_ϕ^{eq} is now replaced by f_ϕ . Thus, the Boltzmann equation for ψ is

$$\frac{dn_\psi}{dt} + 3\mathcal{H}n_\psi = g_\phi \int \frac{d^3k}{(2\pi)^3} \frac{2m_\phi}{2E_k} \Gamma_\phi f_\phi. \quad (\text{A.33})$$

Since ϕ was in equilibrium earlier and goes out of equilibrium after freeze-out, we can write the general form of the Maxwell-Boltzmann distribution function for ϕ with a chemical potential that is nonzero only after the freeze-out of ϕ i.e. $f_\phi = e^{\mu/T} e^{-E_k/T}$.

The chemical potential μ is defined as $\mu = T \ln \left(\frac{n_\phi(T)}{n_\phi^{\text{eq}}(T)} \right)$. Substituting f_ϕ in Eq. (A.33), the Boltzmann equation becomes

$$\frac{dn_\psi}{dt} + 3\mathcal{H}n_\psi = g_\phi e^{\mu/T} \int \frac{d^3k}{(2\pi)^3} \frac{2m_\phi}{2E_k} \Gamma_\phi e^{-E_k/T}. \quad (\text{A.34})$$

The RHS of the equation is same as Eq. (A.8) in case I except for the $e^{\mu/T}$ factor. Hence, following the same procedure as Eq. (A.8) to Eq. (A.10) and replacing μ by number density, we get

$$\begin{aligned} \frac{dn_\psi}{dt} + 3\mathcal{H}n_\psi &= e^{\mu/T} \Gamma_\phi \frac{K_1(m_\phi/T)}{K_2(m_\phi/T)} n_\phi^{\text{eq}}, \\ &= \Gamma_\phi \frac{K_1(m_\phi/T)}{K_2(m_\phi/T)} n_\phi. \end{aligned} \quad (\text{A.35})$$

We can write the above equation in terms of $Y_\psi = n_\psi/s$, $Y_\phi = n_\phi/s$ and $x = m_\phi/T$. In terms of these dimensionless quantities the above equation takes the following form

$$\frac{dY_\psi}{dx} = \frac{\beta}{x\mathcal{H}} \Gamma_\phi \frac{K_1(x)}{K_2(x)} Y_\phi. \quad (\text{A.36})$$

For ν_R energy density:

To find the energy density of ν_R in this case, we will follow the same procedure as in the previous case, the only difference will be that now f_ϕ^{eq} will be replaced by $f_\phi = e^{\mu/T} e^{-E_k/T}$. Hence starting from Eq. (A.28), the R.H.S. of the Boltzmann

equation for ρ_{ν_R} can be written as

$$\begin{aligned}
\frac{d\rho_{\nu_R}}{dt} + 4\mathcal{H}\rho_{\nu_R} &= g_\phi g_\psi g_{\nu_R} \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2}{32\pi^3} \frac{(m_\phi^2 - m_\psi^2)^2}{2m_\phi^4} \int_{m_\phi}^\infty E_k f_\phi \sqrt{E_k^2 - m_\phi^2} dE_k, \\
&= g_\phi g_\psi g_{\nu_R} \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2}{32\pi^3} \frac{(m_\phi^2 - m_\psi^2)^2}{2m_\phi^4} \int_{m_\phi}^\infty E_k e^{\mu/T} e^{E_k/T} \sqrt{E_k^2 - m_\phi^2}, \\
&= g_\phi g_\psi g_{\nu_R} \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2}{32\pi^3} \frac{(m_\phi^2 - m_\psi^2)^2}{2m_\phi^4} e^{\mu/T} \int_{m_\phi}^\infty E_k e^{E_k/T} \sqrt{E_k^2 - m_\phi^2}, \\
&= g_\phi g_\psi g_{\nu_R} \frac{|\mathcal{M}|_{\phi \rightarrow \bar{\nu}_R \psi}^2}{32\pi^3} \frac{(m_\phi^2 - m_\psi^2)^2}{2m_\phi^4} e^{\mu/T} m_\phi^2 T K_2(m_\phi/T), \\
&= \langle E\Gamma \rangle e^{\mu/T} n_\phi^{\text{eq}}, \\
\Rightarrow \frac{d\rho_{\nu_R}}{dt} + 4H\rho_{\nu_R} &= \langle E\Gamma \rangle n_\phi.
\end{aligned} \tag{A.37}$$

Now expressing ρ_{ν_R} by the comoving energy density, $\tilde{Y}$, the above equation in terms of T and $x = m_\phi/T$ are given by

$$\begin{aligned}
\frac{d\tilde{Y}}{dT} &= -\frac{\beta}{\mathcal{H}T s^{1/3}} \langle E\Gamma \rangle Y_\phi, \\
\frac{d\tilde{Y}}{dx} &= \frac{\beta}{\mathcal{H} s^{1/3} x} \langle E\Gamma \rangle Y_\phi.
\end{aligned} \tag{A.38}$$

For comoving number density of non-thermal ϕ :

The calculation of the number density of ϕ will involve two processes : $X(K'_1) + \bar{X}(K'_2) \rightarrow \phi(K_1) + \phi^\dagger(K_2)$ and $\phi(K_1) \rightarrow \psi(P_1) + \bar{\nu}_R(P_2)$. Hence, the differential form of the Boltzmann equation is (X is any SM particle) -

$$\begin{aligned}
\frac{\partial f_\phi}{\partial t} - \mathcal{H} k_1 \frac{\partial f_\phi}{\partial k_1} &= C^{X\bar{X} \rightarrow \phi\phi^\dagger}[f_\phi] - C^{\phi \rightarrow \psi\nu_R}[f_\phi] \\
\int g_\phi \frac{d^3 k_1}{(2\pi)^3} \left(\frac{\partial f_\phi}{\partial t} - \mathcal{H} k_1 \frac{\partial f_\phi}{\partial k_1} \right) &= \int g_\phi \frac{d^3 k_1}{(2\pi)^3} \left(C^{X\bar{X} \rightarrow \phi\phi^\dagger}[f_\phi] - C^{\phi \rightarrow \psi\nu_R}[f_\phi] \right).
\end{aligned} \tag{A.39}$$

The LHS is

$$\int g_\phi \frac{d^3 k_1}{(2\pi)^3} \left(\frac{\partial f_\phi}{\partial t} - \mathcal{H} k_1 \frac{\partial f_\phi}{\partial k_1} \right) = \frac{dn_\phi}{dt} + 3\mathcal{H}n_\phi. \tag{A.40}$$

The first term of RHS is

$$\begin{aligned}
& \int g_\phi \frac{d^3 k_1}{(2\pi)^3} C^{X\bar{X} \rightarrow \phi\phi^\dagger} [f_\phi], \\
&= \int g_\phi \frac{d^3 k_1}{(2\pi)^3} \frac{1}{2E_{k_1}} \int g_X \frac{d^3 k'_1}{(2\pi)^3 2E_{k'_1}} \int g_X \frac{d^3 k'_2}{(2\pi)^3 2E_{k'_2}} \int g_\phi \frac{d^3 k_2}{(2\pi)^3 2E_{k_2}} \\
&\quad \times (2\pi)^4 \delta^4(K'_1 + K'_2 - K_1 - K_2) \times |\mathcal{M}|_{X\bar{X} \rightarrow \phi\phi^\dagger}^2 (f_{k'_1} f_{k'_2} - f_{k_1} f_{k_2}), \\
&= (n_\phi^{\text{eq}})^2 \langle \sigma v \rangle_{\phi\phi^\dagger \rightarrow X\bar{X}} \left(\left(\frac{n_X}{n_X^{\text{eq}}} \right)^2 - \left(\frac{n_\phi}{n_\phi^{\text{eq}}} \right)^2 \right), \quad \left(\because f_i = e^{\mu_i/T} e^{-E_i/T} = \frac{n_i}{n_i^{\text{eq}}} \right), \\
&= \langle \sigma v \rangle_{\phi\phi^\dagger \rightarrow X\bar{X}} \left((n_\phi^{\text{eq}})^2 - (n_\phi)^2 \right), \quad (\because n_X^{\text{eq}} = n_X) \tag{A.41}
\end{aligned}$$

where,

$$\begin{aligned}
\langle \sigma v \rangle &= \frac{1}{(n_\phi^{\text{eq}})^2} \int g_\phi \frac{d^3 k_1}{(2\pi)^3} \frac{1}{2E_{k_1}} \int g_X \frac{d^3 k'_1}{(2\pi)^3 2E_{k'_1}} \int g_X \frac{d^3 k'_2}{(2\pi)^3 2E_{k'_2}} \int g_\phi \frac{d^3 k_2}{(2\pi)^3 2E_{k_2}} \\
&\quad \times (2\pi)^4 \delta^4(K'_1 + K'_2 - K_1 - K_2) |\mathcal{M}|_{X\bar{X} \rightarrow \phi\phi^\dagger}^2 e^{-(E_{k_1} + E_{k_2})/T}, \\
&= \frac{1}{(n_\phi^{\text{eq}})^2} \int g_\phi \frac{d^3 k_1}{(2\pi)^3} \int g_\phi \frac{d^3 k_2}{(2\pi)^3} \frac{1}{4E_{k_1} E_{k_2}} \int g_X \frac{d^3 k'_1}{(2\pi)^3 2E_{k'_1}} \int g_X \frac{d^3 k'_2}{(2\pi)^3 2E_{k'_2}} \\
&\quad \times (2\pi)^4 \delta^4(K'_1 + K'_2 - K_1 - K_2) |\mathcal{M}|_{\phi\phi^\dagger \rightarrow X\bar{X}}^2 e^{-(E_{k_1} + E_{k_2})/T}, \\
&= \frac{g_\phi^2}{(n_\phi^{\text{eq}})^2} \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} (\sigma v)_{\phi\phi^\dagger \rightarrow X\bar{X}} e^{-(E_{k_1} + E_{k_2})/T}, \\
&= \frac{\int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} (\sigma v)_{\phi\phi^\dagger \rightarrow X\bar{X}} e^{-(E_{k_1} + E_{k_2})/T}}{\int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} e^{-(E_{k_1} + E_{k_2})/T}}, \\
&= \frac{1}{8m_\phi^4 T K_2^2 (m_\phi/T)} \int_{4m_\phi^2}^\infty (\sigma)_{\phi\phi^\dagger \rightarrow X\bar{X}} (s - 4m_\phi^2) \sqrt{s} K_1(\sqrt{s}/T) ds. \tag{A.42}
\end{aligned}$$

We have obtained the last expression following the prescription given in [81]. Now the second term in the RHS is

$$\begin{aligned}
\int g_\phi \frac{d^3 k_1}{(2\pi)^3} C^{\phi \rightarrow \psi \nu_R} [f_\phi] &= \int g_\phi \frac{d^3 k_1}{(2\pi)^3} \frac{1}{2E_{k_1}} \int g_\psi \frac{d^3 p_1}{(2\pi)^3 2E_1} \int g_{\nu_R} \frac{d^3 p_2}{(2\pi)^3 2E_2} \\
&\quad \times (2\pi)^4 \delta^4(K_1 - P_1 - P_2) f_\phi. \tag{A.43}
\end{aligned}$$

Here due to non-thermal nature of ψ and ν_R , we have omitted the back reaction term, which otherwise will be there in Eq. (A.43) and is proportional to $f_\psi f_{\nu_R}$. Therefore,

from Eq. (A.35) we obtain

$$\int g_\phi \frac{d^3 k_1}{(2\pi)^3} C^{\phi \rightarrow \psi \nu_R} [f_\phi] = \Gamma_\phi \frac{K_1(m_\phi/T)}{K_2(m_\phi/T)} n_\phi. \quad (\text{A.44})$$

Finally, the full equation for the evolution of n_ϕ is

$$\frac{dn_\phi}{dt} + 3\mathcal{H}n_\phi = -\langle \sigma v \rangle_{\phi\phi^\dagger \rightarrow X\bar{X}} ((n_\phi)^2 - (n_\phi^{\text{eq}})^2) - \Gamma_\phi \frac{K_1(m_\phi/T)}{K_2(m_\phi/T)} n_\phi. \quad (\text{A.45})$$

In terms of comoving number density Y_ϕ ,

$$\begin{aligned} \frac{dY_\phi}{dT} &= -\frac{\beta s}{\mathcal{H}T} \left(-\langle \sigma v \rangle_{\phi\phi^\dagger \rightarrow X\bar{X}} ((Y_\phi)^2 - (Y_\phi^{\text{eq}})^2) - \frac{\Gamma_\phi}{s} \frac{K_1(m_\phi/T)}{K_2(m_\phi/T)} Y_\phi \right), \\ \Rightarrow \frac{dY_\phi}{dx} &= \frac{\beta s}{\mathcal{H}x} \left(-\langle \sigma v \rangle_{\phi\phi^\dagger \rightarrow X\bar{X}} ((Y_\phi)^2 - (Y_\phi^{\text{eq}})^2) - \frac{\Gamma_\phi}{s} \frac{K_1(m_\phi/T)}{K_2(m_\phi/T)} Y_\phi \right). \end{aligned} \quad (\text{A.46})$$

Case III

Distribution function of ϕ :

The case III, where ϕ never attains thermal equilibrium with the SM bath, has the same forms of Boltzmann equations for n_ϕ and ρ_{ν_R} as those are in case I except here we need to replace the thermal distribution function of ϕ by the non-thermal distribution function. The differential form of the Boltzmann equation to find the distribution function of ϕ , f_ϕ is given by [460, 461]

$$\frac{\partial f_\phi}{\partial t} - \mathcal{H}p_1 \frac{\partial f_\phi}{\partial p_1} = C^{h \rightarrow \phi\phi^\dagger} + C^{hh \rightarrow \phi\phi^\dagger} + C^{\phi \rightarrow \bar{\nu}_R \psi}. \quad (\text{A.47})$$

Here $C^{h \rightarrow \phi\phi^\dagger}$ is the collision term for production of $\phi\phi^\dagger$ pair from the decay of the SM Higgs boson $h(K) \rightarrow \phi(P_1) + \phi^\dagger(P_2)$. The expression of $C^{h \rightarrow \phi\phi^\dagger}$ is given by

$$\begin{aligned} C^{h \rightarrow \phi\phi^\dagger} &= \frac{1}{2E_{p_1}} \int \frac{d^3 p_2}{2E_{p_2} (2\pi)^3} \frac{d^3 k}{2E_k (2\pi)^3} (2\pi)^4 \delta^4(K - P_1 - P_2) \\ &\quad \times |\mathcal{M}|_{h \rightarrow \phi\phi^\dagger}^2 (f_h^{\text{eq}}(k) - f_\phi(p_1) f_{\phi^\dagger}(p_2)), \\ &= \frac{1}{2E_{p_1} (2\pi)^2} \int \frac{d^3 p_2}{4E_{p_2} E_{p_1+p_2}} \delta(E_{p_1+p_2} - E_{p_1} - E_{p_2}) \\ &\quad \times |\mathcal{M}|_{h \rightarrow \phi\phi^\dagger}^2 (f_h^{\text{eq}}(k) - f_\phi(p_1) f_{\phi^\dagger}(p_2)). \end{aligned} \quad (\text{A.48})$$

Now we can write $d^3p_2 = p_2^2 dp_2 d(\cos \theta) d\phi$, where θ is the angle between $\vec{p}_1$ and $\vec{p}_2$. Therefore, the Dirac delta function $\delta(E_{p_1+p_2} - E_{p_1} - E_{p_2})$ actually fixes the angle θ . So, from the condition $E_{p_1+p_2} = E_{p_1} + E_{p_2}$, we will get

$$\cos \theta = \frac{2m_\phi^2 - m_h^2 + 2E_{p_1}E_{p_2}}{2p_1p_2} \equiv \cos \theta_0. \quad (\text{A.49})$$

Therefore,

$$C^{h \rightarrow \phi\phi^\dagger} = \frac{1}{2E_{p_1}(2\pi)^2} \int \frac{p_2^2 dp_2 (2\pi)}{4E_{p_2}} \int_{-1}^1 \frac{d(\cos \theta) \delta(\cos \theta - \cos \theta_0)}{E_{p_1+p_2} \left| \frac{df}{d \cos \theta} \right|_{\theta=\theta_0}} \times |\mathcal{M}|_{h \rightarrow \phi\phi^\dagger}^2 \left(f_h^{\text{eq}}(E_{p_1+p_2}) - f_\phi(p_1) f_{\phi^\dagger}(p_2) \right), \quad (\text{A.50})$$

where $f(\cos \theta) = E_{p_1+p_2} - E_{p_1} - E_{p_2}$ with $E_{p_1+p_2} = \sqrt{|\vec{p}_1 + \vec{p}_2|^2 + m_h^2}$ and

$$\left. \frac{df}{d \cos \theta} \right|_{\theta=\theta_0} = \frac{p_1 p_2}{E_{p_1} + E_{p_2}}, \quad (\text{A.51})$$

$$E_{p_1+p_2} \Big|_{\theta=\theta_0} = E_{p_1} + E_{p_2}. \quad (\text{A.52})$$

After some simplification, the collision term takes the following form

$$C^{h \rightarrow \phi\phi^\dagger} = \frac{1}{16\pi E_{p_1} p_1} \int_{p_2^{\min}}^{p_2^{\max}} \frac{p_2 dp_2}{E_{p_2}} |\mathcal{M}|_{h \rightarrow \phi\phi^\dagger}^2 \left(f_\phi^{\text{eq}}(E_{p_1}) f_{\phi^\dagger}^{\text{eq}}(E_{p_2}) - f_\phi(p_1) f_{\phi^\dagger}(p_2) \right). \quad (\text{A.53})$$

The limits of the integration is obtained from the condition $-1 \leq \cos \theta_0 \leq 1$. This condition translates to -

$$p_2^{\min} = \left| \frac{p_1(m_h^2 - 2m_\phi^2) - m_h \sqrt{(m_h^2 - 4m_\phi^2)(p_1^2 + m_\phi^2)}}{2m_\phi^2} \right|, \\ p_2^{\max} = \frac{p_1(m_h^2 - 2m_\phi^2) + m_h \sqrt{(m_h^2 - 4m_\phi^2)(p_1^2 + m_\phi^2)}}{2m_\phi^2}. \quad (\text{A.54})$$

Here we have neglected the inverse decay term in Eq. (A.53) as it is substantially smaller compared to the decay term as long as ϕ is non-thermal. Therefore, the

collision term $C^{h \rightarrow \phi\phi^\dagger}$ becomes

$$\begin{aligned} C^{h \rightarrow \phi\phi^\dagger} &= \frac{1}{16\pi E_{p_1} p_1} \int_{p_2^{\min}}^{p_2^{\max}} \frac{p_2 dp_2}{E_{p_2}} |\mathcal{M}|_{h \rightarrow \phi\phi^\dagger}^2 e^{-E_{p_1}/T} e^{-E_{p_2}/T}, \\ &= \frac{|\mathcal{M}|_{h \rightarrow \phi\phi^\dagger}^2 T e^{-E_{p_1}/T}}{16\pi E_{p_1} p_1} \left(e^{-E_{p_2}^{\min}/T} - e^{-E_{p_2}^{\max}/T} \right), \end{aligned} \quad (\text{A.55})$$

$$\text{and } E_{p_2}^{\max(\min)} = \sqrt{\left(p_2^{\max(\min)}\right)^2 + m_\phi^2}.$$

Now, we will briefly discuss the derivation of the collision term $C^{hh \rightarrow \phi\phi^\dagger}$ for the production of $\phi\phi^\dagger$ pair due the scattering of the Higgs boson $h(K_1) + h(K_2) \rightarrow \phi(P_1) + \phi^\dagger(P_2)$.

$$\begin{aligned} C^{hh \rightarrow \phi\phi^\dagger} &= \frac{1}{2E_{p_1}} \int \frac{d^3 k_1}{2E_{k_1} (2\pi)^3} \frac{d^3 k_2}{2E_{k_2} (2\pi)^3} \frac{d^3 p_2}{2E_{p_2} (2\pi)^3} (2\pi)^4 \delta^4(K_1 + K_2 - P_1 - P_2) \\ &\quad |\mathcal{M}|_{hh \rightarrow \phi\phi^\dagger}^2 (f_h(k_1) f_h(k_2) - f_\phi(p_1) f_{\phi^\dagger}(p_2)), \\ &= \frac{1}{2E_{p_1}} \int \frac{d^3 p_2}{2E_{p_2} (2\pi)^3} \left[\int \frac{d^3 k_1}{2E_{k_1} (2\pi)^3} \frac{d^3 k_2}{2E_{k_2} (2\pi)^3} (2\pi)^4 \delta^4(K_1 + K_2 - P_1 - P_2) \right] \\ &\quad |\mathcal{M}|_{hh \rightarrow \phi\phi^\dagger}^2 (f_h(k_1) f_h(k_2) - f_\phi(p_1) f_{\phi^\dagger}(p_2)). \end{aligned} \quad (\text{A.56})$$

The term inside the square bracket is Lorentz invariant, and we can do that integration easily in the center of momentum frame. Here, for the calculational simplification, we assume that the matrix amplitude square $|\mathcal{M}|_{hh \rightarrow \phi\phi^\dagger}^2$ depends only on the Mandelstam variable s which is true for s -channel scatterings and contact interactions. For a general matrix amplitude square, depending on all three Mandelstam variables one can use the prescription given in [462].

$$I = \int \frac{d^3 k_1}{2E_{k_1} (2\pi)^3} \frac{d^3 k_2}{2E_{k_2} (2\pi)^3} (2\pi)^4 \delta^4(K_1 + K_2 - P_1 - P_2). \quad (\text{A.57})$$

This will give

$$I = \frac{1}{8\pi} \sqrt{1 - \frac{4m_h^2}{s}}. \quad (\text{A.58})$$

Now, since I is a Lorentz invariant quantity, we can use this result in any inertial frame of reference with proper definition of s . In any arbitrary reference frame, the Mandelstam variable $s(p_1, p_2, \cos \alpha) = (P_1 + P_2)^2 = 2m_\phi^2 + 2E_{p_1} E_{p_2} - 2|\vec{p}_1||\vec{p}_2| \cos \alpha$, α is the angle between $\vec{p}_1$ and $\vec{p}_2$ which is π in the center of momentum frame.

Hence, the collision term in an arbitrary inertial frame of reference is given by

$$\begin{aligned}
 C^{hh \rightarrow \phi \phi^\dagger} &= \frac{1}{16\pi E_{p_1}} \int \frac{d^3 p_2}{2E_{p_2}(2\pi)^3} \sqrt{1 - \frac{4m_h^2}{s(p_1, p_2, \cos \alpha)}} \\
 &\quad \times |\mathcal{M}|_{hh \rightarrow \phi \phi^\dagger}^2(s) (f_h(k_1)f_h(k_2) - f_\phi(p_1)f_{\phi^\dagger}(p_2)) , \\
 &= \frac{2\pi}{16\pi E_{p_1} 2(2\pi)^3} \int \frac{p_2^2 dp_2 d(\cos \alpha)}{E_{p_2}} \sqrt{1 - \frac{4m_h^2}{s(p_1, p_2, \cos \alpha)}} \\
 &\quad \times |\mathcal{M}|_{hh \rightarrow \phi \phi^\dagger}^2(s) f_h(k_1)f_h(k_2) , \tag{A.59}
 \end{aligned}$$

where, in the last step we have neglected the back scattering term. Now using the Maxwell-Boltzmann distribution function for the SM Higgs boson and $f_h(k_1)f_h(k_2) = e^{-(E_{k_1}+E_{k_2})/T} = e^{-(E_{p_1}+E_{p_2})/T}$, we obtain

$$\begin{aligned}
 C^{hh \rightarrow \phi \phi^\dagger} &= \frac{e^{-E_{p_1}/T}}{16E_{p_1}(2\pi)^3} \int_0^\infty \frac{p_2^2 dp_2}{\sqrt{p_2^2 + m_\phi^2}} e^{-E_{p_2}/T} \\
 &\quad \times \int_{-1}^{\cos \alpha_{max}} d(\cos \alpha) \sqrt{1 - \frac{4m_h^2}{s(p_1, p_2, \cos \alpha)}} |\mathcal{M}|_{hh \rightarrow \phi \phi^\dagger}^2(s). \tag{A.60}
 \end{aligned}$$

The limit on $\cos \alpha$ will come from the condition that $\sqrt{1 - \frac{4m_h^2}{s(p_1, p_2, \cos \alpha)}}$ is real. This is possible only when $s \geq 4m_h^2$ and therefore

$$\cos \alpha \leq \frac{2m_\phi^2 - 4m_h^2 + 2E_{p_1}E_{p_2}}{2|\vec{p}_1||\vec{p}_2|} \equiv \cos \alpha_0. \tag{A.61}$$

Thus the upper limit of the integration is

$$\cos \alpha_{\max} = \text{Min} [\text{Max} [\cos \alpha_0, -1], 1]. \tag{A.62}$$

And, lastly, the collision term $C^{\phi \rightarrow \bar{\nu}_R \psi}$ is for the decay of ϕ into $\bar{\nu}_R$ and ψ ($\phi(P_1) \rightarrow \bar{\nu}_R(q) + \psi(q')$) and it has the following expression [461]

$$C^{\phi \rightarrow \bar{\nu}_R \psi} = -f_\phi \frac{m_\phi}{\sqrt{p_1^2 + m_\phi^2}} \Gamma_{\phi \rightarrow \bar{\nu}_R \psi}. \tag{A.63}$$

The LHS of Eq. (A.47), can be greatly simplified in we transform the variables from p_1 and T to new variables $r = m_0/T$ and $\xi = \left(\frac{g_{*s}(T_0)}{g_{*s}(T)} \right)^{1/3} \frac{p_1}{T}$ where m_0 is any arbitrary

mass scale. In terms of the two new variables, the LHS of Eq. (A.47) depends only on r [460, 461]

$$\frac{\partial f_\phi}{\partial t} - \mathcal{H} p_1 \frac{\partial f_\phi}{\partial p_1} = r \mathcal{H} \left(1 + \frac{T g'_s(T)}{3 g_{*s}(T)} \right)^{-1} \frac{\partial f_\phi}{\partial r}. \quad (\text{A.64})$$

Therefore, the full Boltzmann equation for f_ϕ is

$$\frac{\partial f_\phi(\xi, r)}{\partial r} = \frac{\left(1 - \frac{r}{3 g_{*s}(r)} \frac{d g_{*s}(r)}{dr} \right)}{r H} (C^{h \rightarrow \phi \phi^\dagger}(\xi, r) + C^{hh \rightarrow \phi \phi^\dagger}(\xi, r) + C^{\phi \rightarrow \bar{\nu}_R \psi}(\xi, r)). \quad (\text{A.65})$$

Now, the number density of ϕ can be written as

$$n_\phi(r) = \frac{g_\phi}{2\pi^2} \mathcal{A}(r)^3 \left(\frac{m_0}{r} \right)^3 \int d\xi \xi^2 f_\phi(\xi, r), \quad (\text{A.66})$$

where

$$\mathcal{A}(r) = \left(\frac{g_{*s}(m_0/r)}{g_{*s}(m_0/T_0)} \right)^{1/3}. \quad (\text{A.67})$$

After solving the Eq. (A.65) for the non-thermal distribution function $f_\phi(\xi, r)$, we can now calculate comoving number density of ψ and $\tilde{Y}$ using the following Boltzmann equations

$$\begin{aligned} \frac{dY_\psi}{dr} &= \frac{g_\phi \beta}{r \mathcal{H} s} \frac{\Gamma_\phi m_\phi}{2\pi^2} \int_0^\infty \frac{\left(\mathcal{A} \frac{m_0}{r} \right)^3 \xi^2 f_\phi(\xi, r)}{\sqrt{(\xi \mathcal{A} \frac{m_0}{r})^2 + m_\phi^2}} d\xi, \\ \frac{d\tilde{Y}}{dr} &= \frac{g_\phi \beta}{r H s^{4/3}} \langle E \Gamma \rangle \frac{1}{2\pi^2} \int_0^\infty \left(\mathcal{A} \frac{m_0}{r} \right)^3 \xi^2 f_\phi(\xi, r) d\xi. \end{aligned} \quad (\text{A.68})$$

A.2 Chapter 3: Approximate analytical solutions:

Case I

The Eqs. (3.6) and (3.7) for case I can be solved analytically neglecting the variation of g_{*s} and g_ρ . The expressions of Y_ψ and $\tilde{Y}$ after freeze-in are

$$\begin{aligned} Y_\psi &= \frac{135 g_\phi}{1.66 \times 8\pi^3 g_{*s} \sqrt{g_\rho}} \frac{M_{pl} \Gamma_\phi}{m_\phi^2}, \\ \tilde{Y} &= \frac{675 g_\phi}{1.66 \times 8\pi^3 g_{*s} \sqrt{g_\rho}} \left(\frac{45}{2\pi^2 g_{*s}} \right)^{1/3} \frac{M_{pl} \langle E\Gamma \rangle}{m_\phi^3}, \end{aligned} \quad (\text{A.69})$$

where g_{*s} and g_ρ are effective number of degrees of freedoms at the freeze-in temperature $T \sim m_\phi$ and

$$\langle E\Gamma \rangle = \frac{m_\phi}{2} \left(1 - \frac{m_\psi^2}{m_\phi^2} \right) \Gamma_\phi. \quad (\text{A.70})$$

With this, the ratio of $\tilde{Y}$ to Y_ψ in the limit $m_\phi \gg m_\psi$ is given by

$$\frac{\tilde{Y}}{Y_\psi} = \frac{675}{270} \left(\frac{45}{2\pi^2} \right)^{1/3} \frac{1}{g_s^{1/3}}. \quad (\text{A.71})$$

Using this ratio, we can easily establish a relation between ΔN_{eff} and $\Omega_{\text{DM}} h^2$ as

$$\Delta N_{\text{eff}} = 3.29 \frac{C_2}{C_1 m_\psi} \frac{\Omega_{\text{DM}} h^2}{g_s^{1/3}} \quad (\text{A.72})$$

where $C_1 = 2 \times 2.755 \times 10^8 \text{ GeV}^{-1}$ and $C_2 = 3 \times 1.16 \times (43/4)^{4/3}$ are constants.

Case II

For case II, we have solved the Eq. (3.9) neglecting its 1st term i.e. after the freeze out of ϕ . This gives

$$Y_\phi = Y_\phi^{fo} e^{-\frac{\Gamma_\phi M_{pl}}{1.66 \times \sqrt{g_\rho^*} m_\phi^2} \left(\frac{x^2}{2} - \frac{(x^f)^2}{2} \right)}. \quad (\text{A.73})$$

Now this expression can be used to solve Eqs. (3.10) and (3.11) analytically (once again we are neglecting the temperature dependence of g_{*s} and g_ρ)

$$\begin{aligned} Y_\psi &\approx Y_\phi^{fo}, \\ \tilde{Y} &\approx \frac{Y_\phi^{fo}}{g_s^{1/3} g_\rho^{1/2}} \frac{M_{pl} \langle E\Gamma \rangle}{m_\phi^3} f_1 \frac{e^{\frac{f_2}{2}(x^f)^2}}{f_2^{3/2}}, \end{aligned} \quad (\text{A.74})$$

where

$$\begin{aligned} f_1 &= \frac{1}{1.66} \sqrt{\frac{\pi}{2}} \left(\frac{45}{2\pi^2} \right)^{1/3} \\ f_2 &= \frac{\Gamma_\phi M_{pl}}{1.66 \sqrt{g_\rho} m_\phi^2} = \frac{\Gamma_\phi}{H(m_\phi)}. \end{aligned} \quad (\text{A.75})$$

Here Y_ϕ^{fo} is the abundance of ϕ just after freeze-out. The expression for $\tilde{Y}$ given in Eq. (A.74) is valid as long as the product $f_2 (x^f)^2 \ll 1$. Now in the limit $m_\phi \gg m_\psi$ the ratio of $\tilde{Y}$ to Y_ψ is given by

$$\frac{\tilde{Y}}{Y_\psi} \approx \frac{1}{g_s^{1/3} g_\rho^{1/2}} \frac{M_{pl} \Gamma_\phi}{2 m_\phi^2} f_1 \frac{e^{\frac{f_2}{2}(x^f)^2}}{f_2^{3/2}}, \quad (\text{A.76})$$

and finally,

$$\Delta N_{\text{eff}} \approx \frac{M_{pl} \Gamma_\phi}{2 m_\phi^2} f_1 \frac{e^{\frac{f_2}{2}(x^f)^2}}{f_2^{3/2}} \frac{C_2}{C_1 m_\psi} \frac{\Omega_{\text{DM}} h^2}{g_s^{1/3} g_\rho^{1/2}}. \quad (\text{A.77})$$

A.3 Chapter 3: Anomaly cancellation in gauged $B - L$ model

$U(1)_{B-L}$ gauge symmetry with only the SM fermions is not anomaly free. This is because the triangle anomalies for both $U(1)_{B-L}^3$ and the mixed $U(1)_{B-L} - (\text{gravity})^2$ diagrams are non vanishing. These triangle anomalies for the SM fermion content are given as

$$\begin{aligned} \mathcal{A}_1 [U(1)_{B-L}^3] &= \mathcal{A}_1^{\text{SM}} [U(1)_{B-L}^3] = -3, \\ \mathcal{A}_2 [(\text{gravity})^2 \times U(1)_{B-L}] &= \mathcal{A}_2^{\text{SM}} [(\text{gravity})^2 \times U(1)_{B-L}] = -3. \end{aligned} \quad (\text{A.78})$$

If three right handed neutrinos are added to the model, they contribute $\mathcal{A}_1^{\text{New}} [U(1)_{B-L}^3] = 3$, $\mathcal{A}_2^{\text{New}} [(\text{gravity})^2 \times U(1)_{B-L}] = 3$ leading to vanishing total of triangle anomalies. This is the most natural and economical $U(1)_{B-L}$ model studied extensively in the literature. However, there exist non-minimal ways of constructing anomaly free versions of $U(1)_{B-L}$ model. For example, it has been known for a few years that three right handed neutrinos with $B-L$ charges 5, -4 , -4 can also give rise to vanishing triangle anomalies [463] as follows.

$$\begin{aligned} \mathcal{A}_1 [U(1)_{B-L}^3] &= \mathcal{A}_1^{\text{SM}} [U(1)_{B-L}^3] + \mathcal{A}_1^{\text{New}} [U(1)_{B-L}^3] = \\ &= -3 + [-5^3 - (-4)^3 - (-4)^3] = 0, \\ \mathcal{A}_2 [(\text{gravity})^2 \times U(1)_{B-L}] &= \mathcal{A}_2^{\text{SM}} [(\text{gravity})^2 \times U(1)_{B-L}] + \mathcal{A}_2^{\text{New}} [(\text{gravity})^2 \times U(1)_{B-L}] , \\ &= -3 + [-5 - (-4) - (-4)] = 0. \end{aligned} \tag{A.79}$$

Another solution to the anomaly cancellation conditions with irrational $B-L$ charges of new fermions was proposed by the authors of [464] where both DM and neutrino mass can have a common origin through radiative linear seesaw. Very recently, another anomaly free $U(1)_{B-L}$ framework was proposed where the additional right handed fermions possess more exotic $B-L$ charges namely, $-4/3$, $-1/3$, $-2/3$, $-2/3$ [465]. These four chiral fermions constitute two Dirac fermion mass eigenstates, the lighter of which becomes the DM candidate having either thermal [465] or non-thermal origins [461]. The light neutrino mass in this model arises from type II seesaw mechanism with the new chiral fermions playing no role in it. In [466], such chiral fermions with fractional charges were also responsible for generating light neutrino masses at one loop level. In the recent work on $U(1)_{B-L}$ gauge symmetry with two component DM [467], the authors considered two right handed neutrinos with $B-L$ number -1 each so that the model still remains anomalous. The remaining anomalies were cancelled by four chiral fermions with fractional $B-L$ charges leading to two Dirac fermion mass eigenstates both of which are stable and hence DM candidates. In [468], type III seesaw was implemented in $U(1)_{B-L}$ model while introducing additional chiral fermions to keep the model anomaly free. Appropriate choice of chiral fermions also leads to multi-component DM in such $U(1)_{B-L}$ symmetric type III seesaw scenario.

While it is possible to study Dirac neutrino portal DM in gauged $U(1)_{B-L}$ frameworks with different anomaly free combinations of chiral fermions, it will require additional scalar fields in order to be consistent with the desired DM phenomenology and non-zero Dirac neutrino mass. For example, if we consider three right handed neutrinos with $B-L$ charges 5, -4 , -4 , we will require two additional Higgs doublets of $U(1)_{B-L}$ charge 6, 3 respectively in order to generate Dirac neutrino masses at tree level. Two singlet scalars $\phi_{1,2}$ and DM ψ , similar to our model, can be incorporated to generate the Dirac neutrino portal of DM and break $U(1)_{B-L}$ symmetry spontaneously. Similar non-minimal scalar content will be required for other chiral fermions mentioned above. While the calculations will be more involved due to more particles, the generic conclusions reached in our work should not change significantly.

A.4 Chapter 3: Calculation of LHC bound on $g_{BL} - m_{Z'}$ plane

We evaluate the constraint on $U(1)_{B-L}$ gauge coupling g_{BL} from dilepton channel: $pp \rightarrow Z' \rightarrow l^+ l^-$. Using the narrow-width approximation, the cross-section $pp \rightarrow Z'$ can be written as

$$\sigma(pp \rightarrow Z') = 2 \sum_{q, \bar{q}} \int dx \int dy f_q(x, Q) f_{\bar{q}}(x, Q) \hat{\sigma}(\hat{s}), \quad (\text{A.80})$$

where $\hat{s} = xys$ with $f_q, f_{\bar{q}}$ being the parton distribution functions for quarks and anti-quark respectively. The narrow-width approximation cross section $\hat{\sigma}(\hat{s})$ is given as

$$\hat{\sigma}(\hat{s}) = \frac{4\pi^2}{3} \frac{\Gamma(Z' \rightarrow q\bar{q})}{m_{Z'}} \delta(\hat{s} - m_{Z'}^2). \quad (\text{A.81})$$

We use the package **CalcHEP** for the computation of $\sigma(pp \rightarrow Z')$ where CTEQ6L is used for parton distribution functions.

In our model, apart from quarks and leptons, three ν_R s and two scalars ϕ_1 and ϕ_2 are charged under $U(1)_{B-L}$. The decay of Z' to both the BSM scalars ϕ_1 and ϕ_2 is not possible due to kinematic restrictions imposed. The decay width of Z' to

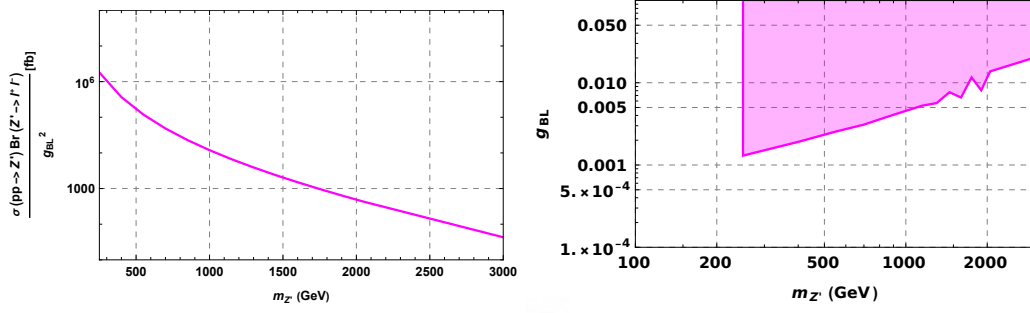


Figure A.1: Left panel: $\frac{\sigma(pp \rightarrow Z') \text{BR}(Z' \rightarrow l^+ l^-)}{g_{BL}^2}$ vs $m_{Z'}$. Right panel: Bound on g_{BL} vs $m_{Z'}$ plane.

quarks, charged leptons, SM neutrinos and ν_R can be written as

$$\begin{aligned} \Gamma(Z' \rightarrow q\bar{q}) &\simeq 12 \times 3 \times \frac{1}{9} \frac{g_{BL}^2}{24\pi} m_{Z'} = 4 \times \frac{g_{BL}^2}{24\pi} m_{Z'} \\ \Gamma(Z' \rightarrow l^- l^+) &\simeq 6 \times \frac{g_{BL}^2}{24\pi} m_{Z'} \\ \Gamma(Z' \rightarrow \nu_L \bar{\nu}_L) &\simeq 3 \times \frac{g_{BL}^2}{24\pi} m_{Z'} \\ \Gamma(Z' \rightarrow \nu_R \bar{\nu}_R) &\simeq 3 \times \frac{g_{BL}^2}{24\pi} m_{Z'} \end{aligned} \quad (\text{A.82})$$

respectively, assuming massless final states. With this, we can calculate the cross-section $\sigma(pp \rightarrow Z' \rightarrow l^+ l^-) \approx \sigma(pp \rightarrow Z') \text{BR}(Z' \rightarrow l^+ l^-)$ for particular values of $m_{Z'}$ and g_{BL} . As $\sigma(pp \rightarrow Z')$ is proportional to g_{BL}^2 , we can normalise it by dividing with g_{BL}^2 .

The left panel plot of Fig. A.1 shows $\frac{\sigma(pp \rightarrow Z') \text{BR}(Z' \rightarrow l^+ l^-)}{g_{BL}^2}$ as a function of $m_{Z'}$ for our model. In the right panel plot, we show the bound on $U(1)_{B-L}$ gauge coupling for our model in magenta region. The bound is obtained by comparing the left plot with the final results from ATLAS collaboration of the LHC Run-2 with 139 fb^{-1} integrated luminosity.

A.5 Chapter 3: Estimation of dark sector temperature

Here we provide an estimation of dark sector temperature using the entropy conservation. Let us assume that g_{*s}^{DS} and g_{*s} are the effective number of relativistic

degrees of freedom of dark sector and SM, respectively. At the epoch of decoupling of dark sector from the standard model bath, the scale factor is taken as a_{dec} . Using the conservation of entropy, we get

$$\begin{aligned} g_{*s,\text{dec}} a_{\text{dec}}^3 (T_{\text{dec}})^3 &= g_{*s,\text{aft}} a_{\text{aft}}^3 (T_{\text{aft}})^3 \\ g_{*s,\text{dec}}^{\text{DS}} a_{\text{dec}}^3 (T_{\text{dec}}^{\text{DS}})^3 &= g_{*s,\text{aft}}^{\text{DS}} a_{\text{aft}}^3 (T_{\text{aft}}^{\text{DS}})^3, \end{aligned} \quad (\text{A.83})$$

where a_{aft} denotes any period after decoupling and T^{DS} and T denote temperature of dark sector and SM bath respectively. At the epoch of decoupling, both the dark sector and SM bath have same temperature, $T_{\text{dec}} = T_{\text{dec}}^{\text{DS}}$. Hence, from the above equations, we get

$$\left(\frac{T_{\text{aft}}^{\text{DS}}}{T_{\text{aft}}} \right)^4 = \left(\frac{g_{*s,\text{dec}}^{\text{DS}} g_{*s,\text{aft}}}{g_{*s,\text{aft}}^{\text{DS}} g_{*s,\text{dec}}} \right)^{4/3}. \quad (\text{A.84})$$

For the situation where dark sector consists of ϕ_1, ψ and ν_R , we have $g_{*s,\text{dec}}^{\text{DS}} = 2 + 6 \times \frac{7}{8} + 4 \times \frac{7}{8} = 10.75$. Taking a_{aft} to a period when both ϕ_1 and ψ annihilate or become non-relativistic, we get $g_{*s,\text{aft}}^{\text{DS}} = 6 \times \frac{7}{8} = 5.25$. If during this period, g_{*s} remains constant, we get

$$\left(\frac{T_{\text{aft}}^{\text{DS}}}{T_{\text{aft}}} \right)^4 = \xi^4 = \left(\frac{10.75}{5.25} \right)^{4/3} = 2.6. \quad (\text{A.85})$$

So, at maximum, we can expect a 2.6 times increase in dark sector temperature from the SM bath. The numerical results shown in Fig. 3.18 are consistent with this analytical estimation.

However, we can have a value of $g_{*s,\text{dec}}^{\text{DS}} < 10.75$ at the epoch of decoupling. This is because before decoupling, some fraction of dark sector particles can become non-relativistic. For example, let us consider $m_{\phi_1} = 60$ GeV, $m_{\psi} = 15$ GeV. Decoupling above $T \sim 60$ GeV gives $g_{*s,\text{dec}}^{\text{DS}} \sim 10.75$ whereas decoupling after $T \sim 60$ GeV (e.g. 10 GeV) gives $g_{*s,\text{dec}}^{\text{DS}} < 10.75$. This results in a smaller increase in dark sector temperature from SM bath. The same behaviour can also be seen from Fig. 3.18.

A.6 Chapter 3: Direct Detection of WIMP type DM

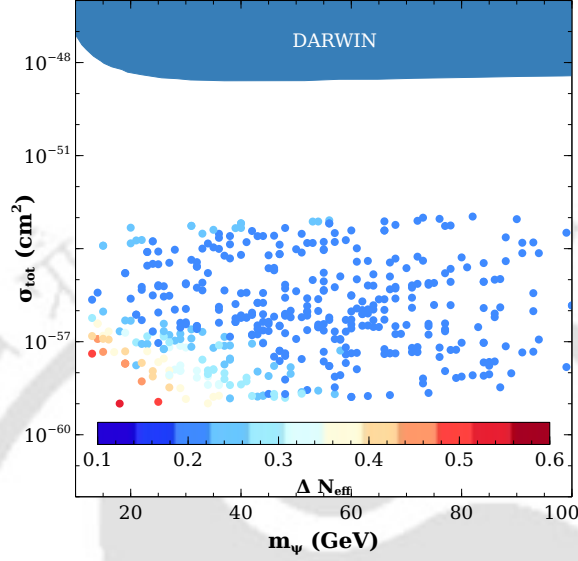


Figure A.2: The DM-nucleon cross-section vs DM mass as a function of effective number of relativistic species. The blue color region shows the sensitivity of DARWIN experiment [469].

In this setup, dark matter ψ does not interact with the nucleus via tree level diagram. However, via one loop diagram, it can scatter off the nucleus. We get two diagrams, one where the mediator is SM Higgs and the other where the mediator is B-L gauged boson. For the Higgs-mediated diagram, the relevant interaction vertices are $g_{\psi\bar{\psi}h}\psi\bar{\psi}h$ and $\frac{m_q}{v}q\bar{q}$. Here q represents the SM quarks. The effective interaction term for Higgs-mediated diagram can be written as

$$\mathcal{L}_{eff}^h = \frac{m_q}{v} \frac{1}{m_h^2} g_{\psi\bar{\psi}h} \psi\bar{\psi}q\bar{q}. \quad (\text{A.86})$$

Similarly, for the Z' mediated diagram, the effective interaction term can be written as

$$\mathcal{L}_{eff}^{Z'} = -\frac{g_{BL}}{3} \frac{1}{m_{Z'}^2} g_{\psi\bar{\psi}Z'} \psi\gamma^\mu\bar{\psi}q\gamma_\mu\bar{q}. \quad (\text{A.87})$$

The interaction vertices $g_{\psi\bar{\psi}h}$ and $g_{\psi\bar{\psi}Z'}$ for one loop diagram are calculated using Package-X[470] and their expressions are given as follows (for zero momentum

transfer)¹

$$g_{\psi\bar{\psi}h} = \frac{i}{16\pi^2} y_{\phi_1}^2 \lambda_{H\phi_1} v \frac{1}{m_\psi} \left[1 + \left(\frac{m_{\phi_1}^2}{m_\psi^2} - 1 \right) \ln \left(1 - \frac{m_\psi^2}{m_{\phi_1}^2} \right) \right]$$

$$g_{\psi\bar{\psi}Z'} = \frac{i}{16\pi^2} y_{\phi_1}^2 g_{BL} \left[\frac{3}{2} \frac{m_{\phi_1}^2}{m_\psi^2} + \left(\frac{m_{\phi_1}^2}{m_\psi^2} - 1 \right) \ln \left(1 - \frac{m_\psi^2}{m_{\phi_1}^2} \right) \left(\frac{3}{2} \frac{m_{\phi_1}^2}{m_\psi^2} + \frac{1}{2} \right) \right]. \quad (\text{A.88})$$

The total DM-nucleon cross-section can be written as [79, 471, 472] -

$$\sigma_{\text{tot}} = \frac{1}{\pi} \frac{m_N^2 m_\psi^2}{(m_N + m_\psi)^2} \left[\frac{m_N}{v} \frac{1}{m_h^2} g_{\psi\bar{\psi}h} f_N + \frac{g_{BL}}{3} \frac{1}{m_{Z'}^2} g_{\psi\bar{\psi}Z'} f_{Z'} \right]^2, \quad (\text{A.89})$$

where $f_N \sim 0.3$, $f_{Z'} = 3$ and m_N is the nucleon mass. In Fig. A.2, we show the DM-nucleon cross-section as a function of m_ψ . To calculate the cross-section, we take the values of different parameters same as for the resultant points we obtained in Fig. 3.19. All the points satisfy correct DM relic abundance and range of different parameters for these points are given by Eq. (3.46). We find that the total cross-section is way below the direct detection limit.

A.7 Chapter 4: Non-standard interactions of dark radiation

We consider the following effective operators for interactions among SM neutrinos and ν_R [255].

$$-\mathcal{L} \supset G_s \bar{\nu}_L \nu_R \bar{\nu}_L \nu_R + G_s^* \bar{\nu}_R \nu_L \bar{\nu}_R \nu_L + G_V \bar{\nu}_L \gamma^\mu \nu_L \bar{\nu}_R \gamma_\mu \nu_R + \tilde{G}_S \bar{\nu}_L \nu_R \bar{\nu}_R \nu_L$$

$$+ G_T \bar{\nu}_L \sigma^{\mu\nu} \nu_R \bar{\nu}_L \sigma_{\mu\nu} \nu_R + G_T^* \bar{\nu}_R \sigma^{\mu\nu} \nu_L \bar{\nu}_R \sigma_{\mu\nu} \nu_L, \quad (\text{A.90})$$

where $G_S, \tilde{G}_S, G_V$ and G_T are effective coupling constants of respective 4-fermion operators. The relevant processes which can lead to thermalisation of ν_R are given

¹We consider only one of the two different one-loop diagrams to calculate the effective $g_{\psi\bar{\psi}Z'}$ vertex for simplicity.

by

$$\begin{aligned}
\nu_R + \nu_R &\leftrightarrow \nu_R + \nu_R, \\
\nu_R + \bar{\nu}_R &\leftrightarrow \nu_L + \bar{\nu}_L, \\
\nu_R + \nu_L &\leftrightarrow \nu_R + \nu_L, \\
\nu_R + \bar{\nu}_L &\leftrightarrow \nu_R + \bar{\nu}_L, \\
\nu_R + \bar{\nu}_L &\leftrightarrow \bar{\nu}_R + \nu_L.
\end{aligned} \tag{A.91}$$

While we have focused on the case of right-handed neutrinos, interactions of other species of dark radiation with Standard Model can also be realised through effective operators. For example, coupling of Goldstone bosons (ϕ) with the SM gauge sector at dimension-5 can be written as [473]

$$\mathcal{L} \supset -\frac{1}{4} \frac{\phi}{\Lambda} \left(c_1 B_{\mu\nu} \tilde{B}^{\mu\nu} + c_2 W_{\mu\nu} \tilde{W}^{\mu\nu} + c_3 G_{\mu\nu} \tilde{G}^{\mu\nu} \right), \tag{A.92}$$

where $\{B_{\mu\nu}, W_{\mu\nu}, G_{\mu\nu}\}$ denotes the field strength tensor associated with SM gauge groups $\{U(1)_Y, SU(2)_L, SU(3)_c\}$, with $\{\tilde{B}^{\mu\nu}, \tilde{W}^{\mu\nu}, \tilde{G}^{\mu\nu}\}$ being their duals. Interaction of Goldstone bosons with the SM can also be realised through Yukawa couplings given by [473]

$$\mathcal{L} \supset -\frac{\partial_\mu \phi}{\Lambda_\psi} \bar{\psi}_1 \gamma^\mu (g_V^{ij} + g_A^{ij} \gamma^5) \psi_j, \tag{A.93}$$

where ψ denotes the SM fermions. Similar analysis performed in Section 4.1.3 for Dirac neutrinos can be carried out to constrain the couplings of other species of dark radiation.

A.8 Chapter 4: Axion field evolution

PBH, after formation, loses its energy via Hawking evaporation. The combined Boltzmann equations for the system can be written as

$$\frac{d\rho_{\text{in}}}{dt} + 3\mathcal{H}\rho_{\text{BH}} = \frac{1}{M_{\text{BH}}} \frac{dM_{\text{BH}}}{dt} \rho_{\text{BH}} \tag{A.94}$$

$$\frac{d\rho_r}{dt} + 4\mathcal{H}\rho_r = -\frac{1}{M_{\text{in}}} \frac{dM_{\text{BH}}}{dt} \rho_{\text{BH}}. \tag{A.95}$$

The PBH mass loss rate is given by

$$\frac{1}{M_{\text{BH}}} \frac{dM_{\text{BH}}}{dt} = -\frac{\mathcal{G}g_H(T_H)}{30720\pi G^2 M_{\text{BH}}^3}. \quad (\text{A.96})$$

Due to PBH evaporation, entropy injection takes place, leading to non-conservation of entropy. The entropy injection can be tracked via

$$\frac{ds}{dt} + 3\mathcal{H}s = -\frac{1}{M_{\text{BH}}} \frac{dM_{\text{BH}}}{dt} \frac{\rho_{\text{BH}}}{T}. \quad (\text{A.97})$$

The evolution of axion field a in the early Universe can be written as

$$\ddot{a} + 3\mathcal{H}\dot{a} + \frac{1}{R^2(t)} \nabla^2 a + \frac{\partial V(a, T)}{\partial a} = 0, \quad (\text{A.98})$$

where

$$V(a, T) = f_a^2 m_a^2(T) \left(1 - \cos \left(\frac{a}{f_a} \right) \right) \quad (\text{A.99})$$

and R denotes the scale factor. The initial axion angle, $\theta = \frac{a}{f_a}$ is frozen in until the oscillation temperature of axion which can be estimated by comparing Hubble expansion rate to the temperature dependent axion mass $\mathcal{H}(T_{\text{osc}}) = m_a(T_{\text{osc}})$. To get the contribution of axion to the current energy budget, we solve the above equations following the procedure in the reference [344]. The procedure is to first estimate the oscillation temperature T_{osc} in presence of PBH. With this, number density of axion at T_{osc} can be calculated following

$$n_a(T_{\text{osc}}) = \frac{\rho_a(T_{\text{osc}})}{m_a(T_{\text{osc}})} = \frac{1}{m_a(T_{\text{osc}})} \left(\frac{1}{2} \dot{a}^2 + \frac{1}{2} |\nabla a|^2 + V(a) \right). \quad (\text{A.100})$$

Due to entropy dilution, the equation for n_a becomes

$$\frac{1}{n_a} \frac{d(n_a/s)}{dt} = \frac{1}{M_{\text{BH}}} \frac{dM_{\text{BH}}}{dt} \frac{\rho_{\text{BH}}}{Ts^2}. \quad (\text{A.101})$$

The above equation is solved from $T \gtrsim T_{\text{osc}}$ to a sufficiently small temperature when PBH completely evaporates.

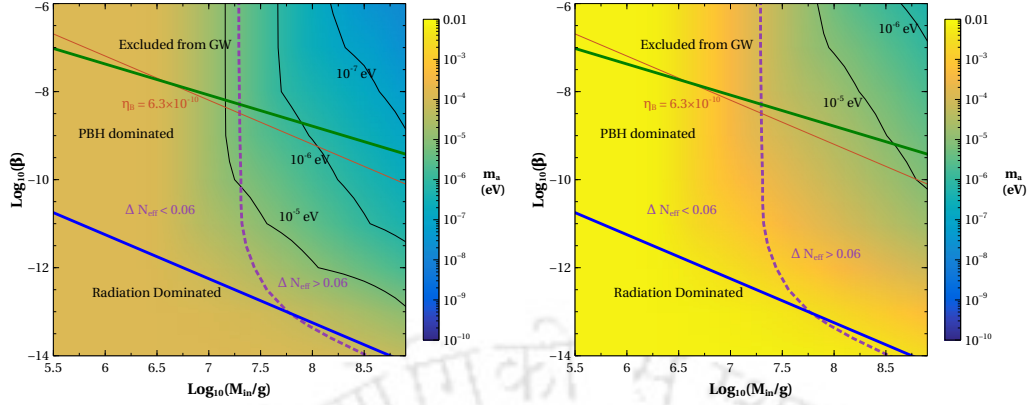


Figure A.3: Constraint on M_{in} versus β plane considering resonant leptogenesis. Axion constitutes 10%, 0.1% of total DM abundance in left and right panels respectively. The red solid lines in the plots denote the parameter space that is consistent with both DM relic and baryon asymmetry. The region excluded from GW is separated by solid green line. The magenta dashed line separate the regions that can be probed by future CMB-S4 experiment due to contribution of hot axion on N_{eff} .

A.9 Chapter 4: Multi-component Dark Matter

If axion does not constitute the entire DM, the allowed parameter space discussed so far can change significantly. The deficit in total DM abundance can be filled up by one of the RHNs, as discussed briefly in Appendix A.10. From the left panel of Fig.4.14, for a particular value of M_{in} and β , one needs a larger value of axion mass (correspondingly smaller value of f_a) to have sub-dominant axion DM. However, at the same time, a smaller value of f_a also reduces the CP asymmetry parameter according to Eq.(4.40). As a result, baryon asymmetry also reduces. Thus, for a particular PBH mass, one needs to consider a smaller value β so that dilution is also less to obtain correct baryon asymmetry. However, it turns out that for this particular scenario, even if one reduces β to β_c (where β_c distinguishes PBH-dominated and radiation-dominated region), the correct baryon asymmetry can not be obtained. The maximum baryon asymmetry that can be obtained is $\sim 4 \times 10^{-10}$ which is less than the observed asymmetry, viz., 6.3×10^{-10} . All the region above the $\beta = \beta_c$ line gives $\eta_B < 4 \times 10^{-10}$. For the sub-dominant axion DM scenario, one therefore must consider the resonant leptogenesis approach to generate the correct BAU. In Fig.A.3 we show the results for resonant leptogenesis by considering the percentage of axion DM as a free parameter. The solid red line in these plots

correspond to the parameter space consistent with observed baryon asymmetry for $\epsilon_1 = 0.1$.

Note that, in the multi-component DM scenario, because the axion DM is a subdominant component, we can have smaller f_a values leading to axion mass range in the solar axion ballpark [379, 474], making this scenario pretty different from the case of axion constituting 100% DM. As an aside, let us mention that there have been efforts to obtain the entire dark matter relic only in the form of axions for a smaller value of decay constant ($f_a \sim 10^8 \text{ GeV}$) via parametric resonance production of axions. see. e.g., Refs.[475–477].

A.10 Chapter 4: Production of RHN Dark Matter

As two RHNs are sufficient to generate neutrino mass and BAU via leptogenesis, the other RHN, if sufficiently long-lived, can constitute some part of the total DM abundance. Due to the presence of PBH and RHN coupling to axion as well as SM leptons, we can have three² different production channels for RHN DM namely, (i) gravitational production from PBH evaporation [156, 157, 159, 315, 325, 479–491], (ii) production via axion portal interactions [492–494], (iii) freeze-in production via Yukawa portal interactions [445, 495].

For the axion decay constant we have in our setup, superheavy RHN DM is more natural which is produced dominantly via PBH evaporation. For the case when $M_{\text{DM}} < T_{\text{BH}}^{\text{in}}$, correct relic abundance needs DM mass in the range of $\sim \text{GeV}$. However, for this mass range of DM produced by PBH evaporation, the DM turns out to be hot and hence disfavored from structure formation constraints. For the other limiting case $M_{\text{DM}} > T_{\text{BH}}^{\text{in}}$, the DM mass is greater than 10^9 GeV , keeping it in the superheavy ballpark, a natural consequence of the high PQ symmetry breaking scale

However, if we allow fine-tuning of RHN DM mass, keeping it much lower than the PQ scale, other production mechanisms like axion portal can dominate.

²It is also possible to generate light RHN DM via active-sterile neutrino oscillation [478], though it faces severe X-ray constraints. See [273] for a review.

Since, axion portal coupling of RHN will be suppressed for allowed mass range of thermal DM below unitarity limit [443], such production is likely to occur via freeze-in mechanism from the SM bath. For $f_a \sim 10^{12}$ GeV and reheat temperature $T_{\text{RH}} \sim 10^{12}$ GeV, we require DM mass to be around 10^5 GeV for observed DM relic to be produced via UV freeze-in. For heavier DM mass, it is likely to be overproduced via axion portal interactions as long as $T_{\text{RH}} > M_{\text{DM}}$. On the other hand, RHN DM of mass $\sim 10^5$ GeV will also be overproduced from PBH evaporation as mentioned above. This pushes RHN DM to low mass where it can be produced efficiently via Yukawa portal interactions while gravitational and axion portal contributions remain sub-dominant. In this regime, correct relic can be achieved via IR freeze-in production of N_1 from the SM bath. As shown in earlier works [445, 495], such freeze-in production is dominated by $W^\pm, Z$ and SM Higgs decays facilitated via active-sterile neutrino mixing θ_1 . The mixing angle can be tuned by choosing the lightest active neutrino mass m_1 . The same active-sterile mixing also leads to decay of N_1 into SM particles. Although the abundance of N_1 only depends on m_1 , the decay rate of N_1 depends on both m_1 and M_1 . This gives a constraint on the N_1 mass, as shown in the left panel of Fig.A.4. The magenta-colored line represents the situation where PBH do not dominate the energy density of the Universe (i.e. no additional entropy dilution). The brown colored line denotes the situation where entropy dilution is present from PBH evaporation. Due to the presence of entropy dilution, N_1 should be produced over-abundantly requiring a larger value of active-sterile mixing angle. As the decay of N_1 is also dependent on θ_1^2 , so M_1 is more tightly constraint in the presence of entropy dilution as seen from the left plot of Fig.A.4. Here both the magenta and the brown color lines give the correct relic density. In the right panel plot, m_1 is shown in the $\beta - M_{\text{in}}$ plane for $\Omega_{N_1} h^2 = 0.12$, correlating neutrino mass with PBH parameters from correct DM relic criteria.

RHN DM with $T_{\text{RH}} < M_{\text{DM}}$: As noticed in the above discussions, the RHN DM parameter space produced from PBH evaporation and axion portal interactions is anti-correlated, forcing the RHN DM mass to sub-GeV ballpark where it can be efficiently produced from Yukawa portal freeze-in mechanisms. However, this requires severe fine-tuning of RHN coupling with the PQ scalar such that $M_1 \equiv M_{\text{DM}} \ll f_a$. Such unnatural fine-tuning can, however, be avoided if we consider the heaviest RHN N_3 to be DM with mass $M_3 = f_a > T_{\text{RH}}$ which can not be produced from the bath. Leptogenesis is obtained from N_1 and N_2 with $M_1 \simeq M_2$. For thermal leptogenesis, we assume $M_1 \simeq M_2 < T_{\text{RH}} \ll M_3 = f_a$. Although the PQ symmetry

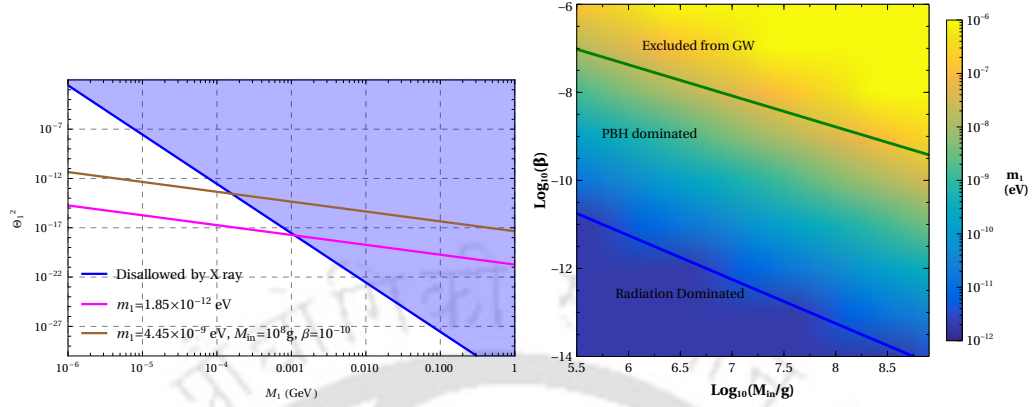


Figure A.4: Left panel: θ_1^2 versus M_1 for two different scenarios: without PBH and with PBH ($M_{in} = 10^8$ g and $\beta = 10^{-10}$) shown by magenta and brown color respectively. The shaded blue color region represents the excluded region by X-ray experiments. Right panel: Lightest neutrino mass m_1 in PBH parameter space $\beta - M_{in}$ with N_1 abundance $\Omega_{N_1} h^2 = 0.12$.

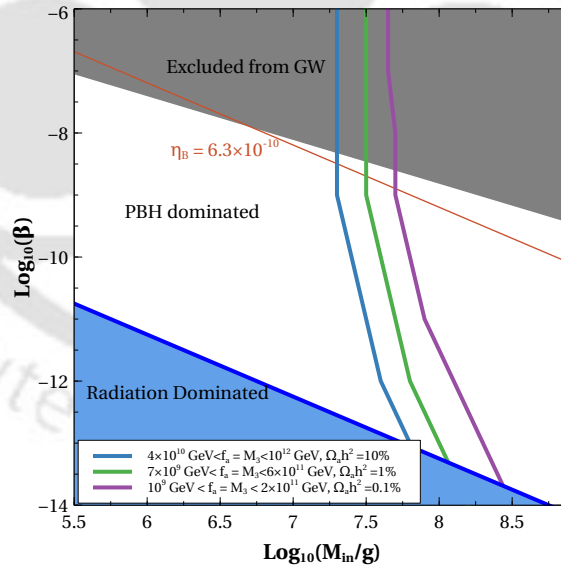


Figure A.5: For $M_3 \gg T_{RH} > M_2 \sim M_3$. Blue, green, and magenta colored lines denote the different situations where $\Omega_a h^2 = 10\%$, $\Omega_a h^2 = 1\%$ and $\Omega_a h^2 = 0.1\%$ respectively and $f_a = M_3$. The RHN N_3 constitutes the rest of DM abundance. The brown-colored solid line denotes the parameter space consistent with the baryon asymmetry for maximal CP asymmetry.

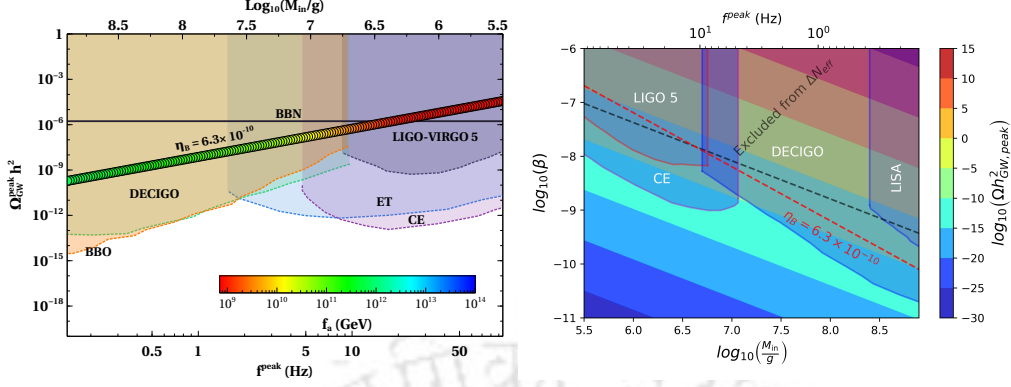


Figure A.6: Left panel: $\Omega_{\text{GW}}^{\text{peak}} h^2$ versus f_{peak} for 0.1% relative abundance of axion DM. Right Panel : Peak Gravitational Waves amplitude as a function of M_{in} and β with different experimental sensitivities. The red dashed line denotes the correct baryon asymmetry value for resonant leptogenesis with maximal CP asymmetry.

gets broken after e.g., slow-roll inflation, a non-instantaneous reheating phase [496] can lead to a reheat temperature $T_{\text{RH}} \ll f_a$. This leads to purely gravitational production of RHN DM from PBH evaporation. The DM mass produced from PBH evaporation is given by

$$M_{\text{DM}} \simeq 4.5 \times 10^3 \left(\frac{0.12}{\Omega_{N_3} h^2} \right) \left(\frac{M_{\text{in}}}{M_{\text{Pl}}} \right)^{-5/2} \frac{M_{\text{Pl}}^2}{\text{GeV}}. \quad (\text{A.102})$$

Considering $M_{\text{DM}} = f_a$, this gives a line in the $\beta - M_{\text{in}}$ plane. In Fig.A.5, blue, green, and magenta colored lines correspond to the situations where $\Omega_a h^2 = 10\%, 1\%$ and 0.1% of total DM abundance respectively. Note that $M_3 = M_{\text{DM}} = f_a$ changes along these contours, following Eq.(A.102). The rest of the DM abundance is constituted by N_3 . The observed BAU can be generated from resonant leptogenesis as discussed earlier. The parameter space satisfying the observed BAU for maximal CP asymmetry $\epsilon_1 = 0.1$ is indicated by the brown-colored solid line in Fig.A.5.

A.11 Chapter 4: GW prospects in multi-component DM scenario

Similar to the right panel of Fig.4.16, we show the $\Omega_{\text{GW}}^{\text{peak}} h^2$ and f_{peak} for resonant leptogenesis scenario in the left panel of Fig.A.6 considering 0.1% relative abundance

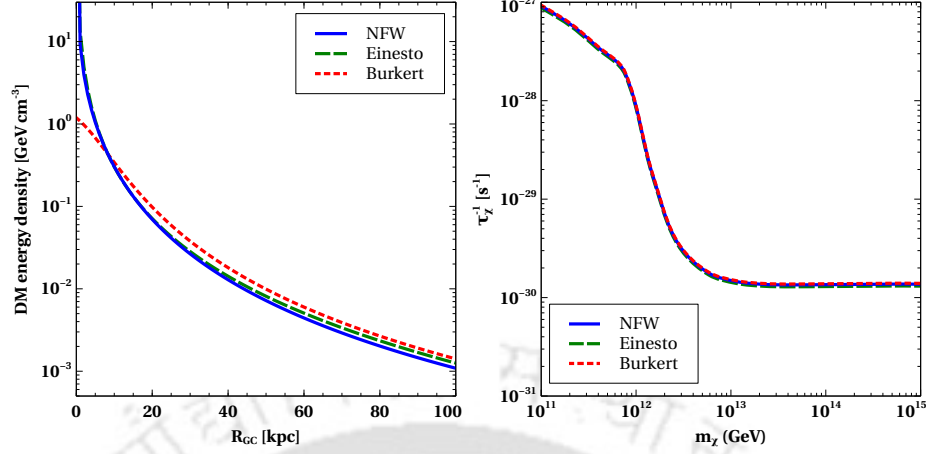


Figure A.7: *Top plot:* DM energy density vs Galactocentric distance for three different DM profiles. *Bottom plot:* Constraints on DM lifetime from UHECR vs DM mass for different DM profiles considering $b\bar{b}$ channel. The constraints are nearly independent of the DM profiles.

of axion DM. In the right panel, we show another representation of the resonant scenario where the peak GWs amplitude are shown as a function of PBH mass and β along with different GWs sensitivities. As expected, the red dashed colored line denoting correct leptogenesis for resonant scenario lies within the future GWs sensitivities.

A.12 Chapter 5: Dependence on DM density profile

We have performed the analysis on SHDM decay considering the NFW DM density profile in the Milky Way throughout Section 5.1. For completeness of the analysis, it is crucial to understand the variation of the results for different DM profiles. Therefore, in this appendix, we present an analysis on the dependence of our results on different DM density profiles. Other than the NFW profile, here we take two other DM profiles: Einasto [205] and Burkert [207]. The equations for DM energy density for these two profiles are

$$\rho_{\text{Ein}}(R_{\text{GC}}) = \rho_c \exp \left[-12.5 \left(\left(\frac{R_{\text{GC}}}{R_c} \right)^{0.16} - 1 \right) \right], \quad (\text{A.103})$$

$$\rho_{\text{Bur}}(R_{\text{GC}}) = \frac{\rho_c R_c^3}{(R_{\text{GC}} + R_c)(R_{\text{GC}}^2 + R_c^2)}, \quad (\text{A.104})$$

respectively.

To illustrate the effect of different DM profiles, we compute the constraints on SHDM lifetime from TA's UHECR observation for the DM profiles discussed above. The results are shown in Fig. A.7. In the top panel of A.7, we show the DM energy density in the Milky Way as a function of Galactocentric distance for the different DM profiles. These DM profiles give different $\mathcal{J}^{\theta_{\text{max}}}$ values in equation (5.3). While for NFW profile, $\mathcal{J}^{\theta_{\text{max}}}$ takes value of 1.99, for Einasto and Burkert profiles, its value is 2.08 and 1.95 respectively. This variation of $\mathcal{J}^{\theta_{\text{max}}}$ gives a maximum of around 7% changes in SHDM lifetime depending on the DM profile as shown in the bottom plot of figure A.7. Hence, the dependence of the constraints on the DM profile is negligible.

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